Quantum Teleportation of Dynamics and Effective Interactions between Remote Systems

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(Received 1 April 2013; published 9 July 2013)

Most protocols for quantum information processing consist of a series of quantum gates, which are applied sequentially. In contrast, interactions between matter and fields, for example, as well as measurements such as homodyne detection of light are typically continuous in time. We show how the ability to perform quantum operations continuously and deterministically can be leveraged for inducing nonlocal dynamics between two separate parties. We introduce a scheme for the engineering of an interaction between two remote systems and present a protocol that induces a dynamics in one of the parties that is controlled by the other one. Both schemes apply to continuous variable systems, run continuously in time, and are based on real-time feedback.

DOI: 10.1103/PhysRevLett.111.020501 PACS numbers: 03.67.Ac, 03.67.Hk, 42.50.−p

Most protocols in quantum information science are discrete in the sense that they consist of a sequence of unitary operations and measurements. Schemes for quantum teleportation or dense coding are typical examples. These elementary protocols are the building blocks of other applications such as quantum repeaters or quantum computing. However, some implementations are intrinsically continuous. The most prominent example are atomic ensembles interacting with light, where schemes based on the continuous detection of quadrature operators are realized [1–3]. In this system, protocols can be performed that are intrinsically deterministic and continuous in time. Here we address the question how this property can be exploited by designing primitives that take advantage of this fact. Continuous schemes have been devised in several subfields of quantum information science, for example for phase estimation [4,5], error correction [6], and the preparation and protection of quantum states [7–10], in particular in the context of dissipative schemes [11–17]. Here we introduce two protocols that achieve a qualitatively new goal—to control and transmit quantum evolutions between remote locations. We consider two separate systems that cannot interact directly but exchange quantum states and classical information. One scheme implements an effective nonlocal dynamics, where the two systems evolve as if they were interacting with each other. The other protocol realizes the quantum teleportation of a time evolution, which uses the dynamics of one system to steer the evolution of the other.

Figure 1(a) shows the setup under consideration. Two spin ensembles interact with a propagating light field, which is constantly measured. By performing real-time feedback on both samples, an effective interaction is established between the systems. Since this is done continuously, the dynamics of the two systems corresponds to the evolution under the desired interaction Hamiltonian at any instant of time. Remarkably, this scheme results ideally in a joint unitary evolution of the two remote systems. This is the case even though the protocol is based on measurements yielding random outcomes and therefore random projections of the states involved [18]. We show that using a quantum nondemolition (QND) interaction between spins and light, any Hamiltonian that is quadratic in the atomic operators can be realized by tuning the feedback operation only, i.e., without variation of the system parameters. In the ideal case, any quadratic Hamiltonian can be implemented perfectly.

The second protocol realizes a continuous teleportation. Teleportation [19] offers a practical solution to the delicate

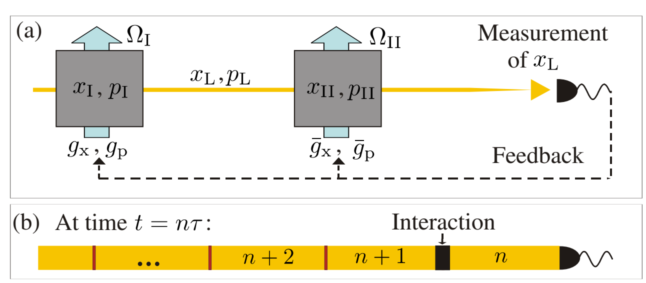

FIG. 1 (color online). Dynamical teleportation and creation of an interaction between two remote systems. (a) The setup consists of two atomic ensembles in constant magnetic fields. A freely propagating light field interacts with both samples and is continuously measured. The result is fed back to the atoms. (b) Illustration of the light-matter interaction in terms of discretized spatially localized light modes.

task of transmitting quantum states [20]. It is a prerequisite for quantum networks [22,23] and a primitive for quantum computation [24]. A standard teleportation scheme consists of three separate steps, which involve (i) establishment of a highly entangled link, (ii) a projective measurement that destroys the state to be teleported, and (iii) the recovery of the state on the receiver's side by applying a feedback operation. Here, we consider a continuous process involving weakly entangled states and measurements that disturb the quantum state only a little at each instant of time. In contrast to previous approaches, which transmit a (static) state using a single feedback operation at the end of the protocol, we consider the transmission of a whole time evolution using real-time feedback. To illustrate this point, we consider the continuous teleportation between two parties, Bob and Charlie, when a time-dependent magnetic field is applied to Charlie's system. The resulting displacements on Charlie's side translate into a corresponding evolution on Bob's system, which evolves as if it was placed in a time-dependent magnetic field: it evolves as if interacting with a field whose time dependence is determined by Charlie's evolution.

