The minimally immersed 4D supermembrane

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In this note we summarize some of the properties found in [1], and its relation with [2]. We comment on the construction of the action of the 11D supermembrane with nontrivial central charges minimally immersed on a 7D toroidal manifold is obtained (MIM2).The transverse coordinates to the supermembrane are maps to a 4D Minkowski space-time. The action is invariant under additional symmetries in comparison to the supermembrane on a 11D Minkowski target space. The hamiltonian in the LCG is invariant under conformal transformations on the Riemann surface base manifold. The spectrum of the regularized hamiltonian is discrete with finite multiplicity. Its resolvent is compact. Susy is spontaneously broken, due to the topological central charge condition, to four supersymmetries in 4D, the vacuum belongs to an $N = 1$ supermultiplet. When assuming the target-space to be an isotropic 7-tori, the potential does not contain any flat direction, it is stable on the moduli space of parameters. Moreover due to the discrete symmetries of the hamiltonian, there are only 7 possible minimal holomorphic immersions of the MIM2 on the 7-torus. When these symmetries are identified on the target space, it corresponds to compactify the MIM2 on a orbifold with G2 structure. Once the singularities are resolved it leads to the compactification of the MIM2 on a G2 manifold as shown in [2].

1 Introduction

There have been interesting advances towards the quantization [3–9] of a sector of M-theory, the 11D minimally immersed supermembrane (MIM2), that may also provide tools to attack the more general problem of M-theory quantization. The goal to achieve is, departing from 11D, to obtain a consistent quantum theory in 4D with an $N = 1$ or $N = 0$ supersymmetries, moduli free, in agreement with the observed 4D physics. Attempts to formulate the theory in 4D have been done mainly in the supergravity approach [10,11], including fluxes [12, 13], but so far no exact formulation has been found. An effective theory when compactified to a 4D model contains many vacua due to the presence of moduli fields. Stabilizacion of these moduli is an important issue to be achieved. For some interesting proposals from M-theory see [12,14]. In the following we will consider the $11D$ M2-brane with irreducible wrapping on the compact sector of the target manifold [1–9] (MIM2). This implies the existence of a non trivial central charge in the supersymmetric algebra. Its spectral properties have been obtained in several papers [5–9]. The MIM2 contains the information about bound states of Dbranes, for example when compactified on 9D, it is equivalent to a bundle of D2-D0

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branes. The MIM2 also contains inside its spectrum nonperturbative string states like (F,Dp) branes and it has been proved in [15] to be the 11D origin of the SL(2,Z) multiplets of IIB theory and it may also be the 11D origin of the nonperturbative dyonic states of type IIA [15] that cannot be seen at perturbative level. This fact may be of relevance since Dp-branes are interesting since they allow to obtain nonabelian gauge groups and are able to reproduce semirealistic models of phenomenology and cosmology. Recently bound states of Dbranes have also enter into the game since they are able to capture nonperturbative effects that could explain very tiny effects inside MSSM in a natural way, i.e. smallness of neutrino masses or Yukawa couplings. The purpose of this note is to show the main aspect of the construction of the action for the supermembrane with nontrivial central charges compactified on a T^7 realized through all of the allowed holomorphic minimal immersions and analyze its physical properties.

2 $D = 11$ Supermembrane with central charges on a $M_5 \times T^6$ target manifold

The hamiltonian of the $D = 11$ Supermembrane [16] may be defined in terms of maps X^M, $M = 0, \ldots, 10$, from a base manifold $R \times \Sigma$, where Σ is a Riemann surface of genus g onto a target manifold which we will assume to be $11 - l$ Minkowski $\times$ l-dim Torus. The canonical reduced hamiltonian to the light-cone gauge has the expression

$$\int_\Sigma \sqrt{W} \left(\frac{1}{2} \left(\frac{P_M}{\sqrt{W}} \right)^2 + \frac{1}{4} \{X^M, X^N\}^2 + \text{ Fermionic terms} \right) \tag{1}$$

subject to the constraints

$$\phi_1 := d \left(\frac{p_M}{\sqrt{W}} dX^M \right) = 0 \tag{2}$$

and

$$\phi_2 := \oint_{C_s} \frac{P_M}{\sqrt{W}} \quad dX^M = 0, \tag{3}$$

where the range of M is now $M = 1, \ldots, 9$ corresponding to the transverse coordinates in the light-cone gauge, $C_s, s = 1, \ldots, 2g$ is a basis of 1-dimensional homology on Σ,

$$\{X^M, X^N\} = \frac{\epsilon^{ab}}{\sqrt{W(\sigma)}} \partial_a X^M \partial_b X^N. \tag{4}$$

