

Reconstruction of the turbulence radial profile from the reflectometry phase root mean square measurements

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Simple expressions allowing estimation of the turbulent density perturbation from the reflectometry phase root mean square (r.m.s.) measurements and accounting for the turbulence correlation length are given. The numerical procedure for precise turbulence radial profile reconstruction for different turbulence wave number spectra and both O-mode and X-mode reflectometry is proposed. It is validated using a full-wave 1D modeling of an ordinary (O) mode reflectometry in the case of arbitrary density fluctuation profiles.

1. Introduction.

Drift wave turbulence is known to be important in the energy and particles losses from hot tokamak plasmas. The reflectometry signal being extremely sensitive to the density fluctuations carries out the information on the turbulence properties. A considerable interest remains in the interpretation of the reflectometry data to obtain this information. The long scale density fluctuations dominant in the drift turbulence spectrum influence mostly the phase of the scattered wave, collected by the receiving antenna. The fluctuating part of the probing wave's phase $\delta\phi = \phi - \langle \phi \rangle$, where ϕ and $\langle \phi \rangle$ are the full random phase and the statistically averaged phase associated with the mean plasma density profile correspondingly, is thought to contain information on the density fluctuation's radial profile $\langle \delta n^2 \rangle^{1/2}$. In order to get out this information the reflectometry phase root mean square (r.m.s.)

$$\sqrt{\langle \delta\phi^2 \rangle} = \sqrt{\langle (\phi - \langle \phi \rangle)^2 \rangle}, \quad (1)$$

is usually investigated under assumption that the cut-off vicinity contributes mainly into the fluctuating part of the signal's phase, so that the phase r.m.s. is proportional to the density fluctuations $\langle \delta n^2 \rangle^{1/2}$ in the cut-off layer ($\sqrt{\langle \delta\phi^2 \rangle} \propto \langle \delta n^2 \rangle^{1/2} \Big|_{\text{cutoff}}$). This simple interpretation of the experimental data however faces with huge difficulties as the reflectometry wave suffers from the poorly localized forward scattering all over its path from the probing antenna to the cut-off and back. In such a case the problem of the density fluctuations radial profile reconstruction from

the phase r.m.s. measurements occurs, as it will be shown below, non local and requiring an additional hypothesis on the density fluctuations radial wave number spectrum.

In this paper in the frame of 1D slab plasma model we propose a numerical procedure of the density fluctuations radial profile reconstruction from the experimental data provided by fixed frequency or frequency hopping reflectometers. This method is applicable for both O- and X-mode reflectometers. For simplicity, the procedure is validated using a full-wave 1D modeling of an ordinary (O) mode reflectometry assuming either Gaussian or exponential turbulence radial wavenumber spectrum, providing reasonable approximation of experimental data both at long and small scales [1-3]. The remainder of this paper is organized as follows: In the Section 2 the theoretical background with the analytical expressions for both the fluctuating part of the phase of the reflectometry signal and the phase r.m.s. for the ordinary (O) and extraordinary (X) probing wave in a case of the non-uniform density fluctuations radial profile are presented. To evaluate the statistical averaging in the phase r.m.s. we consider the Gaussian and the exponential spectrum of the turbulence. Then, in the Section 3 we propose, basing on the results of Section 2, the numerical procedure of the reconstruction of the radial profile of the turbulent density perturbations from the experimental data. We finish with conclusions in the Section 4.

2. Theoretical background.

In the present paper we develop the procedure of the density fluctuations radial profile reconstruction from the fluctuating part of the reflectometry phase collected by the probing wave on its path from the antenna to the cut-off and back. For the sake of simplicity we consider 1D plasma slab model and assume no possibility of multi-reflections of the probing wave from the plasma boundary, metallic walls and the cut-off that is reasonable for the case of large fusion devices. We assume also that only one cut-off point exists in plasma that implies in the case of O-mode limitation on the turbulence density perturbation $|\delta n/n_c| \ll l_c/x_c$, where $n_c = m_e \omega^2 / (4\pi e^2)$ is the critical density expressed in Gaussian units which we use in this paper and x_c is the distance between the antenna and the cut-off. Assuming large turbulence correlation length $l_c \ll l_a$, where $l_a = (c^2 x_c / \omega^2)^{1/3}$ is the Airy wavelength, we justify omitting of backscattering far from the cut-off. Under above assumptions the phase of the reflected wave can be calculated explicitly using the geometrical optics (WKB) approach [4]. It is represented as a sum of two terms

