Gravitational waves from coalescing massive black holes in young dense clusters

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Abstract.
HST observations reveal that young massive star clusters form in gas-rich environments like the Antennæ galaxy which will merge in collisional processes to form larger structures. These clusters amalgamate and if some of these clusters harbour a massive black hole in their centres, they can become a strong source of gravitational waves when they coalesce. In order to understand the dynamical processes that are into play in such a scenario, one has to carefully study the evolution of the merger of two of such young massive star clusters and more specifically their respective massive black holes. This will be a promising source of gravitational waves for both, LISA and the proposed Big Bang Observer (BBO), whose first purpose is to search for an inflation-generated gravitational waves background in the frequency range of $10^{-1} - 1$ Hz. We used high-resolution direct summation $N-$body simulations to study the orbital evolution of two colliding globular clusters with different initial conditions. Even if the final eccentricity is almost negligible when entering the bandwidth, it will suffice to provide us with detailed information about these astrophysical events.

MOTIVATION

Nowadays it is well established that massive stellar clusters form in interacting galaxies. High-resolution Hubble Space Telescope Observations of the Antennæ [1, 2] or Stephan's Quintet [3] show that hundreds of young massive star clusters are lurking in the star forming regions and that they are clustered into larger clusters of a $\sim$ few 100 pc. The images also reveal that only a few of these clusters are reddened, thus suggesting that the gas has been already removed in most of them. Since they harbour $\sim 10^5$ stars within $\sim 1-$ few parsecs and are older than 5 Myr, they are most likely to be bound clusters. Supernova explosions originating from stars with a mass larger than $8M_{\odot}$ only contribute to the total cluster mass -which for $\sim 10^7 - 10^5$ stars is about $\mathscr{M}_{\rm cl} \sim 10^7 - 10^5 M_{\odot}$- in about $\sim$ 10%. Stars more massive than $2M_{\odot}$ contribute in only $\sim$ 25% to the total, so that the clusters are very likely to be bound.

The so-called *clusters complexes* have been employed in the literature as a possibility to build compact dwarf galaxies as a result of the amalgamation of their smaller clusters constituents [4, 5] in collisional processes. On the other hand, it has been studied by different authors how in such a young dense cluster collisional processes among the heaviest stars that segregate to the centre because of dynamical friction might lead to the formation of a very massive star [6, 7, 8, 9]. Such star will become unstable and thus possibly create a massive black hole in their centre with a mass ranging between $10^2 - 10^4 M_{\odot}$ -which therefore receives the surname 'intermediate-mass' black hole (IMBH)-. This means that the young dense clusters in the cluster complexes are

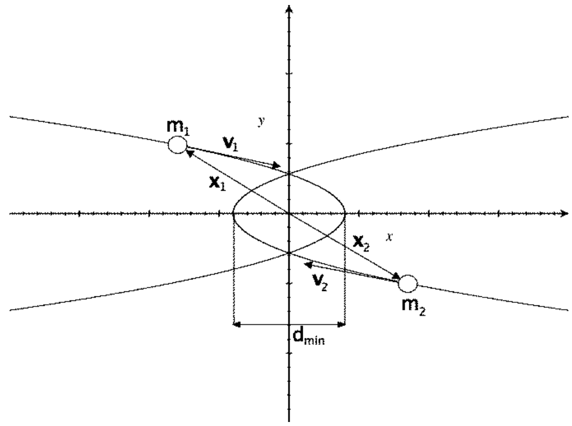

FIGURE 1. Initial set up for a parabolic collision in the centre-of-mass of both clusters

possibly guarding an IMBH in their centres; since the clusters merge with each other, the respective IMBHs will have, at least, the possibility of forming a bound system. Whether or not they will merge within a Hubble time and what the implications for LISA and the BBO would be, is something to be analysed numerically.

