Operational Inefficiencies in Supply Chains: Performance, Coordination and the Role of Information Systems

by
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Abstract

ZEYNEP ALMULA ÇAMDERELI: Operational Inefficiencies in Supply Chains: Performance, Coordination and the Role of Information Systems.
(Under the direction of Jayashankar M. Swaminathan.)

Misplaced inventory is a major operational inefficiency in supply chains and it happens when items are not in their designated places and hard to find in a store. In this thesis, we focus on misplaced inventory and inventory shrinkage, which is a special case of inventory misplacement. Information technology is facilitating significant reductions in these operational inefficiencies. However, such technologies are generally very expensive to implement. This dissertation intends to fill the gap of knowledge about the effects of operational inefficiencies in supply chains of various forms and the role of information systems in mitigating such inefficiencies that often get reflected in poor performance. We do this through three separate essays using analytical research methods.

In the first essay, "Coordination of a Supply Chain under Misplaced Inventory", we consider a supply chain where a proportion of the items ordered become unavailable for sale due to misplacement at the retailer. We investigate the effects of increasing the proportion of inventory availability at the retailer on the profits of the retailer and the manufacturer in vertically integrated and decentralized supply chains where we study uncoordinated and coordinated supply chains.

In the second essay, "Misplaced Inventory and RFID: Information and Coordination", we consider Radio Frequency Identification (RFID) technology to mitigate the negative effects of inventory misplacement. We consider both fixed and variable costs for RFID implementation

and assume that the fixed cost is shared between the two using an arbitrary proportion while the variable cost could be charged to either the retailer or the manufacturer. We characterize the incentives of the parties in the supply chain to invest in RFID in vertically integrated, uncoordinated and coordinated supply chains.

In the third essay, "Inventory Management under Shrinkage and Customer-Driven Search", we consider a supply chain with a single supplier and two retailers under customer search experiencing shrinkage at the retailers, who face nonidentical random demands. Through a single-period model, we explore the effects of shrinkage and competition on the equilibrium stocking decisions and characterize conditions for the parties to benefit from a unilateral reduction in shrinkage.

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Chapter 1

Introduction and Motivation

Operational inefficiencies are prevalent in many industries. An empirical study by Raman, DeHoratius and Ton (2001) show that inventory inaccuracy and inventory misplacement are two major operational inefficiencies in the retail stores. Inventory inaccuracy is defined as a mismatch between the physical inventory in the store and the system record. Transaction error is one reason for inventory inaccuracy to occur. This may happen, for example when a case of diet coke is scanned as regular coke. In such a case, the system will indicate that the store has one more case of regular coke than the store has but, in reality, the store has one more case of diet coke than the system indicates. Hence, the inventory inaccuracies due to transaction errors lead to errors in both positive and negative directions and the amount of physical inventory can be more than the system record. In this thesis, we focus on misplaced inventory cases where the amount of physical inventory available to customers is always less than the system record. Inventory misplacement happens when items are not in their designated places and hard to find in a store; therefore, they are not available for customers. However, one can retrieve them back once the store performs an inventory clean-up. Information technology is enabling reduction in such operational inefficiencies in a significant manner. However, they often are very expensive to implement. This dissertation intends to fill the gap of knowledge about the effects of operational inefficiencies, in particular misplaced inventory, in supply chains of various forms and the role of information systems in mitigating such inefficiencies that often get reflected in poor performance.

The empirical work by DeHoratius and Ton (2001) reveals that inventory misplacement is an important operational inefficiency and the rate of misplacement could be as high as 16% in some cases. Items can be misplaced in storage areas as well as in stores. Inventory misplacement is one of the major causes of stockouts. It was reported that "about 35 percent of store out-of-stocks are caused by physical inventory misplacement" (http://www.privatelabelbuyer.com/). Not only in retailers, this problem is prominent in other settings as well. For example, the U.S. Navy lost more than $3 billion worth of inventories between 1996 and 1998 (McCutcheon, 1999). Inventory misplacement can be expensive even for organizations operating under a small rate of misplacement. For example, Delta Airlines has a high baggage-handling efficiency rate of 99%. However, the 1% of misplaced bags costed Delta about $100 million a year.

The first chapter of this dissertation titled "Coordination of a Supply Chain under Misplaced Inventory" focuses on a single period analysis and is applicable for settings such as inventory in toy stores during weeks prior to Christmas where the items might be misplaced during the season and are retrieved at the end of it. We study two forms of supply chains under misplaced inventory: a vertically integrated supply chain and a decentralized supply chain. Even the vertically integrated case itself is interesting at an operational level. For example, we prove that (counter to intuition) it is not always optimal to order more than before if the firm misplaces more products, i.e. experiences a decrease in her inventory availability level. We show that in the optimum, the firm orders less as more products are misplaced if the products are not attractive to stock. Even if the products are attractive to stock, we prove that the firm still orders less as more products are misplaced if the inventory availability level before further misplacement was already low. This is due to the effective cost of acquiring one more unit becoming too high for the firm, so she prefers to produce less as inventory availability level decreases. Next, we study bilaterally decentralized supply chains under misplaced inventory in two main forms: uncoordinated supply chains where we allow for gaming between the parties and coordinated supply chains. Building on the contracts developed earlier by researchers (Pasternack, 1985; Cachon and Lariviere, 2005), we provide the conditions under which the supply chain under misplaced inventory will be coordinated via buy-back and revenue sharing mechanisms. Contrary to the popular belief in press, we show that a leading manufacturer may indeed benefit from an improvement in availability more than the retailer does under some conditions.

One of the ways to handle operational inefficiencies is through Radio Frequency Identification Devices (RFID) that could provide visibility to track items. Although the history of the RFID technology traces back to World War II, supply chains developed a special interest in RFID when Walmart required her top 100 suppliers to adopt RFID at the pallet level in 2003. RFID is being adopted by a large number of players such as Walmart, Marks & Spencer, Tesco and Metro Group at both retail and warehouse levels although this technology could be expensive to implement. For example, a typical consumer packaged goods manufacturer is estimated to spend from $13 million to $23 million on RFID adoption, including various fixed and variable costs, for shipping 50 million cases per year (Asif and Mandviwalla, 2005). RFID has been publicized as a very promising technology for retail stores. According to sources as Gartner, Accenture and Forrester Research, RFID has increased sales by 3-7% due to stock availability and reduced losses due to theft by 40-50% (http://www.usingrfid.com). However, the costs of adoption and implementation have become a source of supply chain conflict and debate. Who pays for the tagging expenses and who benefits from the technology have become a major issue in the supply chains.

The second chapter of this dissertation titled "Misplaced Inventory and RFID: Information and Coordination " studies incentives of players in supply chain to invest in RFID to remove inefficiencies of misplaced inventory. We consider both fixed and variable costs of adoption of RFID in a vertically integrated and a bilaterally decentralized supply chain, for which we study uncoordinated and coordinated environments. We analyze scenarios where the fixed cost of adoption are shared and study both cases where only the retailer or only the manufacturer pays for the variable costs of adoption, i.e. *tagging expenses*. We show that it does not matter who pays for the tagging expenses since the manufacturer transfers the tag price to the wholesale price if she bears the tagging expenses. However, we prove that relative fixed cost payments of parties lead to interesting conclusions in the characterization of who benefits from this investment. Our results support the current reactions of the supply chains to the RFID and show that unless the fixed cost of RFID is sufficiently low and the tag prices are cheap enough, no party would benefit from an investment in this technology. However, we also find results counter the common opinion that the main beneficiaries of

RFID will be the retailers. We characterize conditions under which the manufacturer, being a leader in our model, may actually benefit more than the retailer from an investment in RFID.

Shrinkage is a special form of inventory misplacement. When items are lost due to misplacement, they are found once an inventory clean-up is performed. However, when items are lost due to shrinkage, they can not be retrieved back and thus, the physical number of items is always less than the system record. Today, worldwide theft is estimated to cost retailers $50 billion a year (http://www.rfidupdate.com). The risk of stock-outs due to shrinkage escalates during peek demand periods. If there is competition, the retailers can be significantly hurt by the out-of-stocks during these periods. The third essay of this dissertation titled "Inventory Management under Shrinkage and Customer-Driven Search" considers a supply chain with one supplier and two retailers subject to inventory shrinkage through a single-period model. In this essay, we study shrinkage caused primarily by shoplifting or employee theft. Here, we explore the effects of shrinkage and competition on the equilibrium stocking decisions. We also characterize conditions for the parties to benefit from a unilateral reduction in shrinkage. Further, we study the effects of market search on the stocking decisions.

Chapter 2

Coordination of a Supply Chain under Misplaced Inventory

2.1. Introduction and Related Literature

Misplaced inventory is a major operational inefficiency in many supply chains. Raman, DeHoratius and Ton (2001) empirically show that 16% of the items were lost in a store due to misplacement of items. Besides commercial firms, losing goods became a huge problem even for the Navy. McCutcheon (1999) reveals that the Navy lost more than $3 billion worth of goods between 1996 and 1998, which is evaluated as "roughly the equivalent of misplacing three destroyers". Misplaced inventory is also present in warehouse operations. G.T Interactive, the creator of computer games like Doom II and Driver, suffered from low productivity due to inventory misplacement in the warehouse. Fundamentally what happens in these settings is that the product is misplaced in the supply chain and is unavailable during the sales period but can be retrieved back when a cleanup is performed. Stores may apply different methods to tackle the problem of misplaced inventory. For example, in some apparel departments of retailers there are signs informing customers not try to return the product to their original location if the customer decides not to buy the product after trying it. Some libraries cope with the misplacement problem by putting signs which tell the customers not to reshelf the books after use while others have designated spaces for returning books taken off the shelf.

In this chapter, we consider an environment where a firm sells a single product. A proportion of the items that are ordered become unavailable for sale due to misplacement at the retailer. In a single period setting, we assume that these misplaced items can be recovered at the end of the period. We find the optimal inventory policy for a centralized firm when the demand is uncertain and compare the optimal ordering quantity under misplacements with the optimal ordering quantity under 100% availability. In addition, we analyze the behavior of the optimal ordering quantity with respect to changes in the level of availability at the retail store. We also study the decentralized case where a retailer and a manufacturer act as different parties. We give conditions for coordinating the channel under buyback and revenue sharing contracts. We characterize situations under which the supply chain is coordinated with the above mechanisms and identify the most profitable contracts for the manufacturer and the retailer. Our research goals include understanding how a vertically integrated supply chain as well as a decentralized supply chain are affected by inventory misplacement; how coordination mechanisms could be used under misplaced inventory setting and how the retailer and the manufacturer are affected by improvements in misplaced inventory. This chapter builds upon three streams of literature: misplaced inventory and inventory inaccuracy, random yield and supply chain contracting.

DeHoratius and Raman (2004) empirically study the drivers of inventory inaccuracy and find that inventory record inaccuracy is positively related with the annual selling quantity of that item, inversely associated with the cost of that item, positively associated with the inventory density and product variety. Inventory management models under stochastic demand have been studied by researchers over a number of years (see Swaminathan and Tayur (2003) for a recent review). A model for inventory inaccuracy was first studied by Iglehart and Morey (1972) who developed an adjusted inventory policy to minimize the expected total cost per unit time subject to the probability of errors due to inventory inaccuracy being below a preset level. More recently, Kök and Shang (2004) develop an effective inspection and replenishment policy for the retailer and show that an inspection adjusted base stock policy (IABS) is optimal for a single period problem. Due to inability to characterize the convexity of the objective function, they construct lower bounds for the finite horizon problem and show that the IABS achieves these lower bounds. In another recent paper, Kang and Gershwin (2004) explore inventory inaccuracy caused by shrinkages using a simulation study. Their results show that even a small rate of undetected shrinkage may lead to serious disruptions in the inventory management system. DeHoratius, Mersereau and Schrage (2005) model inventory inaccuracy through an invisible demand process and study the problem in a periodic review inventory policy system with unobserved lost sales.

RFID technology could act as a solution to inventory inaccuracy. Lee and Özer (2005) study the benefits of RFID technology for inventory inaccuracy. Gaukler, Seifert and Hausman (2004) study coordination issues in RFID implementation to remove inventory inaccuracy. As opposed to the above papers on inventory inaccuracy, in this chapter, we study a particular form of inventory inaccuracy problem that stems from misplacement of inventory at the retail store. Empirical evidence of misplaced inventory can be found in Ton and Raman (2004). They use a four-year longitudinal study to explore the effects of product variety and inventory level per product on misplaced products. They find that product variety and inventory levels are significant drivers of misplacement. Most recently, Rekik, Sahin and Dallery (2005) independently researched misplaced inventory assuming normal demand under three conditions: (1) where the retailer is not aware of the misplacement problem, (2) the retailer knows the misplacement problem and optimizes accordingly and (3) the retailer uses an auto-id system to remove misplacement completely. They computationally study the benefit of such information systems. Our work is distinguished from their model in that we consider a situation where the retailer is aware of misplacement and optimizes the inventory under general demand conditions. In addition to identifying the optimal inventory policy for a centralized retailer, we also study supply chain coordination and gaming issues associated with misplaced inventory and benefits of improving inventory availability at the retailer.

The random yield literature is the second stream related to our work. In this, researchers assume that a proportion of items get lost forever. Henig and Gerchak (1990) provide an analysis of general periodic review model for random yield for both finite and infinite horizons. Bassok and Akella (1991) consider a manufacturing facility that faces stochastic demand and arrival process for the raw materials. The readers are referred to Yano and Lee (1995) for an extensive review of random yield literature. As opposed to random yield, under misplaced inventory, misplaced items during the period can be found when there is a cleanup.

The third stream of related literature is that of supply chain contracting (see Cachon (2003) for a recent review). Revenue sharing contracts are relevant to our research. Dana and Spier (2001) coordinate a supply chain with a monopolist manufacturer and competitive downstream firms with a revenue sharing contract. Cachon and Lariviere (2005) clarify the link between buyback and revenue sharing contracts and show that one can always find a unique buyback contract that generates the same profits for each firm as any revenue sharing contract. We study the revenue sharing contract for the misplaced inventory problem. We can easily extend our findings here for a revenue sharing contract to buyback contracts (Çamdereli 2007).

The remainder of the chapter is organized as follows. We study the effects of inventory misplacement in a vertically integrated supply chain and a decentralized supply chain in Section 2.2 when demand is characterized by a general distribution. In the context of the decentralized supply chains, we analyze supply chains under a wholesale price contract and those under a revenue sharing contract. In Section 2.3, we gain insights on the results that we show in Section 2.2 through closed form solutions for uniformly distributed demand. In addition to revenue sharing, we further analyze supply chain coordination via a buyback contract mechanism in this section. We extend our analysis to inventory misplacement as a random variable and inventory availability as a function of the order size in Section 2.4.

2.2. Basic Model

In this chapter, we consider models that optimize inventory under misplacements for a single product in a single period setting. We study a vertically integrated supply chain in Section 3.2 and a bilaterally decentralized supply chain in Section 3.3. In the context of decentralized supply chains, we analyze those which are under a wholesale price contract and those under a revenue sharing contract. Demand is assumed to occur according to a general distribution with a finite mean in the aforementioned settings.

2.2.1 Centralized firm

In the centralized setting, we assume that the manufacturer owns the retailer. For convenience, we will refer to the centralized manufacturer/retailer as the centralized firm. The centralized firm orders at c per unit and sells to customers at p per unit. In practice, items may get misplaced due to reasons such as customers putting them in the wrong location during shopping, employees labeling products incorrectly and shipments to incorrect retail locations from the warehouse. This process may be happen gradually during the sales season. Although it would be interesting research in its own right to model the gradual misplacement of inventory during the season, our aim in this work is not to study the misplacement dynamics in detail but to address the managerial implications at the retailer and the supply chain of such misplacement. As a result, in our model we will settle for an approximate modeling of misplaced inventory. We will assume that as soon as the order is complete, a fraction l of the inventory gets misplaced and is temporarily lost. Thus, only a $(= 1 - l)$

proportion of total ordered items are available to the end customers. After the selling season, the firm cleans up the store and retrieves the entire misplaced inventory. One could think of this setting as an upper bound on the expected penalty incurred under misplaced inventory since misplacement occurs before demand. In additional to sufficiently simplifying the analysis, this may be a reasonable approximation of misplacement in some real settings as well. An example of such a setting would be inventory in toy stores during the often chaotic Christmas season where one can find a lot of misplaced inventory on the floor but everything gets into an orderly state after a cleanup at the end of the selling season. The assumptions related to a fixed quantity independent proportion a for availability is relaxed for the random availability case as well as quantity dependent availability case in Section 4. Let Q_C be the total items ordered by the centralized firm. It is to be noted that there will be at least lQ_C leftover inventory at the end of the season. The remaining inventory at the end of the period is salvaged at s per unit. A penalty of g_C per unit is assessed for unsatisfied demand to account for the cost of lost goodwill. We assume that Assumption 1 holds for analysis throughout the chapter to make the firm participate and sell the product.

Assumption 1: $s < c < p$

When the centralized firm does not lose any inventory, i.e. when the availability proportion is 1, the problem is a newsvendor formulation. The centralized firm's profit under perfect availability ($a = 1$) and partial availability ($0 < a < 1$) are stated in (2.1) and (2.2), respectively. We will use Q_N and Q_C to denote newsvendor ordering quantity and ordering quantity under partial availability respectively. The expected profits of the firm under perfect availability and partial availability are given by

$$EPC_N(Q_N) = -cQ_N + p\{\mu + \int_{Q_N}^{\infty} (Q_N - \xi) f(\xi) d\xi\} - g_C \int_{Q_N}^{\infty} (\xi - Q_N) f(\xi) d\xi$$
$$+ s \int_{0}^{Q_N} (Q_N - \xi) f(\xi) d\xi \tag{2.1}$$

$$EPC(Q_C) = -cQ_C + p\{\mu + \int_{aQ_C}^{\infty} (aQ_C - \xi) f(\xi) d\xi\} - g_C \int_{aQ_C}^{\infty} (\xi - aQ_C) f(\xi) d\xi$$
$$+ s\{\int_{aQ_C}^{\infty} lQ_C f(\xi) d\xi + \int_{0}^{aQ_C} (Q_C - \xi) f(\xi) d\xi\} \tag{2.2}$$

All of the proofs are given in Appendix A.

Theorem 1

1. *$EPC(Q_C)$ is a concave function of Q_C .*

2. *Let $\tilde{a} = \frac{(c-s)}{(p+g_C-s)}$ and $0 < a < 1$. The optimal ordering amount is*

$$Q_C^* = \begin{cases} \left(\frac{1}{a}\right) F^{-1}\left(1 - \frac{\tilde{a}}{a}\right) > 0 \ \textit{iff} \ a > \tilde{a}. \\ 0 \ \textit{otherwise} \end{cases} \tag{2.3}$$

Theorem 1 characterizes the optimum order, which is positive if and only if availability level is greater than a threshold value. Next, we compare the optimum order under inventory misplacement with the classical Newsvendor quantity ($Q_N^* = F^{-1}(1 - \tilde{a})$).

Corollary 1 *Let us assume that EPC'_N is the classical Newsvendor function where one makes a transformation of $c' = c/a - (1-a)s/a$. Then, $Q^*_C = Q^{*'}_N/a$ where*

$$Q^{*'}_N = \underset{Q_N}{\operatorname{argmax}}\, EPC'_N$$

The above indicates that this problem can be made equivalent to a classical Newsvendor problem with an appropriate adjustment to cost of ordering followed by a deflation of the resulting solution by a fraction a. Since only a proportion of products are available for sale, the cost is deflated by a and then adjusted because there is a guaranteed salvage on $1-a$ proportion at the end of the season.

Corollary 2 *For* $0 < a < 1$, $\left\{ \begin{array}{lll} Q^*_C < Q^*_N & \text{if} & (1-\frac{\tilde{a}}{a}) < F\left(aF^{-1}(1-\tilde{a})\right) \\ Q^*_C > Q^*_N & \text{if} & (1-\frac{\tilde{a}}{a}) > F\left(aF^{-1}(1-\tilde{a})\right) \end{array} \right\}$

It is clear that the values of Q^*_C and Q^*_N are equal when inventory availability is 100% ($a=1$). The ordering of Q^*_C and Q^*_N depends on the demand distribution. We will identify the condition for a uniform demand in Section 2.3.1.

Next we explore how improvements in the inventory availability level affect the optimal ordering quantity and the optimum profit of the centralized firm. Availability can be improved in several ways such as RFID implementation (as studied in Çamdereli and Swaminathan, 2006). In this chapter we only look at benefits neglecting the investments needed to improve availability. However, any linear or convex costs for information availability can be incorporated easily in our analysis.

Lemma 1 *For a general distribution of demand, the following results hold.*

1. $\frac{\partial Q_C^*}{\partial a} > 0$ *if and only if* $g[aQ_C^*] < 1$ *where* $g[x] = \frac{xf(x)}{1-F(x)}$.

2. $\partial EPC(Q_C^*)/\partial a > 0$.

3. *Let us call* $\nu(Q_C^*) = \frac{a}{Q_C^*}\frac{\partial Q_C^*}{\partial a}$. *Optimal centralized profit is*

   (a) *an increasing concave function of availability if* $\nu(Q_C^*) < 1$ *and*

   (b) *an increasing convex function of availability if* $\nu(Q_C^*) > 1$.

$g[x] = xf(x)/(1 - F(x))$ is often called the generalized failure rate (see Lariviere and Porteus, 2001 for an application of the generalized failure rate in inventory). The generalized failure rate is the elasticity of $1-F(x)$. In other words, it approximately shows the percentage decrease in the probability of a stockout from increasing the stocking quantity by 1%. Hence, $g[aQ_C^*] = aQ_C^*f(aQ_C^*)/(1 - F(aQ_C^*))$ is the generalized failure rate evaluated at aQ_C^* and shows the percentage decrease in the stockout probability from increasing the number of available items by 1%. When $g[aQ_C^*] < 1$, the probability of a stockout can still be improved for a percentage increase in the number of available items. In such a case, one would increase the optimal order quantity even if the inventory availability improves. However, if $g[aQ_C^*] > 1$, the probability of a stockout increases by more than 1% as a result of increasing the available items by 1%. In this case, increasing the optimum order quantity leads to a high value of cost of overage and it is optimum to decrease the order quantity when availability improves.

Further, Lemma 1 shows that the firm always benefits from an improvement in inventory availability. We term $\nu(Q_C^*)$ as the availability elasticity of optimum order quantity. Thus, it shows the percentage change in the optimum order for a percentage change in the availability. The behavior of the optimum profit with respect to changes in availability depends on the availability sensitivity of the optimum order. For example, if the optimum order increases in availability and it is sensitive to changes in availability ($\nu(Q_C^*) > 1$), profit function will be an increasing concave function of inventory availability. In this case, if the order is insensitive to changes in availability ($\nu(Q_C^*) < 1$), then the profit function is a convex increasing function. If the order decreases as availability increases, profits always increase concavely.

2.2.2 Decentralized Supply Chain

For the decentralized case, we assume that the retailer and the manufacturer are two independent parties. The manufacturer acts as a Stackelberg leader and reveals the wholesale price to the retailer. The retailer decides the amount to order based on the wholesale price. As soon as the order arrives, $0 < l < 1$ proportion of inventory gets misplaced in the retail store, therefore making $a = 1 - l$ proportion of the order available to end customers. Retail price of the product, p per unit, is assumed to be exogenous. The leftover inventory is salvaged at s per unit. Similar to the model by Pasternack (1985), penalty per unit for unmet demand incurred by the centralized manufacturer is shared between the independent manufacturer and the independent retailer. The retailer incurs g_R per unit and the manufacturer incurs $g_M = g_C - g_R$ per unit of unsatisfied demand.

We will explore two different scenarios under decentralized case. First, we analyze a setting where the supply chain is under a wholesale price contract, i.e. there is no coordination between the manufacturer and the retailer. In the latter setting, we coordinate the supply chain via a revenue sharing contract. The coordination of the supply chain means that there will be no losses in the total supply chain profits as compared to the centralized firm's profit.

Assumption 1 ($s < c < p$) is necessary but not sufficient for the retailer and manufacturer to participate in the selling season. Thus, we modify Assumption 1 as in Assumption 2 for supply a chain under a wholesale price contract. However, the wholesale price being greater than the marginal cost is very restrictive for revenue sharing contracts (Cachon and Lariviere, 2005). Therefore, we will require only Assumption 1 for supply chains under a revenue sharing contract due to the nature of the mechanism. The wholesale price will be denoted by w and w will become subscripted to note different supply chain scenarios.

Assumption 2: $s < c < w < p$

2.2.2.1 No coordination

In this setting, there is no mechanism to coordinate the supply chain. The retailer's and the manufacturer's profit functions under no coordination are shown by (2.4) and (2.5) in that order. We use w_D to indicate the wholesale price announced by the manufacturer and Q_D for the quantity ordered by the retailer under a wholesale price contract.

$$EPR_D(Q_D) = -w_D Q_D + p\left\{\mu - \int_{aQ_D}^{\infty} (\xi - aQ_D) f(\xi) d\xi\right\} - g_R \int_{aQ_D}^{\infty} (\xi - aQ_D) f(\xi) d\xi +$$

$$s\left\{\int_0^{aQ_D}(aQ_D-\xi)f(\xi)d\xi+lQ_D\right\} \tag{2.4}$$

$$EPM_D(w_D)=Q_D(w_D-c)-g_M\int_{aQ_D}^{\infty}(\xi-aQ_D)f(\xi)d\xi \tag{2.5}$$

Since the manufacturer is the leader, she maximizes her profit subject to maximization of the retailer's profit function. Therefore, the optimal ordering quantity, Q_D^* , is actually a function of w_D and is equal to $\arg\max\limits_{Q_D} EPR_D(Q_D)$. In the optimum, the manufacturer's profit function is composed of marginal revenue of producing and selling Q_D^* items and the manufacturer's share of the penalty cost of unsatisfied demand.

Theorem 2 *If demand distribution has an increasing generalized failure rate, the optimum order solves the following where $\tilde{a}$ is given by Theorem 1. The optimum wholesale price is given by (2.7).*

$$F(aQ_D)+aQ_Df(aQ_D)\frac{p+g_R-s}{p+g_C-s}=1-\tilde{a}/a \tag{2.6}$$

$$w_D^*=a(p+g_R-s)[1-F(aQ_D^*)]+s \tag{2.7}$$

Theorem 2 applies to many common demand distributions such as normal, exponential and uniform as these distributions have an increasing generalized failure rate.

Lemma 2 *In an uncoordinated environment, the following findings hold.*

1. $\frac{\partial}{\partial a} EPR_D(Q_D^*(a);a) > 0$ *if and only if* $\frac{\partial w_D^*}{\partial a} < (p + g_R - s)(1 - F(aQ_D^*))$.

2. $\frac{\partial}{\partial a} EPM_D(w_D^*(a);a) > 0$.

We find that the retailer benefits from an increase in availability as long as the wholesale price does not increase by more than $(p + g_R - s)(1 - F(aQ_D^*))$, which is equivalent to $(w_D^* - s)/a$ after substituting (2.7) for w_D^*. Being a leader, the manufacturer always benefits from an improvement in availability as well. In Section 2.3.2.1, we compare the amount of benefits that they can extract from an improvement in the inventory availability when demand is uniformly distributed. Note that $Q_D^* < Q_C^*$; therefore, there will be double marginalization losses in the supply chain. Next, we analyze coordination of a supply chain under inventory misplacement with a revenue sharing contract.

2.2.2.2 Coordination with Revenue Sharing

In this section, we analyze a case where revenue sharing is used as means to coordinate the channel. In this setting, the events take place in the same sequence as in the uncoordinated decentralized case. The retailer agrees to take $0 < \gamma < 1$ proportion of all of her positive earnings, i.e. sales and salvage earnings, and gives the manufacturer (1- γ) proportion of them. We require only Assumption 1 to hold and we do not make any restrictions on the wholesale price.

The retailer's and the manufacturer's profit functions are as formulated in (2.8) and (2.9) in that order. We assume that neither the manufacturer nor the retailer can change γ .

$$EPR_{RS}(Q_{RS}) = -w_{RS}Q_{RS} - g_R \int_{aQ_{RS}}^{\infty} (\xi - aQ_{RS})f(\xi) + \tag{2.8}$$
$$\gamma \left[p \left(\mu - \int_{aQ_{RS}^{\infty}} (\xi - aQ_{RS})f(\xi)d\xi \right) + s \left(\int_0^{aQ_{RS}} (aQ_{RS} - \xi)f(\xi)d\xi + lQ_{RS} \right) \right]$$

$$EPM_{RS}(w_{RS}) = Q_{RS}(w_{RS} - c) - g_M \int_{aQ_{RS}}^{\infty} (\xi - aQ_{RS})f(\xi)d\xi \tag{2.9}$$
$$(1-\gamma) \left[p \left(\mu - \int_{aQ_{RS}}^{\infty} (\xi - aQ_{RS})f(\xi)d\xi \right) + \ s \left(\int_0^{aQ_{RS}} (aQ_{RS} - \xi)f(\xi)d\xi + lQ_{RS} \right) \right]$$

Lemma 3 *The supply chain is coordinated for w_{RS} set to the following.*

$$w_{RS}^* = \frac{(c-s)[\gamma(p-s) + g_R]}{p + g_C - s} + \gamma s \tag{2.10}$$

As seen in (2.10), the supply chain is coordinated for a wholesale price which is independent of the inventory misplacement. This happens due to the reason that the retailer misplaces the inventory and pays the manufacturer for the entire $Q_{RS}^* = Q_C^*$. Mathematically speaking, since we are coordinating the supply chain under inventory misplacement and equating Q_{RS}^* with the optimum order of the centralized firm under inventory misplacement, availability expressions cancel out. We further observe that the optimum wholesale price does not depend on the demand distribution either.

Next, we explore the effects of improving inventory availability on the retailer's and the manufacturer's profits.

Theorem 3

1. *The retailer always benefits from an improvement in inventory availability.*

2. *The manufacturer benefits from an improvement in inventory availability if and only if the following holds.*

$$g[aQ_C^*] < \bar{Y}_1 = \frac{c(p + g_C - s)}{(cg_R + sg_M) + (p - s)[(1 - \gamma)s + \gamma c]} \tag{2.11}$$

   (a) *If* $\bar{Y}_2 < g[aQ_C^*] < \bar{Y}_1$*, then the retailer always benefits more than the manufacturer.*

   (b) *If* $g[aQ_C^*] < \bar{Y}_2$*, then the manufacturer always benefits more than the retailer where* $g[aQ_C^*]$ *is the generalized failure rate evaluated at* aQ_C^**,* Q_C^* *is given by Theorem 1 and*

$$\bar{Y}_2 = \frac{c(p + g_C - s)}{2(c - s)[\gamma(p - s) + g_R] + s(g_C + p - s)} < \bar{Y}_1 \tag{2.12}$$

Theorem 3 shows that an improvement in inventory availability always benefits the retailer. Although the manufacturer is the leader, her extracting benefits from the supply chain is contingent upon the condition specified in Theorem 3.2. The common intuition among the practitioners is that the downstream players will be the main beneficiaries from reductions in inventory misplacement. However, here we show that if the generalized failure rate of the demand distribution evaluated at the amount of the optimum available inventory is smaller than a threshold value, the manufacturer benefits more from an improvement in

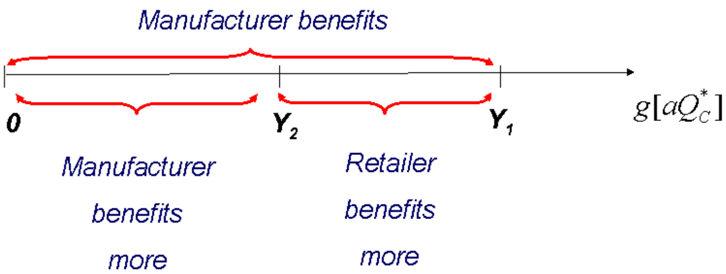


Figure 2.1: Benefits accruing to the retailer versus the manufacturer for an improvement in availability

inventory availability than the retailer does. Figure 2.1 illustrates the regions specified in Theorem 3.2.

2.3. Special Case: Uniform Demand

In order to gain insights through closed form expressions, we assume that demand is distributed uniformly in $[0, \beta]$ throughout this section.

2.3.1 Centralized firm

The profit function of a classical Newsvendor under 100% availability given by Expression (2.1) and that of a centralized firm under inventory misplacement given by expression (2.2) are simplified as follows for uniform demand.

$$EPC_N(Q_N) = Q_N(g_C + p - c) - Q_N^2 \left(\frac{g_C + p - s}{2\beta} \right) - g_C\mu$$

$$EPC(Q_C) = Q_C(a(g_C + p - s) - (c - s)) - Q_C^2 a^2 \left(\frac{g_C + p - s}{2\beta}\right) - g_C\mu$$

Theorem 4

1. *Let* $\tilde{a} = \frac{c-s}{p+g_C-s}$ *and* $0 < a < 1$ *. The optimal ordering quantities are*

   (a) $Q_N^* = (1 - \tilde{a})\beta > 0$

   (b)

$$Q_C^* = \begin{cases} (1/a)(1 - \tilde{a}/a)\tilde{\beta} > 0 & \textit{if and only if } a > \tilde{a} \\ 0 & \textit{otherwise} \end{cases} \tag{2.13}$$

Theorem 4 specifies the optimal ordering quantities for the centralized firm under perfect and partial availability. If availability is less than a threshold value, the firm basically leaves the market. Next, we benchmark the optimal ordering quantity under partial availability with the newsvendor optimal ordering quantity.

Corollary 3

1. $Q_N^* = Q_C^*$ *if* $a = 1$
2. *For* $\tilde{a} < a < 1$, $\begin{cases} Q_N^* \leq Q_C^* \textit{ if and only if } a \geq \frac{\tilde{a}}{1-\tilde{a}} \\ Q_N^* > Q_C^* \textit{ otherwise} \end{cases}$

Generally one would expect that the ordering quantity for the centralized firm under partial availability should be greater than the newsvendor ordering quantity because some items are becoming unavailable to customers during the selling season. However, Corollary 3 leads to an interesting conclusion that the centralized firm under partial availability does not always order more than the newsvendor optimal ordering quantity.

The two optimal ordering quantities become equal to each other at two levels of availability, for $a = 1$ and for $a = \tilde{a}/(1-\tilde{a})$ if $\tilde{a} < .5$ where $\tilde{a}$ is the cost of overage divided by the cost of underage plus overage in the newsvendor setting under perfect availability. Thus, $\tilde{a}$ gives us an intuition about attractiveness for stocking the item. If the items are unattractive, i.e. $\tilde{a} > .5$, $\tilde{a}/(1-\tilde{a})$ will become greater than one and the optimal ordering quantity under partial availability will always be less than the newsvendor optimal ordering quantity for any value of availability. Therefore, the optimal ordering quantity under partial availability can never reach the newsvendor ordering quantity until 100% availability for unattractive items.

We would intuitively think that if the items are more attractive to stock, it is always better to produce more than the newsvendor optimal ordering quantity if the centralized manufacturer is operating under low inventory availability. However, Corollary 3 shows that even if the items are more attractive to stock, i.e. $\tilde{a} < .5$, if the existing availability level is less than a threshold value ($a < \tilde{a}/(1-\tilde{a})$), the effective cost of acquiring an item will be too high. In such a case, it will always be optimal to order less under partial availability than the newsvendor optimal ordering quantity.

Next we explore how improvements in the inventory availability level affect the optimal ordering quantity and the optimum profit of the centralized firm.

Lemma 4 For the centralized firm:

1. $\frac{dQ_C^*}{da}$ *is* $\begin{cases} < 0 \text{ if and only if } a > 2\tilde{a} \\ \geq 0 \text{ otherwise} \end{cases}$

2. $\frac{d^2Q_C^*}{da^2}$ *is* $\begin{cases} > 0 \text{ if and only if } a > 3\tilde{a} \\ \leq \text{ otherwise} \end{cases}$

3. *For* $Q_C^* > 0$, $EPC(Q_C^*)$ *is*

   (a) *an increasing concave function of availability if* $a > 1.5\tilde{a}$ *and*

   (b) *an increasing convex function of availability if* $a < 1.5\tilde{a}$.

One may expect the optimal ordering quantity to decrease with an increase in inventory availability level. As shown by Lemma 4.1, the optimal ordering quantity is not monotonically increasing or decreasing in the availability level for all cases. In Section 2.2.1, we characterized the behavior of the optimum order quantity as inventory availability changes for a general distribution of demand. When demand is distributed uniformly, $2\tilde{a}$ is the value of availability for which $g[aQ_C^*] = 1$. If $a > 2\tilde{a}$, $g[aQ_C^*] > 1$ and vice versa for $a < 2\tilde{a}$. Hence, the optimum order quantity does increase as availability increases if $a < 2\tilde{a}$.

Lemma 4.1 and Lemma 4.2 completely characterize the behavior of the optimum order in 3 different cases: $\tilde{a} < (1/3)$, $(1/3) \leq \tilde{a} < (1/2)$ and $\tilde{a} \geq (1/2)$. Figure 2.2 shows how the optimal ordering quantity changes with respect to availability for all three cases. The profitability of items decreases from case 1 to case 3. For the first two cases, we see that the optimal ordering quantity under partial availability increases in availability till a threshold value, and then it decreases. In case 1, the optimal ordering quantity is a decreasing convex

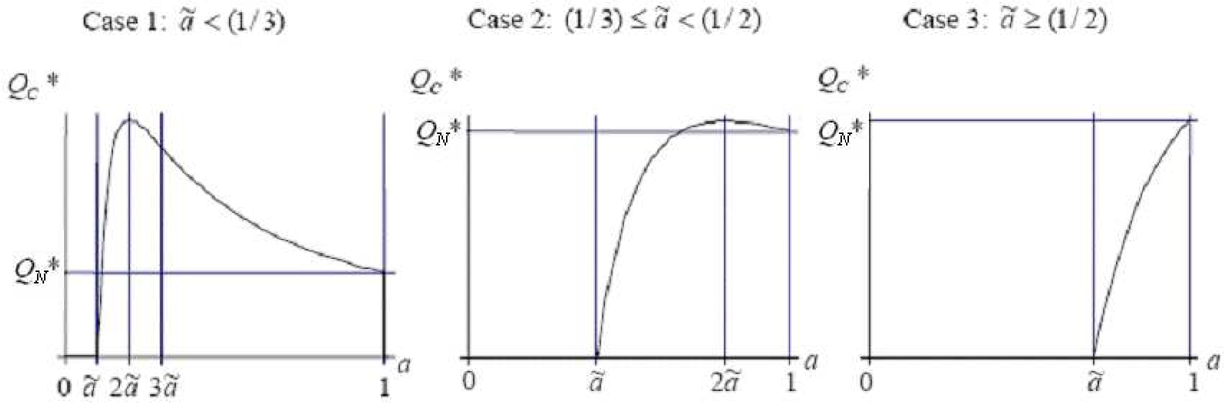


Figure 2.2: The optimal ordering quantity versus availability proportion

function of inventory availability after $3\tilde{a}$. However, in the second case, we do not observe any convex behavior because $3\tilde{a}$ lies beyond 1. The first two cases show that even when the critical fractile ($1 - \tilde{a}$) is high, it is optimal to order less if there is a decrease in current availability level given that the current availability level is sufficiently low (less than $2\tilde{a}$). This happens because of the fact that the existing availability level is too low such that the effective cost of producing one more item becomes too high although the items may be attractive to stock in general. For the same cases, if the existing availability level is sufficiently high (greater than $2\tilde{a}$), then the centralized firm follows our expectations and orders less as availability level improves. As shown in case 3, i.e. when the items are not that attractive, the optimal ordering quantity decreases as availability proportion decreases. The products in case 3 are so unprofitable that the effective cost of acquiring one more item becomes too high if the firm experiences a decrease in availability.

Lemma 4.3 analyzes the effects of changes in inventory availability level on the optimum centralized profit. The profit function increases in availability level for positive optimal ordering quantity. The profit function increases convexly in availability until a reaches a threshold value. After that, it increases concavely. However, this behavior can only be observed if $\tilde{a} < (2/3)$ holds. The profit function portrays an increasing convex behavior if $\tilde{a} > (2/3)$ holds. Figure 2.3 shows the optimal centralized profits versus availability under these two cases. In the first case, the marginal benefit of the centralized firm from an increase in availability level when the current availability level is low is higher than its marginal benefit for an increase in availability level when the current availability level is high. The expected sales will increase or decrease according to the availability proportion. When the availability level is sufficiently high, the firm is already able to satisfy high values of demand. Therefore, an increase in availability level will not affect the profits significantly. On the contrary, when the availability level is sufficiently low, the firm will not be able to accommodate high demand values. Therefore, even a small increase in the availability level will affect the profits by a larger amount when availability is low as compared to the case of high availability proportion.

Although the first case gives us an intuitive explanation for the behavior of the profit function, the second case illuminates an interesting result. When $\tilde{a} > (2/3)$ holds, the marginal benefit of the firm from an improvement in availability level gets larger as the availability level increases because the optimal profit function is convex for all values of availability. In this case, the condition $\tilde{a} > (2/3)$ can be written as $1.5(c - s) > (p + g_C - s)$. We already know that $(p - s)$ is larger than $(c - s)$ due to Assumption 1. Therefore, a positive stock-out cost will only increase the right hand side of the inequality. Ceteris paribus, only a relatively large c can validate the inequality. In other words, the closer the marginal cost is to the sales price, the more likely that this case will hold. This also means

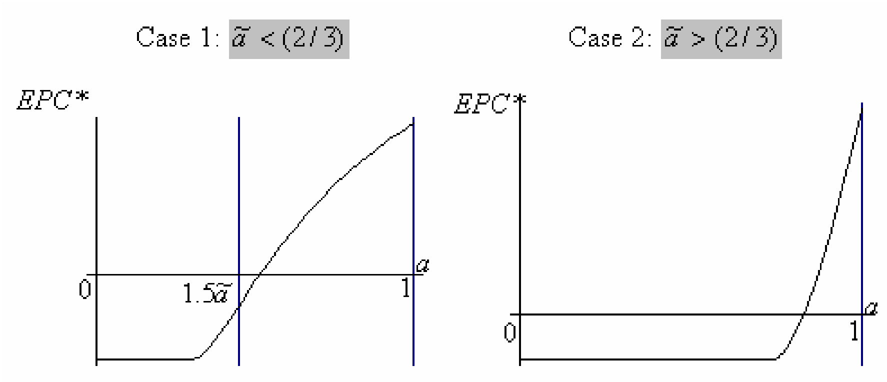


Figure 2.3: Optimal centralized profit versus availability

that the centralized firm applies a low markup to each unit. Since the unit markup is low, the firm makes positive profit as long as he can sell large amounts of inventory during selling season. From that standpoint, the higher the availability level, the more will the centralized firm benefit from an improvement in availability.

We previously showed that inflated value of the optimum classical newsvendor quantity by a, which will be denoted by Q_i^*, does not give the optimum value under inventory misplacement. In the following passages, we compare the optimal profits under Q_N^*, $Q_i^* = (Q_N^*/a)$ and Q_C^* for uniformly distributed demand. Before we present the analytical findings, we illustrate the comparison with two numerical examples.

Let us assume that $\beta = 10000$, $p = 9$, $g_C = 2$, $s = 1$ and $c = 4$ for the first example. As Figure 2.4 shows Q_C^* generates the highest profits. Since $\tilde{a} < .5$, the optimal profits under Q_C^* and Q_N^* are equal to each other at two values of availability: $a = 1$ and $a = \tilde{a}/(1-\tilde{a})$ (see Figure 2.2). The profit under Q_C^* is composed of only expected stock out cost till Q_C^* becomes positive. Q_i^* performs the worst among the three for low values of a since the amount ordered

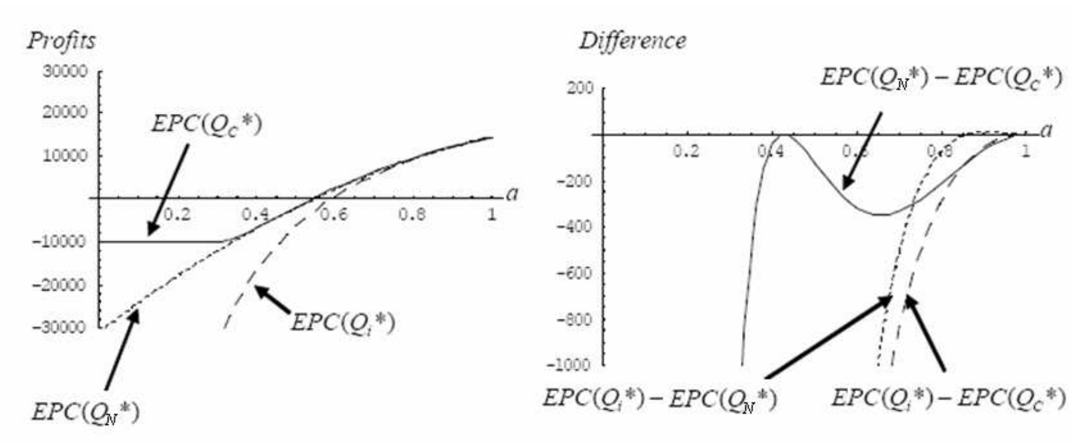


Figure 2.4: Profits versus availability under Q_C^* , Q_N^* and Q_i^* ($\beta = 10000$, $p = 9$, $g_C = 2$, $s = 1$, $c = 4$)

is much higher than the optimum. As availability increases, the performance of Q_i^* improves and after some value of availability, it performs better than Q_N^*. The performance of Q_N^* relative to Q_C^* keeps getting worse after the two quantities become equal to each other at $\tilde{a}/(1-\tilde{a})$ until availability reaches a threshold value. It starts to improve again as availability increases beyond that threshold value till it goes to 1.