$$\langle \phi \rangle = 2 \int_0^{x_c} k_j(x) dx - \frac{\pi}{2} \quad \text{and} \quad \delta\phi = -\frac{\omega^2}{c^2} \int_0^{x_c} \frac{\delta n(x)}{n_c} \frac{h_j dx}{k_j(x)} \quad (2)$$

determined by the unperturbed or averaged density profile ($\langle \phi \rangle$) and density fluctuations ($\delta\phi$) correspondingly.

where the radial wave number $k_j(x)$ and the element h_j for an ordinary ($j=o$) and an extraordinary ($j=e$) wave according to [5] are given correspondingly by

$$k_o = \frac{\omega}{c} \sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}}, \quad k_e = \frac{\omega}{c} \sqrt{\frac{(\omega^2 - \omega_{pe}^2)^2 / \omega^2 - \omega_{ce}^2}{\omega^2 - \omega_{ce}^2 - \omega_{pe}^2}}, \quad h_o = 1, \quad h_e = \frac{[\omega^2 - 2\omega_{pe}^2][\omega^2 - \omega_{ce}^2] + \omega_{pe}^4}{[\omega^2 - \omega_{ce}^2 - \omega_{pe}^2]^2} \quad (3)$$

where $\omega_{pe} = (4\pi e^2 n / m_e)^{1/2}$ and $\omega_{ce} = eH / m_e c$ are the electron plasma and cyclotron frequencies correspondingly.

The reflected wave phase mean square is then given in a form:

$$\langle \delta\phi^2 \rangle = \frac{\omega^4}{c^4} \int_0^{x_c} \int_0^{x_c} dx' dx'' \frac{h_j(x') h_j(x'')}{k_j(x') k_j(x'')} \frac{\langle \delta n(x') \delta n(x'') \rangle}{n_c^2} \quad (4)$$

where $\langle \dots \rangle$ means the statistical averaging. This expression was studied in detail in papers [5, 6] devoted to investigation of radial correlation reflectometry nonlinear regime. The density fluctuations were supposed to be statistically inhomogeneous, however the turbulence inhomogeneity scale L much greater than the turbulence correlation length l_c was assumed satisfying the natural inequality $l_c \ll L$. Under this assumption the density fluctuation correlation function is given by the following expression $\langle \delta n(x') \delta n(x'') \rangle = \delta n_0^2(x' + x'') K(x' - x'', x' + x'')$, where

$$K(x' - x'', x' + x'') = l_c(x' + x'') \int_{-\infty}^{\infty} \frac{d\kappa}{2\pi} \tilde{K}(\kappa, x' + x'') \exp[i\kappa(x' - x'')],$$

$$l_c^{-1} = \int_{-\infty}^{\infty} \frac{d\kappa}{2\pi} \tilde{K}(\kappa, x' + x''), \quad \tilde{K}(0, x' + x'') = 1 \quad (5)$$

δn_0^2 represents the turbulent density perturbation level; $\tilde{K}(\kappa, x' + x'')$ is the spectrum of the density fluctuations and κ stands for their radial wave number. As it was shown [5, 6], under assumption of a weak turbulence statistical inhomogeneity the reflected wave phase mean square can be represented in the form

$$\langle \delta\phi^2 \rangle = \frac{\omega^2}{c^2 n_c^2} \int_0^{x_c} d\xi \cdot l_c \delta n_j^2(x_c - \xi) S(\xi) \quad (6)$$

where new notations have been introduced: $\xi = x_c - x'$, $\delta n_j^2 = h_j^2 \delta n_0^2$ and $S(\xi)$ is the scattering efficiency characterising relative contribution of different regions of probing wave trajectory to the reflected wave phase r.m.s.. Far from the cut-off, at $x_c - x' \ll l_c$, we may put $l_c = 0$ in (5) and considering the density fluctuation correlator in (4) as a delta-function ($K(x' - x'', x' + x'') \ll \delta(x' - x'')$), obtain the scattering efficiency given by simple expression:

