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On the hypothetical utilization of atmospheric potential energy

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Abstract

Atmospheric potential energy is typically divided into an available and a nonavailable part. In this article a hypothetical utilization of a fraction of the nonavailable potential energy is described. This part stems from the water vapor that can be converted into the liquid phase. An energy gain results when the potential energy of the condensate relative to a reference height exceeds the energy necessary to condensate the water vapor. It is shown that this can be the case in a saturated atmosphere without convective available potential energy. Finally, simulations with the numerical cloud model HURMOD are performed to estimate the usability of the device in practice. Indeed, a positive energy output results in a simulation with immediate gathering of the condensate. On the contrary, potential energy gained falls significantly short of the necessary energy for forming the condensate when a realistic cloud microphysical scheme allowing re-evaporation of condensate is applied. Taken together it can be concluded that, a utilization of atmospheric potential energy is hypothetically possible but the practical realization is probably not feasible.

Keywords: available potential energy, atmospheric thermodynamics, cloud physics

1 Introduction

The sum of potential and inner energies of the atmosphere represents the so-called total potential energy (Lorenz, 1955) the quantity of which is huge when compared to the kinetic energy of the atmosphere. However, only a small fraction is thought to be available for conversion into other energy forms. This part is called available potential energy (APE) of the atmosphere. Lorenz (1955) derived an expression for available potential energy, valid only for a stably stratified atmosphere. This is appropriate in the context of the atmospheric general circulation, since only in small areas convective instability exists. On smaller scales more of the total potential energy might be available for conversion by convection. This available energy form was introduced by Moncrieff and Miller (1976) and is called convective available potential energy (CAPE). The remaining, still enormous quantity of total potential energy is believed to be unavailable for conversion.

In this article we examine a theoretical possibility to convert parts of the unavailable potential energy into usable energy by a hypothetical ideal engine. This engine liquefies water vapor by lifting air to a certain height above a reference level where the liquid water is collected in a basin. Then, the potential energy of the liquid water is available for conversion into energy that eventually drives the liquefaction engine (Fig. 1). The processes by which water reaches the reference level again after liquefaction, are not discussed in this study but eventually the liquidized water evaporates and diffuses upward to the reference level. The conversion of potential energy into electric energy can be realized in an efficient way as described by Demirel (2012). Then, the calculations show that the engine can indeed continually utilize non-available potential energy under certain conditions.

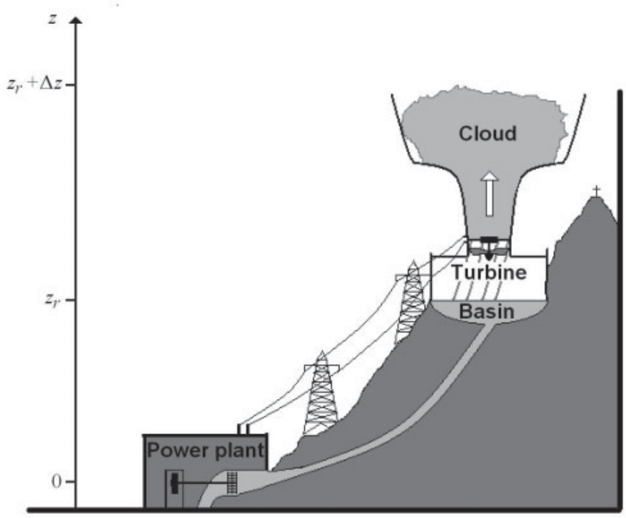

Figure 1: Sketch of the hypothetical liquefaction engine.

The rest of the paper is organized as follows. Section 2 includes a theoretical derivation of the potential efficiency of the engine. Section 3 presents numerical simulations with a cloud model to obtain a more realis-

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tic estimation of the engine's efficiency. The final section contains the conclusions.

2 Theoretical foundations

In the theoretical treatment we assume a horizontally uniform, motionless atmosphere that possesses no APE and no CAPE. The relative humidity of this atmosphere is assumed to be 100 % in order to have optimal conditions to drive the engine.

