Analysis of Neo-Classical Double Tearing Mode

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1. INTRODUCTION

It is well known that double tearing modes (DTM) can exist in a tokamak plasma with a nonmonotonic q-profile. DTM was proposed as a possible mechanism for the observed Mirnov oscillations of the tokamak plasma during the first few milliseconds of discharge[1, 2]. The linear and nonlinear theory and numerical calculations of the resistive DTM have already been developed [3, 4 and reference therein].

Large tokamak plasmas are essentially in the banana regime, and it is known that the neoclassical effect is important for the tearing mode and leads to the neoclassical tearing mode[5, 6]. It is therefore necessary to have an understanding of the DTM in the banana regime, and we will call it the neoclassical double tearing mode hereafter.

In the present work a theory for the nonlinear neoclassical DTM is presented.

2. Island evolution equations of neoclassical DTM

Large aspect-ratio tokamak approximation is used here. We define ψ as the helical flux function, $\mathbf{B}=B_{0z}\mathbf{e_z}-(kr/m)B_{0z}\mathbf{e}_\theta+\nabla\psi\times\mathbf{e_z}$, and $\mathbf{B}$ denotes the magnetic field, m/r and k being the wave vector in $\mathbf{e}_\theta$ (poloidal) and $\mathbf{e_z}$ (toroidal) direction, respectively, and the subscript 0 denoting an equilibrium quantity.

For the case of nonmonotonic q-profile we take $\psi_0'=0$ at locations r_1 and r_2 of the rational surfaces, with $r_1<r_2$, where the prime denote $\partial/\partial r$. The magnetic flux perturbation $\tilde{\psi}$ of the DTM can then be expressed as[3, 4]

$$\tilde{\psi}=\tilde{\psi}_1+\tilde{\psi}_2, \tag{1}$$

where $\tilde{\psi}_1$ is bounded in $0\le r\le r_2$ and satisfies the boundary conditions

$$\tilde{\psi}_1(0)=\tilde{\psi}_1(r_2)=0, \tag{2}$$

and $\tilde{\psi}_2$ is bounded in $r_1\le r\le b$ (b is the conducting wall radius) satisfying

$$\tilde{\psi}_2(r_1)=\tilde{\psi}_2(b)=0. \tag{3}$$

Outside the islands inertia can be neglected. For a given equilibrium $\mathbf{B}$, it can be found from the equation of motion the four outer matching parameters defined by[4, 7]

$$\Delta_{11}=\frac{\tilde{\psi}'_{1(s1+)}-\tilde{\psi}'_{1(s1-)}}{\tilde{\psi}_{1(r_1)}},\quad \Delta_{22}=\frac{\tilde{\psi}'_{2(s2+)}-\tilde{\psi}'_{2(s2-)}}{\tilde{\psi}_{2(r_2)}},$$

$$\Delta_{12}=\frac{\tilde{\psi}'_{2(s1+)}}{\tilde{\psi}_{2(r_2)}},\qquad \Delta_{21}=\frac{-\tilde{\psi}'_{1(s2-)}}{\tilde{\psi}_{1(r_1)}}, \tag{4}$$

where the subscripts s1+(s2+) and s1-(s2-) denote the two sides of the inner (outer) island separatrix around r_1 (r_2) respectively.

Assuming $(r_2-r_1)>>w_1, w_2$, where w_1 (w_2) is the inner (outer) island width, we find the matching parameters from the island regions, Δ_1 and Δ_2,

$$\Delta_1=\frac{dw_1}{\eta_1 dt}+\frac{\Delta_{b1}w_1}{w_1^2+w_{d1}^2}+\frac{iw_1}{2\eta_1}(\omega_1+kv_{z1}+t\frac{d\omega_1}{dt}), \quad (5)$$

$$\Delta_2=\frac{dw_2}{\eta_2 dt}+\frac{\Delta_{b2}w_2}{w_2^2+w_{d2}^2}+\frac{iw_2}{2\eta_2}(\omega_2+kv_{z2}+t\frac{d\omega_2}{dt}), \quad (6)$$