$$S(\xi) \approx \frac{\omega^2}{c^2} \frac{1}{k_j^2(\xi)} \quad (7)$$

It grows towards the cut off thus underlining its contribution into the phase r.m.s., however the efficiency singularity in the cut off predicted by (7) and provided by vanishing probing wave number is not physical. The efficiency growth saturates at distances from the cut off comparable to the turbulence correlation length which was put to zero when deriving (7). In the nearest cut-off vicinity, at $|x_c - x| \leq l_c$ the density profile can be approximated by linear dependence that permits us to represent $S(\xi)$ in a form

$$S(\xi) \approx \frac{L_j}{2} \int_{-\infty}^{\infty} d\kappa \tilde{K}(\kappa, x_c - \xi) J_0(\kappa \xi) \quad (8)$$

where $L_j = \omega^2 / c^2 \cdot \partial k_j^2 / dx|_{x_c}^{-1}$ characterizes the local inhomogeneity of refractive index squared and $J_0(\kappa \xi)$ is the Bessel function of the first kind. The later expression is not singular in the cut off. For the linear background density profile equation (7) is a strict mathematical asymptotic of (8) at $\xi \gg l_c$. The expression (8) for the scattering efficiency can be simplified in two important [1-3] particular cases of the Gaussian and the exponential spectrum of the fluctuations. Begin with consideration of the first one $\tilde{K}(\kappa) = \exp[-\kappa^2 / \kappa_0^2]$, where the spectrum width is related to the correlation length introduced by (5) $l_c = 2\sqrt{\pi} \kappa_0^{-1}$. In this case the scattering efficiency (8) in the cut-off vicinity is given by the following expression:

$$S(\xi) = \sqrt{\pi} \frac{L_j \kappa_0}{2} \exp\left(-\frac{\xi^2 \kappa_0^2}{8}\right) I_0\left(\frac{\xi^2 \kappa_0^2}{8}\right), \quad (9)$$

where $I_0(\dots)$ is the modified Bessel function of the first kind. Using (9) and (6) it is also possible to derive an explicit expression for the reflectometry probing phase r.m.s. for the linear background density profile and statistically homogeneous fluctuations [7]:

$$\langle \delta\phi^2 \rangle \approx \frac{\omega^2 L_j l_c}{c^2} \frac{\delta n_j^2}{n_c^2} \bigg|_{x_c} \left[\ln\left(\frac{8}{\sqrt{\pi}} \frac{L_j}{l_c}\right) + 0.71 \right]. \quad (10)$$

In this case we can find the level of the density fluctuations as a function of the probing wave frequency ω :

$$\frac{\delta n_j}{n_c} \bigg|_{x_c(\omega)} = \frac{\sqrt{\langle \delta\phi^2(\omega) \rangle}}{\sqrt{\frac{\omega^2 L_j(\omega) l_c}{c^2} \left[\ln\left(\frac{8}{\sqrt{\pi}} \frac{L_j(\omega)}{l_c}\right) + 0.71 \right]}}. \quad (11)$$

Proceeding, we consider the exponential spectrum of the fluctuations $K(\kappa) = \exp[-|\kappa|/\kappa_0]$ for which the spectrum width is related to the correlation length as $l_c = \pi\kappa_0^{-1}$. In this case the integral (8) can be evaluated explicitly providing

$$S(\xi) = \frac{L_j}{\sqrt{\xi^2 + \kappa_0^{-2}}}. \quad (12)$$

whereas the phase r.m.s. (6) for the linear density profile and statistically homogeneous turbulence results in expression

$$\langle \delta\phi^2 \rangle \approx \frac{\omega^2}{c^2} L_j l_c \frac{\delta n_j^2}{n_c^2} \Big|_{x_c} \left[\ln \left(\frac{\pi L_j}{l_c} \right) + 2 \ln 2 \right]. \quad (13)$$

The level of the density fluctuations for the exponential spectrum of the density fluctuations can be represented as a function of the probing wave frequency ω in a form:

$$\frac{\delta n_j}{n_c} \Big|_{x_c(\omega)} = \frac{\sqrt{\langle \delta\phi^2(\omega) \rangle}}{\sqrt{\frac{\omega^2 L_j(\omega) l_c}{c^2} \left[\ln \left(\frac{\pi L_j(\omega)}{l_c} \right) + 2 \ln 2 \right]}}. \quad (14)$$

