Essays On Supply Risk In Global Operations

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ABSTRACT

YIMIN WANG: Essays On Supply Risk In Global Operations.
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The prevalence of globalization has created significant operational risks in firms' supply networks. It is, therefore, essential for firms to adopt effective risk management strategies to mitigate supply risks. Of the many operational challenges that globalization poses, this research focuses on the operational risk associated with emerging economies. These operational risks include, for example, leadtime risk, capacity risk, quality risk, intellectual property risk, political risk, regulatory risk, exchange risk, and so on. In this dissertation, we consider three of these risks, namely, leadtime risk, capacity risk, and regulatory trade barrier risk. In the first chapter, we examine the supplier random leadtime issue. In deciding when and how much to order, firms must consider lead time risk and demand risk, i.e., the accuracy of their demand forecast. We characterize a firm's optimal timing and quantity policy. We prove that a firm's optimal procurement time is independent of the demand forecast but that the optimal procurement quantity is not. In the second chapter, we examine the supplier random capacity issue. We relax the assumption that supplier reliability is exogenous and consider a case where a firm may have the opportunity to improve a supplier's reliability. We characterize a firm's optimal procurement policy for both its diversification strategy and its process improvement strategy. We prove that a firm's procurement strategy is significantly influenced by the opportunity for process improvement. In the third chapter, we examine a firm's sourcing strategy under regulatory trade barriers. We contrast four procurement strategies in their effectiveness to mitigate supply risk due to trade barriers.

To my parents and family

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My first impression of North Carolina on my arrival is still vivid: arboreous, hot, and humid. It was the evening of August 23, 2003 and I just flew in from Calgary, Alberta. Accustomed to cool and dry climates, I was in disbelief that I was going to be stranded here for God knows how many years. Fast forward to today: I am glad that I was 'stranded' in North Carolina, a salubrious place full of diversity, openness, friendliness, and serenity. A perfect place for scholarly endeavor, Chapel Hill has discretely instilled her spirit and knowledge into my mind, body, and work. It is difficult to say goodbye.

Contemplating a PhD degree in the middle of my professional career is not easy, let alone completing a dissertation project of this magnitude and complexity. I am grateful that this dissertation project has benefited from a network of strong support and encouragement, both academically and personally. Among these supports and encouragement, I have benefited most immensely from Dr. Brian Tomlin, whose guidance and support are simply invaluable throughout my entire time at Chapel Hill. In retrospect, I realize that it has been a serendipitous opportunity to learn from and work with Dr. Brian Tomlin on a number of interesting research projects, both outside and inside of my dissertation work. These research opportunities form an indispensable foundation for me to conduct rigorous and independent research projects in years ahead. Beyond research activities, it is also a great pleasure to interact with Dr. Brian Tomlin on a personal note. Such amicable relationship makes my entire stay at Chapel Hill much more enjoyable and memorable. Simply put, I am immensely grateful.

The faculty here are extremely friendly to students and immensely helpful. I am very

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Chapter 1

Introduction

Motivated by my own significant professional experience in Asia and North America, this dissertation focuses on global operations management. This is an important area of research in today's business environment because of the significant operational risks associated with global operations, especially in connection with emerging economies. For example, Boston Consulting Group reported that "risks [from low cost countries] have multiple sources. Some, such as shipping delays, unavailability of shipping capacity, and customs issues, are related to the physical transport of goods. Others are linked to procurement issues. Examples include high numbers of rejects, inability of vendors to scale up, and disruption in delivery schedules". Other operational risks include regulatory risk, supply-information risk, technology-transfer risk, and exchange-rate risk etc. The aim of this research is to develop theoretical insights to help companies effectively manage the operational risks associated with global operations.

Of the many operational challenges that globalization poses, this research focuses on the operational risk associated with emerging economies. These operational risks include, for example, leadtime risk, capacity risk, quality risk, intellectual property risk, political risk, regulatory

risk, exchange risk, and so on. In this essay, we focus on three risks: leadtime risk, capacity risk, and regulatory trade barrier risk. These three risks are each addressed in subsequent chapters.

Leadtime risk refers to random leadtimes associated with long and uncertain global supply networks, and capacity risk refers to random quantities associated with unreliable suppliers. While always present, the importance and magnitude of these two forms of uncertainty have increased as a result of some distinct characteristics of emerging economies. First, many suppliers in emerging economies have less stable operational capabilities due to reasons such as energy and raw material shortage, less efficient infrastructure, and less developed technologies. "They [suppliers] have explicitly chosen designs and processes that sacrifice quality for the sake of speed and cost savings" (China's Five Surprises, 2006.) Less stable operational capabilities can lead to uncertainty in production/delivery quantities. Second, long and congested logistics networks can lead to random leadtimes (Agarwal, 2006).

Regulatory trade barriers pose significant threat to the continuity of the firm's supply network. Regulatory trade barriers, such as voluntary export restrictions, are some of the most frequently encountered risk facing firms operating in a global environment.

This dissertation research consists of three chapters. Chapter 2 studies the leadtime risk, Chapter 3 studies the capacity risk, and Chapter 4 studies the regulatory trade barriers risk. All proofs can be found in the appendices (Chapter 5.3). In what follows, we briefly describe each chapter in sequence.

1.1 Leadtime Risk

This chapter investigates the interplay of demand risk and leadtime risk in global sourcing. Specifically, leadtime risk refers to the uncertain supply leadtime associated with the often long and unreliable supply networks in global sourcing and demand risk refers to the accuracy of a firm's demand forecast. In deciding when and how much to procure, firms must consider the trade-off between leadtime risk and demand risk. To improve the accuracy of its demand forecast, a firm may allow customers to place orders before the selling season. It can then use these pre-season orders to update its forecast of overall demand as the selling season approaches. The longer the firm waits before procuring the product, the more information it has on pre-season orders, and, therefore, its demand risk decreases. However, it's leadtime risk, i.e., the chance of not receiving its supply on time, increases as the firm waits closer to the selling season. Thus, the firm faces a trade-off: order earlier to reduce supply risk or order later to reduce demand risk. This research fills the gap in the existing literature by investigating this important trade-off.

We model the firm's pre-season orders as an autoregressive, continuous time-series process with correlated shocks, and we allow for correlations between pre-season and in-season orders. The firm thus faces an optimal stopping problem with a joint timing and quantity decision. We characterize the firm's optimal timing and quantity decision with and without pre-season orders, and characterize the directional effects (on time and cost) of supply and market attributes. We prove that the firm's expected cost decreases as the ratio of pre-season orders to in-season orders increases. We prove that a firm might strictly prefer a leadtime with a higher probability of delay (chance of the delivery not arriving on time) to one with a lower probability of delay. However, we show that the firm becomes less sensitive to lead-time variability as the fraction

of pre-season orders increases. In other words, as a firm increasingly utilizes pre-season orders, it becomes increasingly more resilient to the uncertainty in the supply leadtime. Therefore, a firm can use demand-signal techniques, such as pre-season orders, to partially mitigate the effect of supply uncertainty.

1.2   Capacity Risk

The second chapter investigates a firm's optimal sourcing strategies when they can influence its supply uncertainty. Recent evidence from several industries suggests that supply uncertainty can often be influenced by management practices (GLSCS, 2005). This potential benefit of influencing supplier's reliability is particularly important because virtually every firm seeking to establish operations in emerging economies faces unreliable third party suppliers. A static view of the suppliers' reliability would severely limit the potential range of the firm's risk mitigation strategies.

In contrast to the first chapter of my dissertation, which, as in the existing literature, treats supply uncertainty as exogenous, this chapter introduces another dimension, i.e., operational improvement, to the firm's supply-risk mitigation strategy. To mitigate the supply risk, a firm can either diversify its supplier base or invest in supplier-reliability improvement efforts. The diversification strategy mitigates the supply risk because of the risk pooling benefit. On the other hand, the improvement strategy mitigates the supply risk by directly influencing the supplier's reliability.

This chapter characterizes the firm's optimal procurement quantity under both diversification strategy and improvement strategy. We prove that with diversification strategy, the firm

may, nevertheless, prefer to single source when the demand risk is low. However, the presence of improvement opportunities can influence the firm's preferred supplier. We prove that the firm may make improvement in a supplier that it would not have used if there were no improvement opportunity.

When the firm has the option of either diversification or improvement strategy, the firm's preference of one strategy relative to the other strategy is generally not monotonic in system parameters. While the firm may strictly prefer one strategy over the other, depending on system parameters, we observe that the diversification strategy is more likely to be preferred when the unit procurement cost is very low or very high, but the improvement strategy is more likely to be preferred when the unit procurement cost is moderate.

We found that the volatility of demand has a significant impact on the attractiveness of the improvement strategy. In extreme cases where demand is highly volatile or very stable, the improvement strategy is less likely to be preferred to the diversification strategy. On the other hand, the improvement strategy is more likely to be preferred when demand volatility is moderate. This result is due to the interplay of effective demand and effective supply.

We observed that the distributional shape of the capacity loss has important implications for the attractiveness of the diversification and improvement strategies. We found that the improvement strategy is more likely to be preferred when the unit procurement cost is high and the volatility of the capacity loss is high. A higher unit cost reduces the effective demand which mitigates the capacity disadvantage of the improvement strategy. A higher volatility of capacity loss results in a higher marginal return for the improvement strategy while exerting higher pressure on the diversification strategy.

1.3   Regulatory Trade Barriers Risk

The third chapter investigates the firm's procurement strategy when supply may be subject to regulatory trade barriers. In particular, we consider the situation where the product from the most efficient suppliers is subject to voluntary export restraints[1] (VER). There are several strategies the firm can use to mitigate the negative effect of VER. In particular, the firm may adopt a split procurement strategy by sourcing from two different countries, where one of the countries is not subject to VER. In addition, the firm may take advantage of outward processing agreements[2] (OPA) to eliminate the VER risk.

In this chapter, we contrast four mitigation strategies the firm may use to cope with quota uncertainties. The direct procurement strategies and the split procurement strategies do not eliminate quota uncertainty, while the direct and indirect OPA strategies remove quota uncertainty. We analytically characterize the firm's optimal procurement policies for all four strategies. In addition, we prove that, for the direct procurement and the split procurement strategy, the firm's optimal expected cost is decreasing in the quota cost in terms of the first order stochastic dominance. We also prove that, in the special case of a normally distributed quota cost, the firm's optimal expected profit decreases in the expected quota cost but increases

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[1] "Voluntary export restraints (VER) are arrangements between exporting and importing countries in which the exporting country agrees to limit the quantity of specific exports below a certain level in order to avoid imposition of mandatory restrictions by the importing country. The arrangement may be concluded either at the industry or government level." Organization for Economic Co-operation and Development (OECD). Available at http://stats.oecd.org/glossary/detail.asp?ID=2882. See Chapter 4 for details.

[2] Different entities, such as the US, Hong Kong, and the EU, have somewhat different definitions and procedures of outward processing agreements. This chapter most closely follows the Hong Kong and the EU practices. Here is an excerpt from the EU definition. "Outward Processing is an EU customs duty relief scheme provided for under Council Regulation (EEC) No. 2913/92 and implementing Commission Regulation 2454/93 (amended). It allows Community goods to be temporarily exported from the customs territory of the Community in order to undergo processing operations or repair and the compensating products resulting from these operations to be released for free circulation in the customs territory of the Community with total or partial relief from import duties. Outward processing enables businesses to take advantage of cheaper labour costs outside the EC, while encouraging the use of EC produced raw materials to manufacture the finished products. Goods may be also temporarily exported to undergo processes not available within the Community." See Chapter 4 for details.

in the variance of the quota cost.

We characterize the optimal procurement policies for the direct OPA strategy and we prove that the firm's optimal expected profit is non-decreasing in the leadtime difference, which is the leadtime advantage associated with the domestic production. We capture this leadtime advantage through a general framework of forecast updating process. Furthermore, we characterize the firm's first stage optimal procurement decisions under a general framework of the martingale forecast updating process.

We conducted an extensive numeric study to investigate the relative attractiveness of different mitigation strategies as system parameters change. We observed that, consistent with our analytical results, the firm's optimal expected cost decreases in stochastically increasing quota costs. We also observed that, with the direct OPA strategy, the firm's optimal expected cost increases convexly in the leadtime advantage. We found that an increase in the expected demand does not shift the rankings of different mitigation strategies, but an increase in the demand coefficient of variation (CV) may significantly shift the rankings of the different mitigation strategies. In particular, we found that the direct OPA strategy becomes less attractive as the demand CV increases, if the leadtime difference is small. In this case, an increase in the demand CV can make the direct procurement strategy more appealing than the direct OPA strategy. If, however, the leadtime advantage for the direct OPA strategy is large, then an increase in the demand CV can make the direct OPA strategy significantly more attractive and it can become the preferred strategy, especially when the low cost country unit production cost is relatively high. See chapter 4 for detailed discussions of different mitigation strategies.

Chapter 2

Leadtime Risk

2.1   Introduction

The pressure to reduce direct manufacturing costs has led many firms to outsource production to lower-cost countries such as China. As a result, North American and European firms are faced with longer procurement leadtimes. Moreover, the procurement leadtimes are often uncertain. Delays can occur for many reasons, including transportation-infrastructure issues in rapidly-developing economies, such as China, congestion in foreign and domestic ports, customs inspections, and logistical issues involving export quotas.

Oftentimes, there are two aspects to supply uncertainty. An order might be delayed or not, and if it is delayed, the length of delay may be uncertain. For example, a shipment might be selected for customs inspection and the resulting time spent in inspection might be uncertain. Similarly, a shipment might be delayed due to a port disruption (such as the 2002 US west-coast disruption) and the resulting delay might be uncertain.

Long and uncertain leadtimes are especially problematic for firms with short selling seasons.

My previously work experience in the custom-design drawnwork (textile) industry indicates that, for reasons of cost, U.S. drawnwork wholesalers source from Chinese suppliers. The typical leadtime is on the order of three months but there is significant uncertainty around this for the reasons cited above. The U.S. wholesalers sell the product to domestic customers in the fall but, because of the leadtime, have to source the drawnwork before their customers place orders. In essence, the U.S. wholesalers face a newsvendor-type problem with an uncertain leadtime.

Firms from many industries face a similar problem. The issue of supply uncertainty is of growing managerial concern, as evidenced from the following excerpts from reports by the The Economist and The Boston Consulting Group (BCG):

Last autumn some 80m items of clothing were impounded at European ports and borders because they exceeded the annual import limits that the European Union and China had agreed on only months earlier. Retailers had ordered their autumn stock well before that agreement was signed, and many were left scrambling. (When the Chain Breaks. The Economist. 2006.)

In the run up to Christmas 2004, grid-lock hit the Los Angeles-Long Beach ports, the entry point for almost half the goods coming into the United States. Nearly 100 ships floated around, cooling their keels and waiting to be unloaded - a process that was taking up to twice as long as usual. The results of the dock jam were serious and far reaching. The Sharper Image, for one, claimed that November sales had been adversely affected by reduced inventory resulting from congestion at the ports. Now companies are ordering earlier. (Avoiding Supply Chain Shipwrecks.

BCG. 2005)

Ordering earlier may reduce the risk associated with uncertain leadtimes but it also increases the firms' demand risk, i.e., the potential mismatch between the quantity procured and the realized demand. In industries with short selling-seasons, firms sometimes allow customers to place pre-season orders, that is place orders before the selling season, e.g., Fisher and Raman (1996). This enables the firm to improve the accuracy of its forecast of overall demand (the sum of pre-season and in-season customer orders). The longer the firm waits before procuring the product, the more information it has on pre-season orders, and, therefore, its demand risk decreases. Thus, the firm faces a trade-off: order earlier to reduce supply risk or order later to reduce demand risk. A primary purpose of this research is to investigate this trade-off.

In this chapter, we characterize the optimal procurement timing-and-quantity decision with and without pre-season orders. We also determine how supply attributes (e.g., procurement cost, delay probability, mean delay and delay variability) and market attributes (e.g., pre-season and in-season order volatilities, order correlations and tardiness cost) influence the firm's optimal procurement time and optimal expected cost. This leads to a number of interesting findings.

We prove that the firm can determine its optimal procurement time before receiving any pre-season orders but that its optimal procurement quantity will depend on pre-season order realizations. One might expect that a firm that allows pre-season orders would procure closer to the selling season to take advantage of increasing demand-forecast accuracy. In fact, we prove that this is not necessarily true. There are situations in which, all else being equal, a firm that allows pre-season orders will place its order earlier than a firm allowing only in-season

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orders. We prove that this can only occur if the leadtime is uncertain. We show that a firm that allows pre-season orders is less sensitive to lead-time variability than is a firm that allows for in-season orders only. One might also predict that the firm's expected cost should decrease as the probability of delay decreases. Such intuition is only partially correct. It is true that a firm prefers a perfectly reliable leadtime, i.e., a zero probability of delay, to an unreliable leadtime. However, it is not necessarily true that the firm prefers a leadtime with a lower delay probability to a leadtime with higher delay probability. We characterize when a marginal decrease in delay probability benefits the firm and show that whether a firm benefits or not is largely driven by the variability of the delay distribution.

The rest of this chapter is organized as follows. We discuss the relevant literature in §2.2. Section 2.3 describes the model. In §2.4 we characterize the optimal solution for the model with no pre-season orders. We consider the model with pre-season orders in §2.5. In §2.6 we present a comprehensive numeric study. Concluding remarks are presented in §2.8. Proofs are contained in the appendix.

2.2 Literature

This chapter connects two important streams of literature in supply chain management, namely, the uncertain-supply and dynamic-forecasting streams. We first review the relevant literature on uncertain supply and then turn our attention to the dynamic-forecasting literature. In the end, we briefly discuss the literature on advanced demand information.

Generally speaking, the uncertain-supply literature can be divided into three different but related categories; supply-disruption models, random-yield models, and stochastic lead-time

models. The disruption literature typically models a supplier as alternating between up and down phases, e.g., Meyer et al. (1979), Parlar and Berkin (1991), Gupta (1996), Song and Zipkin (1996), Arreola-Risa and DeCroix (1998) and Tomlin (2006). Orders placed when the supplier is up are received on time and in full. No order can be placed when the supplier is down. The random yield literature considers settings in which the quantity received varies in a random fashion from the quantity ordered. We refer the reader to Yano and Lee (1995) for a review of the random-yield literature. We note that the distinction between disruptions and random yield is less sharp from a manager's perspective, in that stochastically-proportional Bernoulli random-yield models are appropriate models for supply failures, e.g., Anupindi and Akella (1993), Swaminathan and Shanthikumar (1999), Tomlin (2005a,b), and Tomlin and Wang (2005).

This chapter is in the stochastic lead-time category. Stochastic leadtimes have typically been studied in a recurring-demand setting rather than in a newsvendor setting. For the recurring-demand setting, Bagchi et al. (1986) and Song (1994), for example, investigate the impact of leadtime variability in a single-item inventory model. They focus on quantifying the impact of leadtime uncertainty on metrics such as the risk of stock out and the optimal cost. Chu et al. (1994), Mauroy and Wardi (1995), Fujiwara and Sedarage (1997) and Proth et al. (1997) investigate stochastic leadtimes in a multi-item, recurring-demand context. We refer readers to Zipkin (2000) for a comprehensive discussion of stochastic leadtimes in the context of recurring demand.

A question arises in the newsvendor setting that does not arise in the recurring-demand setting; when should the order be placed? This optimal timing decision has been studied in the context of a newsvendor that assembles multiple components into a finished product, e.g.,

Yano (1987), Kumar (1989), Hopp and Spearman (1993), Shore (1995), and Song et al. (2000). We note that, except for Song et al. (2000), these papers assume deterministic demand. This chapter is closely related to Song et al. (2000); the primary difference is that we allow for demand updating (based on pre-season orders) in the context of a single-component product, whereas, they consider the multi-component problem without pre-season orders or demand updating. Our focus on a single-component product allows us to characterize the optimal timing decision with and without pre-season orders. We also introduce the notion of the delay probability, which refers to the probability of a stochastic delay occurring. This gives us a more general model of leadtime uncertainty.

We now turn our attention to dynamic forecast (demand) updating. There is a vast literature on inventory management with forecast updating, but to the best of our knowledge, this research is the first to consider demand updating in an uncertain-supply setting.

A paper that is particularly relevant to our setting is Fisher and Raman (1996), which investigates the value of early-order information. In their model, customer orders are placed in two discrete periods: an early-season period and a primary-season period. Orders not filled by the second period are subject to a shortage cost. Total demand (early- and primary-season orders) is correlated with early-season orders. Our model also allows for two phases of orders. As we will discuss in the model section (§2.3), the demand model of Fisher and Raman (1996) can be viewed as a special case of our demand model. We note that the intent of our work differs from that of Fisher and Raman (1996). We focus on the procurement timing-and-quantity problem under lead-time uncertainty, whereas they focus on the benefit of a second procurement opportunity in a deterministic-supply setting.

The forecast-updating literature can be broadly classified into three categories. The first

category is the Markovian forecast revision approach, as developed by Hausman (1969), Heath and Jackson (1994), and Graves et al. (1998), and recently followed by Güllü (1996), Toktay and Wein (2001), Chod et al. (2004), Chod and Rudi (2005), and Milner and Kouvelis (2005). The second category is the statistical and time series approach, which includes Miller (1986), Lovejoy (1990), Aviv (2003), Kim and Ryan (2003), and references discussed therein. The last category takes a Bayesian approach, which was developed by Scarf (1959) and Iglehart (1964), and followed by Azoury and Miller (1984), Azoury (1985), Fisher and Raman (1996), Eppen and Iyer (1997), and Milner and Kouvelis (2005). This research uses an auto-correlated, continuous time-series approach, where the forecast of total demand (pre-season and in-season orders) is updated based on the evolution of pre-season orders.

Kaminsky and Swaminathan (2004) presents a novel forecast-updating approach by constructing a forecast upper and lower band which narrows as time passes. A number of game-theoretic papers, Gurnani and Tang (1999), Ferguson (2003), and Ferguson et al. (2005), investigate forecast updating in a manufacturer-supplier setting. Their forecast-updating models do not fit naturally into the above categorization. In essence, they assume a second ordering opportunity at which time some of the initial forecast uncertainty has been resolved.

Our work is somewhat related to the literature on advanced demand information. One stream of that literature focuses on the coordination of operations and marketing decisions, i.e., the optimal ordering quantity and level of price discount, respectively. That stream of literature includes Weng (1995), Weng and Parlar (1999), and Tang et al. (2004). Another stream focuses on the operational decision. Hariharan and Zipkin (1995) and Gallego and Özer (2001) focus on the optimality of base stock or (s,S) policies when customer orders do not require instantaneous fulfilment. Therefore, the firm, effectively, has advanced demand

information. Using a queuing model, Lu et al. (2003) investigates an assembly system in which customer orders do not require instantaneous fulfilment.

In closing, we note that while there is an extensive literature on stochastic leadtimes and on demand updating, to the best of our knowledge no existing paper links these two literatures. A key contribution of this research is to provide one such link.

2.3 Model

We consider a firm that sells a seasonal product. Both supply and demand for the product is uncertain. We adopt the convention that 0 is the starting time, the selling season occurs at a known time T, and that $t \geq 0$ measures the distance in time starting from 0. Therefore, $T - t$ represents the remaining time until the selling season. The problem facing the firm is when to procure and what quantity to procure. Before formulating the firm's problem, we describe the demand model, the supply model and the relevant costs.

2.3.1 Demand

Although the selling season occurs at time T, the firm may allow customers to place pre-season orders between 0 and T. Total demand is then the sum of pre-season orders and in-season orders, where in-season orders are those placed at time T.

In modeling the evolution of customer orders, there is the question as to whether the number of orders at a given point in time should depend on past orders. There is ample evidence in the marketing and operations literatures, e.g. Bass (1969), Fisher and Raman (1996) and

Kurawarwala and Matsuo (1996), that customer demand at a given point in time does depend on past orders. In particular, demand often depends on the cumulative number of customers who have ordered the product up to that point of time.

We first discuss pre-season orders and then discuss in-season orders. Let $x_0 > 0$ denote the number of pre-season orders at time 0. We assume that pre-season orders occur continuously between 0 and T. Let $X_{t'}$ denote the cumulative number of pre-season orders placed up to but not including the instant t. Let Z_t denote the instantaneous rate of orders placed at time t. We assume that, $\frac{Z_t}{X_{t'}} = \Delta_t - 1$, or equivalently, $\frac{X_t}{X_{t'}} = \Delta_t$, where $\ln \Delta_t$ is a continuous-time autoregressive time series of order 1, i.e, $\Delta_t = \Delta_{t'}^{\phi} \xi_t^{1-\phi}$, where $0 \leq \phi < 1$ is the autocorrelation parameter and the ξ_t are iid lognormally distributed with parameter μ and σ. We note that (X_t, Δ_t) is a continuous-time Markov processes. The total quantity of pre-season orders just before the season is X_T. In-season orders occur at time T. Let Z_I denote the quantity of in-season orders. Let $X_D = X_T + Z_I$ denote the total demand, i.e., the sum of pre-season and in-season orders. It is reasonable to assume that the quantity of in-season orders will depend on the total number of pre-season orders. In other words, in-season demand will depend on how "hot" the product is. In particular, we assume that, $\frac{Z_I}{X_T} = \Delta_D - 1$, or equivalently, $\frac{X_D}{X_T} = \Delta_D$, where $\Delta_D = \Delta_T^{\phi_D} \xi_D^{1-\phi_D}$, $0 \leq \phi_D < 1$ is the between-season correlation parameter, and ξ_D is log-normally distributed with parameters μ_D and σ_D. We note that setting $\phi = 0$ results in uncorrelated pre-season orders. Similarly, setting $\phi_D = 0$ results in the in-season orders being uncorrelated with pre-season orders.

We note that a two-period discrete-time analog of our model (see Appendix A1.1) would recover the demand model of Fisher and Raman (1996), with the exception that their demand is assumed to be bi-variate normal, whereas ours would be bi-variate lognormal. Fisher and

Raman (1996) note on page 95, however, that "a distribution with fatter tails [than a normal distribution] ... might be more appropriate". We note that the lognormal distribution is such a fat-tailed distribution (Sigman, 1999).

Our formulation implicitly allows customers to cancel previously placed orders. (We note that Fisher and Raman (1996) also implicitly allow this.) However, if $\mu > 1$, then, in expectation, the cumulative number of pre-season orders grows over time. In fact, if $\mu > 1$, then the growth accelerates. However, depending on the relative size of μ_D to μ, the growth of total demand may slow at time T (reminiscent of the S curve in demand diffusion models) or further accelerate.

We use lower case letters, x_t and δ_t, to denote the realizations, at the time t, of the cumulative pre-season order and pre-season order ratio random variables X_t and Δ_t. Let $f_t(\cdot|x_t, \delta_t)$ denote the conditional pdf of X_D, given $X_t = x_t$ and $\Delta_t = \delta_t$ at time t. From the above description of the pre-season and in-season order evolution, we have (see Appendix A1.1)

$$f_t(x|x_t, \delta_t) = \frac{1}{\sqrt{2\pi\psi_\sigma(t)}x} \exp\left(\frac{-\left(\ln x - \psi_\mu(t) - \psi_\delta(t) - \ln x_t\right)^2}{2\psi_\sigma(t)}\right), \qquad (2.1)$$

where

$$\psi_\sigma(t) = \left((T-t) - \left(1 - \phi^{T-t}\right)\left(\frac{\phi}{1-\phi} - \phi_D\right)\left(2 - (\phi - (1-\phi)\phi_D)\frac{1+\phi^{T-t}}{1+\phi}\right)\right)\sigma^2 + (1-\phi_D)^2\sigma_D^2,$$

$$\psi_\mu(t) = \left((T-t) - \left(1 - \phi^{T-t}\right)\left(\frac{\phi}{1-\phi} - \phi_D\right)\right)\mu + (1-\phi_D)\mu_D,$$

$$\psi_\delta(t) = \left(\phi_D\phi^{T-t} + \frac{\phi}{1-\phi}\left(1 - \phi^{T-t}\right)\right)\ln\delta_t.$$

In particular, substituting $t = 0$, $X_0 = x_0$, and $\Delta_0 = \delta_0$ into (2.1), we have

$$f_0(x|x_0, \delta_0) = \frac{1}{\sqrt{2\pi\psi_\sigma(0)}x} \exp\left(\frac{-(\ln x - \psi_\mu(0) - \psi_\delta(0) - \ln x_0)^2}{2\psi_\sigma(0)}\right). \qquad (2.2)$$

Equation (2.2) describes, at time 0, the pdf of the firm's total future demand, i.e., the sum of pre-season and in-season orders.

