Minimal physical model for interaction of MHD instability with plasma

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Introduction

Larger scale plasma instabilities not leading to an immediate termination of a discharge often result in periodic nonlinear perturbations of the plasma. Examples of such behavior are Edge Localized Modes (ELMs), sawteeth crashes and other events. A minimal possible physical model is formulated for description of these system with drive and relaxation processes which have strongly different time scales. This zero-dimension model contains only three parameters: power input, relaxation of the instability and influence of the instability on the heat diffusion coefficient. In spite of its simplicity, the model has rich variety of the possible solutions depending on the model parameters. In the present paper, we have focused on the ELMs but similar formulation could be also derived for other cases.

Definition of the model

The model is based on two equations. The first equation being responsible for the relaxation dynamics and can be derived either from linearization of the energy principle or from linear MHD force balance equation $\left(\rho_{m0}\left(d^2\vec{\xi}/dt^2\right) = -\hat{K}_{MHD}\cdot\vec{\xi}\right)$ and has the following form [1]:

$$\underbrace{\frac{d^2}{dt_n^2}\xi_n = \left(p_n' - 1\right)\cdot\xi_n}_{oscillator} - \underbrace{\delta\cdot\frac{d}{dt_n}\xi_n}_{relaxation/damping} \qquad (1)$$

The second equation represents the energy balance either for edge (ELM case) or for the core (sawtooth case) and derived as well in Ref [1]:

$$\frac{d}{dt_n}p_n' = \eta\cdot\left(\underbrace{h}_{\substack{input\\power}} - \underbrace{p_n'}_{\substack{regular\\loss\\term}} - \underbrace{\beta\cdot\xi_n^2\cdot p_n'}_{\substack{loss\\due\_to\\perturb.[2]}}\right) \qquad (2)$$

where ξ_n is the normalized displacement due to instability, $p_n' = dp_n/dr$ is the normalized radial critical gradient, t_n is the normalized time. There are three quantities which determine the behaviour of the system in the parameter space:

ι) h represents the normalized power input into the system and is responsible for inducing a burst $(h>1)$;

ιι) δ represents dissipation and relaxation of the MHD perturbations;

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iii) η is the characteristic relation between the two heat diffusion coefficients. One is the normal heat diffusion depending on χ_0, the other is the heat diffusion due to MHD perturbations which depends on the growth rate of the instability and system geometry.

The last parameter, $\beta = \chi_{anom} / \chi_0$, is not important if its value is large. It is known from the experiments that β is always larger than 100. For such values, it provides very fast heat diffusion due to the perturbation (χ_{anom}) in the burst phase compared to the normal diffusion (χ_0), and does not lead to any changes with further increase of χ_{anom}.

Pellet injection

The model can be extended to describe the pellet injection and its influence on ELM cycle. In this case, an addition function is introduced in the first equation (1). It produces short periodic bursts which simulate pellet injection. The overall power balance is not affected by the pellet which keeps the equation (2) unchanged.

$$\frac{d^2}{dt_n^2}\xi_n = \left(\left[p_n' + pellet(t)\right] - 1\right)\cdot\xi_n - \delta\cdot\frac{d}{dt_n}\xi_n \qquad (3)$$

$$\frac{d}{dt_n}p_n' = \eta\cdot\left(h - p_n' - \beta\cdot\xi_n^2\cdot p_n'\right) \qquad (4)$$

It is possible to model typical experimental situation using equations 3 and 4. The result is shown in figure 1. Initial phase (0s-1s) has no pellets and natural ELM frequency $(f_{natural})$ sets up after a short transition phase (1s-1.4s). In the next phase (1.4s-3s), the ELM frequency locks to the pellet frequency $(f_{pellets} \approx 5\cdot f_{natural})$. This behavior of the system reproduces experimental situation where the ELM frequency was increased by pellet injection [3]. There are two important experimental findings regarding pellet ELM frequency and size, which could be checked for our model:

I. Pellets are able to increase the ELM frequency if $f_{pellets} > f_{natural}$. (Reduction of the ELM frequency is not possible.)
II. Increase of the ELM frequency is accompanied by reduction of the ELM amplitude, but the total power outflow remains the same. (This is true both for natural ELMs and pellet triggered ELMs).