Since the atmosphere is motionless the pressure p can be determined by the hydrostatic balance equation

$$\frac{\partial p}{\partial z} = -g\rho\,, \tag{2.1}$$

where z denotes the height above the hypothetical power plant, g the acceleration due to gravity and ρ the mass density of the saturated mixture of moist air. By assuming that water vapor and dry air form an ideal mixture the equation of state becomes

$$\rho = \frac{p}{R_d T(1+(R_v/R_d-1)m_v^*)}\,, \tag{2.2}$$

where R_d (R_v) is the specific gas constant for dry air (water vapor), T the temperature and m_v^* the mass fraction of water vapor at saturation. Dalton's law allows us to rewrite the defining relation of the mass fraction of water vapor as follows:

$$m_v^* = \frac{R_d}{R_v}\frac{e^*(T)}{p-(1-R_d/R_v)e^*(T)}\,. \tag{2.3}$$

The water vapor pressure e^* is a function of temperature and it is solution of the approximated Clausius-Clapeyron equation

$$\frac{de^*}{dT} = \frac{L_v e^*}{R_v T^2}\,, \tag{2.4}$$

where L_v is the latent heat of vaporization. Solving this differential equation by neglecting the temperature dependence of L_v gives

$$e^*(T) = e^*(T_0)\exp\left(\frac{L_v}{R_v T_0}\frac{T-T_0}{T}\right)\,. \tag{2.5}$$

where T_0 is a reference temperature. In the subsequent calculations the values for reference temperature and reference saturation pressure are $T_0 = 273.15\,\mathrm{K}$ and $e^*(T_0) = 610.78\,\mathrm{Pa}$, respectively. For simplicity it is assumed that the virtual temperature $T_v := T[1+(R_v/R_d-1)m_v^*]$ decreases linearly with height so that

$$T_v = T_{vr} - \Gamma(z-z_r)\,, \tag{2.6}$$

where Γ denotes the lapse rate and T_{vr} the virtual temperature at the reference level z_r. We assume that the lapse rate is small enough to have a statically stable state so that no CAPE exists. By vertical integration of the hydrostatic balance equation (2.1) we get the pressure as a function of height

$$p(z) = p(z_r)\left[1-\frac{\Gamma(z-z_r)}{T_{vr}}\right]^{\frac{g}{R_d\Gamma}}\,. \tag{2.7}$$

The principal idea is now to drive the engine so as to lift the air from the starting level $z = z_r$ to $z = z_r + \Delta z$ and to use the potential energy of the condensed liquid water, if only we could collect it by some clever means. Work must be supplied to lift a parcel of air in a stably stratified atmosphere. The necessary minimum work can be estimated from the inviscid vertical momentum equation

$$\frac{Dw}{Dt} = -\frac{1}{\rho_p}\frac{\partial p_p}{\partial z} - g + F\,, \tag{2.8}$$

where the index p refers to the lifted air parcel, and F denotes the lifting force of the engine, w the vertical velocity of the parcel, and D/Dt the Lagrangian time derivative. With the perturbation pressure $p_p' = p_p - p$ Eq. (2.8) can be written as

$$\frac{Dw}{Dt} = -\frac{1}{\rho_p}\frac{\partial p_p'}{\partial z} - g\frac{\rho_p-\rho}{\rho_p} + F\,. \tag{2.9}$$

For slow lifting it is possible to neglect both the perturbation pressure and the vertical velocity. Then, the lifting work W to be supplied by the engine to lift the parcel from $z = z_r$ to $z = z_r + \Delta z$ becomes

$$W = \int_{z_r}^{z_r+\Delta z} g\frac{\rho_p-\rho}{\rho_p}dz = \int_{z_r}^{z_r+\Delta z} g\frac{T_v-T_{vp}}{T_v}dz\,. \tag{2.10}$$

This would be identical to the expression for convective inhibition energy (CIN) if the upper level were the level of zero buoyancy. To calculate the work we must determine the parcel temperature. In this analysis we make use of the pseudoadiabatic assumption, that is, the ascending parcel does not exchange heat with the environment and the condensed water is immediately removed by instantaneous precipitation. Then, the parcel temperature T_p results from the following adiabatic form of the first law of thermodynamics

$$c_p dT_p = \frac{1}{\rho_p}dp - L_v dm_{vp}^*\,, \tag{2.11}$$

where c_p is the specific heat capacity of moist air at constant pressure. By using the Clausius-Clapeyron equation (2.4) we obtain

$$\left(c_p + \frac{L_v^2 e_p^*}{R_v T_p^2}\frac{\partial m_{vp}^*}{\partial e^*}\right)dT_p = \left(\frac{R_d T_{vp}}{p} - L_v\frac{\partial m_{vp}^*}{\partial p}\right)dp\,. \tag{2.12}$$