2.3.2 Supply

We now describe the supply system. With probability $0 \leq \theta \leq 1$ the leadtime is $L \geq 0$. With probability $1 - \theta$ the leadtime is $L + \omega$, where ω represents a stochastic, nonnegative delay. For example, a shipment might get selected for inspection by customs with a probability θ and the time spent in inspection might be uncertain. Hereafter we refer to L as the standard leadtime, $1 - \theta$ as the delay probability, and ω as the delay.

Let $G(\cdot)$ be the distribution function for the delay and $g(\cdot)$ be the density. We assume that $G(\cdot)$ is continuous and has a real, non-negative domain. Define $\hat{\omega} = \sup\{\omega : G(\omega) = 1\}$. Note $\hat{\omega} = \infty$ if the domain of $G(\cdot)$ covers $\mathcal{R}^+$. We assume that $E[\omega]$ is positive and bounded. Our model collapses to a constant leadtime case when $\theta = 1$.

If the firm orders at time t, the planned leadtime is defined as $z = T - L - t$. See Figure 2.1. Let $z^+ = \max\{z, 0\}$ and $z^- = \max\{-z, 0\}$. Define

$$A(z) = \theta z^+ + (1 - \theta) \int_0^{z^+} (z^+ - \omega)g(\omega)d\omega, \qquad (2.3)$$

$$B(z) = z^- + (1 - \theta) \int_{z^+}^{\infty} (\omega - z^+)g(\omega)d\omega, \qquad (2.4)$$

where $A(z)$ and $B(z)$ are expected earliness and tardiness respectively. In other words, $A(z)$ is the expected duration that the order spends in inventory before the season and $B(z)$ is the expected duration by which the order is late.
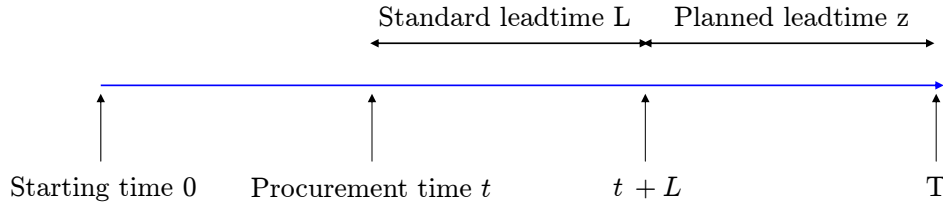
Figure 2.1: Procurement time, leadtime, and planned leadtime illustration.

2.3.3 Costs and Problem Formulation

Let c, h, s, r, and p represent the unit purchasing cost, holding cost, salvage value, revenue, and tardiness penalty respectively. We assume the same tardiness penalty structure as in Song et al. (2000), that is, the incurred tardiness cost is linear in the realized delay. The tardiness penalty might be a tangible cost imposed by customers if the firm sells to corporate customers, e.g., Wal-Mart levies penalties on firms that do not fill its demand on time. Alternatively, the tardiness penalty can be viewed as a proxy to represent the fact that the firm might lose some of its realized demand if its order does not arrive by time 0, i.e., customers might be unwilling to wait or the firm may have to reduce its price to induce customers to wait.

The firm is allowed to procure only once in the horizon. This differs from Fisher and Raman (1996) who assume the existence of a second quick-response supplier. Oftentimes, however, firms that source from emerging economies do not have a contingent domestic supplier that could provide this type of quick response. The firm's problem is to determine the procurement policy that minimizes its expected total cost. A procurement policy specifies, for any pre-season

order evolution, when to procure and the quantity to procure. The order evolution information is fully contained in (X_t, Δ_t) because of the Markov property. If the firm procures at time t, then the expected cost, conditional on the realized pre-order level and ratio being (x_t, δ_t), is given by

$$
\begin{aligned}
v(t, x_t, \delta_t) \quad = \quad \min_{y \geq 0} \Big\{ & cy + rE[(X_D - y)^+ | x_t, \delta_t] - sE[(y - X_D)^+ | x_t, \delta_t] \\
& + hA(T - L - t)y + pB(T - L - t)E[X_D | x_t, \delta_t] \Big\},
\end{aligned}
\tag{2.5}
$$

where y is the procurement quantity. Note that (2.5) is a classic newsvendor formulation with two additional time-based terms that reflect the holding and tardiness costs: the expected holding cost is $hA(T - L - t)y$ and the expected tardiness cost is $pB(T - L - t)E[X_D | x_t, \delta_t]$. It is straightforward to show that (2.5) is equivalent to

$$
\begin{aligned}
v(t, x_t, \delta_t) \quad = \quad \min_{y \geq 0} \Big\{ & (hA(T - L - t) - (r - c))y + (r - s)E[(y - X_D)^+ | x_t, \delta_t] \\
& + (r + pB(T - L - t)) E[X_D | x_t, \delta_t] \Big\}.
\end{aligned}
\tag{2.6}
$$

Let $J(x_0, \delta_0) = E[v(t, X_t, \Delta_t) | x_0, \delta_0]$. The firm's problem is to determine the procurement policy that minimizes $J(x_0, \delta_0)$.

We note that our model is somewhat related to Song et al. (2000), which does not allow for pre-season orders (or demand updating) nor a leadtime delay probability (i.e., $\theta = 0$ in their model) but does consider a product with multiple components. Song et al. (2000) investigate a number of heuristic solutions to the timing-and-quantity problem. The special case of our model in which there is no pre-season orders and in which $\theta = 0$ is similar to the single-component case of Song et al. (2000). Note that, as in Song et al. (2000), we ignore discounting so as to

focus on the primary trade off between supply and demand risks.

2.3.4 Optimum Procurement Quantity

We introduce a useful theorem for the optimal procurement quantity for any given procurement time t and x_t.

Theorem 1. *For any given procurement time t and realized cumulative pre-season orders x_t and pre-season order ratio δ_t, the optimal order quantity is given by*

$$y^*(t, x_t, \delta_t) = F_t^{-1} \left(\frac{r - c - hA(T - L - t)}{r - s} \right), \tag{2.7}$$

where $F_t(\cdot | x_t, \delta_t)$ is the distribution function of X_D at time t.

This theorem establishes that, for any given procurement time t, realized cumulative pre-season orders x_t and realized pre-season order ratio δ_t, the optimal procurement quantity is given by a newsvendor-type expression with an appropriately defined distribution function. Note that $y^*(t, x_t, \delta_t)$ in (2.7) reduces to the standard newsvendor solution if $t \geq T - L$, i.e., if the firm does not procure until it is within a standard leadtime of the selling season. Hereafter, for notational convenience we use $y^*(t)$ rather than $y^*(t, x_t, \delta_t)$, i.e., we suppress the dependence on (x_t, δ_t).

We conclude this section by introducing the following assumptions. These are made for expositional clarity and are assumed to hold throughout the rest of the chapter.

A1 Without loss of generality, we scale $x_0 = 1$ and $\delta_0 = 1$.

A2 We restrict attention to $T > L$, i.e., the starting point is at least a standard leadtime before the selling season.

A3 We assume $\hat{\omega} > \bar{z}$ for $h > 0$, where $\bar{z} = A^{-1}((r-c)/h)$ and $A(\cdot)$ is given by (2.3).

We note that we have solved the case in which assumption A2 does not hold, i.e., $T \leq L$. However, this case is quite straightforward (with the optimal solution being to either order immediately or wait until T.) Assumption A2 allows us to focus on the more interesting case, i.e., $T > L$. We note assumption A3 is a very mild one as any distribution with $\mathfrak{R}^+$ support, such as normal, lognormal, exponential, gamma, and Weibull, satisfies this condition. Using Theorem 1, one can establish that $y^*(z) = 0$ if and only if $z \geq \bar{z}$. Thus, $\bar{z}$ represents the maximum planned leadtime above which the firm would not order a positive quantity of the product. If the firm chooses a planned leadtime $z \geq \bar{z}$, then this is equivalent to the firm deciding not to stock/sell the product. In that case, we say the firm does not participate.

2.4 In-season Orders Only

In this section, we consider the special case where the firm does not allow pre-season orders. This is obtained from our general model by setting $\mu = \sigma = 0$, i.e, no pre-season orders, and $\phi_D = 0$. Therefore, using (2.2), and recalling that (x_0, δ_0) is scaled to $(1, 1)$ [see Assumption A3], we obtain

$$f_t(x) = f_0(x) = \frac{1}{\sigma_D \sqrt{2\pi} x} e^{\frac{-(\ln x - \mu_D)^2}{2\sigma_D^2}}, \tag{2.8}$$

which is independent of the time t. Because of this independence, $J(x_0, \delta_0) = v(t, x_0, \delta_0)$. Therefore, the firm can determine its optimal procurement time and quantity at time 0.

For presentational clarity, we work directly with the planned leadtime $z = T - L - t$ in this section (but revert back to the order time t in subsequent sections). Using (2.6), the expected cost if the firm uses a planned leadtime of z, is

$$v(z) = (hA(z) - (r - c))y^*(z) + (r - s)E[(y^*(z) - X_D)^+] + pB(z)E[X_D], \qquad (2.9)$$

where, using Theorem 1, $y^*(z) = F_0^{-1}\left(\frac{r - c - hA(z)}{r - s}\right)$.

We first briefly discuss the case in which the unit holding cost $h = 0$ and then focus on the $h > 0$ case. Recall that $\hat{\omega} = \sup\{\omega : G(\omega) = 1\}$.

Theorem 2. *If $h = 0$, then any planned leadtime $z \geq \hat{\omega}$ is optimal.*

The above theorem says that, when there is no holding cost, the firm orders as early as possible. This is intuitive because the firm's uncertainty about X_D is independent of the time t.

We now focus on the $h > 0$ case. The following theorem restricts the possible range for the firm's optimal planned leadtime z^*. Recall that $\bar{z} = A^{-1}((r - c)/h)$.

Theorem 3. *The optimal planned leadtime satisfies $0 \leq z^* \leq \bar{z}$.*

We can therefore restrict attention to a nonnegative planned leadtime $z \geq 0$. The following theorem proves an important timing-and-level separation result: one can determine the optimal order time regardless of the expected level of the demand.

Theorem 4. *The optimal procurement time t^* is independent of the expected demand $E[X_D]$.*

Even equipped with Theorems 3 and 4, characterizing the optimal procurement time t^*

is not straight forward because (2.9) is in general not unimodal in the planned leadtime z. For certain classes of the delay distribution function $G(\cdot)$, however, we can exploit structural properties of (2.9) and characterize the optimal time t. We assume that the delay distribution satisfies the following condition throughout the rest of this section.

A4 The density of delay distribution is non-increasing, i.e., $G''(\cdot) \leq 0$,

Note that (A4) is a reasonably mild assumption and is satisfied by a number of distributions, including the uniform, exponential, and certain classes of Weibull and Gamma distributions. In addition, our numeric studies indicate that our later results in the section hold true when (A4) is not satisfied.

The following theorem characterizes the optimal planned leadtime.

Theorem 5. *(i) If $\theta = 1$, then $z^* = 0$. In other words, if the supply is perfectly reliable, then the optimal planned leadtime is zero. (ii) For $\theta < 1$, define*

$$H(z) = y^*(z)\frac{\theta + (1-\theta)G(z)}{(1-\theta)(1-G(z))}.$$

(a) If $\sup(H(z)) < \frac{p}{h}E[X_D]$, then the optimal planned leadtime $z^* = \bar{z}$, i.e., the firm does not participate. (b) If $H(0) > \frac{p}{h}E[X_D]$, then $z^* = 0$. (c) Otherwise, define $\mathcal{Z} = \{z : v'(z) = 0\}$, then $\mathcal{Z} = \{z_1, z_2\}$, $0 \leq z_1 \leq z_2 < \bar{z}$, and $z^* = z_1$. In other words, the objective function $v(z)$ has exactly two stationary points, and the optimal planned leadtime is the smaller of the two.

Given assumption (A4), Theorem 5, combined with Theorem 1, completely characterizes the firm's optimal timing and quantity decisions. Furthermore, it establishes the importance of the ratio of the tardiness penalty cost to the holding cost. Part (a) of Theorem 5 says

that if this ratio is very high, then the firm uses a planned leadtime of $z^* = \bar{z}$, i.e., it does not participate. Part (b) says that if this ratio is very low, then the firm uses a zero planned leadtime, that is, the firm orders a standard leadtime L in advance of the selling season. For intermediate ratios (part (c)), the firm uses a planned leadtime between these two extremes.

We can use Theorem 5 to investigate how various supply and demand characteristics affect the firm's optimal planned leadtime z^*. For example, it follows directly from Theorems 1 and 5 that a firm chooses a planned leadtime of zero, i.e., $z^* = 0$, if

$$F_0^{-1}\left(\frac{r-c}{r-s}\right) > \left(\frac{1-\theta}{\theta}\right)\left(\frac{p}{h}\right) E[X_D].$$

This condition implies that, as one would expect, a firm is more likely to set a zero planned leadtime if the unit holding cost h is large, the unit tardiness penalty cost p is small, and the delay probability $1 - \theta$ is low.

In the following analysis, we first focus on the sensitivity of the firm's timing decision and then discuss the firm's optimal expected cost.

Theorem 6. *Consider case (c) of Theorem 5, i.e., the interior case where the firm's optimal procurement time is not dominated by either the tardiness penalty cost or the holding cost, and so $0 < z^* < \bar{z}$. (a) $\frac{\partial z^*}{\partial p} \geq 0$. (b) $\frac{\partial z^*}{\partial c} \geq 0$. (c) $\frac{\partial z^*}{\partial s} \leq 0$. (d) $\frac{\partial z^*}{\partial r} \leq 0$. (e) $\frac{\partial z^*}{\partial L} = 0$.*

The above theorem tells us that the firm procures earlier as the tardiness penalty cost p or the purchasing cost c increases. On the other hand, the firm procures later, i.e., closer to the selling season, as the salvage value s or the unit revenue r increases. Because z^* is independent of L, the standard leadtime L has no impact on the optimal planned leadtime. We will show in the next section that the standard leadtime can affect the planned leadtime when pre-season

orders are allowed.

Intuitively, one might expect that as the holding cost h increases, the firm would order closer to the season so as to reduce the expected duration over which inventory is held. While we are unable to establish this analytically, our numeric studies indicate that this intuition is indeed correct.

We now turn our attention to the firm's optimal expected cost. The following theorem proves that the optimal expected cost is increasing in the unit cost c, in the holding cost h, and in the penalty cost p; is decreasing in the unit revenue r and in the salvage value s; and is independent of the standard leadtime L. These directional results are quite intuitive.

Theorem 7. (a) $\frac{\partial v(z^*)}{\partial c} \geq 0$. (b) $\frac{\partial v(z^*)}{\partial h} \geq 0$. (c) $\frac{\partial v(z^*)}{\partial p} \geq 0$. (d) $\frac{\partial v(z^*)}{\partial r} \leq 0$. (e) $\frac{\partial v(z^*)}{\partial s} \leq 0$. (f) $\frac{\partial v(z^*)}{\partial L} = 0$.

The directional effect of the delay probability, however, is more nuanced. The following theorem proves that a firm always (at least weakly) prefers a perfectly reliable leadtime, i.e., $\theta = 1$, to an unreliable leadtime, i.e., $\theta < 1$.

Theorem 8. *Everything else being equal, the optimal expected cost for $\theta = 1$ is never higher than the expected cost for $\theta < 1$, i.e., $v(z^*|\theta = 1) \leq v(z'|\theta < 1)$, where z^* and z' represent the optimal planned leadtime with $\theta = 1$ and $\theta < 1$ respectively.*

While the firm always prefers a perfectly-reliable leadtime, a marginal decrease in the delay probability does not always benefit the firm: the firm's optimal expected cost $v(z^*)$ is not necessarily increasing in the delay probability $1 - \theta$. The following theorem proves that the firm's optimal expected cost can, in fact, decrease in the delay probability.

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Theorem 9. *Assume $X_D = 1$ with probability 1. The optimal expected cost $v(z^*)$ is strictly increasing in the delay probability, i.e., $1 - \theta$, if $G(\cdot)$ satisfies*

$$KE[\omega] - (1 - K)G^{-1}[K] \leq \int_0^{G^{-1}[K]} \omega g(\omega)d\omega, \qquad (2.10)$$

where $K = \frac{p}{p+h}$. Otherwise $v(z^*)$ is a convex function of the delay probability, initially decreasing and then increasing in the delay probability.

Note that (2.10) always holds if $G \sim \exp(\cdot)$ but never holds if $G \sim U(0, b)$, unless $h = 0$. For the deterministic demand case, Theorem 9 states that a moderate decrease in the delay probability can increase the firm's overall cost. While surprising, this result can be explained as follows. A decrease in the delay probability reduces the mean leadtime and this benefits the firm. However, a decrease in the delay probability can increase the leadtime variance and this hurts the firm. In fact, one can show that the leadtime variance increases in the delay probability if and only if the delay coefficient of variation (CV) is greater than or equal to $\sqrt{1 - 2\theta}$. Therefore, a decrease in the delay probability benefits the firm if the delay CV is high but may not benefit the firm if the delay CV is low.

We note that there are other possible definitions of leadtime reliability, for example, the delay CV, the leadtime CV, and the probability of the leadtime being less than a certain threshold (PLT). For certain problem instances, the PLT measure largely depends on the delay probability. As such, one can create instances in which the firm's optimal expected cost increases as the PLT lead-time reliability measure improves. The directional effects of the delay CV and the leadtime CV are discussed in §2.6.2.

2.5 Pre-season and In-season Orders

In this section, we consider the general model in which the firm allows both pre-season and in-season orders. Supply uncertainty motivates the firm to procure earlier so as to reduce its expected tardiness penalty. As mentioned in the introduction, firms that allow pre-season orders might wish to delay their procurement order so as to take advantage of increasing accuracy of the forecast for total demand. Recall that x_t and δ_t are the realized cumulative pre-season order and pre-season order ratio at time $t \geq 0$, and σ is the underlying volatility of pre-season orders.

For a given procurement time t, the optimal procurement quantity is given by Theorem 1. Using (2.1), then $y^*(t)$ is given by

$$\frac{\ln y^*(t) - \ln x_t - \psi_\mu(t) - \psi_\delta(t)}{\sqrt{2\psi_\sigma(t)}} = \text{erf}^{-1}\left(2\frac{r - c - hA(T - L - t)}{r - s} - 1\right), \tag{2.11}$$

where $\text{erf}(\cdot)$ is the standard error function. Using (2.11), the firm's expected cost in (2.6) can be simplified (see Lemma A3) to

$$v(t, x_t, \delta_t) = x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right) \cdot$$
$$\left(r + pB(T - L - t) - \frac{1}{2}(r - s)\left(1 + \text{erf}\left(k(t) - \frac{\sqrt{\psi_\sigma(t)}}{\sqrt{2}}\right)\right)\right). \tag{2.12}$$

where

$$k(t) = \text{erf}^{-1}\left(2\frac{r - c - hA(T - L - t)}{r - s} - 1\right).$$

Using (2.12), we first establish an important property of the optimal procurement policy.

Theorem 10. t^* *is independent of the pre-season order evolution* (X_t, Δ_t).

We have now characterized the structure of the optimal procurement policy: (i) the optimal time is independent[1] of the pre-season order evolution (X_t, Δ_t), and (ii) the optimal quantity depends on the pre-season order evolution and is given by (2.7). This result has an important managerial implication: the firm can determine its optimal procurement time in advance but it must wait until that time to determine its procurement quantity. The fact that the firm can determine its procurement time in advance is very beneficial from a planning perspective.

Using Theorem 10, we can then state the firm's problem as $\min_t J(x_0, \delta_0)$ where, as before, $J(x_0, \delta_0) = E[v(t, X_t, \Delta_t)|x_0, \delta_0]$. We denote the optimal expected cost as $J^*(x_0, \delta_0)$. Before characterizing the optimal procurement time, we first explore how the firms' optimal cost is influenced by order volatility and by the mix of pre-season to in-season orders.

Theorem 11. *(a) $J^*(x_0, \delta_0)$ is non-decreasing in the pre-season order volatility σ. (b) $J^*(x_0, \delta_0)$ is non-decreasing in the in-season order volatility σ_D.*

The above theorem states that the firm's optimal expected cost increases in the pre-season order volatility σ as well as the in-season order volatility σ_D, because a higher order volatility reflects a higher degree of demand uncertainty.

The following theorem characterizes how pre-season orders affect the firm's optimal expected cost.

Theorem 12. *Let μ_N and σ_N denote the parameters for the initial forecast, i.e., at time 0, of the total demand. If $\frac{\sigma}{\mu} = \frac{\sigma_D}{\mu_D}$, then define the demand-mix parameter λ, as the unique solution*

---

[1]This independence result relies on the fact that the firm procures the optimal quantity $y^*(t)$ as given by (2.7). We note that $y^*(t)$ does depend on the pre-season order realization (x_t, δ_t).

to

$$\left(T - (1 - \phi^T) \left(\frac{\phi}{1 - \phi} - \phi_D \right) \right) \mu = \lambda \mu_N,$$

$$\left(T - (1 - \phi^T) \left(\frac{\phi}{1 - \phi} - \phi_D \right) \right) \left(2 - (\phi - (1 - \phi)\phi_D) \frac{1 + \phi^T}{1 + \phi} \right) \sigma^2 = \lambda \sigma_N^2,$$

$$(1 - \phi_D)\mu_D = (1 - \lambda)\mu_N, \quad (1 - \phi_D)^2 \sigma_D^2 = (1 - \lambda)\sigma_N^2. \tag{2.13}$$

Then, $v^*(t, x_t, \delta_t)$ is non-increasing in λ.

We first note that λ reflects the relative magnitude of pre-season to in-season orders. At $\lambda = 0$, there are no pre-season orders (which is studied as a special case in §2.4); at $\lambda = 1$, there are no in-season orders; and at $0 < \lambda < 1$, there are both pre-season and in-season orders. The above theorem tells us that the firm's optimal expected cost decreases as the fraction of pre-season orders increases. The reason is that pre-season orders allow the firm to more accurately estimate the total season demand. As time progresses, a firm without pre-season orders, i.e., $\lambda = 0$, retains the same demand forecast. In contrast, the forecast uncertainty reduces over time for a firm with pre-season orders. (In the case of $\lambda = 1$, there is no demand uncertainty by time T.)

Theorem 13. *Define*

$$M(t) = \frac{2p}{r - s} B(T - L - t) - \mathrm{erf}\left(k(t) - \sqrt{\frac{1}{2}\psi_\sigma(t)} \right). \tag{2.14}$$

The optimal procurement time $t^* = \arg\min M(t)$.

The above theorem establishes that the optimal procurement time can be found simply by searching over the $M(t)$ function for the minimum point. We note that $M(t)$ is in gen-

eral not well behaved and can assume very complex shapes. The following theorem partially characterizes the optimal procurement time.

Theorem 14. *If $\phi = \phi_D = 0$, then (i) The firm's optimal procurement time $t^* < T - L$ if $B(0) - B(T-L) > \frac{r-s}{p}$, i.e., the firm uses a strictly positive planned leadtime. (ii) For $x > 0$, define*

$$\Lambda(x) = \Phi\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right),$$

where Φ denote the standard normal cdf. If (a) $(T-L)\sigma^2 + \sigma_D^2 \leq 1$ and $\frac{r-c}{r-s} \leq \Lambda\left((T-L)\sigma^2 + \sigma_D^2\right)$, or (b) $(T-L)\sigma^2 + \sigma_D^2 > 1$, $\sigma_D^2 \leq 1$ and $\frac{r-c}{r-s} \leq \Lambda(1)$, or (c) $\sigma_D^2 > 1$ and $\frac{r-c}{r-s} \leq \Lambda\left(\sigma_D^2\right)$, then $t^* > T - L \Rightarrow t^* = T$. In other words, if it is not optimal to procure by time $T - L$, then it is optimal for the firm not to procure until time T.

Corollary 1. *(a) If $\phi = \phi_D = 0$ and $\frac{r-c}{r-s} < \Phi(2)$, then $t^* > T - L \Rightarrow t^* = T$. (b) It can be optimal for the firm to use a negative planned leadtime.*

Part (i) of the above theorem states that, as one would expect, the firm procures early in the planning horizon if the tardiness cost p is large relative to the marginal revenue $r - s$. Part (ii) of the above theorem and Corollary 1 establish that, under very mild conditions, the firm does not procure between $T - L$ and T. If procurement at $T - L$ is better than procuring earlier, then the firm either procures at $T - L$, i.e., uses a planned leadtime of 0, or procures at T, i.e., uses a planned leadtime of $-L$. This is in sharp contrast with the case of no pre-season orders, in which the firm always uses a non-negative planned leadtime. We note that $\frac{r-c}{r-s}$ is the classic newsvendor critical fractile. Therefore, this result holds as long as the salvage s is not extremely high relative to the procurement cost c.

To further characterize the optimal procurement time, we make the following additional

assumptions for Theorem 15, 16, and Corollary 2.

A5 The delay distribution function $G(\cdot)$ is uniform.

A6 $\eta''(t)$ is monotonic in t, where $\eta(t) = \text{erf}\,(k(t) - \rho(t))$ and $\rho(t) = \sqrt{\frac{1}{2}\psi_\sigma(t)}$.

We note that sets of sufficient conditions that guarantee $\eta''(\cdot)$ to be monotonic increasing or monotonic decreasing are given by Lemma A5 in the appendix. We found that the set of sufficient conditions readily hold for reasonable parameter values, and that $\eta''(t)$ is very often monotonic even if the sufficient conditions do not hold. As the unit holding cost h increases, $\eta''(\cdot)$ changes from being monotonically increasing to monotonically decreasing. However, there can exist an intermediate range of h for which $\eta''(\cdot)$ is not monotonic.

The following theorems characterize potential optimal procurement times.

Theorem 15. *If $\eta''(\cdot)$ is monotonically increasing, then the optimal procurement time $t^* \in \{T, \hat{t}\}$, where*

(i) $\eta''(T - L) \leq \frac{2p}{r-s}(1 - \theta)g(0) \Rightarrow \hat{t} \in \{0, T - L\}$.

(ii) *If $\eta''(0) \geq \frac{2p}{r-s}(1 - \theta)g(0)$, then*

(a) $\eta'(0) \leq \frac{2p}{r-s}(1 - \theta)(1 - G(T - L)) \Rightarrow \hat{t} = 0$.

(b) $\eta'(T - L) \geq \frac{2p}{r-s}(1 - \theta) \Rightarrow \hat{t} = T - L$.