In the next example, we pick a firm with a higher cost of production and change the marginal cost to $c = 6$ while keeping the rest of the parameters the same. Figure 2.4 shows the profits and differences of profits for $\beta = 10000$, $p = 9$, $g_C = 2$, $s = 1$ and $c = 6$. In this case, $\tilde{a} = .5$, as a result the profit generated by Q_N^* catches the profit generated by Q_C^* only at 100% availability level. Although the performance of Q_i^* relative to Q_N^* improves as availability increases, it never becomes equivalent.

Based on these two examples it is clear that the differences between the optimal and simple heuristics such as newsvendor or inflated newsvendor can be substantial. Further,

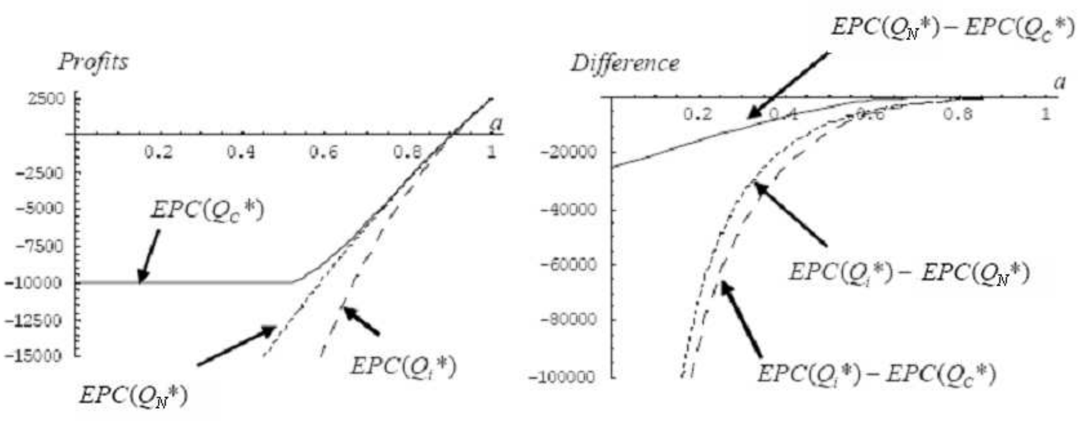


Figure 2.5: Profits versus availability under Q_C^* , Q_N^* and Q_i^* ($\beta = 10000$, $p = 9$, $g_C = 2$, $s = 1$, $c = 6$)

their performance is dependent on the attractiveness to stock the product as well as on the availability level. Next, we analytically characterize the behavior of the profit function under the three ordering quantities and their dependence on availability.

Corollary 4 The following findings hold for $a < 1$

1. *(i)* $EPC(Q_N^*) < EPC(Q_C^*)$, *(ii)* $EPC(Q_i^*) < EPC(Q_C^*)$

2. $EPC(Q_i^*) - EPC(Q_N^*) > 0$ *if and only if* $a > 2\frac{\tilde{a}}{1-\tilde{a}}$

3. (a) $EPC(Q_N^*)$ *and* $EPC(Q_i^*)$ *are concave functions of* a.

   (b) $\partial\left(EPC(Q_i^*) - EPC(Q_N^*)\right)/\partial a > 0$ *if and only if* $\frac{\mathrm{a}^2}{1+\mathrm{a}} < \frac{\tilde{a}}{1-\tilde{a}}$

   (c) $\partial\left(EPC(Q_i^*) - EPC(Q_C^*)\right)/\partial a > 0$

$$\text{(d)}\ \partial\left(EPC(Q_N^*) - EPC(Q_C^*)\right)/\partial a = \left\{ \begin{array}{l} > 0 \ \textit{if } a < \frac{\tilde{a}}{1-\tilde{a}} \ \textit{and } c < \frac{g_C+p+s}{2} \\ > 0 \ \textit{if } a > \sqrt{\frac{\tilde{a}}{1-\tilde{a}}} \ \textit{and } c < \frac{g_C+p+s}{2} \\ > 0 \ \textit{if } c \geq \frac{g_C+p+s}{2} \\ \leq 0 \ \textit{otherwise} \end{array} \right\}$$

The optimal ordering quantity under misplaced inventory is always smaller than the newsvendor ordering quantity for low values of availability (see Corollary 3). Thus, inflating the newsvendor quantity will only worsen the profits if the firm is misplacing most of the products. It is always optimal to order more than the newsvendor optimal ordering quantity if the marginal cost is low and the level of availability is sufficient. Corollary 4.2 confirms Corollary 3 and shows that using the inflated newsvendor optimal ordering quantity instead of the newsvendor optimal ordering quantity will be better for the firm for relatively small values of marginal cost and high values of availability.

The inflated newsvendor optimal ordering quantity will always perform worse than the newsvendor optimal ordering quantity for high values of marginal cost. Since production of one unit is very costly to the firm, the more the firm produces, the worse off she becomes. However, as availability increases, the performance of the inflated newsvendor optimal ordering quantity gets better because the inflated newsvendor quantity decreases. Though, it can never perform as well as the newsvendor optimal ordering quantity.

Although the newsvendor quantity always performs worse than the optimal ordering quantity under misplacement, the performance gap gets narrower for high values of marginal cost as availability improves. If $c \geq (g_C+p+s)/2$, i.e. $\tilde{a} \geq .5$, the optimal ordering quantity under misplacement is always less than the newsvendor optimal ordering quantity but it gets closer to the newsvendor ordering quantity as availability increases (see Figure 2.2). For low values of marginal cost ($\tilde{a} < .5$), Q_C^* gets closer and closer to Q_N^* till availability becomes $\tilde{a}/(1-\tilde{a})$, after which it deviates from the newsvendor quantity more and more and reaches its maximum at $a = 2\tilde{a}$ (see Figure 2.2). Thus, in a low-cost firm, any increase in availability will improve the performance of the newsvendor ordering quantity provided that availability is less than $\tilde{a}/(1-\tilde{a})$. When $c < (g_C + p + s)/2$, i.e. $\tilde{a} < .5$, $\sqrt{\tilde{a}/(1-\tilde{a})}$ always lies between $2\tilde{a}$ and 1. Although Q_C^* starts to decrease after $2\tilde{a}$ and gets closer to Q_N^* , the performance of Q_N^* improves only after availability increases beyond $\sqrt{\tilde{a}/(1-\tilde{a})}$.

Corollary 5

1. *(a)* $\partial\left(EPC(Q_i^*) - EPC(Q_C^*)\right)/\partial c < 0$

   (b) $\partial\left(EPC(Q_i^*) - EPC(Q_C^*)\right)/\partial g_C > 0$

   (c) $\partial\left(EPC(Q_i^*) - EPC(Q_C^*)\right)/\partial s > 0$

   (d) $\partial\left(EPC(Q_i^*) - EPC(Q_C^*)\right)/\partial p > 0$

   (e) $\partial\left(EPC(Q_i^*) - EPC(Q_N^*)\right)/\partial s > 0$

2. *(a)* $\partial\left(EPC(Q_N^*) - EPC(Q_C^*)\right)/\partial c > 0$ *if and only if* $a > \tilde{a}/(1-\tilde{a})$

   (b) $\partial\left(Q_C^* - Q_N^*\right)/\partial c < 0$

3. *(a)* $\partial\left(EPC(Q_N^*) - EPC(Q_C^*)\right)/\partial s > 0$ *if and only if* $a < \tilde{a}/(1-\tilde{a})$,

   (b) $\partial\left(Q_C^* - Q_N^*\right)/\partial s > 0$

We also analyze the behavior of the profit function under the three different ordering quantities with respect changes in parameters. Q_i^* is always greater than Q_C^* . Thus, any increase in the marginal cost of production makes Q_i^* perform even worse. Since the firm will have more products than she would in the optimum, an increase in the sales price, salvage value or stockout cost will improve the performance of Q_i^* relative to Q_C^*. Although Q_N^* can never perform as well as Q_C^* , its performance improves for high values of availability as marginal cost increases if the products are attractive to stock. For profitable products, Q_C^* is greater than Q_N^* for $a > \tilde{a}/(1-\tilde{a})$ (see Corollary 3). An increase in c causes the gap between Q_C^* and Q_N^* to shrink and the performance of the newsvendor ordering quantity to improve. On the contrary, the difference between Q_C^* and Q_N^* increases as salvage value increases. When Q_C^* is greater than Q_N^* , an increase in salvage value causes Q_C^* and Q_N^* to lie further apart. This hurts the performance of the newsvendor ordering quantity relative to the optimum quantity under misplacement. Thus, the performance of Q_N^* improves if $a < \tilde{a}/(1-\tilde{a})$, for which Q_C^* is always less than Q_N^* .

2.3.2 Decentralized Supply Chain

2.3.2.1 No Coordination

In this setting, there is no mechanism to coordinate the supply chain. The retailer's and the manufacturer's profit functions under a wholesale price contract shown by (2.4) and (2.5) are simplified as follows for uniform demand distributed in $[0, \beta]$.

$$EPR_D(Q_D) = Q_D\left(a(g+p-s) - (w_D - s)\right) - Q_D^2 a^2 \left(\frac{g+p-s}{2\beta}\right) - g\mu$$
$$EPM_D(w_D) = Q_D\left(w_D - c\right) - g_M \frac{\left(\beta - aQ_D\right)^2}{2\beta}$$

Theorem 5

1. *$EPR_D(Q_D)$ is strictly concave in Q_D.*

2. *Q_D^* is decreasing in w_D.*

3. *$EPM_D(w_D)$ is strictly concave in w_D.*

4. *The optimal ordering quantity is as follows.*

$$Q_D^* = \begin{cases} \kappa\left(\frac{1}{a}\right)\left(1 - \frac{\tilde{a}}{a}\right)\beta > 0 \text{ if and only if } a > \tilde{a} \\ 0 \text{ otherwise} \end{cases} \quad \text{where } \kappa = \frac{(g+p-s) + g_1}{2\left(g+p-s\right) + g_1} < 1. \tag{2.14}$$

Since the profit functions are concave in their arguments, there exists a unique ordering quantity for the retailer and a unique wholesale price for the manufacturer to maximize their profits. Theorem 5.2 reveals an intuitive result that the optimal ordering amount decreases as the manufacturer charges more. In the uncoordinated supply chains, one of the biggest concerns is the reduction in the optimal ordering amount. Theorem 5.4 shows that the optimal ordering quantity can be written as κQ_C^* where Q_C^* is given by (2.13). Since $\kappa <$

1, the decentralized optimum ordering amount is less than the centralized optimal ordering amount, i.e. $Q_C^* > Q_D^*$. Because κ is a constant, findings of Lemma 4.1 and Lemma 4.2 also hold for the decentralized optimum ordering quantity.

Lemma 5 *In an uncoordinated supply chain, the following findings hold under misplaced inventory given that* $Q_D^* > 0$

1. $dEPR_D(Q_D^*(a); a)/da > 0$ *and* $dEPM_D(w_D^*(a); a)/da > 0$
2. $dEPM_D(w_D^*(a); a)/da > dEPR_D(Q_D^*(a); a)/da$
3. $dw_D^*(a)/da > 0$

Both the retailer and the manufacturer benefit from an increase in availability proportion. However, marginal benefit is higher on the manufacturer's profit than on the retailer's profit for the same amount of increase in the availability. Because the manufacturer knows that the independent retailer's profit function is increasing in the availability, she exploits the advantage of being the leader and charges more for the wholesale price. Due to the increase in wholesale price, the retailer can not benefit as much as the manufacturer. Clearly the position of the manufacturer as the Stackelberg leader helps in her being able to extract the maximum from the system. One could presume that if the cost of implementing RFID (which would reduce misplaced inventory at the retailer) is borne mainly by the manufacturer then our result will be weaker although we believe directionally it might still hold because the manufacturer is the leader (these and other issues related to availability investment are studied in detail in the next chapter).

Corollary 6

1.

$$EPC^{*}-\left(EPR_{D}^{*}+EPM_{D}^{*}\right)=\frac{(g+p-s)^{2}\left(a(g_{C}+p-s)-(c-s)\right)^{2}\beta}{2a^{2}(g_{C}+p-s)\left(2(p+g-s)+g_{1}\right)^{2}}>0 \quad (2.15)$$

2. $d\left(EPC^{*}-\left(EPR_{D}^{*}+EPM_{D}^{*}\right)\right)/da>0 \;\; \textit{for} \;\; Q_{C}^{*},Q_{D}^{*}>0.$

The centralized manufacturer always makes more money than the total supply chain profits in the decentralized setting under no coordination and (2.15) gives us the effect of double marginalization under misplaced inventory. The centralized manufacturer's profit, the manufacturer's and the retailer's profits in an uncoordinated supply chain all increase with improvements in availability. However, due to the increase in the wholesale price for an improvement in the availability level (see Lemma 5), the uncoordinated supply chain can never benefit from an improvement in availability as much as a vertically integrated supply chain. The higher the availability level is, the more severely will the supply chain be affected by double marginalization. Therefore, one can conjecture that the implementation of systems to improve inventory availability at the retailer is more likely to happen in centralized supply chains as compared to decentralized supply chains.

2.3.2.2 Coordination with Revenue Sharing

The retailer's and the manufacturer's profit functions under inventory misplacement are simplified as follows for demand uniformly distributed in $[0,\beta]$.

$$EPR_{RS}(Q_{RS}) = Q_{RS}[s\gamma + a(g + (p-s)\gamma) - w_{RS}] -$$
$$Q_{RS}^2 \frac{a^2(g_R + (p-s)\gamma)}{2\beta} - g_R\mu \tag{2.16}$$
$$EPM_{RS}(w_{RS}) = Q_{RS}[w_{RS} - c + (1-a)(1-\gamma)s + a(g_M + (1-\gamma)p)] -$$
$$Q_{RS}^2 \frac{a^2(g_M + p - s - (p-s)\gamma)}{2\beta} - g_M\mu \tag{2.17}$$

Theorem 6 *1. $EPR_{RS}(Q_{RS})$ is concave in Q_{RS} and the optimal Q_{RS} is*

$$Q_{RS}^* = \frac{\beta}{a^2}\left(\frac{\gamma s + a\,(g_R + (p-s)\gamma) - w_{RS}}{g_R + (p-s)\gamma}\right) \tag{2.18}$$

2. Under a revenue sharing contract, the supply chain is coordinated for

$$w_{RS}^* = \frac{(g_R(c-s) + c\gamma(p-s) + sg_C\gamma)}{p + g_C - s} < c. \tag{2.19}$$

In order to achieve the channel coordination, the manufacturer should choose the wholesale price such that (2.13) and (2.18) are equal to each other. Thus, the wholesale price given in (2.19) coordinates the channel. As observed in (2.19), under a revenue sharing mechanism, the manufacturer charges the retailer a lower wholesale price than her marginal cost of production.

Lemma 6 *In a supply chain coordinated by a revenue sharing contract where the optimal ordering quantity $Q_{RS}^* = Q_C^*$ is greater than zero, the following results hold.*

1. $\partial\left(EPR_{RS}(Q^*_{RS}(a);a)\right)/\partial a > 0$ *and* $\partial\left(EPM_{RS}(w^*_{RS}(a);a)\right)/\partial a > 0$.

2. $\frac{\partial EPM_{RS}(w^*_{RS}(a);a)}{\partial a} > \frac{\partial EPR_{RS}(Q^*_{RS}(a);a)}{\partial a}$ *if and only if* $\gamma < (1/2)\left(1 + \frac{(g_M - g_R)}{(p-s)}\right)$.

Both the retailer and the manufacturer always benefit from an increase in the inventory availability proportion in the retail store. As long as the proportion of the earnings that the manufacturer takes is significantly high, the manufacturer benefits more from an increase in the inventory availability level in the retail store. In case where the stock-out penalties are equal to each other, i.e. $g = g_M$, the manufacturer has to take more than 50% of the earnings of the retailer in order to benefit from an improvement in availability more than the retailer does. Another observation is that if $g_M > p - s + g_R$, then $(1/2)\left(1 + (g_M - g_R)/(p-s)\right)$ becomes greater than 1 and the condition $\gamma < (1/2)\left(1 + (g_M - g_R)/(p-s)\right)$ always holds. Thus, if the manufacturer's stock-out cost per unit is significantly higher than the retailer's stock-out cost per unit, the manufacturer benefits more from an increase in availability than the retailer.

2.3.3 Coordination with Buyback

One possible mechanism to coordinate the supply chain under misplaced inventory is a buyback contract (Pasternack, 1985). As in the uncoordinated case, the retailer misplaces l proportion of goods right after replenishment before demand is realized. According to the contract, at the end of the selling season, the retailer can return at most a fraction R of the purchased goods to the manufacturer at a credit, b per unit. We analyze two situations where according to the contract, the maximum fraction of inventory that the retailer can return is greater than the amount that is misplaced in the store ($R > l$) and the fraction is less than or equal to the amount that is misplaced in the store ($R \leq l$). In addition to Assumption 2, we assume that the retailer can not make profits over leftover inventory.

Assumption 3: $s < b \leq w < p$

2.3.3.1 Buyback Proportion $R > l$

This section focuses on a scenario where the maximum amount of inventory that the retailer can return to the manufacturer according to the contract is greater than the amount of misplaced inventory. Let us call Q_{BB1} the quantity that the retailer orders from the manufacturer under a buyback contract where $(R > l)$. If demand is less than $(1 - R)Q_{BB1}$, the retailer returns the manufacturer the maximum amount that she can return according to the contract and salvages the remainder at s per unit. If demand is greater than $(1 - R)Q_{BB1}$, then all of the remaining inventory is returned to the manufacturer after the selling season. If demand is even greater than the available inventory, then only misplaced inventory remains at the end of the selling season and all of them are returned to the manufacturer. The decentralized manufacturer salvages the entire inventory that is returned from the retailer at the end of the season. The retailer's profit function and the manufacturer's profit function are written as in (2.20) and (2.21), respectively. w_{BB1} indicates the wholesale price under a buyback contract where $(R > l)$.

$$EPR_{BB1}(Q_{BB1}) = -w_{BB1}Q_{BB1} + p\left\{\mu - \int_{aQ_{BB1}}^{\infty}(\xi - aQ_{BB1})f(\xi)d\xi\right\} + \tag{2.20}$$
$$s\int_{0}^{(1-R)Q_{BB1}}[(1-R)Q_{BB1} - \xi]f(\xi)d\xi - g_R\int_{aQ_{BB1}}^{\infty}(\xi - aQ_{BB1})f(\xi)d\xi +$$
$$b\{RQ_{BB1} + \int_{(1-R)Q_{BB1}}^{\infty}((1-R)Q_{BB1} - \xi)\,f(\xi)d\xi -$$
$$\int_{aQ_{BB1}}^{\infty}(aQ_{BB1} - \xi)\,f(\xi)d\xi\}$$

$$EPM_{BB1}(w_{BB1}) = Q_{BB1}(w_{BB1} - c) - \ \mathrm{g_{R1}}\int_{aQ_{BB1}}^{\infty}(\xi - aQ_{BB1})f(\xi)d\xi - \tag{2.21}$$
$$(b-s)\{RQ_{BB1} + \int_{(1-R)Q_{BB1}}^{\infty}[(1-R)Q_{BB1} - \xi]f(\xi)d\xi -$$
$$\int_{aQ_{BB1}}^{\infty}(aQ_{BB1} - \xi)\,f(\xi)d\xi\}$$

Expressions (2.20) and (2.21) are simplified as follows for uniform demand distributed in $[0, \beta]$.

$$EPR_{BB1}(Q_{BB1}) = Q_{BB1}\left(a(g_R + p - b) - (w_{BB1} - b)\right) - Q_{BB1}^2\left(\frac{(b-s)(1-R)^2 + (g_R + p - b)a^2}{2\beta}\right) - g_R\mu$$

$$EPM_{BB1}(w_{BB1}) = Q_{BB1}\left(w_{BB1} - c - (1-a)(b-s)\right) - Q_{BB1}^2\frac{(b-s)\left(a^2 - (1-R)^2\right)}{2\beta} - g_M\frac{(\beta - aQ_{BB1})^2}{2\beta}$$

Lemma 7 *$EPR_{BB1}(Q_{BB1})$ is strictly concave in Q_{BB1} and the optimal ordering quantity is as follows.*

$$Q_{BB1}^{*} = \frac{\beta\left(a(g_R + p - b) - (w_{BB1} - b)\right)}{a^2(g_R + p - b) + (b-s)(1-R)^2} \tag{2.22}$$

The idea behind coordination in a two sided monopoly is to make the retailer order the quantity that the centralized manufacturer would. Thus, the total supply chain profits in the decentralized two sided monopoly will be equal to the profits made if the manufacturer owned the retailer. Since the manufacturer is the leader, she adjusts the wholesale price so the quantity given by (2.13) optimizes the independent retailer's profit. Thereby, the wholesale price that will coordinate the channel must be chosen such that (2.13) and (2.22) become equivalent.

Theorem 7

1. *Allowing for unlimited returns for full credit is never optimal.*

2. *Allowing for unlimited returns for partial credit is system optimal for values chosen according to (2.23).*

$$w^*_{BB1} = \frac{b(g_C + p - c) + (c - s)(p + g_R)}{(p + g_C - s)} \tag{2.23}$$

When the manufacturer allows the retailer to return the remainder of the inventory after the selling season and fully refunds the retailer with the wholesale price for every item that the retailer returns, the channel will never be coordinated because the retailer will always have an incentive to order more than the centralized optimal ordering quantity. However, when the manufacturer allows returns up to 100% of the ordering quantity for partial credit, the contract serves to coordinate the channel. This is similar to an earlier result by Pasternack (1985) for a perfect inventory system that a strategy with unlimited returns with full credit is system suboptimal whereas that with unlimited returns with partial credit is system optimal.

Corollary 7 *For a coordinated supply chain with a buyback contract that allows for unlimited returns, having misplaced inventory at the retailer does not have any impact on the wholesale price decision of the manufacturer.*

Since (2.23) is equivalent to the relationship given by Pasternack (1985) for the policy with unlimited returns with partial credit, we can say that the retailer losing inventory does not have any effect on the wholesale price decision of the manufacturer if the contract allows unlimited returns for partial credit. In this setting, the manufacturer gets back the lost items regardless. For low values of demand, since the return proportion is already greater than the lost inventory proportion, the manufacturer takes back all the lost items and some or all of the leftover items depending on the return proportion.

Theorem 8

1. *Taking $l < R < 1$ fraction of the purchased items back for full credit is system optimal for values chosen according to (2.24).*

$$w^*_{BB1} = b = \frac{(c-s)(g_R+p) + s(p+g_C-s)(1-R)^2\left(\frac{1}{a}\right)\left(1-\frac{\tilde{a}}{a}\right)}{(c-s)+(p+g_C-s)(1-R)^2\left(\frac{1}{a}\right)\left(1-\frac{\tilde{a}}{a}\right)} \tag{2.24}$$

2. *Given $\tilde{a}(p+g_R-b) > (b-s)(1-R)^2\left(\frac{1}{a}\right)\left(1-\frac{\tilde{a}}{a}\right)$, taking $l < R < 1$ fraction of the purchased items back for partial credit is system optimal for values chosen according*

to (2.25)

$$w^*_{BB1} = \frac{c(g_R + p - b) - s(g_R + p) + b(p + g_C)}{(g_C + p - s)} - (b - s)(1 - R)^2 \left(\frac{1}{a}\right)\left(1 - \frac{\tilde{a}}{a}\right) \quad (2.25)$$

Although taking all back at full credit never coordinates the channel, taking a partial fraction of the purchased items back for a full refund of the wholesale price leads to coordination if equation (2.24) is satisfied. Taking a partial fraction of items back for partial credit is also system optimal if the wholesale price is chosen according to (2.25).

2.3.3.2 Buyback Proportion $R \leq l$

When the amount of returnable items is less than or equal to the amount of misplaced inventory at the retailer, the retailer returns the maximum allowed number of items to the manufacturer no matter what. Thus, the retailer's profit function can be formulated as in (2.26). Q_{BB2} represents the quantity that the retailer orders from the manufacturer and w_{BB2} represents the wholesale price under a buyback contract where $(R \leq l)$. Since the retailer returns RQ_{BB2} all the time, the manufacturer always salvages RQ_{BB2} . Thus, his problem can be formulated as in (2.27).

$$EPR_{BB2}(Q_{BB2}) = -w_{BB2}Q_{BB2} + p\left\{\mu + \int_{aQ_{BB2}}^{\infty} (aQ_{BB2} - \xi) f(\xi) d\xi\right\} + \ bRQ_{BB2} + \quad (2.26)$$

$$s\left\{\int_0^{aQ_{BB2}}((1-R)Q_{BB2}-\xi)\,f(\xi)d\xi+\int_{aQ_{BB2}}^{\infty}(l-R)Q_{BB2}f(\xi)d\xi\right\}-$$
$$g_R\int_{aQ_{BB2}}^{\infty}(\xi-aQ_{BB2})f(\xi)d\xi$$

$$EPM_{BB2}(w_{BB2})=Q_{BB2}(w_{BB2}-c)-(b-s)RQ_{BB2}-\mathrm{g}_{\mathrm{R}1}\int_{aQ_{BB2}}^{\infty}(\xi-aQ_{BB2})f(\xi)d\xi \quad (2.27)$$

Expressions (2.26) and (2.27) are simplified as follows for uniform demand distributed in $[0,\beta]$.

$$EPR_{BB2}(Q_{BB2})=Q_{BB2}\left(a(g_R+p-s)+R(b-s)-(w_{BB2}-s)\right)-Q_{BB2}^2\frac{(g_R+p-s)a^2}{2\beta}-g_R\mu$$
$$EPM_{BB2}(w_{BB2})=Q_{BB2}\left(w_{BB2}-c-R(b-s)\right)-g_M\frac{(\beta-aQ_{BB2})^2}{2\beta}$$

Lemma 8 *$EPR_{BB2}(Q_{BB2})$ is concave in Q_{BB2} and the optimal Q_{BB2} is*

$$Q_{BB2}^*=\frac{\beta}{a}\left(1-\frac{(w_{BB2}-s)-R(b-s)}{a(p+g_R-s)}\right). \quad (2.28)$$

The independent retailer will order (2.28) at the optimum. The coordination will be achieved if the manufacturer sets the wholesale price such that (2.28) and (2.13) will be equivalent.

Theorem 9

1. *Allowing for no returns is never system optimal.*

2. *Taking* $0 < R \leq l$ *fraction of the purchased items back for full credit is system optimal for values chosen according to (2.29).*

$$w^*_{BB2} = b = s + \frac{(g_R + p - s)(c - s)}{(p + g_C - s)(1 - R)} \tag{2.29}$$

3. *Given* $c - \frac{g_M(c-s)}{p+g_C-s} > b(1 - R) + Rs$ *, taking* $0 < R \leq l$ *fraction of the purchased items back for partial credit is system optimal for values chosen according to (2.30).*

$$w^*_{BB2} = c - \frac{(g_C - g_R)(c - s)}{p + g_C - s} + (b - s)R \tag{2.30}$$

Allowing for no returns is the same as the uncoordinated decentralized case and never coordinates the channel. However, taking partial fraction of items back coordinates the channel. The amount of inventory purchased back by the manufacturer from the retailer at the end of the season stays the same regardless of the availability level so even for the most general case, the wholesale price in (2.30) is observed to be independent of availability level in the retail store.

Corollary 8 *Given that the retailer orders a positive amount,*

1. *the wholesale price given in (2.30) is less than the wholesale price given in (2.25).*

2. *the manufacturer always makes negative profits in an environment coordinated buy a buyback contract where* $(R \leq l)$ *for any value of availability level.*

Given buyback is less than what the retailer loses, in order to get the retailer to order the centralized ordering quantity, the manufacturer has to give a much lower wholesale price for the same buyback and hence ends up making less profits. Therefore, it is undesirable for a manufacturer to try to use a buyback contract particularly with retailers who have high percentage losses of inventory.

2.3.3.3 Effects of Availability

So far, we have analyzed the impact of availability at the optimum on the manufacturer's profits when the manufacturer owns the retailer and on both the retailer's and the manufacturer's profits when they act uncoordinated. Now, we analyze the effect of a rise in availability in a coordinated environment where the coordination is achieved by a buyback contract. We consider both environments when $R > l$ and $R \leq l$. Note that since supply chain is coordinated, the optimal ordering amounts are equal to the centralized optimal ordering amount ($Q^*_{BB1} = Q^*_{BB2} = Q^*_C$).

Lemma 9 *In a supply chain coordinated by a buyback contract where $l < R < 1$ and the optimal ordering quantity $Q^*_{BB1} = Q^*_C$ is greater than zero, the following hold.*

1. $\partial \left(EPR_{BB1}(Q^*_{BB1}(a); a)\right) / \partial a > 0$

2. $\partial \left(EPM_{BB1}(w^*_{BB1}(a); a)\right) / \partial a > 0$

3. *(i)* $$\begin{array}{ll} \frac{\partial}{\partial a} EPM_{BB1}(w^*_{BB1}(a); a) \geq \frac{\partial}{\partial a} EPR_{BB1}(Q^*_{BB1}(a); a) & \text{if and only if} \quad \Delta \geq 0 \\ \frac{\partial}{\partial a} EPM_{BB1}(w^*_{BB1}(a); a) < \frac{\partial}{\partial a} EPR_{BB1}(Q^*_{BB1}(a); a) & \text{otherwise} \end{array}$$

where $\Delta = \tilde{a}(b + g_M - s) - \left(\tilde{a}(p + g_R - b) + 2(b - s)(2\tilde{a} - a)\left(\frac{1-R}{a}\right)^2\right)$. *(ii)* $\partial\Delta/\partial g_M > 0$

(iii) $\partial\Delta/\partial b > 0$.

When the buyback proportion is greater than the proportion of misplaced inventory ($R > l$), we find that both the retailer's and the manufacturer's profits unconditionally increase in the inventory availability proportion of the retail store. Since ($R > l$), the manufacturer buys back all of the lost items anyway. If demand is at the very low end (even less than $(1 - R)Q_{BB1}$), then an improvement in the availability level will not affect the manufacturer because the retailer will return the maximum number of items that she can according to the contract. On the other hand, for reasonably high demand values, it will always be better for the manufacturer if the retailer puts an item on shelf for customers instead of losing it. In such a case, fewer items will be returned to the manufacturer. Since the retailer is not allowed to make profits on the returned items, it is always a better situation for the retailer to make one item available to sales than returning it back to the manufacturer.

In order to understand expression Δ better, we take the partial derivative of Δ with respect to g_M and b separately. Δ is increasing in g_M meaning that for high values of manufacturer's share of the stock-out cost, the manufacturer is high likely to benefit more than the retailer for a slight increase in availability. We also find that as b increases, Δ increases. Thus, in this case, the manufacturer always prefers the retailer to make as many items available to the customers as possible because she does not want to pay large amounts of buyback credit. Therefore, for high values of buyback credit, the manufacturer is highly likely to benefit more than the retailer for an increase in availability at the retail store.

Lemma 10 *In a supply chain coordinated by a buyback contract where* $0 < R \leq l$ *and the optimal ordering quantity* $Q^*_{BB2} = Q^*_C$ *is greater than zero, the following results hold.*

1. $\partial \left(EPR_{BB2}(Q^*_{BB2}(a); a) \right) / \partial a > 0$

2. $\partial \left(EPM_{BB2}(w^*_{BB2}(a); a) \right) / \partial a > 0$

3. $$\begin{array}{ll} \frac{\partial}{\partial a} EPM_{BB2}(w^*_{BB2}(a); a) \geq \frac{\partial}{\partial a} EPR_{BB2}(Q^*_{BB2}(a); a) & \text{if and only if} \quad s - g_R \geq p - g_M \\ \frac{\partial}{\partial a} EPM_{BB2}(w^*_{BB2}(a); a) < \frac{\partial}{\partial a} EPR_{BB2}(Q^*_{BB2}(a); a) & \text{otherwise} \end{array}$$

Here the availability is allowed to increase as long as $0 < R \leq l$ is preserved. As indicated above, we discover that both the retailer's and the manufacturer's profits are non-decreasing functions of availability at the optimum. Therefore, both parties benefit from an upgrade of availability on the retailer's side in the presence of channel coordination. The manufacturer gains from an increase in the availability at least as much as the retailer benefits if $s - g_R \geq p - g_M$ holds. Since the salvage value is strictly less than the retail price of the product, for the manufacturer to benefit more than the retailer from an enhancement in availability, the manufacturer's share of the penalty cost must be significantly more than the retailer's share.

2.3.4 Comparison of Contracts

Now, we compare the profits of each party under coordinated supply chains which are coordinated by different mechanisms.

Lemma 11 *Given that the supply chain is coordinated and hence* $Q_C^* = Q_{BB1}^* = Q_{BB2}^* = Q_{RS}^* > 0$ *, the decentralized manufacturer*

1. *always prefers a buyback contract where* $R > l$ *to a buyback contract where* $R \leq l$ *.*

2. *always prefers a revenue sharing contract to a buyback contract where* $R \leq l$ *.*

3. *always prefers a buyback contract where* $R > l$ *to a revenue sharing contract if* $\gamma > \frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *where* $\frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *is always less than 1.*

4. *always prefers a revenue sharing contract to a buyback contract where* $R > l$ *if* $\gamma < \frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *where* $\frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *is always less than 1.*

5. *is always indifferent between a buyback contract where* $R > l$ *and a revenue sharing contract if* $\gamma = \frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *where* $\frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *is always less than 1.*

As given by Corollary 8, the manufacturer always makes negative profits in a supply chain coordinated by a buyback contract where $(R \leq l)$. Therefore, between a buyback contract where $(R > l)$ and another buyback contract where $(R \leq l)$, he will always choose the first one. Similarly, when a buyback contract where $(R \leq l)$ is compared to a revenue sharing contract, the manufacturer always chooses the revenue sharing. Cachon and Lariviere (2005) show that there exists a unique buyback contract which generates the same profits as a revenue sharing contract. They do not consider the problem of misplaced inventory and they assume unlimited buyback. We show that for our case with a buyback proportion being greater than the lost inventory proportion and under the problem of misplacement, one can always find a buyback contract which is equivalent to the revenue sharing contract in terms of the profits that it generates. However, one can never find an equivalent buyback contract to the revenue sharing if buyback proportion is less than the proportion of lost inventory. In addition, as long as the proportion of the earnings that the retailer takes is less than a threshold value, the manufacturer always chooses revenue sharing over a buyback contract where $(R > l)$.

It is to be noted that the decentralized retailer's profits are just in the opposite direction of the decentralized manufacturer's (see Lemma 12 in Appendix A).

2.4. Extensions

In the previous sections, we have assumed that inventory misplacement is a constant fraction. We have further assumed that inventory misplacement does not depend on the size of the order. Here, we relax these assumptions one at a time for a general demand distribution.

2.4.1 Inventory Misplacement as a Random Variable

In the previous sections, we have assumed that a constant fraction of items get misplaced in the stores. Although modeling this problem gives us the advantage of analytical tractability, it sacrifices to incorporate the variability of the rate of inventory misplacement. In this section, we assume that availability is a random variable. The sequence of events for the supply chain of interest is exactly as assumed before with the exception of having a random number of items getting misplaced. We assume that a is the actual rate of inventory availability, A

is the random variable and $(.)^+ = max\{0, .\}$. Throughout this section, all random variables are assumed to be nonnegative with finite expected values. We will use $E[A]$ to denote the expected value of inventory availability and μ to denote the expected value of demand. We will use $H(.)$ and $h(.)$ to refer to the probability distribution function of inventory availability and corresponding probability density function, respectively.

2.4.1.1 Centralized firm

The profit function of the centralized firm is as given in (2.31).

$$\begin{aligned} \pi_C &= p\, min\{x, aQ_C\} - cQ_C - g_C(x - aQ_C)^+ + s(aQ_C - x)^+ + \\ &\quad s(1-a)Q_C \end{aligned} \tag{2.31}$$

$$= (p - s)x - Q_C(c - as) - (p + g_C - s)(x - aQ_C)^+ + s(1-a)Q_C \tag{2.32}$$

$$\begin{aligned} EPC(Q_C) &= (p - s)\mu - Q_C(c - E[A]s) + s(1 - E[A])Q_C - \\ &\quad (p + g_C - s)\int_0^\infty \int_0^{x/Q_C} (x - aQ_C)dH(a)dF(x)dx \end{aligned} \tag{2.33}$$

Theorem 10

1. *$EPC(Q_C)$ is concave in Q_C and the optimum order Q_C^* solves (2.34).*

$$\int_0^\infty \int_0^{x/Q_C^*} a dH(a)dF(x)dx = \frac{c - s}{p + g_C - s} \tag{2.34}$$

2. Q_C^* is always larger than the corresponding optimum order under a pure proportional random yield model.

Theorem 10 provides the optimum order for the centralized firm under a random rate of inventory availability. Due to the salvage value that one would get at the end of the period, the optimal stock under inventory misplacement is always higher than than the optimal stock under random yield.

Let $Y \leq_{st} X$ denote Y being stochastically smaller than X. A random variable Y is stochastically smaller than a random variable X provided $P(Y > u) > P(X > u)$ for every real u (see Müller and Stoyan (2002) for an extensive background on stochastic ordering).

Lemma 13 *Suppose two random variables A and $\hat{A}$ where $A \leq_{st} \hat{A}$. If $\hat{Q_C}^* > Q_C^*$, a stochastically larger inventory availability leads to greater profits.*

In general, a stochastically larger yield does not increase optimal profits under random yield. Lemma 13 indicates that such a stochastic ordering increases the profits given an ordering of the optimum values. This finding is in line with the findings of Gupta and Cooper (2005) for a pure random yield.

2.4.1.2 Decentralized Supply Chain-No Coordination

In this section, we assume that the manufacturer is the Stackelberg leader and the sequence of events is the same as given in Section 3.3 except that we assume a random inventory availability here for the retailer. The retailer's and the manufacturer's profits can be written as in (2.35) and (2.36), respectively. Hence, the expected profit function of the retailer $EPR_D(Q_D)$ is equal to (2.33) with the transformation of c to w. The expected profit function of the manufacturer $EPM_D(w)$ is given by (2.37).

$$\pi_D^R = p\, min\{x, aQ_D\} - wQ_D - g_R(x - aQ_D)^+ + s(aQ_D - x)^+ + \tag{2.35}$$
$$s(1-a)Q_D$$
$$\pi_D^M = (w_D - c)Q_D - g_M(x - aQ_D)^+ \tag{2.36}$$
$$EPM_D(w) = (w_D - c)Q_D - g_M \int_0^\infty \int_0^{x/Q_D} (x - aQ_D)dH(a)dF(x)dx \tag{2.37}$$

Theorem 11

1. *$EPR_D(Q_D)$ is concave in Q_D and the optimum order Q_D^* solves the following relation.*

$$w_D^* = s + (p + g_R - s)\int_0^\infty \int_0^{x/Q_D} adH(a)dF(x)dx \tag{2.38}$$

2. *$EPM_D(w_D^*(Q_D))$ is concave in Q_D if and only if the following expression is negative.*

$$(p + g_R - s - g_M)\int_0^\infty \frac{x^2}{Q_D^3} h(x/Q_D)dF(x) + (p + g_R - s)\int_0^\infty \frac{x^3}{Q_D^4} h'(x/Q_D)dF(x) \tag{2.39}$$

3. *If $EPM_D(w_D(Q_D))$ is concave, the optimum Q_D is given by the following.*

$$(p + g_R - s + g_M)\int_0^\infty \int_0^{x/Q_D} adH(a)dF(x)dx - c + s - \tag{2.40}$$
$$(p + g_R - s)\int_0^\infty \frac{x^2}{Q_D^2} h(x/Q_D)dF(x)dx = 0$$

There exists a unique optimum order and a wholesale price if and only if (2.39) is negative. Then, the optimum order is found by solving (2.40).

Corollary 9 *If the inventory availability is uniformly distributed and* $g_M > p + g_R - s$*, then* $EPM_D(w_D^*(Q_D))$ *is concave in* Q_D *.*

Corollary 9 shows that if the manufacturer's goodwill cost is sufficiently high and the inventory availability is characterized by a uniform distribution, the manufacturer's profit function is concave leading to a unique optimum wholesale price and a unique order quantity.

For the subsequent results, we assume that the manufacturer's profit function is always concave.

Lemma 14

1. *Suppose two random variables* A *and* $\hat{A}$ *where* $A \leq_{st} \hat{A}$*. If* $\hat{Q_D}^* > Q_D^*$*, a stochastically larger inventory availability leads to greater profits for the retailer.*

2. *Assume that inventory availability distribution* A *changes to* $\hat{A}$*. The manufacturer benefits from this if and only if the following is positive.*

$$(c-s)(Q_D^* - \hat{Q_D}^*) + (p + g_R + g_M - s)\hat{Q_D}^* \int_0^\infty \int_0^{x/\hat{Q_D}^*} a d\hat{H}(a) dF(x) dx - \quad (2.41)$$
$$(p + g_R + g_M - s)Q_D^* \int_0^\infty \int_0^{x/Q_D^*} a dH(a) dF(x) dx +$$
$$g_M [\int_0^\infty x[H(x/Q_D^*) - \hat{H}(x/\hat{Q}_D^*) dF(x)]$$

We find that a stochastically larger yield has positive effects on the retailer's profit if the corresponding optimum orders are ordered according to Lemma 14. The effects of a change in the inventory availability on the manufacturer's profit is a messy expression and we are unable to derive a general conclusion.

2.4.1.3 Decentralized Supply Chain-Revenue Sharing

Here, we study coordinating the supply chain with a revenue sharing contract as given in Section 2.2.2.2. The retailer's and the manufacturer's profits are given by (2.42) and (2.43). Corresponding expected profits are given by (2.44) and (2.45).

$$\begin{aligned} \pi_{RS}^{R} &= \gamma p\, min\{x, aQ_{RS}\} - w_{RS}Q_{RS} - g_R(x - aQ_{RS})^+ + \gamma s(aQ_{RS} - x)^+ + \\ &\quad \gamma s(1-a)Q_{RS} \\ &= \gamma(p-s)x - Q_{RS}(w_{RS} - \gamma as) - (\gamma p + g_R - \gamma s)(x - aQ_{RS})^+ + \\ &\quad \gamma s(1-a)Q_{RS} \end{aligned} \tag{2.42}$$

$$\begin{aligned} \pi_{RS}^{M} &= (w_{RS} - c)Q_{RS} - g_M(x - aQ_{RS})^+ + (1-\gamma)[p\, min\{x, aQ_{RS}\} + \\ &\quad s(aQ_{RS} - x)^+ + s(1-a)Q_{RS}] \\ &= (w_{RS} - c)Q_{RS} + (1-\gamma)(p-s)x + (1-\gamma)ac_3Q_{RS} - \\ &\quad [(1-\gamma)(p-s) + g_M](x - aQ_{RS})^+ + (1-\gamma)s(1-a)Q_{RS} \end{aligned} \tag{2.43}$$

$$EPR_{RS}(Q_{RS}) = \gamma(p-s)\mu - Q_{RS}(w_{RS} - \gamma E[A]s) + \gamma s(1 - E[A])Q_{RS} - \tag{2.44}$$

$$
\begin{aligned}
EPM_{RS}(w_{RS}) = {} & (\gamma(p-s)+g_R)\int_0^{\infty}\int_0^{x/Q_{RS}}(x-aQ_{RS})dH(a)dF(x)dx \\
& (w_{RS}-c)Q_{RS} - g_M\int_0^{\infty}\int_0^{x/Q_{RS}}(x-aQ_{RS})dH(a)dF(x)dx + \\
& (1-\gamma)[(p-s)\mu + Q_{RS}s] + \\
& (1-\gamma)(p-s)\int_0^{\infty}\int_0^{x/Q_{RS}}(x-aQ_{RS})dH(a)dF(x)dx
\end{aligned} \tag{2.45}
$$

Next, we identify the conditions for the supply chain to be coordinated according to a revenue sharing contract.

