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## Development of a virtual $Z_{\text{eff}}$ diagnostic for the W7-X stellarator

M. Krychowiak, D. Dodt, H. Dreier, R. König, R. Wolf

*Max-Planck-Institut für Plasmaphysik, Euratom Association, Wendelsteinstr. 1, 17491 Greifswald, Germany*

### Abstract

For the W7-X stellarator a diagnostic system for measurement of local  $Z_{\text{eff}}$  values from the visible bremsstrahlung continuum is foreseen. The method is based on passive, absolute measurement of the bremsstrahlung intensity along several lines of sight. In order to eliminate the spurious impact of other radiation sources of different spectral intensity distribution than the bremsstrahlung, like the line radiation, it is intended to spectrally resolve the detected radiation e.g. by use of micro-spectrometers. The visible bremsstrahlung background can be extracted by making use of the  $1/\lambda^2$  dependence of its intensity (expressed in unit  $\text{W}/\text{m}^3\text{-ster-nm}$ ) in the high temperature plasmas by using Bayesian data analysis techniques. In a second step, the local values of  $Z_{\text{eff}}$  as a function of the effective plasma radius are derived by inversion, using different regularization methods, of the line-integrated bremsstrahlung signals with the knowledge of the magnetic flux surfaces. Inversion of the full model based on statistical methods allows taking into account all conceivable uncertainties accompanying  $Z_{\text{eff}}$  measurement and provides uncertainties of the local  $Z_{\text{eff}}$  values and valuable information on other uncertain parameters of the model. In this paper we show the first steps in developing of a virtual  $Z_{\text{eff}}$  diagnostic for W7-X which allows the optimization of the statistical model as well as of the future diagnostic set-up.

### I. INTRODUCTION

One possible method to derive the effective ionic charge  $Z_{\text{eff}}$  of a fusion plasma is based on line-of-sight integrated detection of visible-near infrared bremsstrahlung continuum. Its intensity linearly

depends on  $Z_{\text{eff}}$ , hence, an absolute bremsstrahlung intensity has to be measured. Inversion of the line-integrated bremsstrahlung intensities recorded along several lines of sight covering at least the range between the plasma center and the edge provides local  $Z_{\text{eff}}$  values [2]. This diagnostic method is, however, subject to several uncertainties which have to be treated carefully. On the one hand the measurement of the line-integrated bremsstrahlung level needs an accurate absolute calibration of the optical and the detection system. In addition, it suffers from spurious radiation contributions, like line and recombination radiation of the plasma as well as from thermal radiation of hot divertor plates and the bremsstrahlung itself, reflected at the plasma vessel walls. On the other hand, the bremsstrahlung intensity reveals a strong (quadratic) dependence on the electron density and a slight one on the electron temperature ( $\sim T_e^{-0.35}$ ). A further uncertainty source is given by the mapping of the magnetic flux surfaces to real space coordinates. For this reason it is imperative to find a method of experimental data analysis which can in best way cope with all the uncertainties. A virtual  $Z_{\text{eff}}$  diagnostic provides a way of testing the analysis method: 1) Experimental data are simulated taking into account experimental circumstances with inclusion of all expected uncertainty sources, the statistical and systematic ones. 2)  $Z_{\text{eff}}$  profiles resulting from the inversion of the simulated data should match the ones assumed for the simulation. Uncertainties in the resulting profiles give hints about which design parameters of the future diagnostic are critical and should be optimized in order to reduce the measurement error. In this sense the virtual diagnostic facilitates the design process of the diagnostic set-up [3].

## II. SIMULATION OF THE SPECTRA

For the simulation of the experimental spectra we assume a standard plasma configuration of W7-X with expected plasma profiles as shown in Figure 1. For  $Z_{\text{eff}}$  we assume a parabolic profile with the central value of 1.5 and the edge value of  $\sim 5.5$ . We choose a set of 10 observation lines corresponding to the ones of the Thomson Scattering system [4]. In addition to the bremsstrahlung we assume a number of spectral lines and black body radiation of one temperature. The simulation is done for an overview spectrometer Ocean Optics S2000 which was already used for  $Z_{\text{eff}}$  measurement at the predecessor stellarator experiment W7-AS [1]. Its absolute calibration was determined by use of an

Ulbricht sphere standard light source. We consider in the analysis 1000 pixels of its 2048 pixels CCD detector (12 bit analog-to-digital converter corresponding to a maximum signal of 4096 counts) with the spectral range of 500-840 nm. The thermal radiation is firstly represented in the same way for all lines of sight by a black body radiation of 1800 K (sublimation temperature of graphite). Its amplitude from reflection was assumed to be 0.01% of the intensity which would be seen by the spectrometer if fully illuminating the line of sight. The line radiation, also the same for all channels, is implemented in the form of signal contributions of around 1000 counts (based on W7-AS spectra) in a variable fraction of the total number of pixels: between 20% and 90%. Two of the ten spectra, for the central and the edge line of sight, for the case of the fraction of 20% are shown in Figure 2 (for the edge spectrum the thermal radiation has been omitted in the plot: this reveals the very low level of the bremsstrahlung in the range of  $\sim 10$  counts).