(c) *Otherwise, $\hat{t} = t_e$, where t_e is the unique solution to $\eta'(t_e) = \frac{2p}{r-s}(1 - \theta)(1 - G(T - L - t_e))$.*

(iii) *Otherwise,*

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(a) $\eta'(t_s) \geq \frac{2p}{r-s}(1 - \theta)(1 - G(T - L - t_s)) \Rightarrow \hat{t} = T - L.$

(b) $\eta'(0) \leq \frac{2p}{r-s}(1 - \theta) \Rightarrow \hat{t} \in \{0, T - L\}.$

(c) Otherwise, $\hat{t} \in \{t_{e2}, T - L\}$, where t_{e2} is the unique solution to $\eta'(t_{e2}) = \frac{2p}{r-s}(1 - \theta)(1 - G(T - L - t_{e2}))$ in the region $[0, t_s]$, where t_s is the unique solution to $\eta''(t_s) = \frac{2p}{r-s}(1 - \theta)g(T - L - t_s).$

The following theorem considers the alternative case where $\eta''(\cdot)$ is monotonically decreasing.

Theorem 16. *If $\eta''(\cdot)$ is monotonically decreasing, then the optimal procurement time $t^* \in \{T, \hat{t}\}$, where*

(i) $\eta''(T - L) \geq \frac{2p}{r-s}(1 - \theta)g(0) \Rightarrow \hat{t} \in \{0, T - L\}.$

(ii) If $\eta''(0) \leq \frac{2p}{r-s}(1 - \theta)g(T - L)$, then

  (a) $\eta'(0) \leq \frac{2p}{r-s}(1 - \theta)(1 - G(T - L)) \Rightarrow \hat{t} = 0.$

  (b) $\eta'(T - L) \geq \frac{2p}{r-s}(1 - \theta) \Rightarrow \hat{t} = T - L.$

  (c) Otherwise, $\hat{t} = t_e$, where t_e is the unique solution to $\eta'(t_e) = \frac{2p}{r-s}(1 - \theta)(1 - G(T - L - t_e)).$

(iii) Otherwise,

  (a) $\eta'(t_s) \leq \frac{2p}{r-s}(1 - \theta)(1 - G(T - L - t_s)) \Rightarrow \hat{t} = 0.$

  (b) $\eta'(T - L) \geq \frac{2p}{r-s}(1 - \theta) \Rightarrow \hat{t} \in \{0, T - L\}.$

  (c) Otherwise, $\hat{t} \in \{0, t_{e2}\}$, where $t_{e2} > t_s$ is the unique solution to $\eta'(t_{e2}) = \frac{2p}{r-s}(1 - \theta)(1 - G(T - L - t_{e2}))$ in region $[t_s, T - L]$, where t_s is defined in Theorem 15.

Theorem 15 and 16 establish potential candidates for the optimal order time: 0, t_e, t_{e2}, or $T - L$. They also specify the conditions for which each can be optimal. One can therefore solve for the optimal order time very efficiently.

Corollary 2. *Let t^* denote the optimal procurement time. Suppose the firm has not procured by time $t > t^*$.*

(i) *If $\eta''(t)$ is monotonically increasing, then it is optimal to procure immediately at time t if and only if $v(t, x_t) < \max\{E[v(T-L, X_{T-L})|x_t], E[v(T, X_T)|x_t]\}$. Otherwise it is optimal to procure either at time $T - L$ or time T.*

(ii) *If $\eta''(t)$ is monotonically decreasing, then it is optimal to procure immediately at time t if and only if $v(t, x_t) \leq E[v(T, X_T)|x_t]$. Otherwise it is optimal to procure at time T.*

Corollary 2 establishes that, depending on the system parameter values, knowing the fact that the firm should have made the procurement decision at time t^* does not necessarily imply the firm should procure immediately at time $t > t^*$.

Intuitively, one might expect that given the same starting demand forecast, the optimal procurement time with pre-season orders would be later than that without pre-season orders, because pre-season orders enable the firm to reduce its demand risk by delaying its procurement. This intuition, however, is not true in general. When the unit purchasing cost is low, a firm with pre-season orders may in fact procure earlier than a firm without pre-season orders.

Theorem 17. *For identical initial forecasts of total demand, let t_P^* and t_N^* denote the optimal order time with and without pre-season orders, respectively. (a) If supply is perfectly reliable, i.e., $\theta = 1$, then $t_P^* \geq t_N^*$. (b) If there is no holding cost, i.e., $h = 0$, then $t_P^* \geq t_N^*$. (c) If supply is unreliable, i.e., $\theta < 1$, and $h > 0$, then t_P^* can be smaller than t_N^*.*

In the case of deterministic leadtime, therefore, the intuition is correct. A firm that allows pre-season orders will procure later than a firm that does not. However, in a stochastic-leadtime setting, the firm with pre-season orders may procure earlier than the no pre-season order firm. The reason lies in the interplay of the demand and supply risk. Starting with the same total demand forecast, the pre-season ordering firm's demand risk, i.e., demand uncertainty, reduces over time. For certain cost parameters, a lower demand variance results in a lower procurement quantity for a given procurement time. A lower order quantity reduces the earliness component of the supply risk, i.e., the inventory-related cost of an early arrival is decreasing in the order size. Because of this, it can be optimal under certain circumstances for the pre-season ordering firm to procure earlier than the no pre-season order firm.

Although we can use the Implicit Function Theorem to analytically sign the directional effect of certain system parameters on the firm's optimal timing decision, e.g., $\frac{\partial t^*}{\partial p} \leq 0$, the directional effect for most parameters is analytically ambiguous. We investigate the optimal timing sensitivity in the numeric study in §2.6.2.

2.6   Numeric Study

The goal of our numeric study is fourfold: (1) to investigate the directional effects of the demand and supply characteristics on the optimal procurement time and optimal expected cost; (2) to investigate the value of pre-season orders; (3) to investigate the value of forecast updating; and, (4) to explore the performance of simple heuristics (see Appendix A1.2).

2.6.1 Study Design

In our model, the supply system is characterized by the standard leadtime L, the delay probability $1-\theta$, and the delay distribution $G(\cdot)$. We refer to the expectation, variance and coefficient of variation of $G(\cdot)$ as the mean delay, the delay variance, and the delay CV respectively. In this numeric study, we assume that $G(\cdot)$ has a Weibull distribution, with a shape parameter α and scale parameter β. A higher value of α is associated with a lower delay CV.

One goal of our numeric study is to investigate the value of pre-season orders, i.e., to evaluate the optimal expected cost as the mix of pre-season to in-season orders changes. To ensure a fair comparison, we hold the initial forecast of total demand constant as the mix λ changes (see Theorem 12.) The initial total-demand forecast is parameterized by μ_N and σ_N. Recall that the forecast is lognormally distributed. Therefore, the mean and CV of the initial forecast are $e^{\mu_N + 0.5\sigma_N^2}$ and $\sqrt{e^{\sigma_N^2} - 1}$ respectively. We then set the pre-season and in-season parameters (μ, σ) and (μ_D, σ_D) according to (2.13).

We now specify the base case scenario used for our numeric study. We set $T = 6$, which should be interpreted as six months. The revenue, cost and leadtime parameters were set as follows: the unit revenue $r = 3$, the unit cost $c = 1$, the unit salvage value $s = .05$, the unit tardiness penalty cost $p = 0.3$, the unit holding cost $h = 0.02$, the standard leadtime $L = 2$, the delay probability $1 - \theta = 0.25$, the mean delay equal to 1 and the delay variance equal to 1. We set the demand-mix parameter $\lambda = 0.2$ (See Theorem 12). The initial forecast, i.e., at $t = 0$, of the total demand had an expected value of 100 and a CV of 0.8. Given $T = 6$, this equates to $\mu_N = 4.358$ and $\sigma_N^2 = 0.495$ for the no pre-season order case and $\mu = 0.145$, $\sigma^2 = 0.016$, $\mu_D = 3.486$, and $\sigma_D^2 = 0.396$ for the partial pre-season order case. We set the

pre-season autocorrelation parameter $\phi = 0.05$ and the between-season correlation parameter $\phi_D = 0.05$.

2.6.2 Directional Effects

In §2.4 and §2.5, we established a number of theoretical results for the directional effects (on the optimal procurement time and expected cost) of many of the system parameters. In this section, we investigate the robustness[2] of these earlier results and also explore the directional effects of some parameters, e.g., the delay CV, not considered earlier. Given the Weibull assumption in our numeric study, the delay distribution is completely characterized by the mean delay and the delay CV.

To explore the directional effect of a given parameter, we solved a set of problem instances in which the parameter of interest was changed but all other parameters remained constant at the base-case values specified earlier. We then repeated this exercise by setting $r = 1.5$ or 20, $h = 0.01$ or 0.04, and $\lambda = 0$ or 0.8 for eight different combinations. This gave additional eight sets of parameter values in addition to the base case. The numeric observations were consistent with all of the analytical results established earlier (see Theorems 6, 7, 11, and 12), indicating that our theoretical results are indeed robust. Table 2.1 summarizes the directional effects of the various model parameters on the optimal procurement time and associated optimal expected cost.

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[2]When analytically characterizing the optimal procurement time, we, at times, assumed that the delay distribution satisfied $G''(\cdot) \leq 0$ (for both the pre-season order and no pre-season order cases) and that $\eta''(\cdot)$ was monotonic (for certain results in the pre-season order case). Both of these assumptions are relaxed in our numeric study.

	Optimal order time t^*		Optimal expected cost	
Parameters	No PSO	With PSO	No PSO	With PSO
Unit cost c	D	I or D	I	I
Holding cost h	I	I	I	I
Penalty cost p	D	D	I	I
Salvage value s	I	I or D	D	D
Unit revenue r	I	I	D	D
Standard leadtime L	D	D	C	I
Delay probability $1 - \theta$	I	I	I or D	I or D
Delay mean	D	D	I	I
Delay CV	I or D	I or D	I	I
Autocorrelation ϕ	C	I or D	C	D
Correlation ϕ_D	C	I or D	C	I
Demand CV	I or D	I or D	I	I

Table 2.1: Sensitivity of the optimal order time t^* and expected cost (I = increasing, D = decreasing, C = constant, PSO = pre-season order).

Optimal Procurement Time. Our analytical results regarding the optimal procurement time did not characterize the effect of the mean delay, the delay CV, the leadtime CV, the initial variance of demand forecast, the pre-season order volatility, or the correlation parameters ϕ and ϕ_D. We now discuss these effects.

- Mean delay: As one would expect, the optimal procurement time is decreasing in the

mean delay, i.e., the firm orders earlier as the mean delay increases.

- Delay CV and leadtime CV: We first note that one can prove that an increase in the delay CV increases the leadtime CV. We therefore focus our discussion on the delay CV, with the understanding that the same observations hold for the leadtime CV. The effect of the delay CV is not straightforward. We observed the optimal procurement time to be increasing or decreasing in the delay CV, with it typically being decreasing for a low CV and increasing for a high CV. At a high delay CV, a marginal increase in the delay CV serves to increase the already-high probability of the delay being small, and this allows the firm to further postpone procurement, i.e., the optimal procurement time increases. At a low delay CV, the probability of a small delay is low and the probability of a moderate or large delay is high. A marginal increase in the CV, while increasing the probability of a small delay, also increases the probability of a large delay, and this large-delay effect appears to dominate, with the result that the firm orders earlier.

- Initial variance of demand forecast: We observed the optimal procurement time to be increasing or decreasing in the initial forecast variance. For reasonable cost and revenue parameters, the critical fractile for the order quantity is to the right of the demand distribution's mode, and so an increase in the demand variance results in a larger procurement quantity (for the same procurement time.) A larger quantity increases the expected earliness cost of inventory, with the result that an increase in the demand variance causes the firm to delay its order, i.e., the order time increases. If the product's per-unit revenue is low relative to the purchase cost, then the critical fractile for the procurement quantity can be to the left of the demand's mode and the reverse effect occurs.

- Pre-season order volatility: We observed the optimal procurement time for a firm that

allows pre-season orders to be, typically, increasing in the pre-season order volatility. That is, the firm procures closer to the selling season as the pre-season order volatility increases. While this is intuitive, the result did not always hold, and we observed cases where the order time decreased in the volatility. We believe that this can occur because the shape of the distribution is also affected by the pre-season order volatility, and this shape effect can also play a role through the order-quantity fractile.

- Pre-season correlation parameter ϕ. We observed the optimal procurement time to be increasing in ϕ for very large values of ϕ. In other words, when pre-season orders are highly correlated, the firm orders closer to the season. As ϕ becomes smaller, i.e., pre-season orders are less and less correlated, the firm orders earlier.

- Between-season correlation parameter ϕ_D: As ϕ_D decreases, the optimal procurement time typically increased. The reason is as follows. As in-season orders become less correlated with pre-season orders, the total demand becomes less predictable. The firm must wait closer to the selling season if it wants a more accurate estimation of the total season demand.

Optimal Expected Cost. With regard to optimal expected cost, we focus our discussion on six parameters of particular interest, the leadtime delay probability $1 - \theta$, the leadtime CV, the delay CV, the standard leadtime L, and the correlation parameters ϕ and ϕ_D.

- leadtime delay probability: In §2.4, we established that, while a firm prefers a perfectly reliable leadtime to an unreliable leadtime, the firm does not necessarily benefit from a marginal decrease in the delay probability. In fact, the firm's optimal expected cost can decrease in the leadtime delay probability. We provided a characterization for when this

can occur in the case of deterministic demand. Our numeric investigation revealed that even in the case of stochastic demand and/or pre-season orders, the firm's optimal expected cost can decrease in the leadtime delay probability. Furthermore, the investigation showed that our earlier observation regarding the delay distribution remains valid, that is, a decrease in the delay probability benefits the firm if the delay CV is greater than or equal to 1 but can hurt the firm if the delay CV is less than 1. As we discussed previously, this effect is linked to the overall leadtime variability. At a high delay CV, a decrease in the leadtime delay probability decreases the leadtime variability, but at a low delay CV, a decrease in the leadtime delay probability can increase the leadtime variability.

- Delay CV (leadtime CV): In contrast to the effect of the delay probability, the optimal expected cost was increasing in the delay CV, regardless of whether or not the firm utilizes pre-season orders. We note, however, that as the fraction of pre-season orders increases, the optimal expected cost becomes less sensitive to the delay CV. This is further discussed in §2.6.3.

- Standard leadtime: In §2.4, we proved that the optimal expected cost was independent of the standard leadtime L in the no pre-season order case. This is not true in pre-season order case. With pre-season orders, a reduction in the standard leadtime allows the firm to order later, i.e., when its demand forecast is more accurate. The forecast variance is concave increasing in the distance from the selling season. We observed the optimal expected cost to be concave increasing in the standard leadtime. Therefore, when pre-season orders are allowed, leadtime reduction efforts becomes increasingly valuable as the standard leadtime decreases.

- Correlation parameters ϕ and ϕ_D. The optimal expected cost was decreasing in ϕ. The

reason is that the more correlated pre-season orders are, the more information early orders contain about total demand. We observed the magnitude of ϕ_D on the optimal expected cost to be very small in our problem instances.

2.6.3 Value of Pre-season Orders

We now turn our attention to the value of pre-season orders, that is, we investigate the difference in the optimal expected cost as the mix between pre-season and in-season orders changes. In particular, we investigate how the value of pre-season orders is influenced by supply, product, and market characteristics. We explored this question using the following study design. We varied the mix of pre-season orders versus in-season orders, i.e., the demand-mix parameter λ, from 0 to 1 using a step size of 0.5. In addition, we varied the correlation parameters ϕ and ϕ_D from 0 to 0.5 using a step size of 0.25. We varied the mean delay from 1 to 3 using a step size of 1, the delay CV from 0.2 to 1.8 using a step size of 0.4, and the leadtime delay probability $1 - \theta$ from 0.0 to 1.0 using a step size of 0.25 . We varied the product's unit profit margin by varying the unit purchasing cost c from 0.5 to 2.5 using a step size of 0.5. We varied the demand CV from 0.4 to 2 using a step size of 0.4. We conducted a full factorial study with 16,875 instances.

On average, the cost with only pre-season orders, i.e., $\lambda = 1$, and partial pre-season orders, i.e., $\lambda = 0.5$, was 9.90% and 3.09% lower than the cost with no pre-season orders, respectively. The maximum cost reduction was 39.97% and 16.72% respectively. A detailed review of the data revealed the following observations. As one might expect, the value of pre-season orders (i.e., the relative cost reduction) was increasing in the pre-season order volatility. The value was also increasing in the delay CV, but decreasing in the leadtime delay probability and the mean

delay. As the leadtime delay probability decreases, the firm can better afford to order later and this increases the value of pre-season orders. In contrast, as the mean delay increases, the firm needs to order earlier and this reduces the value. The value of pre-season orders increases in the delay CV primarily because, while the cost without pre-season orders increases in the delay CV, the cost with pre-season orders is quite insensitive to the delay CV.

To further investigate the value of pre-season orders, we conducted an additional study in which we varied the pre-season to in-season mix parameter λ from 0 to 1, keeping all other parameters at the base case setting. See Figure 2.2.
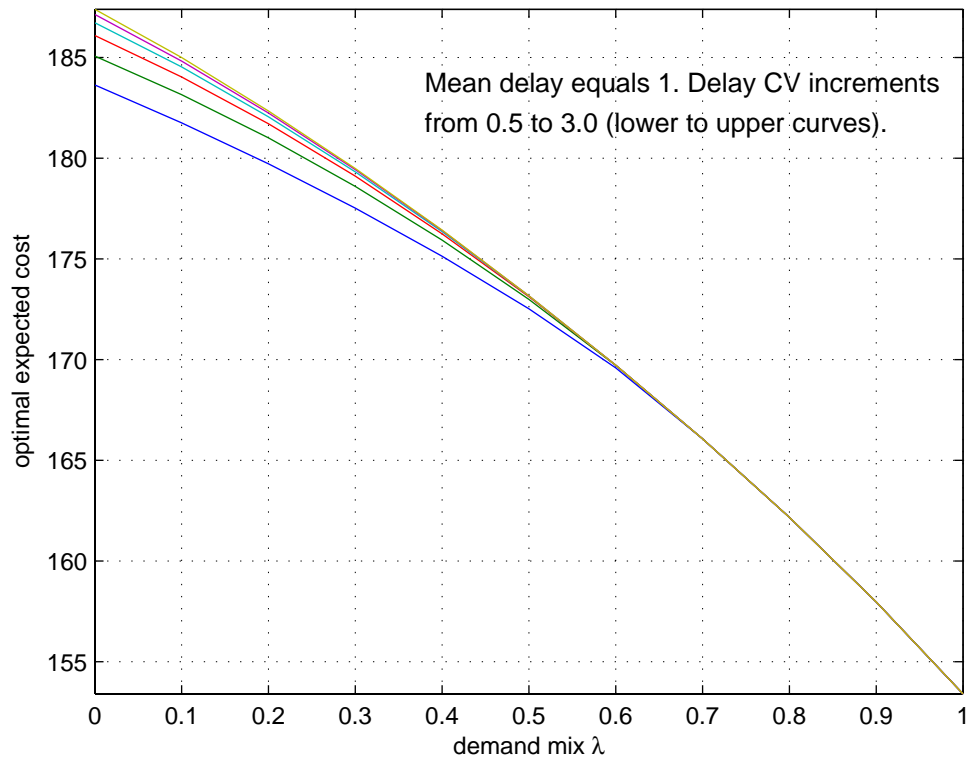
Figure 2.2: Optimal expected cost as a function of the demand-mix parameter λ.

As expected, we observed that as the demand mix shifts towards pre-season orders, the optimal expected cost decreases because the firm has less uncertainty about the final demand. Two further observations are noteworthy. (1) The optimal expected cost decreases more rapidly

as the demand mix becomes more weighted towards pre-season orders. Thus, a marginal increase in the relative fraction of pre-season orders has a larger benefit if the mix is already dominated by pre-season orders. (2) The optimal expected cost becomes less sensitive to the leadtime CV as the demand mix shifts towards pre-season orders. Therefore, a firm with a demand mix dominated by pre-season orders is more robust to variability in the lead-time delay.

2.6.4   Value of Forecast Updating

Pre-season orders enable the firm to update its forecast of the total demand as time progresses. How beneficial is it for the firm to avail of this pre-season order information? To answer this question, we compare the expected cost of two different identical-parameter firms facing identical pre-season and in-season order processes. One firm (U) updates its forecast based on the pre-season order evolution while the other firm (N) does not, i.e., N retains the same total-demand forecast over time. Both the updating firm (U) and the non-updating firm (N) start at time 0 with identical forecasts of total demand, i.e., they have the same initial probability density function for the total demand. Firm U follows the optimal procurement policy (time and quantity) for a firm that updates its forecast over time, whereas firm N follows the optimal procurement policy for a firm that retains a static demand forecast. Firm N will have a higher expected cost than firm U because firm N's policy does not take advantage of the information contained in the pre-order stream. We measure the value of forecast updating as the relative difference in expected cost between N and U.

We conducted a numeric study in which we varied the pre-season to in-season mix parameter λ from 0 to 1 and the correlation parameters ϕ and ϕ_D from 0.1 to 0.9, keeping all other

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parameters at the base case setting as described in §2.6.1. For each instance of λ and (ϕ, ϕ_D), we evaluated the expected cost for firms U and N. Figure 2.3 illustrates the relative difference in expected cost between firms N and U, i.e., the relative cost increase for not updating the demand forecast. The cost of not updating can be very significant. The cost increases as the demand mix shifts towards pre-season orders. This is to be expected. The more pre-season orders dominate total demand, the more information about total demand is contained in the pre-season order stream. Therefore, there is a larger cost to not availing of this information. The cost of not updating also increases in the correlation parameters ϕ and ϕ_D. This is because pre-season orders contain more information about the future as the correlation parameters increase.
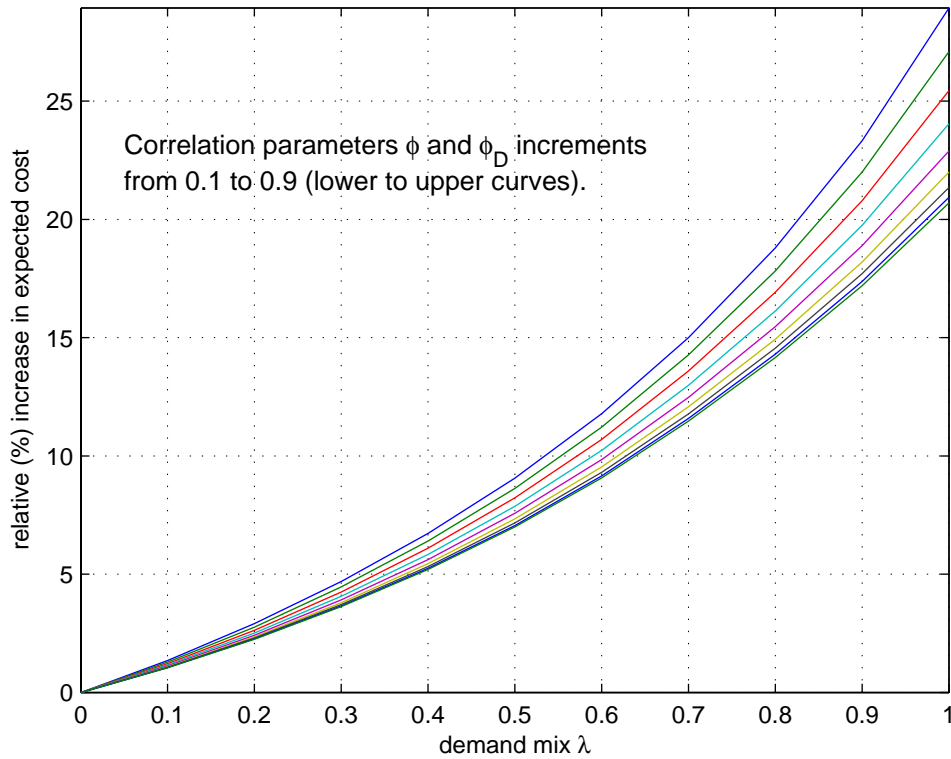
Figure 2.3: Relative cost increase for not updating the demand forecast.

2.7 Extensions - An Alternative Model of Tardiness Penalty Cost

Throughout this chapter, we have adopted the penalty cost function similar to that in Song et al. (2000), where the penalty cost depends on the expected demand. One may argue, however, that the tardiness penalty cost should depend on the minimum of the expected demand and the firm's procurement quantity. After all, why the firm should be held liable for the tardiness penalty cost when it is not the firm's intention to satisfy all the demand? A key argument from this point of view is that the lost sales as represented by the unit revenue already captures the penalty from not satisfying all the demand. While it is debatable whether the firm should bear the tardiness penalty cost for unsatisfied demand, we nevertheless explore whether the alternative model of the tardiness penalty cost will substantially change the insight from this chapter, namely, the separability of the optimal timing decision. In what follows, we prove that the main result of the chapter still holds with the alternative model. Needless to say, the optimal procurement time as well as the optimal procurement quantity will depart from our original model, but these deviations are of secondary concern because they can be computed easily given the main result still holds.

We can rewrite the firm's expected cost function under the alternative tardiness penalty cost model. If the firm procures at time t, then the expected cost, conditional on the realized pre-order level and ratio being (x_t, δ_t), is given by

$$
\begin{aligned}
v(t, x_t, \delta_t) \;=\; \min_{y \geq 0} \Big\{ & cy + rE[(X_D - y)^+ | x_t, \delta_t] - sE[(y - X_D)^+ | x_t, \delta_t] \\
& + hA(T - L - t)y + pB(T - L - t)E[\min\{X_D, y\} | x_t, \delta_t] \Big\}, \quad (2.15)
\end{aligned}
$$

where y is the procurement quantity. Note that the last term of (2.15) is different from that of (2.5). From (2.15), we have

$$v(t, x_t, \delta_t) = \min_{y \geq 0} \left\{ cy + rE[(X_D - y)^+ | x_t, \delta_t] - sE[(y - X_D)^+ | x_t, \delta_t] \right.$$

$$\left. + hA(T - L - t)y + pB(T - L - t)E[X_D - (X_D - y)^+ | x_t, \delta_t] \right\}$$

$$= \min_{y \geq 0} \left\{ cy + (r - pB(T - L - t))E[(X_D - y)^+ | x_t, \delta_t] - sE[(y - X_D)^+ | x_t, \delta_t] \right.$$

$$\left. + hA(T - L - t)y + pB(T - L - t)E[X_D | x_t, \delta_t] \right\}.$$

Simplifying, we have

$$v(t, x_t, \delta_t) = \min_{y \geq 0} \left\{ -(r - pB(T - L - t) - c)y + (r - pB(T - L - t) - s)E[(y - X_D)^+ | x_t, \delta_t] + \right.$$

$$\left. (r - pB(T - L - t))E[X_D | x_t, \delta_t] + hA(T - L - t)y + pB(T - L - t)E[X_D | x_t, \delta_t] \right\}$$

$$= \min_{y \geq 0} \left\{ (hA(T - L - t) - (r - pB(T - L - t) - c))y + \right.$$

$$\left. (r - pB(T - L - t) - s)E[(y - X_D)^+ | x_t, \delta_t] + rE[X_D | x_t, \delta_t] \right\}$$

$$= \min_{y \geq 0} \left\{ (hA(T - L - t) + pB(T - L - t) - (r - c))y + \right.$$

$$\left. (r - pB(T - L - t) - s)E[(y - X_D)^+ | x_t, \delta_t] + rE[X_D | x_t, \delta_t] \right\}. \qquad (2.16)$$

The following theorem characterize the firm's optimal procurement quantity for any given procurement time t and realize x_t and δ_t.