Theorem 12 *The supply chain can be coordinated by a revenue sharing contract if the wholesale price is set to w^*_{RS} given below.*

$$
w^*_{RS} = \frac{(\gamma(p-s)+g_R)(c-s)}{p+g_C-s} + s\gamma \tag{2.46}
$$

Expression (2.46) is equal to the wholesale price that coordinates the supply chain when demand distribution is general and the inventory availability is assumed to be a constant fraction. Further, stochasticity of the inventory availability does not have any effect on the coordination wholesale price.

Lemma 15

1. *Suppose two random variables A and $\hat{A}$ where $A \leq_{st} \hat{A}$. If $\hat{Q_C}^* > Q^*_C$, a stochastically larger inventory availability leads to greater profits for the retailer.*

2. *Assume that inventory availability distribution A changes to $\hat{A}$. The manufacturer benefits from this if and only if the following is positive.*

$$(\frac{(\gamma(p-s)+g_R)(c-s)}{p+g_C-s}+s-c)(\hat{Q}^*_{RS}-Q^*_{RS})-[(1-\gamma)(p-s))+g_M] \tag{2.47}$$
$$[\int_0^\infty \int_0^{x/\hat{Q}^*_{RS}}(x-a\hat{Q}^*_{RS})d\hat{H}(a)dF(x)dx-\int_0^\infty \int_0^{x/Q^*_{RS}}(x-aQ^*_{RS})dH(a)dF(x)dx]$$

Similar to the previous sections, we characterize a condition for which a stochastically larger inventory availability benefits the retailer. However, we are not able to derive a general conclusion for the manufacturer's profits.

2.4.2 Inventory Availability as a Function of Order Size

In this section, we explore settings where inventory availability is an increasing function of the order size. For example, if the misplacement is due to stocking on the shelves, then one might expect a greater percentage of products to be shelved correctly when the quantity is larger. We denote the number of available items during the selling season by $A(Q) \leq Q$. We provide the expressions for the optimum stocking decisions for the centralized firm, for an uncoordinated decentralized supply chain and a coordinated supply chain by a revenue sharing contract. In all of the three settings, we assume that the sequence of events as before. That is, as soon as the order arrives at the firm (retail store), $Q - A(Q)$ of items get misplaced and $A(Q)$ items become available for customers to purchase. At the end of the selling season, the misplaced $Q - A(Q)$ number of items are found and salvaged along with $(x - A(Q))^+$ remaining items after the selling season if any remains where x denotes demand. Further, we require Assumption 1 in all settings and require Assumption 2 in the uncoordinated supply chain.

We first analyze the centralized firm, whose profit function is given by (2.48).

$$EPC(Q) = -cQ + p(\mu + \int_{A(Q)}^{\infty} [A(Q) - x]dF(x)) - g_C \int_{A(Q)}^{\infty} [x - A(Q)]dF(x) + \tag{2.48}$$
$$s\left\{[Q - A(Q)] + \int_{0}^{A(Q)} [A(Q) - x]dF(x)\right\}$$

Theorem 13 *$EPC(Q)$ is concave in Q if $A(Q)$ is an increasing concave function and the optimum Q solves (2.49).*

$$A'(Q)(1 - F(A(Q)) = \frac{c - s}{p + g_C - s} \tag{2.49}$$

A concave increasing function of quantity is sufficient to guarantee the concavity of the profit function. Hence, solving expression (2.49) for Q gives us the optimum order.

Next, we study an uncoordinated decentralized supply chain. The retailer's and the manufacturer's profits are given by (2.50) and (2.51).

$$EPR_D(Q) = -wQ_+p[\mu + \int_{A(Q)}^{\infty} [A(Q) - x]dF(x)] - g_R \int_{A(Q)}^{\infty} [x - A(Q)]dF(x) + \tag{2.50}$$
$$s\left\{[Q - A(Q)] + \int_{0}^{A(Q)} [A(Q) - x]dF(x)\right\}$$
$$EPM_D(w) = (w - c)Q - g_M \int_{A(Q)}^{\infty} [x - A(Q)]dF(x) \tag{2.51}$$

Theorem 14

1. *$EPR_D(Q)$ is concave in Q if the $A(Q)$ is an increasing concave function and the optimal order Q^* solves the following expression.*

$$w^* = A'(Q)\bar{F}(A(Q))(p + g_R - s) + s \tag{2.52}$$

2. *$EPM_D(w^*(Q))$ is concave in Q if and only if the following expression is negative where $\bar{F}(.) = 1 - F(.)$ and $\boldsymbol{R}[Q]$ is given by (2.54).*

$$\boldsymbol{R}[Q] + g_M \left\{A''(Q)\bar{F}(A(Q)) - (A'(Q))^2 f(A(Q))\right\} \tag{2.53}$$

$$\begin{aligned}\boldsymbol{R}[Q] = {} & Q\left\{A'''(Q)\bar{F}(A(Q)) - 3A'(Q)A''(Q)f(A(Q)) - (A'(Q))^2 f'(A(Q))\right\} + \\ & 2\left\{A''(Q)\bar{F}(A(Q)) - (A'(Q))^2 f(A(Q))\right\}(p + g_R - s)\end{aligned} \tag{2.54}$$

A general function of availability complicates concavity check in the uncoordinated supply chain as given by Theorem 14. However, once concavity is assured, the optimum order is found by using the relation given by (2.52).

Next, we analyze a supply chain coordinated by a revenue sharing contract. The profits of the retailer and the manufacturer are given by (2.55) and (2.56), respectively.

$$EPR_{RS}(Q) = -wQ - g_R \int_{A(Q)}^{\infty} [x - A(Q)]dF(x) + \gamma(p\left\{\mu + \int_{A(Q)}^{\infty} [A(Q) - x]dF(x)\right\} + \tag{2.55}$$
$$s\left\{[Q - A(Q)] + \int_{0}^{A(Q)} [A(Q) - x]dF(x)\right\})$$

$$EPM_{RS}(w) = (w - c)Q - g_M \int_{A(Q)}^{\infty} [x - A(Q)]dF(x) + \tag{2.56}$$
$$(1 - \gamma)(p\left\{\mu + \int_{A(Q)}^{\infty} [A(Q) - x]dF(x)\right\}$$
$$+s\left\{[Q - A(Q)] + \int_{0}^{A(Q)} [A(Q) - x]dF(x)\right\})$$

Theorem 15 *The supply chain can be coordinated by a revenue sharing contract if the wholesale price is set to w^*_{RS} given below.*

$$w^*_{RS} = \frac{(\gamma(p - s) + g_R)(c - s)}{p + g_C - s} + s\gamma \tag{2.57}$$

Inventory misplacement does not have any effect on the coordination wholesale price even when we assume inventory availability as a function of quantity.

In this chapter, we study vertically integrated and decentralized supply chains under inventory misplacement. We prove that the behavior of the optimum order quantity with changes in inventory availability depends on the generalized failure rate of the demand distribution evaluated at the optimum number of available items. Further, we give conditions for the supply chains under inventory misplacement to be coordinated by revenue sharing and buyback contracting mechanisms and show that the manufacturer may indeed extract more benefits than the retailer does from a reduction in inventory misplacement.

Chapter 3

Misplaced Inventory and RFID: Information and Coordination

3.1. Introduction

Inventory misplacement is expensive and is prevalent in many industries. For example, Delta Airlines operates at a very high baggage-handling efficiency rate (99%). However, the 1% of misplaced bags costed Delta about $100 million a year (Airline Business Report, 2004). Similarly, the Vatican Library, which has approximately two million books, manuscripts and other valuable items, had to close the library for an entire month each year (Wireless News, 2004). An empirical study by Raman, DeHoratius and Ton (2001) show that the inventory loss proportions can reach significant numbers in retail stores up to 16% in some cases.

When items are misplaced in the store, they become unavailable to the customers until they are found. One of ways to handle misplaced inventory is through Radio Frequency

Identification Devices (RFID). Through this technology, it is possible to track an item on which an RFID tag is attached. Implementation of RFID is beginning on a large scale at the distribution level, i.e. pallets are being tagged. This should reduce misplacement at the warehouse level. Firms started to perceive the RFID technology as a development beyond being only an extension to bar coding when Walmart declared the requirement for her major suppliers to tag each pallet with an RFID tag. It gained momentum with several firms including Unilever, United Biscuits, Fluoroware Inc., Ford Motor Company and Toyota launching pilot projects of RFID (Angeles, 2005). Although pallet-level RFID applications provide some visibility to firms, they may not deliver all the strategic benefits of RFID particularly in the case of retailers'. As a result, Marks & Spencer, Tesco and Metro Group, three European retailers have already started testing RFID at the item-level (Sullivan, 2005). The next level of implementation will be at the store level where RFID should enable item level tracking and identification.

In addition to various variable costs, RFID adopters also have to consider the upfront costs for deploying the new technology in the supply chain (Kambil and Brooks, 2002). For example, a typical consumer packaged goods manufacturer was estimated to spend between \$13 million and \$23 million for shipping 50 million cases per year including various fixed costs and variable costs (Asif and Mandviwalla, 2005).

As these big retailers (along with Walmart) have popularized RFID, a common opinion has been that the main beneficiaries of RFID will be the downstream players in the supply chain (Kevan, 2004; Agarwal, 2001). In addition to being able to track the items, the retailers are expected to benefit from store and warehouse labor reductions and reduced inventory. On the other hand, the benefits of RFID to manufacturers have been anticipated in improved demand planning, stricter quality control, reduced scrap, waste and rework, and more accurate inventory data (Bearing Point, 2005).

3.1.1 Related Literature

Studies related to RFID in inventory management are relatively new. Lee and Özer (2005) provide a review of the literature about the emerging technology RFID by examining the papers under three main categories: misplacement of inventory, shrinkage and transaction errors. As indicated by Raman, Ton and DeHoratius (2003), inventory inaccuracy and misplaced inventory are major operational inefficiencies. Several researchers have developed models for inventory inaccuracy (see Iglehart and Morey (1972), Kök and Shang (2004), Atali, Lee and Özer (2005), DeHoratius, Mersereau and Schrage (2005) and Heese(2006)). Models for inventory misplacement problem have also been studied by researchers (see Çamdereli and Swaminathan (2005) and Rekik, Sahin and Dallery (2005)). In the following passages we will relate our work to other research that have explicitly incorporated RFID in their analysis of inventory inefficiencies.

Gaukler, Seifert and Hausman (2004) study item-level RFID where the retailer uses in-store sales to estimate his demand distribution. The tagging expenses are assumed to be shared and no fixed cost of implementation is taken into account. They research the behavior of the optimal stocking quantity and the profits of the parties for changes in the responsiveness level of the retailer and the tag price. As opposed to their model where RFID improves demand visibility, in our model RFID improves supply visibility. In this chapter, besides tagging expenses, we take the fixed cost of RFID implementation into consideration and allow the parties to share the fixed cost of adoption. We study both scenarios where the tagging expenses are covered by only the manufacturer and only the retailer. Additionally, we also characterize the incentives for the parties to invest in RFID in vertically integrated, uncoordinated and coordinated environments.

Rekik, Sahin and Dallery (2005) computationally study the benefits of using Auto-ID systems for the retailer to completely remove the inventory misplacement by comparing such systems with those where the retailer is not aware of the misplacement problem and where the retailer knows the misplacement problem and optimizes accordingly. Our work is distinguished from their model in that we also consider coordination issues in a supply chain with a retailer and a manufacturer. We analytically derive the conditions for both the retailer and the manufacturer to benefit from the RFID technology under both centralized and decentralized environments.

Heese (2006) studies the effect of inventory inaccuracy on supply chain performance as well as the impact of double marginalization in a decentralized supply chain on the parties' incentives to adopt RFID. Assuming fixed cost of investment zero, the author finds that the manufacturer, who is the Stackelberg leader, is better off with RFID technology if and only if the retailer is better off. In comparison to the above work, we consider a misplaced inventory problem and assume sharing of fixed cost of implementation. We find that the incentives of the retailer and the manufacturer to adopt RFID are not always aligned under inventory misplacement. We also investigate the changes in the incentives of the parties to adopt the technology as the mean and the variance of the demand distribution changes.

Since RFID is a recent development in information technology, literature about adoption of EDI is also relevant to our research. Decisions concerning EDI adoption have been studied in different settings. Wang and Seidmann (1995) showed that while one supplier's EDI adoption can increase her own profit, it may decrease the profits of the other suppliers and a supplier's incentive in EDI adoption decreases in the number of suppliers adopting EDI. While assuming a direct benefit for the buyer, they show that suppliers installing EDI can indirectly benefit from it. In this chapter, we research the effects of RFID adoption on the parties. Instead of assuming a direct benefit to any party, we model the incentives as a result of solving the problem of inventory misplacement. Readers are referred to Narayanan, Marucheck and Handfield (2007) for a research review of EDI literature.

Çamdereli and Swaminathan (2005) is a closely related paper to our work. The main focus of the authors is to characterize the optimal policy for a single product inventory problem under misplaced inventory. They expand the analysis to the two stage supply chain in a single period setting in that paper. Using uniform distribution for demand, they show that behavior of the optimal ordering quantity for an increase in the availability level depends on the existing availability level and attractiveness of the items to stock in vertically integrated supply chains; therefore, it does not monotonically increase or decrease in all cases. Under the decentralized setting, they analyze uncoordinated supply chains as well as those coordinated by buy-back and revenue sharing mechanisms. In their work, they do not consider the costs and incentive issues arising as a result of implementation of an information system such as RFID. In this chapter, we incorporate the cost of removing misplacement using RFID and focus on the incentives of the parties in the supply chain to adopt such a technology.

3.1.2 Our Work and Contributions

In this chapter, we study benefits of RFID implementation on a two stage supply chain experiencing misplaced inventory at the retailer. We consider both fixed and variable costs for RFID implementation and assume that the fixed cost is shared between the two using an arbitrary proportion while the variable cost could be charged to either the retailer or the manufacturer. We study three different settings - (1) a vertically integrated supply chain where the manufacturer owns the retailer, (2) a decentralized and uncoordinated supply chain with a wholesale contract and (3) a supply chain that is coordinated with a revenue sharing contract. In all cases, the manufacturer is assumed to be the leader. We derive conditions under which it is beneficial to invest in the RFID technology and establish threshold on variable tagging cost where the manufacturer or the retailer or both or neither benefit from such investment. We also provide conditions to coordinate the supply chain with a revenue sharing contract. Finally, we establish the effect of changes in mean and variance of a uniform demand on the incentives for investing in RFID.

The remainder of the chapter is organized as follows. In Section 3.2 we study the vertically integrated supply chain. Section 3.3 focuses on the decentralized supply chain. Section 3.3.1 focuses on the uncoordinated supply chain where no mechanism exists for coordination and Section 3.3.2 studies revenue sharing.

3.2. Centralized firm

In this section, we study a vertically integrated supply chain where the manufacturer owns the retailer. For convenience, we will refer to the centralized manufacturer/retailer as the *centralized firm.* We consider a single-seasonal product. We assume that misplacement of inventory occurs immediately on delivery. Hence, as soon as the order is complete, l $(0 < l = 1 - a < 1)$ proportion of the order gets misplaced in the store and only a proportion of the order is available to the end customers during the selling season. Inventory can be misplaced due to various reasons. Customers or employees can misplace items on the shop floor or items can be misplaced in storage areas. In most real life settings, misplacement happens gradually over time. We assume instantaneous misplacement for analytical tractability. Therefore, one can think of our model as an approximation to a real life setting where the costs can be perceived as an upper bound on the negative effects of inventory misplacement. Demand occurs right after the misplacement. When the firm invests in RFID, we assume that each item is tagged with an RFID chip so the availability proportion becomes 100% $(a = 1)$. Since the implementation of RFID in supply chains is relatively new, efficiency rate of the readers can be below 100% in some instances. This can also be incorporated into our model easily by assuming that availability goes to $\hat{a} < 1$ instead of 1. The firm adopting RFID incurs a fixed cost of K and a variable cost of $t > 0$ per unit when she implements RFID. The centralized firm which does not invest in RFID technology orders Q_C and the firm investing in the technology orders $Q_{C,t}$ at c per unit. Each available unit is sold for p during the sales season and any leftover inventory is salvaged at s per unit. If the firm does not invest in the technology, she always salvages the lost inventory (lQ_C) at the end of the season. There may be more leftover inventory if demand is less than the available inventory (aQ_C). If demand is more than the available items (aQ_C when items are misplaced and $Q_{C,t}$ when RFID is implemented), the firm incurs g_C per each unit of excess demand, i.e. a penalty cost for stock out. To gain analytical insights through closed form solutions, we assume that demand is uniformly distributed in $[0, \beta]$. Our results for the firm can be extended to several other generic distributions although the expressions are cumbersome. We make several assumptions on the cost parameters. Assumption 1 holds for both centralized and decentralized settings.

Assumption 1: $s < c < p$

Let Q_N^*, Q_C^* and $Q_{C,t}^*$ denote the optimal quantities. The centralized firm's expected profits without any misplaced inventory, under misplaced inventory and after implementing RFID are formulated as in (3.1), (3.2) and (3.3) respectively where μ is expected value of demand, $E[\,\xi\,]$.

$$\begin{aligned} EPCN(Q_N) = -cQ_N + p(\mu + \int_{Q_N}^{\infty}(Q_N - \xi)f(\xi)d\xi) + s(\int_0^{Q_N}(Q_N - \xi)f(\xi)d\xi \\ -g_C\int_{Q_N}^{\infty}(\xi - Q_N)f(\xi)d\xi \end{aligned} \tag{3.1}$$

$$EPC(Q_C) = -cQ_C + p(\mu + \int_{aQ_C}^{\infty} (aQ_C - \xi) f(\xi) d\xi) + s(lQ_C + \int_0^{aQ_C} (aQ_C - \xi) f(\xi) d\xi$$
$$-g_C \int_{aQ_C}^{\infty} (\xi - aQ_C) f(\xi) d\xi \quad (3.2)$$

$$EPCt(Q_{C,t}) = -(c+t)Q_{C,t} + p(\mu + \int_{Q_{C,t}}^{\infty} (Q_{C,t} - \xi) f(\xi) d\xi) + s \int_0^{Q_{C,t}} (Q_{C,t} - \xi) f(\xi) d\xi$$
$$-g_C \int_{Q_{C,t}}^{\infty} (\xi - Q_{C,t}) f(\xi) d\xi \quad (3.3)$$

The expressions (3.1), (3.2) and (3.3) become the following for demand uniformly distributed in $[0, \beta]$.

$$EPCN(Q_N) = Q_N(p + g_C - c) - Q_N^2(p + g_C - s)/(2\beta) - g_C\mu$$

$$EPC(Q_C) = Q_C[a(p + g_C - s) - (c - s)] - Q_C^2 a^2 (p + g_C - s)/(2\beta) - g_C\mu$$

$$EPCt(Q_{C,t}) = Q_{C,t}(p + g_C - c - t) - Q_{C,t}^2(p + g_C - s)/(2\beta) - g_C\mu - K$$

Theorem 16 (Çamdereli and Swaminathan, 2005)

In a supply chain under inventory misplacement, the following findings hold.

1. *$EPC(Q_C)$ is concave in Q_C.*

2.

$$Q_C^* = \begin{cases} (\beta/a)(1-\tilde{a}/a) > 0 & \textit{if and only if } a > \tilde{a} \textit{ where } \tilde{a} = \frac{c-s}{p+g_C-s} < 1 \\ 0 & \textit{otherwise} \end{cases} \tag{3.4}$$

As shown in Çamdereli and Swaminathan (2005), $EPC(Q_C)$ is concave in Q_C and the optimal order under inventory misplacement is $Q_C^* = (\beta/a)(1-\tilde{a}/a)$ if and only if $a > \tilde{a}$ and is zero otherwise where $\tilde{a} = (c-s)/(p+g_C-s)$. Since $EPCt$ is a newsvendor formulation with a constant setup cost where the marginal cost of product is inflated by t, $EPCt$ is concave in $Q_{C,t}$. Hence, $Q_{C,t}^* = \beta(p+g_C-c-t)/(p+g_C-s)$ and is positive if and only if $t < p+g_C-c$. Due to the tag price, it is clear that the optimal order under RFID is always smaller than the optimal order under no inventory misplacement since $Q_N^* = \beta(1-\tilde{a})$.

All of the proofs are given in the Appendix B.

Lemma 16

1. *Given $Q_{C,t}^* > 0$, there exists a threshold value $\underline{a} < min\{1, \tilde{a}/(1-\tilde{a})\}$ such that $Q_{C,t}^* > Q_C^*$ if and only if $a < \underline{a}$.*

2. (a) *Given $Q_{C,t}^*, Q_C^* > 0$, there exists a threshold value $\underline{t}[a] = \frac{1}{a^2}[c-s-a(p+g_C-s)+a^2(p+g_C-c)]$ such that $Q_{C,t}^* > Q_C^*$ if and only if $t < \underline{t}[a]$.*

   (b) *$\underline{t}[a] > 0$ if and only if $a < min\{1, \tilde{a}/(1-\tilde{a})\}$ where $\tilde{a} = (c-s)/(p+g_C-s)$.*

Lemma 16.1 states that as the inventory availability varies, the value of Q_C^* and $Q_{C,t}^*$ become equivalent at only one value of availability $a < \underline{a} < min\{1, \tilde{a}/(1-\tilde{a})\}$. Figure 3.1

gives two numerical examples to compare Q_C^* with $Q_{C,t}^*$ for the two cases where $\tilde{a} = 3/16 < 0.5$ and $\tilde{a} = 9/16 > 0.5$. As also numerically illustrated by these examples, $Q_{C,t}^*$ crosses Q_C^* only at one value of availability ($\underline{a} \approx .228$ in the graph where $\tilde{a} < .5$ and $\underline{a} \approx .870$ in the graph where $\tilde{a} > .5$), for which Q_C^* is always less than Q_N^*.

Although one may expect to order less under RFID than under inventory misplacement due to 100% inventory availability, it may not be the case if the tag price is sufficiently low. In Lemma 16.2.(a), we provide a threshold value ($\underline{t}[a]$) on the tag price, below which it is always optimal to order more under RFID than under inventory misplacement for appropriate ranges of inventory availability. The expression $\tilde{a}$ is the probability of stocking out in a newsvendor problem where no misplacement occurs. One may also interpret $\tilde{a}$ as how unattractive products are to stock. Ceteris paribus, as the unit cost of order increases, $\tilde{a}$ increases or as the sales price of the product increases, $\tilde{a}$ decreases. When $\tilde{a} < 0.5$, i.e. items are relatively attractive to stock, and $a < \tilde{a}/(1-\tilde{a})$, $\underline{t}[a]$ is positive. Hence, the optimal order under RFID is indeed larger than the optimal order under inventory misplacement if the tag price is less than $\underline{t}[a]$. For the same case ($\tilde{a} < 0.5$), $Q_C^* > Q_N^*$ for $a > \tilde{a}/(1-\tilde{a})$. Thus, $Q_{C,t}^*$ is always smaller than Q_C^* for this range of availability values. Obviously when items are not attractive to stock ($\tilde{a} \geq 0.5$) and the tag price is expensive ($t > \underline{\mathrm{t}}[a]$), the optimum order under RFID is always smaller than under inventory misplacement.

Next, we study the centralized profit under RFID and the incentives of the firm to invest in RFID.

Theorem 17

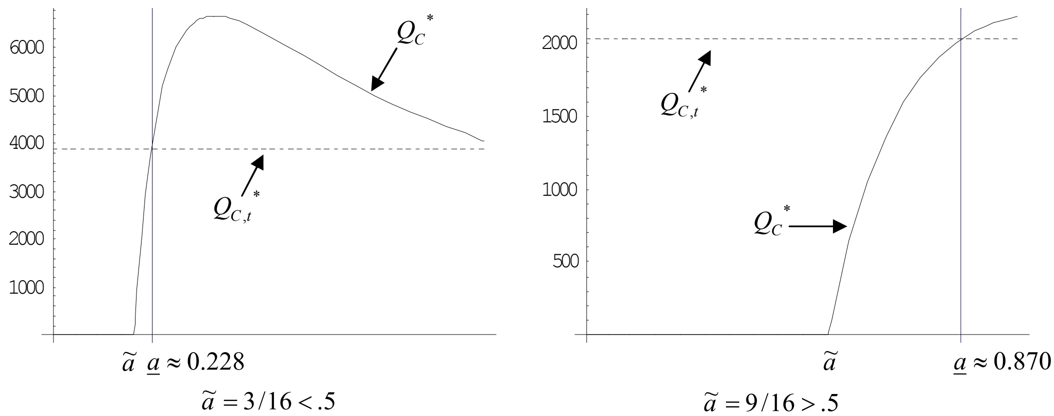


Figure 3.1: $Q_{C,t}^*$ and Q_C^* with changes in a

1. *For a given K, $EPCt(Q_{C,t}^*) - EPC(Q_C^*) > 0$ if and only if $t < t_C^*$ where*

$$t_C^* = (p + g_C - c) - \sqrt{\delta_C}$$

and $\delta_C = \frac{2a^2K(p+g_C-s)+(a(p+g_C-s)-(c-s))^2\beta}{a^2\beta} > 0.$

2. *For a given t, $EPCt(Q_{C,t}^*) - EPC(Q_C^*) > 0$ if and only if $K < K_C^*$ where*

$$K_C^* = \frac{[(1-a)(c-s) - at][a(2(p+g_C) - (c+s)) - (c-s) - at]\beta}{2a^2(p+g_C-s)}$$

An investment in this technology always brings in more profits if and only if the tag price is less than t_C^*. Note that t_C^* can also be negative, in which case it is never optimal to invest in the technology. For a given tag price, the centralized firm benefits from an investment in RFID if and only if the fixed cost of adoption is less than K_C^*. Figure 3.2 illustrates an example where the centralized firm has an incentive to invest in the RFID technology. We also provide two numerical examples to gain more insights on the incentives of the vertically integrated firm to adopt RFID when demand distribution is not uniform. Figure 3.3(a)

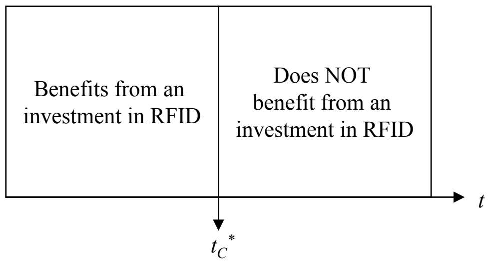
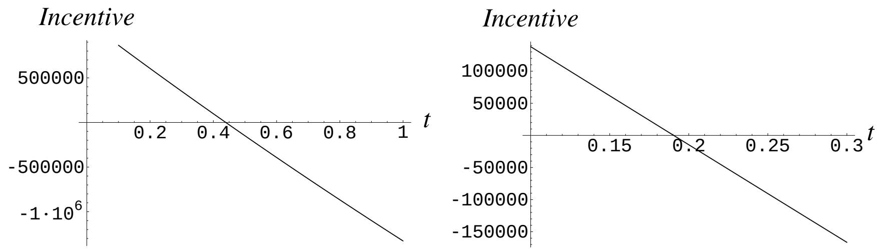


Figure 3.2: Investment in RFID for Centralized Manufacturer



(a) Demand is distributed with Exponential[λ]

(b) Demand is distributed with Normal[μ, σ]-doubly truncated at 2σ

Figure 3.3: Changes in the incentives of the vertically integrated supply chain under inventory misplacement to adopt RFID when demand distribution is Exponential[λ] and Normal[μ, σ]-doubly truncated at 2σ ($\lambda = 10^{-6}, \mu = 10^6, \sigma = 4 \times 10^5, p = 25, c = 5, g_C = 4, s = 2, K = 10^6, a = 0.7$)

and Figure 3.3(b) show the incentives of the centralized firm to adopt the technology when demand is distributed with Exponential[λ] and Normal[μ, σ], respectively. We observe the same threshold-type of behavior in the incentives of the firm as the tag price increases.

Lemma 17

1. *(i)* $\partial t_C^*/\partial K < 0$ *(ii) Given* $Q_C^* > 0$, $\partial t_C^*/\partial a < 0$
2. $K_C^* > 0$ *if and only if* $t < t_1$ *given* $Q_C^*, Q_{C,t}^* > 0$.
3. *If* $K = 0$, *given* $Q_C^*, Q_{C,t}^* > 0$, $t_C^* = t_1$ *where* $t_1 = (1-a)(c-s)/a > 0$

Lemma 17 explains how the thresholds provided in Theorem 17 change with some parameters. If the upper bound on the tag price for the centralized firm to benefit from an investment in RFID (t^*_C) is positive, an increase in the fixed cost makes it tighter. Furthermore, if K increases beyond some value, t^*_C can no longer be positive. Hence, it is essential to consider both tag prices and fixed costs of investment while deciding on the RFID implementation.

Note that t^*_C is decreasing in availability. In other words, the higher the availability level is, the lower the firm is willing to pay for the tag price. The centralized firm's optimal profit increases in the availability level. Thus, if an improvement in availability can be achieved without the new technology, the firm will be less willing to pay high prices for RFID tags because the firm is already making more profits without any investment; therefore, only a lower tag price can make the new technology attractive to the firm. This is similar to the result by Gaukler, Seifert and Hausman (2004), who found that the threshold value of the tag price for the firm to benefit from the technology also decreases with the value of responsiveness of the firm when RFID improves demand visibility.

A special case of our model is where there is no fixed cost of RFID adoption. In such a case, the upper bound on the maximum tag price $t^*_C = t_1 = (1-a)(c-s)/a$ is always positive.

Next, we study how the incentives of the centralized firm change as the mean or the variance of the uniformly distributed demand increases. We use a more general demand uniformly distributed in $[\underline{\beta}, \overline{\beta}]$ where $\underline{\beta}$ is allowed to be positive. We will use $'$ (prime) to denote the optimal values when $\underline{\beta} \geq 0$. We find the effects of changes in mean by shifting the range of demand distribution while keeping the width of the range constant. To capture the effects of changes in variance, we change the width of the range while keeping the mean constant. In the below results, we only consider the case where $t < t_1$, which is equivalent to saying that the firm may invest in RFID.

Theorem 18 *The following findings hold for uniformly distributed demand in* $[\underline{\beta}, \overline{\beta}]$ *if* $t < t_1$.

1. *The incentive of a centralized firm under inventory misplacement to adopt RFID decreases as inventory availability increases.*
2. *Given the variance of demand is fixed, as the expected value of demand increases:*
   - (a) $Q_C'^*$ *and* $Q_{C,t}'^*$ *increase.*
   - (b) *The incentive of a centralized firm under inventory misplacement to adopt RFID increases*

   where t_1 *is given by Lemma 17.*

One would expect to stock more when demand is expected to be larger and our results follow this intuition. Further, the firm has more incentives to adopt the new technology as the mean of demand increases. Hence, the greatest benefits from RFID accrue to firms under low inventory availability and in expectation of higher demand.

Theorem 19 *The following findings hold for uniformly distributed demand in* $[\underline{\beta}, \overline{\beta}]$ *if* $t < t_1$ *given the expected value of demand is fixed, as the variance of demand increases.*

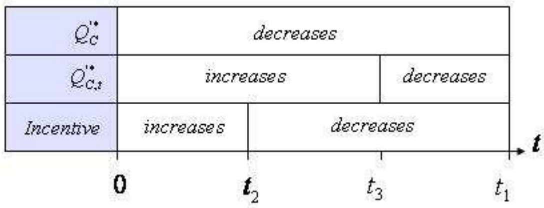


Figure 3.4: Changes in $Q_C'^*, Q_{C,t}'^*$ and the incentive of a centralized firm under inventory misplacement to adopt RFID when $a < 2\tilde{a}$ and $t_2 > 0$ as variance of demand increases.

1. *(a)* $Q_C'^*$ *decreases if* $a < 2\tilde{a}$. *If* $a > 2\tilde{a}$, *it increases.*

   (b) $Q_{C,t}'^*$ *decreases if* $t > t_3$. *If* $t < t_3$, *it increases.*

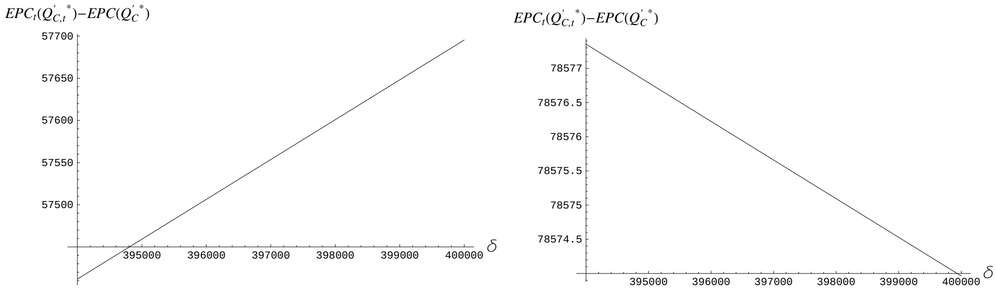
   (c) If $t_2 < t < t_1$ *holds, the incentive of a centralized firm under inventory misplacement to adopt RFID decreases. Otherwise, it increases.*

2. $t_2 < t_1$ *if and only if* $a < 2\tilde{a}$.

where $t_2 = p + g_C - c - \frac{(c-s)}{a}$, $t_3 = (t_1 + t_2)/2$ *and* t_1 *and* $\tilde{a}$ *are given by Lemma 17 and Theorem 16, respectively.*

One may also anticipate the firm to order more as demand becomes more variable. However, as shown in Theorem 19.1, the optimal order under misplaced inventory does decrease in demand variability if $a < 2\tilde{a}$ because the effective cost of acquiring an item becomes too costly as variance increases. For example, when the products are relatively unattractive to stock ($\tilde{a} \geq 0.5$), the cost of overage is higher than the cost of underage. Hence, as demand becomes more variable, one would decrease the amount of order.

As indicated by Theorem 19.1.(c), the incentive of a firm to adopt RFID does decrease as the variance of demand increases except under one condition, which can be valid only if $a < 2\tilde{a}$ holds; thus, $t_2 < t_1$. Figure 3.4 graphically illustrates the results of Theorem 19 for



(a) $t < t_2$ (b) $t > t_2$

Figure 3.5: Changes in the incentives of the vertically integrated supply chain under inventory misplacement to adopt RFID as variance of uniformly distributed demand increases (variance increases as δ increases)

$0 < t_2 < t_1$. When t_2 is negative, the incentive of the firm to adopt RFID always decreases with increasing variance. A tag price greater than t_3 increases the overage cost such that the firm under RFID decreases the order size with increasing variance. In the same region, the incentive of the firm under inventory misplacement to adopt RFID decreases as well. When $t_2 < t < t_3$, although the optimal order of the firm under RFID increases, the tag price is too high for the firm under inventory misplacement to benefit more from the technology as variance increases. Hence, if the firm is already operating under low inventory availability level ($a < 2\tilde{a}$) and the tag price is sufficiently high ($t > t_2$), RFID becomes a more beneficial technology as demand lies in a smaller interval of values, i.e. with a smaller variance. We can observe this behavior in Figure 3.5 where δ shows the symmetric distance from the mean to the left and to the right for which demand can take positive values.

3.3. Decentralized Supply Chain

We consider a supply chain consisting of one retailer and one manufacturer. We assume that the manufacturer acts as a Stackelberg leader and announces the wholesale price to the retailer. The retailer orders taking the wholesale price into consideration. If the supply chain adopts RFID technology, there will be no loss of inventory. Otherwise, the retailer misplaces $0 < l = 1 - a < 1$ proportion of the items as soon as the order arrives. Therefore, as in the centralized firm setting, only a proportion of the order will be available to the customers during the selling season. In both cases, products are sold for p and the leftover inventory (remaining items after the selling season and also lost items under inventory misplacement) is salvaged at s per unit by the retailer. If demand is larger than the available items in the retail store (a proportion of the order if the supply chain is not implementing RFID and the order itself if RFID is being implemented), both parties incur a stock-out cost. The stock-out cost incurred is shared between the manufacturer and the retailer in the decentralized settings. The retailer incurs g_R and the manufacturer incurs g_M for each unit of excess demand where $g_R + g_M = g_C$. If the supply chain adopts RFID technology, each item will be tagged with one RFID chip. Each RFID chip costs t per unit. For all cases, we study both scenarios where either the retailer or the manufacturer pays for the tagging expenses. We also assume that there is a fixed cost of K of the RFID adoption and it is shared between the manufacturer and the retailer. Thus, the retailer pays θK and the manufacturer pays $(1 - \theta)K$ where $0 \leq \theta \leq 1$. We first study an uncoordinated supply chain. Then, we analyze a supply chain that is coordinated by a revenue sharing contract. Since Assumption 1 is necessary but not sufficient for the uncoordinated decentralized supply chain, we also make

Assumption 2 for Section 3.3.1.

Assumption 2: $s < c < w < p$

Note that w (wholesale price) will be represented by w_D in Section 3.3.1 under misplaced inventory. In the existence of RFID, we will denote the wholesale prices by $w_{D,tR}$ if the retailer pays for the tagging expenses and $w_{D,tM}$ if the manufacturer pays for them. Therefore, Assumption 2 will be transformed to $s < c < w_D, w_{D,tR}, w_{D,tM} < p$ in Section 3.3.1.

3.3.1 No coordination

In this section, we study a supply chain where no mechanism exists for coordination and the manufacturer acts as a Stackelberg leader. The manufacturer announces the wholesale price, w_D under misplaced inventory and $w_{D,ti}$ under RFID where i stands for the party paying for the tagging expenses ($i = M$ if the manufacturer and $i = R$ if the retailer pays for the tagging expenses), the retailer orders Q_D under inventory misplacement and $Q_{D,ti}$ under RFID.

We will denote the retailer's and manufacturer's profits under misplaced inventory in an uncoordinated supply chain by EPR_D and EPM_D, respectively. EPR_D and EPM_D are as given below.

$$EPR_D(Q_D) = -w_D Q_D + p(\mu + \int_{aQ_D}^{\infty} (aQ_D - \xi) f(\xi) d\xi +$$

$$s(lQ_D + \int_0^{aQ_{D,tR}} (aQ_{D,tR} - \xi) f(\xi) d\xi) - g_R \int_{aQ_D}^{\infty} (\xi - aQ_D) f(\xi) d\xi \quad (3.5)$$

$$EPM_D(w_D) = (w_D - c)Q_D - g_M \int_{aQ_D}^{\infty} (\xi - aQ_D) f(\xi) d\xi \quad (3.6)$$

Expressions (3.5) and (3.6) are as follow for demand distributed in $[0, \beta]$.

$$EPR_D(Q_D) = Q_D(a(p + g_R - s) - (w_D - s)) - Q_D^2 a^2 (p + g_R - s)/(2\beta) - g_R \mu$$

$$EPM_D(w_D) = Q_D(w_D - c) - g_M(\beta - aQ_D)^2/2\beta$$

Theorem 20 (Çamdereli and Swaminathan, 2005)

In a supply chain under inventory misplacement (without RFID), the following findings hold.

1. *$EPR_D(Q_D)$ is concave in Q_D and $EPM_D(w_D)$ is concave in w_D .*

2. *$w_D^* = (1/4)[2(c+s) - 2a(p+g_R-s) - ag_M + g_M[ag_M - 2(c-s)]/[2(p+g_R-s)+g_M]$*

3. *$Q_D^* = \kappa(1/a)(1-\tilde{a}/a)\beta = \kappa Q_C^*$ where $\kappa = [(p+g_R-s)+g_M]/[2(p+g_R-s)+g_M] < 1$.*

Theorem 20 specifies the wholesale price that optimizes the manufacturer's profit and the order that optimizes the retailer's profit without RFID. The optimal order in an uncoordinated supply chain is the optimal centralized order multiplied by a constant κ, which is less than 1. Therefore, the order in an uncoordinated supply chain is always less than the optimal centralized order and there will be double marginalization losses in effect. In a recent paper, Heese (2006) considers inventory inaccuracies where the number of physical items in the store can be more than the system record and finds that the optimal decentralized order under inventory inaccuracy is at most 50% of the optimal order of a vertically integrated supply chain under inventory inaccuracy. The expression κ in Theorem 20 is greater than 0.5 if $g_M > 0$. Hence, the optimal decentralized order under inventory misplacement is at least 50% of the order quantity of a vertically integrated supply chain. Next, we study the optimal wholesale price and the ordering quantity under the implementation of RFID technology.

The retailer's and the manufacturer's profits in an uncoordinated supply chain inclusive of the fixed cost of adoption but exclusive of the tagging expenses under RFID are given by $\Gamma_D(Q_{D,ti}, w_{D,ti})$ and $\Lambda_D(Q_{D,ti}, w_{D,ti})$ where $i\epsilon\{R, M\}$, respectively. We use $EPR_{D,ti}$ and $EPM_{D,ti}$ to represent the retailer's profit and the manufacturer's profit where $i\epsilon\{M, R\}$.

$$\begin{aligned}
\Gamma_D(x,y) &= -yx - \theta K + p(\mu + \int_x^\infty (x-\xi)f(\xi)d\xi) + s\int_0^x (x-\xi)f(\xi)d\xi \\
&\quad -g_R\int_x^\infty (\xi - x)f(\xi)d\xi \\
\Lambda_D(x,y) &= (y-c)x - (1-\theta)K - g_M\int_x^\infty (\xi - x)f(\xi)d\xi
\end{aligned}$$

$$EPR_{D,tR}(Q_{D,tR}) = \Gamma_D(Q_{D,tR}, w_{D,tR}) - tQ_{D,tR} \tag{3.7}$$

$$EPM_{D,tR}(w_{D,tR}) = \Lambda_D(Q_{D,tR}, w_{D,tR}) \tag{3.8}$$

$$EPR_{D,tM}(Q_{D,tM}) = \Gamma_D(Q_{D,tM}, w_{D,tM}) \tag{3.9}$$

$$EPM_{D,tM}(w_{D,tM}) = \Lambda_D(Q_{D,tM}, w_{D,tM}) - tQ_{D,tM} \tag{3.10}$$

The retailer's problem is to find $Q^*_{D,ti} = \arg\max_{Q_{D,ti}} EPR_{D,ti}$ and the manufacturer's problem is $w^*_{D,ti} = \arg\max_{w_{D,ti}} EPM_{D,ti}$. Expressions (3.7), (3.8), (3.9) and (3.10) are as follow for demand distributed in $[0, \beta]$.

$$EPR_{D,tR}(Q_{D,tR}) = -Q^2_{D,tR}(p + g_R - s)/(2\beta) + Q_{D,tR}(p + g_R - t - w_{D,tR}) - g_R\mu - \theta K$$

$$EPM_{D,tR}(w_{D,tR}) = -(Q_{D,tR} - \beta)^2 g_M/(2\beta) + Q_{D,tR}(w_{D,tR} - c) - (1 - \theta)K$$

$$EPR_{D,tM}(Q_{D,tM}) = -Q^2_{D,tM}(p + g_R - s)/(2\beta) + Q_{D,tM}(p + g_R - w_{D,tM}) - g_R\mu - \theta K$$

$$EPM_{D,tM}(w_{D,tR}) = -(Q_{D,tM} - \beta)^2 g_M/(2\beta) + Q_{D,tM}(w_{D,tM} - c - t) - (1 - \theta)K$$

Theorem 21

1. *$EPR_{D,tR}(Q_{D,tR})$ and $EPR_{D,tM}(Q_{D,tM})$ are strictly concave in $Q_{D,tR}$ and $Q_{D,tM}$, respectively.*

2. *$EPM_{D,tR}(w_{D,tR})$ and $EPM_{D,tM}(w_{D,tM})$ are strictly concave in $w_{D,tR}$ and $w_{D,tM}$, respectively.*

3. *The optimal wholesale prices in an uncoordinated supply chain in the existence of RFID are as follow.*

$$w^*_{D,tR} = [(g_R + p - s)(g_R + p + c) + sg_M - t(p + g_C - s)]/(2(g + p - s) + g_M) \tag{3.11}$$

$$< w_D^* \textit{ if and only if } t > (1-a)(p+g_R-s)^2/(p+g_C-s)$$

$$\begin{aligned} w_{D,tM}^* &= [(g_R+p-s)(g_R+p+c)+sg_M+t(p+g_R-s])/(2(g+p-s)+g_M) \\ &= w_{D,tR}^* + t > w_D^* \end{aligned} \quad (3.12)$$

4. *The optimal ordering quantities in an uncoordinated supply chain in the existence of RFID are as follow.*

$$\begin{aligned} Q_{D,tR}^* &= Q_{D,tM}^* \\ &= \begin{cases} (p+g_C-t-c)\beta/(2(p+g_R-s)+g_M) > 0 & \textit{if and only if} \\ t < p+g_C-c & \\ 0 & \textit{otherwise} \end{cases} \end{aligned} \quad (3.13)$$

Theorem 21 gives the optimal wholesale prices and the ordering quantities in the existence of RFID. Gaukler, Seifert and Hausman (2004) show that when RFID improves demand visibility, the optimum wholesale price under RFID is at least as much as the one with no RFID. However, when RFID improves supply visibility, as shown by Theorem 21, the wholesale price under RFID is indeed smaller than the one with no RFID if the retailer covers for the tagging expenses and the tag price is sufficiently high.