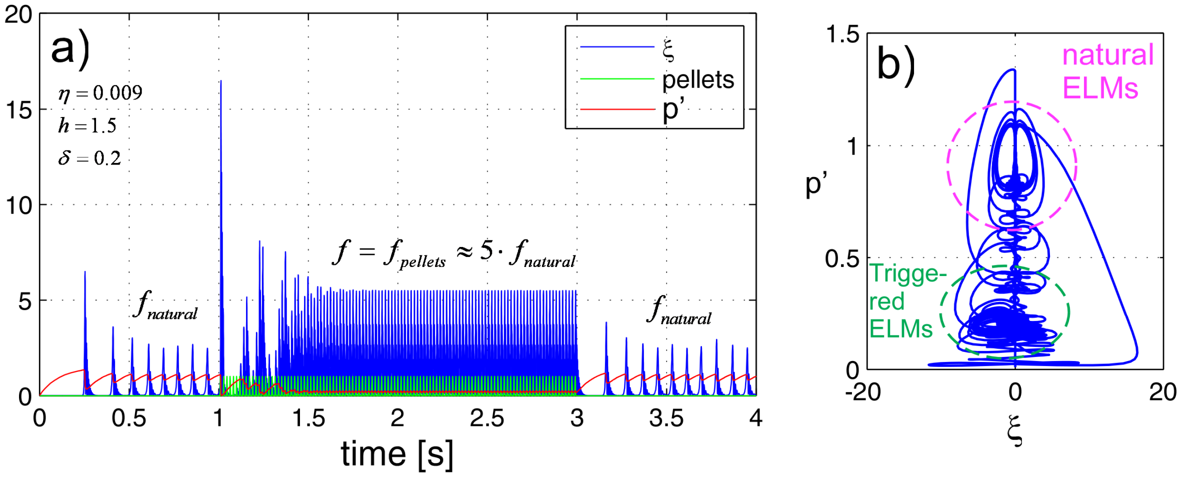

Figure 1. Experimental situation is modelled by solving the system of equations (3,4). Pellets are introduced into the system between 1s and 3s (figure 1a). After a short transition phase (1s-1.4s), the ELM frequency is locked to the pellet frequency. The phase space evolution is shown on figure 1b.

The figure 2a shows that reduction of the ELM frequency could not be achieved if $f_{pellets} \le f_{natural}$. ELM frequency is about natural ELM frequency in this phase. The exact ELM behavior depends on the initial conditions (natural ELM frequency, pellet frequency, start of the pellet time). Increase of the pellet frequency leads to a locking of the ELM frequency to the pellet frequency. Thus, the first experimental observation is reproduced.

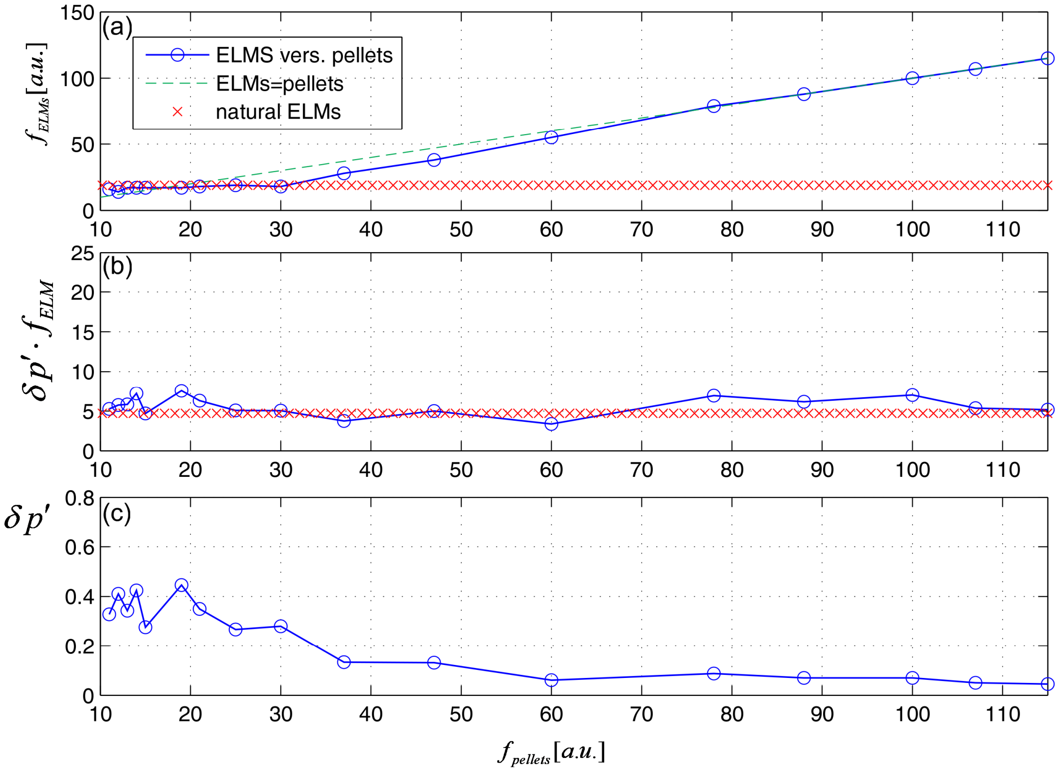

Figure 2. (a) ELM frequency dependence on the pellet frequency, (b) average total energy release per ELM, (c) Energy per ELM.