$a, b = 1, 2$ and σ^a are local coordinates over Σ. $W(\sigma)$ is a scalar density introduced in the light-cone gauge fixing procedure. ϕ_1 and ϕ_2 are generators of area preserving diffeomorphisms. That is

$$\sigma \to \sigma' \quad \to \quad W'(\sigma) = W(\sigma).$$

The $SU(N)$ regularized model obtained from (1) [17] was shown to have continuous spectrum from $[0, \infty)$, [17–19]. In what follows we will impose a topological restriction on the configuration space. It characterizes a $D = 11$ supermembrane with non-trivial central charges generated by the wrapping on the compact sector of the target space [5–7, 9]. Following [8] we may extend the original construction on a $M_9 \times T^2$ to $M_7 \times T^4$, $M_5 \times T^6$ target manifolds by considering genus $1, 2, 3$ Riemann surfaces on the base respectively. We are interested in reducing the theory to a 4 dimensional model, we will then assume a target manifold $M_4 \times T^6 \times S^1$. The configuration maps satisfy:

$$\oint_{c_s} dX^r = 2\pi S_s^r R^r \quad r, s = 1, \ldots, 6, \quad \oint_{c_s} dX^m = 0 \quad m = 8, 9 \tag{5}$$

and

$$\oint_{c_s} dX^7 = 2\pi L_s R, \tag{6}$$

where S^r_s, L_s are integers and R^r, $r = 1, \ldots, 6$ are the radius of $T^6 = S^1 \times \cdots \times S^1$ while R is the radius of the remaining S^1 on the target. We now impose the central charge condition

$$I^{rs} \equiv \int_\Sigma dX^r \wedge dX^s = (2\pi R^r R^s)\omega^{rs} n \tag{7}$$

where ω^{rs} is a symplectic matrix on the T^6 sector of the target and n denotes an integer representing the irreducible winding. The topological condition (7) does not change the field equations of the hamiltonian (1). In addition to the field equations obtained from (1), the classical configurations must satisfy the condition (7). In the quantum theory, the space of physical configurations is also restricted by the condition (7) [3, 4].

We consider now the most general map satisfying condition (7):

$$dX^r = M^r_s d\widehat{X}^s + dA^r \tag{8}$$

where $d\widehat{X}^s$, $s = 1, \ldots, 2g$ is a basis of harmonic one-forms over Σ and impose the constraints (2),(3). It turns out that M^r_s can be expressed in terms of a matrix $S \in Sp(2g, Z)$, [1]. The natural election for $\sqrt{W(\sigma)}$ in this geometrical setting is define

$$\sqrt{W(\sigma)} = \frac{1}{2} \partial_a \widehat{X}^r \partial_b \widehat{X}^s \omega_{rs}. \tag{9}$$

$\sqrt{W(\sigma)}$ is then invariant under the change

$$d\widehat{X}^r \to S^r_s d\widehat{X}^s, \quad S \in Sp(2g, Z) \tag{10}$$

We thus conclude that the theory is invariant not only under the diffeomorphisms generated by ϕ_1 and ϕ_2 but also under the diffeomorphisms, biholomorphic maps, changing the canonical basis of homology by a modular transformation. The theory of supermembranes with central charges in the light cone gauge (LCG) we have constructed depends then on the moduli space of compact Riemanian surfaces M_g only. In addition when compactify in 9D there has been proved in [15] the hamiltonian is also invariant under a second SL(2,Z) symmetry associated to the T^2 target space that transform the Teichmuller parameter of the 2-torus.T^2.

3   Compactification on the remaining S^1

We will discuss two approaches for the analysis of the compactification on the remaining S^1. In the first case, we may solve the condition (6), we obtain

$$dX^7 = RL_s d\widehat{X}^s + d\widehat{\phi} \tag{11}$$

where $d\widehat{\phi}$ is an exact 1-form and $d\widehat{X}^s$ as before are a basis of harmonic 1-forms over Σ. We may analyze the contribution of the dX^7 field to the potential of the hamiltonian, we call it V_7. It is bounded from below

$$V_7 \geq \left\langle (\mathcal{D}_r \phi)^2 + \{X^m, X^7\}^2 \right\rangle \tag{12}$$

which directly shows that the winding corresponding to dX^7 does not affect the qualitative properties of the spectrum of the hamiltonian [8]. The inequality (12) will ensure that the discretness property of