When the manufacturer pays for the tagging expenses, she can adjust the wholesale price by t as compared to the wholesale price in the scenario where the retailer pays for the tagging expenses. As a result, the optimal ordering quantity does not change whether the manufacturer or the retailer is charged for the tagging expenses. Since $w^*_{D,tM} = w^*_{D,tR} + t$ and $Q^*_{D,tR} = Q^*_{D,tM}$, the optimal profits of both the retailer and the manufacturer stay the same regardless of the party paying for the tagging expenses. Thereby, the following corollary follows from Theorem 21.

Corollary 10

1. $EPR_{D,tR}(Q^*_{D,tR}) = EPR_{D,tM}(Q^*_{D,tM})$

2. $EPM_{D,tR}(w^*_{D,tR}) = EPM_{D,tM}(w^*_{D,tM})$

Following Corollary 10, the rest of the results related with profits of the retailer or the manufacturer holds regardless of the party paying for the tagging expenses.

Lemma 18 *The following findings hold in an uncoordinated supply chain and in the existence of RFID for $Q^*_{D,ti} > 0$ where $i \,\epsilon \{R, M\}$.*

1. (a) *For $0 < \theta \leq 1$ and a given t, $EPR_{D,ti}(Q^*_{D,ti}) > 0$ if and only if $K < \rho^*_1$ where*

$$\rho^*_1 = (2\theta)^{-1}[(p + g_R - s)(\frac{Q^{*2}_{D,ti}}{\beta}) - g_R\beta]$$

and $\rho^*_1 \geq 0$ if and only if $t \leq (p+g_C-c) - [2(p+g_R-s)+g_M]\sqrt{g_R/(p+g_R-s)}$.

   (b) *For $\theta = 0$, $EPR_{D,ti}(Q^*_{D,ti}) > 0$ if and only if $t < (p + g_C - c) - [2(p + g_R - s) + g_M]\sqrt{g_R/(p + g_R - s)}$.*

2. (a) *For* $0 \leq \theta < 1$ *and a given* t, $EPM_{D,ti}(w^*_{D,ti}) > 0$ *if and only if* $K < \rho^*_2$ *where*

$$\rho^*_2 = (1-\theta)^{-1}[\frac{(p+g_C-s)Q^{*2}_{D,tR}}{\beta} - g_M(\beta/2 + Q^{*2}_{D,tR}/(2\beta))]$$

and $\rho^*_2 \geq 0$ *if and only if* $t \leq (p+g_C-c) - \sqrt{g_M[2(p+g_R-s)+g_M]}$.

   (b) *For* $\theta = 1$, $EPM_{D,ti}(w^*_{D,ti}) > 0$ *if and only if* $t < (p+g_C-c) - \sqrt{g_M[2(p+g_R-s)+g_M]}$.

3. *When* $\theta = 0.5$ *and* $g_R = g_M$, $EPR_{D,ti}(Q^*_{D,ti}) < EPM_{D,ti}(w^*_{D,ti})$.

As seen in Lemma 18, although the profit functions are optimized at a positive value, the sign of the value they take depends on the fixed cost as well as the tag price. Lemma 18 shows that when tag expenses are low, the parties in the supply chain will incur negative profits if the fixed cost of investment is greater than a threshold value (ρ^*_1 for the retailer and ρ^*_2 for the manufacturer). Thus, considering only the tagging expenses will not suffice in characterizing the benefits of investment in RFID technology. If the parties share the fixed cost of investment and the stock-out cost as half and half, being leader, the manufacturer always extracts higher profits than the retailer.

Lemma 19 *In an uncoordinated supply chain, the following findings hold given* $Q^*_{D,ti}, Q^*_D > 0$ *where* $i \in \{R, M\}$.

1. $EPR_{D,ti}(Q^*_{D,ti}) - EPR_D(Q^*_D) > 0$ *if and only if* $t < t^*_{D,R}$,
2. $EPM_{D,ti}(w^*_{D,ti}) - EPM_D(w^*_D) > 0$ *if and only if* $t < t^*_{D,M}$ *where*

$$t^*_{D,R} = (p+g_C-c) - \delta_{D,R1}\sqrt{\delta_{D,R2}},$$

$$
\begin{aligned}
t^*_{D,M} &= (p + g_C - c) - \sqrt{\delta_{D,M}}, \\
\delta_{D,R1} &= [4(p + g_C - s) + g_M^2/(p + g_R - s)] > 0, \\
\delta_{D,R2} &= \frac{[(a(p + g_C - s) - (c - s))(p + g_R - s)]^2 + \frac{2a^2K[2(p+g_R-s)+g_M]^2\theta(p+g_R-s)}{\beta}}{a^2[2(p + g_R - s) + g_M]^4} > 0, \\
\delta_{D,M} &= \frac{(a(p + g_C - s) - (c - s))^2 + \frac{2a^2K[2(p+g_R-s)+g_M](1-\theta)}{\beta}}{a^2} > 0.
\end{aligned}
$$

The retailer and the manufacturer benefit from an investment in RFID as long as the tag price is less than $t^*_{D,R}$ and $t^*_{D,M}$, respectively. When a threshold is negative, there is never an incentive for the corresponding party to adopt this technology.

Lemma 20 *In an uncoordinated supply chain, the following findings hold given $Q^*_{D,ti}, Q^*_D > 0$ where $i \in \{R, M\}$.*

1. *For $0 < \theta < 1$,*
   (a) *$EPR_{D,ti}(Q^*_{D,ti}) - EPR_D(Q^*_D) > 0$ if and only if $K < K^*_{D,R}$*
   (b) *$EPM_{D,ti}(w^*_{D,ti}) - EPM_D(w^*_D) > 0$ if and only if $K < K^*_{D,M}$*
   (c) *$K^*_{D,R}, K^*_{D,M} > 0$ if and only if $t < t_1$*
2. *For $\theta = 0$, $EPR_{D,ti}(Q^*_{D,ti}) - EPR_D(Q^*_D) > 0$ if and only if $t < t_1$*
3. *For $\theta = 1$, $EPM_{D,ti}(w^*_{D,ti}) - EPM_D(w^*_D) > 0$ if and only if $t < t_1$*

   where t_1 is as given in Lemma 17,

$$
\begin{aligned}
K^*_{D,R} &= \frac{(p + g_R - s)[(1 - a)(c - s) - at][a(2(p + g_C) - (c + s)) - (c - s) - at]\beta}{2a^2[2(p + g_R - s) + g_M]^2\theta}, \\
K^*_{D,M} &= \frac{[-(1 - a)(c - s) + at][a(2(p + g_C) - (c + s)) - (c - s) - at]\beta}{2a^2[2(p + g_R - s) + g_M]^2(1 - \theta)}.
\end{aligned}
$$

As shown by Lemma 20, given a tag price, there exits a threshold value for the fixed cost beyond which, it is never beneficial to adopt RFID for a party. If the tag price is greater than t_1, both parties will always be worse off under RFID for any positive fixed cost. This threshold is the same threshold value for the centralized manufacturer to benefit from this technology in the absence of any fixed cost of investment.

When the manufacturer covers for the entire fixed cost of investment, i.e. $\theta = 0$, the retailer will be worse off under RFID if the tag price is greater than the same value as above, t_1. The case where the retailer does not pay for any fixed cost is the same for him as $K = 0$. If the tag price is more than t_1, the investment is never beneficial for him even if $K = 0$, since $K^*_{D,R}$ is negative for $t > t_1$. The same argument explains the threshold for the manufacturer to benefit from this investment when the retailer covers for the entire fixed cost of investment. Hence, in the absence of any fixed cost of investment, the threshold value of the tag price for either the manufacturer or the retailer to benefit from this investment would be the same. Thereby, it is the same threshold (t_1) for the whole supply chain to benefit from the RFID investment when $K = 0$.

Having characterized the incentives of the parties independent from each other, we next analyze the benefits of RFID for one party relative to the other's.

Theorem 22 *In an uncoordinated supply chain, the following findings hold if* $Q^*_D > 0$.

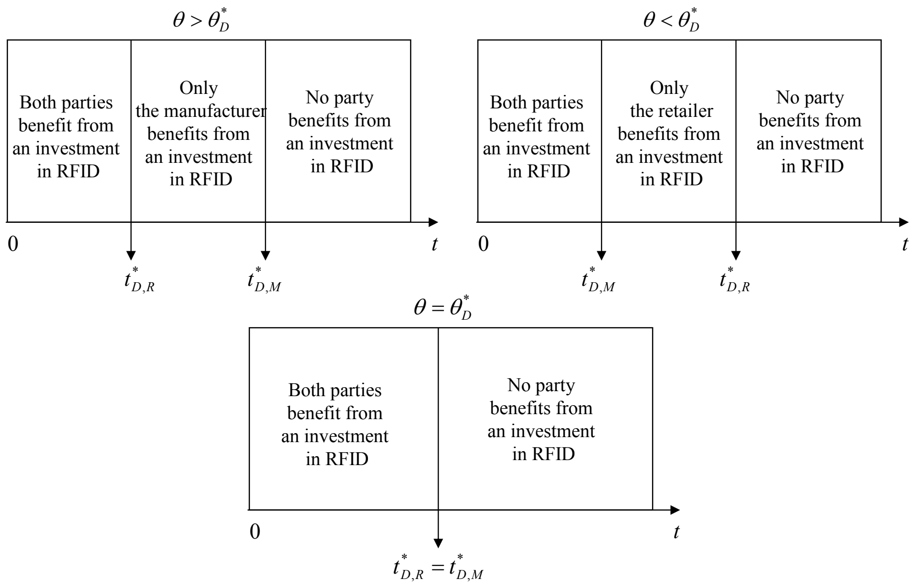


Figure 3.6: Investment in RFID in an Uncoordinated Supply Chain for positive threshold values

1. $t^*_{D,R}, t^*_{D,M} < t_1$.

2.

$$t^*_{D,R} is \begin{cases} < t^*_{D,M} & if \;\; \theta > \theta^*_D \\ > t^*_{D,M} & if \;\; \theta < \theta^*_D \\ = t^*_{D,M} & otherwise \end{cases}$$

3. Given $K^*_{D,R}, K^*_{D,M} > 0$*:*

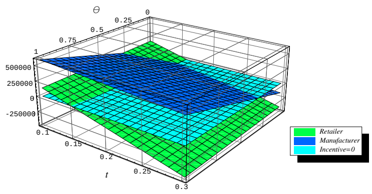
$$K^*_{D,R} is \begin{cases} < K^*_{D,M} & if \;\; \theta > \theta^*_D \\ > K^*_{D,M} & if \;\; \theta < \theta^*_D \\ = K^*_{D,M} & otherwise \end{cases}$$

where $t^*_{D,R}$ *and* $t^*_{D,M}$ *are given by Lemma 19,* $K^*_{D,R}$ *and* $K^*_{D,M}$ *are given by Lemma 20,* t_1 *is given by Lemma 17 and* $\theta^*_D = (p + g_R - s)/[3(p + g_R - s) + g_M] \leq 1/3$.

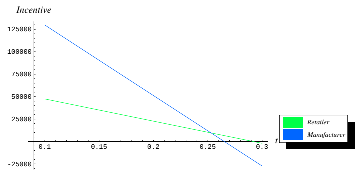
Benefits of parties from making an investment in RFID technology in comparison to each other depend on the relative fixed cost of investment as well as the tag prices. One would expect the thresholds to be different due to the fixed cost of investment. As illustrated by Theorem 22, the relation between $t^*_{D,R}$ and $t^*_{D,M}$ depends on the value of θ. When the retailer covers for exactly $\theta^*_D K$, these thresholds are equal to each other. Therefore, in such a case, there exist only two regions regarding the RFID-adoption: where both parties benefit from the adoption for $t < t^*_{D,R} = t^*_{D,M}$ and where no party benefits. Figure 3.6 shows these regions.

When $\theta \neq \theta^*_D$, the incentives of the parties to adopt RFID are not always perfectly aligned. If the tag price is smaller than $min\{t^*_{D,R}, t^*_{D,M}\}$, both parties benefit from the investment. If the tag price is greater than $max\{t^*_{D,R}, t^*_{D,M}\}$, no party benefits from the investment. An interesting situation arises when the tag price is in between these two values because only one party benefits in that case. If the retailer covers for more than $\theta^*_D \leq 1/3$, then $t^*_{D,R}$ is less than $t^*_{D,M}$; thereby, only the manufacturer benefits for tag prices falling in the middle region ($t^*_{D,R} < t < t^*_{D,M}$). However, if the retailer covers for less than θ^*_D, $t^*_{D,R}$ is greater than $t^*_{D,M}$ and thereby only the retailer benefits for the tag prices falling in the middle region ($t^*_{D,M} < t < t^*_{D,R}$).

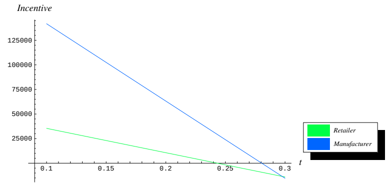
We know from Lemma 20 that $t > t_1$ is a sufficient condition for none of the parties to adopt RFID. Further, Theorem 22 shows that both threshold values, $t^*_{D,R}$ and $t^*_{D,M}$, are less than t_1. Hence, one can conclude that although a given tag price can induce both parties to adopt RFID in the absence of fixed cost, in reality, it may make only one party benefit from this investment depending on the relative payments of fixed cost. When fixed cost of investment is considered, the parties' RFID-adoption decisions are not perfectly aligned unless the retailer pays exactly θ^*_D of the fixed cost of investment.



(a) Incentives of parties to invest in RFID as θ and t change



(b) Incentives of parties to invest in RFID as t changes when $\theta = 0.23$

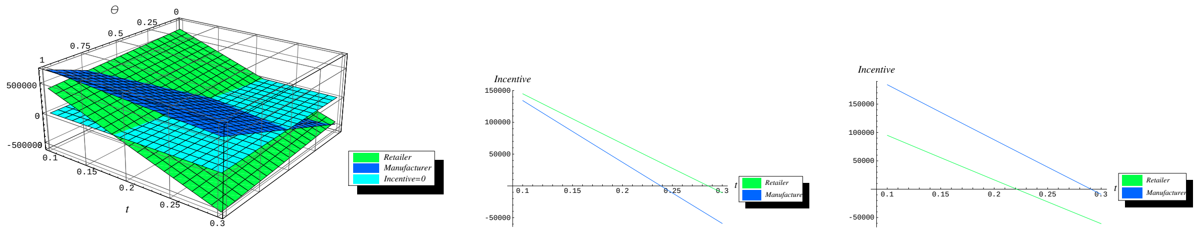


(c) Incentives of parties to invest in RFID as t changes when $\theta = 0.25$

Figure 3.7: Changes in the incentives of the parties in an uncoordinated supply chain under inventory misplacement to adopt RFID when demand distribution is Normal$[\mu, \sigma]$-doubly truncated at 2σ ($\mu = 10^6, \sigma = 4 \times 10^5, p = 25, c = 5, g_R = 1, g_M = 3, s = 2, K = 6 \times 10^5, a = 0.7$)

An interesting observation from Theorem 22 is that even if the parties share the fixed cost as half and half, if the tag price falls into $[t^*_{D,R}, t^*_{D,M})$, contrary to popular belief in press about retailers being the main beneficiaries of the RFID investment, only the manufacturer benefits from this investment. Further, as the manufacturer's goodwill cost (g_M) increases, θ^*_D decreases. Thus, as the manufacturer gets more concerned about the stockouts, the retailer has to cover for even lower proportions of the fixed cost to be the only beneficiary of this investment for certain values of tag prices. As given by the relation between $K^*_{D,R}$ and $K^*_{D,M}$, a similar setting of regions exist for ranges of fixed cost as well.

Theorem 22 specifies regions of incentives when demand is uniformly distributed in $[0, \beta]$. Figure 3.7 and Figure 3.8 show two numerical examples where the incentives of the parties in an uncoordinated supply chain under inventory misplacement to adopt RFID are illustrated for Normal$[\mu, \sigma]$ and Exponential$[\lambda]$ demand (refer to Appendix B for the proofs regarding concavity). Both of these examples lend support to Corollary 10; thus, incentive of a party to adopt RFID is the same regardless of who pays for the tagging expenses. Further, they



(a) Incentives of parties to invest in RFID as θ and t change

(b) Incentives of parties to invest in RFID as t changes when $\theta = 0.42$

(c) Incentives of parties to invest in RFID as t changes when $\theta = 0.47$

Figure 3.8: Changes in the incentives of the parties in an uncoordinated supply chain under inventory misplacement to adopt RFID when demand distribution is Exponential[λ] ($\lambda = 10^{-6}, p = 25, c = 5, g_R = 1, g_M = 3, s = 2, K = 10^6, a = 0.7$)

support the results of Theorem 22. As shown in Figure 3.7(c) and Figure 3.8(c), the manufacturer still benefits from the investment for a while after the retailer's incentive becomes zero.

Lemma 21 *In an uncoordinated supply chain, the following findings hold given $Q^*_{D,ti}, Q^*_D > 0$ where $i \,\epsilon\, \{R, M\}$.*

1. (a) i. $\partial t^*_{D,R}/\partial K < 0$ ii. $\partial t^*_{D,R}/\partial a < 0$ iii. $\partial t^*_{D,R}/\partial \theta < 0$

   (b) i. $\partial t^*_{D,M}/\partial K < 0$ ii. $\partial t^*_{D,M}/\partial a < 0$ iii. $\partial t^*_{D,M}/\partial \theta > 0$

2. (a) i. $\partial K^*_{D,R}/\partial t < 0$ ii. $\partial K^*_{D,R}/\partial a < 0$ iii. When $K^*_{D,R} > 0$, $\partial K^*_{D,R}/\partial \theta < 0$.

   (b) i. $\partial K^*_{D,M}/\partial t < 0$ ii. $\partial K^*_{D,M}/\partial a < 0$ iii. When $K^*_{D,M} > 0$, $\partial K^*_{D,M}/\partial \theta > 0$.

Next, we study how the threshold values given in the previous lemmas behave with changes in the fixed cost, the retailer's (or the manufacturer's) share of the fixed cost, and availability proportion under misplaced inventory. As the fixed cost of investment or the proportion of the fixed cost that a party is charged increases, the new technology becomes so costly that the maximum amount of tag price the firm can afford decreases. Additionally, neither party in a supply chain that is already operating under high availability level can afford high tag prices to benefit from RFID. An improvement in the availability level will increase both parties' profits. Since the firms can make more profits with a costless improvement in availability, they will not be willing to pay for RFID as much as they could before the improvement. Similar effects on the threshold values of the fixed cost of investment take place as a result of changes in the tag price, availability level and the sharing of fixed cost.

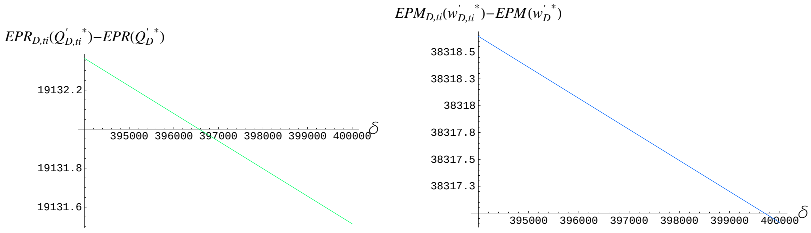
Theorem 23 *The following findings hold for uniformly distributed demand in* $[\underline{\beta}, \overline{\beta}]$ *if* $t < t_1$.

1. *Regardless of the party paying for the tagging expenses, the incentives of the retailer and the manufacturer under inventory misplacement to adopt RFID decreases as inventory availability increases.*

2. *The following results hold for each party as the expected value of demand changes given the variance is fixed or vice versa regardless of the party paying for the tagging expenses.*

   (a) *The results given in Theorem 18 and Theorem 19 for* $Q_C^{\prime *}$ *and* $Q_{C,t}^{\prime *}$ *are exactly valid for* $Q_D^{\prime *}$ *and* $Q_{D,ti}^{\prime *}$*, respectively where* $i \epsilon \{R, M\}$.

   (b) *The results given in Theorem 18 and Theorem 19 regarding the incentives of a centralized firm under inventory misplacement to adopt RFID are exactly valid for both the retailer and the manufacturer under inventory misplacement to adopt RFID.*



(a) Retailer's incentive (b) Manufacturer's incentive

Figure 3.9: Changes in the incentives of the parties in an uncoordinated supply chain under inventory misplacement to adopt RFID as variance of uniformly distributed demand increases when $t > t_2$ (variance increases as δ increases)

In the above results, we only consider cases where $t < t_1$ since each firm may invest in RFID for such tag prices. Theorem 23 shows how the changes in the expected value or the variance of demand affect stocking decisions and the incentives of the firms in an uncoordinated supply chain to adopt RFID. Both Q'^*_D and $Q'^*_{D,tR} = Q'^*_{D,tR}$ can be expressed as $Q'^*_D = \kappa Q'^*_C$ and $Q'^*_{D,ti} = \kappa Q'^*_{C,t}$ where $\kappa < 1$ is given by Theorem 20. Hence, the results that are given by Theorem 18 and Theorem 19 related with corresponding order quantities of a centralized firm apply to those in the uncoordinated supply chain. Furthermore, we show that the incentive of each party to adopt RFID changes in the same manner. Thus, both firms' incentives to adopt the technology decreases with variance of demand if the existing inventory availability level of the retailer is low and the value of tag price is high. Figure 3.9 shows a numerical example where the incentives of the firms decrease as variance of demand increases.

3.3.2 Revenue Sharing

Sometimes supply chains are coordinated through a revenue sharing mechanism (see Cachon and Lariviere, 2005). In this section, we study the implications of RFID implementation in such situations. The sequence of events is the same as in Section 3.3.1 except for that according to the revenue sharing contract, the retailer takes γ of the positive earnings (sales and salvage value earnings) and the manufacturer takes $1-\gamma$ of them where $0 < \gamma < 1$. As in the previous decentralized setting, if the supply chain is subject to inventory misplacement, $0 < l = 1 - a < 1$ proportion of the order released to the manufacturer is misplaced in the retail store. We will use Q_{RS} and w_{RS} to denote the ordering quantity and the wholesale price under misplaced inventory, respectively. The retailer's and the manufacturer's profits under inventory misplacement are given by (3.14) and (3.15) in that order.

$$EPR_{RS}(Q_{RS}) = -w_{RS}Q_{RS} + \gamma[p(\mu + \int_{aQ_{RS}}^{\infty}(aQ_{RS} - \xi)f(\xi)d\xi) + s(lQ_{RS} + \int_{0}^{aQ_{RS}}(aQ_{RS} - \xi)f(\xi)d\xi) - g_R\int_{aQ_{RS}}^{\infty}(\xi - aQ_{RS})f(\xi)d\xi \tag{3.14}$$

$$EPM_{RS}(w_{RS}) = (w_{RS} - c)Q_{RS} + (1-\gamma)[p(\mu + \int_{aQ_{RS}}^{\infty}(aQ_{RS} - \xi)f(\xi)d\xi) + s(lQ_{RS} + \int_{0}^{aQ_{RS}}(aQ_{RS} - \xi)f(\xi)d\xi) - g_M\int_{aQ_{RS}}^{\infty}(\xi - aQ_{RS})f(\xi)d\xi \tag{3.15}$$

Expressions (3.14) and (3.15) are as follow for demand uniformly distributed in $[0, \beta]$.

$$
\begin{aligned}
EPR_{RS}(Q_{RS}) = & -Q_{RS}^2 a^2 (g_R + (p-s)\gamma)/(2\beta) + \\
& Q_{RS}(s\gamma + a(g_R + (p-s)\gamma) - w_{RS}) - g_R\mu \\
EPM_{RS}(w_{RS}) = & -Q_{RS}^2 a^2 (g_M + (p-s)(1-\gamma))/(2\beta) + \\
& Q_{RS}(w_{RS} - c + (1-a)(1-\gamma)s + a(g_M + (1-\gamma)p))
\end{aligned}
$$

Theorem 24 (Çamdereli and Swaminathan, 2005)

1. *$EPR(Q_{RS})$ is concave in Q_{RS}.*
2. *A supply chain under inventory misplacement is coordinated by a revenue sharing contract for the wholesale price chosen according to $w_{RS}^* = [g_R(c-s) + c\gamma(p-s) + sg_C\gamma]/[p + g_C - s] < c$.*

To coordinate a supply chain, the manufacturer sets the wholesale price such that the retailer orders the centralized optimum order. Theorem 24 specifies the wholesale price that will lead to coordination under revenue sharing. We assume that γ is set such that the retailer's profit is at least as much as that before the coordination. In a supply chain subject to inventory misplacement, the coordination wholesale price is less than the marginal cost of production (see Çamdereli and Swaminathan (2005) for details).

The retailer's and the manufacturer's profits under a revenue sharing contract in the existence of RFID are given by $EPR_{RS,ti}$ and $EPM_{RS,ti}$ where $i \,\epsilon \{R, M\}$. The expressions $\Gamma_{RS}(Q_{RS,ti}, w_{RS,ti})$ and $\Lambda_{RS}(Q_{RS,ti}, w_{RS,ti})$, where $Q_{RS,ti}$ is the order and $w_{RS,ti}$ is the wholesale price, show the retailer's and the manufacturer's profits exclusive of the tagging expenses under a revenue sharing contract, respectively.

$$
\begin{aligned}
\Gamma_{RS}(x,y) = {} & -yx - \theta K + \gamma[p(\mu + \int_x^\infty (x-\xi)f(\xi)d\xi) + s\int_0^x (x-\xi)f(\xi)d\xi] \\
& -g_R \int_x^\infty (\xi - x)f(\xi)d\xi \\
\Lambda_{RS}(x,y) = {} & (y-c)x - (1-\theta)K + (1-\gamma)[p(\mu + \int_x^\infty (x-\xi)f(\xi)d\xi) + s\int_0^x (x-\xi)f(\xi)d\xi] \\
& -g_M \int_x^\infty (\xi - x)f(\xi)d\xi
\end{aligned}
$$

$$EPR_{RS,tR}(Q_{RS,tR}) = \Gamma_{RS}(Q_{RS,tR}, w_{RS,tR}) - tQ_{RS,tR} \tag{3.16}$$

$$EPM_{RS,tR}(w_{RS,tR}) = \Lambda_{RS}(Q_{RS,tR}, w_{RS,tR}) \tag{3.17}$$

$$EPR_{RS,tM}(Q_{RS,tM}) = \Gamma_{RS}(Q_{RS,tM}, w_{RS,tM}) \tag{3.18}$$

$$EPM_{RS,tM}(w_{RS,tM}) = \Lambda_{RS}(Q_{RS,tM}, w_{RS,tM}) - tQ_{RS,tM} \tag{3.19}$$

Expressions (3.16), (3.17), (3.18) and (3.19) are as follow for demand uniformly distributed in $[0, \beta]$.

$$
\begin{aligned}
EPR_{RS,tR}(Q_{RS,tR}) &= -Q^2_{RS,tR}[\gamma(p-s)+g_R]/(2\beta)+ \\
&\quad Q_{RS,tR}(\gamma p + g_R - w_{RS,tR} - t) - g_R\mu - \theta K \\
EPM_{RS,tR}(w_{RS,tR}) &= -Q^2_{RS,tR}[(1-\gamma)(p-s)+g_M]/(2\beta)+ \\
&\quad Q_{RS,tR}[(1-\gamma)p + g_M + w_{RS,tR} - c] - g_M\mu - (1-\theta)K \\
EPR_{RS,tM}(Q_{RS,tM}) &= -Q^2_{RS,tM}[\gamma(p-s)+g_R]/(2\beta)+ \\
&\quad Q_{RS,tM}(\gamma p + g_R - w_{RS,tM}) - g_R\mu - \theta K \\
EPM_{RS,tM}(w_{RS,tM}) &= -Q^2_{RS,tM}[(1-\gamma)(p-s)+g_M]/(2\beta)+ \\
&\quad Q_{RS,tM}[(1-\gamma)p + g_M + w_{RS,tM} - c - t] - g_M\mu - (1-\theta)K
\end{aligned}
$$

Next, we specify the wholesale prices that will coordinate the channel under RFID technology.

Theorem 25 *The supply chain is coordinated by a revenue sharing contract in the existence of RFID where the retailer takes $0 < \gamma < 1$ of the earnings as follow.*

1. *When the retailer pays for the tagging expenses, the coordination is achieved by*

$$w^*_{RS,tR} = \frac{c(\gamma(p-s)+g_R) - t[(1-\gamma)(p-s)+g_M] - s(g_R - \gamma g_C)}{p + g_C - s} < w^*_{RS} \quad (3.20)$$

2. *When the manufacturer pays for the tagging expenses, the coordination is achieved by*

$$\begin{aligned} w^*_{RS,tM} &= \frac{g_R(c-s+t)+\gamma[c(p-s)+s(g_C-t)+pt]}{p+g_C-s} > w^*_{RS} \\ &= w^*_{RS,tR}+t \end{aligned} \tag{3.21}$$

3. *Given* $Q^*_{RS,ti}>0$ *where* $i\,\epsilon\{R,M\}$, *the following findings hold.*

   (a) $w^*_{RS,tR}$ *is always less than* c.

   (b) $w^*_{RS,tM}>c$ *if and only if* $t>\frac{c(g_M+(1-\gamma)(p-s))+s(g_R-\gamma g_C)}{g_R+\gamma(p-s)}>0$.

Unlike the case of inventory misplacement, the coordination wholesale price is always higher than the marginal cost of production if the tag price is expensive ($t>\frac{c(g_M+(1-\gamma)(p-s))+s(g_R-\gamma g_C)}{g_R+\gamma(p-s)}$) and the manufacturer pays for the expenses.

Corollary 11

1. $EPR_{RS,tR}(Q^*_{RS,tR})=EPR_{RS,tM}(Q^*_{RS,tM})$

2. $EPM_{RS,tR}(w^*_{RS,tR})=EPM_{RS,tM}(w^*_{RS,tM})$

The manufacturer makes the retailer pay for the tag price through the wholesale price when she is charged for the tagging expenses. Thus, the profit of each party under RFID is the same as well as the benefits accrued to the same party from investing in RFID whoever covers for the tagging expenses.

Similar to Lemma 19 and Lemma 20, we can show the following results for a supply chain under a revenue sharing contract (refer to Lemma 22 and Lemma 23 in Appendix B for the mathematical representation of the following results and their proofs):

1. RFID benefits the retailer (manufacturer) in a supply chain coordinated by a revenue sharing contract under misplaced inventory as long as the tag price is less than $t^*_{RS,R}$ ($t^*_{RS,M}$) for a given fixed cost and as long as the fixed cost is less than $K^*_{RS,R}$ ($K^*_{RS,M}$) for given a tag price.

2. If the tag price is greater t_1, the technology is never beneficial to any of the parties for any fixed cost of investment.

3. When one party covers for the entire fixed cost, the other party has an incentive to invest in the technology if and only the tag price is lower than t_1. Thus, one can conclude that when the fixed cost of investment is ignored, each party in the supply chain; therefore, the supply chain under a revenue sharing contract has an incentive to adopt the technology if and only if the tag price is less than t_1. With respect to incentives to adopt the RFID technology, the supply chain under a revenue sharing contract behaves as a centralized firm under no fixed cost due to the coordination.

4. When the tag price is sufficiently small (lower than t_1), the retailer's incentive to invest in the technology increases in γ. The manufacturer's incentive changes in the opposite direction by the same magnitude because the summation of the incentives of parties in the coordinated supply chain is always equal to that of a centralized firm.

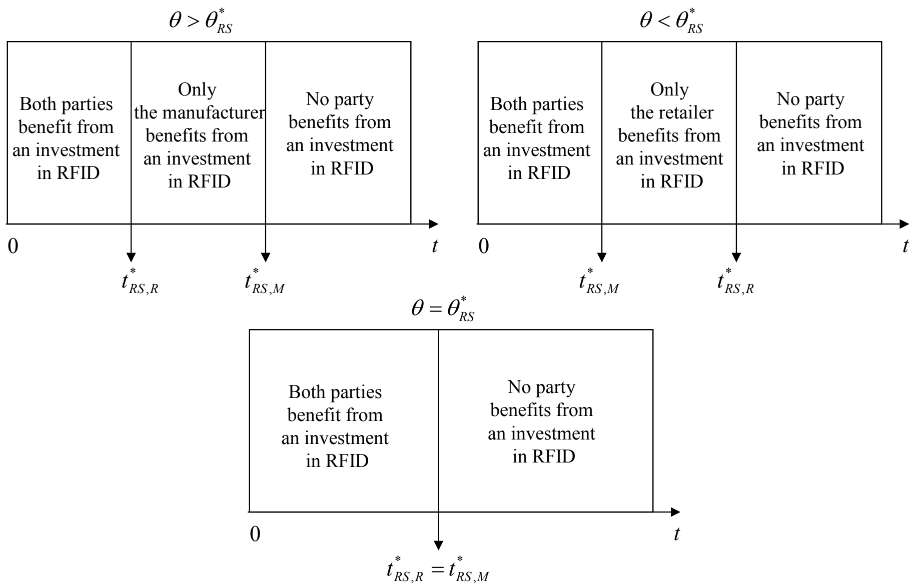
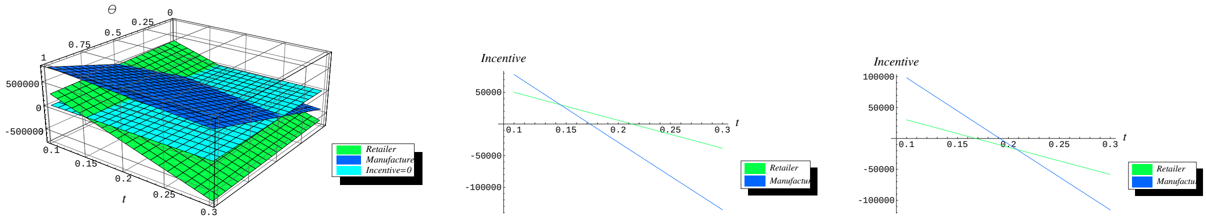


Figure 3.10: Investment in RFID in a supply chain coordinated by a revenue sharing contract for positive threshold values

Next, we demonstrate how incentive of each party to adopt RFID under a revenue sharing contract changes in comparison to the other's.

Theorem 26 *In a supply chain coordinated by a revenue sharing contract, the following findings hold provided that* $Q^*_{RS} > 0$.

1. $t^*_{RS,R}, t^*_{RS,M} < t_1$.
2. $$t^*_{RS,R} is \begin{cases} < t^*_{RS,M} & \text{if } \theta > \theta^*_D \\ > t^*_{RS,M} & \text{if } \theta < \theta^*_D \\ = t^*_{RS,M} & \text{otherwise} \end{cases}$$
3. *Given* $K^*_{RS,R}, K^*_{RS,M} > 0$*:*
$$K^*_{RS,R} is \begin{cases} < K^*_{RS,M} & \text{if } \theta > \theta^*_D \\ > K^*_{RS,M} & \text{if } \theta < \theta^*_D \\ = K^*_{RS,M} & \text{otherwise} \end{cases}$$



(a) Incentives of parties to invest in RFID as θ and t change

(b) Incentives of parties to invest in RFID as t changes when $\theta = 0.30$

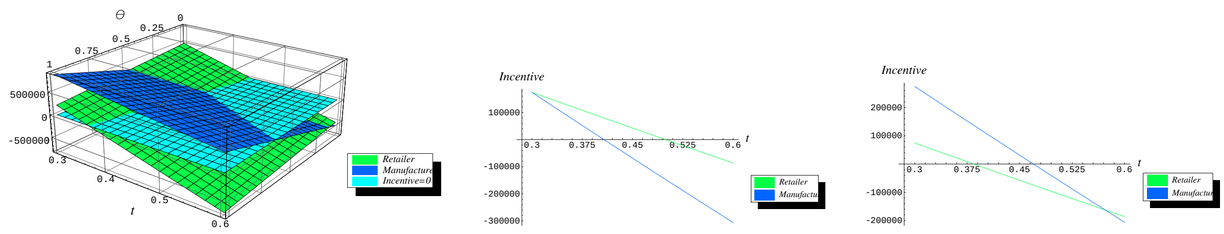
(c) Incentives of parties to invest in RFID as t changes when $\theta = 0.40$

Figure 3.11: Changes in the incentives of the parties in a coordinated supply chain by a revenue sharing contract under inventory misplacement to adopt RFID when demand distribution is Normal$[\mu, \sigma]$-doubly truncated at 2σ ($\mu = 10^6, \sigma = 4 \times 10^5, \gamma = 0.30, p = 25, c = 5, g_R = 1, g_M = 3, s = 2, K = 10^6, a = 0.7$)

where $t^*_{RS,R}$ and $t^*_{RS,M}$ are given by Lemma B, $K^*_{D,R}$ and $K^*_{D,M}$ are given by Lemma B, t_1 is given by Lemma 17 and $\theta^*_{RS} = [g_R + \gamma(p - s)]/(p + g_C - s) < 1$.

As mentioned above, when $t > t_1$, the parties do not adopt RFID technology. We show that $t_{RS,R}$ and $t_{RS,M}$ are less than t_1. Thus, although one can make this supply chain adopt RFID for $t < t_1$ when $K = 0$, the same tag price may not make both of them adopt the technology when $K > 0$. Figure 3.10 demonstrates the incentives of parties under a revenue sharing contract. Theorem 26 is explained similar to Theorem 22. When $\theta > \theta^*_{RS}$, $min\{t^*_{RS,R}, t^*_{RS,M}\}$ is $t^*_{RS,R}$. Thus, the manufacturer is the only party benefiting from this technology for tag prices falling into $[t^*_{RS,R}, t^*_{RS,M})$. Only the retailer benefits from RFID if he covers for less than θ^*_{RS} of the fixed cost for tag prices falling in $[t^*_{RS,M}, t^*_{RS,R})$.

Figure 3.11 and Figure 3.12 show incentives of the parties in a coordinated supply chain by a revenue sharing contract under inventory misplacement to adopt RFID when demand distribution is Normal$[\mu, \sigma]$ and Exponential $[\lambda]$, respectively (refer to Appendix B for the



(a) Incentives of parties to invest in RFID as θ and t change

(b) Incentives of parties to invest in RFID as t changes when $\theta = 0.30$

(c) Incentives of parties to invest in RFID as t changes when $\theta = 0.40$

Figure 3.12: Changes in the incentives of the parties in a coordinated supply chain by a revenue sharing contract under inventory misplacement to adopt RFID when demand distribution is Exponential[λ] ($\lambda = 10^{-6}, \gamma = 0.40, p = 25, c = 5, g_R = 1, g_M = 3, s = 2, K = 10^6, a = 0.7$)

proofs regarding concavity). These graphs lend support to our findings regarding the incentives of the parties in such a supply chain where demand distribution is uniform. We observe that these examples also follow Corollary 11, hence the incentive of a party to adopt the technology is the same regardless of the party paying for the tagging expenses.

Lemma 22 *In a supply chain coordinated by a revenue sharing contract, the following findings hold given $Q^*_{RS,ti} = Q^*_{C,t} > 0$ where $i\,\epsilon\{R, M\}$.*

1. *(a) i. $\partial t^*_{RS,R}/\partial K < 0$ ii. $\partial t^*_{RS,R}/\partial a < 0$ iii. $\partial t^*_{RS,R}/\partial \theta < 0$ iv. $\partial t^*_{RS,R}/\partial \gamma > 0$.*

   (b) i. $\partial t^*_{RS,M}/\partial K < 0$ ii. $\partial t^*_{RS,M}/\partial a < 0$ iii. $\partial t^*_{RS,M}/\partial \theta > 0$ iv. $\partial t^*_{RS,M}/\partial \gamma < 0$.

2. *(a) i. $\partial K^*_{RS,R}/\partial t < 0$ ii. $\partial K^*_{RS,R}/\partial a < 0$ iii. When $K^*_{RS,R} > 0$, $\partial K^*_{RS,R}/\partial \theta < 0$ iv. When $K^*_{RS,R} > 0$, $\partial K^*_{RS,R}/\partial \gamma > 0$.*

   (b) i. $\partial K^*_{RS,M}/\partial t < 0$ ii. $\partial K^*_{RS,M}/\partial a < 0$ iii. When $K^*_{RS,M} > 0$, $\partial K^*_{RS,M}/\partial \theta > 0$ iv. When $K^*_{RS,M} > 0$, $\partial K^*_{RS,M}/\partial \gamma < 0$.

Maximum tag prices and fixed costs that each party can afford to benefit from the technology portray similar behavior to their counterparts in an uncoordinated supply chain with changes in tag price, availability level, fixed cost and sharing of fixed cost. Additionally, the retailer taking more of the supply chain earnings puts tighter bounds on the threshold values related with the manufacturer and vice versa.

The optimum order quantity in a coordinated supply chain is equal to that of a centralized firm. Hence, it is obvious that optimal stocking decisions in a coordinated supply chain react to changing mean or variance of demand in the exact same way as in the centralized firm's. We further prove that the incentives of each firm in a coordinated supply chain is affected by the same way as the centralized firm is by such a change (Refer to Theorem 27 in Appendix B).

In this chapter, we study incentives of parties in vertically integrated, uncoordinated and coordinated supply chains under inventory misplacement to adopt RFID. Our results lend support to the current reactions of the supply chains in that if the fixed costs or the variable costs of this investment are not sufficiently low, it is not beneficial to adopt this technology for neither party. Depending on the sharing of the fixed costs of investment, we show that the manufacturer may indeed be the only party benefiting from this technology.

Chapter 4

Inventory Management under Shrinkage and Customer-Driven Search

4.1. Introduction

> "When we think of how we can compete against larger retailers, being a small and regional player, we realized we should be able to manage our inventory a lot better, securing it and making sure people are paying for it."
>
> *Chris Dorsey, CIO of Chase-Pitkin (Demery, 2005)*

Chase-Pitkin Home & Garden Centers of New York is just one of the retailers who face the "shrinkage" problem, which is the reduction of physical inventory caused primarily by shoplifting or employee theft. They claim that they have historically lost 2% of their sales (amounting to \$4 million) due to shrinkage and they are not alone (Demery, 2005). According to an Ernst & Young survey, in the United States, the average loss in the retail industry due to shrinkage ranged from 0.8% to 4.7% of sales in 2002 (Loss Prevention Survey, 2003).

In this chapter, we explicitly focus on shrinkage as a special case of inventory inaccuracy. Our aim is to identify the impact of shrinkage on retailers' order placement policies in a competitive environment. As suggested by the quote, shrinkage is a more serious problem under competition since a customer who cannot find the product at one store is likely to search for it at the competitor. Retail-industry surveys also suggest that shrinkage is a big problem for "hot" products, especially during peak demand seasons. A great example is a fad toy during the before-Christmas shopping season. The retailers would like to sell as much as possible and do not want to lose customers to their competitors during this period because demand goes down dramatically at the end of the season. At the same time, shrinkage is highest during during peak demand; therefore the risk of stock-out due to shrinkage is highest right when it will hurt the retailers most. In this work, we study a supply chain with a single reliable manufacturer and two retailers subject to shrinkage. We study a single product model where the product has a short life cycle. We assume that demand each retailer faces is stochastic and independent from each other. A fraction of customers can not find the product at one retailer searches for it at the other retail store. We explore the following research questions: When faced with both shrinkage and competition, how much inventory should the retailers stock? If the retailers have the ability to reduce shrinkage, how will their order quantities and expected profits change? How will this impact a competitor? What is the effect of such improvements on the manufacturer?

The remainder of the chapter is organized as follows. Section 4.2 gives an overview of related literature and Section 4.3 gives our analytical model.