We consider two separate systems that are characterized in terms of continuous variables $x_{\rm I}$, $p_{\rm I}$ and $x_{\rm II}$, $p_{\rm II}$, which commute canonically $[x, p] = i$. We assume that both systems can be rotated locally and interact with a propagating auxiliary bosonic system via a QND interaction [Eq. (2)]. For concreteness, we consider two atomic spin ensembles interacting with coherent light [25]. The free evolution of the atomic system,

$$H_{\rm A} = \frac{\Omega_{\rm I}}{2}(x_{\rm I}^2 + p_{\rm I}^2) + \frac{\Omega_{\rm II}}{2}(x_{\rm II}^2 + p_{\rm II}^2),$$

describes the atomic rotation (Larmor precession) in homogeneous magnetic fields along $\hat{x}$ with Larmor frequencies $\Omega_{\rm I}$ and $\Omega_{\rm II}$ [see Fig. 1(a)]. In the following, we use transformed atomic variables

$$\begin{aligned}\begin{pmatrix}\tilde{x}_{\rm I/II}\\ \tilde{p}_{\rm I/II}\end{pmatrix} &= R(\Omega_{\rm I/II})\begin{pmatrix}x_{\rm I/II}\\ p_{\rm I/II}\end{pmatrix},\\ R(\Omega_{\rm I/II}) &= \begin{pmatrix}\cos(\Omega_{\rm I/II}t) & -\sin(\Omega_{\rm I/II}t)\\ \sin(\Omega_{\rm I/II}t) & \cos(\Omega_{\rm I/II}t)\end{pmatrix}\begin{pmatrix}x_{\rm I/II}\\ p_{\rm I/II}\end{pmatrix},\end{aligned} \tag{1}$$

which rotate at the Larmor frequencies of the respective fields. The light propagates along $\hat{z}$, passing both ensembles. We adopt here a discretized one-dimensional model. The interaction time T is divided into N infinitesimally small time steps of length τ, and the light is accordingly described in terms of N short pulse pieces. The quadratures associated with the nth localized light mode are denoted by $x_{{\rm L},n}$ and $p_{{\rm L},n}$ with $[x_{{\rm L},n}, p_{{\rm L},n'}] = i\delta_{n,n'}$. The nth pulse piece interacts with the atoms during the time window from $t = (n-1)\tau$ to $t = n\tau$ [Fig. 1(b)] according to the interaction Hamiltonian [27]

$$H_{{\rm QND},n} = \frac{\kappa}{\sqrt{N}\tau}(p_{\rm I}(n\tau) + p_{\rm II}(n\tau))p_{{\rm L},n}, \tag{2}$$

where κ is a dimensionless coupling constant. Such a QND interaction can, for example, be realized using a Faraday interaction in atomic vapors [28,29] or in optomechanical systems [30]. The total Hamiltonian is given by $H = H_{\rm L} + H_{\rm A} + \sum_{n=1}^{N} H_{{\rm QND},n}$, where $H_{\rm L}$ accounts for the free propagation of the light. By virtue of the consecutive QND interactions shown in Fig. 1(a), the quantum state of the atoms is mapped to the x quadrature of the light field. The corresponding input-output relation for an infinitesimal time step is given by

$$x_{{\rm L},n}^{\rm out} = x_{{\rm L},n}^{\rm in} + \frac{\kappa}{\sqrt{N}}\vec{V}_{\rm L}^T(n\tau)\vec{R}_{\rm A}([n-1]\tau), \tag{3}$$

with

$$\vec{V}_{\rm L}^T(t) = (-\sin(\Omega_{\rm I}t), \cos(\Omega_{\rm I}t), -\sin(\Omega_{\rm II}t), \cos(\Omega_{\rm II}t)),$$
$$\vec{R}_{\rm A}^T(t) = (\tilde{x}_{\rm I}(t), \tilde{p}_{\rm I}(t), \tilde{x}_{\rm II}(t), \tilde{p}_{\rm II}(t)),$$