HOW TO MERGE TWO YOUNG DENSE CLUSTERS

The clusters were set on to a parabolic orbit so that the minimum distance at which they pass by is $d_{\rm min}$ of Fig. (1) if they are considered to be a point particle at that moment. In the centre of mass reference frame of both clusters, as shown in the figure, we have that $\mathbf{x}_1 = \lambda_2\,\mathbf{d}$, $\mathbf{x}_2 = -\lambda_1\,\mathbf{d}$, $\mathbf{v}_1 = \lambda_2\,v_{\rm rel}$ and $\mathbf{v}_2 = -\lambda_1\,\mathbf{v}_{\rm rel}$; where $\mathbf{v}_{\rm rel}$ is the relative velocity of the clusters, $\mathbf{x}_{1,2}$ their positions (if we regard them to be a point mass, or to their centres) and $\lambda_{1,2} = m_{1,2}/(m_1+m_2)$

From the reduced particle standpoint, we have to determine $\mathbf{d}$ and $\mathbf{v}_{\rm rel}$ when the separation is d (which is given as an initial condition). Since the reduced mass is $\mu = m_1\,m_2/(m_1+m_2)$, for a parabolic orbit we have that the energy at the pericentre

$$E = \frac{-G\,m_1\,m_2}{d} + \frac{1}{2}\mu\,v_{\rm rel}^2 = \frac{-G\,m_1\,m_2}{d} + \frac{1}{2}\mu\,v_{\rm max}^2 = 0. \tag{1}$$

Since the specific angular momentum per unit μ is $l = l_z = -v_{\rm max}\cdot d_{\rm min} = |\mathbf{d}\wedge v_{\rm rel}|_z = x\,v_y - y\,v_x$, we obtain $v_{\rm max} = \sqrt{2\,G(m_1+m_2)/d_{\rm min}}$, for a given specific angular momentum $l = \sqrt{2\,G\,d_{\rm min}(m_1+m_2)}$.

As for the components of the velocities, for a given relative velocity of $v_{\rm rel} = \sqrt{2\,G\,d_{\rm min}(m_1+m_2)/d}$ and with the help of the relations $-l = x\cdot v_y - y\cdot v_x$, $v_{\rm rel}^2 = v_x^2+v_y^2$ and $d^2 = x^2+y^2$, we can infer that the velocity components are

$$v_x = \frac{l\cdot y}{d^2}\left[1+\sqrt{1+\left(\frac{d}{y}\right)^2(x^2\frac{v_{\rm rel}^2}{l^2}-1)}\right]\,, v_y = -\sqrt{v_{\rm rel}^2-v_x^2}. \tag{2}$$

We resort finally to the definition of parabola to obtain the required expressions for x and y, $2\,d_{\rm min} - x = d$, so that $x = d - 2\,d_{\rm min}$ and $y = \sqrt{d^2 - x^2}$.

For the model presented in this work, we chose a system with $d_{\rm min} = 2$ pc, corresponding to a relative velocity at pericentre of 23.3 km/s. The clusters will always merge and form a larger cluster, because they are initially set in a parabolic orbit. The number of stars used for each cluster is $\mathscr{N}_{\star} = 6.3 \times 10^4$, the masses of they set to $6.3 \times 10^4 M_{\odot}$ and we used for the initial distribution a King model of concentration $W_0 = 7$ (cluster 1) and $W_0 = 6$ (cluster 2). The central velocity dispersions were set to $\sigma_{\rm core\,1} = 8.41$ km/s and $\sigma_{\rm core\,2} = 8.29$ km/s (hereafter the subscripts 1 and 2 stand for the cluster 1 and 2), with core radii of $R_{\rm core\,1} = 0.203$ pc and $R_{\rm core\,2} = 0.293$ pc. Both clusters host additionally an IMBH of mass $300 M_{\odot}$ in their centres.