4.2. Literature Review

There has been recent empirical and analytical work on inventory inaccuracy, which has been reported as one major operational inefficiency in an empirical paper by Raman, Ton and DeHoratius (2003). Inventory inaccuracy is defined as a mismatch between the physical inventory in a store and what the inventory record shows. Shrinkage is one reason of inventory inaccuracy among others such as transaction errors. Readers are referred to Lee and Özer (2005) for a review of analytical work related with inventory inaccuracies. Most of the analytical work in this area account for inaccuracies due to transaction errors and model inventory inaccuracy through a random error with zero mean, partially observable markov chains or an invisible demand process (see Kök and Shang, 2005; Bensoussan, Cakanyildirim and Sethi, 2005a; Bensoussan, Cakanyildirim and Sethi, 2005b; DeHoratius, Merserau and Schrage, 2005). Inventory shrinkage, however, can be interpreted as a type of inventory inaccuracy where the physical inventory is always less than the inventory record.

Another stream of research closely related with shrinkage is inventory misplacement. Under inventory misplacement, although inventory records are accurate, a fraction of the inventory is not available for sale because they are not at their proper locations. Misplaced items can be retrieved back whenever a cycle count is performed. Çamdereli and Swaminathan (2005) and Rekik, Sahin and Dallery (2005) study such systems. Contrary to systems subject to inventory misplacement, items lost due to shrinkage can never be retrieved.

Even though shrinkage is clearly a big problem for the retail industry, to our knowledge, it has not received much attention in the operations management literature. Different from all of the papers above, we study an environment where the items are lost primarily by shoplifting or employee theft. Kang and Gershwin (2004) computationally study the effects of inventory discrepancies due to shrinkages on the lost sales. Their simulation results show that replenishment process and out-of-stocks are highly sensitive to inventory discrepancies. Our setting is different in that we analytically study a game theoretic environment with two retailers under customer search subject to shrinkage.

Several papers address mitigating the negative effects of inventory inaccuracies through new technologies such as Radio Frequency Identification (RFID). Gaukler, Seifert and Hausman (2004), Çamdereli and Swaminathan (2006) and Heese (2006) consider two-player supply chains subject to different types of inventory inaccuracies in the context of RFID adoption. In this chapter, we explicitly research shrinkage in a two-retailer setting where customers are allowed to search for the product they could not find in the other retail store. We also consider the cost of reducing shrinkage and model it as a two-stage game, where in the first stage, the retailers set their availability levels and in the second stage, the retailers set their order quantities through a non-cooperative game.

Papers which study competitive inventory models under demand substitution are closely related with our work (the readers are referred to Mahajan and van Ryzin (1999) for a more comprehensive review). Parlar (1988) studies a two-firm competitive newsvendor model and shows that a unique Nash Equilibrium exists in inventory levels when demands independent and random. Lippman and McCardle (1997) study a number of allocation mechanisms to split the industry demand among competitive firms. Netessine and Rudi (2003) study a consumer-driven demand substitution problem in an arbitrary number of products where demand vector is assumed to follow a known continuous multivariate distribution. In a similar setting to the one presented in this work, Anupindi and Bassok (1999) consider a two-retailer supply chain free of any inaccuracy and concentrate on the interests of the manufacturer and the retailers with regard to centralization of stocks. Different from the papers in this area, in addition to demand substitution, we assume that a fraction of the inventory in each retailer becomes unavailable for sale in each retailer. We study the effects of inventory shrinkage on stocking decisions. Further, we research the incentives of the manufacturer and the retailers to reduce shrinkage. Contrary to the common belief regarding the suppliers not benefiting from improvements in inventory availability, we identify conditions for which the manufacturer may actually benefit from such an improvement.

Another relevant stream of research is random yield literature (refer to Yano and Lee (1995) for an extensive review). Many of the papers which study a single-product and a single-firm setting subject to stochastic demand assume random yield (e.g. Anupindi and Akella, 1993; Ciarallo, Akella and Morton, 1994; Gupta, Kooper, 2005; Gerchak, Wang and Yano, 1994; Gurnani, Akella, Lehoczky, 2000; Henig and Gerchak, 1990; Hojati, 2006; Inderfurth, 2004; Rekik, Sahin and Dallery, 2006). In this chapter, we assume that a constant fraction of the items are lost. However, we analyze a multi-retailer setting where a portion of excess demand is substitutable. Some work on random yield considering multi-product settings assume known demand (Gerchak, Tripathy and Wang, 1996). We assume stochastic demand here. Some other random yield papers which study multi-product settings also include stochastic demand in their models (Hsu, Bassok, 1999; Karaesmen, Van Ryzin, 2004; Kazaz, 2004). However, in our model, demand substitution occurs in both ways and we analyze a supply chain under shrinkage in a game theoretic environment. The analysis of such a system subject to stochastic demand is analytically complex even for deterministic stock losses. Allowing shrinkage to be random would impose analytical tractability issues on the model. One may think of shrinkage rates as expected values. We provide a numerical study to gain insights on the case where stock loss is allowed to be random.

4.3. The Model

4.3.1 Model Overview

We consider a supply chain with a single manufacturer and two retailers. The retailers sell a single product with a short life cycle and they purchase the product from the same manufacturer. Due to the short life cycle of the product, we model this problem as a single period problem. We let $i = 1, 2$ denote the retailers. Each retailer faces continuously distributed stochastic demand (d_i) with cumulative distribution function $F_i(\cdot)$ and probability distribution function $f_i(\cdot)$. The demand at each of the retailers is independent; however a percentage of customers who cannot find the product at their first retailer of choice will search for the product at the other retailer. We use α to denote the percentage of customers who are willing to search for the product in case of a stock-out and the search percentage is the same for both retailers. The willingness-to-search depends more on the product type than the retailer selling it, so we believe this assumption is a reasonable representation of reality. Retailers first fulfill demand from their own customers and then, if any inventory is available, they fulfill the demand from the other retailer's customers. The retail price is p per item, the wholesale price is w per item and the manufacturer's cost is c per item. The retailers incur a cost g per item if they fail to fulfill demand. However this shortage cost is incurred only if a retailer fails to fulfill demand from his own customers, i.e. failure to fulfill a searching customer's demand does not result in shortage cost. Left-over inventory is disposed of at a salvage price of s per item.

We model shrinkage as follows: if retailer i orders q_i units from the manufacturer, only a percentage of those q_i units will be available for sale and we call this percentage "availability percentage" denoted by a_i for retailer $i = 1, 2$. Hence we assume that shrinkage takes place all at once, at the time the order is received. This assumption is consistent with our focus on products that are sold over a short period.

Both retailers are profit maximizers and they set their order quantities to maximize their expected profits. Without loss of generality, the expected profit of retailer 1 (denoted by π_1) is given by (4.1). Expression $H_1[q_1, q_2; \alpha] = min[a_1q_1, d_1] + (min[a_1q_1 - d_1, \alpha(d_2 - a_2q_2)])^+$ denotes the number of items sold by retailer 1.

$$\pi_1 = E[pH_1[q_1, q_2; \alpha] + s(a_1q_1 - H_1[q_1, q_2; \alpha])^+ - g(d_1 - a_1q_1)^+ - wq_1] \tag{4.1}$$

where $x^+ = max(0, x)$.

Due to customer-driven search, the order decisions of the retailers are dependent and the order quantities will be determined as a result of a non-cooperative game (on the order quantities) between the retailers. In the next section, we show the existence and the uniqueness of the Nash equilibrium. Before we start discussing our results, it is important to note that the manufacturer's profit (π_s) is deterministic once the order quantities of the retailers are set; hence π_s is simply as in (4.2).

$$\pi_s = w\ [q_1 + q_2] \tag{4.2}$$

where without loss of generality we have normalized the per unit cost of the manufacturer to zero.

4.3.2 Nash Equilibrium

We first establish the existence and the uniqueness of the Nash equilibrium. Theorem 1 states our result.

Theorem 28 *A pure-strategy Nash equilibrium for the order-quantity-game between the two retailers exists when $a_1, a_2 > \frac{w}{p+g}$ and is unique. The equilibrium is found by solving the set of equations given by the following best-response function for $i = 1$ and $i = 2$.*

$$ga_iF_i(a_iq_i) + (p - s)\int_0^{a_iq_i} a_iF_j(\frac{a_iq_i - x_i}{\alpha} + a_jq_j)f_i(x_i)dx_i = (p + g)a_i - w \tag{4.3}$$

Theorem 28 shows that the order-quantity-game has a unique Nash Equilibrium. The equilibrium quantities are obtained by solving expression (4.3) for each retailer.

Lemma 25 *Let (q_1^*, q_2^*) be the solution to the set of equations given by expression (4.3) for $i = 1$ and $i = 2$ and let $(q_{1,N}^*, q_{2,N}^*)$ be the equilibrium quantities of a game when $a_1, a_2 = 1$.*

1. *Using $(\frac{q_{1,N}^*}{a_1}, \frac{q_{2,N}^*}{a_2})$ instead of (q_1^*, q_2^*) generates suboptimal results.*
2. *Let $(\frac{q_{1,N}^{'*}}{a_1}, \frac{q_{2,N}^{'*}}{a_2})$ be the equilibrium of the game when one makes the transformation $w_i' = \frac{w_i}{a_i}$ in each retailer's profit function facing customer search under no shrinkage (when $a_i = 1$ for $i \epsilon \{1, 2\}$). Then, $(\frac{q_{1,N}^{'*}}{a_1}, \frac{q_{2,N}^{'*}}{a_2}) = (q_1^*, q_2^*)$.*

An intuitive approach is to solve the problem for $a_1, a_2 = 1$ to obtain the newsvendor quantities $q_{i,N}^*$ (see Parlar, 1988) and to inflate them by the corresponding availability rate ($q_{i,INF}^* = q_{i,N}^*/a_i$). However, this will generate a non-equilibrium result. If one adjusts the wholesale price of the newsvendor facing market search under no-shrinkage by the rate of inventory availability at the corresponding retailer, inflated newsvendor quantities $(q_{1,INF}^{'*}, q_{2,INF}^{'*})$ indeed constitute the Nash Equilibrium. Ceteris paribus, as the shrinkage problem escalates at one of the retailers, the equivalent newsvendor under no-shrinkage, should offer a higher wholesale price so that we can still use new inflated newsvendor equilibrium quantities as solutions to set of equations given by (4.3).

Next, we compare the equilibrium stocking decisions for four different cases, which are defined in Table 4.1. Case 1 is the most generalized case where there is customer search and shrinkage at both retailers. Case 2 is where there is customer search but no shrinkage at

Table 4.1: Definition of cases

Case number	Definition
Case 1	$\alpha > 0$ and $a_i, a_j < 1$
Case 2	$\alpha > 0$ and $a_i, a_j = 1$
Case 3	$\alpha = 0$ and $a_i, a_j < 1$
Case 4	$\alpha = 0$ and $a_i, a_j = 1$

any of the retail stores. Case 3 occurs when each retailer acts as an independent party and is under shrinkage. Finally, Case 4 is where there is neither shrinkage nor customer search; therefore, each retailer acts as a classical newsvendor.

Theorem 29 *Let us call $q^*_{i,k}$ where $i \epsilon \{1, 2\}$ the equilibrium order of retailer i under Case k, definition of which is given in Table 4.1.*

1. $q^*_{i,1} < \frac{q^*_{i,2}}{a_i}$ *holds for at least one i.*

2. $\frac{q^*_{1,2}}{a_1} > q^*_{1,3}$ *and* $\frac{q^*_{2,2}}{a_2} > q^*_{2,3}$.

3. $q^*_{1,3} < \frac{q^*_{1,4}}{a_1}$ *and* $q^*_{2,3} < \frac{q^*_{2,4}}{a_2}$.

4. $q^*_{1,1} > q^*_{1,3}$ *and* $q^*_{2,1} > q^*_{2,3}$.

5. $q^*_{1,2} > q^*_{1,4}$ *and* $q^*_{2,2} > q^*_{2,4}$.

Theorem 29 compares equilibrium quantities of the four cases. In Lemma 25, we showed that the equilibrium of the case where retailers are subject to shrinkage will never be equal to the equilibrium under no shrinkage. With this theorem, we further prove that in the existence of customer search, at least one of the equilibrium quantities under shrinkage is always less than the corresponding inflated equilibrium order under no shrinkage. The second and third results of Theorem 29 show that simply inflating the equilibrium quantities of the cases where there is no shrinkage never gives the equilibrium for the cases where there is shrinkage. In fact, inflated equilibrium orders of the Case 2 and Case 4 are always higher than the corresponding equilibrium orders of Case 3. We explore the effects of having customer search in a system where there exists no search. The last two results of the theorem states that whether there is shrinkage or not, equilibrium quantity of each retailer is always higher than the corresponding equilibrium order of the case where there is no customer search.

Having compared the equilibrium quantities of the four different cases with respect to existence of shrinkage and/or customer search, we next explore the effects of changes in customer search in the equilibrium stocking decisions and the manufacturer's profit in the equilibrium.

Lemma 26 *The total equilibrium order increases as α increases if and only if*

$$a_2(N1[2] + N2[2] + N3[2] - N1[1]\alpha) + a_1(N1[1] + N2[1] + N3[1] - N1[2]\alpha) > 0$$

where

$$N1[j] = a_i a_j (p-s)^2 \left(\int_{a_j q_j}^{a_j q_j + a_i q_i/\alpha} (x_j - a_j q_j) f_i(\psi_i) f_j(x_j) dx_j\right)\left(\int_{a_i q_i}^{a_i q_i + a_j q_j/\alpha} f_i(x_i) f_j(\psi_j) dx_i\right) > 0,$$

$$N2[j] = a_i a_j (p-s)^2 F_i(a_i q_i) f_j(a_j q_j)\left(\int_{a_j q_j}^{a_j q_j + a_i q_i/\alpha} (x_j - a_j q_j) f_i(\psi_i) f_j(x_j) dx_j\right) > 0,$$

$$N3[j] = gN2[j]/[(p-s)F_i(a_i q_i)] > 0,$$

$$N1[i] = a_i a_j (p-s)^2 \left(\int_{a_i q_i}^{a_i q_i + a_j q_j/\alpha} (x_i - a_i q_i) f_j(\psi_j) f_i(x_i) dx_i\right)\left(\int_{a_j q_j}^{a_j q_j + a_i q_i/\alpha} f_j(x_j) f_i(\psi_i) dx_j\right) > 0,$$

$i,\ j \in \{1,2\}$, $\psi_i = a_i q_i - \alpha(x_j - a_j q_j)$ and $\psi_j = a_j q_j - \alpha(x_i - a_i q_i)$.

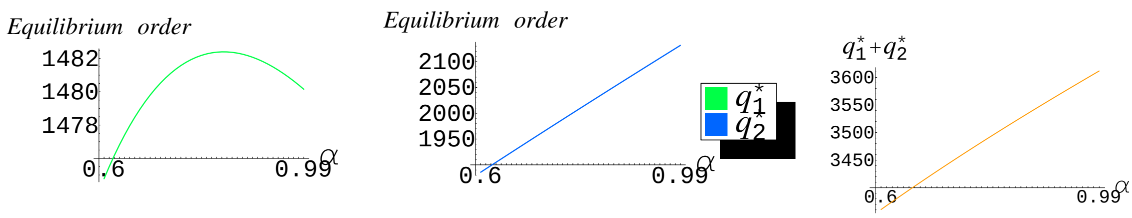
Lemma 26 gives the necessary and sufficient condition for the total order in the equilibrium to increase as customer search increases. As the condition is complicated, next we derive sufficient conditions to characterize the behavior of the total order as α changes.

Lemma 27

1. *If $\alpha < 1/2$, $q_1^* + q_2^*$ increases in α.*
2. *Without loss of generality, let us assume that $a_2 \geq a_1$. If $\alpha < \frac{a_1}{a_2}$, $q_1^* + q_2^*$ increases in α.*

Not being able to derive a general conclusion about the effects of changes in alpha on the summation of the optimum orders, we show that as the customer search increases up to 50%, the total quantity ordered from the manufacturer always goes up. Therefore, if less than 50% of the customers search for the product in the other retail store, the manufacturer always benefits from an increase in α. Further, we show that when $\alpha < \frac{a_1}{a_2}$, the total order always increases as well. This result is also in line with the findings of Anupindi and Bassok (1999). The authors prove that in a two-retailer supply chain under customer search and no operational inefficiency, the total order always increases as more disappointed customers of a retail store search for the product in the other retailer. Lemma 27 also indicates that when $a_1 = a_2 = 1$, the total order always increases as α increases.

Next, we analyze the effects of customer search on each equilibrium order.



(a) Effects of α on each equilibrium quantity

(b) Effects of α on the summation of equilibrium quantities

Figure 4.1: Effects of α on the equilibrium order quantities when each retailers' demand distribution is Exponential[λ] ($\lambda = 10^{-3}, p = 300, w = 100, g = 20, s = 40, a_1 = 0.6, a_2 = 0.99$)

Lemma 28

1. $\frac{\partial q_i^*}{\partial \alpha} < 0$ *if and only if* $\alpha > \alpha_i^*$ *where*

$$\alpha_i^* = \frac{N1[j] + N2[j] + N3[j]}{N1[i]}, \tag{4.4}$$

N1[j], N2[j], N3[j] and N1[i] are given by Lemma 26, i and j $\epsilon\{1,2\}$ and $i \neq j$.

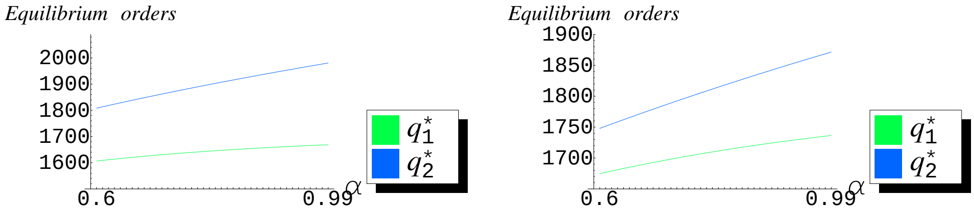
2. *As α increases, q_1^* and q_2^* do not decrease simultaneously.*

Lemma 28 shows the effects of changes in the search parameter on the equilibrium stocking decisions. If more of the customers who could not find the item they are looking for in one retailer search for it in the other retail store, one could expect the retailers to increase their orders in the equilibrium. However, Lemma 28.1 implies that if the search parameter is greater than a threshold value, an equilibrium quantity may indeed decrease as search increases. Note that this threshold also depends on the value of the search parameter itself.

As customer search intensifies, keeping the quantities constant, spill over demand increases in both directions. If one of the equilibrium quantities decreases, spill over demand towards the other retailer increases even further. Hence, the two can never decrease at the same time. This result also leads to the fact that if demands are identically distributed and $a_1 = a_2$, each equilibrium quantity increases as search increases.

Because expression (4.4) depends on the values of q_1^* and q_2^*, one has to solve for the equilibrium orders first to calculate the value of α_i^*. Here, we gain insights about this result through numerical examples. Anupindi and Bassok (1999) show that when demand distributions are identical, each equilibrium quantity always increases as search intensifies. Although not being able to prove it analytically for nonidentical demand distributions, they state that in all of the numerical experiments they have conducted, they do not encounter a case where an equilibrium quantity decreases as the search parameter decreases. However, as we numerically illustrate below, in the existence of shrinkage, this may not be the case even for identical demands.

Figure 4.1 shows an example where one of the order quantities decreases as α increases when demands are identically distributed according to Exponential distribution. While the first retailer loses 40% of his order ($0.6 \times q_1^*$ is available for sales) due to shrinkage, the second retailer loses only 1% ($0.99 \times q_2^*$ is available for sales). Figure 4.1(a) shows plots of q_1^* and q_2^* as α increases. When search parameter is sufficiently low, both quantities increase as α increases. However, when the search parameter is higher than a threshold value, q_1^* decreases while q_2^* continues to increase. In this example, retailer 1 has a significantly lower inventory availability rate relative to the other retailer. Hence, knowing that the second retailer has



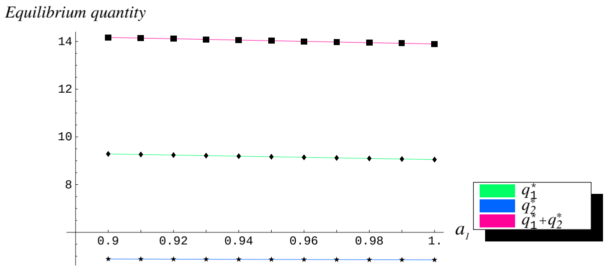
(a) $a_1 = 0.60$ and $a_2 = 0.70$ (b) $a_1 = 0.85$ and $a_2 = 0.99$

Figure 4.2: Effects of α on the equilibrium order quantities when each retailers' demand distribution is Exponential[λ] ($\lambda = 10^{-3}, p = 300, w = 100, g = 20, s = 40$)

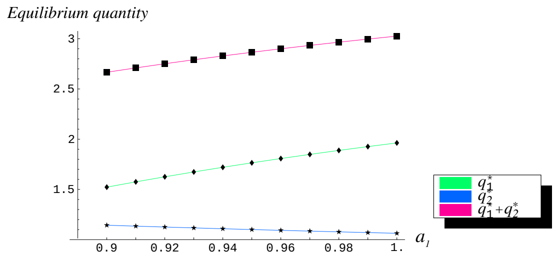
a very high inventory availability level, the first retailer anticipates to have less spill over demand from the second retailer as search intensifies. Hence, he orders less for such values of α.

Next, we give a numerical example, which shows the effects of α on the stocking decisions when inventory availability values are close to each other. Using the same parameter values, Figure 4.2 plots the effects of α for the two cases: where inventory availability levels of both retailers are low ($a_1 = 0.60$ and $a_2 = 0.70$) and where inventory availability levels of both retailers are high ($a_1 = 0.85$ and $a_2 = 0.99$). We observe that the equilibrium orders show the same type of behavior in both of these examples.

In order to judge who will benefit from a unilateral improvement of availability at one of the retailers, we next analyze how the equilibrium profit levels at the retailers change as the availability improves at one of the retailers. First we consider the case, where availability improvement is costless. Our motivation is to understand which player(s) have the incentive to invest in availability improvement.



(a) High-margin product ($w = 2$)



(b) Low-margin product ($w = 7$)

Figure 4.3: Changes in the equilibrium order quantities as a_1 increases when retailer 1 and retailer 2 face demand distributed exponentially with parameter λ_1 and λ_2, respectively. ($\lambda_1 = 2 \times 10^{-1}, \lambda_1 = 4 \times 10^{-1}, \alpha = 0.20, p = 9, g = 0.5, s = 0.5$ and $a_2 = 0.9$)

Theorem 30 *The following results hold where i and j $\epsilon\{1, 2\}$ and $i \neq j$.*

1. *As the availability level of retailer j increases, the equilibrium quantity of retailer i decreases.*

2. *If $w > q_i|\frac{\partial^2 \pi_i}{\partial q_i^2}|$ holds, the equilibrium quantity of retailer i increases in the availability level of retailer i.*

We show that as the availability level at a retailer improves, the equilibrium order quantity of the other retailer decreases. As availability improves at a retailer, the spill-over demand at the other retailer drops and he responds to this by ordering less under the new equilibrium. In this case, we would also expect the retailer whose availability improves to order less. However, due to results for supermodular games, his order quantity may increase when his profit is decreasing in his order quantity and his availability level. A sufficient condition for this to happen is $w > q_i|\frac{\partial^2 \pi_i}{\partial q_i^2}|$. Further, $\frac{\partial^2 \pi_i}{\partial q_i \partial a_i} > 0$ is more likely to hold when the difference between the retailer's selling price and the wholesale price is small, or in other words this condition is more likely to hold when the retailer's margin on the product is low. In general, the retailer will respond to low availability by a large order quantity and when availability improves, he will lower his order quantity. However, if the product has a low margin, it does not pay to order it in large quantities to make up for the shrinkage problem. In this case, the retailer increases the order quantity only when availability improves and the product becomes more profitable. Figure 4.3 illustrates a numerical example where we observe this behavior.

Theorem 31

1. *The equilibrium profit of a retailer increases as her inventory availability level increases.*
2. *The equilibrium profit of a retailer decreases as the other retailer's inventory availability level increases.*

As long as the availability improvement is costless, we would expect a retailer to benefit from an improvement in her own inventory availability level. As she makes this improvement, the other retailer's equilibrium profit decreases. It is easy to see this result when the equilibrium order quantity of the retailer making the improvement, say retailer i, increases in a_i. In the region where q_i^* decreases, we find that the equilibrium profit of retailer j increases if and only if $\frac{\partial q_i^*}{a_i} \frac{a_i}{q_i} < -1$ (see Appendix C); that is, a percentage increase in a_i causes a larger decrease in q_i^*. However, we can show that this can never happen and hence retailer j's profit always decreases.

Table 4.2: Payoffs of the sequential game

Retailer 1	Retailer 2
$u_1^*[0,00] = u_1^*[0,01] = u_1^*[00]$	$u_2^*[0,00] = u_2^*[0,01] = u_2^*[00]$
$u_1^*[0,10] = u_1^*[0,11] = u_1^*[01]$	$u_2^*[0,10] = u_2^*[0,11] = u_2^*[01] - C[a_2]$
$u_1^*[1,00] = u_1^*[1,10] = u_1^*[10] - C[a_1]$	$u_2^*[1,00] = u_2^*[1,10] = u_2^*[10]$
$u_1^*[1,01] = u_1^*[1,11] = u_1^*[11] - C[a_1]$	$u_2^*[1,01] = u_2^*[1,11] = u_2^*[11] - C[a_2]$

Next, we analyze a game where each retailer can completely solve the problem of shrinkage and increase his inventory availability level to 100% at a cost of investment, $C[a_i]$ where $i\epsilon\{1,2\}$. Hence $C[a_i]$ is the cost of raising a_i to one. We do not assume any form for $C[a_i]$. Without loss of generality, we assume that retailer 1 moves the first in the sequential game, which is described below.

Definition 1

Definition of the sequential game: Retailer 1 moves the first, the strategy sets of retailer 1 and retailer 2 are $S_1 = \{0,1\}$ and $S_2 = \{00, 01, 10, 11\}$, respectively, and the payoffs to the retailers are shown by $u_i^*[s_i, s_j]$ where $i\epsilon\{1,2\}$ as given by Table 4.2.

$$\begin{bmatrix} 0 \equiv & \text{keeping the inventory availability level at the existing level (less than 100\%)} \\ 1 \equiv & \text{raising the inventory availability level to 100\%} \end{bmatrix}$$

$$\begin{bmatrix} 00 \equiv & \text{playing 0 whether retailer 1 chooses 0 or 1} \\ 01 \equiv & \text{playing 0 if retailer 1 chooses 0 and playing 1 otherwise} \\ 10 \equiv & \text{playing 1 if retailer 1 chooses 0 and playing 0 otherwise} \\ 11 \equiv & \text{playing 1 whether retailer 1 chooses 0 or 1} \end{bmatrix}$$

Theorem 32 *In a sequential game which is described by Definition 1, the following results* *hold.*

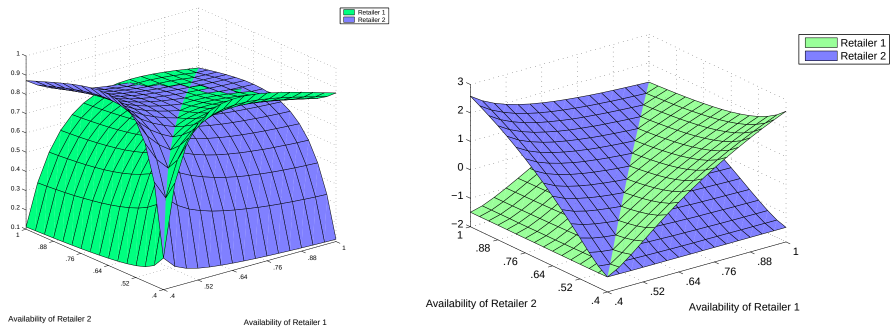
1. $C[a_2] < u_2^*[11] - u_2^*[10]$ *and* $C[a_1] < u_1^*[11] - u_1^*[00]$ *are sufficient for either* $\{1, [01]\}$ *or* $\{1, [11]\}$ *to be the Nash Equilibrium.*

2. *If* $C[a_2] > u_2^*[11] - u_2^*[10]$*, at least one retailer decides to keep the existing availability level.*

3. *If* $C[a_2] < u_2^*[01] - u_2^*[00]$*, at least one retailer always invests to remove the shrinkage.*

Theorem 32.1 gives sufficient conditions for both of the parties to invest to completely remove the problem of shrinkage. Since retailer 2 makes the second move, his decision is a function of the 1st retailer's. Hence, his sufficient condition to make an investment compares the cost of such an investment with the difference between the highest and the lowest profits that he can make when the 1st retailer chooses to invest. On the other hand, having the advantage of being a leader in the game, the first retailer compares his cost of investment with the difference between his profits if he decides to invest and the best profits that he could get if he does not.

As stated by Theorem 32.2, if the cost of such an investment to the second retailer is higher than what he could gain over the worst case provided that the first retailer makes an investment, then both retailers making an investment can never be a Nash equilibrium. On the contrary, if $C[a_2] < u_2^*[01] - u_2[00]$ holds, then the second retailer always invests when the first one chooses not to. As a result, at least one retailer always invests to reduce remove the problem of shrinkage.



(a) Equilibrium quantities (b) Equilibrium profits

Figure 4.4: Equilibrium quantities and profits when each retailers' demand distribution is Uniform[0,1] ($p = 10, w = 5, \alpha = 0.6, g = 3, s = 1$)

Theorem 32 analyzes a game where each retailer has a dichotomous strategy set: whether a retailer completely removes shrinkage by making an investment to increase the inventory availability level to 100% or does nothing. However, improvements can be experienced in increments as well. We next analyze a simultaneous game where strategy set of each player is a continuous.

Theorem 33 *Let us assume that the retailers are allowed to increase their inventory availabilities free of charge. In a game where the retailers simultaneously decide on inventory availability level and then the order quantity, a unique equilibrium occurs where* $a_1 = a_2 = 1$.

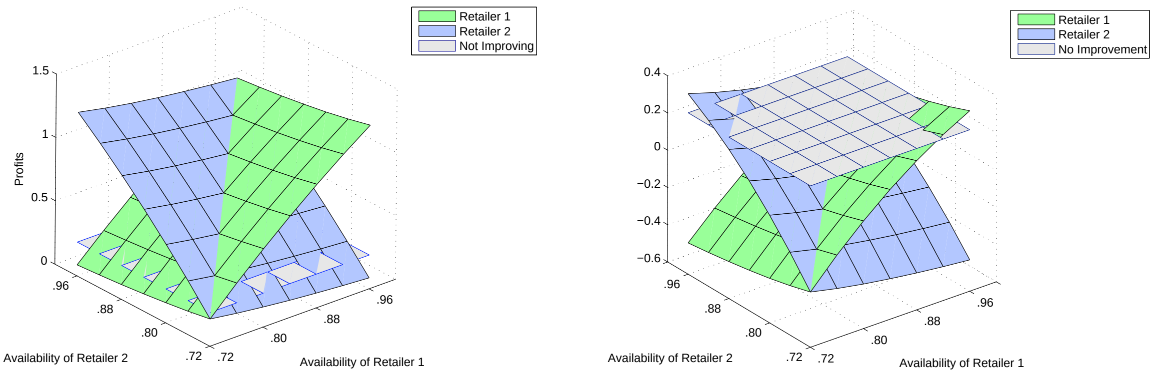
Theorem 33 states that if one retailer chooses to improve inventory availability, the other always responds to it if inventory improvement can be accomplished at a negligible cost. Further, the retailers do not settle for inventory availability less than 100%.

Actions of the retailers are clear when reducing shrinkage is free of charge. Inventory availability can be improved through creating awareness of the problem (Raman, DeHoratius and Ton, 2003), closed monitoring, securing inventory, RFID, and so on. The equilibrium becomes interesting when costs of investment are included. We gain insights about this problem through numerical examples.

Figure 4.4(a) plots optimum orders with respect to inventory availabilities at the retail stores when demands are i.i.d. distributed with uniform distribution in [0,1] ($p = 10, w = 5, \alpha = 0.6, g = 3, s = 1$). Figure 4.4(b) shows the profits of these retailers in the equilibrium for different inventory availability values. This plot also confirms Theorem 33 in that if improving availability were free, the two would end up at $a_1 = a_2 = 1$.

Next, we use a convex cost $C[a_i] = (K \times a_i^2)$ to improve an availability of a_i by one unit. We assume that both parties already have an inventory availability of 40%. Figure 4.5 shows the net profits after incurring the costs of improvement. One unit corresponds to 0.44 in this example. Therefore, C[0.96] corresponds to improving inventory availability to 100%. We assume that both retailers start at an inventory availability level of 0.72. As we see in Figure 4.5(a), when the cost of improving inventory availability is low ($K = 0.1$), retailers choose to improve inventory availabilities up to 100%. However, when the cost of improving inventory availability is high, retailers choose to keep their existing inventory availabilities by Figure 4.5(b).

In this chapter, we study a two-retailer supply chain under consumer search and inventory shrinkage. We identify the unique equilibrium for the quantity-game of the retailers. We further show that simply inflating the newsvendor equilibrium quantities under customer search and no shrinkage does not yield optimum results unless one makes an adjustment in



(a) K=0.1

(b) K=1.1

Figure 4.5: Effects of improving availability at $C[a_i] = Ka_i^2 \; where \; i\epsilon\{1,2\}$ when each retailers' demand distribution is Uniform[0,1] $(p = 10, w = 5, \alpha = 0.6, g = 3, s = 1)$

the wholesale price. We study the effects of consumer search in the stocking decisions of the retailers and find that each retailer can decrease his order quantity as search intensifies in the existence of shrinkage. This is an interesting result given that Anupindi and Bassok (1999) does not encounter such a situation in a similar setting to the one studied in this chapter but where no operational inefficiency is taken into account. We also show that a reduction in shrinkage at one retail store leads to a decrease in the other retailer's equilibrium profit.

Chapter 5

Conclusions

This dissertation studies the effects of operational inefficiencies, specifically inventory misplacement and shrinkage, on various forms of supply chains and the role of information technology to mitigate the negative effects of such operational inefficiencies through three separate essays. The first essay titled "Coordination of a Supply Chain under Misplaced Inventory", studies inventory management under misplaced inventory in centralized and decentralized supply chains. We first model inventory misplacement as a constant fraction. Then, we relax this assumption by extending our study to modeling inventory misplacement as a random variable and a function of order quantity.

When the supply chain is vertically integrated and subject to stochastic demand distributed according to a general continuous distribution with a finite mean, we find that the changes in the optimum order under inventory misplacement with respect to changes in inventory availability depends on the increasing generalized failure rate of the demand distribution evaluated at the optimum available number of items. One may chose to decrease the order size when less items are misplaced in the store, i.e. more items become available to customers. However, we show that when the value of the generalized failure rate evaluated at the optimum available number of items is less than one, the optimum order does increase eventhough more items become available. When the value of the generalized failure rate is more than one, then a reduction in inventory misplacement leads to a decrease in the optimum order. We are able to observe this interesting behavior of the optimum order when demand distribution is uniform in $[0, \beta]$. We prove that as inventory availability increases, the optimum order first increases until inventory availability equals to a threshold value, where the the aforementioned value of the generalized failure rate becomes one, then decreases thereafter. When demand is distributed according to a general distribution, we further prove that reduction in inventory misplacement unconditionally benefits the centralized firm, the manufacturer in an uncoordinated supply chain and both of the parties in a supply chain coordinated by a revenue sharing contract. Moreover, we prove that the manufacturer indeed extracts more benefits than the retailer does from a reduction in inventory misplacement in the retailer's store if the value of the generalized failure rate evaluated at the optimum number of available inventories is less than a threshold value.

We derive closed form solutions for the special case of uniform demand, and additionally extend the analysis by Pasternack (1985) of a buyback contracting mechanism to a two sided supply chain with misplaced inventory (refer to Table D1 in Appendix D). Surprisingly, we find that leading manufacturers may benefit more as compared to retailers from an improvement in retail inventory availability. This is in contrast to popular belief that retailers are more likely to benefit from misplacement reduction at their stores. Finally, we make a pair wise comparison of contracts for both the retailer and the manufacturer. We show that one can find a revenue sharing contract which generates the same profits as a buyback contract where the return proportion is greater than the proportion of inventory misplacement. However, one can never find a revenue sharing contract which can generate the same profits as a buyback contract where return proportion is less than the proportion of inventory misplacement. A summary of comparison of contracts is given in Table D2 and Table D3 in Appendix D for the decentralized manufacturer and the decentralized retailer, respectively.

The second essay of this dissertation titled "Misplaced Inventory and RFID: Information and Coordination", studies incentives of players in supply chains to invest in RFID to remove inefficiencies of misplaced inventory. We consider both fixed and variable costs of adoption of RFID in vertically integrated and bilaterally decentralized supply chains, for which we study uncoordinated and coordinated environments by revenue sharing contracts. We analyze scenarios where the fixed cost of adoption is shared and study both cases where only the retailer or only the manufacturer pays for the variable costs of adoption, i.e. *tagging expenses*.

We find that in both decentralized settings, when the manufacturer pays for the tagging expenses, she inflates the wholesale price by the tag price as compared to the wholesale price in the scenario where the retailer pays for the tagging expenses. Hence, incentive of a party to adopt RFID is the same regardless of the one paying for the tagging expenses. Further, we show that the manufacturer charges a higher wholesale price under RFID than she would under no RFID if she covers for the tagging expenses. However, we also prove that the wholesale price under RFID is unconditionally smaller than the one under inventory misplacement in coordinated supply chains and can indeed be smaller than that in uncoordinated environments for high tag prices if the retailer pays for the tagging expenses.

One common result in all of our settings support the current reactions of the supply chains to the RFID technology and show that unless the fixed cost of RFID is sufficiently low and the tag prices are cheap enough, no party would benefit from an investment in this technology. However, our findings regarding the incentives of the parties in the decentralized settings counter the common opinion that the main beneficiaries of RFID will be the retailers. We show that for a given fixed cost of investment (or a tag price of an RFID chip), the characterization of incentives depend on relative investments of fixed cost and the tag price (or the fixed cost of investment). If the retailer covers for $\theta_j^* K$ ($j = D$ in an uncoordinated supply chain, $j = RS$ in a coordinated supply chain by a revenue sharing contract), the incentives of the parties are perfectly aligned such that both parties will benefit from the investment if and only if the tag price is smaller than a threshold value. Otherwise, no party will benefit from such an investment. If the retailer pays for an amount other than $\theta_j^* K$, the incentives of the parties are characterized by two threshold values of tag prices (or fixed cost of investment) for a given fixed cost of investment (or a given tag price). Given that these threshold values are positive, no party benefits from this investment if the tag price is higher than the maximum of the two and both parties benefit if the tag price is smaller than the minimum of the two. An interesting situation arises when the tag price is in between these two values and only one party benefits from the investment. If the retailer pays for more than $\theta_j^* K$, the manufacturer is indeed the only party benefiting from this investment for such tag prices. Although our analytical results are proven for uniform demand distributions, our numerical examples with truncated Normal and Exponential demand distributions lend support to our analytical findings regarding the threshold behavior of the incentives of the parties.

We show that greatest incentives to adopt the technology occur for lower values of availability. We study the effects of changes in the mean and the variance of demand independent from each other on the incentives of the centralized firm and parties in uncoordinated and coordinated decentralized supply chains. Our findings regarding the effects of changes in the mean of the distribution for fixed variance comply with intuition. The more the expected value of demand is, the more beneficial the RFID technology becomes for each firm. However, we find an interesting result regarding the effects of a change in the variance of the distribution. We show that the firms' incentives in the aforementioned supply chains can indeed decrease with increasing variance of demand if the existing availability level is less than a threshold value and the tag price is sufficiently high. Furthermore, those threshold values happen to be identical for each firm and all types of supply chains studied in this paper.

The third essay of this dissertation titled "Inventory Management under Shrinkage and Customer-Driven Search" studies a two-retailer supply chain under customer search where the retailers are subject to inventory shrinkage. A customer who cannot find the product at one of the retailers may search for the product at the other retailer; hence, retailer competition is modeled through a customer-driven search. A unique equilibrium exists in this game and the retailers participate in the game if and only if their inventory availability levels are sufficiently high. Otherwise, the effective cost of stocking positive inventory is higher than not participating. One approach to this problem could be to simply inflate the newsvendor stocking quantities under customer search without any shrinkage by the amount of inventory availability. However, we show that this will never yield the equilibrium under shrinkage unless one makes a wholesale price transformation.

Our findings indicate that, without thorough analysis, it is not easy to predict how the equilibrium stocking decisions of the retailers and the profit of the manufacturer will change if more customers perform market search. The manufacturer benefits from increasing customer search if the total quantity ordered is also increasing. Not being able to derive a general conclusion, we show that if the customer search increases up to 50%, the manufacturer always benefits from an increase in search. We also show that if the percentage of the customers searching for the product is less than the ratio of the inventory availabilities of the retailers, then the total quantity always increases as well. As search intensifies, one may expect each retailer to increase his order size as well. In a similar setting to ours but in the absence of any operational inefficiencies, Anupindi and Bassok (1999) show that when demand distributions are identical, each equilibrium quantity always increases as search intensifies. Further, they state that in all of the numerical experiments they have conducted, they do not encounter a case where an equilibrium quantity decreases as the market search intensifies. However, we identify analytical conditions for each retailer to actually decrease his order size as search increases. Since the condition is messy and not insightful, we illustrate with a numerical example that such a reaction can indeed become optimum even for i.i.d. demand distributions.

We find that as one retailer reduces shrinkage, the other retailer decreases his order size in the equilibrium. This occurs as a result of decreasing spill over demand with increasing inventory availability. We further observe that if a retailer improves his inventory availability level, he is likely to increase his order size if the product has low margin. Recent articles on technologies such as Radio Frequency Identification (RFID), which have the potential to reduce shrinkage, suggest that availability improvement may leave the manufacturer worse off. Our analysis shows that the manufacturer may actually benefit from such an improvement.

Our analysis shows that a reduction in shrinkage at one retail store always causes reduction in the profits of the other retail store. Our findings further indicate that, without thorough analysis, it is not easy to predict how the equilibrium profits of the manufacturer will change if the shrinkage rate at one of the retailers goes down. Further, we study games where each retailer is allowed to respond to the retailer reducing shrinkage in his store. First, we study a sequential game where each retailer can keep the existing inventory availability levels or completely remove shrinkage at a cost of investment. We identify conditions for both retailers to make such an investment and for at least one retailer to invest or not to invest. Then, we study a simultaneous game where each retailer is free to set his inventory availability level to any number at an ignorable cost. We show that the retailers do not settle for an inventory availability level less than 100%. We also provide a numerical example where each retailer can improve his inventory availability level at a cost, which is a convex function of his existing inventory availability. Our findings lend support to the current reactions of the supply chains in that both firms invest to reduce shrinkage unless the cost of improvement is sufficiently high. Otherwise, they do not take any action.

Although we model only misplaced inventory in the first two essays of this dissertation, shrinkages due to theft can easily be incorporated. A generalized version of cleanup process where x% of the products are retrieved and 100-x % are stolen or lost forever can take care of both misplaced inventory as well as shrinkages. The methodology to follow will be similar to the one studied here.

We studied contracts where the sharing of the fixed costs of RFID investment is exogeneous. It would be interesting to study contracts which endogenizes the sharing of the fixed costs of RFID investment.

We further assume that the ordered quantity is misplaced immediately whereas in reality they may be misplaced over time. Modeling gradual misplacement will need a different model of customer demand, particularly the type where customers coming into the store misplace an item with a given probability. Analysis of such a model is likely to be different.

Our last essay shows that the manufacturer may benefit from an increase in inventory availability. Hence, an extension of this work could identify whether or not cooperating with the retailers to improve their inventory availability level is beneficial for the manufacturer. One could employ the techniques of cooperative game theory to study this problem.

We also assume that each retailer is aware of the other retailer's inventory availability level. One possible extension could employ information asymmetry. One can benefit from economics models where one party receives a signal about the other party's inventory availability.

Appendix

A. Appendix for Chapter 2

Theorem 1:

1. Manufacturer's profit function at (2.2) can be organized as follows where μ is $E[\xi]$.

   $EPC(Q_C) = -cQ_C + p\mu + p\int_{aQ_C}^{\infty}(aQ_C - \xi)f(\xi)d\xi + slQ_C + s\int_0^{aQ_C}(aQ_C - \xi)f(\xi)d\xi - g_C\int_{aQ_C}^{\infty}(\xi - aQ_C)f(\xi)d\xi$

   The second derivative of $EPC(Q_C)$ with respect to Q_C is $\frac{\partial^2 EPC}{\partial(Q_C)^2} = -a^2 f(aQ_C)(p + g_C - s) \leq 0$ by Assumptions 1 and 2.

