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Robust control of resistive wall modes in ITER: comparing different feedback coil systems

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Introduction

A future fusion power plant must reach a sufficiently high $= 2\ {}_0\langle p\rangle/\langle B^2\rangle$ in order to be efficient and economically attractive. In advanced tokamak scenarios, however, ***external kink instabilities*** impose a hard limit on the achievable . While a superconducting wall sufficiently close to the plasma would stabilize these kink modes and thus substantially increase the -limit, the modes become unstable again if the wall has non-zero resistivity. These instabilities are called ***resistive wall modes (RWMs)*** and grow sufficiently slowly so that their active stabilization using magnetic field sensors, additional control coils, and suitable feedback controller logics becomes technologically feasible. Here, a novel methodology for the computational design of robust RWM controllers is introduced and applied to an ITER-relevant scenario. The control system is represented by a parametrized matrix, whose robust stability properties are optimized under variations of the parameters. The robust stability concept is based upon matrix pseudospectra and thereby accounts for the sensitivity of eigenvalues. Futhermore, the possible transient growth of perturbations is diagnosed. Four different feedback coil systems proposed for ITER are analyzed for their capability of stabilizing RWMs robustly while preventing too large transient amplifications.

Method

The basic computational tool is the 3D ideal magnetohydrodynamics (MHD) stability code STARWALL, which is specialized to resistive wall modes [1, 3]. The plasma dynamics is linearized about a prescribed ideal MHD equilibrium. The dynamics of the entire system composed of the plasma, the conducting wall, and the feedback coils is governed by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}, \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^N$ is the state vector of the system; the components of $\mathbf{x}$ describe the coil currents and the current potential values at the nodes of the wall mesh. The system matrix is given by $\mathbf{A} = (\mathbb{1} + \ \mathbf{F})^{-1}(\mathbf{A}_0 + \mathbf{F})$, i. e., it is composed of an "open-loop" part $\mathbf{A}_0$ and a feedback part $\mathbf{F}$ while accounting for a small, technically caused time delay between sensors and actuators.

Substituting a time dependence $\mathbf{x} \sim \mathrm{e}^{\gamma t}$, one obtains the eigenvalue problem

$$\mathbf{A}\mathbf{x}_i = \;{}_i\mathbf{x}_i, \qquad i = 1,\ldots,N. \tag{2}$$

The system is stable if and only if all eigenvalues of $\mathbf{A}$ have negative real parts. Without feedback ($\mathbf{F} = 0$), all eigenvalues are real, and positive ones belong to unstable RWMs. The control problem consists in choosing proper feedback logics ($\mathbf{F} \neq 0$) so that $\mathbf{A}$ becomes stable.

The feedback logics is implemeted in terms of a gain matrix which maps the sensor signal vector onto a vector of voltages to be applied to the coils. To construct the gain matrix, all coils and sensors, respectively, are subdivided into one or more toroidal arrays. Each coil array k ($k = 1,\ldots,K$) is linked to each sensor array l ($l = 1,\ldots,L$) via a gain submatrix $\mathbf{G}^{kl}$. As a means to increase the coil's time constants, the current flowing in each coil in the k-th array is fed back onto that coil's voltage by an additional gain factor $-\tilde{R}_k$ which is equal for each member of the array. Summarizing, for $k = 1,\ldots,K$, the voltage vector

$$\mathbf{u}^k = \sum_{l=1}^{L} \mathbf{G}^{kl}\mathbf{s}^l - \tilde{R}_k\mathbf{i}^k, \tag{3}$$

is applied to the coils in the k-th toroidal array, where $\mathbf{i}^k$ is the vector of currents already flowing in these coils, and $\mathbf{s}^l$ is the signal vector measured by the l-th sensor array.

The gain submatrices $\mathbf{G}^{kl}$, $k = 1,\ldots,K$, $l = 1,\ldots,L$ are constructed in such a way that each coil array produces a field with toroidal Fourier index n in response to measured perturbations with the same n:

$$\mathbf{G}^{kl} = \sum_n \left(\;{}^{kl}_{n}\mathbf{G}^{kl}_{n,\alpha} + \;{}^{kl}_{n}\mathbf{G}^{kl}_{n,\beta} \right), \tag{4}$$

where the sum runs over all n's to be controlled. The elements of $\mathbf{G}^{kl}_{n,\alpha}$ and $\mathbf{G}^{kl}_{n,\beta}$ are given by

$$(\mathbf{G}^{kl}_{n,\alpha})_{ij} = \cos(n \;{}^{kl}_{ij}), \qquad (\mathbf{G}^{kl}_{n,\beta})_{ij} = \sin(n \;{}^{kl}_{ij}), \tag{5}$$

where ${}^{kl}_{ij}$ is the toroidal angle between coil i of coil array k and sensor j of sensor array l. The values of the free parameters ${}^{kl}_{n}$, ${}^{kl}_{n}$, $\tilde{R}_k$ uniquely determine the feedback matrix $\mathbf{F}$ and need to be optimized to achieve robust stabilization of $\mathbf{A}$.