where $\Omega\tau \ll 1$ has been assumed [31]. The p quadrature is conserved $p_{{\rm L},n}^{\rm out} = p_{{\rm L},n}^{\rm in}$. The atomic evolution is given by

$$\begin{pmatrix}\tilde{x}_{\rm I/II}(n\tau)\\ \tilde{p}_{\rm I/II}(n\tau)\end{pmatrix} = \begin{pmatrix}\tilde{x}_{\rm I/II}([n-1]\tau)\\ \tilde{p}_{\rm I/II}([n-1]\tau)\end{pmatrix} + \frac{\kappa}{\sqrt{N}}\begin{pmatrix}\cos(\Omega_{\rm I/II}n\tau)\\ \sin(\Omega_{\rm I/II}n\tau)\end{pmatrix}p_{{\rm L},n}^{\rm in}.$$

We assume that $x_{\rm L}$ is continuously measured and that the result is instantaneously fed back onto the atoms by applying a conditional displacement. We apply here feedback operations with temporally modulated gain factors $(1/\sqrt{N})g_{x,{\rm I/II}}(t)$ and $(1/\sqrt{N})g_{p,{\rm I/II}}(t)$ such that

$$\begin{pmatrix}\tilde{x}_{\rm I/II}^{\rm fin}(n\tau)\\ \tilde{p}_{\rm I/II}^{\rm fin}(n\tau)\end{pmatrix} = \begin{pmatrix}\tilde{x}_{\rm I/II}(n\tau)\\ \tilde{p}_{\rm I/II}(n\tau)\end{pmatrix} + \frac{1}{\sqrt{N}}\begin{pmatrix}g_{x,{\rm I/II}}(n\tau)\\ g_{p,{\rm I/II}}(n\tau)\end{pmatrix}x_{{\rm L},n}^{\rm out}. \tag{4}$$

We show now how this setup can be used to establish an arbitrary quadratic interaction between two ensembles. Using suitable local rotations [33], any interaction Hamiltonian for two continuous variable systems, which is quadratic in the system operators, can be expressed as

$$H = \gamma(\mu H_{\rm A} + \nu H_{\rm P}) = \gamma\left(Zx_1p_2 + \frac{1}{Z}p_1x_2\right), \tag{5}$$

where $\mu = (1/2)(Z + (1/Z))$ and $\nu = (1/2)(Z - (1/Z))$ [34]. γ characterizes the overall coupling strength of the interaction while Z parametrizes the imbalance between the active (entanglement creating) and the passive (energy conserving) components, $H_{\rm A}$ and $H_{\rm P}$ [35]. After the light-matter interactions, the measured quadrature $x_{\rm L}$ contains information on both ensembles [see Eq. (3)]. Feedback of $x_{\rm L}$ according to Eq. (4) leads therefore to terms that correspond to the evolution under both local and interaction Hamiltonians. The feedback of information of each ensemble onto itself leads to an effective evolution according to local squeezing Hamiltonians. In order to suppress these contributions, we use the fact that the quadratures of each ensemble are mapped to the light with an oscillatory time dependence. For implementing a purely nonlocal evolution, the gain function for ensemble I (II) is chosen to oscillate with Ω_{II} (Ω_{I}), such that information is transferred with high efficiency between the ensembles, while contributions due to the feedback from the samples to themselves are out of phase and average out. This becomes apparent by considering the time varying gain functions $g_{x,\mathrm{I/II}}(t) = (1/\sqrt{N}) g_{a/b} \sin(\Omega_{a/b} t)$ and $g_{p,\mathrm{I/II}}(t) = (1/\sqrt{N}) g_{b/a} \cos(\Omega_{a/b} t)$. In the continuous limit,

$$\begin{pmatrix} \dot{\tilde{x}}_{\mathrm{I}}^{\mathrm{fin}}(t) \\ \dot{\tilde{p}}_{\mathrm{I}}^{\mathrm{fin}}(t) \end{pmatrix} = \frac{\kappa}{T}\left[M_{g_a,g_b}^{\Omega_a,\Omega_{\mathrm{I}}}(t) \begin{pmatrix} \tilde{x}_{\mathrm{I}}(t) \\ \tilde{p}_{\mathrm{I}}(t) \end{pmatrix} + M_{g_a,g_b}^{\Omega_a,\Omega_{\mathrm{II}}}(t) \begin{pmatrix} \tilde{x}_{\mathrm{II}}(t) \\ \tilde{p}_{\mathrm{II}}(t) \end{pmatrix} \right] + \begin{pmatrix} \mathcal{N}_{x,\mathrm{I}}(t) \\ \mathcal{N}_{p,\mathrm{I}}(t) \end{pmatrix},$$