**III. INVERSION** We perform the inversion of the problem in two different ways (see Figure 3). Within the first approach we first fit the bremsstrahlung background of all ten spectra and obtain ten line-of-sight-integrated bremsstrahlung emission coefficients with well defined uncertainties (see below). The line-integrated bremsstrahlung coefficients are subsequently inverted by use of a  $X^2$  minimization with or without regularization (additional constraints on the fitted parameters).

The second method is based on an inversion of the full forward model (i.e. a model describing the raw spectra of all lines of sight) using a Markov-Chain Monte-Carlo sampling. The application of Bayesian probability theory allows to obtain the uncertainty of the  $Z_{\text{eff}}$  reconstruction resulting from the error statistics of the spectroscopic measurement and in particular the uncertainty caused by parameters of the model which are not known precisely, e.g. the electron density profile.

### III.A Two-Step Inversion

The fit of the bremsstrahlung background from a spectrum in the presence of spectral lines, as needed for the first inversion method, also makes use of Bayesian probability theory. The possibility of outliers in the spectrum caused by line radiation is taken into account by a so-called mixture model [1]. As a result, we obtain the line-of-sight-integrated bremsstrahlung intensity and its uncertainty due to

the statistical error of the signals and due to the contamination of the spectrum by line radiation. One example of a bremsstrahlung background with its  $1/\lambda^2$  dependence, fitted to an experimental spectrum recorded at W7-AS is shown in Figure 4. By fitting the bremsstrahlung background to simulated spectra, we can investigate the impact of the number of lines in the spectrum on the uncertainty of the fitted bremsstrahlung intensity. This dependence is shown in Figure 5 for all 10 lines of sight (l.o.s). The case with a fraction of 20%, 70% and 90% of pixels contaminated with line radiation is compared to the fit of the pure bremsstrahlung. We see that the relative error of the bremsstrahlung intensity lies in the range of 1% except for the edge l.o.s (#10). The error is only weakly dependent on the number of spectral lines even for a fraction of 90%. This demonstrates the robustness of the mixture model in handling the line radiation.

Furthermore, we superimpose ten simulated spectra with a black body radiation contribution, as mentioned above and include its spectral dependence in the forward function. The temperature and amplitude of this contribution is fitted simultaneously to the bremsstrahlung intensity. For the fraction of 20% of pixels contaminated with line radiation, we see that the existence of the black body radiation also only marginally effects the uncertainty of the bremsstrahlung fit. The one-temperature model assumed here is, of course, not sufficient to describe the divertor radiation reflected on the vessel walls, which can contribute to measured spectra. This is subject of future investigation.

The l.o.s.-integrated bremsstrahlung intensities and their uncertainties, as obtained from the background fitting procedure, are subsequently inverted by using the  $X^2$  minimization method with regularization. We use the Maximum Entropy as well as the Minimum Fisher regularization [5]. For the choice of the Lagrange parameters we use the criterion of  $X^2$  equaling the number of the l.o.s. In the case of taking into account only the uncertainty resulting from the background fit (including statistical noise and the line radiation contribution) the resulting  $Z_{\text{eff}}$  profile is nearly identical to the one used for the simulation of the spectra even if no regularization is used. This is because of the low error level in the line-integrated bremsstrahlung signals. If we assume an uncertainty of only 5% in the absolute calibration factors of the spectrometers and increase or decrease (alternating for adjacent l.o.s.) the bremsstrahlung intensities taken into the inversion, the Maximum Entropy regularization, using only the information content of the data, results in a rugged  $Z_{\text{eff}}$  profile (Figure 6 lhs). In such a

case additional physical information can be introduced into the inversion procedure e.g. in form of the smoothness of the  $Z_{\text{eff}}$  profile. This is done within the Minimum Fisher regularization (Figure 6 rhs) the result of which is much more satisfactory. However, inversion methods based on the  $X^2$  minimization provide no means to evaluate the uncertainty of the resulting local  $Z_{\text{eff}}$  values and for proper treatment of the uncertain parameters included in the forward function. For this reason it is very desirable to use the probabilistic method of inversion based on the Bayesian data analysis.