In the case of arbitrary turbulence spectrum, as it was shown in [6], the relation between the phase r.m.s. and the density perturbation r.m.s. is given by expression

$$\langle \delta\phi^2 \rangle \approx \frac{\omega^2}{c^2} L_j l_c \frac{\delta n_j^2}{n_c^2} \Big|_{x_c} \left[\ln \left(\frac{L_j}{l_c} \right) + \alpha \right]$$

which looks similar to (10) and (13) and differs only by a factor α determined by specific form of the turbulence spectrum. Expressions (10), (11) and (13), (14) generalize relations between the phase perturbation and the coherent fluctuation amplitude derived in [8] to the case of statistically homogeneous turbulent density fluctuations. In the general case of arbitrary both the background density profile and the density fluctuations radial profile, the expressions (11) and (14) provide the fluctuations level estimated by ignoring the non-local contributions to the reflectometry phase all over the plasma far from the cut off. Nevertheless, if the effects of non-locality are not excessively strong, we can use (11) and (14) as the zero order approximation for the numerical procedure being described in the forthcoming section.

3. Numerical modelling of O-mode reflectometry.

In this paper we analyze numerically the case of O-mode reflectometry and the Gaussian spectrum of the fluctuations $K(\kappa) = \exp[-\kappa^2/\kappa_0^2]$. To develop the semi-analytical procedure of the density

fluctuations radial profile reconstruction we solve the Helmholtz equation by using a 4th order Numerov scheme [9]. We assume the plasma density consisting of two pieces: the unperturbed regular part $\bar{n}(x)$ and the random part $\delta n(x)$ given by a sum of harmonic fluctuations

$$\delta n(x) = \frac{\delta n_0}{2\pi} \sum_{m=-N_k}^{m=N_k} \Delta\kappa \sqrt{\tilde{n}_{mq}^2} e^{im\Delta\kappa x} e^{i\varphi_m} \quad (15)$$

where the wave numbers are defined by $\kappa = m\Delta\kappa$, random phases φ_m , and amplitude distributed in accordance with the input turbulence spectrum. The value of the wave number resolution $\Delta\kappa$ thus defines also the wavelength of the longest mode that is present in the simulation. The simulated scattered wave phase r.m.s. $\langle \delta\phi^2 \rangle$ is obtained from the variance of $\delta\phi$ in a significant number ($N \approx 50000$) of independent random density perturbation samples.

The described simulation procedure was first benchmarked against the case of linear density profile and statistically homogeneous turbulence characterized by Gaussian wavenumber spectrum for which both the level δn_0^2 and correlation length $l_c = 2\sqrt{\pi\kappa_0^{-1}}$ are not dependent on coordinate. In this case, according to analytical theory, the scattered wave phase r.m.s. $\langle \delta\phi^2 \rangle$ should be given by expression (10). However with our choice of the simulation setup $\langle \delta\phi^2 \rangle$ appears to be substantially dependent on $\Delta\kappa$, as it is seen in figure 1. This dependence is explained by much stronger sensitivity of reflected wave phase fluctuations to long wavelength density fluctuations [10] leading also to non ideal localization of the fluctuation reflectometry signal by the cut off position. Nevertheless, we observe that $\langle \delta\phi^2 \rangle$ is within 10% of its final (at $\Delta\kappa \rightarrow 0$) value, if the condition $\Delta\kappa L < 1$ holds. Already in 1D modeling, to say nothing about the 2D case, this criterion can be demanding in terms of computing resources, given the plasma dimension is large and the vacuum wave length is small. It should be underlined that at $\Delta\kappa \rightarrow 0$ the simulated value of $\langle \delta\phi^2 \rangle$ is found to be in excellent agreement with the analytical result (10) demonstrating its potential for estimation of turbulent density perturbation based on the reflected wave phase r.m.s.. After this short introduction of the numerical scheme and the way of the phase r.m.s. calculation used for treatment of the problem we proceed with consideration of the procedure of the density fluctuations radial profile reconstruction from the experimental data for realistic density profile.