2. The first order condition gives $\frac{\partial EPC}{\partial(Q_C)} = (p+g_C-s)a - aF(aQ_C)(p+g_C-s) + (s-c) = 0$ or $F(aQ_C^*) = 1 - \frac{(c-s)}{a(p+g_C-s)}$. Thus, the second term in $F(aQ_C^*)$ must be < 1 and > 0 for $Q_C^* > 0$. It is > 0 by Assumption 1. If we let $\tilde{a} = \frac{(c-s)}{(p+g_C-s)}$, we know that $\tilde{a} < 1$ by Assumption 1. Hence, $\tilde{a} < a$ must hold for Q_C^* to have a positive value.

Corollary 1: Let us call $\frac{c'-s}{p+g_C-s}$ by $\tilde{a}'$, which is equal to $\tilde{a}/a$. The result follows.

Corollary 2:

For $0 < a < 1$, $Q_C^* > Q_N^* \Rightarrow \frac{1}{a}F^{-1}(1 - \frac{\tilde{a}}{a}) > F^{-1}(1 - \tilde{a})$. Since $\tilde{a}$ can never become 0, the arguments of the survival functions in the inequality will never become 1. Therefore, taking the F of both sides of the inequality will not affect the strict inequality. $F\left(F^{-1}(1 - \frac{\tilde{a}}{a})\right) > F\left(aF^{-1}(1 - \tilde{a})\right) \Rightarrow (1 - \frac{\tilde{a}}{a}) > F\left(aF^{-1}(1 - \tilde{a})\right)$. The same approach is taken for $Q_C^* < Q_N^*$.

$F(Q_N^*) = 1 - \tilde{a}$ and $F(aQ_C^*) = 1 - \tilde{a}/a$. If we substitute c by $c' = c/a - (1-a)s/a$ in $\tilde{a}$, we obtain $\tilde{a}/a$. Hence, the result follows.

Lemma 1:

1. $\frac{\partial Q_C^*}{\partial a} = \frac{1}{a}(\frac{\tilde{a}}{a^2 f(aQ^*)} - Q_C^*)$. Thus, if $\frac{\tilde{a}}{a} > (aQ_C^*)f(aQ_C^*)$, then $\frac{\partial Q_C^*}{\partial a} > 0$ (or equivalently $g[aQ_C^*] < 1$ after substituting $(1 - F(aQ_C^*))$ for $\tilde{a}/a$) and if $\frac{\tilde{a}}{a} < (aQ_C^*)f(aQ_C^*)$, otherwise.

2. We use Envelope Theorem for unconstrained optimization (Simon and Blume, Theorem 19.4, 1994) for the second part and $\frac{d}{da}EPC(Q_C^*(a); a)$ can be expressed as $\frac{d}{da}EPC(Q_C^*(a); a) = Q_C^*(p + g_C - s)\frac{\tilde{a}}{a} > 0$ for $Q_C^* > 0$.

3. We take the derivative of the previous result and obtain $\frac{d^2 EPC(Q_C^*(a);a)}{da^2} = (p + g_C - s)\frac{\frac{dQ^*}{da}\tilde{a}a - Q_C^*\tilde{a}}{a^2}$ where the first multiplier is positive by Assumption 1. Hence, the results (a) and (b) follow.

Theorem 2: $\partial^2 EPR_D(Q_D)/\partial Q_D^2 = -(p + g_R - s)f(aQ_D) < 0$. We obtain $w_D^* = a(p + g_R - s)(1 - F(a(Q_D))) + s$ from the FOC $\partial EPR_D(Q_D)/\partial Q_D = s - w_D + a(p + g_R - s) - F(aQ_D)a(p + g_R - s) = 0$. Let us call $R(Q_D) = w_D^* + w_D^* Q_D$.

$$\partial EPM_D(Q_D)/\partial Q_D = R(Q_D) + g_M a\bar{F}(aQ_D) \tag{A1}$$

$$\partial^2 EPM_D(Q_D)/\partial Q_D^2 = R'(Q_D) - g_M a^2 f(aQ_D) \tag{A2}$$

where

$$R(Q_D) = a(p + g_R - s)\bar{F}(aQ_D)[1 - \frac{aQ_D f(aQ_D)}{\bar{F}(aQ_D)} + s] \tag{A3}$$

Let us call $q = aQ_D$. Hence we can write $R(q) = a(p + g_R - s)\bar{F}(q)[1 - g[q]] + s$

and $\partial R(q)/\partial Q_D = -a^2(p + g_R - s)(f(q)(1 - g[q]) + \bar{F}(q)g'[q])$. The rest of the proof for concavity follows from the proof of Lemma 2 of Lariviere and Porteus (2001). The optimum quantity solves $EPM_D(Q_D)/\partial Q_D = 0$.

Lemma 2:

1. We derive $\frac{\partial(EPR_D(Q_D^*(a);a))}{\partial a} = -\frac{\partial w_D^*}{\partial a}Q_D^* + (p + g_R - s)Q_D^*(1 - F(aQ_D^*))$ by Envelope Theorem for the retailer.

2. $\partial EPM_D(Q_D^*(a);a)/\partial a = Q_D^*\bar{F}(aQ_D^*)(p + g_R - s)(1 - g[aQ_D^*]\frac{p+g_R-s}{p+g_C-s}] > 0$ where $g[aQ_D^*]$ is the generalized failure rate evaluated at aQ_D^* where $g[aQ_D^*] < 1$ (see Theorem 2). Hence, the result follows.

Lemma 3: $\partial EPR_{RS}(Q_{RS})/\partial Q_{RS} = -w_{RS} + \gamma s + a[\gamma(p - s) + g_R] - F(aQ_{RS})a[\gamma(p - s) + g_R]$ and $\partial^2 EPR_{RS}(Q_{RS})/\partial Q_{RS}^2 = -a^2 f(aQ_{RS})[\gamma(p - s) + g_R] < 0$. When the channel is coordinated, $\frac{w_{RS}^* - \gamma s}{a[\gamma(p-s)+g_R]} = \frac{c-s}{a[(p-s+g_C]}$.

Theorem 3

1. $\partial EPR_{RS}(Q_{RS}^*(a);a)/\partial a = \frac{\tilde{a}}{a}Q_R S^*[\gamma(p - s) + g_R] > 0$ from Envelope Theorem.

2. $\partial EPM_{RS}(w_{RS}^*(a);a)/\partial a = (\partial Q_{RS}^*/\partial a)[w_{RS}^* + (1-\gamma)s] + [(1-\gamma)(p-s) + g_M]\bar{F}(aQ_{RS}^*)(Q_{RS}^* + a\partial Q_{RS}^*/\partial a)$. Substituting $\partial Q_{RS}^*/\partial a$ and $F(aQ_{RS}^*) = 1 - \tilde{a}/a$, we obtain the following for $\partial EPM_{RS}(w_{RS}^*(a);a)/\partial a$.

$$\frac{c(c - s) - a^2 Q_{RS}^*(cg_R + (p + g_M - s)s + (c - s)(p - s)\gamma)f(aQ_{RS}^*)}{a^3(p + g_C - s)f(aQ_{RS}^*)} \quad \text{(A4)}$$

The expression above is greater than zero if and only the numerator is positive. Hence, the result follows after simplification.

3. $(\partial EPM_{RS}(w^*_{RS}(a);a)/\partial a - \partial EPR_{RS}(w^*_{RS}(a);a)/\partial a = \frac{1}{a}(\frac{-YQ^*_{RS}}{p+g_C-s} + \frac{c\tilde{a}}{a^2 f(aQ^*_{RS})})$ where $Y = 2(c-s)[\gamma(p-s)+g_R] + s(p+g_C-s)$. $(\frac{-YQ^*_{RS}}{p+g_C-s} + \frac{c\tilde{a}}{a^2 f(aQ^*_{RS})})$ is equivalent to $g[aQ^*_{RS}] < \bar{Y}_2$ after some algebra.

   Note that the numerators of $\bar{Y}_1$ and $\bar{Y}_2$ are the same. One can show that the denominator of $\bar{Y}_1$ is always smaller than Y, i.e. the denominator of $\bar{Y}_2$. Hence, $\bar{Y}_2 < \bar{Y}_1$.

Theorem 4 :

1. $\frac{dEPC_N}{d(Q_N)} = (g_C+p-c) - \frac{Q_N}{\beta}(g_C+p-s)$ and $\frac{dEPC}{d(Q_C)} = a(g_C+p-s)-(c-s)-\frac{Q_C}{\beta}a^2(g_C+p-s)$. The second order conditions lead to $\frac{d^2EPC_N}{d(Q_N)^2} = -\frac{(p+g_C-s)}{\beta}$ and $\frac{d^2EPC_C}{d(Q_C)^2} = -a^2\frac{(p+g_C-s)}{\beta}$, which are negative by Assumption 1.

2. Solving the first order conditions ($\frac{dEPC_N}{d(Q_N)} = 0$ and $\frac{dEPC}{d(Q_C)} = 0$) for Q_N and Q_C gives $Q^*_N = (1-\tilde{a})\,\beta > 0$ and $Q^*_C = \begin{cases} \frac{1}{a}\left(1-\frac{\tilde{a}}{a}\right)\beta > 0 & \text{if and only if} \quad a > \tilde{a} \\ 0 & \text{otherwise} \end{cases}$ where $\tilde{a} = \frac{(c-s)}{(p+g_C-s)}$. $\tilde{a} < 1$ holds by Assumption 1.

Corollary 3: Directly follows from Theorem 1.2.

Lemma 4:

1. Taking the derivative of Q^*_C with respect to a , we find $\frac{dQ^*_C}{da} = \frac{(2\tilde{a}-a)\beta}{a^3}$. Thus, $\frac{dQ^*_C}{da} > 0$ if and only if $a < 2\tilde{a}$ and $\frac{dQ^*_C}{da} \leq 0$ otherwise.

2. Taking the second derivative of Q^*_C with respect to a , we obtain $\frac{d^2Q^*_C}{da^2} = \frac{2(a-3\tilde{a})\beta}{a^4}$. Thus, $\frac{d^2Q^*_C}{da^2} > 0$ if and only if $a > 3\tilde{a}$ and $\frac{d^2Q^*_C}{da^2} \leq 0$ otherwise.

3. $\frac{dEPC^*}{da} = (c-s)\frac{\beta}{a^2}(1-\frac{\tilde{a}}{a}) = \frac{(c-s)}{a}Q^*_C \geq 0$ by Assumption 1. Taking the second derivative leads to $\frac{d^2EPC^*}{da^2} = \frac{(c-s)(3\tilde{a}-2a)\beta}{a^4}$ where $(c-s) > 0$ by Assumption 1. Thus,

$\frac{d^2EPC*}{da^2}$ is positive if and only if $3\tilde{a} > 2a$ and less than or equal to zero otherwise.

Corollary 4:

1. Directly follows from Theorem 1.1.

2. $EPC(Q_i*) - EPC(Q_N^*) = \frac{(1-a)^2(g_C+p-c)(a(g_C+p-c)-2(c-s))\beta}{2a(p+g_C-s)} > 0$ if and only if $(a(g_C + p - c) - 2(c - s)) > 0$, i.e. $a > \frac{2(c-s)}{p+g_C-c} = \frac{2\tilde{a}}{1-\tilde{a}}$ since the remainder is positive by Assumption 1.

3. (a) $\frac{d^2EPC(Q_N^*)}{da^2} = -\frac{(g_C+p-c)^2\beta}{(g_C+p-s)} < 0$ and $\frac{d^2EPC(Q_i^*)}{da} = -\frac{2(c-s)(g_C+p-c)\beta}{a^3(g_C+p-s)} < 0$ by Assumption 1.

   (b) $\frac{\partial\left(EPC(Q_i^*)-EPC(Q_N^*)\right)}{\partial a} = \frac{(1-a)(g_C+p-c)\left((c-s)(1+a)-a^2(g_C+p-c)\right)\beta}{a^2(p+g_C-s)} > 0$ if and only if $((c-s)(1 + a) - a^2(g_C + p - c)) > 0$, i.e. $\frac{\mathrm{a}^2}{1+\mathrm{a}} < \frac{\tilde{a}}{1-\tilde{a}}$, since the remainder is positive by Assumption 1.

   (c) $\frac{\partial\left(EPC(Q_i^*)-EPC(Q_C^*)\right)}{\partial a} = \frac{(1-a)(c-s)^2\beta}{a^3(p+g_C-s)} > 0$ by Assumption 1.

   (d) $\frac{\partial\left(EPC(Q_N^*)-EPC(Q_C^*)\right)}{\partial a} = \frac{(1-a)(a(g_C+p-c)-(c-s))\left(a^2(g_C+p-c)-(c-s)\right)\beta}{a^3(p+g_C-s)} > 0$, which is greater than zero if the following conditions are satisfied: either ($a > \frac{\tilde{a}}{1-\tilde{a}}$ and $a > \sqrt{\frac{\tilde{a}}{1-\tilde{a}}}$)or ($a < \frac{\tilde{a}}{1-\tilde{a}}$ and $a < \sqrt{\frac{\tilde{a}}{1-\tilde{a}}}$). For $c < \frac{p+g_C+s}{2}$, i.e. $\frac{\tilde{a}}{1-\tilde{a}} < 1$, the first condition reduces to $a > \sqrt{\frac{\tilde{a}}{1-\tilde{a}}}$ and the second condition reduces to $a < \frac{\tilde{a}}{1-\tilde{a}}$. For $c \geq \frac{p+g_C+s}{2}$, i.e. $\frac{\tilde{a}}{1-\tilde{a}} \geq 1$, the 1st condition is never satisfied but the latter always holds.

Corollary 5:

1. (a) $\frac{\partial\left(EPC(Q_i^*)-EPC(Q_C^*)\right)}{\partial c} = -\frac{(1-a)^2(c-s)\beta}{a^2(p+g_C-s)} < 0$ by Assumption 1.

   (b) $\frac{\partial\left(EPC(Q_i*)-EPC(Q_C^*)\right)}{\partial g_C} = \frac{(1-a)^2(c-s)^2\beta}{2a^2(p+g_C-s)^2} > 0$ by Assumption 1.

(c) $\frac{\partial\left(EPC(Q_i*)-EPC(Q_C^*)\right)}{\partial s} = \frac{(1-a)^2(c-s)(g_C+p-c+g_C+p-s)\beta}{2a^2(p+g_C-s)^2} > 0$ by Assumption 1.

(d) $\frac{\partial\left(EPC(Q_i*)-EPC(Q_C^*)\right)}{\partial p} = \frac{(1-a)^2(c-s)^2\beta}{2a^2(p+g_C-s)^2} > 0$ by Assumption 1.

(e) $\frac{\partial\left(EPC(Q_i^*)-EPC(Q_N*)\right)}{\partial s} = \frac{(1-a)^2(2+a)(g_C+p-c)^2\beta}{2a(p+g_C-s)^2} > 0$ by Assumption 1.

2. (a) $\frac{\partial\left(EPC(Q_N^*)-EPC(Q_C^*)\right)}{\partial c} = \frac{(1-a)^2(1+a)(a(g_C+p-c)-(c-s))\beta}{a^2(p+g_C-s)}$ is positive as long as $a > \frac{(c-s)}{p+g_C-c} = \frac{\tilde{a}}{1-\tilde{a}}$ since the remainder is positive by Assumption 1.

   (b) $\frac{\partial\left(Q_C^*-Q_N^*\right)}{\partial c} = -\frac{(1-a^2)\beta}{a^2(p+g_C-s)} < 0$ by Assumption 1.

3. (a) $\frac{\partial\left(EPC(Q_N^*)-EPC(Q_C^*)\right)}{\partial s} = \frac{(1-a)^2((1+a)(p+g_C-c)+(p+g_C-s))((c-s)-a(g_C+p-c))\beta}{2a^2(p+g_C-s)^2}$ is positive as long as $a < \frac{(c-s)}{p+g_C-c} = \frac{\tilde{a}}{1-\tilde{a}}$ since the remainder is positive by Assumption 1.

   (b) $\frac{\partial\left(Q_C^*-Q_N^*\right)}{\partial s} = \frac{(1-a^2)(p+g_C-c)\beta}{a^2(p+g_C-s)^2} > 0$ by Assumption 1.

Theorem 5:

1. $\frac{dEPR_D}{d(Q_D)} = a(p+g_R-s)-(w_D-s)-a^2\frac{(p+g_R-s)}{\beta}Q_D$ and $\frac{d^2EPR_D}{d(Q_D)^2} = -a^2\frac{(p+g_R-s)}{\beta} < 0$ by Assumption 1.

2. The optimal ordering quantity solves $\frac{dEPR_D}{d(Q_D)} = 0$ and is equal to $Q_D^* = \frac{\beta}{a}(1-\frac{w_D-s}{a(p+g_R-s)})$. Thus, Q_D^* is decreasing in w_D .

3. By substituting Q_D^* in $EPM_D(w_D)$ and taking the first derivative, we get $\frac{dEPM_D}{d(w_D)} = \left(\frac{\left(a(g_R+p)^2+c(g_R+p-s)+s(p+g_C-2a(g_R+p))-s^2(1-a)\right)}{a^2(g_R+p-s)^2} - \frac{(2p+2g_R+g_M-2s)}{a^2(p+g_R-s)^2}w_D\right)\beta$. The second order condition gives $\frac{d^2EPM_D}{d(w_D)^2} = -\frac{(2p+2g_R+g_M-2s)\beta}{a^2(p+g_R-s)^2} < 0$ by Assumption 1.

4. Solving $\frac{dEPM_D}{d(w_D)} = 0$ for the optimal wholesale price leads to $w_D^* = \frac{1}{4}(2(c+s)-2a(p+g_R-s)-ag_1+\frac{g_M(ag_1-2(c-s))}{2(p+g_R-s)+g_M})$ after some algebra. Substituting the optimum wholesale price back into Q_D^* gives $Q_D^* = \begin{cases} \kappa\left(\frac{1}{a}\right)\left(1-\frac{\tilde{a}}{a}\right)\beta > 0 & \text{if and only if } a > \tilde{a} \\ 0 & \text{otherwise} \end{cases}$ where $\kappa = \frac{(g_R+p-s)+g_1}{2(g_R+p-s)+g_M} < 1$ and positive by Assumption 1.

Lemma 5:

1. $\frac{dEPR_D(Q_D^*(a);a)}{da} = \frac{(c-s)(g_R+p-s)(a(g_C+p-s)-(c-s))}{a^3(2(g_R+p-s)+g_M)} > 0$ and $\frac{dEPM_D(w_D^*(a);a)}{da} = \frac{(c-s)(a(g_C+p-s)-(c-s))}{a^3(2(g_R+p-s)+g_M)} > 0$ by Assumption 1 and $Q_D^* > 0$. Since $Q_D^* > 0$, $a > \tilde{a}$ and therefore $(a(g_C+p-s)-(c-s)) > 0$.

2. $\frac{dEPM_D(w_D^*(a);a)}{da} - \frac{dEPR_D(Q_D^*(a);a)}{da} = \frac{(c-s)(g_C+p-s)(a(g_C+p-s)-(c-s))}{a^3(2(g_R+p-s)+g_M)} > 0$ by Assumption 1 and $Q_D^* > 0$.

3. Taking the derivative of w_D^* with respect to a gives $\frac{dw_D^*}{da} = \frac{(g_R+p-s)^2}{2(g_R+p-s)+g_M} > 0$ by Assumption 1.

Corollary 6

1. Substituting the optimal values back into profit functions and subtracting gives $EPC^* - (EPR_D^* + EPM_D^*) = \frac{(g_R+p-s)^2(a(g_C+p-s)-(c-s))^2\beta}{2a^2(g_C+p-s)(2(p+g_R-s)+g_M)^2}$, which is positive by Assumption 1.

2. $\frac{d\left(EPC^* - (EPR_D^* + EPM_D^*)\right)}{da} = \frac{(c-s)(g_R+p-s)^2(a(g_C+p-s)-(c-s))\beta}{a^3(g_C+p-s)(2(g_R+p-s)+g_M)} > 0$ for $Q_C^*, Q_D^* > 0$ by Assumption 1.

Theorem 6:

1. $\frac{dEPR_{RS}}{d(Q_{RS})} = s\gamma + a(g_R + (p-s)\gamma) - w_{RS} - Q_{RS}\frac{a^2(g_R+(p-s)\gamma)}{\beta}$; hence, $\frac{d^2EPR_{RS}}{d(Q_{RS})^2} = -\frac{a^2(g_R+(p-s)\gamma)}{\beta} < 0$ by Assumption 1.

2. Q_{RS}^* solves $\frac{dEPR_{RS}}{d(Q_{RS})} = 0$ and is $Q_{RS}^* = \frac{\beta}{a^2}\left(\frac{\gamma s + a(g_R+(p-s)\gamma) - w_{RS}}{g_R+(p-s)\gamma}\right)$. Equating 21 to 3 gives $w_{RS}^* = \frac{(g_R(c-s)+c\gamma(p-s)+sg_C\gamma)}{p+g_C-s}$. Computation of $w_{RS}^* - c$ leads to $w_{RS}^* - c = \frac{s(-g_R+g_C\gamma)+c(g_R+\gamma(p-s)-g_C-p+s)}{p+g_C-s}$, which can be simplified to $\frac{s(-g_R+g_C\gamma)-c((p-s)(1-\gamma)+g_M)}{p+g_C-s}$. The denominator is always positive by Assumption 1. Therefore, $s(-g_R + g_C\gamma) - c((p-s)(1-\gamma) + g_M) < 0$ will be sufficient for $w_{RS}^* - c < 0$. The second term in paren-

theses is always positive by Assumption 1 and $\gamma < 1$. Now, there are two possibilities: one possibility is $(-g_R + g_C\gamma) < 0$ and the other one is $(-g_R + g_C\gamma) > 0$. The first one automatically leads to $w^*_{RS} < c$. In order to prove $w^*_{RS} < c$, showing $(p-s)(1-\gamma)+g_M > (-g_R+g_C\gamma)$ will suffice since $s < c$ by Assumption 1. After further simplification, $(p-s)(1-\gamma)+g_M > (-g_R+g_C\gamma)$ becomes $(p+g_C-s)(1-\gamma) > 0$, which is always true by Assumption 1 and $\gamma < 1$.

Lemma 6:

1. $\frac{\partial}{\partial a}EPR_{RS}(Q^*_{RS}(a);a) = \frac{(c-s)(g_R+(p-s)\gamma)(a(g_C+p-s)-(c-s))\beta}{a^3(p+g_C-s)^2} > 0$ and $\frac{\partial}{\partial a}EPM_{RS}(w^*_{RS}(a);a) = \frac{(c-s)(g_M+(p-s)(1-\gamma))(a(g_C+p-s)-(c-s))\beta}{a^3(p+g_C-s)^2} > 0$ by Assumption 1, $Q^*_{RS} = Q^*_C > 0$ and $\gamma < 1$.

2. $\frac{\partial}{\partial a}EPM_{RS}(w^*_{RS}(a);a) - \frac{\partial}{\partial a}EPR_{RS}(Q^*_{RS}(a);a) = \frac{(c-s)(a(g_C+p-s)-(c-s))\beta(g_M-g_R+p-s-2(p-s)\gamma)}{a^3(p+g_C-s)^2}$ is positive if and only if $(g_M - g_R + p - s - 2(p-s)\gamma)$ is positive since the remainder of the expression is positive by Assumption 1 and $Q^*_{RS} = Q^*_C > 0$. The condition is further simplified to $\gamma < (1/2)\left(1 + \frac{(g_M-g_R)}{(p-s)}\right)$.

Lemma 7: $\frac{dEPR_{BB1}}{dQ_{BB1}} = a(g_R+p-b)-(w_{BB1}-b)-Q_{BB1}\frac{a^2(p+g_R-b)+(b-s)(1-R)^2}{\beta}$ and $\frac{d^2EPR_{BB1}}{dQ^2_{BB1}} = \frac{-a^2(p+g_R-b)-(b-s)(1-R)^2}{\beta} < 0$ by Assumption 3. Q^*_{BB1} solves $\frac{dEPR_{BB1}}{d(Q_{BB1})} = 0$ and is $Q^*_{BB1} = \frac{\beta(a(g_R+p-b)-(w_{BB1}-b))}{a^2(g_R+p-b)+(b-s)(1-R)^2}$.

Theorem 7:

1. If $R = 1$ and $b = w_{BB1}$, the wholesale price that coordinates the channel and hence makes $Q^*_{BB1} = Q^*_C$ is $w_{BB1} = g_R+p$. However, w_{BB1} is always less than p by Assumption 2.

2. Since w_{BB1} is equal to the expression derived by Pasternack (1985) for the wholesale price for unlimited returns, the proof directly follows from Pasternack (1985).

Corollary 7: Directly follows from Theorem 7.

Theorem 8:

1. When $b = w_{BB1}$, solving w_{BB1} for b and simplifying gives $\frac{(c-s)(g_R+p)+s(p+g_C-s)(1-R)\frac{(Q^*_{BB1}(1-R))}{\beta}}{(c-s)+(p+g_C-s)(1-R)\frac{(Q^*_{BB1}(1-R))}{\beta}}$. Therefore, $b-s = \frac{(c-s)(g_R+p-s)}{(c-s)+(p+g_C-s)(1-R)\frac{(Q_{BB1}(1-R))}{\beta}} > 0$ by Assumption 1 and $R < 1$. This indicates that $b > s$.

2. Solving $Q^*_{BB1} = Q^*_C$ gives $w^*_{BB1} = \frac{c(g_R+p-b)-s(g_R+p)+b(p+g_C)}{(g_C+p-s)} - (b-s)(1-R)^2\left(\frac{1}{a}\right)\left(1-\frac{\tilde{a}}{a}\right)$

.

If $w_{BB1} > b$, then $w_{BB1} - b = \frac{(p+g_R-b)(c-s)}{p+g_C-s} - (b-s)(1-R)\frac{\left(Q^*_{BB1}(1-R)\right)}{\beta} > 0$ holds. If $\frac{(p+g_R-b)(c-s)}{p+g_C-s} - (b-s)(1-R)\frac{\left(Q^*_{BB1}(1-R)\right)}{\beta} > 0$ holds, then $w_{BB1} > b$. Further simplification on the condition leads to $\tilde{a}(p+g_R-b) > (b-s)(1-R)^2\left(\frac{1}{a}\right)\left(1-\frac{\tilde{a}}{a}\right)$. One can always find a solution such that this inequality is verified.

Given that $w_{BB1} > b$, now we show that one can always find a b satisfying (2.25) such that $s < b$. Solving w_{BB1} for b and simplifying gives

$b = s - \frac{c(g_R+p-s)-(g_C+p)w_{BB1}+s(w_{BB1}+g_C-g_R)}{(g_C+p-c)-(1-R)(g_C+p-s)\frac{(Q_{BB1}(1-R))}{\beta}}$. If the ratio is negative, then we can find a $b > s$. We take the denominator first and show that it is positive. Thus, this leads to $(g_C+p-c) > (1-R)(g_C+p-s)\frac{\left(Q^*_{BB1}(1-R)\right)}{\beta}$. In this case, we know that $R > l$ which also means that $a > 1-R$. Thus the right hand side of the inequality is always less than $a(g_C+p-s)\frac{\left(aQ^*_{BB1}\right)}{\beta}$ which is equal to $a(g_C+p-s)(1-\frac{c-s}{a(g_C+p-s)})$ when the channel is coordinated. Thus, if the left hand side of the inequality is even greater than that value, then we know that the denominator will always be positive. After some algebra, we obtain that $(g_C+p-c) > a(g_C+p-s)\left(1-\frac{c-s}{a(g_C+p-s)}\right) \Rightarrow (g_C+p-c) > a(g_C+p-s)-(c-s)$, which is always true. Therefore, the numerator must be negative to ensure $b > s$. So, $c(g_R+p-s)-(g_C+p)w_{BB1}+s(w_{BB1}+g_C-g_R) < 0$. Substitution of $g_C = g_R+g_M$ gives

$-(w_{BB1}-c)(g_R+p-s)-(w_{BB1}-s)g_M<0$ and the inequality always holds. Therefore, $b>s$. Hence, a b satisfying (2.25) can be found to coordinate the channel such that $s<b<w_{BB1}$ if w_{BB1} is chosen such that $c<w_{BB1}<p$.

Lemma 8: $\frac{dEPR_{BB2}}{d(Q_{BB2})} = a(g_R+p-s)+R(b-s)-(w_{BB2}-s)-Q_{BB2}a^2\frac{(g_R+p-s)}{\beta}$ and $\frac{d^2EPR_{BB2}}{dQ_{BB2}^2} = -\frac{a^2(p+g_R-s)}{\beta} < 0$ by Assumption 1. Q^*_{BB2} solves $\frac{dEPR_{BB2}}{d(Q_{BB2})} = 0$ and is $Q^*_{BB2} = \frac{\beta}{a}\left(1-\frac{(w_{BB2}-s)-R(b-s)}{a(p+g_R-s)}\right)$

Theorem 9:

Equating (2.13) to (2.28), we get a general relationship for the channel to be coordinated. $\frac{(w^*_{BB2}-s)-R(b-s)}{a(p+g_R-s)} = \frac{(c-s)}{a(p+g_C-s)}$ leads to $w^*_{BB2} = c-\frac{(g_C-g_R)(c-s)}{p+g_C-s}+(b-s)R$.

1. $w^*_{BB2} = c-\frac{(g_C-g_R)(c-s)}{p+g_C-s} = c-\frac{g_M(c-s)}{p+g_C-s} < c$ which contradicts with Assumption 1.

2. $w^*_{BB2} = b = s+\frac{(g_R+p-s)(c-s)}{(p+g_C-s)(1-R)}$ is greater than s since the second term is positive by Assumption 1 and w^*_{BB2} leads to coordination with right selection of parameters.

3. If $w^*_{BB2} > b$, $w^*_{BB2}-b>0$ and in turn $c-\frac{g_M(c-s)}{p+g_C-s} > b(1-R)+Rs$. Given that $w^*_{BB2}>b$ holds, solving $Q^*_{BB2}=Q^*_C$ for b gives $b = s+\frac{(w_{BB2}*-c)+\frac{(c-s)g_M}{(p+g_C-s)}}{R}$, which is greater than s due to Assumption 1 and Assumption 2. Right hand side of the inequality above is less than b and greater than s. Left hand side of the inequality is greater than s due to Assumption 1. One can always find numbers such that the left hand side is greater than the right hand side. Thus, a b satisfying (2.30) can be found to coordinate the channel such that $s<b<w_{BB2}$ if w_{BB2} is chosen such that $c<w_{BB2}<p$.

Corollary 8:

1. Let us call w_{BB} the coordination wholesale price for buyback contract.

$$w^*_{BB} = \left\{ \begin{array}{ll} w^*_{BB1} = \frac{c(g_R+p-b)-s(g_R+p)+b(p+g_C)}{(g_C+p-s)} - (b-s)(1-R)^2\left(\frac{1}{a}\right)\left(1-\frac{\tilde{a}}{a}\right) & \text{if } l < R \leq 1 \\ w^*_{BB2} = c - \frac{(g_C-g_R)(c-s)}{p+g_C-s} + (b-s)R & \text{if } \quad 0 < R \leq l \end{array} \right\}$$

Computation of left and right limits at l leads to $\lim\limits_{R\to l^-} w_{BB} = \lim\limits_{R\to l^+} w_{BB} = c - \frac{g_M(c-s)}{p+g_C-s} + l(b-s)$ after simplification and shows that w^*_{BB} is a continuous function of R. Taking the derivatives of w^*_{BB1} and w^*_{BB2} with respect R leads to $\frac{2(b-s)(a(p+g_C-s)-(c-s))(1-R)}{a^2(p+g_C-s)}$ and $(b-s)$, both of which are positive by Assumption 1, Assumption 3 and $Q^*_{BB1} = Q^*_{BB2} = Q^*_C > 0$. Since w^*_{BB} is increasing in R, $w^*_{BB2} < w^*_{BB1}$.

2. Plugging Q_{BB2} and w_{BB2} back into EPM_{BB2} and further simplification give $\frac{\tilde{a}(\tilde{a}-2a)}{2a^2}g_M\beta < 0$.

Lemma 9:

1. $\frac{\partial}{\partial a}EPR^*_{BB1}(Q^*_{BB1}(a);a)$ can be simplified as $Q^*_{BB1}\left((p+g_R-b)\frac{\tilde{a}}{a} + \frac{(b-s)(1-R)^2}{\beta}\frac{dQ^*_{BB1}}{da}\right)$. Further algebra shows that if the retailer benefits from availability, $\frac{dQ^*_{BB1}}{da} > -\frac{\tilde{a}}{a}\frac{(p+g_R-b)\beta}{(b-s)(1-R)^2}$ must hold. Simplification of the inequality gives $(a-2\tilde{a})(b-s)(1-R)^2 < \tilde{a}(p+g_R-b)a^2$. After further simplification, we obtain $\left(\frac{a}{(1-R)}\right)^2 - \frac{(a-2\tilde{a}))(b-s)}{\tilde{a}(p+g_R-b)} > 0$. We see from Theorem 8.2. that $w^*_{BB1} \geq b$ holds as long as $\left(\frac{a}{(1-R)}\right)^2 \geq \frac{(a-\tilde{a})(b-s)}{\tilde{a}(p+g_R-b)}$. In order for the retailer not to benefit from improvements in availability, $\left(\frac{a}{(1-R)}\right)^2 < \frac{(a-2\tilde{a})(b-s)}{\tilde{a}(p+g_R-b)}$ must be true. However, $\frac{(a-\tilde{a})(b-s)}{\tilde{a}(p+g_R-b)} \leq \left(\frac{a}{(1-R)}\right)^2 < \frac{(a-2\tilde{a})(b-s)}{\tilde{a}(p+g_R-b)}$ is never possible.

2. $\frac{\partial}{\partial a}EPM^*_{BB1}(Q^*_{BB1}(a);a)$ can be simplified as $Q^*_{BB1}\left((b-s+g_M)\frac{\tilde{a}}{a} + \frac{(b-s)(1-R)^2}{\beta}\frac{dQ^*_{BB1}}{da}\right)$. Further algebra shows that if the manufacturer benefits from availability, $\frac{dQ^*_{BB1}}{da} < \frac{\tilde{a}}{a}\frac{(b-s+g_M)\beta}{(b-s)(1-R)^2}$

must hold. Simplification of the inequality leads to $a^2\tilde{a}(b-s+g_M) > (1-R)^2(b-s)(-a+2\tilde{a})$. In this inequality, $a^2 > (1-R)^2$, $(b-s+g_M) \geq (b-s)$ and $\tilde{a} > -a+2\tilde{a}$ (or equivalently $a > \tilde{a}$ due to $Q^*_{BB1} = Q^*_C > 0$). Thus, the inequality holds all the time.

3. (i) If the manufacturer gains at least as much as the retailer, then $\frac{d}{da}EPM_{BB1}(w^*_{BB1}(a);a) - \frac{d}{da}EPR_{BB1}(Q^*_{BB1}(a);a) \geq 0$ must hold. Simplifying the subtraction gives $\frac{\beta(a-\tilde{a})}{a^3}\left[\tilde{a}(b+g_M-s) - \left(\tilde{a}(p+g_R-b) + 2(b-s)(2\tilde{a}-a)\left(\frac{1-R}{a}\right)^2\right)\right]$. The first term is positive, thus, the marginal benefit for the manufacturer is more than the retailer if the term in square brackets is positive.

(ii) $\frac{\partial\Delta}{\partial g_M} = -\frac{2(c-s)\left(-a^2(g_R+p)+b(a^2-2(1-R)^2)+2s(1-R)^2\right)}{a^2(g_C+p-s)^2}$ is positive as long as $(-a^2(g_R+p) + b(a^2-2(1-R)^2) + 2s(1-R)^2) < 0$ holds. The inequality can be organized as $-a^2(g_R+p-b) < 2(b-s)(1-R)^2$, which is always true by Assumption 3.

(iii) $\frac{\partial\Delta}{\partial b} = 2\tilde{a} + 2\left(\frac{1-R}{a}\right)^2(a-2\tilde{a}) = 2\frac{\tilde{a}a^2+(1-R)^2(a-2\tilde{a})}{a^2}$, which is positive as long as $\tilde{a}a^2 + (1-R)^2(a-2\tilde{a}) > 0$. The condition can be organized as $\tilde{a}a^2 > (1-R)^2(2\tilde{a}-a)$. Since $R > l = (1-a)$, $a^2 > (1-R)^2$. In addition, $\tilde{a} > (2\tilde{a}-a)$ holds if and only if $\tilde{a} < a$, which is the condition for the retailer to order a positive amount under a coordinated supply chain.

Lemma 10:

1. $\frac{\partial}{\partial a}EPR_{BB2}(Q^*_{BB2}(a);a) = \frac{(c-s)(p+g_R-s)(a(g_C+p-s)-(c-s))\beta}{a^3(p+g_C-s)^2} > 0$ by Assumption 1 and $Q^*_{BB2} = Q^*_C > 0$.

2. $\frac{\partial}{\partial a}EPM_{BB2}(w^*_{BB2}(a);a) = \frac{(c-s)g_M(p+g_R-s)(a(g_C+p-s)-(c-s))\beta}{a^3(p+g_C-s)^2} > 0$ by Assumption 1 and $Q^*_{BB2} = Q^*_C > 0$.

3. If the manufacturer gains at least as much as the retailer, then $\frac{\partial}{\partial a}EPM_{BB2}(w^*_{BB2}(a);a) -$

$\frac{\partial}{\partial a}EPR_{BB2}(Q^*_{BB2}(a);a) \geq 0$ must hold. $\frac{\partial}{\partial a}EPM_{BB2}(w^*_{BB2}(a);a) - \frac{\partial}{\partial a}EPR_{BB2}(Q_{BB2}*(a);a) = -\frac{\beta(c-s)(s-g_R+g_M-p)(c-s-a(p+g_C-s))}{a^3(p+g_C-s)^2}$. From Assumption 1 and $Q^*_{BB2} = Q^*_C > 0$, $-\frac{\beta(c-s)(c-s-a(p+g_C-s))}{a^3(p+g_C-s)^2} > 0$. Thus, if $(s - g_R + g_M - p) \geq 0$, the manufacturer benefits at least as much as the retailer. Otherwise, the retailer benefits more.

Lemma 11:

Let us assume that R_1 is $\in (l, 1]$ and R_2 is $\in (0, l]$. Thus, in a supply chain coordinated by a buyback contract where $(R > l)$, the coordination profits for the manufacturer will be EPM^*_{BB1} and the return proportion R will be replaced by R_1 . Similarly, in a supply chain coordinated by a buyback contract where $(R \leq l)$, the coordination profits for the manufacturer will be EPM^*_{BB2} and the return proportion R will be replaced by R_2 . The manufacturer's profits in a supply chain coordinated by a revenue sharing contract will be represented by EPM^*_{RS}.

1. $EPM^*_{BB2} - EPM^*_{BB1} = -\frac{(b-s)(a(g_C+p-s)-(c-s))^2\left(a^2-(1-R_1)^2\right)\beta}{2a^4(p+g_C-s)^2} < 0$ since all the terms are positive by Assumption 1, Assumption 3, $Q^*_C > 0$ and $R_1 \in (l, 1]$.

2. $EPM^*_{RS} - EPM^*_{BB2} = \frac{(p-s)(a(g_C+p-s)-(c-s))^2(1-\gamma)\beta}{2a^2(p+g_C-s)^2} > 0$ since all the terms are positive by Assumption 1, $Q^*_C > 0$ and $\gamma < 1$.

3. $EPM^*_{BB1} - EPM^*_{RS} = \frac{(a(g_C+p-s)-(c-s))^2\left(s+sR_1(R_1-2)+b\left(a^2-(1-R_1)^2\right)+a^2(\gamma(p-s)-p)\right)\beta}{2a^4(p+g_C-s)^2}$ is positive as long as $(s + sR_1(R_1 - 2) + b\,(a^2 - (1 - R_1)^2) + a^2\,(\gamma(p - s) - p)) > 0$, which can be simplified as $(a^2\gamma(p - s) - (1 - R_1)^2(b - s) - a^2(p - b)) > 0$. After further algebra, the condition leads to $\gamma > \frac{(1-R_1)^2(b-s)+a^2(p-b)}{a^2(p-s)}$,which is less than 1 since $R_1 \in (l, 1]$.

4. In order for the manufacturer to prefer a revenue sharing contract to a buyback contract where $R_1 > l$, $EPM^*_{BB1} - EPM^*_{RS} < 0$ must hold. This condition is satisfied

if $(s + sR_1(R_1 - 2) + b(a^2 - (1 - R_1)^2) + a^2(\gamma(p - s) - p))$ is negative. Thus, following the proof for Lemma 11.3., $\gamma < \frac{(1-R_1)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ must hold.

5. In order for the manufacturer to be indifferent between a revenue sharing contract and a buyback contract where $R_1 > l$, $EPM^*_{BB1} - EPM^*_{RS} = 0$ must hold. Following the proof for Lemma 11.3., we obtain $\gamma = \frac{(1-R_1)^2(b-s)+a^2(p-b)}{a^2(p-s)}$.

Lemma 12: *Given that the supply chain is coordinated and hence* $Q^*_C = Q^*_{BB1} = Q^*_{BB2} = Q^*_{RS} > 0$ *, the decentralized retailer*

1. *always prefers a buyback contract where* $R \leq l$ *to a buyback contract where* $R > l$ *.*
2. *always prefers a buyback contract where* $R \leq l$ *to a revenue sharing contract.*
3. *always prefers a buyback contract where* $R > l$ *to a revenue sharing contract if* $\gamma < \frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *where* $\frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *is always less than 1.*
4. *always prefers a revenue sharing contract to a buyback contract where* $R > l$ *if* $\gamma > \frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *where* $\frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *is always less than 1.*
5. *is always indifferent between a buyback contract where* $R > l$ *and a revenue sharing contract if* $\gamma = \frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *where* $\frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ *is always less than 1.*

Proof of Lemma 12:

We will follow the same line of proof as in the proof of Lemma 11.

1. $EPR^*_{BB2} - EPR^*_{BB1} = \frac{(b-s)(a(g_C+p-s)-(c-s))^2\left(a^2-(1-R_1)^2\right)\beta}{2a^4(p+g_C-s)^2} > 0$ since all the terms are positive by Assumption 1, Assumption 3, $Q^*_C > 0$ and $R_1 \in (l, 1]$.

2. $EPR^*_{RS} - EPR^*_{BB2} = -\frac{(p-s)(a(g_C+p-s)-(c-s))^2(1-\gamma)\beta}{2a^2(p+g_C-s)^2} < 0$ since all the terms are positive by Assumption 1, $Q^*_C > 0$ and $\gamma < 1$.

3. $EPR^*_{BB1} - EPR^*_{RS} = -\frac{(a(g_C+p-s)-(c-s))^2\left(s+sR_1(R_1-2)+b\left(a^2-(1-R_1)^2\right)+a^2(\gamma(p-s)-p)\right)\beta}{2a^4(p+g_C-s)^2}$

$= -(EPM^*_{BB1} - EPM^*_{RS})$ Thereby, the retailer prefers a buyback contract where $R > l$ to a revenue sharing contract if $\gamma < \frac{(1-R_1)^2(b-s)+a^2(p-b)}{a^2(p-s)}$, which is less than 1 since $R_1 \in \ (l, 1]$.

4. Following the proof of Lemma 12.3., in order for the retailer to prefer a revenue sharing contract to a buyback contract where $R_1 > l$, $\gamma > \frac{(1-R_1)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ must hold.

5. In order for the retailer to be indifferent between a revenue sharing contract and a buyback contract where $R_1 > l$, $EPR^*_{BB1} - EPR^*_{RS} = 0$ must hold. Following the proof for Lemma 9.3., we obtain $\gamma = \frac{(1-R_1)^2(b-s)+a^2(p-b)}{a^2(p-s)}$.

Theorem 10:

1. $\partial EPC(Q_C)/\partial Q_C = -c + s + (p + g_C - s)\int_0^\infty \int_0^{x/Q^*_C} adH(a)dF(x)dx$ and $\partial^2 EPC(Q_C)/\partial Q^2_C = -(p + g_C - s)\int_0^\infty (x^2/Q^3_C)h(x/Q_C)dF(x) < 0$. Therefore the result follows from the first order condition.

