AdS/CFT Duality and the Higgs Branch of $\mathcal{N} = 2$ SYM

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Abstract

We construct the AdS description of the Higgs branch of the finite $\mathcal{N} = 2$ $Sp(N)$ gauge theory with one antisymmetric hypermultiplet and four fundamental hypermultiplets. Holography, combined with the non-renormalization of the metric on the Higgs branch, leads to novel constraints on unknown terms in the non-abelian Dirac-Born-Infeld action. These terms include non-minimal couplings of D-branes to bulk supergravity fields.

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1 Introduction

Many generalisations of the original AdS/CFT duality have been proposed and in particular there have been numerous articles discussing supergravity duals of theories with matter in the fundamental representation, including examples with confinement and chiral symmetry breaking. We will discuss the supergravity description of the Higgs branch of a finite four-dimensional $\mathcal{N} = 2$ gauge theory with fundamental representations [1]. This article is based on work which is expounded in more detail in [1]. Besides allowing one to study the Higgs phase of strongly coupled large N gauge theories, this work leads to constraints on unknown terms coupling D-branes to supergravity, as well as possible cosmological applications [2].

The AdS description of the Higgs branch involves a supergravity background with probe D-branes. We will specifically consider a finite $\mathcal{N} = 2$ $\mathrm{Sp}(N)$ gauge theory which is conformal at the origin of moduli space, and is dual to string theory in $\mathrm{AdS}_5 \times S^5/Z_2$, with D7-branes wrapping the Z_2 fixed subspace with geometry $\mathrm{AdS}_5 \times S^3$. This background is the near horizon limit of a D3-D7-O7 system, with the D7-branes treated as probes.

The $\mathcal{N} = 2$ theory we consider has one hypermultiplet in the antisymmetric representation and four in the fundamental representation. For this theory, there is a known exact correspondence between the Higgs branch and the moduli space of Yang-Mills instantons [3, 4] (see [5] for a review). We will show that the equations of motion obtained from the D7-brane effective action at leading order in the α' expansion admit solutions which are the usual Yang-Mills instantons, despite the curved background. The existence of these solutions is due to a conspiracy between the Yang-Mills and Wess-Zumino terms in the D7-brane action.

At higher orders in α', little is known about the non-Abelian DBI action in flat space, with the exception of a few low order terms [6–9]. Even less is known about non-minimal couplings between the world-volume gauge fields on D-branes and bulk fields (curvature, p-forms and dilaton) which appear at higher orders in α', although some terms of the form $R^2\,\mathrm{tr}F^2$ have been studied [10, 11]. We will show that the F^4 corrections in the D-brane effective action do not modify the leading order solution, but without knowing all the coupling to bulk fields with non-zero background value we can not explicitly show that conventional Yang-Mills instantons remain solutions at higher orders in α'. However the exact correspondence between instantons and the Higgs branch implies that instantons must be solutions to all orders in the α' expansion. This leads, reversing the point of view, to constraints on the unknown couplings. We find for example that all terms containing bulk fields which are quadratic in the D7-brane field strength must sum to zero when the bulk fields are set equal to their background values. This constraint is similar in spirit to constraints on the flat space DBI action which follow from requiring that stable holomorphic bundles solve the equations of motion [9, 12, 13].

The metric on the Higgs branch moduli space can be computed at large 't Hooft coupling by considering slowly varying instantons on the probe D-branes. The non-renormalization of the metric on the Higgs branch [14] implies that the leading term in the strong coupling expansion generated by the AdS/CFT duality must be the only

[1]In related work, the AdS description of the Higgs branch of a $(4, 4)$ defect CFT was constructed in [15].

non-zero term, and should equal the weak coupling tree level result. We will show that the leading term in our construction gives rise to the correct metric on the Higgs branch and that higher order corrections vanish assuming that certain bulk − brane couplings sum to zero.