4. Reconstruction of the radial profile of the fluctuations.

In this section based on theoretical relations (6) – (9) we develop the semi-analytical procedure of the density fluctuations radial profile reconstruction from the measured phase r.m.s. $\langle \delta\phi_{\text{exp}}^2 \rangle$ dependence on the probing frequency. At the first step, we use equation (11), derived for the homogeneous fluctuations radial profile, to estimate the absolute value of the density fluctuations at

the cut-off of the probing wave at each frequency ω from the corresponding phase r.m.s. $\langle \delta\phi_{\text{exp}}^2 \rangle(x_c(\omega))$. Then, we use the obtained estimation $\sqrt{\langle \delta n_0^2(x_c) \rangle}$ to calculate a set of phase r.m.s. $\langle \delta\phi_0^2 \rangle$. Thus, we substitute $\sqrt{\langle \delta n_0^2(x) \rangle}$ determined by (11) with $L_o(x)$ being the local gradient length at the actual cut-off position into equation (6) with the kernel having been defined in the whole x - coordinate space according equations (7) and (9) to recalculate $\langle \delta\phi^2 \rangle$. Naturally, as we neglect the non-locality of the phase perturbations formation entirely, when estimating the density fluctuation $\sqrt{\langle \delta n_0^2(x) \rangle}$ profile, the calculated phase perturbation profile $\langle \delta\phi_0^2(x_c) \rangle$ doesn't coincide with the measured one. However the error can be used to correct the density perturbation value using the iterative procedure based upon the least-square fit procedure, a key element of which is a hypothesis of a weak non-locality due to the small angle scattering (the main contribution to the phase perturbations comes from the cut-off vicinity). We make the difference of $\langle \delta\phi_0^2(x_c) \rangle$ to $\langle \delta\phi_{\text{exp}}^2 \rangle$ as a subject to minimization that affords to precise $\sqrt{\langle \delta n_0^2(x_c) \rangle}$ for the fixed $x_c(\omega)$.

We illustrate the procedure for the unperturbed density profile and the radial profile of the density fluctuations $\sqrt{\langle \delta n_0^2(x) \rangle}$ shown in figure 2, which experiences sharp growth when approaching plasma edge. For the sake of simplicity we neglect the effect of the turbulence inhomogeneity on the spectrum of the fluctuations. Then, we consider a set of $\langle \delta\phi_{\text{exp}}^2 \rangle$ numerically calculated for different ω using full-wave Helmholtz equation for O-mode reflectometry and shown in figure 3 by circles, as the experimental data. At the first step of the reconstruction procedure described above we get an estimation $\sqrt{\langle \delta n_0^2(x_c) \rangle}$ for this phase r.m.s. profile using the ‘‘local’’ expression (11). The corresponding distribution is shown by crosses in figure 4. It is worth to mention that this estimation of the actual density fluctuation profile shown by solid line there is already rather accurate (less than 15% difference). Based upon this distribution we calculate the phase r.m.s. using expression (6) and compare it to the ‘‘experimental’’ one $\langle \delta\phi_{\text{exp}}^2 \rangle$ in figure 3, where it is shown by crosses. The obtained phase r.m.s. distribution resembles the experimental one, however exceeding it in the internal plasma region by up to 25%. To obtain more accurate results, a nonlinear least-squares method based on the interior-reflective Newton method described in [11, 12] is used in which the density fluctuations amplitudes $\delta n(x_i)$ are the unknowns. The non-linear data-fitting process is initiated using an input set of density fluctuations amplitude provided by the simplified formula (11). An illustration of this iterative method is shown on the Fig. 4 where the reconstructed density fluctuations profile corresponding to squares exhibits an improved agreement

with the input density fluctuations profile and becomes very close to it. To examine the influence of the phase r.m.s. error, caused by not perfect statistical averaging, on the $\sqrt{\langle \delta n_0^2(x) \rangle}$ reconstruction we vary one element of the 'measured' set $\langle \delta \phi_{\text{exp}}^2 \rangle$ at the plasma edge where the phase r.m.s. is the highest (see figure 5), as well as the probability of giant phase fluctuations caused by single events (blobs, filaments etc.), and then, study the effect on the reconstructed fluctuation profile after the fitting procedure in figure 6. As it is seen there, the error effect on the fluctuation profile is only strong at the cut off of the frequency for which the erroneous phase r.m.s. measurement was taken. Figure 6 illustrates the limited spatial range of the density fluctuation profile which is affected by the error. Keeping in mind that the probability to find giant single density perturbation decreases in the plasma core we may conclude that the density perturbation reconstruction in this region is more robust.