the latest is also valid for the original complete hamiltonian. We will assume the dual formulation to the hamiltonian [1] when dX^7 is restricted by the condition (6) ensuring that X^7 takes values on S^1. We follow [21]. We notice that A_s is not a connection in a line bundle over Σ. In fact the condition

$$\int_\Sigma F_{ab} d\sigma^a \wedge d\sigma^b = 2\pi n \qquad (13)$$

is not necessarily satisfied. In order to have a connection on line bundle over Σ one should require a periodic euclidean time on the functional integral formulation. In that case the condition (6), ensures that $F_{\mu\nu}$ is the curvature of a one-form connection over the three dimensional base manifold. Under this assumption the condition (6) for any L_s implies summation over all $U(1)$ principle bundles.

The final expression of the dual formulation of the hamiltonian when X^7 is wrapped on a S^1, condition (6), is

$$H_d = \int \sqrt{w} d\sigma^1 \wedge d\sigma^2 \left[\frac{1}{2}\left(\frac{P_m}{\sqrt{W}}\right)^2 + \frac{1}{2}\left(\frac{\Pi^r}{\sqrt{W}}\right)^2 + \frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{2}(\mathcal{D}_r X^m)^2 \right.$$

$$\left. + \frac{1}{4}(\mathcal{F}_{rs})^2 + \frac{1}{2}(F_{ab}\frac{\epsilon^{ab}}{\sqrt{W}})^2 + \frac{1}{8}(\frac{\Pi^c}{\sqrt{W}}\partial_c X^m)^2 + \frac{1}{8}[\Pi^c\partial_c(\widehat{X}_r + A_r)]^2 \right]$$

$$+ \Lambda\left(\left\{\frac{P_m}{\sqrt{W}}, X^m\right\} - \mathcal{D}_r\Pi^r - \frac{1}{2}\Pi^c\partial_c\left(F_{ab}\frac{\epsilon^{ab}}{\sqrt{W}}\right)\right) + \lambda\partial_c\Pi^c] + susy$$

where $\mathcal{D}_r X^m = D_r X^m + \{A_r, X^m\}$, $\mathcal{F}_{rs} = D_r A_s - D_s A_r + \{A_r, A_s\}$,
$D_r = 2\pi R^r \frac{\epsilon^{ab}}{\sqrt{W}}\partial_a \widehat{X}^r \partial_b$ and P_m and Π_r are the conjugate momenta to X^m and A_r respectively. $\mathcal{D}_r$ and $\mathcal{F}_{rs}$ are the covariant derivative and curvature of a symplectic noncommutative theory [4, 6], constructed from the symplectic structure $\frac{\epsilon^{ab}}{\sqrt{W}}$ introduced by the central charge. The integral of the curvature we take it to be constant and the volume term corresponds to the value of the hamiltonian at its ground state. The physical degrees of the theory are the X^m, A_r, X_7 together with its supersymmetric extension. They are single valued fields on Σ.

4 $N = 1$ supersymmetry

The topological condition associated to the central charge determines an holomorphic minimal immersion from the g-Riemann surface to the 2g-torus target manifold. This minimal immersion is directly related to the BPS state that minimizes the hamiltonian. When we start with the $g = 1$ and T^2 on the target space the ground state preserves $\frac{1}{2}$ of the original supersymmetry with parameter a 32-component Majorana spinor. When we consider our construction for a $g = 2, 3$ and T^4, T^6 torus on the target, the analysis of the SUSY preservation becomes exactly the same as when considering orthogonal intersection of 2-branes with the time direction as the intersecting direction [22]. The SUSY of the ground state preserves $\frac{1}{4}, \frac{1}{8}$ of the original SUSY. The preservation of the ground state implies the breaking of the supersymmetry. In the light cone gauge, we end up, when $g = 3$, with $\frac{1}{8}$ of the original SUSY, that is one complex grassmann parameter corresponding to a $N = 1$ light-cone SUSY multiplet. The action is invariant under the whole light-cone SUSY. However when the vacuum is spontaneously fixed to one of them, the SUSY is broken at the quantum level up to $N = 1$ when the target is $M_5 \times T^6$. There is no further breaking when we compactify the additional S^1, to have a target $M_4 \times T^6 \times S^1$.