2 Holography for an $\mathcal{N} = 2$ gauge theory with fundamental representations

We will consider the $\mathcal{N} = 2$ theory which describes the low energy dynamics of a D3-D7-O7 system. This system consists of 4 D7-branes coincident with an O7-plane, such that one has a consistent tadpole free string background, and N D3-branes within the D7-O7-plane. It arises considering the near horizon geometry on a stack of D3-branes in the vicinity of a fixed point in the type IIB orientifold $T^2/(-1)^F \Omega I$. At low energies, this system is described by a four dimensional gauge theory with $\mathrm{Sp}(N)$ gauge symmetry and $\mathrm{SO}(8)$ flavor symmetry. There is one hypermultiplet in the anti-symmetric representation and four in the fundamental representation. The latter arise from open strings stretched between the D3- and D7-branes, and have non-zero expectation values on the Higgs branch.

At the origin of the moduli space, the theory is conformal and is dual to string theory in $\mathrm{AdS}_5 \times S^5/Z_2$ with a D7-brane wrapping the $\mathrm{AdS}_5 \times S^3$ fixed surface [16, 17]. The near horizon geometry on the D3-branes is $\mathrm{AdS}_5 \times S^5/Z_2$, with metric

$$\mathrm{d}s^2 = \frac{r^2}{L^2}\left(-\mathrm{d}x_0^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2\right) + \frac{L^2}{r^2}\left(\mathrm{d}r^2 + r^2 \mathrm{d}\hat{\Omega}_5^2\right), \tag{2.1}$$

where L is the radius of both the AdS_5 and the S^5 factors and as usual $L^4 = 4\pi g_{\mathrm{s}} N \alpha'^2$. In (2.1) we have denoted with x_μ, $\mu = 0, 1, 2, 3$, the coordinates on the AdS_5 boundary and with r the radial coordinate transverse to the D3-branes, $r^2 = X_4^2 + \cdots + X_9^2$. In (2.1) $\mathrm{d}\hat{\Omega}_5^2$ denotes the metric on S^5/Z_2 given by

$$\mathrm{d}\hat{\Omega}_5^2 = \mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2 + \cos^2\theta\, \mathrm{d}\Omega_3^2, \tag{2.2}$$

where the range of ϕ is $[0, \pi]$ instead of $[0, 2\pi]$ as for an ordinary S^5.

The D7-branes are at a fixed point of the orientifold, $X_8 = X_9 = 0$. After taking the near horizon limit they fill AdS_5 and wrap the S^3 corresponding to $\theta = 0$ in (2.2), which is fixed under Z_2. The induced metric on the D7-branes is

$$\mathrm{d}s^2 = \frac{U^2}{L^2}\, \mathrm{d}x_\parallel^2 + \frac{L^2}{U^2}\left(\mathrm{d}U^2 + U^2 \mathrm{d}\Omega_3^2\right) = \upsilon^2\, \mathrm{d}x_\parallel^2 + \frac{1}{\upsilon^2}\, \mathrm{d}X_\perp^2, \tag{2.3}$$

where

$$U^2 = r^2\big|_{X_8 = X_9 = 0} = X_4^2 + X_5^2 + X_6^2 + X_7^2$$

and

$$\mathrm{d}x_\parallel^2 = -\mathrm{d}x_0^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2, \qquad \mathrm{d}X_\perp^2 = \mathrm{d}X_4^2 + \mathrm{d}X_5^2 + \mathrm{d}X_6^2 + \mathrm{d}X_7^2.$$

For convenience of notation in (2.3) we have also defined the dimensionless variable υ related to U by $\upsilon^2 = U^2/L^2$.