Discussing the limitations which are put on the proposed fluctuation profile reconstruction procedure by 2D effects caused by plasma curvature and finite probing beam width we would like to mention that as it was shown in [13], the phase measurements are less sensitive to 2D effects and can be described by 1D simulations. A similar conclusion has been obtained by [14] when performing turbulence radial wave number spectrum reconstruction from the reflected wave phase fluctuations.

5. Conclusions.

In this paper we propose a simple looking expression for estimation of the turbulent density perturbation from the measured reflected wave phase r.m.s. Moreover, we developed the semi-analytical procedure for the turbulence density perturbation radial profile reconstruction from the phase of the wave reflected from the cut-off. Using a 1D full-wave O-mode code we have successfully validated analytical expressions which relate the phase fluctuations in fluctuation reflectometry to the absolute value of the density fluctuations. If the profile of the fluctuation level is inhomogeneous the phase fluctuations can be described accurately by an integral expression. With this expression we demonstrated that the reconstruction of the fluctuation level profile can be obtained from a set of phase variances measured at different frequencies. Furthermore, the integral expression allowed us to study how uncertainty of the fluctuation level at the plasma edge can contribute to the error of the fluctuation level measured at the plasma center. We can note also that the proposed procedure utilizes the data of the phase perturbations of the reflected wave and is based on the WKB approximation valid both for O-mode and X-mode reflectometry. This method is complementary to those giving the density fluctuations profile obtained using ultrafast sweep frequency reflectometer data running on X- or O-mode [15, 16]. Therefore, as the analytical

expressions in section 2 were derived within the WKB approximation we believe that they work well in the case of the X-mode fluctuation reflectometry just as for the O-mode fluctuation reflectometry which was confirmed by comparison to results of numerical modeling in an inhomogeneous turbulence case.

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- [1] L Vermare, P Hennequin, Ö D Gürçan et al. (2011) *Phys. Plasmas* **18** 012306
- [2] A D Gurchenko, E Z Gusakov, D V Kouprienko et al. (2010) *Plasma Phys. Control. Fusion* **52** 035010
- [3] A D Gurchenko and E Z Gusakov (2010) *Plasma Phys. Control. Fusion* **52** 124035
- [4] V L Ginsburg (about WKB method)
- [5] E Z Gusakov and A Yu Popov (2004) *Plasma Phys. Control. Fusion* **46** 1393
- [6] E Z Gusakov and A Yu Popov (2002) *Plasma Phys. Control. Fusion* **44** 467
- [7] G Leclert, S Heuraux, E Z Gusakov et. al. (2006) *Plasma Phys. Control. Fusion* **48** 1389
- [8] N Bretz (1992) *Phys. Fluids B* **4** 2414
- [9] C Fanack, I Boucher, F Clairet et al. (1996) *Plasma Phys. Control. Fusion* **38** 1915
- [10] E Z Gusakov, N V Kosolapova (2011) *Plasma Phys. Control. Fusion* **53** 045012
- [11] T F Coleman and Y Li (1994) *Mathematical Programming* **67** 189-224
- [12] T F Coleman and Y Li (1996) *SIAM Journal on Optimization* **6** 418-445
- [13] Lin (About curvature effects)
- [14] S Hacquin, F da Silva, S. Heuraux et al. 28th EPS Conf. CFPP ECA Vol. 25A (2001) p. 3.078
- [15] L Vermare, S Heuraux, F Clairet et. al. (2006) *Nucl. Fusion* **46** S743
- [16] T Gerbaud, F Clairet, A Sirinelli et. al. (2006) *Rev. Sci. Instrum.* **77** 10E928