The pressure differential can be evaluated with the hydrostatic balance equation (2.1) so that the vertical parcel temperature gradient becomes

$$\frac{dT_p}{dz} = -\frac{g}{c_p}\frac{T_{vp}}{T_v}\frac{1+\frac{L_v p}{R_d T_{vp}}\frac{m^*_{vp}}{p-(1-R_d/R_v)e^*_p}}{1+\frac{L_v^2}{c_p R_v T_p^2}\frac{m^*_{vp} p}{p-(1-R_d/R_v)e^*_p}}. \quad (2.13)$$

From this result we can deduce the vertical gradient of the parcel's virtual temperature

$$\frac{dT_{vp}}{dz} = -\Gamma_m = \frac{dT_p}{dz}\left\{1+\left(\frac{R_v}{R_d}-1\right)\times\right. \quad (2.14)$$
$$\left.\times\left[m^*_{vp}+\frac{L_v}{R_v T_p}\frac{p m^*_{vp}}{p-(1-R_d/Rv)e^*_p}\right]\right\}$$
$$+\frac{gT_p}{R_d T_v}\left(\frac{R_v}{R_d}-1\right)\frac{m^*_{vp} p}{p-(1-R_d/R_v)e^*_p},$$

where Γ_m is the pseudoadiabatic lapse rate of virtual temperature. The lapse rate cannot be calculated analytically. Therefore, it must be determined by a numerical iteration scheme. However, at the starting level $z = z_r$ the environmental temperature can be used in Eqs. (2.13) and (2.14) so that an analytical evaluation becomes possible there. For a small lifting distance we can assume a constant lapse rate to approximate the lifting work by

$$W \approx \int_0^{\Delta z} g\frac{(\Gamma_{mr}-\Gamma)z'}{T_{vr}-\Gamma z'}dz' \quad (2.15)$$
$$= -\frac{g(\Gamma_{mr}-\Gamma)}{\Gamma}\left[\Delta z+\frac{T_{vr}}{\Gamma}\ln\left(1-\frac{\Gamma}{T_{vr}}\Delta z\right)\right],$$

where $z' = z - z_r$ and the subindex r denotes evaluation at $z = z_r$. The purpose of the machine is to gain more potential energy by condensed liquid water than the invested lifting work. The amount of condensed water can be calculated by assuming that the lifted parcel conserves a relative humidity of 100 %. Then, the specific amount of condensed water becomes

$$-\Delta m_{vp} = -\Delta m^*_{vp} = m^*_{vp}(z_r) - m^*_{vp}(z_r+\Delta z). \quad (2.16)$$

The specific saturation humidity at $z = z_r + \Delta z$ can be evaluated analytically by using the approximation $T_p(z_r+\Delta z) \approx T(z_r) - \Gamma_{mr}\Delta z$. For the specific value of gained potential energy P we get

$$P = gz_r\left[m^*_{vp}(z_r) - m^*_{vp}(z_r+\Delta z)\right], \quad (2.17)$$

if all condensed water could be gathered in a basin at $z = z_r$.

For an energy surplus the engine must fulfill the criterion $P > W$ leading to the following inequality

$$z_r > -\frac{(\Gamma_{mr}-\Gamma)\left[\Delta z+\frac{T_{vr}}{\Gamma}\ln\left(1-\frac{\Gamma}{T_{vr}}\Delta z\right)\right]}{\Gamma\left[m^*_{vp}(z_r)-m^*_{vp}(z_r+\Delta z)\right]}. \quad (2.18)$$

The inequality holds in the limit $\Delta z \to 0$ for arbitrarily small z_r. However, we cannot expect that enough usable condensate is produced for a small lifting distance Δz.