3 The Higgs branch

There is a well know exact map between the moduli space of Yang-Mills instantons and the Higgs branch of the $p+1$ dimensional theories describing Dp – D(p+4) brane systems [3, 4]. The Higgs branch corresponds to Dp-branes which are not pointlike, but which have instead been dissolved in the D(p+4)-branes. Dissolved Dp-branes can be viewed as instantons in the $p+5$ dimensional world-volume theory on the D(p+4)-branes [18], due to the Wess-Zumino coupling

$$S_{WZ} = \mu_p \int \mathrm{d}^{p+5}\xi \, C^{(p+1)} \wedge \mathrm{tr}\,(F \wedge F) \; . \tag{3.1}$$

The low energy degrees of freedom on the D7-branes are described by an eight-dimensional gauge theory, but due to the curved geometry resulting from the embedding in the near horizon geometry (2.1) of the D3-branes the existence of instanton solutions in such a theory is far from obvious. Moreover the inclusion of higher order corrections gives rise to an infinite number of higher dimensional couplings which could modify the solutions of the leading order equations of motion. We will find however that despite the curved geometry the theory admits ordinary instanton solutions which under certain assumptions are not corrected by the inclusion of higher derivative interactions. The D7 action takes the form,

$$\begin{aligned} S =& \frac{1}{(2\pi)^7 g_\mathrm{s} \alpha'^4} \int \sum_q C^{(q)} \wedge \, \mathrm{tr}\,(\mathrm{e}^{2\pi\alpha' F}) \\ &+ \frac{1}{(2\pi)^7 g_\mathrm{s} \alpha'^4} \int \mathrm{d}^8 x \, \sqrt{-g}\, (2\pi\alpha')^2 \, \frac{1}{4}\, \mathrm{tr}(F_{AB}F^{AB}) + \cdots \; . \end{aligned} \tag{3.2}$$

We have not written terms involving world-volume fermions or scalars. This action is the sum of a Wess-Zumino term, a Yang-Mills term, and an infinite number of α' corrections represented by "$\cdots$". Very little is known about the latter. Nevertheless, the correspondence between instantons and the Higgs branch suggest that the equations of motion should be solved by field strengths which are self dual with respect to a flat four-dimensional metric.

Let us first consider the equations of motion to leading order in the α' expansion[2]. With non-trivial field strengths only in the directions $X_\perp$, the leading order action for D7-branes embedded in (2.1) with induced metric (2.3) is

$$\begin{aligned} S &= \frac{1}{(2\pi)^5 g_\mathrm{s} \alpha'^2} \left(\int C^{(4)} \wedge \, \mathrm{tr}\, F \wedge F + \int \mathrm{d}^8 x \, \sqrt{-g}\, \frac{1}{4}\, \mathrm{tr}(F_{ab}F^{ab}) \right) \\ &= \frac{N}{(2\pi)^4 \lambda L^4} \int \mathrm{d}^4 x_\parallel \, \mathrm{d}^4 X_\perp \, v^4 \, \frac{1}{2}\, \mathrm{tr}(\frac{1}{2}\epsilon_{mnrs} F_{mn} F_{rs} + F_{mn}F_{mn}) \\ &= \frac{N}{(2\pi)^4 \lambda L^4} \int \mathrm{d}^4 x_\parallel \, \mathrm{d}^4 X_\perp \, v^4 \, \frac{1}{4}\, \mathrm{tr} F_+^2 \, , \end{aligned} \tag{3.3}$$

where the lowercase latin indices m label the $X_\perp = X^{4,5,6,7}$ directions and, to arrive at the last line, we have used the explicit form of the Ramond-Ramond four-form in

[2]In the AdS setting, the α' expansion effectively becomes a large 't Hooft coupling expansion.

$\mathrm{AdS}_5 \times S^5/Z_2$,

$$C^{(4)}_{0123} = \frac{U^4}{L^4}\,. \tag{3.4}$$

Thus, at leading order in α', field strengths for which $F^+ = 0$ (anti-self-dual with respect to the flat metric $\mathrm{d}X_\perp^2$) solve the equations of motion due to a conspiracy between the Wess-Zumino and Yang-Mills terms.

