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Tokamak equilibria with reversed magnetic shear and sheared flow

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1. Introduction and conclusion.

There has been established in tokamaks that sheared flows can reduce turbulence either in the edge region (L-H transition) or in the central region [internal transport barrier (ITB)] and produce mass and energy barriers which under certain conditions can extend up to the whole plasma [1]. The ITB's are usually associated with reversed magnetic shear profiles while in the L-H transition the q-profiles are monotonically increasing from the center to the edge. In both improved confinement regimes the pressure profile becomes steep in the barrier region and sheared electric fields related to sheared flows are produced. They exhibit an extremum close to the separatrix in the case of the L-H transition and close to the minimum-q position in the case of ITB. The transport barrier formation mechanism is far from completely understood. It is believed that the flow, the electric field and their shear and, probably most important, the $\mathbf{E}\times\mathbf{B}$-velocity shear play a role in the barrier formation by reducing transport through mode decorrelation. Also, in the case of ITB it is not clear whether the reversed magnetic shear or the sheared flow plays a more important role.

The subject of a recent publication [2] was the study of tokamak equilibria with incompressible sheared flows and monotonically increasing q-profiles in cylindrical and axisymmetric geometries by constructing analytic solutions to the pertinent equations [3]. The electric fields associated with the above mentioned solutions have characteristics similar to those observed in the L-H transition. The aim of the present work is to extend the study to the case of cylindrical equilibria with reversed magnetic shear. Unlike the axisymmetric configurations in cylindrical geometry the q-profile can be easily prescribed. The analysis is conducted within the framework of the MHD theory. The results indicate that the reversed magnetic shear and the sheared velocity have synergetic effects in the formation of ITB's with the velocity and its shear playing a more important role. Also, as in the case of the L-H transition, the poloidal velocity is important. The equilibrium equations together with the prescribed and calculated quantities are presented in Sec. 2. The main results are summarized in Sec. 3. The study will be reported in detail elsewhere.

2. Cylindrical equilibria with reversed magnetic shear

The equilibrium of a cylindrical plasma with flow satisfies (in convenient units) the relation

$$\frac{d}{dr}\left(P+\frac{B_\theta^2+B_z^2}{2}\right)+\left(1-M_\theta^2\right)\frac{B_\theta^2}{r}=0 \tag{1}$$

stemming from the radial component of the force-balance equation $\rho(\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{j}\times\mathbf{B} - \nabla P$. Here, P is the plasma pressure, B_θ (B_z) is the poloidal (axial) component of the magnetic field; $M_\theta^2 = (v_\theta^2 \varrho)/B_\theta^2$ is the square of the Mach number defined as the ratio of the poloidal velocity to the poloidal-magnetic-field Alfvén velocity. Because of the symmetry any equilibrium quantity depends only on the radial distance r and the axial velocity v_z as well as the velocity shear do not appear in (1); also, the flow is incompressible. In addition to v_z four out of the five quantities in (1) can be prescribed. On account of typical experimental ITB profiles we prescribed the quantities q B_z, v_θ, v_z and ϱ as follows:

weakly reversed shear profile (WRS) [see Fig. 10 of Ref. [5]]

$$q(\rho) = q_c\left(1 - \frac{3\Delta q}{q_c}\frac{r_0^2}{r_{\rm min}^2}\rho^2 + \frac{2\Delta q}{q_c}\frac{r_0^3}{r_{\rm min}^3}\rho^3\right) \tag{2}$$

or alternatively strongly reversed shear profile (SRS)

$$q(\rho) = q_c\left(1 - \frac{2\Delta q}{q_c}\frac{r_0}{r_{\rm min}}\rho + \frac{\Delta q}{q_c}\frac{r_0^2}{r_{\rm min}^2}\rho^2\right), \tag{3}$$

where $\rho = r/R_0$ with r_0 defining the plasma surface, $q_c = q(r=0)$, $r_{\rm min}$ is the position of minimum q, and $\Delta q = q_c - q_{\rm min}$. The WRS-profile (2) exhibits a maximum at the plasma center $r = 0$ in addition to the minimum at $r = r_{\rm min}$;

$$B_z = B_{z0}\left[1 + \delta(1-\rho^2)\right]^{1/2}, \tag{4}$$

where B_{z0} is the vacuum magnetic field and the parameter δ is related to the magnetic properties of the plasma, i.e. for $\delta < 0$ the plasma is diamagnetic;

Gaussian-like poloidal velocity profile

$$v_\theta = 4v_{\theta 0}\rho(1-\rho)\exp\left[-\frac{(\rho - \rho_{min})^2}{h}\right], \tag{5}$$

where the parameter h determines its broadness and $v_{\theta 0}$ is the maximum of v_θ; either peaked axial velocity profile $v_z = v_{z0}(1-\rho^3)^3$ or Gausssian-like v_z profile similar to that of (5); and density profile

$$\varrho = \varrho_0(1-\rho^3)^3. \tag{6}$$

The following quantities can then be calculated: the poloidal magnetic field $B_\theta = \epsilon\rho B_z/q$ where $\epsilon = r_0/R_0$ is the inverse aspect ratio with $2\pi R_0$ associated with the length of the plasma column; the magnetic shear $s = (r/q)(dq/dr)$; the current density via Ampere' s law; the electric field via Ohm' s law; and the $\mathbf{E}\times\mathbf{B}$ shear

$$\omega_{\mathbf{E}\times\mathbf{B}} = \left|\frac{d}{dr}\frac{\mathbf{E}\times\mathbf{B}}{B^2}\right|. \tag{7}$$

Also, integration of (1) yields the pressure. The integration has been performed analytically with the integration constant being fixed so that $P(r = r_0) = 0$.