**III.B Bayesian inversion of the full forward model** The inversion of a simplified problem using the Bayesian data analysis method in which only the statistical error and the impact of line radiation are included returns the assumed  $Z_{\text{eff}}$  profile very well. In the second step we treat all ten  $n_e$  points as uncertain with a standard deviation of 20% and fit the values of the electron density simultaneously to the  $Z_{\text{eff}}$  profile. No prior information on  $Z_{\text{eff}}$  has been included. The result is shown in Figures 7 lhs. The  $Z_{\text{eff}}$  profile contains two points which considerably deviate from the assumed profile. However, the preliminary results of the Bayesian analysis shown in this paper have to be interpreted cautiously. This is due to the fact that the Monte Carlo sampling is not yet sufficiently optimized. Nevertheless, what can be seen from the inversion result in Figure 7 lhs is that the uncertainties of the  $Z_{\text{eff}}$  and  $n_e$  points in the middle of the profiles are smaller than the central and the edge values. This is expected since the main information on the central and the edge values is contained only in the signals of the central and the edge line of sight. In Figure 7 rhs we show an inversion result for the case of further extension of the number of uncertain model parameters. Here we additionally insert a relative uncertainty of 5% of the absolute calibration factors of the spectrometers (concerning the whole spectrum of each spectrometer). As result we also see rather rugged  $Z_{\text{eff}}$  and  $n_e$  profiles but the same tendency of larger errors for the central and the edge points is visible. However, the results have to be seen as preliminary and no final statement about the amplitude of the errors can be made yet.

#### IV. OUTLOOK

The inversion of the full problem by means of the Bayesian data analysis allows a proper treatment of all uncertain parameters of the diagnostic system and provides reliable estimation of resulting

uncertainties of local  $Z_{\text{eff}}$  values. Hence, it is planned to use this tool to optimize the diagnostic setup in such points as the configuration of the lines of sight and of the detector system. Moreover, the spurious radiation contribution to the measured spectra needs a careful treatment. A more detailed model of the divertor thermal radiation is needed which could possibly be tested with experimental spectra recorded during the divertor operation of the stellarator W7-AS. Furthermore, the impact of the bremsstrahlung reflected on the plasma vessel walls should be investigated as well as possible methods of its discrimination from the bremsstrahlung fraction arriving at the detector on a direct way. In the final step the  $Z_{\text{eff}}$  diagnostic will become a part of the integrated data analysis framework for W7-X containing diagnostics like Thomson scattering, interferometry, ECE and soft-x-ray [6].

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**FIGURES**

**FIG. 1.** W7-X Plasma profiles used for the simulation of the spectra and inversion

**FIG. 2.** Spectra simulated for the central and the edge lines of sight (light integration: 7 ms).

**FIG. 3.** Two ways of inversion of the simulated spectra.

**FIG. 4.** Absolute calibrated bremsstrahlung spectrum contaminated with line radiation measured at W7-AS.

**FIG. 5.** Relative error of the bremsstrahlung fit to the simulated spectra of all ten lines of sight depending on the amount of spectral lines and the existence of black body radiation of one temperature.

**FIG. 6.**  $Z_{\text{eff}}$  profiles retrieved by inversion of line-of-sight-integrated bremsstrahlung intensities obtained by baseline fit of the spectra (separately for all lines of sight). lhs: Maximum Entropy regularization, rhs: Minimum Fisher regularization.

**FIG. 7.**  $Z_{\text{eff}}$  profiles retrieved by inversion of the full Bayesian model using Monte Carlo method. lhs: result for  $Z_{\text{eff}}$  and electron density for the case of fitting 10  $Z_{\text{eff}}$  and  $n_e$  values. rhs: additional fit of 10 absolute calibration factors included.