$$\begin{pmatrix} \dot{\tilde{x}}_{\mathrm{II}}^{\mathrm{fin}}(t) \\ \dot{\tilde{p}}_{\mathrm{II}}^{\mathrm{fin}}(t) \end{pmatrix} = \frac{\kappa}{T}\left[M_{g_b,g_a}^{\Omega_b,\Omega_{\mathrm{II}}}(t) \begin{pmatrix} \tilde{x}_{\mathrm{II}}(t) \\ \tilde{p}_{\mathrm{II}}(t) \end{pmatrix} + M_{g_b,g_a}^{\Omega_b,\Omega_{\mathrm{I}}}(t) \begin{pmatrix} \tilde{x}_{\mathrm{I}}(t) \\ \tilde{p}_{\mathrm{I}}(t) \end{pmatrix} \right] + \begin{pmatrix} \mathcal{N}_{x,\mathrm{II}}(t) \\ \mathcal{N}_{p,\mathrm{II}}(t) \end{pmatrix},$$

where the coupling matrix $M_{g_1,g_2}^{\Omega_1,\Omega_2}(t)$ is defined by

$$M_{g_1,g_2}^{\Omega_1,\Omega_2}(t) = \begin{pmatrix} -g_1 \sin(\Omega_1 t)\sin(\Omega_2 t) & g_1 \sin(\Omega_1 t)\cos(\Omega_2 t) \\ -g_2 \cos(\Omega_1 t)\sin(\Omega_2 t) & g_2 \cos(\Omega_1 t)\cos(\Omega_2 t) \end{pmatrix}.$$

$\mathcal{N}_{x,\mathrm{I/II}}$ and $\mathcal{N}_{p,\mathrm{I/II}}$ are noise terms due to the mapping of the input light field onto the atomic systems,

$$\begin{pmatrix} \mathcal{N}_{x,\mathrm{I/II}}(t) \\ \mathcal{N}_{p,\mathrm{I/II}}(t) \end{pmatrix} = \frac{1}{\sqrt{T}} \begin{pmatrix} g_{a/b} \sin(\Omega_{a/b} t) & \kappa \cos(\Omega_{\mathrm{I/II}} t) \\ g_{b/a} \cos(\Omega_{a/b} t) & \kappa \sin(\Omega_{\mathrm{I/II}} t) \end{pmatrix} \begin{pmatrix} \bar{x}_{\mathrm{L}}(ct,0) \\ \bar{p}_{\mathrm{L}}(ct,0) \end{pmatrix}.$$

We use here continuous light modes with quadratures $\bar{x}_{\mathrm{L}}(ct,0) = x_{\mathrm{L},n}^{\mathrm{in}}/\sqrt{\tau}$ and $\bar{p}_{\mathrm{L}}(ct,0) = p_{\mathrm{L},n}^{\mathrm{in}}/\sqrt{\tau}$ [36]. As shown in the Supplemental Material (SM), the equations above can be approximated by their coarse-grained version using coarse-graining time intervals $\Delta t \gg \Omega_{\mathrm{I/II}}^{-1}$, $|\Omega_{\mathrm{I}} - \Omega_{\mathrm{II}}|^{-1}$. In this limit, the coupling matrices $M_{g_1,g_2}^{\Omega_1,\Omega_2}$ lead to a negligible contribution for $\Omega_1 \neq \Omega_2$, since their matrix elements average out. Similarly, they can be approximated by a constant diagonal matrix for $\Omega_1 = \Omega_2$ [37]. The noise terms $\mathcal{N}_{x,\mathrm{I/II}}(t)$ and $\mathcal{N}_{p,\mathrm{I/II}}(t)$ give rise to noise modes, which are approximately independent for $\Delta t \gg \Omega_{\mathrm{I/II}}^{-1}$, $|\Omega_{\mathrm{I}} - \Omega_{\mathrm{II}}|^{-1}$ and can therefore be squeezed simultaneously such that their contributions become negligible. A detailed analysis is provided in [37]. For establishing an interaction according to Eq. (5) with $\gamma = \kappa g/2T$, we consider the case $\Omega_{a/b} = \Omega_{\mathrm{II/I}}$, $g_{a/b} = -gZ^{\mp 1}$, which leads to