The simulation was performed on a special-purpose hardware GRAPE-6A single PCI card with a peak performance of 130 Gflops [10], roughly equivalent to 100 single PCs, with the direct-summation NBODY4 code of Aarseth [11]. This choice was made for the sake of the accuracy of the study of the orbital parameters evolution of the binary of IMBHs; for this numerical tool includes both the *KS regularisation* and *chain regularisation*, which means that when two or more particles are tightly bound to each other or the separation among them becomes very small during a hyperbolic encounter, the system becomes a candidate to be regularised in order to avoid problematical small individual time steps. The basis of direct NBODY4 codes relies on an improved Hermit integrator scheme [12] for which we need not only the accelerations but also their time derivative. The computational effort translates into accuracy and this way we can reliably follow track of the orbital evolution of every single particle in our system. Other alternative codes that add a softening to the gravitational forces (i.e. substituting the $1/r^2$ factor with $1/(r^2 + \varepsilon^2)$, where r is the radius and ε the softening parameter) in order to avoid them to become too large are to be discarded if we want to befittingly make a highly accurate estimate of the orbital evolution of the IMBHs system (for at a certain point in the evolution of the binary the separation $\sim \varepsilon$) which is the final purpose of our numerical study.

DYNAMICS AND GEOMETRY OF THE SYSTEM

In Fig. (2) we show nine snapshots of the merger of the two clusters and the position of the IMBHs marked with a black dot. The nine snapshots show the evolution of the clusters merger for $T = 0$, 54, 66, 76, 110, 128, 146, 152 and 192 $\mathscr{U}_{\rm TNB}$ $N-$body time units, which correspondingly are $T = 0, 2.77$, 3.19, 4.62, 5.37, 6.13, 6.38 and 8.06 Myrs. We can clearly observe an exchange of stars between the clusters already after the first cluster interaction (the corresponding "red" stars do not show up in the "blue" cluster because the blue colour overwrites the red). As shown in Fig. (3), the binary of intermediate-mass black holes hardens steadily. We obtain the *classical* value of the *hardening constant* $H \simeq 16$ [13, 14].

In Fig. (4) we show the evolution of the triaxiality of the cluster formed as a result of the merger of the two cluster for our fiducial model. In the figure, a, b and c –with $a > b > c$ ab definitio– are the semi-major axes of the ellipsoid of inertia, determined by

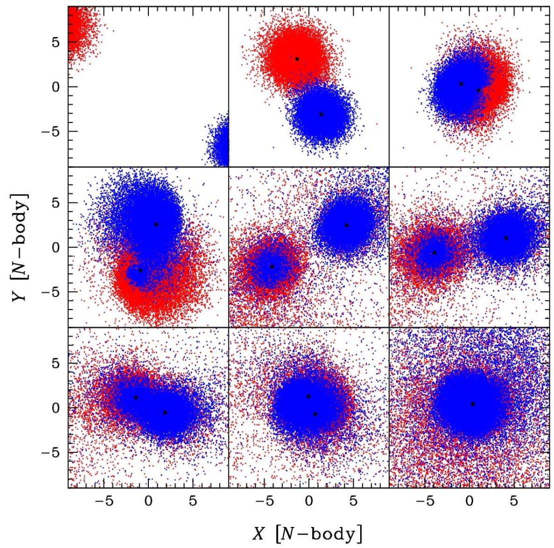

FIGURE 2. Projection in the X-Y plan of the trajectory of every single particle in the process of merger of the two young dense clusters. See text for further details