Figure 1

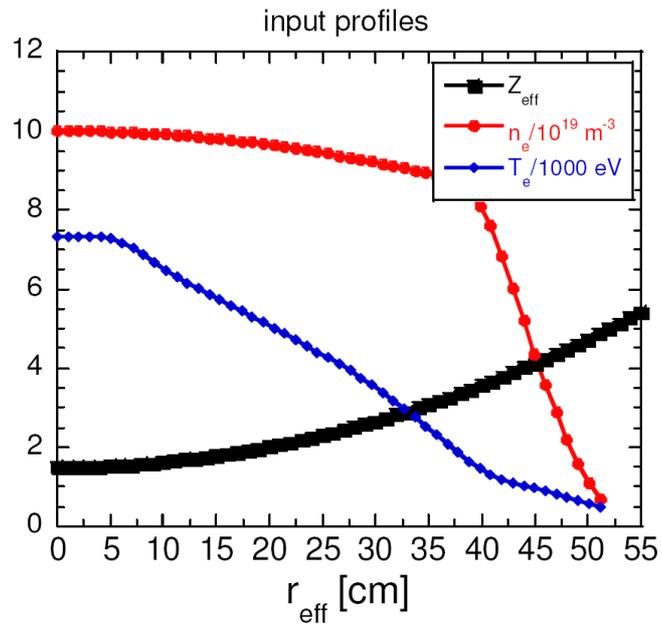


Figure 2

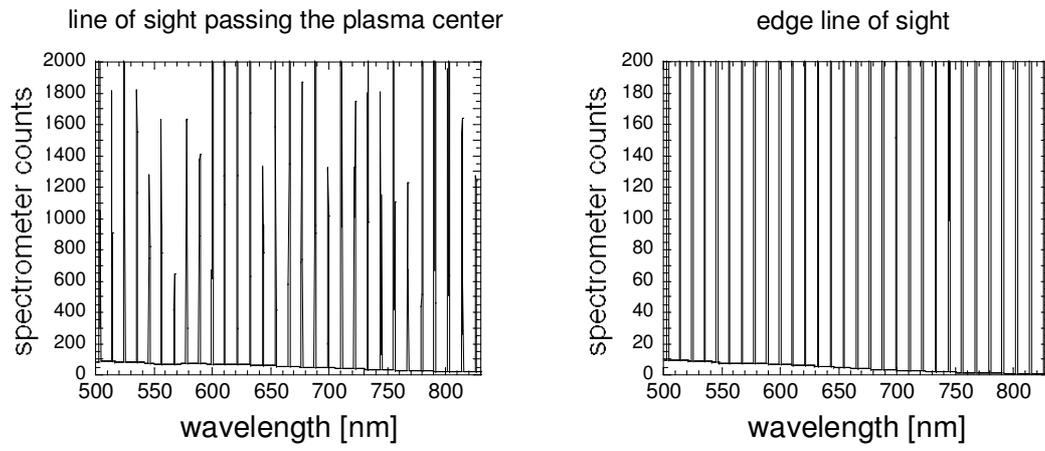


Figure 3

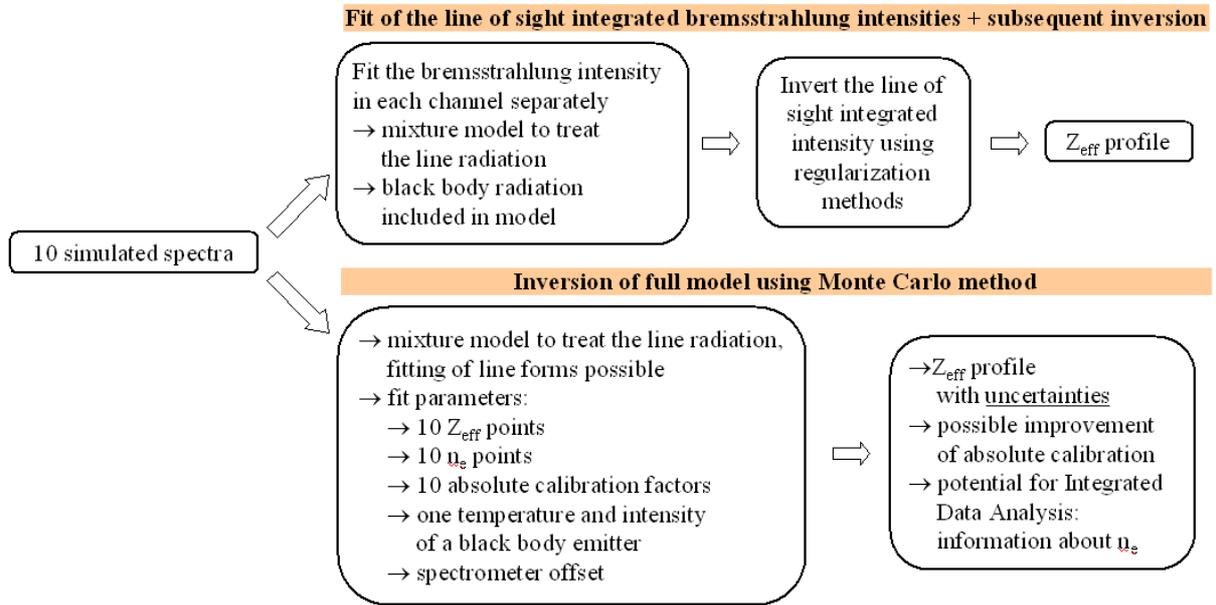