Using a reduced state-space model of $\mathbf{A}$, a robustly stabilizing parameter set is determined in two steps using the eigenvalue optimization code OPTIM [2]. First, $\mathbf{A}$ is stabilized by minimizing the *spectral abscissa*

$$(\mathbf{A}) = \max_{i=1,\ldots,N} \mathrm{Re} \;{}_i, \tag{6}$$

under parameter variations. As soon as $\mathbf{A}$ becomes stable, the optimization of $(\mathbf{A})$ is stopped, and the stability is made robust by maximizing the *complex stability radius*

$$(\mathbf{A}) = \sup\{ \;: \mathbf{A} + \mathbf{E} \text{ is stable } \forall\, \mathbf{E} \in \mathbb{C}^{N\times N} \text{ with } ||\mathbf{E}||_2 < \; \}. \tag{7}$$

Finally, after an optimal $\mathbf{A}$ has been found, the possible transient amplification of initial perturbations is diagnosed by computing $||e^{t\mathbf{A}}||_2$ curves over an appropriate time interval $0 \leq t \leq T$. Furthermore, *-pseudospectra*

$$_{\varepsilon}(\mathbf{A}) = \{z \in \mathbb{C} \, : \, z \text{ is an eigenvalue of } \mathbf{A}+\mathbf{E} \text{ for some } \mathbf{E} \in \mathbb{C}^{N\times N} \text{ with } ||\mathbf{E}||_2 < \quad \} \qquad (8)$$

are plotted. By means of $_{\varepsilon}(\mathbf{A})$, the sensitivity of the spectrum of $\mathbf{A}$ with respect to system perturbations (model uncertainties, imperfections, etc.) of “strength” can be inferred. Both $(\mathbf{A})$ and $||e^{t\mathbf{A}}||_2$ are closely related to $_{\varepsilon}(\mathbf{A})$ [4].

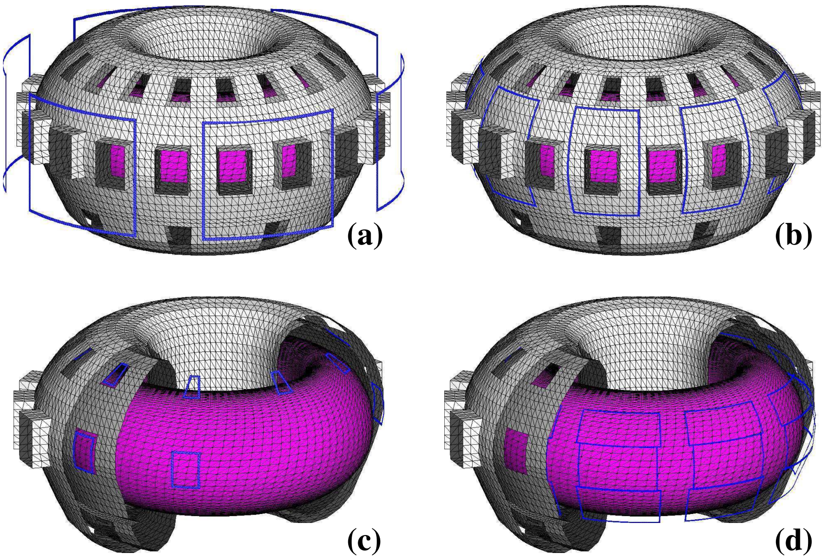

Figure 1: ITER conducting wall model and the side correction coil set (a), the alternative RWM coil set (b), the port plug coil set (c), and the in-vessel coil set (d).