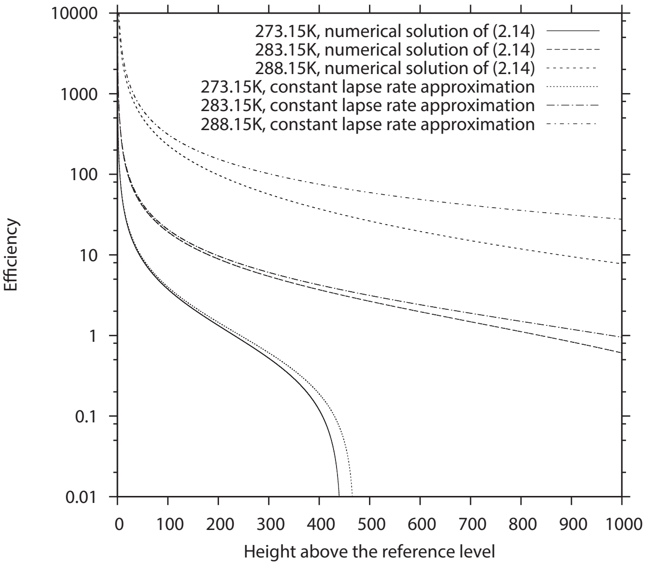

Figure 2: Efficiency of the liquefaction engine as a function of the lifting distance for different virtual temperatures at the reference level $z_r = 1000$ m. The figure also displays the efficiencies for the approximated analytical estimation.

Increasing Δz leads to an increasing value for the minimum reference height of the device. The maximum possible energy conversion efficiency of the engine we define as

$$\eta = \frac{P-W}{W} \quad (2.19)$$
$$= \frac{\Gamma z_r\left[m^*_{vp}(z_r)-m^*_{vp}(z_r+\Delta z)\right]}{(\Gamma_{mr}-\Gamma)\left[\Delta z+\frac{T_{vr}}{\Gamma}\ln\left(1-\frac{\Gamma}{T_{vr}}\Delta z\right)\right]} - 1.$$

It is also possible to deduce the efficiency η without the approximation (2.15) by a numerical integration of the differential equation (2.13). Then, the parcel temperature profile follows a pseudoadiabat.

To judge whether the liquefaction engine could potentially work we choose the realistic temperature values $T_{vr} = 273.15$ K, 283.15 K and 288.15 K as well as a lapse rate of $\Gamma = 0.05$ K/100 m. Fig. 2 displays the efficiency as a function of Δz for a reference level at $z_r = 1000$ m. The figure reveals a significant increase for a decreasing lifting distance and increasing temperatures. It can also be seen that the approximation overestimates efficiency, the more so for large lifting distances.

3 Model simulations

The theoretical treatment in section 2 employs assumptions that are not valid in the real atmosphere. One assumption concerns the neglect of turbulent exchange. This would lead to an underestimation of the work by neglecting turbulent momentum diffusion, and an overestimation of condensation by omission of turbulent moisture diffusion. Another assumption is that all condensed water could be utilized in the engine. In reality, droplets do not gather as bulk water and only large droplets fall down to the surface while the remaining liquid water moves away with the flow and eventually evaporates again. Therefore, the efficiency of the hypothetical engine will be much smaller or even negative due to these effects.

The consideration of turbulent exchange and rain formation requires the use of a hydrodynamical cloud model. For this purpose we adopt the mass-consistent cloud model HURMOD which is described in detail by Frisius and Wacker (2007) and Frisius and Hasselbeck (2009). It solves the axisymmetric Reynolds-averaged Euler equations in which turbulent exchange is parameterized with a flux gradient closure following Lilly (1962). The parameterization of cloud microphysics is based on the scheme by Kessler (1969) in which the ice phase is ignored. This cloud microphysical scheme is rather simple but it can provide a rough estimation of the obstacles for rain formation.

A cylinder with a height of $H_C = 2500\,\text{m}$ and a radius of $R_C = 1250\,\text{m}$ radius defines the model domain. It has 100 height and 50 radius levels so that the gridpoint distance becomes 25 m. The cylinder has insulating free slip boundaries except for the lower boundary where the transfer coefficients for momentum, moisture and heat are $C_T = 0.0011$. The turbulent exchange coefficient K is determined by the relation

$$K = l_0^2 S\sqrt{1-\text{Ri}}, \qquad (3.1)$$

where $l_0 = 12.5\,\text{m}$ is the length scale parameter of the turbulent eddies, S the deformation of the flow and Ri the Richardson number (for more details see Frisius and Wacker (2007)). The initial surface pressure is $p_s = 880\,\text{hPa}$ which is a typical value at a height of 1000 m. The surface temperature amounts to $T_s = 288.15\,\text{K}$. The initial stratification of the saturated atmosphere is stable with a constant lapse rate of $\Gamma = 0.0045\,\text{K/m}$. All other model parameters are chosen as in Frisius and Hasselbeck (2009).