Theorem 18. *For any given procurement time t and realized cumulative pre-season orders x_t and pre-season order ratio δ_t, the optimal order quantity is given by*

$$y^*(t, x_t, \delta_t) = F_t^{-1} \left(\frac{r - c - hA(T - L - t) - pB(T - L - t)}{r - s - pB(T - L - t)} \right), \qquad (2.17)$$

where $F_t(\cdot|x_t, \delta_t)$ is the distribution function of X_D at time t.

Note that under the alternative tardiness penalty cost model, the optimal procurement quantity explicitly depends on both the earliness and tardiness cost, whereas the optimal procurement quantity explicitly depends only on the earliness cost. Clearly, with this alternative model, the optimal procurement quantity is, every thing else being equal, less than that under our original model.

We next prove that the separability result still holds in this case. From (2.16), we have

$$v(t, x_t, \delta_t) = (hA(T - L - t) + pB(T - L - t) - (r - c))y^*(t)+$$

$$(r - pB(T - L - t) - s)\int_0^{y^*(t)}(y^*(t) - \xi)f_t(\xi|x_t, \delta_t)d\xi + rx_t \exp(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)).$$

We can simplify the above equation as follows.

$$v(t, x_t, \delta_t) = (hA(T - L - t) + pB(T - L - t) - (r - c))y^*(t) + rx_t \exp(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t))$$

$$+ (r - pB(T - L - t) - s)y^*(t)\int_0^{y^*(t)} f_t(\xi|x_t, \delta_t)d\xi$$

$$- (r - pB(T - L - t) - s)\int_0^{y^*(t)} \xi f_t(\xi|x_t, \delta_t)d\xi$$

$$= (hA(T - L - t) + pB(T - L - t) - (r - c))y^*(t) + rx_t \exp(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t))$$

$$+ (r - pB(T - L - t) - s)y^*(t)\frac{r - c - hA(T - L - t) - pB(T - L - t)}{r - s - pB(T - L - t)}$$

$$- (r - pB(T - L - t) - s)\int_0^{y^*(t)} \xi f_t(\xi|x_t, \delta_t)d\xi$$

$$= rx_t \exp(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t))$$

$$- (r - pB(T - L - t) - s)\int_0^{y^*(t)} \xi f_t(\xi|x_t, \delta_t)d\xi.$$

But we know

$$\int_0^{y^*(t)} \xi f_t(\xi|x_t, \delta_t)d\xi = \frac{1}{2}x_t \exp(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)) \left(1 + \operatorname{erf}\left(k(t) - \sqrt{\frac{\psi_\sigma(t)}{2}}\right)\right).$$

Substitute the above equation into the previous equation, we have

$$v(t, x_t, \delta_t) = x_t \exp(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)) \cdot$$
$$\left(r - \frac{1}{2}(r - s - pB(T - L - t))\right) \left(1 + \operatorname{erf}\left(k(t) - \sqrt{\frac{\psi_\sigma(t)}{2}}\right)\right).$$

Clearly, the separation result still holds, although the optimal procurement time function is slightly different. Therefore, whether the penalty cost depends only on the expected demand or the minimum of the expected demand and the procurement quantity do not affect the fundamental insight for this chapter.

2.8 Conclusions

In this chapter, we investigate a key trade-off that will be faced by an increasing number of firms as more goods are sourced from distant suppliers. In particular, we study the optimal timing-and-quantity problem for a newsvendor-type firm facing supply and demand risk. The supply risk, i.e., leadtime uncertainty, motivates the supplier to order earlier. In contrast, the opportunity to reduce its demand risk through pre-season orders motivates the firm to order later. We study this timing-and-quantity problem in a quite general setting and establish a number of interesting technical and managerial results.

We prove an important timing-and-level separation result. In the no pre-season order case,

the optimal procurement time is independent of the expected demand. In the pre-season order case, the optimal procurement time is independent of the realization of pre-season orders (but the optimal quantity is not.) We characterize the optimal procurement time and quantity in both cases and analytically establish the directional effect of many important supply and market attributes. While, one might expect that, all else being equal, a firm with pre-season orders would procure closer to the selling season (to take advantage of increasing demand-forecast accuracy) than would a firm without pre-season orders, this is not necessarily true. We prove that this intuition is correct in the case of a deterministic leadtime. However, in a stochastic-leadtime setting, the firm with pre-season orders may procure earlier than the no pre-season order firm. The reason lies in the interplay of the demand and supply risk. One might also expect that a firm should benefit from a decrease in the leadtime delay probability, that is, the firm's expected cost should decrease as the probability of delay decreases. We prove that this intuition is only partially correct. While the firm prefers a perfectly reliable leadtime to an unreliable leadtime, it does not necessarily prefer a leadtime with lower delay probability to a leadtime with higher delay probability. In fact it may strictly prefer a leadtime with higher delay probability. We prove that the firm's optimal expected cost decreases as the demand mix shifts towards pre-season orders. We show that the marginal value of pre-season orders increases as the demand-mix shifts from in- to pre-season orders. That is, the firm realizes increasingly significant cost savings as it encourages more pre-season orders. We show that pre-season orders makes the firm less sensitive to the variance of the leadtime delay.

The research in this chapter can be expanded in several directions. The most immediate extension is the incorporation of the supplier leadtime information updating. As the firm updates its demand, it is also possible for the firm to learn about the supplier's leadtime over time. Such additional information on the supplier leadtime uncertainty may also affect the

firm's decision process. Another interesting direction for extension is the interaction between the firm and its customers. As the firm encourages its customers to place pre-season orders, the customers may not necessarily be truth telling in the order size. In other words, if the customers anticipate a potential shortage of the product, they may inflate their orders, which will dilute the firm's ability to learn the real demand from its pre-season orders. In addition, it is also interesting to investigate how the potential price fluctuation affects the firm's optimal timing decision. Finally, one may explore the situation when the firm has multiple procurement opportunities. It is of interest to understand, for example, whether multiple procurement opportunities will affect the firm's optimal procurement decisions.

Chapter 3

Capacity Risk

3.1 Introduction

Global sourcing has created unprecedented opportunities for firms to reduce procurement cost by sourcing from developing economies. At the same time, however, increased competition and heightened complexity of the global supply chain network present significant challenges for firms to maintain competitive advantage. Among many different challenges of managing a complex global supply chain, ensuring a consistent and stable supply network is one of the top challenges facing many firms. A recent survey by McKinsey indicates that "Executives believe they face growing risks from disruption to their supply chains - yet many are unprepared to manage those risks." (McKinsey Quarterly. Global survey of business executives. 2006.) In fact, reliability of suppliers is ranked in the survey as one of the top three supply chain risks facing firms across the globe. Many factors can influence the reliability of a particular supplier, but one important common factor that influences supplier reliability is the supplier's operational capability.

As mentioned in the introduction, many suppliers in developing economies have less stable

operational capabilities due to reasons such as energy and raw material shortages, less efficient infrastructure, and less developed technologies. Less stable operational capabilities can often lead to uncertainty in production/delivery quantities. Oftentimes, significant uncertainty in production/delivery quantities can have a detrimental effect on the firm's performance, which can offset any cost savings the firm may have achieved through global sourcing in the first place. In this research, we explore potential strategies that the firm can use to mitigate the adverse effect of uncertain production quantities associated with global sourcing due to suppliers' operational capability.

One common strategy that firms use to mitigate supply related risk is to diversify the supplier base. While a large supplier base can help the firm to mitigate its supply risk, such an arrangement introduces significant complexities to managing the supplier base, especially in a global setting. In addition, the practice of having a large supplier base can be at odds with the lean and agile supply chain structure. On the other hand, having a single supplier generally puts the firm under significant risk exposure due to supply uncertainty. Therefore, as a compromise, many firms adopt a dual sourcing strategy to mitigate their supply risk. Much industry evidence suggests, however, that dual sourcing does not have to be the only strategy for firms to mitigate supply uncertainty.

As firms become increasingly dependent on key suppliers in their supply chain, many firms start to adopt the practice of supplier relationship management (SRM) (Sako, 2004). SRM practice allows the firm to better communicate with suppliers and therefore can significantly reduce procurement cost and improve product availability. For a more detailed discussion of supplier relationship management, see Krause et al. (1998) and the references mentioned therein. Early SRM practice emphasized the communication and monitoring activities between

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the firm and suppliers, such as establishing shared information exchange platforms and efficient monitoring systems in terms of inventory level, production rate, and sales forecast. Recent evidence suggests that firms are making strategic decisions to assist their suppliers to achieve better supply chain performance. The following quote is from the Toyota's supplier support website[1].

> Toyota is committed to helping our suppliers ... . We dispatch experts to work with suppliers who ask for assistance in devising and implementing necessary improvements.

Similarly, Valeo, one of the largest auto parts suppliers, "helps suppliers to make progress" as one of the four key strategic priorities[2]. "We are developing a continuous improvement approach with our suppliers in order to meet world-class standards of competitiveness and quality."

Recently, many firms have started to consider monitoring and helping suppliers to be a key risk mitigation strategy. United Technologies, one of the largest manufacturers of a wide range of industrial and commercial products, for example, adopts supplier insight strategy to mitigate its supply risk.

> UTC's lean experts also guided the supplier through several opportunities for new efficiencies in manufacturing, order processing and logistics. ... Within a year of UTC's intervention, the supplier ... experienced vastly improved quality and on-time delivery performance.

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[1]http://www.toyotasupplier.com/sup_guide/sup_compete.asp.

[2]http://www.valeo.com/automotive-supplier/Jahia/pid/1113.

Through the supplier monitoring and supporting initiative, UTC gained three key advantages: cost reductions, effective deployment of resources and risk mitigation (GLSCS, 2005). This trend is also echoed by Agarwal (2006), who notes that "Many companies today are leveraging their supply base. ... They seek to help their suppliers implement Six Sigma quality efforts." Clearly, helping a supplier to improve its process capabilities can be a cost effective alternative to mitigate supply risk. In fact, "When firms can procure from proven alliances [suppliers] they conduct single sourcing as much as possible" (Agarwal, 2006).

Therefore, besides using dual sourcing to mitigate supply uncertainty, the firm may adopt an alternative approach by improving the suppliers' reliability. We note that the firm may not necessarily limit itself to procuring from the supplier where it has made process improvement efforts. There are four canonical structures that the firm can use to mitigate its supply uncertainty. See Figure 3.1.
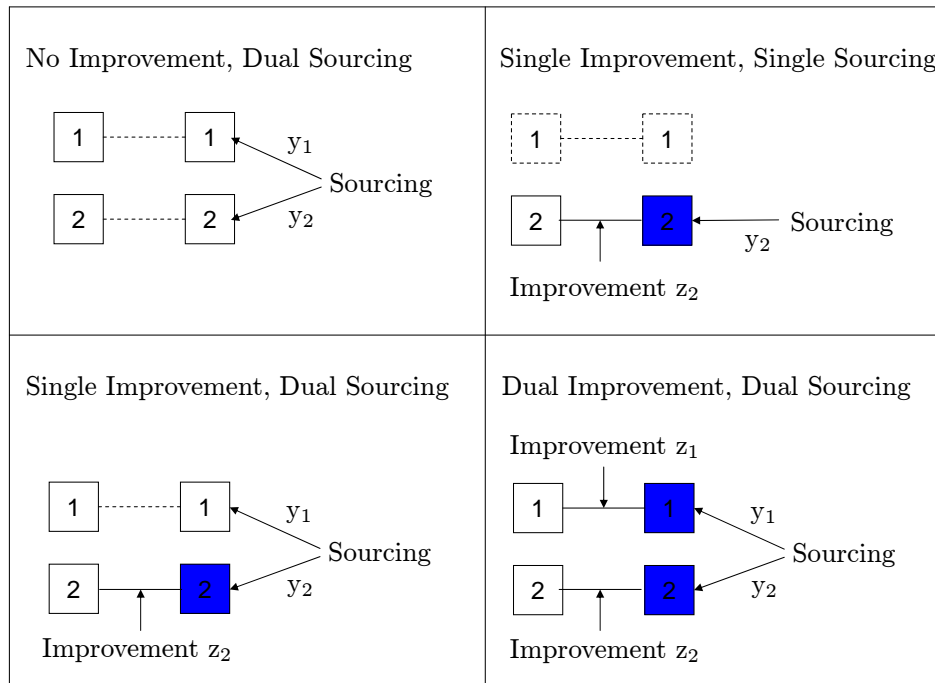
Figure 3.1: Four mitigation structures under supply uncertainty

Note that the top left structure is the classical dual-sourcing approach to mitigate supply risk. In this case, there is no improvement effort involved and the firm simply diversifies its supply uncertainty. On the other hand, the top righthand structure is the improvement strategy where the firm makes an improvement effort first and then procures from this particular supplier. The bottom two structures are extensions of the basic improvement strategy. The bottom left structure extends the basic improvement strategy by considering the option of dual sourcing after improvement effort. Clearly, this strategy dominates both classical dual-sourcing strategy and the basic improvement strategy. The bottom right structure is the most general structure. In fact, all the other three structures can be seen as special cases of this general structure. It is important to recognize that our focus is on short term improvement efforts and therefore we do not consider the time lag of the improvement decision. In essence, we assume that the time for the improvement effort is sufficiently short as compared with production time.

The classical dual-sourcing strategy implicitly treats suppliers' reliability as exogenous. The above mentioned evidence indicates, however, that suppliers' reliability need not be exogenous. A firm could make investment in its suppliers to improve their process reliability. The operational improvement strategy can be an effective alternative to mitigate the supply uncertainty. A key focus of this research is to contrast the diversification (dual-sourcing) versus improvement strategies. In particular, we seek to understand under what circumstances one strategy is preferred to the other, and which supplier the firm prefers to make the improvement effort. We also seek to understand how the opportunity for an improvement effort alters the firm's optimal procurement strategy.

The rest of the chapter is organized as follows. Section 3.2 surveys the relevant literature. In §3.3, we describe the model setup. Section 3.4 and 3.5 establish the optimality structure of the

diversification strategy and improvement strategy, respectively. We contrast the diversification and improvement strategy in §3.6. Section 3.7 discusses various extensions to the model, and §3.9 provides concluding remarks. All proofs can be found in the appendix.

3.2 Literature

Our research extends the diversification literature by relaxing the implicit assumption that supplier reliability is exogenous. From this perspective, our research is related to the supplier development as well as the process improvement literature, although our focus is quite different from the main thrust of either the supplier development or the process improvement literature. In what follows, we discuss the diversification literature first and then briefly discuss the supplier development and process improvement literature.

The relevant diversification literature studies the firm's optimal procurement problem when suppliers are unreliable. The reliability of suppliers has been modeled in three contexts: random capacity (Ciarallo et al., 1994; Erdem, 1999), random yield (Gerchak and Parlar, 1990; Parlar and Wang, 1993; Anupindi and Akella, 1993; Agrawal and Nahmias, 1997; Swaminathan and Shanthikumar, 1999), and random disruption (Parlar and Perry, 1996; Gürler and Parlar, 1997; Tomlin, 2006). Note that Dada et al. (2007) investigate a more general construct of supplier unreliability, where the random capacity and random yield models can be viewed as special cases of the general reliability construct. All these papers, however, treat supplier reliability as exogenous and therefore the improvement strategy is not relevant.

A central contribution of our research is that by relaxing the exogenous assumption for supplier reliability, we provide insights into the relative attractiveness of diversification versus

improvement strategy. In addition, our model of supplier reliability allows us to explicitly consider the supplier capacity constraint, which is an important aspect in the context of global sourcing where supplier capacity is often limited and scarce. Note that in the context of random capacity, the firm's procurement problem is in general not concave (convex) in the procurement quantity. The current literature establishes that the firm's procurement problem is unimodal in procurement quantity when there is a single supplier. Ours is the first study to establish the joint-unimodality property in situations where there are two suppliers and their capacities are explicitly constrained.

There is a large stream of literature on supplier development, most of which is concerned with inter-organizational cooperation between the buyer and the suppliers. A detailed discussion of the relevant literature can be found in the literature survey in Krause et al. (1998). Note that most of the above literature consists of case-studies or empirical based research, investigating the relative benefits and trade-offs of various supplier relationship programs. This literature establishes a concrete basis for viability of the supplier development program. Our research builds upon this literature, but our focus lies in the relative attractiveness of supplier improvement to the diversification strategy when suppliers are unreliable.

Our research is also somewhat related to the process improvement literature. Process improvement in normative analysis has been studied in terms of cost improvement (Fine, 1986; Fine and Porteus, 1989; Li and Rajagopalan, 1998) and effective capacity improvement (Spence and Porteus, 1987; Carrillo and Gaimon, 2000, 2004). A detailed review of the process improvement literature can be found in Carrillo and Gaimon (2002). Note that the above literature investigates the trade-off between unit cost reduction or effective capacity improvement and process improvement efforts. These papers focus on the relative effectiveness of particular pro-

cess improvement efforts or a particular policy for process improvement efforts. This research treats process improvement as a viable option for firms to mitigate supply uncertainty, and we focus on the relative attractiveness of the diversification versus the improvement strategy. Ours is the first study to bridge the gap between the diversification and the process improvement literature.

3.3 Model

We study a firm that needs to procure a single product from unreliable suppliers to satisfy a future stochastic demand. Suppliers are unreliable in their realized capacity because of various operational disruptions, which can be attributed to less efficient process management or insufficient technical expertise. Demand is uncertain when the firm makes its procurement decision. Therefore, the firm faces both supply and demand uncertainty when deciding the procurement quantity. To mitigate such supply uncertainty, a firm may procure from multiple sources, i.e., diversify across suppliers.

With the diversification strategy, the firm's decision is to determine the optimal procurement quantity from each unreliable supplier. With the improvement strategy, the firm's decision is to determine 1) with which supplier to invest in process improvement, 2) the amount of investment efforts, and 3) the optimal procurement quantity. In what follows, we first describe the supplier model and then describe the firm's procurement problem. Note that we use $\tilde{y}$ to signify that y is a random variable and $\vec{y}$ to signify that y is a (random) vector. In addition, we adopt the notation $y^+ = \max(0, y)$. We use ∇_x to denote the partial derivative with respect to x.

3.3.1 Supplier Model

3.3.1.1 Capacity and Reliability

Suppliers can differ in their capacity and reliability. Let K_i denote the supplier i's total capacity. The supplier i's total capacity K_i can be unreliable due to various disruptions in the production process. Let $\tilde{S}_i$ denote the total capacity loss due to process disruptions; then the supplier i's effective capacity is given by $\left(K_i - \tilde{S}_i\right)^+$. We assume that $\tilde{S}_i$ is a non-negative random variable, which depends on the supplier i's process capabilities, such as competence in production technology and process management. Now, let u_i denote the supplier i's process capabilities; then the distribution function of $\tilde{S}_i$ can be expressed by $G_i(\cdot) = G(\cdot, u_i)$, with the density function $g_i(\cdot) = g(\cdot, u_i)$. Note that u_i can be viewed as a composite process capability index, where a higher value is associated with a more competent process capability. If two suppliers have different capability indices, e.g., $u_i > u_j$, then $\tilde{S}_i \leq_{st} \tilde{S}_j$, i.e., supplier i is stochastically more reliable than supplier j. To explicitly express the dependence of $\tilde{S}_i$ on u_i, we define $\tilde{S}_i = \tilde{S}(u_i)$.

3.3.1.2 Capability Improvement

Oftentimes, the firm may have the opportunity to improve the supplier i's process capability by investment and/or knowledge transfer. To have a firm grasp on the concept of process improvement, it is important to consider various dimensions of process improvement.

Naturally, one may consider that a process is improved if the random shock / capacity loss is reduced. Therefore, one may be tempted to consider that either a reduction in the mean or

variance is an improvement. It is, however, questionable whether such measurement is indeed coherent. For example, if the variance of a random shock is reduced but the mean is kept at the same level, do we consider the process improved? The answer is in general considered no, and at the least this is not appealing. This is because before the "improvement," the process may incur a small or large capacity loss, but after the "improvement" the process is guaranteed to have a medium level of capacity loss. This resulting phenomenon should hardly be considered as an improvement. On the other hand, a reduction in the mean of the capacity loss is almost always considered good, because in expectation the capacity loss is reduced. A reduction in mean, however, is in general accompanied by a reduction in variance. It is in fact difficult to conjecture a situation where the mean is reduced but the variance is kept at the same level.

A parsimonious approach to model the process improvement is to use the concept of stochastic dominance. One variable is stochastically dominated by another variable if the first variable is less random then the second variable. The appeal of this approach lies in the fact that it allows us to model a variety of different concepts of process improvement. For example, if the stochastic loss is exponentially distributed, then the stochastic dominance improvement implies that both the mean and the variance are improved, which is a very natural and appealing feature in process improvement. On the other hand, if the stochastic loss is normally distributed (truncated), then the stochastic dominance improvement implies that the mean (approximately) is reduced, which also at least agrees with our common perception of improvement. In fact, the stochastic dominance approach allows us to model a large family of distributions, which makes our model very general and yet appealing in a variety of different scenarios.

Let $u_i(0)$ represent the supplier i's current process capability and let $u_i(z)$ represent the process capability after $z \geq 0$ units of improvement efforts. We assume that the resulting

capability after improvement, $u_i(z)$, is a function of $u_i(0)$ and z, i.e.,

$$u_i(z) = h\left(u_i(0), z\right), \tag{3.1}$$

where $h(\cdot)$ satisfies the following conditions:

T1. $h(u_i(0), 0) = u_i(0)$,

T2. $\nabla_z h\left(u_i(0), z\right) \geq 0$,

T3. $\nabla_{zz} h\left(u_i(0), z\right) \leq 0$.

Note that the above conditions are readily satisfied by many functions. For example, $h(u_i(0), z_i) = u_i(0)(1 + \sqrt{z_i})$, or $h(u_i(0), z_i) = u_i(0) + \sqrt{z_i}$, or $h(u_i(0), z_i) = u_i(0) + \ln(1 + z_i)$, $h(u_i(0), z_i) = u_i(0)(1 + \ln(1 + z_i))$. Note that the above three conditions guarantee that the resulting process capability is non-decreasing in improvement efforts and the marginal return on improvement efforts is non-increasing. These conditions are consistent with the existing process improvement literature.

The process improvement effort may not always be successful and hence the outcome of the process improvement may not be certain. We therefore assume that with probability θ, the improvement effort is successful and with probability $1 - \theta$, the improvement effort is not successful. It follows that

$$u_i(z) = \begin{cases} h\left(u_i(0), z\right), & \text{w.p. } \theta; \\ \\ u_i(0), & \text{w.p. } 1 - \theta. \end{cases} \tag{3.2}$$

Let m denote the unit cost of the process improvement efforts; then, given the firm having

committed z unit of improvement efforts, the total improvement cost incurred is $m \cdot z$. Note that although the total improvement cost is linear in the process improvement efforts, the cost is non-linear in the capability improvements because of conditions T1 - T3. In fact, these conditions imply that the improvement cost is convex increasing in the capability improvements.

3.3.1.3 State Space

It is convenient to think of $\vec{u}(0) = (u_1(0), u_2(0), \ldots)$ as the initial state space of the supplier system. The firm's process improvement effort can be viewed as an attempt to alter the initial state space of the system. For a given supplier i and z units of improvement efforts, we have $\tilde{S}(u_i(z)) \leq_{st} \tilde{S}(u_i(0))$. Equivalently, we have $G(x, u_i(z)) \geq G_i(x, u_i(0))$ for any x. It follows immediately that $E\left[\tilde{S}(u_i(z))\right] \leq E\left[\tilde{S}(u_i(0))\right]$, that is, process improvement leads to, in expectation, a larger effective capacity. Recall that we define $\tilde{S}_i = \tilde{S}(u_i(\cdot))$. When there is no ambiguity, we at times suppress the state variable $u_i(\cdot)$ and simply write $\tilde{S}_i$.

3.3.2 Procurement Cost

Let y_i denote the order quantity placed with supplier i. The delivered quantity $\tilde{y}_i$ is limited by the minimum of the order quantity and the supplier's effective capacity, i.e., $\tilde{y}_i = \min\{y_i, (K_i - \tilde{S}_i)^+\}$. Note that $\tilde{y}_i$ implicitly depends on the firm's operational capability level $u_i(\cdot)$ and may be strictly less than the order quantity y_i. The firm therefore faces a supplier-specific procurement risk, i.e., the probability of not receiving its full ordered quantity from a particular supplier.

The supplier-specific procurement risk may be partially offset by paying the supplier only for the quantity delivered. Oftentimes, however, the firm incurs some procurement cost even

if a fraction of the order is not delivered because of the significant cost incurred with the procurement process. Such a situation arises frequently in global sourcing, when the suppliers' production capability may be unreliable and the shipping and custom clearance processes are costly. Therefore, the firm's procurement cost will in general depend on the ordered quantity y_i and the delivered quantity $\tilde{y}_i$.

Let c_i denote the unit procurement cost associated with supplier i. Then the firm pays a fraction $0 \leq \eta_i \leq 1$ of the unit procurement cost for the ordered quantity, and a fraction $(1 - \eta_i)$ for the delivered quantity. It follows that the firm's total procurement cost associated with supplier i is given by

$$\tilde{C}(y_i) = (\eta_i y_i + (1 - \eta_i)\tilde{y}_i) c_i. \tag{3.3}$$

Note that if $\eta_i = 0$, then the procurement risk is transferred to the supplier. Consequently, the cost of the procurement risk associated with any undelivered quantity is fully born by the supplier. On the other hand, if $\eta_i = 1$, then the procurement risk stays with the firm. For $0 < \eta_i < 1$, the procurement risk is shared between the supplier and the firm. We note that even at $\eta_i = 0$, the firm may still have incentive to mitigate its procurement risk through alternative sourcing strategies. The firm may find it desirable to minimize any undelivered quantity to maximize its expected profit. Note that here we do not explore the contracting/negotiation issue between the firm and the supplier. We therefore treat η_i as exogenous for a given supplier i.

3.3.3 Problem Formulation

When making the procurement decision, the firm faces a stochastic future demand $\tilde{D}$, of which we denote the distribution and the density function as $F(\cdot)$ and $f(\cdot)$, respectively. Let r, v, and p represent the product's unit revenue, salvage value and penalty cost, respectively. The firm's decision problem is to decide, given the supplier's process capability, the procurement quantity from each supplier so as to maximize the total expected profit.

3.3.3.1 Diversification

The firm may choose to source from different suppliers to mitigate its procurement risk. For any given procurement quantity $\vec{y} = (y_1, y_2, \ldots)$ and supplier process capability $\vec{u} = (u_1, u_2, \ldots)$, the firm's profit function is given by

$$\tilde{\pi}(\vec{y}, \vec{u}) = -\sum_k \tilde{C}(y_k) + r \min\left\{\tilde{D}, \sum_k \tilde{y}_k\right\} + v\left(\sum_k \tilde{y}_k - \tilde{D}\right)^+ - p\left(\tilde{D} - \sum_k \tilde{y}_k\right)^+, \quad (3.4)$$

where $\tilde{C}(\cdot)$ is given by (3.3). Note that $\tilde{y}_k$, the delivered quantity, depends on the process capability u_k.