Coil systems comparison

The methodology is applied to an ITER steady state scenario 4 equilibrium with $_N$=2.67. In absence of a conducting wall, the equilibrium is unstable with respect to an $n = 1$ external kink, but stable to higher n's. A realistic conducting wall geometry and four different sets of feedback coils, denoted (a) through (d) hereafter, are used (see Fig. 1). In all cases, a single toroidal array of 18 equidistant sensors, positioned between the equatorial ports and inside, but very close to the interior wall and measuring the vertical component of the magnetic field perturbation, is used. For each coil set, the optimization of the objective functions (6) and (7) is carried out subject to constraints limiting the voltage gains, the “saturation current gains”, and the coil time constant reciprocals. The current gain limit is set approximately equal to twice the

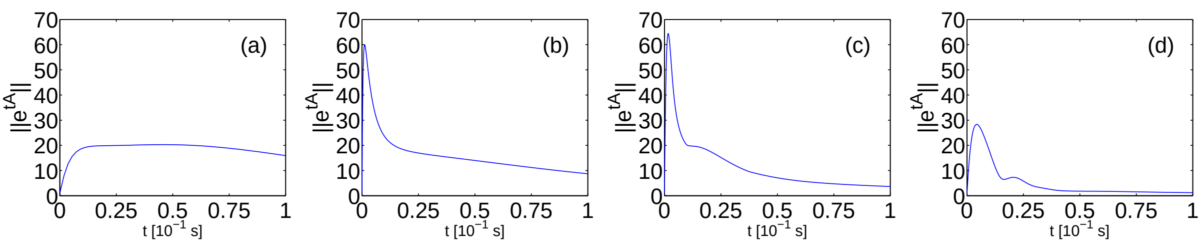

Figure 2: $||e^{t\mathbf{A}}||_2$ curves for the four coil sets (a) - (d).

current gain necessary for stabilization, respectively. A time delay $\;= 0.1$ ms is used.

The optimal $\;(\mathbf{A})$ values are 0.40 s^{-1} (a), 0.57 s^{-1} (b), 1.29 s^{-1} (c), and 2.95 s^{-1} (d). It follows that the in-vessel coils provide by far the most robust stabilization. As visible from the $||e^{t\mathbf{A}}||_2$ plots (Fig. 2), they also produce the most favorable transient behavior, although there is still an amplification by a factor of about 28. There are, however, possibilities to reduce the transient peak, in general [2]. For all four coil systems, respectively, the $_{\varepsilon}(\mathbf{A})$ plots (Fig. 3) provide insight into the sensitivity of individual eigenvalues as well as into the sensitivity of the system's stability as a whole. Namely, $_{\varepsilon}(\mathbf{A})$ overlaps the right complex halfplane for $\;> \;(\mathbf{A})$.

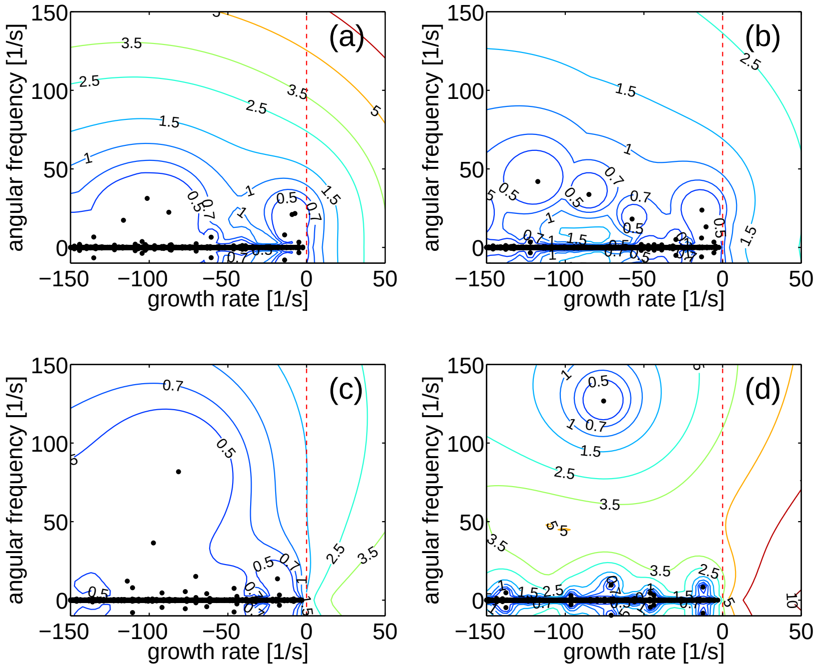

Figure 3: Boundaries of $\gamma_{\varepsilon}(\mathbf{A})$ for the cases (a) - (d), with ε values as given by the contour labels; black dots correspond to eigenvalues.

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