3. Results

We have employed the following values for some of the parameters: $B_{z0} = 5$ Tesla, $v_{\theta 0} = 3 \cdot 10^4$ m/sec, $v_{z0} = 3 \cdot 10^5$ m/sec, $\varrho_0 = 8.35\ 10^{-8}$kgr/m^3, $\rho_{\min} = 0.5$, $\epsilon \approx 1/6$, and $\delta = -0.0975$; Consequently, it is guaranteed that $M_\theta^2 \approx M_z^2$, where $M_z^2 = (v_z^2 \varrho)/B_z^2$, a scaling typical in tokamaks because $B_z \approx 10\ B_\theta$ and $v_z \approx 10\ v_\theta$. It is also noted here that since in tokamaks $M_\theta < 0.1$ the flow term in (1) is perturbative around the "static" equilibrium $M_\theta = 0$.

For reversed magnetic shear profiles we came to the following conclusions:

1. For $s < 0$, increase of $|s|$ makes the pressure profile steeper. Indeed, substitution of B_θ and its derivative in terms of q and s in (1) yields

$$P' = -B_z B_z' \left[1 + \left(\epsilon \frac{r_0}{R_0}\right)^2\right] + r_0 \rho \left[M_\theta^2 + (s-2)\right] \left(\frac{B_z}{R_0 q}\right)^2, \qquad (8)$$

   where the prime denotes a radial derivative. Also, (8) implies that the P-profile becomes steeper when the plasma is more diamagnetic, i.e. B_z'-related to the parameter δ in (4)- takes larger values.

2. The axial current density profile becomes hollow. In particular, very large values of Δq on the order of 10^2 result in the formation of j_z profiles with "holes" in the central region- $j_z \approx 0$- inside the $\rho_{\min}$ position, a result consistent with experimental evidence [4].

3. The electric field has an extremum nearly at $\rho = \rho_{\min}$ and vanishes at $\rho = 0$ and $\rho = 1$. A percentage increase of the absolute value of the velocity and its shear, respectively, result in nearly the same percentage increase of $|E_r|$ and $|E_r'|$. The impact of the poloidal velocity shear on E_r' is shown in Fig. 1.

4. For a given value of $|s|$, $\omega_{\mathbf{E}\times\mathbf{B}}$ takes larger values in the $s < 0$ region than in the $s > 0$ region. In particular, for the position at which $E_r' = 0$ for $\delta = 0$ one obtains

$$\omega_{\mathbf{E}\times\mathbf{B}} = \left| \frac{(1-s)\left(\epsilon \frac{\rho v_z}{q} - v_\theta\right)}{R_0 q \left[1 + \left(\epsilon \frac{\varrho}{q}\right)^2\right]} \right|. \qquad (9)$$

   Also, in the majority of the cases considered the local maximum of $\omega_{\mathbf{E}\times\mathbf{B}}$ in the $s < 0$ region is slightly larger than that in the $s > 0$ region.

5. A percentage increase of either the poloidal or the toroidal velocity or their shear causes nearly the same percentage increase of $\omega_{\mathbf{E}\times\mathbf{B}}$.

6. For extended flows (large values of the parameter h) a percentage increase of $|s|$ in the ITB region results: (a) in approximately the same percentage increase of $\omega_{\mathbf{E}\times\mathbf{B}}$ if the velocity is purely toroidal (see fig. 2), (b) nearly does not affect the value of $\omega_{\mathbf{E}\times\mathbf{B}}$ if $v_\theta \neq 0$.

7. In the majority of the profiles considered the contribution of the poloidal velocity to $\omega_{\mathbf{E}\times\mathbf{B}}$ is nearly 70 % larger than that of either the toroidal velocity or the magnetic shear.

In addition, irrespective of the reversal of the magnetic shear, there is a critical distance ρ_{cr} at which j_z reverses. In particular, for $B_z = B_{z0} = \text{const.}$ one obtains

$$j_z = \frac{1}{r}\frac{d}{dr}(rB_\theta) = \frac{B_{z0}}{R_0 q}(2-s). \tag{10}$$

Therefore, for $s > 2$ j_z reverses. The radial distances at which $j_z = 0$ for the WRS [Eq. (2)] and the SRS [Eq. (3)] q-profiles are $\rho_{cr}^{WRS} = \rho_{\min}(q_c/\Delta q)^{1/3}$ and $\rho_{cr}^{SRS} = \rho_{\min}(q_c/\Delta q)$, respectively.

Finally, we note that it is interesting to extend the present study within the framework of the two fluid theory in which the electric field appears explicitly in the momentum equations and the plasma current density is related self-consistently with the ion and electron velocities.

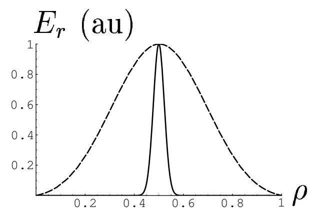

Figure 1: E_r profiles with WRS [Eq. (2)] for purely poloidal flow and $\Delta q = 4$. The continuous curve and dashed curve, respectively, indicate localized flow ($h = 0.001$) and extended flow ($h = 0.1$).

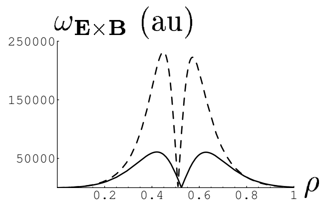

Figure 2: $\mathbf{E}\times\mathbf{B}$-velocity shear profiles for WRS [Eq. (2)] and extended purely toroidal flow ($h = 0.1$) with $\Delta q = 4$ (continuous line) and $\Delta q = 14$ (dashed line).

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