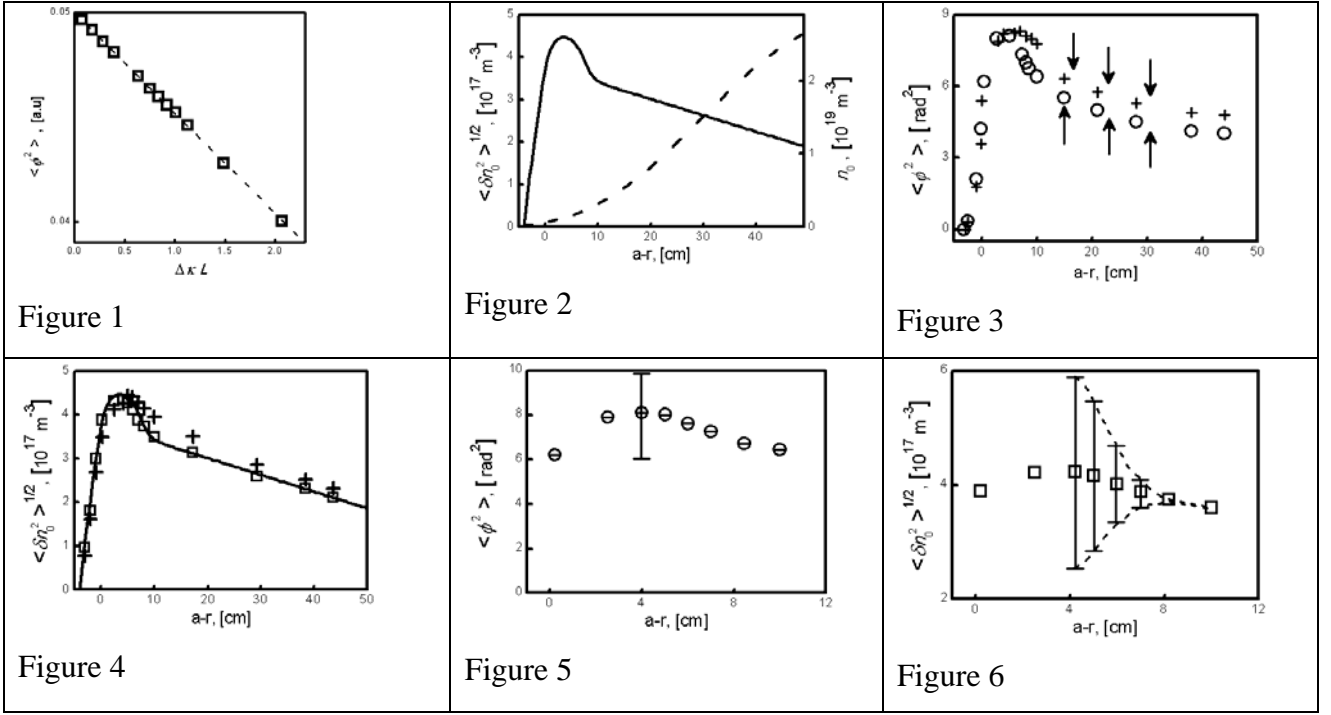


Figure 1. The phase variance $\langle \phi^2 \rangle$ as a function of $\Delta\kappa L$ for $f = 47\text{GHz}$, $L = 58\text{cm}$, $l_c = 0.75\text{cm}$, $\sqrt{\langle \delta n_0^2 \rangle} / n_c = 0.001$. Note that $\Delta\kappa$ stands for both the wavenumber resolution and the wavenumber of the longest mode that is present in the simulation box.

Figure 2. The background density profile (dashed curve) and the fluctuations level (solid curve); $x_c = a$.

Figure 3. $\langle \delta \phi^2 \rangle$ versus coordinate of the cut-off (17 different frequencies). Open circles - $\langle \delta \phi^2 \rangle$ full-wave simulation for the profiles shown in figure 2, crosses - $\langle \delta \phi^2 \rangle$ obtained using expression (10).

Figure 4. $\sqrt{\langle \delta n_0^2 \rangle}$ versus the coordinate of the cut-off. Solid curve - input fluctuations level, crosses - initial guess reconstructed from (10), squares – calculated from least-squares fitting of the $\langle \delta \phi^2 \rangle$ data.

Figure 5. Error in the $\langle \delta \phi^2 \rangle$ data located at the plasma edge.

Figure 6. Effect of the error on the reconstructed $\sqrt{\langle \delta n_0^2 \rangle}$.