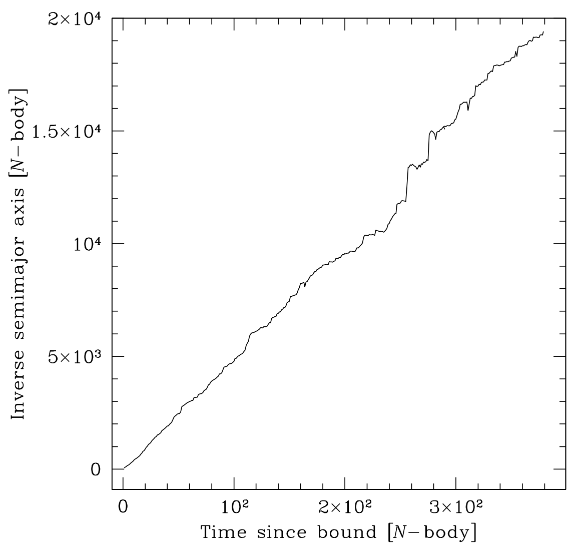

FIGURE 3. Evolution of the semi-major axis of the massive black hole binary

four different mass fractions of the stars which were distributed according to the amount of gravitational energy. This means that the lower the mass fraction is, the closer we are to the centre of the resulted merged system. We can clearly see that the system resulting of a realistic parabolic cluster merger has a shorter axis and two approximately equal longer axes, but larger than the third one, $a \sim b > c$, which means that it is manifestly oblate and not prolate. We can also observe that the binary of IMBHs makes the system more spherical in the centre than at larger distances, where it is flatter. This is so because the flattening depends on the gravitational and centrifugal forces of the cluster, its size,

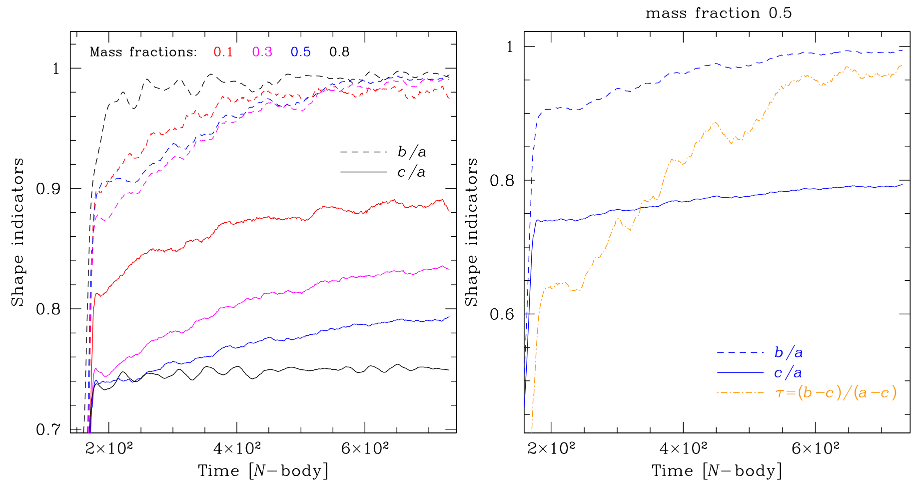

FIGURE 4. Triaxiality of the resulting merged cluster for different mass fractions (left panel) and for the mass fraction 0.5 (right panel)

rotation and density of stars. For the case of two galactic nuclei harbouring massive black holes, this would translate into a clear danger of hang-up of the binary or stalling; i.e. there would not be enough stars to strongly interact with the binary of massive black holes because the loss-cone -a phase space region where stars can interact with the central object/s in one crossing time, [15, 16, 17]- would remain depleted because of the lack of centrophilic orbits in an axially symmetric potential. In the case studied here, though, this is not the case, because the ratio of relaxational to dynamical timescales is small due to the lower number of stars in the system.

IMPLICATIONS FOR LISA AND THE BBO: GENERAL DISCUSSION ON NBODY4 PROSPECTS FOR LOWER-FREQUENCY GRAVITATIONAL-WAVES ASTROPHYSICS