Define $\psi_k := \frac{-\eta_k c_k}{r+p-v}$ and $\phi_k := \frac{r+p-(1-\eta_k)c_k}{r+p-v}$; then (3.4) can be simplified (see Lemma A9) into a more compact form.

$$\tilde{\pi}(\vec{y}, \vec{u}) = (r + p - v)\left(\sum_k \psi_k y_k + \sum_k \phi_k \tilde{y}_k - \left(\sum_k \tilde{y}_k - \tilde{D}\right)^+\right) - p\tilde{D}. \quad (3.5)$$

Note that the terms $(r + p - v)$ and $p\tilde{D}$ in (3.5) do not affect the optimization problem. For

simplicity, we scale $p = 0$ and $r + p - v = 1$ such that

$$\tilde{\pi}(\vec{y}, \vec{u}) = \sum_k \psi_k y_k + \sum_k \phi_k \tilde{y}_k - \left(\sum_k \tilde{y}_k - \tilde{D} \right)^+ .$$

Therefore, the firm's expected profit function is

$$\pi(\vec{y}, \vec{u}) = \sum_k \psi_k y_k + E_{\vec{S}} \left[\sum_k \phi_k \tilde{y}_k \right] - E_{\vec{S}, D} \left[\left(\sum_k \tilde{y}_k - \tilde{D} \right)^+ \right]. \qquad (3.6)$$

For a given supplier process capability vector $\vec{u}$, the firm's decision problem is to find an optimal procurement quantity vector $\vec{y}$ to maximize (3.6), i.e.,

$$\pi^*(\vec{u}) = \max_{\vec{y} \geq 0} \pi(\vec{y}, \vec{u}). \qquad (3.7)$$

3.3.4 Process Improvement

The firm may make investment to improve a supplier i's process capability from $u_i(0)$ to $u_i(z)$ with $z \geq 0$ units of efforts. After the realization of the supplier's process capability, the firm decides the procurement quantity from each supplier. Therefore, the firm's problem can be formulated as a two-stage stochastic program.

In the second stage, after the realization of the state space vector $\vec{u}(z)$, the firm decides the optimal procurement quantity from each supplier. This second-stage decision problem is identical to (3.7), but optimized over a different state space vector.

In the first stage, the firm chooses the amount of process improvement effort $\vec{z}$ to maximize

the objective function

$$g(\vec{z}, \vec{u}(0)) = \max_{\vec{z} \geq 0} \left\{ -m \sum_k z_k + E_{\tilde{\theta}} \left[\pi^* \left(\vec{u}(\vec{z}) \right) \right] \right\}. \tag{3.8}$$

Note that the optimization in (3.8) is over all different suppliers, i.e., the firm needs to decide among all suppliers how much process improvement effort to invest in.

3.4 Diversification Strategy With Two Suppliers

In this section, we characterize the structure of the firm's optimal procurement policy for any given supplier state $\vec{u} = (u_1, u_2)$. Many structural results in this section also provide important building blocks for characterizing the process improvement policy. In what follows, we first derive the necessary conditions for the optimal ordering quantity (y_1, y_2). We then show that the objective function is submodular in (y_1, y_2), which makes it difficult to obtain a global optimum procurement quantity (y_1, y_2). Nevertheless, we prove that the expected profit function is jointly unimodal in y_1 and y_2, such that the optimal solution can be efficiently obtained.

3.4.1 The Optimum Procurement Quantity

Using (3.6), we have

$$\nabla_{y_i} \pi(\vec{y}, \vec{u}) = \psi_i + \phi_i E_{\vec{S}} \left[\frac{\partial \tilde{y}_i}{\partial y_i} \right] - E_{\vec{S}} \left[\frac{\partial \tilde{y}_i}{\partial y_i} F \left(\sum_{k=1}^{2} \tilde{y}_k \right) \right]. \tag{3.9}$$

Recall that the delivered quantity $\tilde{y}_i = \min\{y_i, K_i - \tilde{S}_i\}$. It then follows that

$$\frac{\partial \tilde{y}_i}{\partial y_i} = \begin{cases} 1, & K_i - \tilde{S}_i \geq y_i; \\ \\ 0, & K_i - \tilde{S}_i < y_i. \end{cases} \tag{3.10}$$

Substitute (3.10) into (3.9), we have

$$\nabla_{y_i} \pi(\vec{y}, \vec{u}) = \psi_i + \phi_i G_i(K_i - y_i, u_i) - E_{\vec{S}} \left[\int_0^{K_i - y_i} F\left(y_i + \sum_{k \neq i} \tilde{y}_k \right) dG_i(s_i, u_i) \right]$$

$$= \psi_i + G_i(K_i - y_i, u_i) \left(\phi_i - E_{\vec{S}} \left[F\left(y_i + \sum_{k \neq i} \tilde{y}_k \right) \right] \right). \tag{3.11}$$

Using (3.11), we have

$$\nabla_{y_i y_i} \pi(\vec{y}, \vec{u}) = - g_i(K_i - y_i, u_i) \left(\phi_i - E_{\vec{S}} \left[F\left(y_i + \sum_{k \neq i} \tilde{y}_k \right) \right] \right)$$

$$- G_i(K_i - y_i, u_i) E_{\vec{S}} \left[f\left(y_i + \sum_{k \neq i} \tilde{y}_k \right) \right], \tag{3.12}$$

and

$$\nabla_{y_i y_j} \pi(\vec{y}, \vec{u}) = -G_i(K_i - y_i, u_i) G_j(K_j - y_j, u_j) E_{\vec{S}} \left[f\left(y_i + y_j + \sum_{k \neq i,j} \tilde{y}_k \right) \right]$$

$$= -G_i(K_i - y_i, u_i) G_j(K_j - y_j, u_j) f\left(y_i + y_j \right). \tag{3.13}$$

Note that (3.11) indicates that the optimal ordering quantity from supplier i is not separable from the ordering quantity from supplier j. In addition to this non-separability problem, the following lemma further proves that the expected profit function is submodular in y_1 and y_2.

Lemma 1. *The firm's expected profit function* (3.6) *is submodular in* y_1 *and* y_2. *That is, for*

any given $i \neq j$, $\nabla_{y_i}\pi(\vec{y}, \vec{u}) - \nabla_{y_j}\pi(\vec{y}, \vec{u})$ is decreasing in y_i and is increasing in y_j.

The above lemma tells us that, everything else being equal, an increase in the procurement quantity from supplier i will result in a decrease in the optimal procurement quantity from supplier j. The firm's expected profit function, therefore, is in general not jointly concave in the procurement quantity y_i and y_j. The following lemma demonstrates, however, that the expected profit function is component-wise unimodal in the procurement quantity y_i.

Lemma 2. *The firm's expected profit function (3.6) is component-wise unimodal in the procurement quantity y_i. That is, for any given i, $\nabla_{y_i y_i}\pi(\vec{y}) < 0$ if $\nabla_{y_i}\pi(\vec{y}) = 0$.*

The component-wise unimodality property helps to obtain the optimal procurement quantity from supplier i - given that the procurement quantity from the other supplier has been determined. In what follows, we shall establish that the above result can be further strengthened, which allows us to obtain global optimum conditions for the procurement quantity problem.

Theorem 19. *(a) There exists a unique vector $\vec{y} = (y_1, y_2)$ such that $\nabla_{y_i}\pi(\vec{y}, \vec{u}) \leq 0$ and $y_i \nabla_{y_i}\pi(\vec{y}, \vec{u}) = 0$ for $i = 1, 2$. (b) This unique vector $\vec{y}$ maximizes $\pi(\cdot, \vec{u})$.*

Theorem 19 proves that the firm's expected profit function is jointly unimodal in the procurement quantity y_1 and y_2. This guarantees that the first order condition is sufficient to ensure a global optimum solution. If follows from Theorem 19 that the optimal procurement quantity satisfies

$$\psi_1 + G_1(K_1 - y_1, u_1)\left(\phi_1 - E_{\tilde{S}_2}\left[F(y_1 + \tilde{y}_2)\right]\right) = 0,$$

$$\psi_2 + G_2(K_2 - y_2, u_2)\left(\phi_2 - E_{\tilde{S}_1}\left[F(y_2 + \tilde{y}_1)\right]\right) = 0. \tag{3.14}$$

3.4.2 The Distribution of Procurement Risk

Recall that even if the firm can shift the procurement risk to its suppliers by paying only for what is delivered, the firm may still have incentive to further mitigate its procurement risk by sourcing from multiple suppliers. Such is the case when there is economic incentive to utilize, for example, a cheaper supplier. Note that the procurement risk is composed of two important components: the cost risk and the variance risk. The cost risk is captured by the fraction of the procurement cost incurred for non-delivered quantity. The variance risk is the reduction in the expected profit caused by the uncertainty in the delivered quantity. A natural question arises as to how these two distinct aspects of procurement risk are distributed across suppliers.

Note that in (3.14), the procurement risk is interrelated with the demand risk. It is therefore difficult to directly compare the procurement risk distributed between the two suppliers. To elucidate the balance of the procurement risk across different suppliers, we remove the demand risk by assuming a deterministic demand. Note that we relax this assumption and explore the more general case in §3.7.1 . For notational clarity, we suppress the state space vector $\vec{u}$ in the distribution function $G(\cdot)$.

When the demand is deterministic, denoted as D, the optimality condition (3.14) can be simplified as follows. If $y_1 + y_2 \leq D$, then we have

$$(r - (1 - \eta_1)c_1)\overline{G}_1(K_1 - y_1) = c_2 - c_1 + (r - (1 - \eta_2)c_2)\overline{G}_2(K_2 - y_2), \qquad (3.15)$$

where $\overline{G}(\cdot) = 1 - G(\cdot)$. Note that $\overline{G}_i(K_i - y_i)$ is the probability that the firm will not get the last unit from supplier i. In other words, the $\overline{G}_i(K_i - y_i)$ function measures the firm's procurement risk exposure from supplier i. The left hand side of (3.15) therefore measures the underage cost

of not being able to get the last unit from supplier 1. Similarly, the right hand side measures the underage cost from supplier 2, adjusted by the unit cost differential between supplier 1 and 2. Note that if $c_1 = c_2$ and $\eta_1 = \eta_2$, then the firm's procurement risk is equally split between the two suppliers. On the other hand, if $c_1 \neq c_2$ and/or $\eta_1 \neq \eta_2$, then the firm faces a higher level of procurement risk from the cheaper supplier. However, as the procurement cost risk, i.e., η, increases, the firm's procurement risk becomes increasingly balanced between the two suppliers.

Now if $y_1 + y_2 > D$, then we have

$$G_2(K_2 - (D - y_1)) = \frac{r - c_1\,(1 + \eta_1 H_1)}{[r - v]},$$

$$G_1(K_1 - (D - y_2)) = \frac{r - c_2\,(1 + \eta_2 H_2)}{[r - v]},$$

where $H_i = \frac{\overline{G}_i(K_i - y_i)}{G_i(K_i - y_i)}$, i.e., H_i measures the odds that the supplier i will not fully deliver the ordered quantity. Note that the above equations collapse to the standard newsvendor result when $\eta_i = 0$. For example, with probability $G_2(K_2 - (D - y_1))$, supplier 2 is sufficiently reliable such that the last unit from supplier 1 incurs an overage cost of $(c_1 - v)$. On the other hand, with probability $1 - G_2(K_2 - (D - y_1))$, supplier 2 is unreliable such that the order quantity from supplier 1 incurs an underage cost of $(r - c_1)$. Combine the above equations, we have

$$[r - v]\overline{G}_1(K_1 - (D - y_2)) = c_2\,(1 + \eta_2 H_2) - c_1\,(1 + \eta_1 H_1) + [r - v]\overline{G}_2(K_2 - (D - y_1)),$$

$$(3.16)$$

Note that the distribution of procurement risk is balanced by both the underage and overage cost in (3.16), while the distribution of procurement risk is only balanced by the underage

cost in (3.15). This is intuitive because there are possibilities of both overage and underage in (3.16), whereas there is no possibility of overage in (3.15). Consistent with (3.15), the firm faces a higher procurement risk exposure from the cheaper supplier. In addition, an increase in the procurement cost risk η makes the procurement risk more equally balanced between the two suppliers.

3.4.2.1 The Case of Pure Procurement Risk

When $\eta_1 = \eta_2 = 0$, the firm pays only for what is delivered. In this section, we assume that, without loss of generality, $\phi_1 G_1(K_1) \geq \phi_2 G_2(K_2)$. Recall that we assume that demand is deterministic and we denote demand as D.

Theorem 20. *Assume $\phi_1 G_1(K_1) \geq \phi_2 G_2(K_2)$. If $\eta_1 = \eta_2 = 0$, then (a) The optimal procurement quantity satisfies $y_1^* < D$, $y_2^* < D$, and $y_1^* + y_2^* \geq \min\{D, K_1 + K_2\}$. (b) The optimal procurement strategy is as follows.*

- *Single source exact D from supplier 1, i.e., $y_1^* = D$ and $y_2^* = 0$, if $D \in \omega_1$;*

- *Dual source exact D, i.e., $y_1^* > 0$, $y_2^* > 0$, and $y_1^* + y_2^* = D$, if $D \in \omega_2$;*

- *Dual source more than D, i.e., $y_1^* > 0$, $y_2^* > 0$, and $y_1^* + y_2^* > D$, if $D \in \omega_3$;*

- *Dual source more than D, utilize 100% of one supplier, e.g., $0 < y_1^* < K_1$, $y_2^* = K_2$, and $y_1^* + y_2^* > D$, if $D \in \omega_4$;*

- *Dual source more than D and utilize 100% of both suppliers, i.e., $y_1^* = K_1$ and $y_2^* = K_2$, and $y_1^* + y_2^* \geq D$ if $D \in \omega_5$,*

- *Dual source less than D and utilize 100% of both suppliers, i.e., $y_1^* = K_1$ and $y_2^* = K_2$, and $y_1^* + y_2^* < D$ if $D \in \omega_6$,*

where $\omega_i, i = 1, \ldots, 6$, defines the demand space:

$$\omega_1: \qquad 0 \leq D \leq K_1 - G_1^{-1}\left(\frac{r - c_2}{r - c_1}G_2(K_2)\right),$$

$$\omega_2: \qquad K_1 - G_1^{-1}\left(\frac{r - c_2}{r - c_1}G_2(K_2)\right) < D \leq K_1 + K_2 - \left(G_1^{-1}(r - c_2) + G_2^{-1}(r - c_1)\right),$$

$$\omega_3: \qquad K_1 + K_2 - \left(G_1^{-1}(r - c_2) + G_2^{-1}(r - c_1)\right) < D \leq K_1 + K_2 - \min(G_1^{-1}(r - c_2), G_2^{-1}(r - c_1))$$

$$\omega_4: \qquad K_1 + K_2 - \min(G_1^{-1}(r - c_2), G_2^{-1}(r - c_1)) < D \leq K_1 + K_2 - \max(G_1^{-1}(r - c_2), G_2^{-1}(r - c_1)),$$

$$\omega_5: \qquad K_1 + K_2 - \max(G_1^{-1}(r - c_2), G_2^{-1}(r - c_1)) < D \leq K_1 + K_2,$$

$$\omega_6: \qquad D > K_1 + K_2.$$

Note that not all demand regions will exist for all parameter values. For example, if the two suppliers are identical, then ω_1 will not exist. The above theorem tells us that the firm's procurement risk mitigation strategy is intimately related to the supplier's effective capacity relative to the magnitude of the demand. When the supplier's effective capacity is very high relative to the demand, then the firm will not use any procurement risk mitigation strategy. The firm simply single sources from the most attractive supplier - the lower cost one if both suppliers have identical process reliability. As the demand increases, the firm will first switch to a dual sourcing strategy. Note that as long as the demand is not too high, the firm's total procurement quantity will not change, whether it single sources or dual sources. The risk pooling benefit of dual sourcing is sufficient to mitigate the potential procurement risk.

As demand becomes sufficiently high, however, the firm not only uses dual sourcing, but also procures more than the total demand. In this case, the firm needs both dual sourcing and quantity hedge to help mitigate the supply risk. As demand continues to increase, then the firm becomes constrained by the supplier's effective capacity, which makes quantity hedge less viable. In the extreme case of very large demand, the firm fully utilizes both suppliers' capacity.

Note that in contrast to sourcing from multiple suppliers, when the firm sources only from one supplier, then the firm will never source more than D when the demand is deterministic. In fact, the firm always sources exact D with a single supplier, and may source less than D when $\eta > 0$, i.e., when there is cost risk.
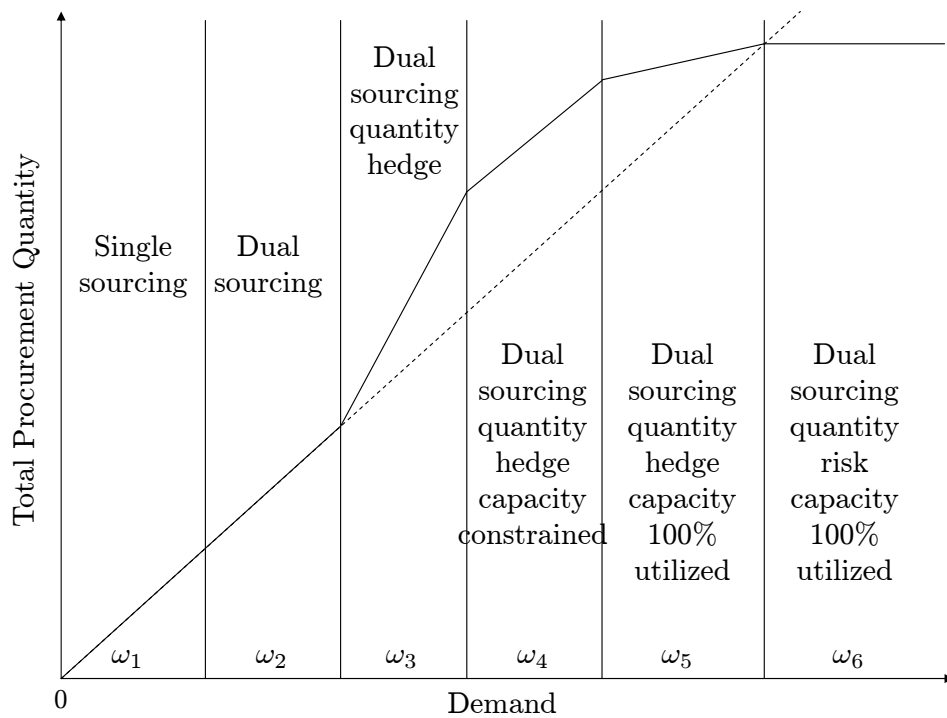
Figure 3.2: Optimal procurement quantity as a function of demand

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3.5 Improvement Strategy

Recall that the supplier i's operational capability is captured by the state variable u_i. The firm may be able to improve the supplier's process capability by making investment and/or knowledge transfer. Note that the firm's second stage problem is a special case of the diversification strategy. In what follows, we consider the firm's investment problem with a single supplier. An extension of the investment problem with multiple suppliers is discussed in §3.7.2.

By (3.8),

$$g(z, u(0)) = \max_{z \geq 0} \left\{ -mz + E_{\tilde{\theta}} \left[\pi^* \left(u(z) \right) \right] \right\}$$

$$= \max_{z \geq 0} \left\{ -mz + \theta \pi^* \left(u(z) \right) + (1 - \theta) \pi^* \left(u(0) \right) \right\}. \tag{3.17}$$

Instead of optimizing (3.17) over investment effort z directly, it is convenient to work with the desired level of $u(z)$. Given $u(0)$ and $u(z)$, the amount of improvement effort, z, can be derived by (3.1),

$$z = h^{-1}(u(z), u(0)). \tag{3.18}$$

We can therefore rewrite (3.17) as

$$g(u(z), u(0)) = \max_{u(z) \geq u(0)} \left\{ -m h^{-1} \left(u(z), u(0) \right) + \theta \pi^* \left(u(z) \right) + (1 - \theta) \pi^* \left(u(0) \right) \right\}. \tag{3.19}$$

The following theorem proves that an improvement in the supplier's process capability will increase the firm's procurement quantity from the supplier.

Lemma 3. *The optimal procurement quantity is non-decreasing in the supplier's process capability, i.e., given $u(z) \geq u(0)$, $y^*(u(z)) \geq y^*(u(0))$.*

Note that the above result holds even if there are multiple suppliers. This lemma tells us that, everything else being equal, the firm will procure more from the supplier with a higher process capability. The following lemma further proves that the firm's expected profit is increasing in the supplier's process capability.

Lemma 4. *For any given supplier, the firm's optimal expected revenue is non-decreasing in the supplier's process capability.*

Note that Lemma 4 is a very general result, which holds with multiple suppliers and general process capability distributions.

Theorem 21. *If the supplier's marginal reliability improvement is non-increasing in the process capability, i.e., $\frac{\partial^2 G_i(\cdot, u)}{\partial u^2} \leq 0$, then the firm's optimal profit is a concave function of the supplier's process capability $u(z)$, i.e., $\frac{\partial^2 g(u(z), u(0))}{\partial u(z)^2} \leq 0$. Furthermore, the firm's optimal improvement effort z^* satisfies*

$$\theta \int_{K-y^*}^{K} \left((\phi - F(K-s)) \frac{\partial G(s, u(z^*))}{\partial u(z^*)} \right) ds = \frac{m}{h'(u(0), z^*)}. \tag{3.20}$$

The above theorem establishes that the firm's expected revenue function is well behaved in terms of the process improvement efforts.

Corollary 3. *The firm's optimal improvement effort is (a) decreasing in the improvement cost m, (b) increasing in the improvement success probability θ, (c) decreasing in the unit procurement cost c, (d) increasing in the unit revenue r, and (e) decreasing in the existing*

capacity K if $\frac{\partial g(\cdot, u)}{\partial u} > 0$ and non-decreasing in the existing capacity otherwise.

If the improvement is more likely to succeed then the firm is more likely to engage in more improvement efforts. When the unit procurement cost is higher, however, the firm is more likely to reduce its improvement efforts.

3.5.1 Improvement Selection

When the firm has the opportunity to choose with which supplier to make improvement efforts, it is desirable to understand which supplier the firm should engage. In what follows, we consider the case where there are two suppliers, but the firm decides to use only one of the suppliers.

Lemma 5. *Suppose there are two suppliers which are identical in every aspect except for one parameter value. If the firm has an opportunity to invest in one supplier, then the firm will choose to invest in the supplier which the firm would have used as its single-sourcing supplier without investment.*

Proof of Lemma 5. Note that if two suppliers are identical except for one parameter, the firm will always choose the more desirable supplier for its single-sourcing supplier. In other words, the firm would choose to source from the supplier which has a lower cost c, a lower fraction of upfront cost η, a lower unit investment cost m, a higher probability of investment success θ, a larger capacity K, and a higher process capability $u(0)$. The lemma statements can be proved by contradiction. Suppose $c_1 < c_2$ but the firm choose to make investment z^* in supplier 2. Since all other parameters are identical, the optimal expected profit $\pi_2^*(z^*) < \pi_1^*(z^*)$ because the $\frac{\partial \pi^*}{\partial c} > 0$. The lemma statement for other parameters can be similarly proved. $\square$

When the two suppliers differ in more than one dimension, however, the firm's investment selection strategy is not as straightforward. In other words, it is not necessarily true that the firm will choose to make investment in the supplier it would have used if there were no improvement opportunity.

Lemma 6. *Suppose there are two suppliers which differ in more than one dimension, e.g., one supplier is cheaper but less reliable and the other supplier is more expensive but more reliable. If the firm has an opportunity to invest in one supplier, then the firm may choose to invest in the supplier which the firm would NOT have used as its single-sourcing supplier without investment.*

The above lemma proves that the opportunity to make improvement efforts can significantly change the dynamics of the firm's procurement strategy. In absence of improvement opportunities, the firm may choose to single source from a more reliable supplier because of the higher supplier reliability. However, if the firm has the flexibility to change supplier reliability, then the firm may choose to improve the less reliable supplier and then procure from the improved supplier. In this case, it is worthwhile for the firm to take advantage of the lower cost supplier by making the initial investment.

Remark 1. Even if the firm finds it optimal to single source from one supplier (given that the firm has the opportunity to dual source) in the first place, the firm may still want to make an improvement effort. Therefore, even if the demand risk does not warrant dual sourcing, as long as the improvement costs are not prohibitively expensive, improvement strategy can be attractive. In fact, improvement strategy can be strictly preferred to dual sourcing strategy under low demand risk. (Note that with high demand risk, both dual sourcing strategy and improvement strategy are more likely to be employed simultaneously.)

Remark 2. Even if the firm finds it optimal to single source from supplier i, $i = 1, 2$, the firm may find it optimal to improve the supplier $3 - i$ and only single source from the supplier $3 - i$ after the improvement efforts. Therefore, the opportunity for improvement can drastically reverse the firm's preference of one supplier over the other.

3.6 Comparison of Diversification and Investment Strategies

Note that the general investment strategy (weakly) dominates the diversification strategy if the firm's second stage procurement decision is not limited to the supplier in which the firm made process improvement efforts. Consistent with industry evidence, however, it is of interest to investigate the attractiveness[3] of the two pure strategies: pure diversification strategy versus pure investment strategy, where the firm selects one supplier to make a process improvement and then procures solely from this particular supplier.

3.6.1 Pure Strategies

In this section, we focus on the relative benefits of improvement and diversification strategies. We therefore restrict attention to the case where the two suppliers are identical in every way. In addition, we note that depending on parameter values, either improvement or diversification strategy can dominate the other strategy. In what follows, we consider how the firm's preference of either improvement or diversification strategy changes as suppliers' characteristics change.

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[3]Note that we use 'attractiveness' to simply indicate the potential preference of one strategy over the other. For example, we may use the word 'attractiveness' to refer to strategy A relative to strategy B, even though strategy A is never preferred, or attractive, for a given set of parameter values.

3.6.1.1   Unit Cost

The following lemma partially characterizes how the firm's preference of diversification strategy relative to investment strategy changes as the unit procurement cost changes.

Lemma 7. *Let π_I^* and π_D^* represent two pure strategies: pure investment and pure diversification strategies, respectively. If (i) the demand $F(\cdot)$ and reliability $G(\cdot)$ are uniformly distributed and (ii) the firm pays for only what is delivered, i.e., $\eta = 0$, then we have*

(a) *If the relative benefit of diversification over improvement strategy is decreasing in unit procurement cost c, then the relative benefit is convex decreasing in c, i.e., $\frac{\partial(\pi_D^* - \pi_I^*)}{\partial c} < 0 \Rightarrow \frac{\partial^2(\pi_D^* - \pi_I^*)}{\partial c^2} > 0$.*

(b) *If the relative benefit of diversification over improvement strategy is a concave function of the procurement cost c, then the relative benefit is concave increasing in c, i.e., $\frac{\partial^2(\pi_D^* - \pi_I^*)}{\partial c^2} < 0 \Rightarrow \frac{\partial(\pi_D^* - \pi_I^*)}{\partial c} > 0$.*

The above lemma tells us that the attractiveness of the diversification strategy is either convex decreasing or concave increasing in the unit procurement cost. In general, one can show that the firm's preference is not monotonic with respect to the unit procurement cost. Our numeric study indicates that the firm is more likely to prefer the diversification strategy when the unit cost is either very high or very low, and the firm is more likely to prefer the investment strategy when the procurement cost is moderate.

3.6.1.2 Heterogeneity of Suppliers

In this section, we consider how the firm's preference of diversification strategy relative to improvement strategy changes as the heterogeneity of the two suppliers becomes more significant. The following lemma proves that the diversification strategy becomes less attractive as the suppliers become more heterogenous, i.e., as they differ more and more in unit cost, total capacity, and process capability.

Lemma 8. *Let π_I^* and π_D^* represent two pure strategies: (best) pure investment and pure diversification strategies, respectively.*

- *Let δ represent the procurement cost differential, such that supplier 1's cost is $c - \delta$ and supplier 2's cost is $c + \delta$.*

- *Let δ represent the fraction of procurement cost differential, such that supplier 1's cost is $\eta - \delta$ and supplier 2's cost is $\eta + \delta$.*

- *Let δ represent the capacity differential, such that supplier 1's capacity is $K + \delta$ and supplier 2's cost is $K - \delta$. Iff (i) there is no probability of complete capacity loss, i.e., $\overline{G}(K - \delta, u) = 0$, or (ii) $G(\cdot, u)$ is uniformly distributed.*

- *Let δ represent the capability differential, such that supplier 1's capability is $u + \delta$ and supplier 2's capability is $u - \delta$.*

Then, everything else being equal, $\pi_I^* - \pi_D^*$ is increasing in δ.