In our model, the power per pellet is characterized by changes in the pressure gradient $\left(\delta p' = p'_{beforeELM} - p'_{afterELM}\right)$. This change is much smaller in pellet phase compared to natural ELM case (see figure 1a). Scan of the pellet frequency shows that increase of the pellet frequency leads to reduction of $\delta p'$ (figure 2c). At the same time, the total power , $\delta p' \cdot f_{ELM}$, remains constant (figure 2b). This verifies the second observation for the pellet case. It is also interesting that no upper frequency limit for pellet triggering was found. The maximal applied frequency ($f_{pellet} \approx 6 f_{natural}$) leads to oscillations of the pressure gradient.

One can change the natural ELM frequency without pellets by changing the plasma stability and/or relaxation of the MHD perturbations (in our case parameter δ). A scan similar to figure 2 was performed for natural ELMs. The result is similar to the ELM case and agrees with experimental observations that the total power per ELM, $\delta p' \cdot f_{ELM}$, remains the same (see figure 3).

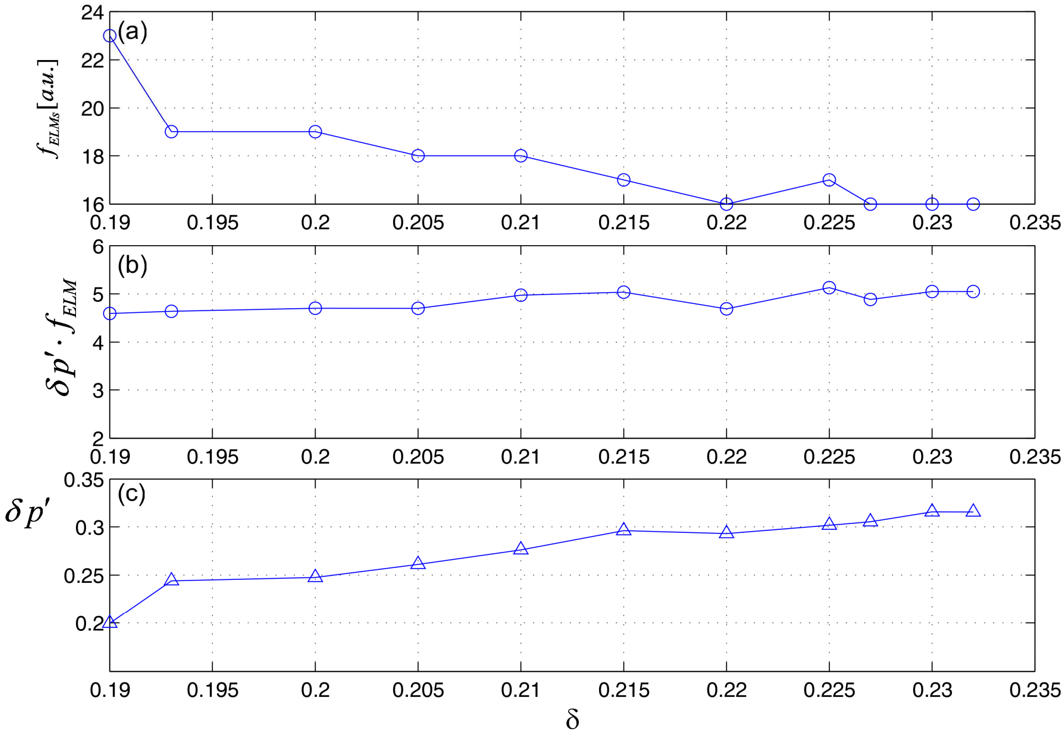

Figure 3. (a) ELM frequency dependence on δ, (b) Average total energy release per ELM, (c) Energy per ELM versus pellet frequency.

One can see that the presented simple model describes surprisingly well the experimental findings. This points out that, the main physical features are indeed included in the model. It is actually the energy balance equation (second equation) which is responsible for the experimental observations (I and II). Any changes in the first equation only change the way of the energy outflow but the total flux remains constant (it is given by the second equation). The proposed zero dimension model does not qualify for a complete description of the plasma phenomena such as ELM or sawtooth, which require full scale nonlinear simulations. However, it provides useful tools for understanding the basic physics and allows to estimate relative quantities.

Reference

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