2. The optimum order under random yield, Q^*, solves $\int_0^\infty \int_0^{x/Q^*} adH(a)dF(x)dx = \frac{c-sE[A]}{p+g_C-s}$. Here, $\frac{c-sE[A]}{p+g_C-s} > \frac{c-s}{p+g_C-s}$ for $E[A] < 1$. Thus, $Q^* < Q^*_C$.

Lemma 13: Let us assume that $\hat{A}$ has a distribution of $\hat{H}(.)$.

$EPC(\hat{Q}^*_C) - EPC(Q^*_C) = (p+g_C-s)\int_0^\infty x[H(x/Q^*_C) - \hat{H}(x/\hat{Q}^*_C)]dF(x)dx$. If $\hat{Q}^*_C > Q^*_C$, $H(x/Q^*_C) - \hat{H}(x/\hat{Q}^*_C) > 0$ for $A \leq_{st} \hat{A}$ since $H(x/Q^*_C) \geq \hat{H}(x/\hat{Q}^*_C)$.

Theorem 11:

1. $\partial^2 EPR_D(Q_D)/\partial Q^2_D = -(p + g_R - s)\int_0^\infty (x^2/Q^3_D)h(x/Q_D)dF(x) < 0$. Hence, the optimum order solves the FOC, which is $\partial EPR_D(Q_D)/\partial Q_D = -w_D + s + (p + g_R - s)\int_0^\infty \int_0^{x/Q^*_D} adH(a)dF(x)dx = 0$. Therefore, $EPR_D(Q_D)$ is concave in Q_D and the

optimum order Q^*_D solves the following relation.

$$w_D = s + (p + g_R - s)\int_0^\infty \int_0^{x/Q_D} adH(a)dF(x)dx \tag{A5}$$

2. $\partial EPM_D(Q_D)/\partial Q_D = (w_D - c) + (\partial w_D/\partial Q_D)Q_D + g_M \int_0^\infty \int_0^{x/Q_d} adH(a)dF(x)$ and $\partial^2 EPM_D(Q_D)/\partial Q_D^2 = (\partial^2 w_D/\partial Q_D^2)Q_D + 2(\partial w_D/\partial Q_D) - g_M \int_0^\infty (x^2/Q_D^3)h(x/Q_D)dF(x)$. We substitute (A5) for w_D in $\partial^2 EPM_D(Q_D)/\partial Q_D^2$ where

$$\partial w_D/\partial Q_D = -(p + g_R - s)\int_0^\infty (x^2/Q_D^3)h(x/Q_D)dF(x) < 0 \tag{A6}$$

$$\partial^2 w_D/\partial Q_D^2 = (p + g_R - s)[3\int_0^\infty (x^2/Q_D^4)h(x/Q_D)dF(x) + \int_0^\infty (x^3/Q_D^5)h'(x/Q_D)dF(x)] \tag{A7}$$

   After plugging the expressions above in $\partial^2 EPM_D(Q_D)/\partial Q_D^2$ and further simplifying we receive $(p+g_R-s-g_M)\int_0^\infty \frac{x^2}{Q_D^3}h(x/Q_D)dF(x) + (p+g_R-s)\int_0^\infty \frac{x^3}{Q_D^4}h'(x/Q_D)dF(x)$.

3. If the manufacturer's function is concave, the optimum order solves the FOC of the manufacturer $\partial EPM_D(Q_D)/\partial Q_D = 0$ where w_D is given by (A5). Further simplification yields the result.

Corollary 9: If A is uniformly distributed, then $h'(.) = 0$. Hence, EPM_D is concave if and only if $(p + g_R - s - g_M)\int_0^\infty \frac{x^2}{Q_D^3}h(x/Q_D)dF(x) < 0$.

Lemma 14: We follow the same line of proof for this result as we did for Lemma 13.

1. $EPR_D(\hat{Q}^*_D) - EPR_D(Q^*_D) = (p + g_R - s)\int_0^\infty x[H(x/Q^*_D) - \hat{H}(x/\hat{Q}^*_D)]dF(x)dx$ after substituting (A5) for w and simplification.

2. $EPM_D(\hat{Q}_D^*) - EPM_D(Q_D^*) = (c-s)(Q_D^* - \hat{Q_D}^*) +$

$(p + g_R + g_M - s)\hat{Q_D}^* \int_0^\infty \int_0^{x/\hat{Q_D}^*} ad\hat{H}(a)dF(x)dx -$

$(p+g_R+g_M-s)Q_D^* \int_0^\infty \int_0^{x/Q_D^*} adH(a)dF(x)dx + g_M[\int_0^\infty x[H(x/Q_D^*) - \hat{H}(x/\hat{Q}_D^*)dF(x)]$

after substituting (A5) for w_D and simplification.

Theorem 12: $\partial EPR_{RS}(Q_{RS})/\partial Q_{RS} = [\gamma(p-s)+g_R]\int_0^\infty \int_0^{x/Q_{RS}} adH(a)dF(x) - w_{RS} + \gamma s$ and $\partial^2 EPR_{RS}(Q_{RS})/\partial Q_{RS}^2 = -[\gamma(p-s)+g_R]\int_0^\infty (x^2/Q_{RS}^3)h(x/Q_{RS})dF(x) < 0$. Hence, the FOC of the retailer yields: $[\gamma(p-s)+g_R]\int_0^\infty \int_0^{x/Q_{RS}} adH(a)dF(x) = w_{RS} - \gamma s$. If $Q_{RS}^* = Q_C^*$, the channel will be coordinated. Therefore, solving (A8) for w_{RS} gives us the result.

$$\begin{aligned} \int_0^\infty \int_0^{x/Q_C^*} adH(a)dF(x)dx &= \int_0^\infty \int_0^{x/Q_RS^*} adH(a)dF(x)dx \\ \frac{c-s}{p+g_C-s} &= \frac{w_{RS}-\gamma s}{\gamma(p-s)+g_R} \end{aligned} \tag{A8}$$

Lemma 15: We follow the same line of proof for this result as we did for Lemma 13.

1. $EPR_{RS}(\hat{Q}_{RS}^*) - EPR_D(Q_{RS}^*) = [\gamma(p-s)+g_R]\int_0^\infty x[H(x/Q_C^*) - \hat{H}(x/\hat{Q}_C^*)]dF(x)dx$

   after substituting (2.46) for w_{RS} and simplification.

2. $EPM_{RS}(\hat{Q}_{RS}^*) - EPM_{RS}(Q_{RS}^*)$ is equal to (2.47) after substituting (2.46) for w_{RS}

   and further simplification.

Theorem 13: $\partial EPC/\partial Q = -c + s + A'(Q)[1 - F(A(Q))](p + g_C - s)$ and $\partial^2 EPC/\partial Q^2 = \{A''(Q)[1 - F(A(Q))] - (A'(Q))^2 f(A(Q))\}(p + g_C - s)$ and is negative if $A''(Q) < 0$ and $A'(Q) > 0$. The result follows from the FOC.

Theorem 14

1. $\partial EPR_D/\partial Q = -w + s + A'(Q)[1 - F(A(Q))](p + g_R - s)$ and $\partial^2 EPR_D/\partial Q^2 = \{A''(Q)[1 - F(A(Q))] - (A'(Q))^2 f(A(Q))\}(p + g_R - s)$ and is negative if $A''(Q) < 0$ and $A'(Q) > 0$. The result follows from the FOC.

2. $\partial EPM_D/\partial Q = -(w^{*\prime}\, Q + w^*) - c + A'(Q)[1 - F(A(Q))]g_M$ and $\partial^2 EPM_D/\partial Q^2 = -(w^{*\prime\prime}\, Q + w^{*\prime}) + g_M\{A''(Q)[1 - F(A(Q))] - (A'(Q))^2 f(A(Q))\}$ where

$$w^{*\prime} = \left\{A''(Q)\bar{F}(A(Q)) - (A'(Q))^2 f(A(Q))\right\}(p + g_R - s) < 0 \tag{A9}$$

$$w^{*\prime\prime} = \left\{A'''(Q)\bar{F}(A(Q)) - 3A'(Q)A''(Q)f(A(Q)) - (A'(Q))^2 f'(A(Q))\right\} (p + g_R - s) \tag{A10}$$

Theorem 15: $\partial EPR_{RS}/\partial Q = -w + \gamma s + A'(Q)\bar{F}(A(Q))[\gamma(p - s) + g_R]$ and $\partial^2 EPR_{RS}/\partial Q^2 = (A''(Q)\bar{F}(A(Q)) - (A'(Q))^2 f(A(Q)))[\gamma(p - s) + g_R]$ which is negative for increasing concave $A(Q)$. Hence, the coordination wholesale price is found by solving the following for w.

$$\frac{c - s}{p + g_C - s} = \frac{w - \gamma s}{\gamma(p - s) + g_R} \tag{A11}$$

B. Appendix for Chapter 3

Lemma 16:

1. Çamdereli and Swaminathan (2005) show that when $\tilde{a} < 0.5$, $Q_C^* = Q_N^*$ for $a = 1$ and $a = \tilde{a}/(1 - \tilde{a})$, $Q_C^* < Q_N^*$ for $a < \tilde{a}/(1 - \tilde{a})$ and $Q_C^* > Q_N^*$ otherwise. Further,

when $\tilde{a} \geq 0.5$, $Q_C^* = Q_N^*$ for $a = 1$ and $Q_C^* < Q_N^*$ for all $a < 1$. Given this and $Q_N^* > Q_{C,t}^*$, the result follows.

2. (a) $Q_{C,t}^* = Q_C^*$ for $\underline{\mathrm{t}}[a] = \frac{1}{a^2}[c - s - a(p + g_C - s) + a^2(p + g_C - c)]$, which is smaller than $(p + g_C - c)$ iff $a < \tilde{a}$. The difference $Q_{C,t}^* - Q_C^*$ is linearly decreasing in t. Hence, accompanied by Theorem 16, the lemma follows.

   (b) Let us write $\underline{\mathrm{t}}[a] = \breve{t}[a]/a^2$. The function $\breve{t}[a]$ is strictly convex in a ($\partial^2\breve{t}[a]/\partial a^2 = 2(p + g_C - c) > 0$). Solving $\breve{t}[a] = 0$ for a yields $a = 1$ and $a = \tilde{a}/(1 - \tilde{a})$. The result follows given that $a < 1$. Additionally, $\underline{\mathrm{t}}[a] < p + g_C - c$ iff $Q_C^* > 0$.

Theorem 17:

1. $\partial^2[EPCt(Q_{C,t}^*) - EPC(Q_C^*)]/\partial t^2 = \beta/(p + g_C - c) > 0$. Solving $EPCt(Q_{C,t}^*) - EPC(Q_C^*)] = 0$ for t yields $t = (p + g_C - c) - \sqrt{\delta_C}$ and $t = (p + g_C - c) + \sqrt{\delta_C}$ where $\delta_C = \frac{2a^2K(p+g_C-s)+(a(p+g_C-s)-(c-s))^2\beta}{a^2\beta} > 0$. The first root of t is viable by $Q_{C,t}^* > 0$.

2. $EPCt(Q_{C,t}^*) - EPC(Q_C^*)]$ is linearly decreasing in K. Solving $EPCt(Q_{C,t}^*) - EPC(Q_C^*)] = 0$ for K yields: $K = \frac{[(1-a)(c-s)-at][a(2(p+g_C)-(c+s))-(c-s)-at]\beta}{2a^2(p+g_C-s)}$.

Lemma 17:

1. (i) Let us call $\wp = a^2\beta[2a^2K(p + g_C - s) + (a(p + g_C - s) - (c - s))^2\beta] > 0$. $\partial t_C^*/\partial K = -a^2(p + g_C - s)/\sqrt{\wp} < 0$. (ii) $\partial t_C^*/\partial a = -(c - s)(a(p + g_C - s) - (c - s))/(a\sqrt{\wp}) < 0$ iff $a > \tilde{a}$.

2. Let us write K_C^* given by Theorem 17 as $K_C^* = \beta\upsilon_1\upsilon_2/[2a^2(p + g_C - s)]$ where $\upsilon_1 = [(1 - a)(c - s) - at]$ and $\upsilon_2 = [a(2(p + g_C) - (c + s)) - (c - s) - at]$. Hence,$K_C^* > 0$ iff $\upsilon_1\upsilon_2 > 0$, which is strictly convex in t since $\partial^2[\upsilon_1\upsilon_2]/\partial t^2 = 2a^2 > 0$. Further, solving $\partial[\upsilon_1\upsilon_2]/\partial t = 0$ for t yields $t = p + g_C - c$. The roots of $\upsilon_1\upsilon_2$ are found

by solving $\upsilon_1 = 0$ and $\upsilon_2 = 0$ for t. Solving the first equality yields $t = t_1$ where $t_1 = (1-a)(c-s)/a > 0$, which is less than $p + g_C - c$ iff $a < \tilde{a} \Leftrightarrow Q_C^* > 0$.

3. $\partial^2[EPCt(Q_{C,t}^*) - EPC(Q_C^*)]/\partial t^2 = \beta/(p + g_C - s) > 0$. Solving $\partial[EPCt(Q_{C,t}^*) - EPC(Q_C^*)]/\partial t = 0$ yields $t = p + g_C - c$. When $K = 0$, $[EPCt(Q_{C,t}^*) - EPC(Q_C^*)]$ is equal to zero for two values of t, one of which is t_1, which is less than $p + g_C - c$ iff $a < \tilde{a} \Leftrightarrow Q_C^* > 0$.

Theorem 18: One can show that the profit functions under inventory misplacement and under RFID are concave in $Q_C^{'*}$ and $Q_{C,t}^{'*}$, respectively. Hence, solving the first order conditions give us the following: $Q_C^{'*} = (1/a)[(1 - \tilde{a}/a)(\bar{\beta} - \underline{\beta}) + \underline{\beta}] > 0$ iff $a > \tilde{a}$ and $Q_{C,t}^{'*} = (1 - \frac{c+t-s}{p+g_C-s})(\bar{\beta} - \underline{\beta}) + \underline{\beta} > 0$ if and only if $t < p + g_C - c$. Now, we show that $t > t_1$ is a sufficient condition for the firm to not to benefit from RFID. Solving $EPCt(Q_{C,t}^{'*}) - EPC(Q_C^{'*}) = 0$ for K, we receive the following: $K_C^{'*} = \frac{[(1-a)(c-s)-at][a\underline{\beta}t - a(-2(p+g_C)+t)\bar{\beta} - (1+a)c(\bar{\beta}-\underline{\beta}) - s((1+a)\underline{\beta} - (1-a)\bar{\beta})]}{2a^2(p+g_C-s)}$. The numerator of this expression is convex in t and the smaller root $t = t_1$ is viable when $Q_C^{'*}, Q_{C,t}^{'*} > 0$ where t_1 is given by Lemma 17.

1. $\partial[EPCt(Q_{C,t}^{'*}) - EPC(Q_C^{'*})]/\partial a = -(c-s)[(c-s)\underline{\beta} + [a(p+g_C-s) - (c-s)]\bar{\beta}]/[a^3(p+g_C-s)] < 0$ for $Q_C^{'*} > 0$

2. We set the lower bound of the distribution to δ and the upper bound of the distribution to $\bar{\beta} + \delta$. Hence, any change in δ will capture the effects of changes in the mean of the distribution. The mean of the distribution is $0.5(\bar{\beta} + 2\delta)$ while the variance of the distribution is fixed at $\bar{\beta}^2/12$.

   (a) $\frac{\partial Q_C^{'*}}{\partial \delta} = 1/a$ and $\frac{\partial Q_{C,t}^{'*}}{\partial \delta} = 1$.

(b) $\frac{\partial[EPCt(Q'^{*}_{C,t})-EPC(Q'^{*}_{C})]}{\partial\delta} = \frac{(1-a)(c-s)-at}{a} > 0$ iff $t < t_1$.

Theorem 19: Refer to the proof of Theorem 18 for Q'^{*}_{C} and $Q'^{*}_{C,t}$ and $t > t_1$ being a sufficient condition for the firm to not to benefit from the technology.

1. We set the lower bound of the distribution to δ and the upper bound of the distribution to $\bar{\beta} - \delta$. Hence, any change in δ will capture the effects of changes in the variance of the distribution in the opposite direction. The variance of the distribution is $(\bar{\beta} - 2\delta)^2/12$ while the mean of the distribution is fixed at $\bar{\beta}/2$.

   (a) $\frac{\partial Q'^{*}_{C}}{\partial\delta} = (-a + 2\tilde{a})/a^2 > 0$ iff $a < 2\tilde{a}$.

   (b) $\frac{\partial Q'^{*}_{C,t}}{\partial\delta} = (2(t + c) - (p + g_C + s))/(p + g_C - s) > 0$ iff $t > t_3$ where $t_3 = (p + g_C + s)/2 - c$.

   (c) $\frac{\partial[EPCt(Q'^{*}_{C,t})-EPC(Q'^{*}_{C})]}{\partial\delta} = -\frac{[(1-a)(c-s)-at]\varphi}{a^2(p+g_C-s)} > 0$ where $\varphi = -(1 + a)c + s + a(p + g_C - s) < 0$ and iff $t > t_2$ where $t_2 = (p+g_C-c)-(c-s)/a$. Hence, if $t_2 < t < t_1$ holds, it decreases with the variance.

2. $t_2 < t_1$ is equivalent to $a < 2\tilde{a}$ after simplification.

Theorem 21:

1. $\partial^2 EPR_{D,ti}(Q_{D,ti})/\partial Q^2_{D,ti} = -(p + g_R - s)/\beta < 0$ where $i \,\epsilon \{R, M\}$.

2. Solving $\partial EPR_{D,ti}(Q_{D,ti})/\partial Q_{D,ti} = 0$ yields $Q^{*}_{D,ti} = \beta(p+g_R-X)/(p+g_C-s)$ where $X = w_{D,tR} + t$ when $i = R$ and $X = w_{D,tM}$ when $i = M$. after substitution of these values back to the corresponding profit functions of the manufacturer, we receive the following: $\partial^2 EPM_{D,ti}(w_{D,ti})/\partial w^2_{D,ti} = -[2(p + g_R - s) + g_M]/(p + g_R - s)^2 < 0$ where $i \,\epsilon \{R, M\}$.

3. First Order Conditions of the manufacturer's profit functions yield $w^*_{D,tR} = [(g_R + p - s)(g_R + p + c) + sg_M - t(p + g_C - s)]/(2(g + p - s) + g_M)$ and $w^*_{D,tM} = [(g_R + p - s)(g_R + p + c) + sg_M + t(p + g_R - s])/(2(g + p - s) + g_M)$. The expression $w^*_{D,tR} - w^*_D = -[(p + g_C - s)t - (1 - a)(p + g_R - s)^2]/(2(g + p - s) + g_M) > 0$ iff $t > (1 - a)(p + g_R - s)^2/(p + g_C - s)$. Further, $w^*_{D,tM} - w^*_{D,tR} = t$.

4. $Q^*_{D,tR} = Q^*_{D,tM}$ since $w^*_{D,tM} - w^*_{D,tR} = t$ and $Q^*_{D,ti} = \beta(p + g_R - X)/(p + g_C - s)$ where $X = w_{D,tR} + t$ when $i = R$ and $X = w_{D,tM}$ when $i = M$. Substituting the $w^*_{D,ti}$ back into $Q_{D,ti}$ gives us the result.

Corollary 10: Follows from Theorem 21.

Lemma 18: Following Corollary 10, the following proofs are valid for $i = R$ and $i = M$.

1. (a) $EPR_{D,ti}(Q^*_{D,ti})$ is linearly decreasing in K. Solving $EPR_{D,ti}(Q^*_{D,ti}) = 0$ for K and simplifying yields: $K = \rho^*_1$ where $\rho^*_1 = (2\theta)^{-1}[(p + g_R - s)(\frac{Q^{*2}_{D,ti}}{\beta}) - g_R\beta]$. Substituting (3.13) into ρ^*_1, we obtain $\rho^*_1 = -(2\theta)^{-1}\beta(g_R - (p + g_R - s)(p + g_C - c - t)^2/[2(p + g_R - s) + g_M]^2)$. Hence, $\rho^*_1 \geq 0$ if and only if $t \leq (p + g_C - c) - [2(p + g_R - s) + g_M]\sqrt{g_R/(p + g_R - s)}$.

   (b) $\partial^2 EPR_{D,ti}(Q^*_{D,ti})/\partial t^2 = \beta(p + g_R - s)/[2(p + g_R - s) + g_M] > 0$. Solving $\partial EPR_{D,ti}(Q^*_{D,ti})/\partial t = 0$ yields $t = p + g_C - c$. The roots of $EPR_{D,ti}(Q^*_{D,ti}) = 0$ when $\theta = 0$ are $t = (p + g_C - c) \pm \sqrt{g_M[2(p + g_R - s) + g_M]}$. The positive root is not viable since $Q^*_{D,ti} < 0$ for $t < p + g_C - c$.

2. (a) $EPM_{D,ti}(w^*_{D,ti})$ is linearly decreasing in K. Solving $EPM_{D,ti}(wD, ti^*) = 0$ for K and simplifying yields: $K = \rho^*_2$ where $\rho^*_2 = (1 - \theta)^{-1}[\frac{(p+g_C-s)Q^{*2}_{D,tR}}{\beta} - g_M(\beta/2 + Q^{*2}_{D,tR}/(2\beta))]$. Substituting (3.13) into ρ^*_2, we obtain $\rho^*_2 = -\beta g_M/2 +$

$\beta(p+g_C-c-t)^2/[2[2(p+g_R-s)+g_M]]$. Hence, $\rho_2^* \geq 0$ if and only if $t \leq (p+g_C-c)-\sqrt{g_M[2(p+g_R-s)+g_M]}$.

(b) $\partial^2 EPM_{D,ti}(w^*_{D,ti})/\partial t^2 = \beta/[2(p+g_R-s)+g_M] > 0$. Solving $\partial EPM_{D,ti}(w^*_{D,ti})/\partial t = 0$ yields $t = p+g_C-c$. The roots of $EPM_{D,ti}(w^*_{D,ti}) = 0$ when $\theta = 1$ are $t = (p+g_C-c) \pm \sqrt{g_M[2(p+g_R-s)+g_M]}$. The positive root is not viable since $Q^*_{D,ti} < 0$ for $t < p+g_C-c$.

3. When $\theta = 0.5$ and $g_R = g_M$, $EPM_{D,ti}(w^*_{D,ti}) - EPR_{D,ti}(Q^*_{D,ti}) = (2g_R+p-s)(2g_R+p-c-t)^2/[2(2p+3g_R-2s)^2] > 0$.

Lemma 19: Following Corollary 10, the following proofs are valid for $i = R$ and $i = M$.

1. $EPR_{D,ti}(Q^*_{D,ti}) - EPR_D(Q^*_D)$ is convex in t (see the proof of Lemma 18.1.(b). Solving $EPR_{D,ti}(Q^*_{D,ti}) - EPR_D(Q^*_D) = 0$, we find $t = (p+g_C-c) \pm \delta_{D,R1}\sqrt{\delta_{D,R2}}$ where $\delta_{D,R1} = [4(p+g_C-s)+g_M^2/(p+g_R-s)] > 0$ and $\delta_{D,R2} = [(a(p+g_C-s)-(c-s))(p+g_R-s)]^2 + \frac{2a^2K[2(p+g_R-s)+g_M]^2\theta(p+g_R-s)}{\beta}]/[a^2[2(p+g_R-s)+g_M]^4] > 0$. The positive root is not viable due to $Q^*_{D,ti} < 0$ for $t > p+g_C-c$. Hence, the result follows from convexity.

2. $EPM_{D,ti}(w^*_{D,ti}) - EPM_D(w^*_D)$ is convex in t (see the proof of Lemma 18.2.(b). Solving $EPM_{D,ti}(w^*_{D,ti}) - EPM_D(w^*_D) = 0$, we find $t = (p+g_C-c) \pm \sqrt{\delta_{D,M}}$ where $\delta_{D,M} = [(a(p+g_C-s)-(c-s))^2 + \frac{2a^2K[2(p+g_R-s)+g_M](1-\theta)}{\beta}]/a^2 > 0$. The positive root is not viable due to $Q^*_{D,ti} < 0$ for $t > p+g_C-c$. Hence, the result follows from convexity.

Lemma 20: Following Corollary 10, the following proofs are valid for $i = R$ and $i = M$.

1. (a) $EPR_{D,ti}(Q^*_{D,ti}) - EPR_D(Q^*_D)$ is linearly decreasing in K. Hence, $K^*_{D,R}$ solves

$EPR_{D,ti}(Q^*_{D,ti}) - EPR_D(Q^*_D) = 0.$

(b) $EPM_{D,ti}(w^*_{D,ti}) - EPM_D(w^*_D)$ is linearly decreasing in K. Hence, $K^*_{D,M}$ solves $EPM_{D,ti}(w^*_{D,ti}) - EPM_D(w^*_D) = 0.$

(c) $K^*_{D,R} > 0$ if and only if $\upsilon_1\upsilon_2 > 0$ (See Lemma 17.2). Hence, the result follows from the proof of Lemma 17.2. The proof for $K^*_{D,M} > 0$ is the same since $K^*_{D,M} > 0$ if and only if $\upsilon_1\upsilon_2 > 0$.

2. $EPR_{D,ti}(Q^*_{D,ti}) - EPR_D(Q^*_D)$ is convex in t (see the proof of Lemma 18.1.(b)). When $\theta = 0$, solving the roots of this equality for t yields two values of t, one of which is t_1 and is less than $p + g_C - c$ iff $a < \tilde{a} \Leftrightarrow Q^*_D > 0$ (See Theorem 20).

3. The proof is exactly the same as the one in Lemma 20.2.

Theorem 22:

1. $t_1 > t^*_{D,R}$ is equivalent to $\sqrt{\Omega_R} > \frac{\beta[a(p+g_C-s)-(c-s)](p+g_R-s)}{a[2(p+g_R-s)+g_M]^2}$ where $\Omega_R = \delta_{D,R2}/\beta$. Since $\delta_{D,R2} > 0$ and the right hand side of the inequality is also positive if $Q^*_D > 0$, we can write this inequality as $\Omega_R - (\frac{\beta[a(p+g_C-s)-(c-s)](p+g_R-s)}{a[2(p+g_R-s)+g_M]^2})^2 > 0$, and this is equivalent to $\frac{2K\beta(p+g_R-s)\theta}{(2(p+g_R-s)+g_M)^2} > 0$ after simplification and it is always true. Similarly, $t_1 > t^*_{D,M}$ is equivalent to $\sqrt{\beta^2\delta_{D,M}/(2(p+g_R-s))^2} > \frac{\beta[a(p+g_C-s)-(c-s)]}{a[2(p+g_R-s)+g_M]}$. Since $\delta_{D,M} > 0$ and the right hand side of the inequality is also positive if $Q^*_D > 0$, we can proceed with simplifying this expression as above. We receive $\frac{2K\beta(1-\theta)}{2(p+g_R-s)+g_M} > 0$ after simplification and it is always true (Refer to Lemma 19 for $\delta_{D,R2}, \delta_{D,M}, t^*_{D,R}$ and $t^*_{D,M}$.

2. Following Corollary 10, the following proofs are valid for $i = R$ and $i = M$. $t^*_{D,M} > t^*_{D,R}$ is as follows after simplification $2K[2(p + g_R - s) + g_M]\upsilon_3/[(p + g_R - s)\beta] < 0$

where $\upsilon_3 = [p + g_R - (3g_R + g_M + 3p)\theta + s(-1 + 3\theta)]$. Hence, $t^*_{D,M} - t^*_{D,R} > 0$ iff $\upsilon_3 < 0$, which is equivalent to $\theta > \theta^*_D = (p + g_R - s)/[3(p + g_R - s) + g_M] \le 1/3$.

3. Similarly, $K^*_{D,R} - K^*_{D,M}$ can be written as $\beta\upsilon_1\upsilon_2\upsilon_3/[2a^2(2(p+g_R-s)+g_M)^2(1-\theta)\theta]$ after simplification (See the proof for Theorem 22.2 for υ_3 and Lemma 17.2 for $\upsilon_1\upsilon_2$). $K^*_{D,R}, K^*_{D,M} > 0$ iff $\upsilon_1\upsilon_2 > 0$ (See the proof for Lemma 17.2). Hence, the result follows.

Lemma 21: Given $Q^*_{D,ti}, Q^*_D > 0$ and following Corollary 10, the following proofs are valid for $i = R$ and $i = M$.

1. (a) *i.* $\partial t^*_{D,R}/\partial K = -\theta/[\beta\sqrt{\delta_{D,R2}}] < 0$ where $\delta_{D,R2} > 0$ is given by Lemma 19. *ii.* $\partial t^*_{D,R}/\partial a = -(c - s)(p + g_R - s)[a(p + g_C - s) - (c - s)]/[a^3[2(p + g_R - s) + g_M]^2\sqrt{\delta_{D,R2}}] < 0$ *iii.* $\partial t^*_{D,R}/\partial\theta = -K/[\beta\sqrt{\delta_{D,R2}}] < 0$.

   (b) *i.* $\partial t^*_{D,M}/\partial K = -(1 - \theta)(2(p + g_R - s) + g_M)/[\beta\sqrt{\delta_{D,M}}] < 0$ where $\delta_{D,M} > 0$ is given by Lemma 19. *ii.* $\partial t^*_{D,R}/\partial a = -(c-s)[a(p+g_C-s)-(c-s)]/[a^3\sqrt{\delta_{D,M}}] < 0$ *iii.* $\partial t^*_{D,R}/\partial\theta = -K(2(p + g_R - s) + g_M)/[\beta\sqrt{\delta_{D,M}}] < 0$.

2. (a) *i.* $\partial K^*_{D,R}/\partial t = -(p + g_R - s)(p + g_C - c - t)\beta/[\theta(2(p + g_R - s) + g_M)^2] < 0$. *ii.*$\partial K^*_{D,R}/\partial a = -(c-s)(a(p+g_C-s)-(c-s))(p+g_R-s)\beta/[a^3\theta(2(p+g_R-s)+g_M)^2] < 0$ *iii.* $\partial K^*_{D,R}/\partial\theta = -(p + g_R - s)\upsilon_1\upsilon_2/[2a^2\theta^2(2(p + g_R - s) + g_M)] < 0$ when $K^*_{D,R} > 0$ since $\upsilon_1\upsilon_2 > 0$ (see the proof of Lemma 17.2)

   (b) *i.* $\partial K^*_{D,M}/\partial t = -(p+g_C-c-t)\beta/[(1-\theta)(2(p+g_R-s)+g_M)] < 0$. *ii.*$\partial K^*_{D,M}/\partial a = -(c - s)(a(p + g_C - s) - (c - s))\beta/[a^3(1 - \theta)(2(p + g_R - s) + g_M)] < 0$ *iii.* $\partial K^*_{D,M}/\partial\theta = \upsilon_1\upsilon_2/[2a^2(1 - \theta)^2(2(p + g_R - s) + g_M)] > 0$ when $K^*_{D,M} > 0$ since $\upsilon_1\upsilon_2 > 0$ (see the proof of Lemma 17.2)

Theorem 23: One can show that the profit functions are concave in their choice variables. Hence, solving for the first order conditions for the wholesale prices first and plugging the resulting expressions into the retailer's profit function give us the optimum orders. We find $Q_D^{'*} = \kappa Q_C^{'*}$ and $Q_{D,tR}^{'*} = Q_{D,tM}^{'*} = \kappa Q_{C,t}^{'*}$ where $\kappa < 1$ is given by Theorem 20 and $Q_C^{'*}$ and $Q_{C,t}^{'*}$ are given by the proof of Theorem 18. We also find that $EPR_{D,ti}(Q_{D,ti}^{'*})$ is the same for $i = R$ and $i = M$ and $EPM_{D,ti}(w_{D,ti}^{'*})$ is the same for $i = R$ and $i = M$. solving $EPR_{D,ti}(Q_{D,ti}^{'*}) - EPR_D(Q_D^{'*}) = 0$ and $EPM_{D,ti}(w_{D,ti}^{'*}) - EPM_D(w_D^{'*}) = 0$ for K yield $K_{D,R}^{'*} = \frac{(p+g_R-s)(p+g_C-s)}{(2(p+g_R-s)+g_M)^2\theta} K_C^{'*}$ and $K_{D,M}^{'*} = \frac{(p+g_C-s)}{(2(p+g_R-s)+g_M)(1-\theta)} K_C^{'*}$ where $K_C^{'*}$ is given by Theorem 18. Hence, this proves that $t < t_1$ is a sufficient condition for the parties not to benefit from the technology (See Theorem 18 for the proof).

1. $\partial[EPR_{D,ti}(Q_{D,ti}^{'*}) - EPR_D(Q_D^{'*})]/\partial a = -(c-s)(p+g_R-s)[(c-s)\underline{\beta} + [a(p+g_C-s) - (c-s)]\bar{\beta}]/[a^3(2(p+g_R-s)+g_M)^2] < 0$ and $\partial[EPM_{D,ti}(w_{D,ti}^{'*}) - EPM_D(w_D^{'*})]/\partial a = -(c-s)[(c-s)\underline{\beta} + [a(p+g_C-s)-(c-s)]\bar{\beta}]/[a^3(2(p+g_R-s)+g_M)] < 0$ for $Q_D^{'*} > 0$.

2. (a) The result follows the fact that $Q_D^{'*} = \kappa Q_C^{'*}$ and $Q_{D,tR}^{'*} = Q_{D,tM}^{'*} = \kappa Q_{C,t}^{'*}$.

   (b) We use the same method as we used in Theorem 18 and Theorem 19 to capture the effects of mean independent from the effects of variance and vice versa. Effects of mean: $\frac{\partial[EPR_{D,ti}(Q_{D,ti}^{'*}) - EPR_D(Q_D^{'*})]}{\partial\delta} = \frac{(p+g_R-s)(p+g_C-s)[(1-a)(c-s)-at]}{a(2(p+g_R-s)+g_M)^2} > 0$ and $\frac{\partial[EPM_{D,ti}(w_{D,ti}^{'*}) - EPM_D(w_D^{'*})]}{\partial\delta} = \frac{(p+g_C-s)[(1-a)(c-s)-at]}{a(2(p+g_R-s)+g_M)} > 0$ for $t < t_1$.
   Effects of variance: $\frac{\partial[EPR_{D,ti}(Q_{D,ti}^{'*}) - EPR_D(Q_D^{'*})]}{\partial\delta} = -\frac{(p+g_R-s)\varphi[(1-a)(c-s)-at]}{a^2(2(p+g_R-s)+g_M)^2}$ and $\frac{\partial[EPM_{D,ti}(w_{D,ti}^{'*}) - EPM_D(w_D^{'*})]}{\partial\delta} = -\frac{\varphi[(1-a)(c-s)-at]}{a^2(2(p+g_R-s)+g_M)}$ where φ is given by Theorem 19. Hence, the results of the centralized firm are valid for the retailer and the manufacturer.

Theorem 25: $EPR_{RS,ti}(Q_{RS,ti})$ where $i\epsilon\{R, M\}$ is a modified version of $EPR_{D,ti}(Q_{D,ti})$ where the revenues are multiplied by a constant γ. Hence, $EPR_{RS,ti}(Q_{RS,ti})$ is concave in $Q_{RS,ti}$ and $Q^*_{RS,ti}$ solves $\partial EPR_{RS,ti}(Q_{RS,ti})/\partial Q_{RS,ti} = 0$. Hence, we obtain $Q^*_{RS,ti} = (p\gamma + g_R - Z)/(\gamma(p-s) + g_R)$ where $Z = w_{RS,tR} + t$ when $i = R$ and $Z = w_{RS,tM}$ when $i = M$.

1. The coordinating wholesale price $w^*_{RS,tR}$ is found by solving $Q^*_{RS,tR} = Q^*_{C,t}$. Hence, $w^*_{RS,tR} = \frac{c(\gamma(p-s)+g_R)-t[(1-\gamma)(p-s)+g_M]-s(g_R-\gamma g_C)}{p+g_C-s}$, which is greater than w^*_{RS} since $w^*_{RS,tR} - w^*_{RS} = -t[(1-\gamma)(p-s) + g_M]/(p + g_C - s) < 0$.

2. The coordinating wholesale price $w^*_{RS,tM}$ is found by solving $Q^*_{RS,tM} = Q^*_{C,t}$. Hence, $w^*_{RS,tM} = \frac{g_R(c-s+t)+\gamma[c(p-s)+s(g_C-t)+pt]}{p+g_C-s} = w^*_{RS,tR} + t$, which is greater than w^*_{RS} since $w^*_{RS,tM} - w^*_{RS} = t(\gamma(p-s) + g_R)/(p + g_C - s) > 0$.

3. (a) $w^*_{RS,tR} > c$ is equivalent to $t < [s(\gamma g_C - g_R) - c(g_M + (1-\gamma)(p-s)]/[g_M + (1-\gamma)(p-s)]$. However, this can never be true because $[s(\gamma g_C - g_R) - c(g_M + (1-\gamma)(p-s)] < 0$ for $\gamma < 1$. Hence, $w^*_{RS,tR} < c$.

   (b) $w^*_{RS,tM} > c$ is equivalent to $t > [c(g_M+(1-\gamma)(p-s))+s(g_R-g_C\gamma)]/[g_R+\gamma(p-s)]$. For this values to be positive, the numerator must be positive. Hence, $[c(g_M + (1-\gamma)(p-s))+s(g_R-g_C\gamma)] > 0 \Leftrightarrow \gamma < 1 < (c(p+g_M-s)+sg_R)/(sg_C+c(p-s))$, which is always true. The expression $[s(\gamma g_C - g_R) - c(g_M + (1-\gamma)(p-s)]/[g_M + (1-\gamma)(p-s)]$ is less than $p + g_C - c$ iff $c < g_R + \gamma p$. Hence, this threshold value is a feasible one in a coordinated supply chain where the order quantity is positive.

Corollary 11: The corollary follows from $w^*_{RS,tM} = w^*_{RS,tR} + t$ and $Q^*_{RS,tR} = Q^*_{RS,tM}$.

Theorem 26:

1. $t_1 > t^*_{RS,M}$ is equivalent to $\sqrt{\delta_{RS,M}} > a(p+g_C-s)-(c-s)$. Since $\delta_{D,M} > 0$ and the right hand side of the inequality is also positive if $Q^*_{RS} > 0$, we can write this inequality as $\delta_{RS,M} - (a(p+g_C-s)-(c-s))^2 > 0$, and this is equivalent to $\frac{2a^2K(p+g_C-s)^2(1-\theta)}{\beta(g_M+(1-\gamma(p-s))} > 0$ after simplification and it is always true. We show $t_1 > t^*_{RS,R}$ in the exact same way and $t_1 > t^*_{RS,R}$ is equivalent to $\frac{2a^2K(p+g_C-s)^2\theta}{\beta(g_R+\gamma(p-s))} > 0$ after simplification and it is always true.

2. $t^*_{RS,R} < t^*_{RS,M}$ is equivalent to $2K(p+g_C-s)^2(g_R+\gamma(p-s)-(p+g_C-s)\theta)/[\beta(g_M+(1-\gamma)(p-s))(g_R+\gamma(p-s))] < 0$, which is true iff $(g_R+\gamma(p-s)-(p+g_C-s)\theta) < 0$. This inequality is the same as $\theta > \theta^*_{RS} = [g_R+\gamma(p-s)]/(p+g_C-s) < 1$.

3. Similarly, $K^*_{RS,R} < K^*_{RS,M}$ is equivalent to $\beta\upsilon_1\upsilon_2(g_R+\gamma(p-s)-(p+g_C-s)\theta)/[2a^2(2(p+g_R-s)+g_M)^2(1-\theta)\theta] < 0$ after simplification (See the proof for Lemma 17.2 for $\upsilon_1\upsilon_2$). $K^*_{RS,R}, K^*_{RS,M} > 0$ iff $\upsilon_1\upsilon_2 > 0$ (See the proof for Lemma 17.2). Hence, the result follows.

Lemma 22: Refer to Lemma B for $\delta_{RS,R}$ and $\delta_{RS,M}$.

1. (a) *i.* $\partial t^*_{RS,R}/\partial K = -(p+g_C-s)^2\theta/[\beta(g_R+\gamma(p-s))\sqrt{\delta_{RS,R}/a^2}] < 0$ *ii.* $\partial t^*_{RS,R}/\partial a = -2(c-s)(a(p+g_C-s)-(c-s))/(2a^3\sqrt{\delta_{RS,R}/a^2}) < 0$ *iii.* $\partial t^*_{RS,R}/\partial\theta = -K(p+g_C-s)^2/[\beta(g_R+\gamma(p-s))\sqrt{\delta_{RS,R}/a^2}] < 0$ *iv.* $\partial t^*_{RS,R}/\partial\gamma = K(p-s)(p+g_C-s)^2\theta/[\beta(g_R+\gamma(p-s))^2\sqrt{\delta_{RS,R}/a^2}] > 0$.

   (b) *i.* $\partial t^*_{RS,M}/\partial K = -(p+g_C-s)^2(1-\theta)/[\beta(g_M+(1-\gamma)(p-s))\sqrt{\delta_{RS,M}/a^2}] < 0$ *ii.* $\partial t^*_{RS,M}/\partial a = -2(c-s)(a(p+g_C-s)-(c-s))/(2a^3\sqrt{\delta_{RS,M}/a^2}) < 0$ *iii.* $\partial t^*_{RS,M}/\partial\theta = K(p+g_C-s)^2/[\beta(g_M+(1-\gamma)(p-s))\sqrt{\delta_{RS,M}/a^2}] > 0$ *iv.*

$\partial t^*_{RS,M}/\partial\gamma = -K(p-s)(p+g_C-s)^2(1-\theta)/[\beta(g_M+(1-\gamma)(p-s))^2\sqrt{\delta_{RS,R}/a^2}] < 0$.

2. (a) *i.* $\partial K^*_{RS,R}/\partial t = -\beta(p+g_C-c-t)(g_R+\gamma(p-s))/[(p+g_C-s)^2\theta] < 0$ *ii.* $\partial K^*_{RS,R}/\partial a = -\beta(c-s)(a(p+g_C-s)-(c-s))(g_R+\gamma(p-s))/[a^3(p+g_C-s)^2\theta] < 0$ *iii.* $\partial K^*_{RS,R}/\partial\theta = -\beta\upsilon_1\upsilon_2((p-s)\gamma+g_R)/[2a^2(p+g_C-s)^2\theta^2] < 0$ iff $\upsilon_1\upsilon_2 > 0$ (see the proof for Lemma 17.2 for $\upsilon_1\upsilon_2$). Hence, the partial derivative is negative iff $K_{RS,R} > 0$ (see Lemma B.1.(c)). *iv.* $\partial K^*_{RS,R}/\partial\gamma = \beta\upsilon_1\upsilon_2(p-s)/[2a^2(p+g_C-s)^2\theta] > 0$ iff $K_{RS,R} > 0$ is proven exactly as in the proof of Lemma 22.2.(a).iii.

   (b) *i.* $\partial K^*_{RS,M}/\partial t = -\beta(p+g_C-c-t)(g_M+(1-\gamma)(p-s))/[(p+g_C-s)^2(1-\theta)] < 0$ *ii.* $\partial K^*_{RS,M}/\partial a = -\beta(c-s)(a(p+g_C-s)-(c-s))(g_M+(1-\gamma)(p-s))/[a^3(p+g_C-s)^2(1-\theta)] < 0$ *iii.* $\partial K^*_{RS,M}/\partial\theta = \beta\upsilon_1\upsilon_2((p-s)(1-\gamma)+g_M)/[2a^2(p+g_C-s)^2(1-\theta)^2] > 0$ iff $\upsilon_1\upsilon_2 > 0$ (see the proof for Lemma 17.2). Hence, the partial derivative is positive iff $K_{RS,M} > 0$ (see Lemma B.1.(c)). *iv.* $\partial K^*_{RS,M}/\partial\gamma = -\beta\upsilon_1\upsilon_2(p-s)/[2a^2(p+g_C-s)^2(1-\theta)] < 0$ iff $K_{RS,M} > 0$ is proven exactly as in the proof of Lemma 22.2.(b).iii.