$$\begin{pmatrix} \dot{\tilde{x}}_{\mathrm{I}} \\ \dot{\tilde{p}}_{\mathrm{I}} \end{pmatrix} = \frac{\kappa g}{2T} \begin{pmatrix} \frac{1}{Z}\tilde{x}_{\mathrm{II}} \\ -Z\tilde{p}_{\mathrm{II}} \end{pmatrix}, \qquad \begin{pmatrix} \dot{\tilde{x}}_{\mathrm{II}} \\ \dot{\tilde{p}}_{\mathrm{II}} \end{pmatrix} = \frac{\kappa g}{2T} \begin{pmatrix} Z\tilde{x}_{\mathrm{I}} \\ -\frac{1}{Z}\tilde{p}_{\mathrm{I}} \end{pmatrix}.$$

By tuning the feedback parameters $g_{a/b}$, it is therefore possible to realize any time evolution that corresponds to a quadratic interaction Hamiltonian.

Using a modified configuration, a continuous teleportation can be realized. A teleportation scheme [19] involves three parties—Alice, Bob, and Charlie. It allows Alice to teleport an unknown quantum state provided by Charlie to Bob. Here, Charlie's state is stored in ensemble II and teleported to ensemble I, representing Bob, while the light field plays the role of Alice. Step (i) in the standard protocol outlined in the Introduction corresponds to the interaction between the light and ensemble I, resulting in an entangled state. The distribution of entanglement between the remote sites is realized by the free propagation of the light. Step (ii) corresponds to the interaction of the light with ensemble II and the measurement of x_{L}. Step (iii) is implemented in the form of a feedback operation realizing a conditional displacement on ensemble I, which can be done using magnetic fields. Using this protocol, the deterministic teleportation of a quantum state between two ensembles has been demonstrated recently [38]. We present now an extension to a time-continuous operation, which facilitates the teleportation of quantum dynamics. For this purpose, we consider the special case of the scheme above where $\Omega_{\mathrm{I}} = \Omega_{\mathrm{II}} = \Omega$ and feedback is applied only to the first ensemble $g_{x,\mathrm{I}}(t) = (1/\sqrt{N})\bar{g}_x \sin(\Omega t)$, $g_{p,\mathrm{I}}(t) = (1/\sqrt{N})\bar{g}_p \cos(\Omega t)$. With this choice, Eq. (4) yields

$$\begin{pmatrix} \dot{\tilde{x}}_{\mathrm{I}}^{\mathrm{fin}}(t) \\ \dot{\tilde{p}}_{\mathrm{I}}^{\mathrm{fin}}(t) \end{pmatrix} = \frac{\kappa}{T} M_{\bar{g}_x,\bar{g}_p}^{\Omega,\Omega}(t) \begin{pmatrix} \tilde{x}_{\mathrm{I}}(t) \\ \tilde{p}_{\mathrm{I}}(t) \end{pmatrix} + \frac{\kappa}{T} M_{\bar{g}_x,\bar{g}_p}^{\Omega,\Omega}(t) \begin{pmatrix} \tilde{x}_{\mathrm{II}}(t) \\ \tilde{p}_{\mathrm{II}}(t) \end{pmatrix} + \frac{1}{\sqrt{T}} \begin{pmatrix} \bar{g}_x \sin(\Omega t) & \kappa\cos(\Omega t) \\ \bar{g}_p \cos(\Omega t) & \kappa \sin(\Omega t) \end{pmatrix} \begin{pmatrix} \bar{x}_{\mathrm{L}}(ct,0) \\ \bar{p}_{\mathrm{L}}(ct,0) \end{pmatrix}. \tag{6}$$