The hypothetical engine accelerates the air vertically at the center of the domain. For this purpose we include an additional vertical force per unit mass of the form

$$f_E(r,z) = f_0 \exp\left(-\frac{r^2}{\Delta r^2}\right)\exp\left(-\frac{(z-z_0)^2}{\Delta z^2}\right), \qquad (3.2)$$

where $f_0 = 0.05\,\text{m/s}^2$, $\Delta r = 200\,\text{m}$, $z_0 = 50\,\text{m}$, $\Delta z = 30\,\text{m}$ and r denotes the distance to the center. With this additional force per unit mass the invested power of the engine becomes

$$P_W = 2\pi\int_0^{H_C}\int_0^{R_C}\rho f_E w r\,dr\,dz. \qquad (3.3)$$

The potential energy gain per time unit on the other hand results from the precipitation rate at the surface

$$P_P = 2\pi g z_r\int_0^{R_C}\rho m_r W_r r\,dr, \qquad (3.4)$$

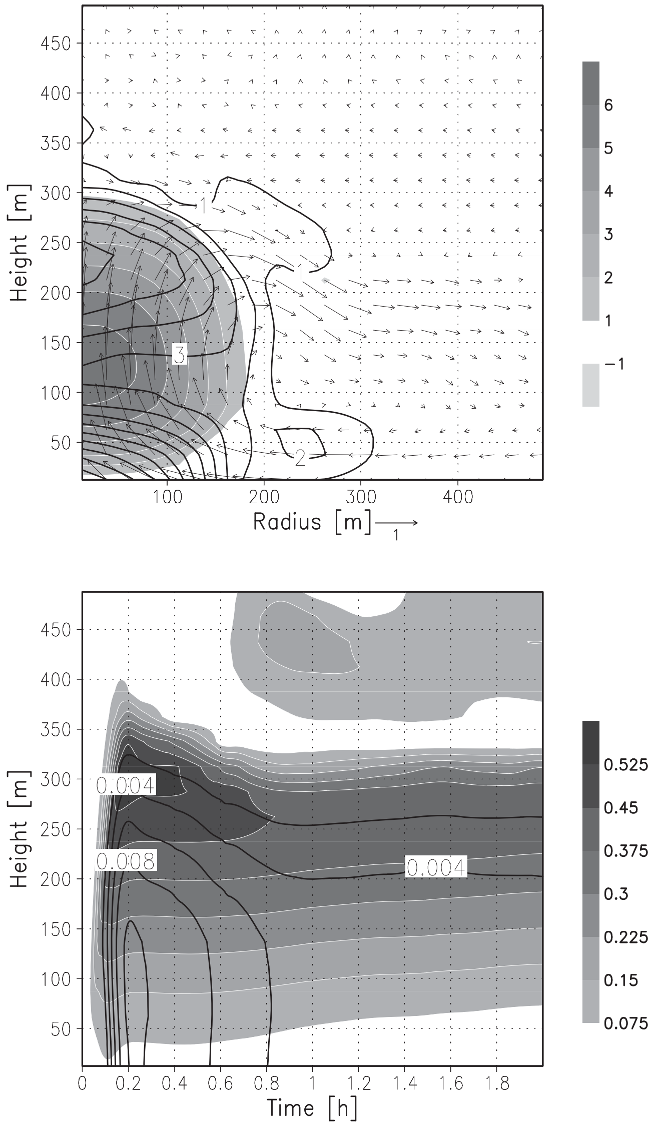

Figure 3: Upper panel: latent heating rate (shadings, W/kg), wind vectors and exchange coefficient for turbulence (contour interval $1\,\text{m}^2$) as a function of radius and height at $t = 0.5\,\text{h}$. Lower panel: mass fraction of nonprecipitating liquid water (shadings, g/kg) and mass fraction of rain (contour interval 0.002 g/kg) at $r = 0$ as a function of time and height.

where m_r is the mass fraction of rain and W_r the sedimentation velocity of rain. We assume $z_r = 1000\,\text{m}$.