The correspondence with the Higgs branch moduli space of the $\mathcal{N}=2$ SYM theory requires that the $F^+=0$ configurations remain solutions when higher order corrections are included in the D7-brane effective action. Temporarily neglecting terms which involve non-minimal couplings to bulk supergravity fields, the action to order α'^2 is given by

$$S = \frac{N}{\lambda^2}\frac{1}{(2\pi)^5}\int \mathrm{tr}\left[\frac{1}{2}x_\perp^4 F^+_{mn}F^+_{mn} - \frac{x_\perp^8}{4\pi\lambda}\frac{1}{384}\left(2F^+_{mn}F^+_{mn}F^-_{rs}F^-_{rs} + F^+_{mn}F^-_{rs}F^+_{mn}F^-_{rs}\right)\right]\,, \tag{3.5}$$

where we have written the (known [6–8]) F^4 terms in terms of self dual and anti-self-dual field strengths. Since the F^4 terms are quadratic in F^+, anti-self-dual field strengths are still manifestly solutions of the equations of motion of (3.5). However, even to this order in the α' expansion, not all the couplings to bulk fields needed for a complete proof are known. There may also be terms of the general form $R^2\mathrm{tr}F^2, \mathcal{F}^4_{(5)}\mathrm{tr}F^2, R\mathcal{F}^2_{(5)}\mathrm{tr}F^2$ which effect the equations of motion in the AdS background, for which the curvature R and Ramond-Ramond five-form $\mathcal{F}_{(5)}$ are non-vanishing. Rather than proving that self-dual field strengths solve the equations of motion, we will show that the existence of such solutions leads to constraints on the unknown couplings.

The CP odd Wess-Zumino term proportional to $\upsilon^4\epsilon_{mnrs}F_{mn}F_{rs}$ is exact, with no corrections at any order in α'. In order to preserve the $F^+=0$ solutions, the quadratic CP even term must be $\upsilon^4\,\frac{1}{2}\,\mathrm{tr}F_{mn}F_{mn}$ with exactly the same coefficient. As discussed above, this is already the case at leading order in α'. Thus, at every order in the α' expansion, the terms of the form $f(R,\mathcal{F}_{(5)})\mathrm{tr}F^2$ must sum to zero when the bulk fields are set equal to their AdS values.

Some terms of the form R^2F^2 have appeared in the literature. These are [10, 11]

$$\begin{aligned}S_{R^2F^2} = -\mu_p(2\pi\alpha')^2\int\sqrt{g}\,\frac{1}{4}\mathrm{tr}F_{\alpha\beta}F^{\alpha\beta}\bigg[\frac{1}{24}\frac{(4\pi^2\alpha')^2}{32\pi^2}\Big(&(R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta}\\ &-2(R_T)_{\alpha\beta}(R_T)^{\alpha\beta} - (R_N)_{ab\alpha\beta}(R_N)^{ab\alpha\beta} + 2\bar{R}_{ab}\bar{R}^{ab}\Big)\bigg]\,.\end{aligned} \tag{3.6}$$

The curvature terms appearing after $F_{\alpha\beta}F^{\alpha\beta}$ are the same as the pure R^2 terms computed in [19]. The various curvature tensors appearing in (3.6) are defined in [19]. For the special case of an embedding with vanishing second fundamental form, the tensors $(R_T)_{\alpha\beta\gamma\delta}$ and $(R_N)_{ab\alpha\beta}$ are just pull-backs of the bulk Riemann tensor to the tangent and normal bundle, indicated by greek and latin indices respectively (we emphasize that this is a change of notation from the previous sections). The tensors $(R_T)_{\alpha\beta}$ and $\bar{R}_{ab}$ are not pull-backs of the bulk Ricci tensor, but are obtained from contractions of tangent indices in the pull-backs of the Riemann tensor. Specifically, for vanishing second fundamental form,