5   Discretness of the spectrum

We consider a gauge fixing procedure on a BFV formulation of the theory. We consider, in the usual way, a decomposition of all scalar fields over Σ in terms of an orthonormal discret basis $Y_A(\sigma^1, \sigma^2)$ and

$$\int_\Sigma \{Y_A, Y_B\} Y_C = f_{ABC},$$

f_{ABC} is consequently completely antisymmetric. We then replace those expressions into the hamiltonian density and integrate the σ^1, σ^2 dependence. We obtain then a formulation of the operator in terms of the τ dependent modes only. We now consider a truncation of the operator, that is we restrict the range of the indices A, B, C to a finite set N and introduce constants $f_{AB}^N \ {}^C$ such that

$$\lim_{N \to \infty} f_{AB}^N \ {}^C = f_{AB}^C \tag{14}$$

$f_{AB}^N \ {}^C$ are the structure constants of $SU(N)$. In [7] the truncated supermembrane with central charges compactified on a T^2 was shown to have a $SU(N)$ a gauge symmetry. The algebra of first class constraints is isomorphic to the algebra of a $SU(N)$ gauge theory. We proceed to the analysis of the spectrum of the truncated Schröedinger operator associated to $\widehat{H}$ without further requirements on the constants f^N:

i) The potential of the Schröedinger operator only vanishes at the origin of the configuration space,
ii) There exists a constant $M > 0$ such that

$$V(X, A, \phi) \geq M ||(X, A, \phi))||^2 \tag{15}$$

The Schröedinger operator is then bounded by an harmonic oscillator. Consequently it has a compact resolvent. We now use Theorem 2 [23] to show that: i) The ghost and antighost contributions to the effective action assuming a gauge fixing condition linear on the configuration variables, ii) the fermionic contribution to the susy hamiltonian, do not change the qualitative properties of the spectrum of the hamiltonian. The regularized hamiltonian compactified on the target space $M_4 \times T^6 \times S^1$ has then a compact resolvent and hence a discrete spectrum with finite multiplicity. We expect the same result to be valid for the exact theory.

6   Physical properties

Another of the characteristics of the theory is that due to the topological condition the fields acquire mass. In here, the fields of the theory X^m, A_r, ϕ acquire mass via the vector fields $\widehat{X}_r$ defined on the supermembrane. There is no violation of Lorentz invariance. It is important to point out that the number of degrees of freedom in 11D and in 4D is preserved, but just redistributed. This fact has the a advantage for many phenomenological purposes of mantaining the number of fields small. At classical level, generically the analysis of moduli fields have been performed in a supergravity approach [12–14]. Since our approach is exact these terms do not appear, however the action posseses scalars that may lead to flat directions in the potential. We are going to analyze the two types of classical moduli. This decoupling approach is only justified iff the scales of stabilization (the masses of the moduli) are clearly different, otherwise the minimization with respect to the whole set of moduli (geometrical and of matter origin) should be performed. The theory does not contain any string-like configuration, this is because the scalar fields parametrizing the position of the supermmbrane gets all mass, so these type of moduli gets fixed. With respect to the geometrical moduli parametrizing the manifold if we assume that we compactify on a 7 isotropic tori it can be rigurosuly proved that all of the moduli gets fixed. An heuristical argument to understand better this effect of moduli stabilization is the following: We are dealing in our construction with nontrivial gauge bundles that can be represented as worldvolume fluxes [25]. Since for construction the mapping represent a minimal immersion on the target space they induce a similar effect that the one induced by the generalized

calibration, that is, there is an associated flux effect on the target space. Minimal immersions take also into account the dependence on the base manifold, the Rieman surface chosen Σ. The condition of the generalized calibration -which shows the deformation of the cycles that are wrapped by the supermembrane-represent a condition for minimizing the energy [26]. It happens the same with the minimal inmersions. For a given induced flux, one may expect the volume to be fixed [27].

7   Conclusion

We obtained the action of the $D = 11$ supermembrane compactified on $T^6 \times S^1$ with nontrivial central charge induced by a topological condition invariant under supersymmetric and kappa symmetry transformations. The hamiltonian in the LCG is invariant under conformal transformations on the Riemann surface base manifold. The susy is spontaneously broken, by the vacuum to $1/8$ of the original one. It corresponds in 4D to a $N = 1$ multiplet. Classicaly the hamiltonian does not contain singular configurations and at the quantum level the regularized hamiltonian has a discrete spectrum, with finite multiplicity. Its resolvent is compact. The potential does not contain any flat direction on configuration space nor on the moduli space of parameters. The hamiltonian is stable on both spaces. It is stable as a Schrodinger operator on configuration space and it is structurable stable on the moduli space of parameters. The discrete symmetries of the theory restrict the allowed minimal immersions to those corresponding to an orbifold with G2 structure. When the symmetries are identified on the target space they lead to a compactification on a true G2 manifold [2].

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