The intuition behind this lemma lies in the fact that the value of diversification diminishes as the suppliers differ more and more from each other. As the suppliers differ more and more

along one dimension, the firm shifts more and more procurement quantity to the more desirable firm. This, however, increases the supply risk exposure because the other supplier covers an increasingly smaller fraction of the firm's total procurement quantity.

3.7 Extensions

In this chapter, we at times restrict attention to deterministic demand in order to gain better insight into the distribution of procurement risk. In addition, the previous analysis of the improvement strategy focuses on the pure improvement strategy, i.e., the firm chooses a supplier for improvement and subsequently procures exclusively from this particular supplier. In this section, we relax these assumptions and study the more general case.

3.7.1 Procurement Risk with Stochastic Demand

Using (3.11), we have

$$
E_{\tilde{S}_1}\left[\Pr[\tilde{y}_1 < \tilde{D} - y_2]\right] = c_2\left(1 + \eta_2 H_2\right) - c_1\left(1 + \eta_1 H_1\right) + E_{\tilde{S}_2}\left[\Pr[\tilde{y}_2 < \tilde{D} - y_1]\right], \qquad (3.21)
$$

where the $\Pr(\cdot)$ is the probability function associated with the demand and H_i is similarly defined as in (3.16). Note the similarities between (3.16) and (3.21), which show that the firm's procurement risk is balanced by the supplier cost differentials. Again, if $\eta_1 = \eta_2 = 0$, the firm generate a higher risk exposure from the cheaper supplier. It is important to note, however, that the procurement quantity placed with the higher risk supplier may strictly be less than that placed with the lower risk supplier. It follows that a cheaper supplier may not get a higher

proportion of the firm's procurement quantity.

3.7.2 Improvement Strategy with Two Suppliers

When there are two suppliers, following similar logic as (3.19), we have

$$g(\vec{u}(z), \vec{u}(0)) = \max_{\vec{u}(z) \geq \vec{u}(0)} \left\{ -m \sum_{i=1}^{2} h^{-1}\left(u_i(z_i), u_i(0)\right) + \sum_{i=1}^{2}\sum_{j=1}^{2} q_{1i}q_{2j}\pi^*\left(u_1((i-1)z_1), u_2((j-1)z_2)\right) \right\},$$

(3.22)

where $q_{k1} = 1 - \theta_k$ and $q_{k2} = \theta_k$ for $k = 1, 2$. The following lemma proves that the firm's expected profit function is submodular in $(u_1(z_1), u_2(z_2))$.

Lemma 9. *The firm's expected profit function* $g(\vec{u}(z), \vec{u}(0))$ *is a submodular function in* $\vec{u}(z)$.

Although the above lemma partially characterizes the firm's profit function as submodular in operational improvement efforts, and the profit function is individually concave in improvement efforts, such results do not in general guarantee a global optimum. In fact, the firm's profit function is in general not jointly concave in improvement efforts. In what follows, we provide a sufficient condition such that the firm's profit function is jointly well-behaved in improvement efforts.

Theorem 22. *If the supplier's reliability distribution is a concave function of the improvement efforts, i.e.,* $\frac{\partial G_i(\cdot, u_i)}{\partial u_i} \leq G_i(\cdot, u_i)$, *then a sufficient condition for the firm's optimal profit function to be jointly unimodal in* $\vec{u}(z)$ *is the following.*

$$m\frac{h''\left(u_i(0), z_i\right)}{\left(h'\left(u_i(0), z_i\right)\right)^3} \leq -\theta_1\theta_2\phi_i K_i, \quad i = 1, 2.$$

(3.23)

Note that condition (3.23) is readily satisfied. If, for example, $h(u_i(0), z_i) = u_i(0) + \sqrt{z_i}$, then condition (3.23) is equivalent to $2m \geq \theta_1 \theta_2 \phi_i K_i$, which can also be readily satisfied. As another example, if $h(u_i(0), z_i) = u_i(0) + \ln(1 + z_i)$, then condition (3.23) is equivalent to $me^{u_i(z_i) - u_i(0)} \geq \theta_1 \theta_2 \phi_i K_i$, which can also be readily satisfied as $u_i(z_i) - u_i(0) \geq 0$.

3.8   Numeric Studies

While §3.6 partially characterizes the relative attractiveness of the diversification versus the improvement strategy, it is important to further investigate the directional change of the firm's preference of one strategy over the other under a wide range of system parameters. In particular, it is desirable to understand under what system conditions the firm is more likely to prefer one strategy over the other. It is also of interest to investigate the relative benefit of the preferred strategy. The following is a list of the notations used in our numeric study.

μ: Demand mean

σ: Demand standard deviation

r: Unit revenue

η: Fraction of the unit cost paid for ordered product

θ: The probability that an improvement effort is successful

α: The shape parameter for the capacity loss distribution

c: Unit procurement cost

m: Unit improvement cost

E_K: Expected capacity for a supplier

CV_K: Coefficient of variation of the capacity for a supplier

3.8.1 Study Design

In our numeric study, the base case scenario is set up as follows. The demand is normally distributed with a mean of 100 and a standard deviation of 30. We scale the unit revenue $r = 1$, the salvage value $v = 0$ and the penalty cost $p = 0$. Recall that scaling $r - v + p = 1$ does not affect the optimization problem. In addition, we set the fraction of the cost paid for ordered quantity $\eta_1 = \eta_2 = 0$, i.e., the firm pays only for what is delivered in the base case scenario. In addition, for the improvement strategy, we set the probability of success $\theta = 1$. The rest of the system parameters are set up such that they have low, medium, and high values. In particular, we set the unit cost c equal to 0.3, 0.6, and 0.9, which gives a newsvendor ratio of 0.7, 0.4, and 0.1, respectively. Note that, given the base case demand parameter values, the above newsvendor ratios correspond to effective demand of 116, 92, and 62, respectively. We set the unit improvement cost m equal to 0.1, 0.3, and 0.5, respectively. In view of the relative demand, we set the expected capacity (E_K) for each supplier equal to 60, 90, and 120, respectively. In addition, we set the coefficient of variation of the expected capacity (CV_K) equal to 0.1, 0.3, and 0.5, respectively.

We adopted the Weibull distribution family for the capacity loss distribution. The Weibull distribution is a very flexible family of distributions that can take on a variety of different shapes, depending on the shape parameter α and the scale parameter β. Here it is important to clarify what an improvement entails in this context. Our definition of improvement states

that, for any x, the probability of obtaining x units of capacity is higher than that before the improvement. Based on this definition, an improvement corresponds to a reduction in β. Consequently, given the Weibull family of distribution, an improvement does not affect the shape of the distribution, but does affect the mean and the variance of the distribution. With Weibull distribution, both the mean and variance are reduced after an improvement. In fact, this is a more reasonable representation of process improvement. It is difficult to justify a reduction in variance but not in the mean as an improvement. For the functional form of the effect of the improvement effort on the process capabilities, we adopt the log functional form, i.e., $\beta(m) = \frac{\beta(0)}{1+\log(1+m)}$. Note that the log functional form has been adopted in the literature, e.g., Porteus (1986).

For the base case scenario, we set $\alpha = 1$, i.e., the capacity loss is exponentially distributed. Given the specification of E_K and CV_K, the capacity loss parameter for the Weibull distribution, as well as the capacity upper bound, is completely specified. For example, with $E_K = 90$ and $CV_K = .1$, we have $K = 99$ and $\beta = 9$. Note that the specification of K and β uniquely depends on a particular shape parameter value α (besides E_K and CV_K). Table 3.1 summarizes the setup of the basic parameter values.

Scenarios	μ	σ	r	η	θ	α	c	m	E_K	CV_K
Low	-	-	-	-	-	-	0.3	0.1	60	0.1
Medium	100	30	1	0	1	1	0.6	0.3	90	0.3
High	-	-	-	-	-	-	0.9	0.5	120	0.5

Table 3.1: Numeric study base case parameter setup

In summary, there are $3^4 = 81$ base case scenarios. When we investigate the directional

effect of a particular system parameter, we repeat the computation for all 81 scenarios. In the case where the parameter of interest is used in the definition of the 81 scenarios, we have $3^3 = 27$ scenarios. Note that we set the two suppliers identical in every way and the random capacity losses are independent of each other. Given this setup, the diversification strategy has an inherent capacity advantage over the improvement strategy. Our intention, however, is not to quantify the absolute value of one strategy over the other. Instead, our goal is to understand how the relative difference between the diversification and investment strategy changes over different system parameters. From this perspective, whether the firm prefers one strategy over the other at the starting point is of less concern.

3.8.2 Directional Effects

In this section, we discuss how each individual parameter value affects the attractiveness of the improvement strategy over the diversification strategy. Note that we use π_I^* and π_D^* to denote the optimal profit for the improvement strategy and the diversification strategy, respectively.

3.8.2.1 Demand volatility

We varied the demand standard deviation σ from 5 to 50, with a step size of 5, and calculated the optimal expected profit under both the improvement and the diversification strategy. This gives us a total of 810 observations. As expected, the optimal expected profit for both the improvement and the diversification strategy decreases in the demand volatility. The relative attractiveness[4] of the improvement strategy over the diversification strategy, defined as $(\pi_I^* -$

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[4]Recall that we use 'attractiveness' simply to refer to the potential preference of one strategy over the other. For many parameter values, even though the improvement strategy is never preferred, we nevertheless use the word 'attractiveness' for consistency.

$\pi_D^*)/\pi_D^*$, however, is, on average, a concave function of the demand volatility. See Figure 3.3[5]. Note that the difference[6] in the optimal expected profit between the improvement strategy and the diversification strategy also follows a similar pattern.

Figure 3.3: Relative profit difference of the improvement strategy over the diversification strategy as demand uncertainty increases

We see that the improvement strategy is more likely to be preferred[7] when the demand uncertainty is neither too high nor too low. The intuition is as follows. When the demand

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[5]For presentational clarity, we label, along the vertical axis, the improvement strategy as IMP and the diversification strategy as DIV in all our figures within this chapter.

[6]Note that here we use 'difference' to indicate the difference in the magnitude of the optimal expected profit between the two strategies. We use 'relative difference' to indicate the percentage difference between the two strategies.

[7]Note that Figure 3.3 indicates that the improvement strategy is, on average, never preferred. Such a result is mainly driven by the inherent capacity advantage of the diversification strategy. Nevertheless, of the 810 cases, the improvement strategy is preferred in 31.9% of the cases.

volatility is low, the effective demand (given the newsvendor ratio) is large in most of the cases. Therefore, available capacity is a more significant driver for the firm's optimal profit. The diversification strategy is preferred because of its inherently larger capacity. On the other hand, when the demand volatility is high, the effective demand is low and therefore the diversification strategy can safely satisfy the effective demand. The improvement strategy becomes attractive when demand volatility is moderate, where an improvement in the process capability allows the firm to adequately satisfy demand with one supplier only. This is particularly true when the CV_K is high, where the diversification strategy is not able to efficiently mitigate the supply risk.

Note that the effect of the demand uncertainty is also closely linked with the newsvendor ratio, because an increase in the demand uncertainty may result in either an increase or a decrease in the effective demand, which in turn affects the trade-off between the capacity advantage versus the risk disadvantage. One may therefore conjecture that a higher newsvendor ratio, i.e., a lower procurement cost, will result in the improvement strategy becoming less attractive as the demand uncertainty increases. This conjecture is indeed correct. Setting the unit procurement cost $c = 0.1$, the unit improvement cost $m = 0.1$, the capacity coefficient of variation $CV_K = 0.5$, and all other parameter values at the medium base case values, we obtain Figure 3.4. We can see that the attractiveness of the improvement strategy is strictly decreasing as the demand uncertainty increases. In this case, an increase in the demand uncertainty results in a larger effective demand, which makes the improvement strategy more capacity constrained.
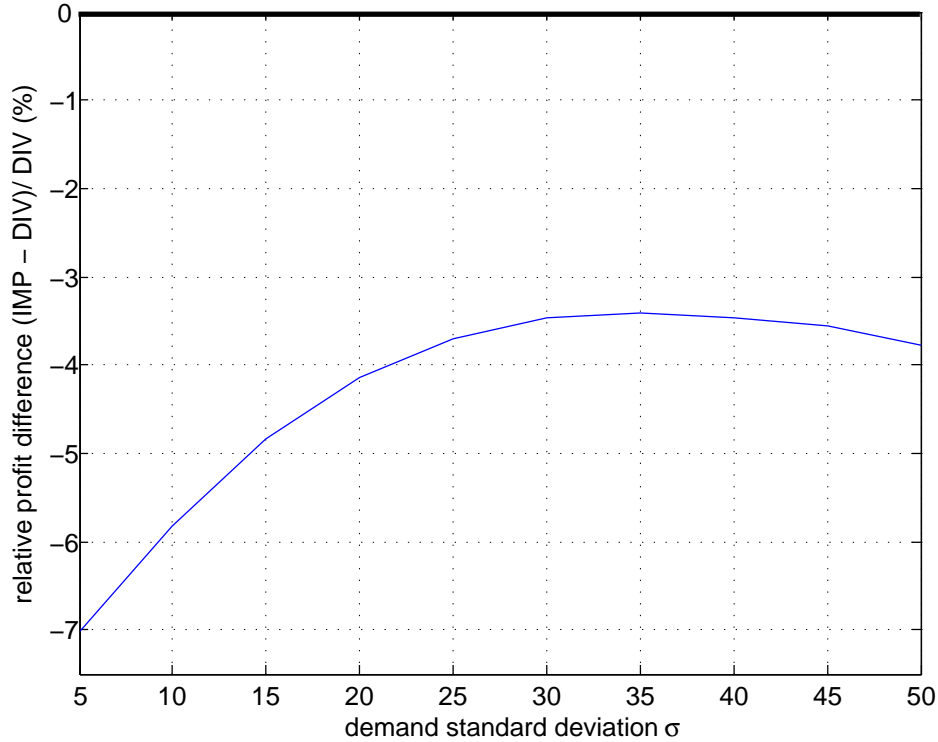
Figure 3.4: Relative profit difference of the improvement strategy over the diversification strategy as demand uncertainty increases (with a higher newsvendor ratio)

3.8.2.2   Unit Cost

We varied the unit cost c from 0.1 to 0.9 with a step size of 0.1. This gives a total of 243 observations, which is less than 810 because the unit cost is used in the definition of the base case scenarios. We observed that, on average, the relative attractiveness of the improvement strategy over the diversification strategy is concave increasing in the unit cost. See Figure 3.5.

Figure 3.5: Relative profit difference of the improvement strategy over the diversification strategy as the unit cost increases

We note, however, that the relative attractiveness of the improvement strategy over the diversification strategy can be strictly concave, especially when the CV_K is high, i.e., the supply risk is high. The intuition for this concave behavior is similar to that of the demand uncertainty. A low unit procurement cost induces a higher effective demand, where the inherent capacity advantage of the diversification strategy plays a dominant role. On the other hand, a high unit cost results in low effective demand, in which case the supply risk is insignificant. The value of improvement strategy is most apparent when the unit cost is moderate, such that a reduction in the supply risk may outweigh the benefit of diversification.

Note that Lemma 7 proves that, if demand and supply uncertainty are uniformly dis-

tributed, the difference in the optimal expected profit between the diversification strategy and the improvement strategy, i.e., $\pi_D^* - \pi_I^*$, is either convex decreasing or concave increasing. Our numeric observation confirms this result with more general demand and supply distributions. Figure 3.6 plots the value difference between the diversification and the improvement strategy. Note that the parameter values used in Figure 3.6 are such that the unit improvement cost is 0.5, the expected capacity is 90, the CV_K is 0.5, and all other parameters are set at the base case scenarios. Figure 3.6 as well as our numeric observation indicate that the result in Lemma 7 is indeed robust.
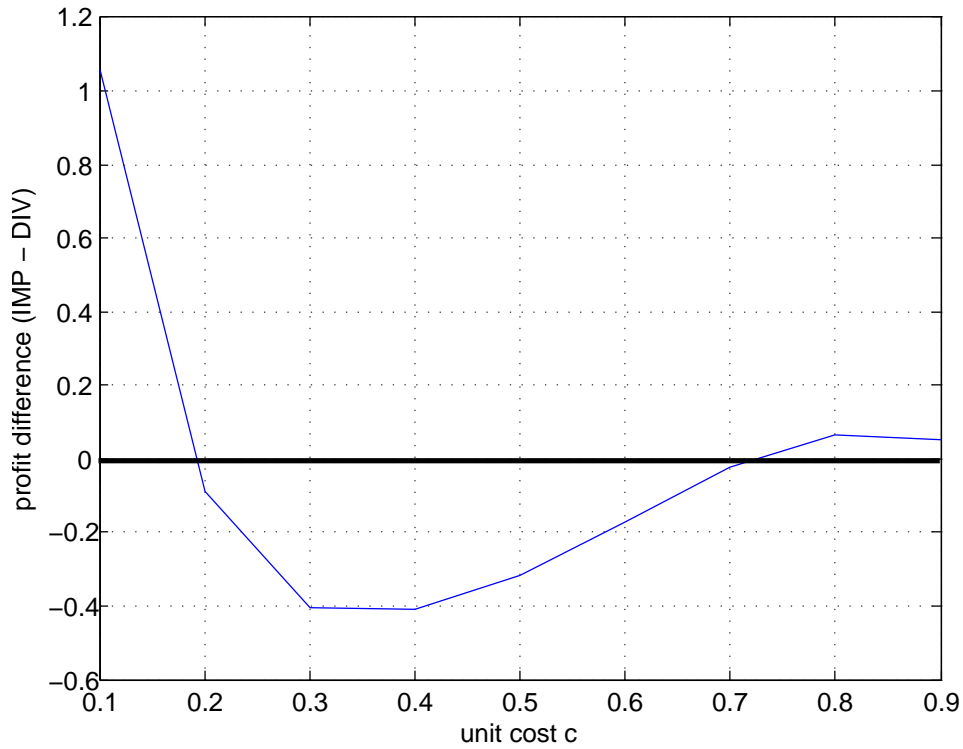
Figure 3.6: Profit difference between the improvement strategy and the diversification strategy as the unit cost increases

3.8.2.3 Unit Improvement Cost

We varied the unit improvement cost m from 0.1 to 1 with a step size of .1. This gives a total of 270 observations. As expected, the optimal expected profit for the improvement strategy is decreasing in m. What is interesting, however, is that the optimal expected profit is convex decreasing in m. See Figure 3.7.
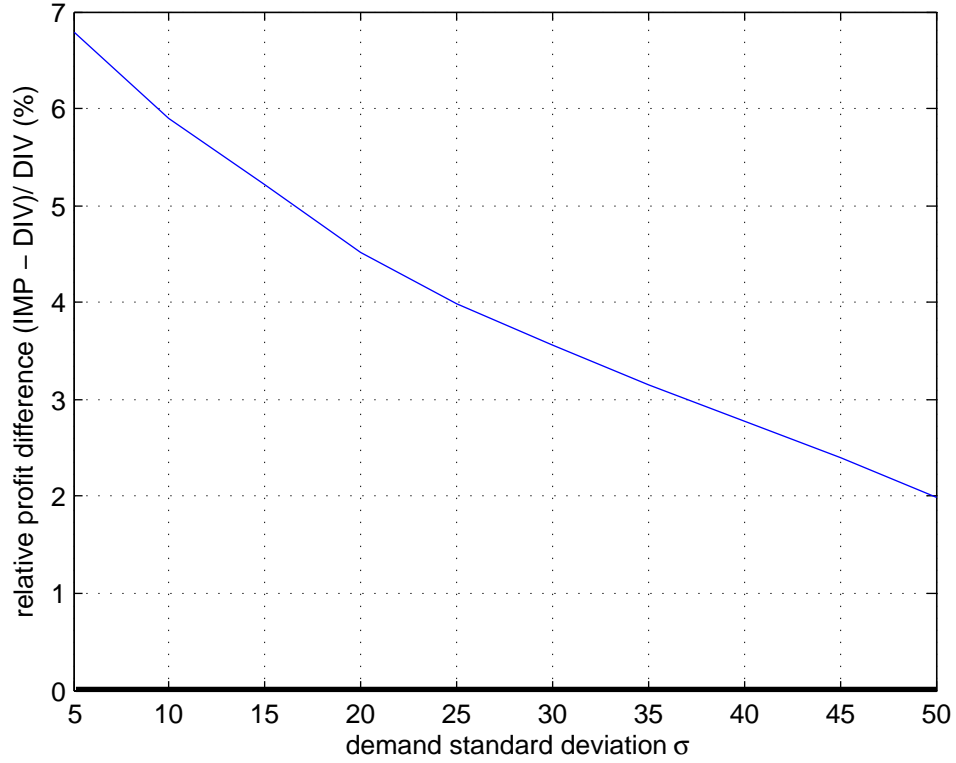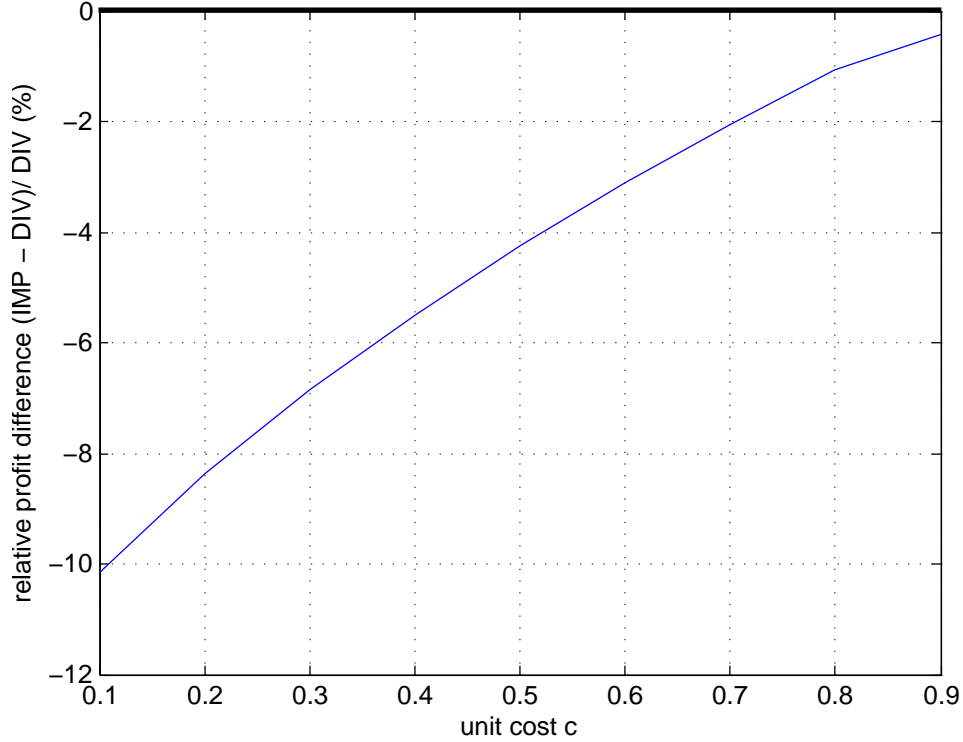

Figure 3.7: Relative profit difference of the improvement strategy over the diversification strategy as the unit improvement cost increases

Therefore, a marginal increase in the unit improvement cost m becomes less critical in the attractiveness of the improvement strategy as the unit improvement cost increases. Note that in the limiting case where the unit improvement cost is sufficiently high, then the firm will not make any improvement and the optimal profit with improvement approaches that of the single

sourcing case.

3.8.2.4 Fraction of Cost for Ordered Quantity

We varied the fraction of the procurement cost incurred for the quantity ordered, i.e., η, from 0 to 1 with a step size of 0.1. This gives a total of 891 observations. We observed that attractiveness of the improvement strategy over the diversification strategy is convex increasing in η. See Figure 3.8.
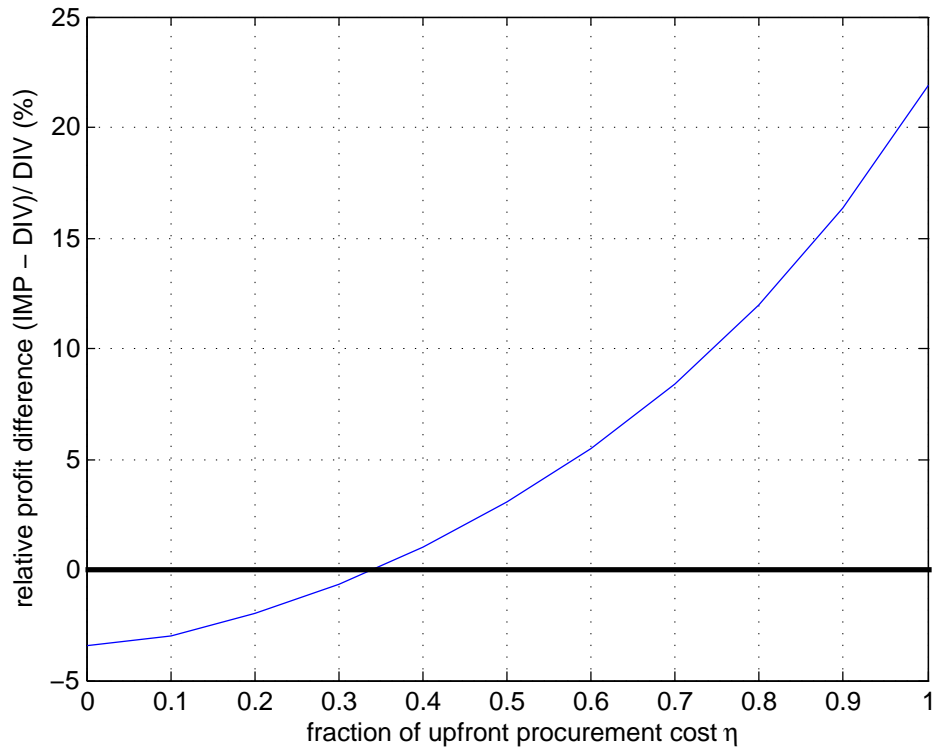
Figure 3.8: Relative profit difference of the improvement strategy over the diversification strategy as the fraction of the upfront procurement cost η increases

This convex increasing behavior is consistent across different scenarios, regardless of the demand and supply uncertainty or the unit procurement cost. This finding has significant

practical implications: the improvement strategy becomes increasingly attractive as the firm incurs a higher fraction of upfront procurement cost. In many global sourcing environments, a firm will typically incur a fraction of the procurement cost upfront, which may include inspection cost, paperwork cost, and audit cost, etc. Whether these costs are accounted for in the total procurement cost can significantly influence the attractiveness of the improvement or diversification strategy.

3.8.2.5   Improvement Success Probability

We varied the improvement success probability θ from 0 to 1 with a step size of 0.1, which gives a total of 891 observations. As one might expect, the optimal profit under improvement strategy is increasing in θ. Our numeric observations indicate that the relative attractiveness of the improvement strategy over the diversification strategy is approximately linear in the success probability θ. See Figure 3.9. Note that the magnitude of the difference is also approximately linear in θ.