After a strong dynamical evolution with the surrounding stars of the merged cluster in which the IMBHs are embedded, the evolution is dominated by gravitational wave emission during the last 10^8 yrs. The evolution from the moment the binary is bound takes $\sim$ 160 Myrs and enters the LISA bandwidth (i.e. has an orbital period of less than 10^4 sec) on an almost circular orbit (for a more detailed description of the final stage of the evolution see [18]). For the work presented here we performed also a parallel test simulation including the possibility that stars are tidally disrupted by the IMBHs binary if they enter the tidal disruption radius of one of the IMBHs but these effects are almost insignificant in terms of the description of the parameter space of the IMBH binary and thus negligible for the main purpose of this study. In any case, the "final" eccentricity of the binary (the eccentricity it achieves by entering the LISA band) is almost zero; i.e. the binary has almost totally circularised. We can expect an event rate of between four and five of these events per year for LISA [18]. As for the detectability of the gravitational waves emitted by the IMBH binary as it inspirals and merges, in [18] we show the amplitude ratio assuming a coherent integration (matched filtering) over the observation time. For both observatories, LISA and the BBO, only the n = 2 harmonic of the quadrupolar emission is clearly detectable during the last few years of inspiral at 1 Gpc, since it is dominant for low eccentricity binaries; some higher (odd) harmonics depend on the mass ratio and are zero for equal mass (for zero eccentricity orbits). The eccentricity is so low (10^{-3}) that the source should be closer than $\sim 50-100$ Mpc to be detectable.

For the data analysis of these sources, it is very important to know the phase of the gravitational wave; a missmatch even in one single cycle can reduce the signal to noise ratio (SNR). In practice, since we do not know the parameters of the gravitational wave signal, we will search over a large parameter space (including the masses of the objects, the eccentricity etc) and the template with largest SNR will be chosen. The highest SNR will be achieved by resorting to templates with parameters close to the actual ones. However, there are some correlations between the parameters, so that one can compute the level of correlation and the error bar on the parameter determination. For strong signals with long duration within the detector band, we need to know the waveform (more specifically the phase) very accurately. The eccentricity affects the phase as well as the amplitude. Since the analysis is very sensitive to even a tiny change in phase, we can expect that one will be able to subtract the necessary information of the waveforms of the event to identify it, due to the induced detectable phase shift of the residual eccentricity (Bernard Schutz personal communication).

The space-borne LISA mission will fly in $\sim$ 10 yrs and critical design choices affecting the ability to detect this kind of events will have to be made soon. It is of big importance to produce robust estimates for the rates and typical orbital parameters of these and other events interesting for the detection of gravitational waves (extreme mass-ratio inspirals, for instance) in order to develop a detection template family. This detection template family consists of a bank of waveforms. It is relatively fast to generate waveforms depending on the initial conditions and mass ratio, but not fast enough if one wants to have a bank of them. If we want to rigorously explore the parameter space of these events, we need realistic Astrophysical estimates of the eccentricity, mass ratio etc at the beginning of the final merger, “one step before” the objects enter the LISA band. An assumption for the initial parameter space is necessary in order to develop waveform banks. For instance, in the case of an inspiral search, we know that majority -if not all- of binaries entering the bandwidth of ground-based detectors like LIGO will be circular. This allows us to significantly simplify the study of the parameter space. Similarly, in the case of space-born detectors like LISA, we need to better understand what we will observe and so focus our efforts on the part of the parameter space that from which we expect the largest contribution, even if our final aim is to cover the whole parameter space. For this, an Astrophysical understanding of the scenario is of paramount importance. The high-accuracy, Astrophysical numerical simulations performed with a direct-summation NBODY4 will shed light on many of these aspects by studying realistic astrophysical scenarios with relativistic corrections, which have been added recently, and arbitrary geometries [19, 11]. The biggest impact on data analysis is a realistic estimation of the event rate above a fixed SNR and possibly also as a function of the SNR itself. The models one can develop with these numerical tools have no precedents because of the inclusion of non-symmetry, rotation and relativistic corrections. A realistic estimation of the parameter space is crucial for the data analysis of LISA gravitational waves. A too small event rate and signals with smaller SNR will require a method similar to ground based gravitational waves analysis. This will imply the need for digging out of signals from the noise with only a few overlapping signals in the data stream. On the other hand, in the case of having a large even rate, many overlapped signals would probably require different data analysis algorithms to make a parameter estimation.

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