Figure 4

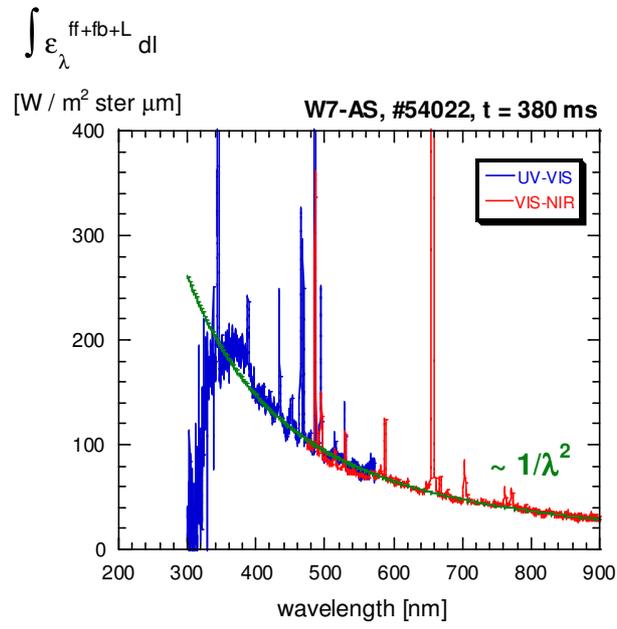


Figure 5

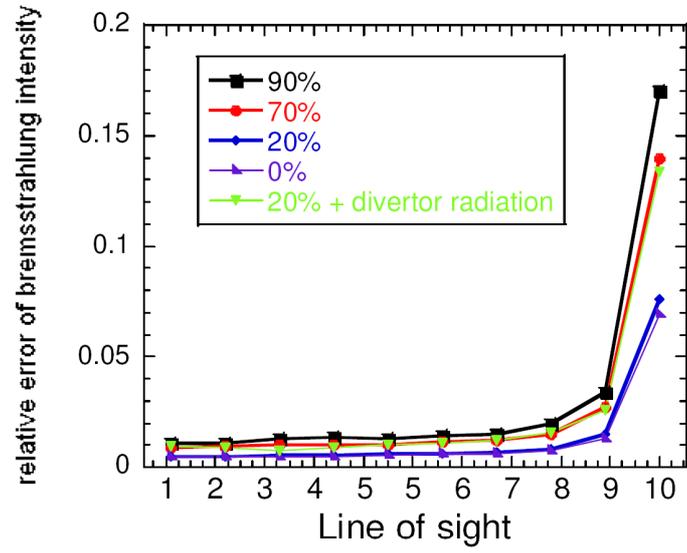


Figure 6

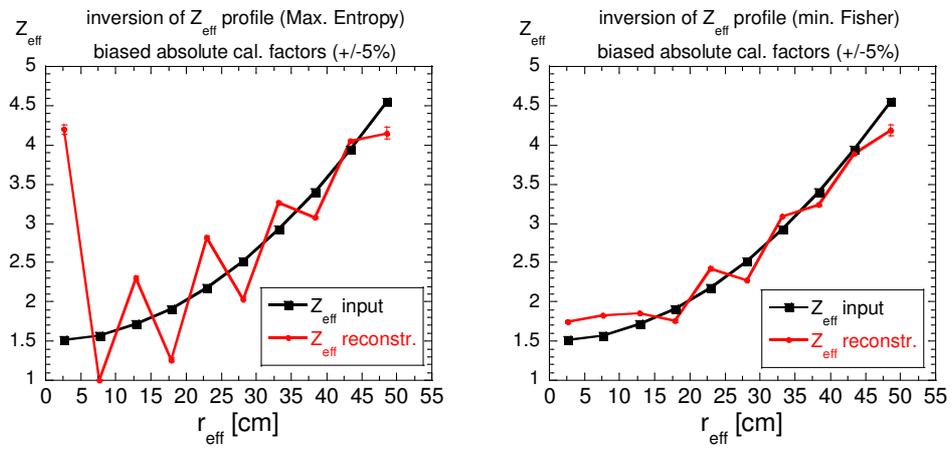


Figure 7