Proofs of concavity for Demand distributed according to Normal$[\mu,\sigma]$-doubly truncated at $\Theta_L < \mu$ on the left handside and at $\Theta_R > \mu$ on the right handside where $\Theta_L, \Theta_R > 0$:

1. Centralized firm: When $\Theta_L < aQ_C < \Theta_R$, $\partial^2 EPC(Q_C)/\partial Q_C^2 = -a^2(p+g_C-s)f[aQ_C] < 0$ where $f[\xi]$ is the probability density function of the truncated normal distribution and $EPC(Q_C)$ is given by (3.2). Hence, the function is concave in $\Theta_L < aQ_C < \Theta_R$. When $\Theta_L > aQ_C$ or $\Theta_R < aQ_C$, expression (3.2) is a linear

function of Q_C.

$$\frac{\partial EPC(Q_C)}{\partial Q_C} = \begin{cases} -c+s+a(p+g_C-s) & ; aQ_C < \Theta_L \\ -c+s & ; aQ_C > \Theta_R \\ (-c+s)+a(p+g_C-s)-a(p+g_C-s)F[aQ_C] & ; \text{otherwise} \end{cases}$$

Therefore, when $a < \tilde{a}$ where $\tilde{a} = (c-s)/(p+g_C-s)$, it is optimal to order $aQ_C^* = F^{-1}[1-\tilde{a}/a] > 0$. Otherwise $Q_C^* = 0$. One can show that a unique optimum solution exists under RFID in a similar way.

2. Uncoordinated supply chain: When $\Theta_L < aQ_D < \Theta_R$, $\partial^2 EPR_D(Q_D)/\partial Q_D^2 = -a^2(p+g_R-s)f[aQ_D] < 0$ where $EPR_D(Q_D)$ is given by (3.5). Hence, the function is concave in $\Theta_L < aQ_D < \Theta_R$. When $\Theta_L > aQ_D$ or $\Theta_R < aQ_D$, expression (3.5) is a linear function of Q_D.

$$\frac{\partial EPR_D(Q_D)}{\partial Q_D} = \begin{cases} -w_D+s+a(p+g_R-s) & ; aQ_D < \Theta_L \\ -w_D+s & ; aQ_D > \Theta_R \\ (-w_D+s)+a(p+g_R-s)-a(p+g_R-s)F[aQ_D] & ; \text{otherwise} \end{cases}$$

Therefore, when $w_D < a(p+g_R-s)+s$, it is optimal to order $aQ_D^* = F^{-1}[1-\frac{w_D-s}{a(p+g_R-s)}] > 0$. Otherwise $Q_D^* = 0$. Hence, the manufacturer has to offer a wholesale price less than $a(p+g_R-s)+s$ to get the retailer order a positive quantity. The expression (3.6) is a concave function of w_D for $\Theta_L < aQ_D < \Theta_R$ as follows: $\partial^2 EPM_D(w_D)/\partial w_D^2 = e^{\frac{(\mu-aQ_D)^2}{2\sigma^2}}\sqrt{\pi/2}\sigma\{Erf[\frac{(\Theta_L-\mu)}{(\sqrt{2}\sigma)}] + Erf[\frac{(\mu-\Theta_R)}{(\sqrt{2}\sigma)}]\}/(a^2(p+g_R-s)) < 0$ where $Erf[\xi] = (2/\sqrt{\pi})\int_0^z(e^{-t^2})f(\xi)d\xi$. Hence, one can find a unique w_D^*.

One can prove that a unique optimum solution exists under RFID in a similar way.

3. Revenue Sharing: Similar to the previous proof, one can show that it is optimal for the retailer to order $aQ^*_{RS} = F^{-1}[1 - \frac{w_{RS}-\gamma s}{a(\gamma(p-s)+g_R)}] > 0$ iff $w_{RS} < a(\gamma(p-s)+g_R)+s\gamma$ and zero otherwise. When the manufacturer sets w_{RS} to $w^*_{RS} = (c-s)(\gamma(p-s)+g_R)/(p+g_C-s)+\gamma s$, $Q^*_C = Q^*_{RS}$. One can show that $w^*_{RS} < a(\gamma(p-s)+g_R)+s\gamma$ iff $Q^*_C > 0$. Hence, the channel is coordinated. One can also prove that a unique optimum solution exists under RFID in a similar way.

Lemma 22: In a coordinated supply chain by a revenue sharing contract, the following findings hold given $Q^*_{RS,ti}, Q^*_{RS} > 0$ where $i \epsilon \{R, M\}$.

1. $EPR_{RS,ti}(Q^*_{RS,ti}) - EPR_{RS}(Q^*_{RS}) > 0$ if and only if $t < t^*_{RS,R} = (p+g_C-c) - (1/a)\sqrt{\delta_{RS,R}}$

2. $EPM_{RS,ti}(w^*_{RS,ti}) - EPM_{RS}(w^*_{RS}) > 0$ if and only if $t < t^*_{RS,M} = (p+g_C-c) - (1/a)\sqrt{\delta_{RS,M}}$ where

$$\delta_{RS,R} = (a(p+g_C-s)-(c-s))^2 + \frac{2a^2K(p+g_C-s)^2\theta}{\beta(g_R+\gamma(p-s))} > 0,$$
$$\delta_{RS,M} = (a(p+g_C-s)-(c-s))^2 + \frac{2a^2K(p+g_C-s)^2(1-\theta)}{\beta(g_M+(1-\gamma)(p-s))} > 0.$$

Proof of Lemma 22: Following Corollary 11, the following proofs are valid for $i = R$ and $i = M$.

1. $\partial^2 EPR_{RS,ti}(Q^*_{RS,ti})/\partial t^2 = \beta(g_R+\gamma(p-s))/(p+g_C-s)^2 > 0$.

Solving $\partial EPR_{RS,ti}(Q^*_{RS,ti})/\partial t = 0$ yields $t = p + g_C - c$. Solving $EPR_{RS,ti}(Q^*_{RS,ti}) - EPR_{RS}(Q^*_{RS}) = 0$ yields $t = (p + g_C - c) \pm (1/a)\sqrt{\delta_{RS,R}}$. Only the negative root is viable since $Q^*_{C,t} \leq 0$ otherwise.

2. $\partial^2 EPM_{RS,ti}(w^*_{RS,ti})/\partial t^2 = \beta(g_M + (1-\gamma)(p-s))/(p + g_C - s)^2 > 0$.

   Solving $EPM_{RS,ti}(w^*_{RS,ti}) - EPM_{RS}(w^*_{RS}) = 0$ yields $t = (p + g_C - c) \pm (1/a)\sqrt{\delta_{RS,M}}$. Only the negative root is viable since $Q^*_{C,t} \leq 0$ otherwise.

Lemma 23: In a coordinated supply chain by a revenue sharing contract, the following findings hold given $Q^*_{RS,ti}, Q^*_{RS} > 0$ where $i \,\epsilon\, \{R, M\}$.

1. For $0 < \theta \leq 1$,

   (a) $EPR_{RS,ti}(Q^*_{RS,ti}) - EPR_{RS}(Q^*_{RS}) > 0$ if and only if $K < K^*_{RS,R}$

   (b) $EPM_{RS,ti}(w^*_{RS,ti}) - EPM_{RS}(w^*_{RS}) > 0$ if and only if $K < K^*_{RS,M}$

   (c) $K^*_{RS,R}, K^*_{RS,M} > 0$ if and only if $t < t_1$

2. For $\theta = 0$, $EPR_{RS,ti}(Q^*_{RS,ti}) - EPR_{RS}(Q^*_{RS}) > 0$ if and only if $t < t_1$

3. For $\theta = 1$, $EPM_{RS,ti}(w^*_{RS,ti}) - EPM_{RS}(w^*_{RS}) > 0$ if and only if $t < t_1$

   where t_1 is as given in Lemma 17,

$$K^*_{RS,R} = \frac{(\gamma(p-s) + g_R)[(1-a)(c-s) - at][a(2(p+g_C) - (c+s)) - (c-s) - at]\beta}{2a^2(p + g_C - s)^2\theta},$$
$$K^*_{RS,M} = \frac{[((1-\gamma)(p-s) + g_M)[(1-a)(c-s) - at][a(2(p+g_C) - (c+s)) - (c-s) - at]\beta}{2a^2[2a^2(p + g_C - s)^2(1-\theta)}.$$

4. $\partial(EPR_{RS,ti}(Q^*_{RS,ti}) - EPR_{RS,ti}(Q^*_{RS}))/\partial\gamma = -\partial(EPM_{RS,ti}(w^*_{RS,ti}) - EPM_{RS,ti}(w^*_{RS}))/\partial\gamma > 0$ if and only if $t < t_1$ where t_1 is as given by Lemma 17.

Proof of Lemma 23: Following Corollary 11, the following proofs are valid for $i = R$ and $i = M$.

1. For $0 < \theta \leq 1$,

   (a) $EPR_{RS,ti}(Q^*_{RS,ti}) - EPR_{RS}(Q^*_{RS})$ is linearly decreasing in K. Hence $K = K^*_{RS,R}$ is the value of K for which $EPR_{RS,ti}(Q^*_{RS,ti}) - EPR_{RS}(Q^*_{RS}) = 0$.

   (b) The line of proof is the same as the proof of Lemma B.1.(a).

   (c) $K^*_{RS,R}$ is positive iff $\upsilon_1\upsilon_2 > 0$ (see the proof of Lemma 17.2). Hence the proof of this result is the same as in the proof of Lemma 17.2. $K^*_{RS,M} > 0$ is also proven in the same way.

2. $EPR_{RS,ti}(Q^*_{RS,ti}) - EPR_{RS}(Q^*_{RS})$ is convex in t (see the proof of Lemma 11.1) and is minimized for $t = p + g_C - c$. When $\theta = 0$, solving the roots of this equality for t yields two values of t, one of which is t_1 and is less than $p + g_C - c$ iff $a < \tilde{a} \Leftrightarrow Q^*_C > 0$ (See Theorem 20).

3. The result follows the same line of proof as in the one in Lemma B.2.

4. $\partial(EPR_{RS,ti}(Q^*_{RS,ti}) - EPR_{RS,ti}(Q^*_{RS}))/\partial\gamma = -\partial(EPM_{RS,ti}(w^*_{RS,ti}) - EPM_{RS,ti}(w^*_{RS}))/\partial\gamma = (p - s)\upsilon_1\upsilon_2\beta/[2a^2(p + g_C - s)] > 0$ where υ_1, υ_2 are given by the proof of Lemma 17.2. Hence, $\partial(EPR_{RS,ti}(Q^*_{RS,ti}) - EPR_{RS,ti}(Q^*_{RS}))/\partial\gamma$ is positive iff $\upsilon_1\upsilon_2 > 0$ and this is valid when $t < t_1$ where t_1 is as given by Lemma 17 (see the proof of Lemma 17.2).

Theorem 27: The following findings hold for uniformly distributed demand in $[\underline{\beta}, \overline{\beta}]$ if $t < t_1$.

1. Regardless of the party paying for the tagging expenses, the incentives of the retailer and the manufacturer under inventory misplacement to adopt RFID decreases as inventory availability increases.

2. The results given in Theorem 18 and Theorem 19 regarding the incentives of a centralized firm under inventory misplacement to adopt RFID are exactly valid for both the retailer and the manufacturer under inventory misplacement in a coordinated supply chain by revenue sharing to adopt RFID.

Proof of Theorem 27: Profit functions of the retailer under RFID and inventory misplacement are concave in the order quantity. The wholesale prices are set such that $Q'^*_{RS} = Q^*_C$ and $Q'^*_{RS,ti} = Q^*_{C,t}$ for $i = R$ and $i = M$. We find that $EPR_{RS,ti}(Q'^*_{RS,ti})$ is the same for $i = R$ and $i = M$ and $EPM_{RS,ti}(w'^*_{RS,ti})$ is the same for $i = R$ and $i = M$. Further, solving $EPR_{RS,ti}(Q'^*_{RS,ti}) - EPR_{RS}(Q'^*_{RS}) = 0$ and $EPM_{RS,ti}(w'^*_{RS,ti}) - EPM_{RS}(w'^*_{RS}) = 0$ for K yield $K'^*_{RS,R} = \frac{(g_R+(p-s)\gamma)}{(p+g_C-s)\theta} K'^*_C$ and $K'^*_{RS,M} = \frac{(g_M+(p-s)(1-\gamma))}{(p+g_C-s)(1-\theta)} K'^*_C$ where K'^*_C is given by Theorem 18. Hence, this proves that $t < t_1$ is a sufficient condition for the parties not to benefit from the technology (See Theorem 18 for the proof).

1. $\partial[EPR_{RS,ti}(Q'^*_{RS,ti}) - EPR_{RS}(Q'^*_{RS})]/\partial a = -(c-s)(g_R + (p-s)\gamma)[(c-s)\underline{\beta} + [a(p+g_C - s) - (c-s)]\bar{\beta}]/[a^3(p+g_C-s)^2] < 0$ and $\partial[EPM_{RS,ti}(w'^*_{RS,ti}) - EPM_{RS}(w'^*_{RS})]/\partial a = -(c-s)(g_M + (p-s)(1-\gamma))[(c-s)\underline{\beta} + [a(p+g_C-s) - (c-s)]\bar{\beta}]/[a^3(p+g_C-s)^2] < 0$ for $Q'^*_C > 0$.

2. Refer to Theorem 18 to see how the effects of mean are captured independent from the effects of variance and Theorem 19 for the reverse.
Effects of mean: $\frac{\partial[EPR_{RS,ti}(Q'^{*}_{RS,ti})-EPR_{RS}(Q'^{*}_{RS})]}{\partial\delta} = \frac{(g_R+(p-s)\gamma)(1-a)(c-s)-at}{a(p+g_C-s)} > 0$ and $\frac{\partial[EPM_{RS,ti}(w'^{*}_{RS,ti})-EPM_{RS}(w'^{*}_{RS})]}{\partial\delta} = \frac{(g_M+(p-s)(1-\gamma))[(1-a)(c-s)-at]}{a(p+g_C-s)} > 0$ for $t < t_1$.
Effects of variance: $\frac{\partial[EPR_{RS,ti}(Q'^{*}_{RS,ti})-EPR_{RS}(Q'^{*}_{RS})]}{\partial\delta} = -\frac{(g_R+(p-s)\gamma)\varphi[(1-a)(c-s)-at]}{a^2(p+g_C-s)^2}$ and $\frac{\partial[EPM_{RS,ti}(w'^{*}_{RS,ti})-EPM_{RS}(w'^{*}_{RS})]}{\partial\delta} = -\frac{(g_M+(p-s)(1-\gamma))\varphi[(1-a)(c-s)-at]}{a^2(p+g_C-s)^2}$ where φ is given by Theorem 19. Hence, the results of the centralized firm are valid for the retailer and the manufacturer.

C. Appendix for Chapter 4

Theorem 28:

$$\begin{aligned}
\pi_i = p\{&\int_0^{a_iq_i}\int_0^{a_jq_j} x_i f_i(x_i) f_j(x_j) dx_j dx_i + \\
&\int_0^{a_iq_i}\int_{a_jq_j}^{\frac{a_iq_i-x_i}{\alpha}+a_jq_j} (x_i+\alpha(x_j-a_jq_j)) f_i(x_i) f_j(x_j) dx_j dx_i \\
&\int_0^{a_iq_i}\int_{\frac{a_iq_i-x_i}{\alpha}+a_jq_j}^{\infty} (a_iq_i) f_i(x_i) f_j(x_j) dx_j dx_i + \int_{a_iq_i}^{\infty}\int_0^{a_jq_j} (a_iq_i) f_i(x_i) f_j(x_j) dx_j dx_i + \\
&\int_{a_iq_i}^{\infty}\int_{a_jq_j}^{\infty} (a_iq_i) f_i(x_i) f_j(x_j) dx_j dx_i\} + s\{\int_0^{a_iq_i}\int_0^{a_jq_j} (a_iq_i - x_i) f_i(x_i) f_j(x_j) dx_j dx_i + \\
&\int_0^{a_iq_i}\int_{a_jq_j}^{\frac{a_iq_i-x_i}{\alpha}+a_jq_j} (a_iq_i - x_i - \alpha(x_j-a_jq_j)) f_i(x_i) f_j(x_j) dx_j dx_i\} - \\
&g\int_o^{a_iq_i} (x_i - a_iq_i) f_i(x_i) dx_i - wq_i
\end{aligned} \tag{C1}$$

The expected profit of retailer i is expressed above where i and $j\ \epsilon\{1,2\}$ and $i \neq j$.

$$\begin{aligned}\partial\pi_i/\partial q_i = (p+g)a_i - ga_iF_i(a_iq_i) - w - \\ (p-s)\int_0^{a_iq_i} a_iF_j(\frac{a_iq_i - x_i}{\alpha} + a_jq_j)f_i(x_i)dx_i = 0\end{aligned} \tag{C2}$$

$$\begin{aligned}\partial^2\pi_i/\partial q_i^2 = -ga_i^2f_i(a_iq_i) - a_i^2(p-s) \\ [F_j(a_jq_j)f_i(a_iq_i) + \int_0^{a_iq_i}(1/\alpha)f_j(\frac{a_iq_i - x_i}{\alpha} + a_jq_j)f_i(x_i)dx_i] < 0\end{aligned} \tag{C3}$$

As seen in (C3), π_i is concave in q_i for a given q_j. The lower bound $\underline{q_i}$ in (C4) is obtained when $q_j \to \infty$ in (C2) and the upper bound on $\bar{q}_i$ in (C5) is obtained when q_j is 0 in (C2). Hence, there exists a pure-strategy Nash Equilibrium by continuity and concavity of payoff functions and the strategy sets of players being nonempty, convex and compact.

$$ga_iF_i(a_i\underline{q_i}) + (p-s)\int_0^{a_i\underline{q_i}} a_if_i(x_i)dx_i = a_i(p+g) - w \tag{C4}$$

$$ga_iF_i(a_i\bar{q}_i) + (p-s)\int_0^{a_i\bar{q}_i} a_iF_j(\frac{a_i\bar{q}_i - x_i}{\alpha}) = a_i(p+g) - w \tag{C5}$$

The best-response function of each retailer is derived from $\partial\pi_i/\partial q_i = 0$. Hence, $a_i > w/(p+g)$ should hold to have $q_i^*(q_j) > 0$. We prove that $\partial\pi_i/\partial q_i = 0$ is a strictly decreasing curve in the (q_1, q_2) plane. Let dq_{21}/dq_1 and dq_{22}/dq_1 be the derivative of $\partial\pi_1/\partial q_1 = 0$ and $\partial\pi_2/\partial q_2 = 0$ at (q_1, q_2), respectively. Using the implicit function theorem, we show that $\partial q_{21}/\partial q_1 = -\frac{a_1}{\alpha a_2} - \frac{a_1^2L + a_1^2K}{a_1a_2M} < 0$ and $\partial q_{22}/\partial q_1 = -\frac{a_1a_2R}{a_2^2Z + a_2^2R/\alpha + a_2^2T} <$

0 where

$$
\begin{aligned}
K &= (p-s)F_2(a_2q_2)f_1(a_1q_1) > 0,\\
L &= gf_1(a_1q_1) > 0,\\
M &= (p-s)\int_0^{a_1q_1} f_2(\frac{a_1q_1 - x_1}{\alpha} + a_2q_2)f_1(x_1)dx_1 > 0,\\
R &= (p-s)\int_0^{a_2q_2} f_1(\frac{a_2q_2 - x_2}{\alpha} + a_1q_1)f_2(x_2)dx_2 > 0,\\
T &= (p-s)F_1(a_1q_1)f_2(a_2q_2) > 0,\\
Z &= gf_2(a_2q_2) > 0.
\end{aligned}
$$

After some algebra, one can show that $dq_{21}/dq_1 < dq_{22}/q_1$. Hence, the derivative of the first retailer's best-response function is always strictly smaller than the derivative of the second retailer's. Therefore, this proves the uniqueness of the Nash Equilibrium.

Lemma 25:

1. When $a_1, a_2 = 1$, expression (C6) gives the best-response functions of the retailers facing customer search for $i\epsilon\{1,2\}$.

$$gF_i(q_i) + (p-s)\int_0^{q_i} F_j(\frac{q_i - x_i}{\alpha} + q_j)f_i(x_i)dx_i = p + g - w \tag{C6}$$

$$gF_i(Q_i) + (p-s)\int_0^{Q_i} F_j(\frac{Q_i - x_i}{\alpha} + Q_j)f_i(x_i)dx_i = (p+g) - \frac{w}{a_i} \tag{C7}$$

Expression (4.3) can be organized as in (C7) after algebraic manipulation and letting $Q_i = a_iq_i$. Since the right hand sides of equations above are inequivalent for $a_i < 1$, (Q_1^*, Q_2^*) that solves the set of equations given by (C7) for $i\epsilon\{1,2\}$ can not be equal to $(q^*_{1,N}, q^*_{2,N})$ that solves the set of equations given by (C6).

2. If w in (C6) is changed with w/a_i, expressions (C6) and (C7) become equivalent. Hence, $(q^*_1, q^*_2) = (Q^*_1, Q^*_2)$ holds where $Q_i = a_i q_i$.

Theorem 29: The following are the set of equations which give the optimum orders in each case.

Case 1:

$$gF_1(a_1q_1) + (p-s)\int_0^{a_1q_1} F_2(\frac{a_1q_1 - x_1}{\alpha} + a_2q_2)f_1(x_1)dx_1 = p + g - \frac{w}{a_1} \tag{C8a}$$

$$gF_2(a_2q_2) + (p-s)\int_0^{a_2q_2} F_1(\frac{a_2q_2 - x_2}{\alpha} + a_1q_1)f_2(x_2)dx_2 = p + g - \frac{w}{a_2} \tag{C8b}$$

Case 2:

$$gF_1(q_1) + (p-s)\int_0^{q_1} F_2(\frac{q_1 - x_1}{\alpha} + q_2)f_1(x_1)dx_1 = p + g - w \tag{C9a}$$

$$gF_2(q_2) + (p-s)\int_0^{q_2} F_1(\frac{q_2 - x_2}{\alpha} + q_1)f_2(x_2)dx_2 = p + g - w \tag{C9b}$$

Case 3:

$$gF_1(a_1q_1) + (p-s)F_1(a_1q_1) = p + g - \frac{w}{a_1} \tag{C10a}$$

$$gF_2(a_2q_2) + (p-s)F_2(a_2q_2) = p + g - \frac{w}{a_2} \tag{C10b}$$

Case 4:

$$gF_1(q_1) + (p - s)F_1(q_1) = p + g - w \tag{C11a}$$

$$gF_2(q_2) + (p - s)F_2(q_2) = p + g - w \tag{C11b}$$

In the following passages, we give the proofs in the same order as given in the Theorem.

1. Case 1 vs. Case 2: We make $Q_1 = a_1 q_1$ and $Q_2 = a_2 q_2$ transformation in (C8). Righthand side of each equality in (C8) is always less than the value of the righthand side of the corresponding equality in (C9). Hence, the same applies to the lefthand sides of the equalities. For a given $Q_2 = \bar{q}$ in (C8a) and $q_2 = \bar{q}$ in (C9a), $a_1 q^*_{1,1} < q^*_{1,2}$. Similarly, the same applies to $a_2 q^*_{2,1} < q^*_{2,2}$. Therefore, the result follows.

2. Case 2 vs. Case 3: $p + g - w > p + g - w/a_i$ for $a_i < 1$ and

$$\int_0^{q_1} F_2(\frac{q_1 - x_1}{\alpha} + q_2) f_1(x_1) dx_1 < F_1(q_1) \text{ for all } q_2$$
$$\int_0^{q_2} F_1(\frac{q_2 - x_2}{\alpha} + q_1) f_2(x_2) dx_2 < F_2(q_2) \text{ for all } q_1.$$

   Hence, the result follows.

3. Case 3 vs. Case 4: Let us make $Q_1 = a_1 q_1$ and $Q_2 = a_2 q_2$ transformation in (C10) so the lefthand sides of (C10) and (C11) will become similar. The result directly follows from the fact that $p + g - w > p + g - w/a_i$ for $a_i < 1$.

4. Case 1 vs. Case 3: Because each expression in (C8) and (C10) is equal to $p + g - w/a_i$, the lefthand sides of (C8a) and (C10a) (and (C8b) and (C10b)) are the same.

However,

$$\int_0^{a_1q_1} F_2(\frac{a_1q_1 - x_1}{\alpha} + a_2q_2)f_1(x_1)dx_1 < F_1(a_1q_1) \text{ for all } q_2$$
$$\int_0^{a_2q_2} F_1(\frac{a_2q_2 - x_2}{\alpha} + a_1q_1)f_2(x_2)dx_2 < F_2(a_2q_2) \text{ for all } q_1.$$

Hence, the result follows.

5. Case 2 vs. Case 4: Because each expression in (C9) and (C11) is equal to $p+g-w$, the lefthand sides of (C9a) and (C11a) (also (C9b) and (C11b)) are the same. However,

$$\int_0^{q_1} F_2(\frac{q_1 - x_1}{\alpha} + q_2)f_1(x_1)dx_1 < F_1(q_1) \text{ for all } q_2$$
$$\int_0^{q_2} F_1(\frac{q_2 - x_2}{\alpha} + q_1)f_2(x_2)dx_2 < F_2(q_2) \text{ for all } q_1.$$

Lemma 26:

$$H = \begin{vmatrix} \frac{\partial^2\pi_1}{\partial q_1^2} & \frac{\partial^2\pi_1}{\partial q_1 \partial q_2} \\ \frac{\partial^2\pi_2}{\partial q_2 \partial q_1} & \frac{\partial^2\pi_2}{\partial q_2^2} \end{vmatrix}$$

Partial derivatives necessary to derive the result are as follow where i and j $\epsilon\{1, 2\}$ and $i \neq j$.

$$\frac{\partial^2\pi_i}{\partial q_i^2} = -ga_i^2 f_i(a_iq_i) - (p-s)a_i^2 f_i(a_iq_i)F_j(a_jq_j) - \tag{C12}$$
$$(p-s)a_i^2 \int_{a_jq_j}^{a_iq_i/\alpha + a_jq_j} f_i(a_iq_i - \alpha(x_j - a_jq_j))dF_j(x_j)$$
$$\frac{\partial^2\pi_i}{\partial q_i \partial q_j} = -(p-s)a_ia_j\alpha \int_{a_jq_j}^{a_iq_i/\alpha + a_jq_j} f_i(a_iq_i - \alpha(x_j - a_jq_j))dF_j(x_j) \tag{C13}$$
$$\frac{\partial^2\pi_i}{\partial q_i \partial \alpha} = (p-s)a_i \int_{a_jq_j}^{a_iq_i/\alpha + a_jq_j} f_i(a_iq_i - \alpha(x_j - a_jq_j))(x_j - a_jq_j)dF_j(x_j) \tag{C14}$$

$$\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} = -(p-s)a_i a_j \int_0^{a_i q_i} f_j(\frac{a_i q_i - x_i}{\alpha} + a_j q_j) f_i(x_i) dx_i < 0 \tag{C15}$$

$$\frac{\partial(q_1^* + q_2^*)}{\partial \alpha} = \frac{-\frac{\partial^2 \pi_1}{\partial q_1 \partial \alpha}\frac{\partial^2 \pi_2}{\partial q_2^2} + \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2}\frac{\partial^2 \pi_2}{\partial q_2 \partial \alpha} - \frac{\partial^2 \pi_2}{\partial q_2 \partial \alpha}\frac{\partial^2 \pi_1}{\partial q_1^2} + \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}\frac{\partial^2 \pi_1}{\partial q_1 \partial \alpha}}{|H|} \tag{C16}$$

$\partial(q_1^* + q_2^*)/\partial\alpha$ is given by (C16). We first show that $|H| = \frac{\partial^2 \pi_1}{\partial q_1^2}\frac{\partial^2 \pi_2}{\partial q_2^2} - \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2}\frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}$, which is equivalent to (C17) after some algebra, is positive. K, L, M, R, T and Z are as given by the proof of Theorem 28.

$$|H| = a_1^2 a_2^2 [\frac{(T+Z)(M+\alpha(K+L))}{\alpha} + \frac{(1-\alpha)M + \alpha(K+L)}{\alpha^2}] > 0 \tag{C17}$$

Hence the sign of $\frac{\partial(q_1^*+q_2^*)}{\partial\alpha}$ depends on the sign of its numerator, which is equivalent to $a_2(N1[2]+N2[2]+N3[2]-N1[1]\alpha)+a_1(N1[1]+N2[1]+N3[1]-N1[2]\alpha)$ after simplification.

Lemma 27: Without loss of generality, let us write $a_2 = a_1 + \delta$ where $\delta \geq 0$.

1. The numerator of $\partial(q_1^* + q_2^*)/\partial\alpha$ (see the proof of Lemma 26) is equivalent to (C18), which can be written as $a_1 U_1 + \delta U_2 + a_1 N1[1][1 - \alpha(1+\delta)]$ where $U_1, U_2 > 0$. Hence, $\alpha < \frac{1}{1+\delta} = \frac{1}{1+a_2-a_1}$ is a sufficient condition for the numerator to be positive. We find a lower bound on $\frac{1}{1+a_2-a_1}$ using $\frac{1}{1+a_2-a_1} > \frac{1}{1+1-w/(p+g)} = \frac{p+g}{2(p+g)-w} > 1/2$.

$$\begin{aligned} & a_1[(1-\alpha)(N1[1] + N1[2]) + N2[1] + N2[2] + N3[2]] \\ & \quad + \delta[N1[2] + N2[2] + N3[2] - \alpha N1[1]] \end{aligned} \tag{C18}$$

2. $\frac{a_1}{a_2} \leq \frac{1}{1+\delta} = \frac{1}{1+a_2-a_1}$ is equivalent to $a_2 \geq a_1$ which always holds for $\delta \geq 0$.

Lemma 28: Refer to Lemma 26 for the partial derivatives which are not given in this proof.

1.

$$\frac{\partial q_i^*}{\partial \alpha} = -\frac{\frac{\partial^2 \pi_i}{\partial q_i \partial \alpha}\frac{\partial^2 \pi_j}{\partial q_j^2} - \frac{\partial^2 \pi_i}{\partial q_i \partial q_j}\frac{\partial^2 \pi_j}{\partial q_j \partial \alpha}}{|H|} \tag{C19}$$

$$\frac{\partial^2 \pi_i}{\partial q_i \partial \alpha} = \frac{(p-s)a_i}{\alpha^2}\left(-\int_0^{a_i q_i} x_i f_j\left(\frac{a_i q_i - x_i}{\alpha} + a_j q_j\right) f_i(x_i) dx_i + a_i q_i\left(\int_0^{a_i q_i} f_j\left(\frac{a_i q_i - x_i}{\alpha} + a_j q_j\right) f_i(x_i) dx_i\right)\right) \tag{C20}$$

$\frac{\partial q_i^*}{\partial \alpha}$ is equal to (C19) by the Implicit Function Theorem where $|H| > 0$ is given by (C17). Hence, the sign of $\frac{\partial q_i^*}{\partial \alpha}$ is determined by the sign of the numerator, which is equivalent to $a_j(N1[j] + N2[j] + N3[j] - \alpha N1[i])$ after simplification where $N1[j], N2[j], N3[j]$ and $N1[i]$ are given by Lemma 26. The numerator is negative if and only if $\alpha > \alpha_i^*$ where $\alpha_i^* = \frac{N1[j]+N2[j]+N3[j]}{N1[i]}$.

2. In the first result of this lemma we found that $\frac{\partial q_1^*}{\partial \alpha} < 0$ if and only if $\alpha > \alpha_1^*$ and $\frac{\partial q_2^*}{\partial \alpha} < 0$ if and only if $\alpha > \alpha_2^*$. Here, we show that α_1^* and α_2^* can not be less than 1 simultaneously. $\alpha_1^* < 1$ and $\alpha_2^* < 1$ if and only if the conditions given by (C21) are satisfied. Therefore, $N1[2] - N2[1] - N3[1] > N1[2] + N2[2] + N3[2]$ must hold. This is equivalent to saying $-N2[1] - N3[1] > N2[2] + N3[2]$ and it is never true because $N2[1], N3[1], N2[2], N3[2] > 0$.

$$\left[\begin{array}{l} N1[2] + N2[2] + N3[2] < N1[1] \\ N1[1] < N1[2] - N2[1] - N3[1] \end{array}\right. \tag{C21}$$

Theorem 30: See Theorem 25 and Lemma 26 for the partial derivatives which are required but not given in this proof.

1.

$$\frac{\partial q_i^*}{\partial a_j} = -\frac{\frac{\partial^2 \pi_i}{\partial q_i \partial a_j}\frac{\partial^2 \pi_j}{\partial q_j^2} - \frac{\partial^2 \pi_i}{\partial q_i \partial q_j}\frac{\partial^2 \pi_j}{\partial q_j \partial a_j}}{|H|} \tag{C22}$$

$\partial q_i^*/\partial a_j$ where $i,\ j\ \epsilon\{1,2\}$ is given in (C22) where $|H| > 0$ (see Lemma 26).

$$\frac{\partial^2 \pi_i}{\partial q_i \partial a_j} = -(p-s)a_i q_i \int_0^{a_i q_i} f_j(\frac{a_i q_i - x_i}{\alpha} + a_j q_j) f_i(x_i) dx_i < 0 \tag{C23}$$

$$\begin{aligned}\frac{\partial^2 \pi_j}{\partial q_j \partial a_j} &= \{-g a_j q_j f_j(a_j q_j) - (p-s)a_j[\int_0^{a_j q_j} \frac{q_j}{\alpha} f_i(\frac{a_j q_j - x_j}{\alpha} + \\ &\quad a_i q_i) f_j(x_j) dx_j + q_j f_j(a_j q_j) F_i(a_i q_i)]\} + \{-g F_j(a_j q_j) + (p+g) - \\ &\quad (p-s)\int_0^{a_j q_j} F_i(\frac{a_j q_j - x_j}{\alpha} + a_i q_i) f_j(x_j) dx_j\}\end{aligned} \tag{C24}$$

The numerator of $\partial q_i^*/\partial a_j$ is given by (C25) and is positive (refer to Theorem 25 for M). Hence, $\partial q_i^*/\partial a_j$ is always a negative value.

$$\begin{aligned}a_1 a_2 M[p\ (1 - \int_0^{a_j q_j} F_i(\frac{a_j q_j - x_j}{\alpha} + a_i q_i) f_j(x_j) dx_j) + g(1 - F_j(a_j q_j)) \\ +s\int_0^{a_j q_j} F_i(\frac{a_j q_j - x_j}{\alpha} + a_i q_i) f_j(x_j) dx_j] > 0\end{aligned} \tag{C25}$$

2.

$$\frac{\partial q_i^*}{\partial a_i} = -\frac{\frac{\partial^2 \pi_i}{\partial q_i \partial a_i}\frac{\partial^2 \pi_j}{\partial q_j^2} - \frac{\partial^2 \pi_i}{\partial q_i \partial q_j}\frac{\partial^2 \pi_j}{\partial q_j \partial a_i}}{|H|} \tag{C26}$$

In (C26), $|H| > 0$, $\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} \frac{\partial^2 \pi_j}{\partial q_j \partial a_i} > 0$ and $\frac{\partial^2 \pi_j}{\partial q_j^2} < 0$ (see the proof of Theorem 28 and Theorem 30.1). Hence, if $\frac{\partial^2 \pi_i}{\partial q_i \partial a_i} > 0$, $\frac{\partial q_i^*}{\partial a_i} > 0$.

$\frac{\partial^2 \pi_i}{\partial q_i \partial a_i}$ is given by (C27) and can be organized as in (C28). Hence, the result follows.

$$\frac{\partial^2 \pi_i}{\partial q_i \partial a_i} = \{-g a_i q_i f_i(a_i q_i) - (p-s) a_i [\int_0^{a_i q_i} \frac{q_i}{\alpha} f_j(\frac{a_i q_i - x_i}{\alpha} + a_j q_j) f_i(x_i) dx_i + q_i f_i(a_i q_i) F_j(a_j q_j)]\} + \{-g F_i(a_i q_i) + (p+g) - (p-s) \int_0^{a_i q_i} F_j(\frac{a_i q_i - x_i}{\alpha} + a_j q_j) f_i(x_i) dx_i\} \tag{C27}$$

$$\frac{\partial^2 \pi_i}{\partial q_i \partial a_i} = \frac{\partial^2 \pi_i}{\partial q_i^2} \frac{q_i}{a_i} + w/a_i \tag{C28}$$

Theorem 31: The following results hold for $i, j \in \{1, 2\}$ and $i \neq j$.

1. We use envelope theorem to show the result. Therefore, $d\pi_i^*(q_i^*)/da_i = \partial \pi_i^*(q_i^*)/\partial a_i$ and is given by (C29) where $\frac{\partial q_j^*}{\partial a_i} < 0$ by Theorem 30.1.

$$\partial \pi_i^*(q_i^*)/\partial a_i = (p+g) \int_{a_i q_i^*}^{\infty} q_i^* f_i(x_i) dx_i + p \int_0^{a_i q_i^*} \int_{\frac{a_i q_i^* - x_i}{\alpha} + a_j q_j^*}^{\infty} q_i^* f_i(x_i) f_j(x_j) dx_j dx_i + s \int_0^{a_i q_i^*} \int_0^{\frac{a_i q_i^* - x_i}{\alpha} + a_j q_j^*} q_i^* f_i(x_i) f_j(x_j) dx_j dx_i + (p-s)\alpha \int_0^{a_i q_i^*} \int_{a_j q_j^*}^{\frac{a_i q_i^* - x_i}{\alpha} + a_j q_j^*} (-a_j \frac{\partial q_j^*}{\partial a_i}) f_i(x_i) f_j(x_j) dx_j dx_i > 0 \tag{C29}$$

2. We use envelope theorem to derive $d\pi_j^*(q_j^*)/da_i = \partial \pi_j^*(q_j^*)/\partial a_i$ as given by (C30). Hence, $d\pi_j^*(q_j^*)/da_i > 0$ if and only if $(q_i^* + a_i \frac{\partial q_i^*}{\partial a_i}) < 0$. However, this condition can never be satisfied since $q_i^* + a_i \frac{\partial q_i^*}{\partial a_i} = \Delta_1/\Delta_2 > 0$ as shown in (C31) and (C32) where

$A = \frac{a_j q_j^* - x_i}{\alpha} + a_i q_i^*$, $B = \frac{a_i q_i^* - x_j}{\alpha} + a_j q_j^*$.

$$\partial \pi_j^*(q_j^*)/\partial a_i = -\alpha(p-s) \int_0^{a_j q_j^*} \int_{a_i q_i^*}^{\frac{a_j q_j^* - x_j}{\alpha} + a_i q_i^*} (q_i^* + a_i \frac{\partial q_i^*}{\partial a_i}) f_j(x_j) f_i(x_i) dx_i dx_j \tag{C30}$$

$$\Delta_1 = \alpha[\alpha(g + (p-s)F_i(a_i q_i^*) + (p-s)\int_0^{a_j q_j^*} f_i(A) f_j(x_j) dx_j] \tag{C31}$$
$$(p + g - gF_i(a_i q_i^*) - (p-s)\int_0^{a_i q_i^*} f_i(x_i) f_j(B) dx_i) > 0$$

$$\Delta_2 = a_i^2[\alpha f_i(a_i q_i^*)(g + (p-s)F_j(a_j q_j^*))(\alpha(g + (p-s)F_i(a_i q_i^*))f_j(a_j q_j^*) + \tag{C32}$$
$$(p-s)\int_0^{a_j q_j^*} f_i(A) f_j(x_j) dx_j) + (p-s)\int_0^{a_i q_i^*} f_j(B) f_i(x_i) dx_i]$$
$$(\alpha(g + (p-s)F_i(a_i q_i^*))f_j(a_j q_j^*) + (1-\alpha^2)(p-s)\int_0^{a_j q_j^*} f_i(A) f_j(x_j) dx_j) > 0$$

Theorem 32: Refer to Definition 1 for the description of the game.

1. We prove by using backwards induction. We want the second retailer to play 01 or 11. Condition given by (C35) make him play 01 and those given by (C34) make him play 11.

$$\left[\begin{array}{l} u_2^*[00] > u_2^*[01] - C[a_2] \\ u_2^*[11] - C[a_2] > u_2^*[10] \end{array}\right. \tag{C33}$$

$$\left[\begin{array}{l} u_2^*[00] < u_2^*[01] - C[a_2] \\ u_2^*[11] - C[a_2] > u_2^*[01] \end{array}\right. \tag{C34}$$

If retailer 2 plays 01, $u_1^*[00] < u_1^*[11] - C[a_1]$ and if retailer 2 plays 11, $u_1^*[01] < u_1^*[11] - C[a_1]$ are sufficient for the first retailer to play 1. Thus, the following conditions are sufficient to have the first player invest to remove shrinkage and the second player invest as well.

$$u_2^*[11] - C[a_2] > u_2^*[10] \tag{C35a}$$

$$C[a_1] < u_1^*[11] - u_1^*[00] \tag{C35b}$$

$$C[a_1] < u_1^*[11] - u_1^*[01] \tag{C35c}$$

$u_1^*[01] < u_1^*[00]$ by Theorem 31. Hence, inequality (C35b) implies expression (C35c).

2. If $C[a_2] > u_2^*[11] - u_2^*[10]$, the second retailer always deviates from making an investment if the first retailer plays 1. Thus, the result follows.

3. $C[a_2] < u_2^*[01] - u_2^*[00]$ is a sufficient condition for the second retailer to play 1 if the 1st retailer plays 0. Hence, the result follows.

Theorem 33: We derive this result by assuming that the retailers simultaneously chose the availabilities and then for that given availability, play a quantity-game and chose the equilibrium orders. Therefore, the strategy space of each player is $S_i = \{0 \leq s_i \leq 1\}$ where s_i denotes the value to which inventory availability is set and $i \epsilon \{1, 2\}$. However, decreasing the current inventory level is a strictly dominated strategy for each retailer since equilibrium profit of a retailer increases in her inventory availability level for a given inventory availability of the other retailer by Theorem 31. Hence, no player i plays $s_i < a_i$. Let us call $\pi_i^*[s_i, s_j]$ the equilibrium profit of retailer i as a result of playing the quantity game after retailer i chooses to set the inventory availability level to s_i and retailer j chooses to set his inventory availability level to s_j. We can write the following by Theorem 31.

$$\pi_1^*[1, s_j] > \pi_1^*[s_i, s_j] \text{ for all } a_i \leq s_i \leq 1$$

$$\pi_2^*[s_i, 1] > \pi_1^*[s_i, s_j] \text{ for all } a_j \leq s_j \leq 1$$

Hence, iterated elimination of strictly dominated strategies eliminates all but setting inventory availability to 100%. Therefore, setting inventory availability levels to 100% is the unique Nash equilibrium.

D. Tables

Table D1: Coordination results based on different cases

		$R > l$	$R \leq l$
$R = 1$	$b = w_{BB}$	Never coordinated	Not feasible
	$b \neq w_{BB}$	Coordinated for $w_{BB1} = \frac{b(g_C+p-c)+(c-s)(p+g_R)}{(p+g_C-s)}$	Not feasible
$R = 0$		Not feasible	Never coordinated
$0 < R < 1$	$b = w_{BB}$	Coordinated for $w_{BB1} = b = \frac{(c-s)(g_R+p)+s(p+g_C-s)\frac{(1-R)^2 Q_{BB1}*}{\beta}}{(c-s)+(p+g_C-s)\frac{(1-R)^2 Q_{BB1}*}{\beta}}$	Coordinated for $w_{BB2} = b = s + \frac{(g_R+p-s)(c-s)}{(p+g_C-s)(1-R)}$
	$b \neq w_{BB}$	Coordinated for $w_{BB1} = \frac{c(g_R+p-b)-s(g_R+p)+b(p+g_C)}{(g_C+p-s)} - (b-s)\frac{(1-R)^2 Q_{BB1}*}{\beta}$	Coordinated for $w_{BB2} = c - \frac{(g_C-g_R)(c-s)}{p+g_C-s} + (b-s)R$

Table D2: Comparison of contracts for the decentralized manufacturer

Decentralized Manufacturer	Buyback-($R > l$)	Buyback-($R \leq l$)	Revenue Sharing
Buyback-($R > l$))	–	Buyback-($R > l$)	Buyback - ($R > l$) if $\gamma > \psi$ Revenue Sharing if $\gamma < \psi$ Indifferent if $\gamma = \psi$
Buyback-($R \leq l$)	Buyback-($R > l$)	–	Revenue Sharing
where $\psi = \frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ and $l = 1 - a$			

[1] : Manufacturer

[2] : Retailer

Table D3: Comparison of contracts for the decentralized retailer

Decentralized Retailer	Buyback-($R > l$)	Buyback-($R \leq l$)	Revenue Sharing
Buyback-($R > l$))	–	Buyback-($R \leq l$)	Buyback - ($R > l$) if $\gamma < \psi$ Revenue Sharing if $\gamma > \psi$ Indifferent if $\gamma = \psi$
Buyback-($R \leq l$)	Buyback-($R \leq l$)	–	Revenue Sharing
where $\psi = \frac{(1-R)^2(b-s)+a^2(p-b)}{a^2(p-s)}$ and $l = 1 - a$			

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