Figure 3.9: Relative profit difference of the improvement strategy over the diversification strategy as the probability of improvement success θ increases

3.8.2.6 Expected Capacity

We varied the expected capacity E_K from 50 to 150 with a step size of 10. This gives us a total of 297 observations. On average, the relative attractiveness of the improvement strategy over the diversification strategy is concave increasing in the expected capacity. See Figure 3.10.
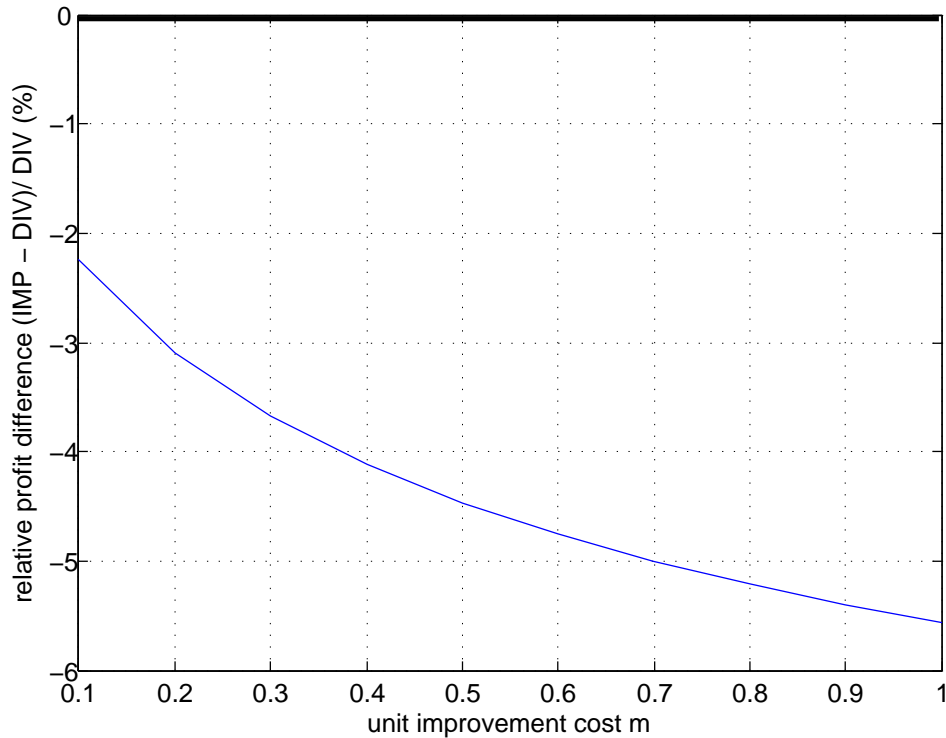
Figure 3.10: Relative profit difference of the improvement strategy over the diversification strategy as the expected capacity increases

This does not imply, however, that for any given system parameters, the relative attractiveness of the diversification strategy over the improvement strategy is increasing in E_K. We observed that when the unit procurement cost is high, i.e., $c = 0.9$, and/or the supply uncertainty is high, i.e., $CV_K = 0.5$, the relative attractiveness of the diversification strategy over the improvement strategy is initially concave increasing in E_K, but is convex decreasing in E_K as E_K becomes large. See Figure 3.11. Note that we set $c = 0.9$, $m = 0.1$, $CV_K = 0.3$, and all other parameters at the base case scenarios to obtain Figure 3.11. When the expected capacity is low, an increase in the expected capacity benefits the improvement strategy more than the diversification strategy because the improvement strategy is more constrained. As

the expected capacity increases, the marginal benefit to the improvement strategy decreases. As the expected capacity becomes very high, neither improvement nor diversification strategy faces significant supply risk and the difference between the two strategies diminishes.
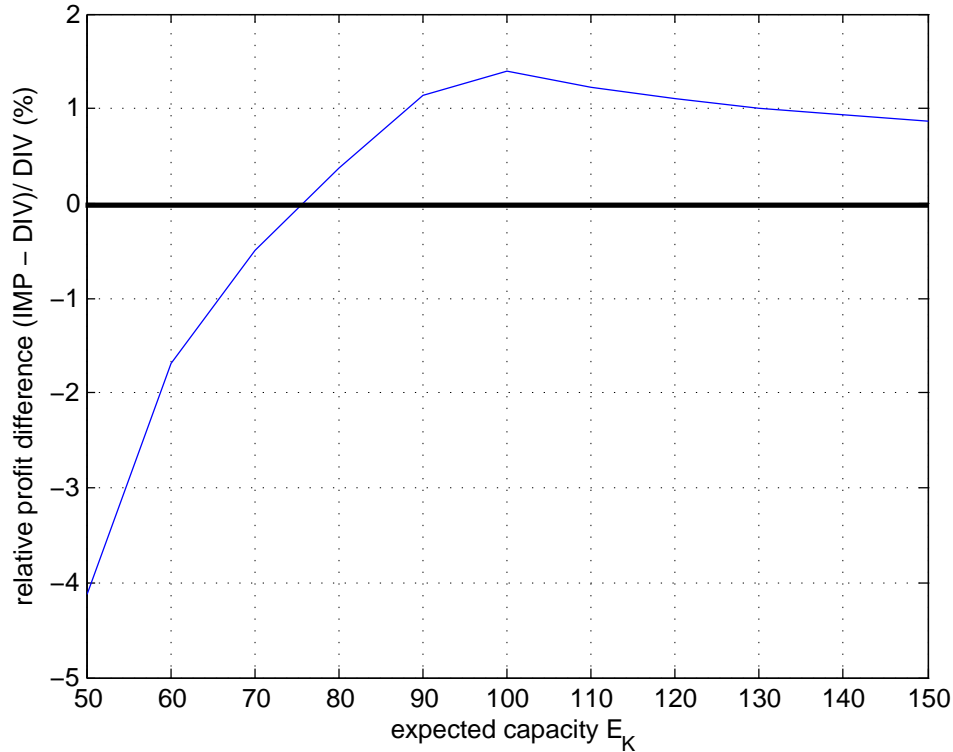
Figure 3.11: Relative profit difference of the improvement strategy over the diversification strategy as the expected capacity increases (with higher unit cost)

We note that in the limiting case as the expected capacity grows large, the relative difference in the optimal expected profit between the diversification strategy and the improvement strategy will approach zero. The reason is that with sufficiently high capacity, the improvement strategy will degenerate into a single sourcing case where the firm makes no improvement effort. On the other hand, the diversification strategy will also degenerate into a sole sourcing case - even though the firm has the option of dual sourcing. Essentially, as E_K grows large,

the supply risk diminishes and sole sourcing becomes optimal.

3.8.2.7 Capacity Coefficient of Variation

We varied the CV_K from 0.05 to 0.5 with a step size of 0.05. This gives a total of 270 observations. We observed that, on average, the relative attractiveness of the improvement strategy over the diversification strategy is a convex function in the CV_K. See Figure 3.12.
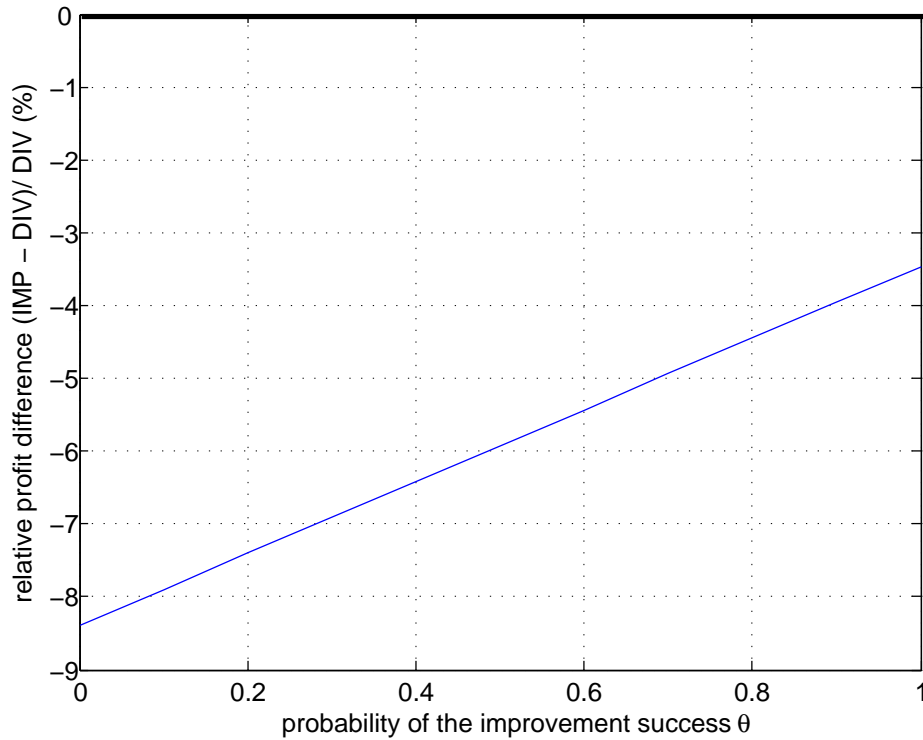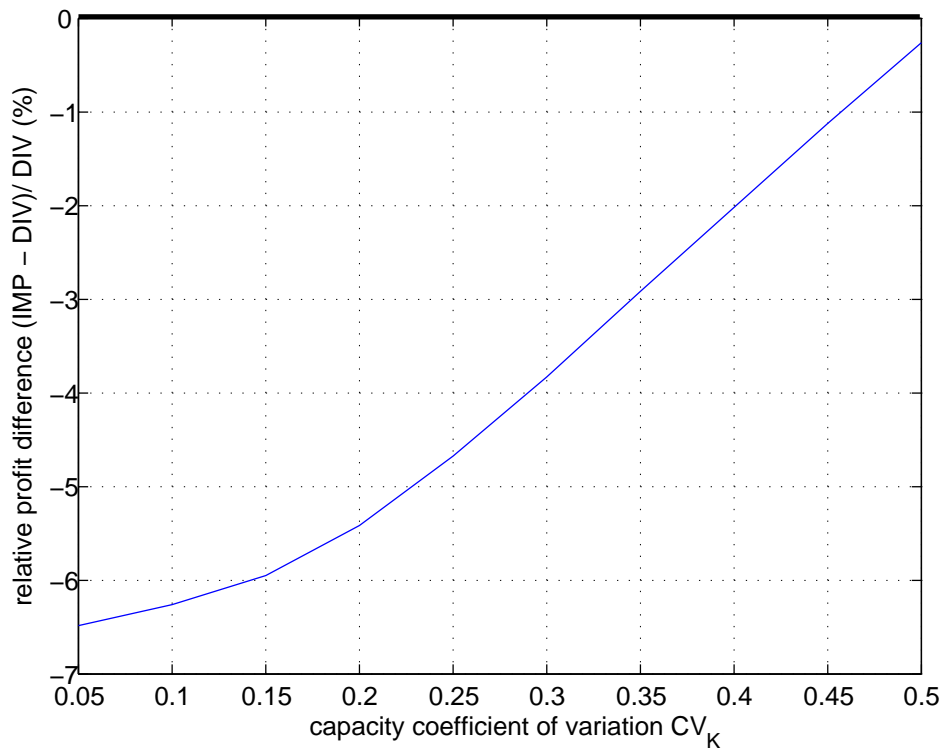

Figure 3.12: Relative profit difference of the improvement strategy over the diversification strategy as the capacity coefficient of variation increases

We observed, however, that the relative attractiveness of the improvement strategy over the diversification strategy is in general not monotonically increasing. In fact, the relative attractiveness of the improvement strategy over the diversification strategy is monotonically

increasing only when the expected capacity is low and the unit procurement cost is low. Otherwise, the ratio is initially decreasing and then increasing in the CV_K. This indicates that the attractiveness of the improvement strategy is initially decreasing in CV_K when CV_K is low, but then is increasing in CV_K as CV_K increases. See Figure 3.13, where the figure is obtained by setting $c = 0.6$, $m = 0.1$, $E_K = 90$ and all other parameters at the medium base case scenario.
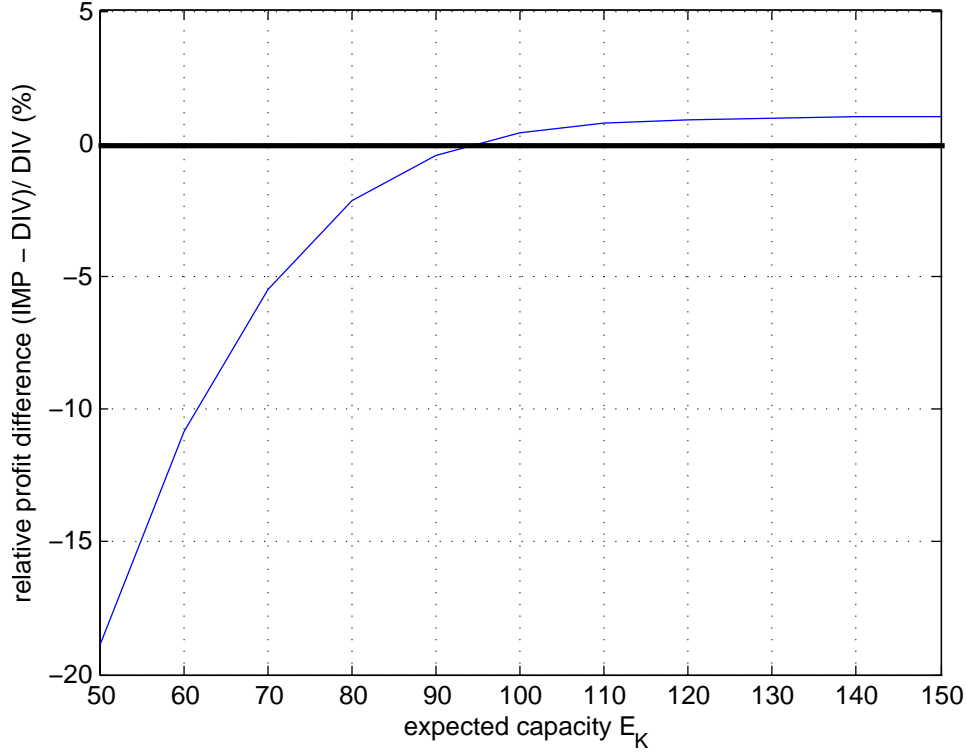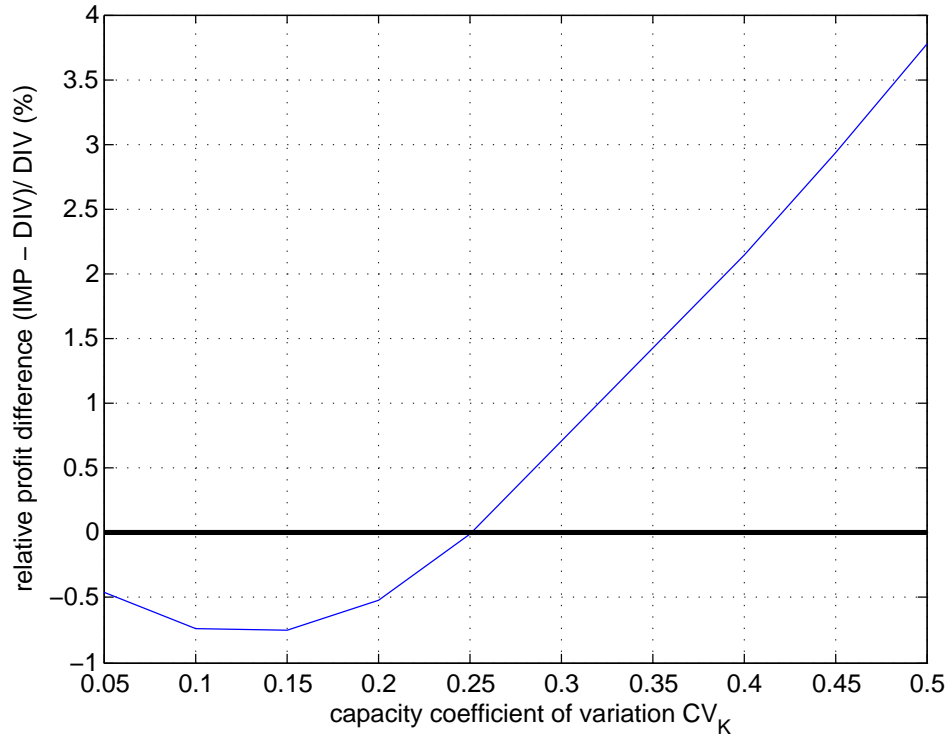

Figure 3.13: Relative profit difference of the improvement strategy over the diversification strategy as the capacity coefficient of variation increases (base case scenario)

Note that the difference in the optimal expected profit between the improvement strategy and the diversification strategy also follows a similar trend. When the CV_K is low, a marginal increase in the CV_K does not affect the diversification strategy as much as the improvement strategy, because the diversification strategy has sufficient supply capacity. When CV_K is high, however, the improvement strategy becomes much more attractive because the diversification

strategy can no longer effectively provide sufficient supply, whereas the improvement strategy can effectively reduce the supply risk.

3.8.3   The Effect of Distributional Shape in Supply Uncertainty

In our numeric study, we assumed a special class of the Weibull family of distribution, i.e., exponential distribution, for the supply uncertainty. It is of interest to understand how the distributional shape of the random shock to the supplier capacity affect the attractiveness of the improvement versus the diversification strategy. To investigate this effect, we set all parameter values at the base case scenarios (medium cases) described in §3.8.1 and vary the shape parameter α from 0.6 to 4.0. Because the density of the Weibull distribution is more sensitive for $\alpha <= 1$ and less sensitive for $\alpha > 1$, we use a step size of 0.1 for $\alpha \leq 1$ and a step size of 0.2 for $\alpha > 1$. Note that the shape of the Weibull distribution changes significantly when varying α from 1 to any point greater than 1; the density function becomes unimodal for $\alpha > 1$ and approximates normal distribution when α gets larger.

We observed that relative attractiveness of the improvement strategy over the diversification strategy is in general not monotonic in the shape of the supply uncertainty distribution. See Figure 3.14.
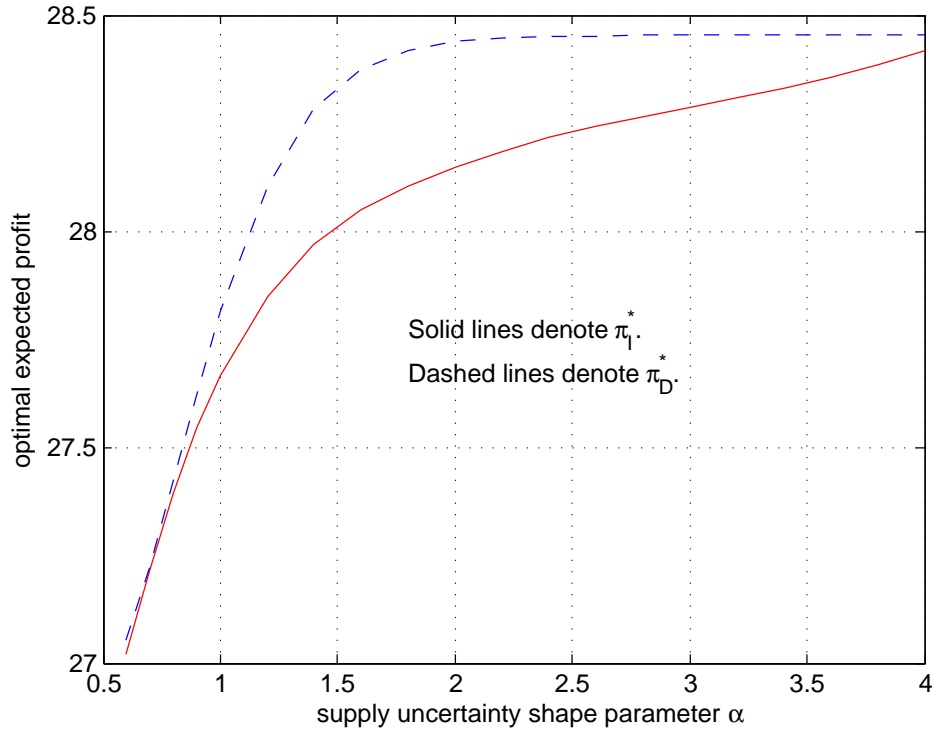
Figure 3.14: The optimal expected profit for the diversification and the improvement strategy as the supply uncertainty shape parameter α increases

Note that when the unit procurement cost is high, e.g., $c = 0.9$, then $\pi_I^* - \pi_D^*$ approaches 0 as α increases. One may conjecture that, given the shape of the π_I^* and π_D^* in Figure 3.14, with appropriate parameter values $\pi_I^* - \pi_D^*$ may cross 0 more than once. Our numeric study indeed confirms this conjecture. We repeat the above study by setting the unit procurement cost $c = 0.9$, the unit improvement cost $m = 0.1$, and all other parameters at the medium base case scenarios. We observed that, when the α value is low, i.e., $\alpha \leq 1.5$, the improvement strategy is strictly preferred. As α increases beyond 1.5, however, the diversification strategy becomes more attractive. As the α value further increases, $\pi_I^* - \pi_D^*$ approaches 0, i.e., the firm is indifferent between the diversification and the investment strategy. In this case, the firm makes no improvement effort as the supply risk is sufficiently low. In essence, an increase in

the α value reduces the uncertainty about the capacity loss, but the expected capacity loss remains the same. When the unit procurement cost is high, the effective demand is lower than the expected capacity and therefore the supply risk becomes lower. See Figure 3.15.
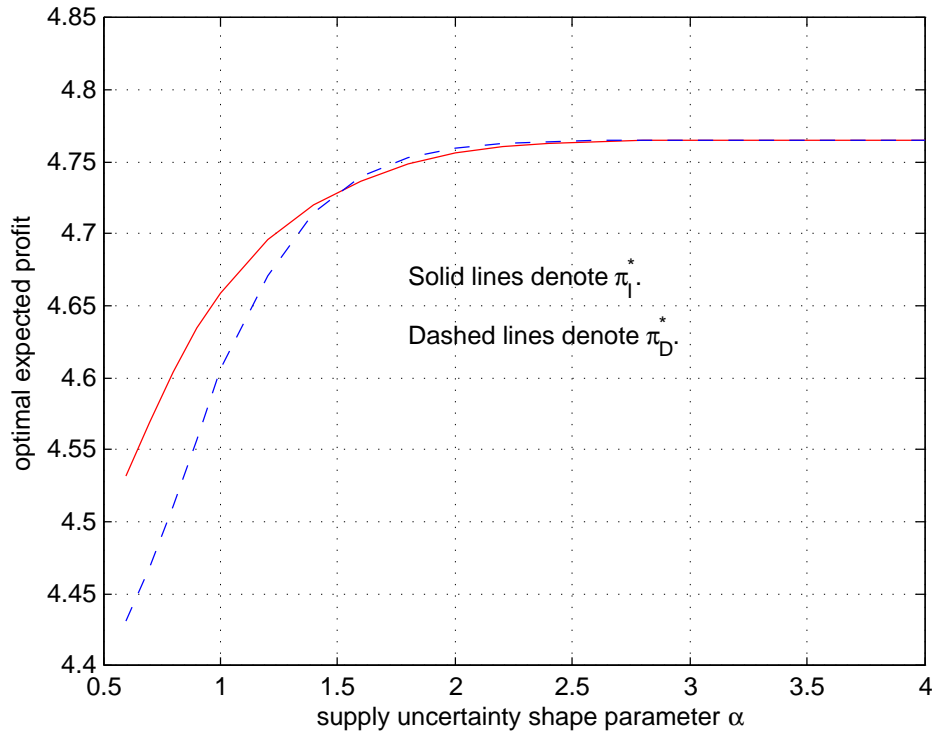
Figure 3.15: The optimal expected profit for the diversification and the improvement strategy may cross twice as the supply uncertainty shape parameter α increases

3.8.4 Improvement with Dual Sourcing

It is of interest to understand whether with improvement strategy dual sourcing is still of significant value and if so, under what conditions dual sourcing is still a valuable option. We investigate this question using the basic study design described in §3.8.1. This gives us a total of 81 observations for the improvement strategy with the option of dual sourcing. We then compare the optimal profit under this hybrid strategy, i.e., improvement with dual sourcing,

with that of the improvement strategy only. Given our earlier numeric study result, one may conjecture that the dual sourcing option after the improvement strategy should be of significant value. This is indeed correct. We observed that the dual sourcing option after the improvement strategy still gives the firm between 1% and 9% additional benefit. See Figure 3.16. The dual sourcing option is most valuable when the unit procurement cost is low. The intuition is that, when the unit procurement cost is low, the effective demand is high and therefore the improvement strategy alone is more likely to be capacity constrained. In this case, the inherent benefit of dual sourcing effectively compensates for this capacity shortage.
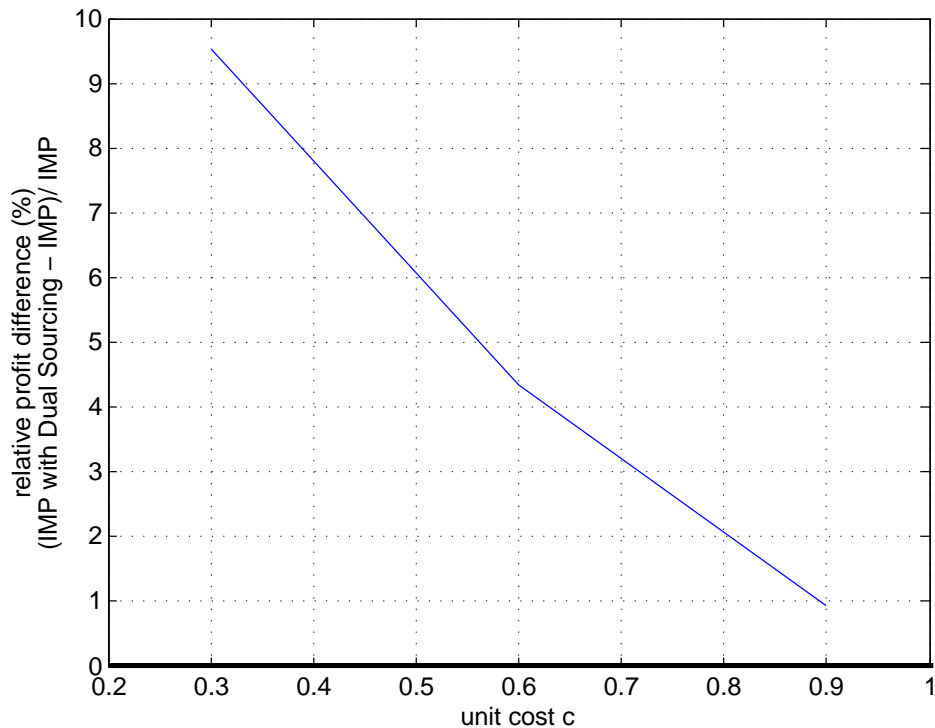
Figure 3.16: Relative profit difference of the improvement w/ dual sourcing strategy over the pure improvement strategy as the unit procurement cost increases

While the dual sourcing option after the improvement strategy can give the firm significant benefit, it is unclear whether this hybrid strategy is significantly better than the best of the

improvement and the dual sourcing strategy. To investigate this question, we repeat the above numeric study, but calculate the relative percentage increase in the optimal profit under the hybrid strategy over the maximum of the pure improvement and the dual sourcing strategy. We observe that the benefit of the hybrid strategy is significantly diminished. In fact, we found that the maximum benefit of the hybrid strategy is less than 0.07% over the maximum of the pure improvement and the dual sourcing strategy. When the unit procurement cost increases, the benefit of the hybrid strategy approaches zero. See Figure 3.17.
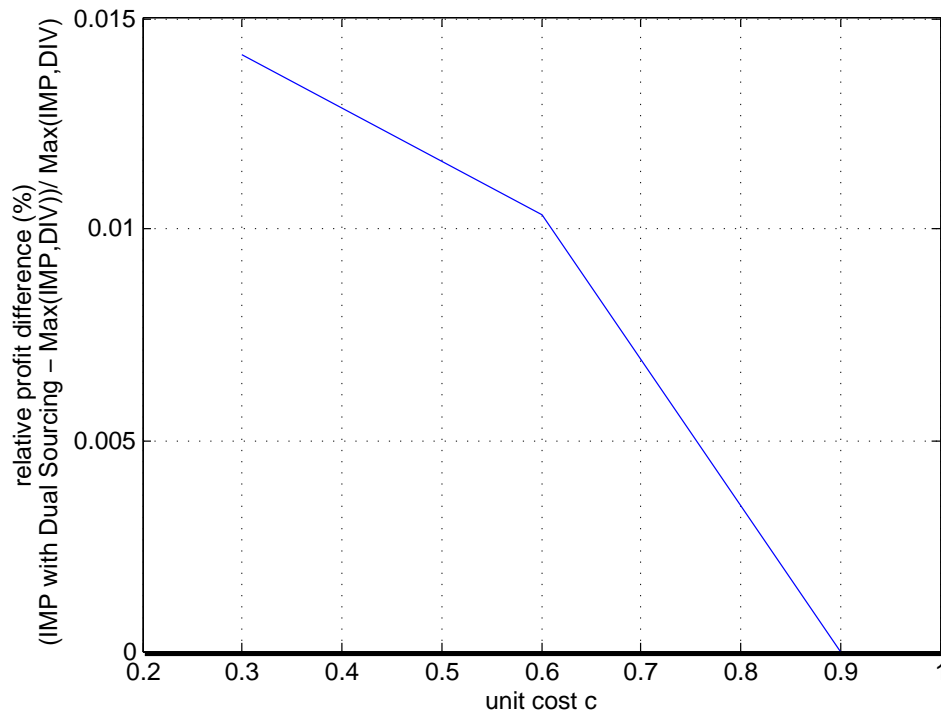
Figure 3.17: Relative profit difference of the improvement w/ dual sourcing strategy over the best of the pure improvement or diversification strategy as the unit procurement cost increases

Note that the above finding offers significant managerial insights. If the firm chooses the best strategy, either the dual sourcing or the improvement strategy, for its particular operating environment (such as unit cost and demand volatility), then it is of negligible value to

employ both improvement and dual sourcing strategy. The additional benefit of the combined strategy is simply not sufficient to warrant its implementation. On the other hand, however, if the firm's operating environment is subject to frequent change, then the hybrid strategy of both improvement and dual sourcing can be of significant value. A caveat, however, is that the firm may not be able to adopt and switch to alternative strategies frequently, either due either to operational constraints or strategic considerations. In such cases, the hybrid strategy can be adopted as a stable strategy, as opposed to switching between dual sourcing and the improvement strategy as the industry environment changes.