The second term in Eq. (6) allows us to control Bob's dynamics using Charlie's system. For demonstrating the transmission of a time evolution, we add an extra Hamiltonian corresponding to a time-dependent transverse magnetic field on Charlie's side $H_{\mathrm{extra,II}} = \alpha_x(t)x_{\mathrm{II}} + \alpha_p(t)p_{\mathrm{II}}$, where $\alpha_{x/p}(t)$ are real time-dependent functions. Accordingly,

$$\begin{pmatrix}\tilde{x}_{\rm II}(t)\\ \tilde{p}_{\rm II}(t)\end{pmatrix} = \int_0^t dt' \begin{pmatrix}\tilde{\alpha}_p(t')\\ -\tilde{\alpha}_x(t')\end{pmatrix} + \begin{pmatrix}\tilde{x}_{\rm II}^{\rm in}\\ \tilde{p}_{\rm II}^{\rm in}\end{pmatrix} + \frac{\kappa}{\sqrt{T}}\int_0^t dt'\begin{pmatrix}\cos(\Omega t')\\ \sin(\Omega t')\end{pmatrix}\bar{p}_{\rm L}(ct',0), \quad (7)$$

where $\tilde{\alpha}_{x/p}(t)$ describe the time dependence of $H_{\rm extra,II}$ in the rotating frame [see Eq. (1)]. The resulting time evolution on Bob's side can be evaluated by inserting Eq. (7) into Eq. (6). As in the general case above, the resulting differential equation can be approximated by the coarse-grained equation with $M_{g,-g}^{\Omega,\Omega} \rightarrow -(1/2)\mathbb{1}$ for $\Omega\Delta t \gg 1$, where Δt is the coarse graining time interval. This yields

$$\begin{pmatrix}\dot{\tilde{x}}_{\rm I}^{\rm fin}(t)\\ \dot{\tilde{p}}_{\rm I}^{\rm fin}(t)\end{pmatrix} = \frac{-\kappa}{2T}\int_0^t dt'\begin{pmatrix}\bar{g}_x\tilde{\alpha}_p(t')\\ \bar{g}_p\tilde{\alpha}_x(t')\end{pmatrix} + \frac{\kappa}{2T} \times \begin{pmatrix}-\bar{g}_x[\tilde{x}_{\rm I}^{\rm in}+\tilde{x}_{\rm II}^{\rm in}]\\ \bar{g}_p[\tilde{p}_{\rm I}^{\rm in}+\tilde{p}_{\rm II}^{\rm in}]\end{pmatrix} + \frac{\kappa^2}{T^{3/2}} \times \int_0^t dt'\begin{pmatrix}-\bar{g}_x\cos(\Omega t')\\ \bar{g}_p\sin(\Omega t')\end{pmatrix}\bar{p}_{\rm L}(ct',0) + \frac{1}{\sqrt{T}} \times \begin{pmatrix}\bar{g}_x\sin(\Omega t) & \kappa\cos(\Omega t)\\ \bar{g}_p\cos(\Omega t) & \kappa\sin(\Omega t)\end{pmatrix}\begin{pmatrix}\bar{x}_{\rm L}(ct,0)\\ \bar{p}_{\rm L}(ct,0)\end{pmatrix}. \quad (8)$$

The first term on the right is equivalent to a field experienced by Bob. Bob's ensemble evolves as if it was placed in a magnetic field, whose time dependence is given by Charlie's evolution [see Eq. (7)]. This way, the effect of the magnetic field applied at Charlie's side is teleported to Bob. This is possible, since entanglement generation, measurement, and feedback are performed continually. In contrast to a traditional teleportation, Charlie's quantum state is therefore not destroyed in a single step and subsequently restored on Bob's side, but rather transmitted continuously, which offers the possibility to include the effect of a time evolution.

In the following, we discuss imperfections and consider the case, where real-time feedback is applied to both ensembles. This discussion can also be applied for evaluating the added noise in the teleportation protocol. For analyzing the performance of the scheme, we introduce a figure of merit which quantifies the deviation of the realized time evolution from the desired one using the Jamiolkowsi isomorphism between quantum maps and states. Both the imperfect map and the ideal one are transformed into their corresponding states and then compared as explained in the SM. The time evolution ε_T acting on the atomic system is described in terms of an entangled state of twice the system size that can be used to teleport a given input state $\rho_{\rm in}$ through ε_T, such that the output $\varepsilon_T(\rho_{\rm in})$ is obtained. This entangled state consists of two copies of a two-mode squeezed state with squeezing parameter R. For $R \rightarrow \infty$, which corresponds to a quantum state with infinite energy, any input state can be teleported through ε_T. For