$$\bar{R}_{ab} \equiv g^{\alpha\beta}R_{\alpha ab\beta}, \qquad (R_T)_{\alpha\beta} \equiv g^{\lambda\mu}R_{\lambda\alpha\mu\beta}\,, \tag{3.7}$$

where $g_{\alpha\beta}$ is the induced metric on the D-brane. For the $\mathrm{AdS}_5 \times S^3$ embedding in $\mathrm{AdS}_5 \times S^5/Z_2$, the second fundamental form vanishes. In this background,

$$(R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta} - 2(\hat{R}_T)_{\alpha\beta}(\hat{R}_T)^{\alpha\beta} - (R_N)_{ab\alpha\beta}(R_N)^{ab\alpha\beta} + 2\bar{R}_{ab}\bar{R}^{ab} = -\frac{6}{25}L^2 \,, \quad (3.8)$$

where $L^2 = \sqrt{\lambda}\alpha'$ is the square of the AdS_5 (or S^5) curvature radius. Thus (3.6) can not be the only term of the form $f(R, \mathcal{F}^{(5)})F^2$ at order α'^2, which must collectively sum to zero in the AdS background.

4 The metric on the Higgs Branch

To two derivative order, the effective action on the Higgs branch of the four-dimensional $\mathcal{N} = 2$ theory we are considering is equivalent to the action describing slowly varying "instantons" in eight-dimensional super Yang-Mills (see [5] for a review). This action has the form

$$S = \int \mathrm{d}x^0 \cdots \mathrm{d}x^3 G_{ij}(\mathcal{M})\partial_\mu \mathcal{M}^i \partial^\mu \mathcal{M}^j \,, \quad (4.1)$$

where $\mathcal{M}_i(x^\mu)$ are either Higgs branch or instanton moduli. From the point of view of the eight-dimensional theory the instantons we are considering are solitons for which the gauge fields only depend on the four Euclidean coordinates $x^{4,5,6,7}$ and have a self-dual field strength with respect the flat metric in these directions. These solutions depend on moduli $\mathcal{M}_i$. The metric (4.1) is obtained in the moduli space approximation in which the parameters $\mathcal{M}_i$ are allowed to depend on the coordinates $x^{0,1,2,3}$, but are slowly varying. The metric $G_{ij}(\mathcal{M})$ is also known to be tree level exact in the four-dimensional $\mathcal{N} = 2$ theory.

In the AdS dual description of the $\mathcal{N} = 2$ theory, one can compute the metric on the Higgs branch by finding the action for slowly varying instantons of the D7-brane theory (3.2). To two derivative order, the effective action must be the same as that for slowly varying instantons in conventional super Yang-Mills theory in eight-dimensional flat space, which gives the exact un-renormalized metric on the Higgs branch.

The metric on the Higgs branch is determined by inserting the instanton solution into the action, letting the moduli depend on the coordinates $x^0, \ldots, x^3$, which we indicate by greek indices. The instantons are localized in the directions $x^m = x^{4,5,6,7}$ and depend on moduli $\mathcal{M}_i$, $A_m = A_m^{\mathrm{inst}}(x^n, \mathcal{M}_i)$. Configurations in which the moduli are coordinate dependent, $A_m = A_m^{\mathrm{inst}}(x^n, \mathcal{M}_i(x^\alpha))$, are approximate solution in the limit of slowly varying moduli. More precisely, the metric on the Higgs branch can be extracted from the equations which configurations

$$\begin{aligned} A_m &= A_m^{\mathrm{inst}}(x^n, \mathcal{M}_i(x^\alpha)) \\ A_\mu &= \Omega_i \partial_\mu \mathcal{M}_i(x^\alpha) \end{aligned} \quad (4.2)$$

must satisfy in order to solve the full equations of motion to leading (two derivative) order in a derivative expansion. The relevant terms in the Dirac Born Infeld action are those involving two greek indices. Two order α'^2 (or equivalently $\mathcal{O}(1/\lambda)$), the relevant terms are (neglecting bulk couplings)