where η_1 and η_2 are neoclassical resistivity at r_1 and r_2, $\Delta_{b1}=(k_1\sqrt{\varepsilon}\beta_{pe}L_s/L_p)_{r=r_1}$, $\Delta_{b2}=(k_2\sqrt{\varepsilon}\beta_{pe}L_s/L_p)_{r=r_2}$, $L_s=q/q'$, q being the safety factor, $L_p=p_e/p_e'$, p_e being electron pressure, β_{pe} is the ratio between the local electron pressure and poloidal magnetic field pressure, $\varepsilon=r/R$ is the inverse aspect ratio, k_1 and k_2 are parameters depending on the equilibrium and are of the order of unity[5, 6], w_{d1} and w_{d2} are of the order of one percent of the minor radius for large tokamak plasmas[8], ω_1 (ω_2) is the oscillating frequency of $\tilde{\psi}_1$ ($\tilde{\psi}_2$), v_{z1} (v_{z2}) is the toroidal plasma rotation velocity at r_1 (r_2).

Matching the inner and outer solution it is found that[7]

$$\Delta_1=\Delta_{11}+\Delta_{12}e^{-i\omega t}\tilde{\psi}_2(r_2)/\tilde{\psi}_1(r_1), \quad (7)$$

$$\Delta_2=\Delta_{22}+\Delta_{21}e^{i\omega t}\tilde{\psi}_1(r_1)/\tilde{\psi}_2(r_2), \quad (8)$$

where $\omega=\omega_1-\omega_2$.

Using Eqs. (4) - (6), the imaginary parts of Eqs. (7) and (8) becomes

$$t\frac{d\omega}{dt}+(\omega+kv_z)=-2(\eta_1|c|\Delta_{12}\frac{w_2^2}{w_1^3}+\eta_1|c|^{-1}\Delta_{21}\frac{w_1^2}{w_2^3})\sin(\omega t), \quad (9)$$

where $v_z=v_{z1}-v_{z2}$. If kv_z is assumed to be much larger than the coupling terms on the right hand side of Eq. (9), the asymptotic solution of Eq. (9) is $\omega=-kv_z$.

The real parts of Eqs. (7) and (8) are

$$\frac{dw_1}{\eta_1 dt}=\Delta_{11}(w_1)+|c|\Delta_{12}\frac{w_2^2}{w_1^2}\cos(\omega t)-\frac{\Delta_{b1}w_1}{w_1^2+w_{d1}^2}, \quad (10)$$

$$\frac{dw_2}{\eta_2 dt}=\Delta_{22}(w_2)+|c|^{-1}\Delta_{21}\frac{w_1^2}{w_2^2}\cos(\omega t)-\frac{\Delta_{b2}w_2}{w_2^2+w_{d2}^2}, \quad (11)$$

where $c=\psi_0''(r_2)/\psi_0''(r_1)$. Equations (9)-(11) describe the nonlinear evolution of the magnetic islands with different rotation frequencies. The first terms on the right hand side of Eqs.(10) and (11) are the conventional Δ' term resulting from plasma current gradient, the second terms result from the coupling between the two islands, and the third terms result from the perturbed bootstrap current. Since $L_s/L_p>0$ ($L_s/L_p<0$) at r_1 (r_2), the perturbed bootstrap current has a stabilizing (destabilizing) effect on the inner (outer) island.

When the poloidal electromagnetic torque between the two islands is larger than plasma viscous torque, the relative phase between the two island will be locked. The phase locking condition is found to be[7]

$$w_1^2 w_2^2 (w_1 + Qw_2) > 16^2 R^2 \mu_1 \frac{r_1^2}{r_2} \frac{L_{s1} L_{s2} \omega}{n r_1 \Delta_{12} v_{A1}^2}, \tag{12}$$

where $v_{A1}=B_{0z}/(\mu_0 \rho_1)^{1/2}$ being the local Alfven velocity at r_1, $\omega=(v_{z1}-v_{z2})/R$, $Q=r_1^3 \mu_1 \rho_1/(r_2^3 \mu_2 \rho_2)$, μ is plasma viscosity, ρ is plasma mass density, and the subscripts 1 and 2 denote taking values at r_1 and r_2 respectively. It is seen that the island width is the key parameter in determining the island phase locking.

3. Magnetic island width

3.1 Outer island width

For most circumstances, the approximation $\Delta_{22}(w_2) w_2^2 >> c^{-1} \Delta_{21} w_1^2$ is valid, and it is found from Eq. (11) that

$$\Delta_{22}(w_2) \approx \frac{\Delta_{b2} w_2}{w_2^2 + w_{d2}^2}. \tag{13}$$

The saturation of the outer island is due to the balance of the destabilizing effect from the perturbed bootstrap current and the stabilising effect from the negative $\Delta_{22}(w_2)$.