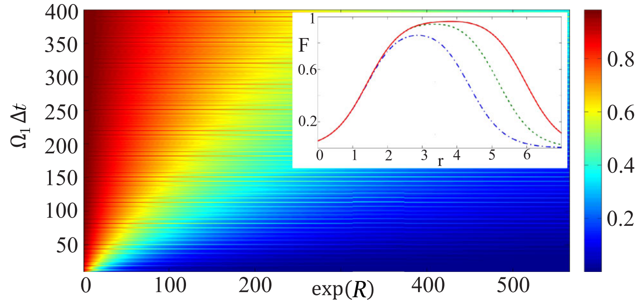

FIG. 2 (color online). Fidelity for realizing $H_{\rm A}$ for $\Omega_2 = 2\Omega_1$ and $g = \kappa = 1$. The results for implementing $H_{\rm P}$ are provided in [37]. Any evolution under a quadratic Hamiltonian can be realized by combining these two interactions with local rotations [33,39]. The main panel displays the fidelity for optimal squeezing $r_{\rm opt}$ versus e^R and $\Omega_1\Delta t$. The inset shows the fidelity versus the squeezing of the light for $R = 3$. The three curves correspond to $\Omega_1\Delta t = 100$ (dash-dotted line), $\Omega_1\Delta t = 300$ (dashed line), and $\Omega_1\Delta t = 500$ (solid line).

finite R, this holds for a restricted set of input states. We start by considering the fidelity for fixed R. As explained in the SM, the scheme involves two types of imperfections, fast rotating terms in the evolution and light noise added to the atomic system. If a time window $\Delta t \gg \Omega_1$, Ω_2, $|\Omega_1 - \Omega_2|$ is considered, the former are negligible and the latter can be suppressed using squeezed input light fields. The inset in Fig. 2 shows the fidelity for fixed R versus the squeezing of the light field r for $\Omega_2 = 2\Omega_1$. For increasing $\Omega_1\Delta t$, the optimal squeezing parameter $r_{\rm opt}$ increases, which leads to an increased accuracy of the scheme. For $\Omega_1\Delta t \rightarrow \infty$, $r_{\rm opt} \rightarrow \infty$, and $F \rightarrow 1$. For increasing values of R, correspondingly high values of Δt are required to obtain a good fidelity. The required value of Δt increases with e^R as shown in Fig. 2, which displays the fidelity F for fixed Larmor frequencies Ω_1 and $\Omega_2 = 2\Omega_1$. This graph features a fine substructure since for time windows $\Delta t = 2\pi/\Omega_1$, local maxima are obtained, which gives rise to stripelike regions of high fidelity. In general, realizing a desired time evolution with high temporal resolution requires high Larmor frequencies. For fixed Ω_1 and Ω_2, the attainable precision depends on the temporal resolution as shown in Fig. 2. Higher fidelities can be obtained if a stroboscopic interaction is implemented where points of interest in time are chosen to coincide with the local maxima.

In conclusion, we proposed a dynamical teleportation scheme, where entangling operations, measurement, and feedback are performed continuously and simultaneously. Moreover, we demonstrated how a generalized version of this protocol can be used to implement arbitrary quadratic interactions between remote systems.

We thank Karl Gerd Vollbrecht and Géza Giedke and acknowledge support by the EU project MALICIA, the Alexander von Humboldt Foundation, and the Spanish program TOQATA (FIS2008-00784). Additional support by the ERC grants INTERFACE and QUAGATUA, the Danish National Science Foundation Center QUANTOP, and the DARPA program QUASAR are also acknowledged.

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[35] The two mode squeezing Hamiltoninan H_{A} is given by $H_{\mathrm{A}} \propto i(a_1 a_2 - a_1^\dagger a_2^\dagger)$ where $a_j = (1/\sqrt{2})(x_j + i p_j)$, with $j = 1, 2$. $H_{\mathrm{P}} \propto i(a_1 a_2^\dagger - a_1^\dagger a_2)$ describes a beamsplitter-like interaction.
[36] The continuous light modes with quadratures $\bar{x}_{\mathrm{L}}(\xi, t)$, $\bar{p}_{\mathrm{L}}(\xi, t)$ are obtained by considering spatially localized modes $x_{\mathrm{L}}(z, t)$, $p_{\mathrm{L}}(z, t)$ with $[x_{\mathrm{L}}(z, t), x_{\mathrm{L}}(z', t)] = ic\delta(z - z')$ and performing the variable transformation $\xi = ct - z$ (such that $\bar{x}_{\mathrm{L}}(\xi, t) = x_{\mathrm{L}}(ct - \xi, t)$).
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