$$\begin{aligned}
S = &\frac{N}{\lambda}\frac{1}{(2\pi)^5}\int \mathrm{d}^4X_\perp \, \frac{1}{4}\mathrm{tr}(F_{\mu m}F_{\mu m}) \\
&+ \frac{N}{\lambda^2}\frac{1}{(2\pi)^6}\int \mathrm{d}^4X_\perp \, X_\perp^4 \, \frac{1}{12}\mathrm{tr}\left[F_{s\mu}F_{\mu n}\left(\{F_{nr},F_{rs}\} - \frac{1}{2}\delta_{sn}F_{tu}F_{ut}\right)\right. \\
&\left. + \frac{1}{2}\left(F_{\mu n}F_{nr}F_{s\mu}F_{rs} + F_{\mu n}F_{rs}F_{s\mu}F_{nr} - \frac{1}{2}F_{\mu n}F_{rs}F_{n\mu}F_{sr}\right)\right] . \qquad (4.3)
\end{aligned}$$

Note that the leading term is just that of Yang-Mills theory in eight flat dimensions; the warp factors appearing in the $AdS^5 \times S^3$ metric cancel in this term. It is useful to rewrite the field strengths F_{mn} in the subleading term in terms of self-dual and anti-self-dual parts, giving

$$\begin{aligned}
S = &\frac{N}{\lambda}\frac{1}{(2\pi)^5}\int \mathrm{d}^4X_\perp \, \frac{1}{4}\mathrm{tr}(F_{\mu m}F_{\mu m}) \\
&+ \frac{N}{\lambda^2}\frac{1}{(2\pi)^6}\int \mathrm{d}^4X_\perp \, X_\perp^4 \, \frac{1}{48}\mathrm{tr}\left[F_{s\mu}F_{\mu n}\left(\{F^+_{nr},F^-_{rs}\} + \{F^-_{nr},F^+_{rs}\}\right)\right. \\
&\left. + \frac{1}{2}\left(F_{\mu n}F^+_{nr}F_{s\mu}F^-_{rs} + F_{\mu n}F^-_{nr}F_{s\mu}F^+_{rs} + F_{\mu n}F^+_{rs}F_{s\mu}F^-_{nr} + F_{\mu n}F^-_{rs}F_{s\mu}F^+_{nr}\right)\right] . \qquad (4.4)
\end{aligned}$$

The equations of motion $\frac{\delta}{\delta A_\mu}S = 0$ give

$$\partial_\mu \mathcal{M}_i \left(D_m \frac{\delta A_m}{\delta \mathcal{M}_i} - D_m D_m \Omega_i\right) = 0\,, \qquad (4.5)$$

which has a unique solution for Ω_i as a function of x^m and $\mathcal{M}_i$. Taking this Ω_i and inserting (4.2) into the action (4.4) gives the metric on the Higgs branch via the relation

$$\partial_\mu \mathcal{M}^i \partial_\mu \mathcal{M}^j G_{ij}(\mathcal{M}) = \frac{N}{\lambda}\frac{1}{(2\pi)^5}\int d^4X_\perp \, \frac{1}{4}\mathrm{tr}(F_{\mu m}F_{\mu m})\,, \qquad (4.6)$$

where the higher order term vanishes because the configuration (4.2) satisfies $F^+ = 0$. One therefore gets the same result from instantons on the D7-brane embedded in AdS as one gets from Yang-Mills theory in flat space. We emphasize that this result assumes that bulk couplings of the form $h_{nm\mu\nu}(R, \mathcal{F}^{(5)})F_{\mu n}F_{\nu m}$ sum to zero when R and $\mathcal{F}^{(5)}$ are set to their AdS background values. With this assumption, the non-renormalization of the metric on the Higgs branch is realized in the strong coupling expansion obtained using holography. The leading and only term is the same as the exact tree level result.

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