3.2 Inner island width

The inner island could have a variety of behavior depending on plasma parameters as it can be seen from Eq. (10) and (11). Following are three typical cases.

For the rotating island with $\omega >> \kappa$, where $\kappa=3a^2 \Delta_{b1}/(w_{d1}^2 \tau_R)$, it is found that

$$w_1 \approx [A_1 \sin(\omega t)]^{1/3} \tag{14}$$

when $\Delta_{11} w_{d1}^2/\Delta_{b1} << w_1 << w_{d1}$, where $A_1 \approx |c| \Delta_{12}\, \omega_{2s}^2\, 3a^2/(\tau_R \omega)$. It is seen that $w_1 \sim \omega^{-1/3}$.

When the relative phase between the two island is locked, for a high-β_{pe} plasma with $\Delta_{11} w_{d1}^2/\Delta_{b1} << w_1 << w_{d1}$, we have

$$w_{1s} = [\frac{c \Delta_{12} w_{2s}^2 w_{d1}^2}{\Delta_{b1}}]^{1/3}. \tag{15}$$

For a low-β_{pe} plasma with $w_1 >> w_{d1}$, when

$$w_c = 4c \Delta_{11} \Delta_{12} w_{2s}^2/\Delta_{b1}^2 > 1, \tag{16}$$

w_1 will continues its growth without saturation, and the hollow current profile will be destroyed.

The island width of the m/n=3/1 mode is calculated for two examples using the safety factor profile $q = q_0 r^2 / \int_0^{r^2} (1-t)^{g_1} (1+\alpha t)^{g_2} dt$.

One q-profile is obtained with $q_0=4.0$, $g_1=4.0$, $g_2=0.4$ and $\alpha=150$. This q-profile is close to that achieved on TFTR shown on Fig. 2 of Ref. [9]. The q=3 surfaces are at $r_1=0.13a$ and $r_2=0.68a$. With the experimental and calculated values of $a\Delta_{11}=-25$, $a\Delta_{22}=-2.0$, $a\Delta_{12}=0.71$, $a\Delta_{21}=0.14$, $\Delta_{b1}=12$, $\Delta_{b2}=-0.11$, $c=3.5$, and $w_{d1}=w_{d2}=0.02a$ [7, 9], it is found that $w_{2s} \approx 4.8 \times 10^{-2} a$ and $w_{1s} \approx 5.7 \times 10^{-3} a$.

Another q-profile is obtained with q_0=4.0, g_1=4.0, g_2=1.0 and α=22. For r<0.9a this q-profile is close to that achieved on DIII-D shown on Fig. 3 of Ref. [10]. The q=3 surfaces are at r_1=0.20a and r_2=0.82a. With the experimental and calculated values of $a\Delta_{11}$=-12, $a\Delta_{22}$=-2.6, $a\Delta_{12}$=0.85, $a\Delta_{21}$=0.21, Δ_{b1}=5.2, Δ_{b2}=-0.16, c=8.2, and $w_{d1}=w_{d2}$=0.02a[7, 10], it is found that $w_{2s} \approx 5.3\times 10^{-2}a$ and $w_1 \approx A_1^{1/3} \approx 6.9\times 10^{-3}a$.

The calculated inner island width for both examples is small than $10^{-2}a$, which agrees with the experimental results that no coherent MHD activity was observed in the reversed shear region[9, 10]. More detailed analysis of Eq. (9)-(11) are given in Ref. [7].

4. Summary

(1) For a high β_{pe} plasma, the inner magnetic island width will be greatly reduced by the perturbed bootstrap current. This makes the island coupling not important for the outer island, and the outer island behaves like a single tearing mode with the inner rational surface acting as a conducting wall.

(2) The relative rotation between the two islands mainly has a stabilising effect on the inner island, while the outer island is essentially not affected. Periodical burst of the inner island can be excited by the outer island via the coupling effect when there is a differential rotation.

(3) The island width is the key parameter in determining the phase locking.

(4) For the locked islands, when $w_c=4c\Delta_{11}\Delta_{12}(w_{2s}^2/\Delta_{b1}^2>1$, the inner island will continue its growth, and this will eventually destroy the nonmonotonic q-profile.

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