Fig. 3 displays the results of this experiment. After half an hour a marked radial overturning circulation has developed (upper panel of Fig. 3). In the updraft region condensation takes place that leads to the displayed latent heat release. The circulation is accompanied by turbulence that can be seen as contours of the turbulent exchange coefficient with values up to $10\,\text{m}^2/\text{s}$. The time development of rain (lower panel of Fig. 3) reveals very inefficient formation of precipitation. Only up to $0.01\,\text{g/kg}$ rain occurs at the surface. The nonprecipitating liquid water content is much larger which eventually diffuses radially by turbulence. The secondary liquid water maximum seen at a height of $z = 430\,\text{m}$ results

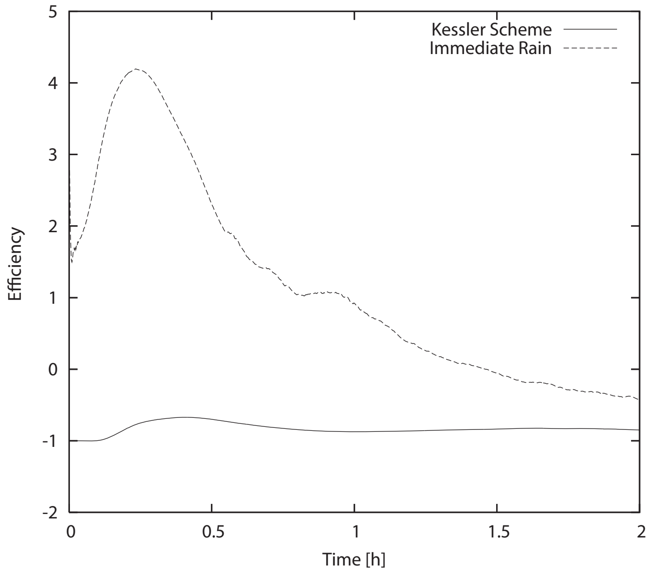

Figure 4: Efficiency evolution of the liquefaction engine as simulated by HURMOD. The solid line displays the result of the experiment based on the Kessler microphysical scheme while the dashed line shows the results of the experiment in which all condensed water immediately falls out.

from an excited gravity wave. Due to the low amount of rain formed we expect that the efficiency of the liquefaction engine is negative. This is indeed the case as can be seen in Fig. 4 that displays the efficiency as a function of time. It is negative during the complete time integration period of two hours. However, we may wonder what efficiency would result if we hypothetically assume that the device is able to collect all condensed liquid water. For this purpose a second experiment has been performed in which all condensed water immediately falls down into the basin. The dashed curve in Fig. 4 displays the efficiency that results in this additional experiment. Now, efficiencies up to 4 come up. Finally, however, the efficiency becomes negative because more and more dry air is entrained into the updraft. What these experiments suggest is that we could draw non-available potential energy from the atmosphere if indeed it would be possible to collect condensed liquid water. An engine that is entirely driven by naturally formed rain, on the other hand, seems to be rather difficult to realize.

4 Conclusion

In this article we discussed a hypothetical possibility of extracting non-available potential energy from the atmosphere by condensation of water vapor. The utilization of this energy is performed by a hypothetical device that uses the potential energy of the condensed liquid water with a generator that drives an engine to accomplish the required lifting work. The analytical and numerical calculations reveal substantial efficiencies under highly idealized conditions. Numerical model simulations by the cloud model HURMOD show, on the other hand, that the generation of rain is too inefficient for a real operation of this liquefaction engine. Positive efficiencies result only when it is possible to collect all condensed liquid water immediately.

One may ask if the liquefaction engine could potentially work under conditions of thermodynamic equilibrium. Then, the second law of thermodynamics appears questionable. A try to tackle this issue was made by considering an isothermal atmosphere with a vanishing vertical gradient of water vapor and saturation at the surface ($z = 0$). The results hint at the impossibility of drawing off mechanical energy in this situation but it cannot be substantiated with a mathematical proof. Furthermore, it has not been presumed that the device can work in a periodic way without changing the system. Therefore, a conversion of potential energy into other energy forms is not necessarily in contradiction with the second law even if the atmosphere appears to be at thermodynamic equilibrium.

Even if we have our doubts that the device discussed could operate in reality, we do not want to discard the possibility entirely. Our main aim is rather to present a thought experiment to make the point that there is more potential energy available than might be thought when reasoning in terms of global circulations.

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