3.8.5   Dual Improvement with Dual Sourcing

In the most general case, the firm may choose to invest in and procure from both suppliers. It is of interest to understand whether it is worthwhile to make improvement efforts to both suppliers, and if so, under what conditions the dual improvement strategy is most valuable. To investigate this problem, we repeat a study similar to that in §3.8.4, but allow the firm to make improvement to both suppliers. We therefore have a total of 81 observations. We found that, compared with the single improvement dual sourcing strategy (SID), the dual improvement dual sourcing strategy (DID) is only marginally better. On average, the DID strategy is 0.03% better than the SID strategy, with a maximum improvement of 0.12%. We observed that the DID strategy is relatively more desirable when the CV_K is high. This is as expected because the diversification strategy alone is not efficient when the supply risk is highly volatile. The SID strategy is an asymmetric strategy and only partially enhances one supplier's reliability state. In these situations, the DID strategy provides additional, although marginal, improvements over the SID strategy. Based on the above numeric observations, it is

in general not in a firm's best interest to engage in dual improvement strategy. The finding has a significant managerial implication: the presence of the improvement opportunity will diminish the benefit of diversification strategy and along the way, marginalize the supplier which has not been selected for the improvement effort.

3.9   Conclusion

In this research, we study a firm's optimal procurement problem when suppliers are unreliable. We relax the common assumption that the suppliers' reliability is exogenous and investigate how the presence of the improvement opportunity influences a firm's preference of diversification strategy versus improvement strategy. We use the general stochastic dominance concept to model the capacity loss to the supplier. We fully characterize the firm's diversification problem and the improvement problem. We analytically prove that both the pure improvement strategy and the improvement with dual sourcing strategy are well behaved in the improvement effort. In addition, we analytically prove that a firm's preference of the diversification strategy is decreasing in the heterogeneity of the supplier characteristics.

We also analytically investigate how a firm's risk exposure changes as the demand risk increases. We find that, when the demand risk is low, the firm will use the single sourcing strategy, even if the firm has the opportunity to dual source. However, if the two suppliers are identical, then the firm will never use single sourcing strategy. As the demand risk increases, the firm will use dual sourcing strategy to cope with the demand risk. As the demand risk further increases, the firm will use both dual sourcing strategy as well as quantity hedge to cope with the supply risk. Note that the firm will not use quantity hedge unless it uses the

dual sourcing strategy.

We found that the firm's preference of the diversification strategy relative to the improvement strategy is in general not monotonic in the supplier characteristics. We found that a firm is more likely to prefer the diversification strategy when the procurement cost is very low or is very high, but the firm is more likely to prefer the improvement strategy when the procurement cost is moderate. We also found that the volatility of the demand has significant impact on the attractiveness of the improvement strategy. In extreme cases where the demand is highly volatile or very stable, then the improvement strategy is less likely to be preferred to the diversification strategy. On the other hand, the improvement strategy is more likely to be preferred when the demand volatility is moderate. This result is due to the interplay of the effective demand and the effective supply.

We found that the distributional shape of the capacity loss has important implications for the attractiveness of the diversification and improvement strategies. We found that the improvement strategy is more likely to be preferred when the unit procurement cost is high and the volatility of the capacity loss is high. A higher unit cost reduces the effect demand, which mitigates the capacity disadvantage of the improvement strategy. A higher volatility of the capacity loss results in a higher marginal return for the improvement strategy while exerting higher pressure on the diversification strategy.

Our numeric study indicates that the improvement strategy is increasingly preferred as the fraction of the upfront procurement cost increases. If the firm pays only for what is delivered, then everything else being equal, the diversification strategy is more likely to be preferred. However, as the firm incurs a higher fraction of the upfront procurement cost, the improvement strategy becomes more attractive.

For future research, we conjecture that the improvement strategy is more likely to be preferred in a multi-period setting, where the benefit of the improvement strategy can be accrued over multiple periods. It is also of interest to investigate the incentive issues between suppliers and the buying firm. The buying firm, for intellectual property rights concerns, for example, may hesitate to invest in a supplier. The buying firm may decide to first learn about the supplier's true capacity performance before investing in capacity improvement efforts. On the other hand, with competing suppliers, each supplier may have its own incentive to improve its capacity performance.

Chapter 4

Regulatory Trade Barriers Risk

4.1   Introduction

With accelerating globalization, firms have become increasingly dependent on suppliers across
the globe to maintain competitive advantage. However, this increased dependency on global
suppliers also creates myriads of potential risks in the firm's supply network. A well imple-
mented supply risk management strategy is essential for firms to capture the advantage of
global sourcing without falling prey to the perils of potential supply risks. Clearly, to develop
effective risk management strategies, firms must consider particular types of risk that they face
in their supply network. In general, the supply risk includes capacity, leadtime, and quality
risk, as well as regulatory risk, geo-political, and foreign exchange risk. Regulatory trade risk
is unique to global sourcing and can pose significant risk to a broad range of firms engaged in
global sourcing. While the World Trade Organization (WTO) consistently promotes free trade
and discourages any regulatory trade barriers, authorities in individual countries or economic
entities have considerable latitude in imposing regulatory trade barriers to protect domestic
markets.

With the WTO's free trade effort, for example, the Multi Fibre Agreement (MFA), a framework for international textile trade for the past 30 years, was dismantled in 2005. The expiration of MFA was heralded as the beginning of an era where firms can freely import and export textile supplies and products. Within months, however, many countries, including the US and the EU, invoked the safeguard clause and essentially rectified quantity-based trade barriers. While these safeguard measures are slated to expire in a few years, it is widely believed that some forms of trade barriers will be maintained for the foreseeable future.

Many industries, such as paper, chemical, steel, and agriculture, face similar types of trade barriers. Note that non-quantity based trade barriers, such as antidumping (AD) rules, are also in widespread use. In fact, Blonigen and Prusa (2001) note that "since 1980 ... more AD duties are now levied in any one year worldwide than were levied in the entire period of 1947-1970." Therefore, regulatory trade barriers pose a significant risk to firms operating in a global environment, and an effective strategy to mitigate such risk is essential for firms to maintain competitive advantage.

The quantity based trade barriers, commonly known as voluntary export restraints (VER), are implemented against products by the country of origin. These VERs are usually implemented bi-laterally, where the importing country sets an upper limit on the total amount of the specific product (for example, a specific product can be classified under the Harmonized System Codes) that the exporting country can sell for a fixed period of time (usually on an annual basis). The following is a formal definition of VER from the Organization for Economic Co-operation and Development (OECD).

"Voluntary export restraints (VER) are arrangements between exporting and im-

porting countries in which the exporting country agrees to limit the quantity of specific exports below a certain level in order to avoid imposition of mandatory restrictions by the importing country. The arrangement may be concluded either at the industry or government level."

Such quantity restrictions are often targeted against countries that have a compelling advantage in the specific category of restricted products. While sourcing from these targeted countries would bring the most economic benefit to the buying firm, such sourcing activity also poses the most significant risk due to uncertainty around the VER. The suppliers in the targeted country may not be able to secure a sufficient amount of VER allocation to export the product. Even if the supplier were able to secure the necessary amount of VER allocation, the associated cost of obtaining the necessary amount of VER allocation may make it uneconomical to continue to supply the product.

4.1.1   Mitigation Strategies

Suppose a buying firm in the domestic country, denoted as DOM, is interested in procuring a particular product globally. Suppose a low cost country, denoted as LCC, is most efficient at manufacturing the product and is subject to VER control. Note that LCC need not to be the only country that can supply the product. In fact, suppliers in a medium cost country, denoted as MCC, which are not subject to VER, can also supply the product, but at a higher cost because of their higher labor costs or less efficient operations. In the absence of VER, it is cheaper, on a per-unit basis, for the firm to procure the product from a supplier in the LCC. However, sourcing solely from LCC brings significant risk to the firm's supply because the supplier may fail to obtain a sufficient amount of VER allocation.

One often used approach to mitigate the VER control is outward processing arrangements (OPA). Different entities, such as the US, Hong Kong, and the EU, have somewhat different definitions and procedures of outward processing arrangements. This chapter most closely follows Hong Kong (for indirect OPA) and the EU (for direct OPA) practices. The following is a formal definition by the Council of the European Union.

Outward Processing is an EU customs duty relief scheme provided for under Council Regulation (EEC) No. 2913/92 and implementing Commission Regulation 2454/93 (amended). It allows Community goods to be temporarily exported from the customs territory of the Community in order to undergo processing operations or repair and the compensating products resulting from these operations to be released for free circulation in the customs territory of the Community with total or partial relief from import duties. Outward processing enables businesses to take advantage of cheaper labour costs outside the EC, while encouraging the use of EC produced raw materials to manufacture the finished products. Goods may be also temporarily exported to undergo processes not available within the Community.

Therefore, to mitigate the supply risk under quantity based trade barriers, firms may adopt several different strategies in their supply chain design. Figure 4.1 illustrates the structure of several alternative mitigation strategies.
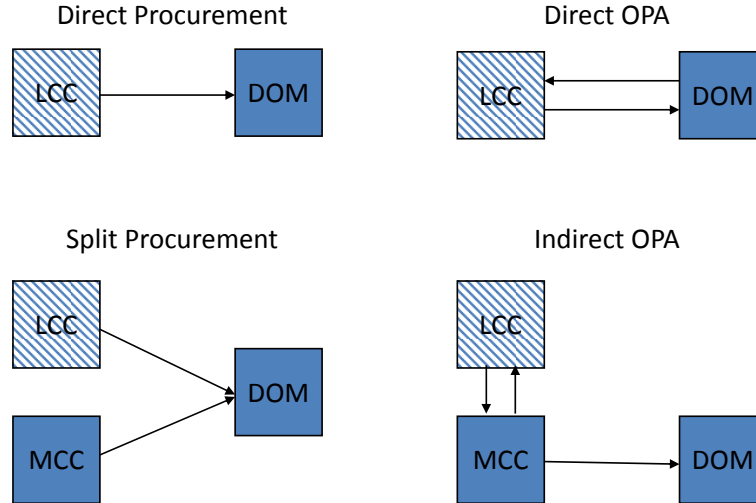
Figure 4.1: Four mitigation strategies under regulatory quota restrictions

Before we describe in detail each of the four strategies in Figure 4.1, we first list the most salient features associated with each strategy.

- Direct Procurement: passively accepts full quota risk; incurs the lowest pre-quota, per-unit procurement cost.

- Split Procurement: partially mitigates the quota risk; the average per-unit procurement cost is moderate, but less than the domestic procurement cost.

- Direct OPA: completely removes the quota risk; must maintain domestic production to qualify; have a leadtime advantage due to domestic production.

- Indirect OPA: completely removes the quota risk; the per-unit procurement cost depends on the third party outward processing arrangements.

The direct procurement strategy is a basic strategy that pursues the cheapest per-unit procurement cost. While this strategy completely ignores the quota risk, its per-unit cost

advantage can be so significant that, at times, the cost advantage outweighs the potential quota risk. On the down side, however, if the quota risk turns out to be significant, the firm that uses this strategy may find itself having no supply to serve its customers.

An intuitive approach to mitigate the quota related supply risk is to split source from both LCC and MCC. Since a supplier in MCC is not subject to VER, the firm can partially guarantee its supply, even if the supplier in LCC fails to obtain the sufficient amount of VER. The downside of such an approach is that the average procurement cost may be higher. Note that the strategy of sole sourcing from LCC or MCC can be considered as a special case of the split sourcing strategy.

An alternative approach to reduce or eliminate the supply uncertainty is to utilize the outward processing arrangement (OPA)[1]. Many countries, including the US, the EU, and Hong Kong, have OPAs with other countries. Under OPA, the firm is able to perform certain production activity domestically and let suppliers in a VER controlled country perform additional or full processing. The finished product, subject to specific regulations, will not be subject to VER control[2] or full tariffs. Firms may employ either the direct OPA or indirect OPA approach to remove the VER control.

To qualify for direct OPA, the buying firm has to 1) maintain a full domestic production line for the finished product, where the domestic production must be no less than a certain fraction[3] of the firm's total procurement quantity, and 2) utilize domestically sourced raw materials for

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[1]Also known as outward processing trade (OPT) in the EU.

[2]The product under OPA may subject to additional quantity based restrictions, but such restrictions are normally satisfied before the product undergoes OPA processing.

[3]The minimum fraction of the domestic production quantity is regulated by competent authorities on a fixed time period basis, typically on an annual basis. The minimum fraction is defined relatively to the firm's total procurement quantity, such that the fraction of the domestic production must not fall below a threshold of the firm's total procurement.

the LCC outward processed product. See DTI (2006). For example, knitted cotton shirts from China are subject to VER control for export to the EU. However, a firm based in Poland, for example, can avoid the VER control by sourcing the fabric from the EU, shipping it to China for production and importing the finished product back to the EU without being subject to VER control, as long as the firm maintains a full production line in Poland that satisfies the minimum production quantity.

Firms do not have to use the direct OPA program from their domestic countries. Instead, the firm may use third party OPA programs. For example, a firm based in the EU may source the product from Hong Kong, which has an OPA program with China. While China is under VER control for export to the EU, Hong Kong is not subject to VER control. Therefore, the supplier in Hong Kong can split its manufacturing operation between Hong Kong and China such that the finished product is considered to originate in Hong Kong. The firm in the EU can, therefore, procure the product without the risk associated with VER, although such an arrangement typically increases the total procurement cost. "Producing this way costs roughly 35% to 40% more than if we make everything directly in China," says the managing director of the TAL Group and vice chairman of the Hong Kong Textile Council, Dr. Harry Lee (Windle, 2005).

In summary, we note that the direct procurement strategy has the highest risk exposure due to VER. The split procurement strategy can significantly reduce the risk exposure, but cannot completely eliminate the risk exposure except in the degenerate case where the firm sole sources from MCC. On the other hand, both the direct and indirect OPA approaches eliminate the risk exposure, but in different ways. The direct approach requires the firm to maintain a certain level of domestic production (which incurs a shorter leadtime). Under the direct OPA

approach, therefore, products are produced both domestically and in the foreign LCC country. On the other hand, the indirect approach does not maintain any domestic production. Under the indirect OPA approach, each unit of the product is split processed between the LCC and the MCC. Therefore, the accounting of the per-unit procurement cost is quite different between the two OPA strategies. See the model section (§4.3) for details.

4.1.2 List of Acronyms

In closing, we list the acronyms frequently used throughout the chapter.

AD Antidumping Duties

DOM Domestic country

LCC Low cost country that is subject to VER

MCC Medium cost country that is not subject to VER

MFA Multi Fibre Agreement

OPA Outward Processing Arrangement

VER Voluntary Export Restraints

WTO World Trade Organization

4.2   Literature

To date, research on the implications of trade barriers on the firm's sourcing strategies is relatively sparse in the operations management literature. There exists, however, a stream of literature on global operations, where specific issues, such as global transportation and/or foreign exchange rate, are explicitly modeled.

Arntzen et al. (1995) study global supply chain management issues at Digital Equipment Corporation. They developed a large scale mixed-integer program to solve the multiple location and transportation problem. Their mathematical program explicitly incorporates tariff rules in different countries. In a similar vein, Munson and Rosenblatt (1997) also use the mixed-integer linear programming approach to solve global sourcing problems when the local content rule must be satisfied. Li et al. (2007) study a raw material sourcing problem in the presence of the local content rule as well as value-added requirements. The focus of all the above-mentioned papers is to solve practical, large scale optimization problems. In contrast, the purpose of this chapter is to explore different regulatory-risk mitigation strategies so as to provide managerial insights.

A recent paper that is also related to local content rules is Kouvelis et al. (2004), which uses a linear programming model to investigate a global plant location problem. Their mathematical programming model incorporates global location related constraints, such as subsidy, tariff/tax, and local content rules. They explore how these global location related constraints influence the optimal plant locations. Note that the local content rule is a hard constraint on every unit of product. This is different from some other types of trade barriers, such as quotas, where the constraint is binding only when the total production exceeds the pre-set limit.

Because most trade barriers, such as quotas, can also be modeled alternatively as stochastic cost problems, this research is also related to the stochastic cost (price) models in the operations literature. In an earlier paper, Kalymon (1971) studies a multi-period purchasing problem where the future purchasing prices are Markovian. The paper proves that a state-dependent (s, S) policy is optimal. Similar research that incorporates the stochastic purchasing cost/price includes Özekici and Parlar (1999); Erdem and Özekici (2002); Berling and Rosling (2005). Note that all these papers investigate the form of the optimal purchasing policies for a given procurement configuration. They do not contrast different mitigation strategies.

This research is also tangentially related to the large stream of literature in contracting theory. In this stream of research, a typical model involves either the supplier, e.g., (Araman and Özer, 2003), or the buyer, e.g., (Akella et al., 2002; Martinea-de-Albeniz and Simchi-Levi, 2003; Seifert et al., 2004; Sethi et al., 2004), that designs a portfolio of contracts to manage supply or demand uncertainties. These papers are related to our research because they consider the role of the spot market, which provides an instantaneous access to inventories or products at an expected price premium. Typically, these spot markets exist in commodity market contexts, where products are standard but prices are subject to significant variation over any period of time. Here the spot market serves as an emergency source, which will be used if other sources of supply are insufficient to meet demand. The use of the spot market, however, is not a planned decision a priori. In this chapter, the use of a supplier in the VER controlled LCC is not a result of insufficient supply and the procurement quantity is an active decision made by the firm. In closing, we note that Kouvelis and Li (2006) also consider the existence of spot market in hedging leadtime risks.

4.3 Model

Before developing models for each specific strategy, we first discuss higher level modeling considerations and then describe each specific model. One important modeling consideration is concerned with the key drivers that affect the attractiveness of each different mitigation strategy. The following is a list of high level drivers that may influence a firm's preference for a particular strategy.

A1. Landed unit cost. The landed unit cost includes the unit procurement cost, transportation cost, and tariff.

A2. VER uncertainty. The likelihood that a supplier is able to secure sufficient allocation to clear VER control.

A3. Demand uncertainty. The level of demand uncertainty at the time of procurement.

A4. Leadtime. The production and transportation leadtime.

A5. Foreign exchange rate. The stability of foreign exchange.

In this chapter, we do not focus on the foreign exchange rate risk. Instead, we focus on the effect of A1-A4 on the attractiveness of each mitigation strategy. Note that, even in the absence of the production and transportation leadtime as well as the foreign exchange rate risk, the relative attractiveness of the indirect OPA structure versus the direct OPA structure cannot be unambiguously determined. Therefore, with transportation and production leadtime differences, the direct OPA strategy may be more attractive than the indirect OPA approach.

We model the four mitigation strategies as the newsvendor type of model, where the firm's objective is to maximize its expected profit. The unit procurement cost is assumed to be exogenous, and the firm's decision is to determine the optimal procurement quantity. For the direct procurement and the split procurement strategy, we model the VER uncertainty as follows. Because the allocation of VER is tradable among the suppliers, any supplier can, theoretically, obtain sufficient allocation of VER for exportation. However, the unit price of VER is uncertain. If the price is too high, then it is not economical to ship the finished product. If the price of VER is acceptable, then the cost of VER is either borne by the supplier or shared with the buying firm. For simplicity, we assume that the buying firm absorbs the price of VER and makes the decision of whether to ship the finished product. Intuitively, if the price of VER is above a certain threshold, the firm will not ship the final product. The firm's decision of whether to ship the finished product is based on the realization of the price of VER. In addition, we assume the firm also observes the realized market demand, because the production leadtime is much longer than the transportation leadtime. Such an approach allows us to model the VER uncertainty as a stochastic cost newsvendor type of model. See Figure 4.2.
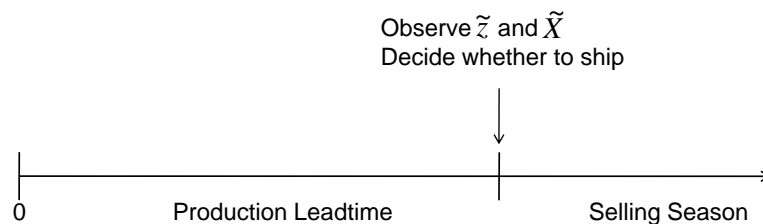
Figure 4.2: Time line for the stochastic cost model ($\tilde{z}$: Price of VER. $\tilde{X}$: Demand)

Note that the MCC in the split procurement strategy need not to be the same as the MCC in the indirect procurement strategy. For purely expositional reasons, we assume hereafter that the two MCCs share similar cost structures.

4.3.1 Notation

We now introduce the notation for those system parameters common to all four strategies. Other strategy-specific notation will be introduced, as necessary, when describing a particular strategy. Let r and p represent the unit revenue and the unit penalty cost, respectively. Let c_0, c_1, and c_2 represent the unit procurement cost from LCC[4], MCC and DOM, respectively. The unit procurement cost should be interpreted as total landed cost, which includes the production cost as well as the transportation cost. We assume without loss of generality that $c_0 \leq c_1 \leq c_2$. Similarly, we use s_0, s_1, and s_2 to represent the unit salvage values within LCC, MCC, and DOM, respectively. In addition, let y_0, y_1, and y_2 to denote the firm's procurement quantity from LCC, MCC and DOM, respectively. We adopt the convention that LCC is a foreign country subject to quota restrictions. We use MCC to represent the foreign country that is not subject to quota restrictions.

For the quota related cost, let $\tilde{z}$ represent the unit stochastic quota cost. In addition, we use $G(\cdot)$ and $g(\cdot)$ to represent the distribution and the density function of the quota cost, respectively.

On the demand side, let $\tilde{x}$ denote the random seasonal demand. We use $F(\cdot)$ and $f(\cdot)$ to represent the distribution and the density function of the seasonal demand, respectively. In addition, we use μ_0 and σ_0 to denote the mean and the standard deviation of the demand.

In what follows, we characterize each procurement strategy and discuss relevant sensitivity analysis on the optimal procurement solutions.

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[4]The LCC cost c_0 may be slightly different across different strategies. For example, the direct OPA strategy may incur a higher LCC cost because of the raw material procurement and shipment cost. However, many suppliers in LCC also source raw materials from overseas. To focus on the main effect of the quota issues, we assume the LCC unit cost is identical across different strategies.

4.3.2 Direct Procurement

Using our notational convention discussed earlier, we use r, p, and $\tilde{z}$ to represent the unit revenue, shortage penalty and stochastic quota cost, respectively. Recall that we use c_0 and y_0 to denote the unit procurement cost and the firm's procurement quantity, respectively. Note that if the firm salvages the product in LCC (because of the prohibitive quota price, for example), the unit salvage value is s_0. If, however, the firm salvages the product in DOM, the unit salvage value is s_2. We do not make assumptions on the relative magnitude of s_0 and s_2. We use $\tilde{x}$ denote the random demand. At the second stage, the firm observes the realized unit quota cost z as well as the market demand x. In what follows, we first characterize the firm's second stage basic decision problem. We then drive the optimal procurement quantity for the direct procurement strategy.

At the second stage, the firm observes the realized demand x and the unit quota price z. The firm's objective is to determine the shipment quantity $\hat{y}_0$ to maximize its revenue, i.e.,

$$\pi(\hat{y}_0|y_0, z, x) = \max_{0 \leq \hat{y}_0 \leq y_0} \left\{ r \min(\hat{y}_0, x) - z\hat{y}_0 + s_2(\hat{y}_0 - x)^+ + s_0(y_0 - \hat{y}_0)^+ - p(x - \hat{y}_0)^+ \right\},$$

$$(4.1)$$

subject to the condition that

$$\pi(\hat{y}_0|y_0, z, x) \geq -px + s_0 y_0.$$

Note that the above condition ensures that, if the firm decides to make the shipment, it is indeed optimal. In what follows, we analyze four difference cases and derive the firm's optimal shipment decision in each case.

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If the realized demand is less than the total procurement quantity, i.e., $x \leq y_0$, then (4.1) is increasing in $\hat{y}_0$ for any $\hat{y}_0 \leq x$. In addition, if $z \leq s_2 - s_0$, then (4.1) is increasing in $\hat{y}_0$ for any $\hat{y}_0 \leq y_0$. On the other hand, if $z > s_2 - s_0$, then (4.1) is decreasing in $\hat{y}_0$ for any $\hat{y}_0 > x$. Combining these observations, we have

$$\pi_l(y_0, z, x) = \begin{cases} rx - zy_0 + s_2(y_0 - x), & z \leq s_2 - s_0; \\ rx - zx + s_0(y_0 - x), & z > s_2 - s_0; \\ -px + s_0 y_0, & \text{abandon shipment.} \end{cases} \tag{4.2}$$

Note that when the realized demand is less than the firm's original procurement quantity y_0, the firm may or may not ship the full amount, depending on the realized quota cost as well as the salvage value s_0 and s_2. By (4.2), it is optimal to make the shipment if and only if

$$rx - zx + s_0(y_0 - x) \geq -px + s_0 y_0 \Rightarrow z \leq r + p - s_0.$$

Clearly, the firm's decision of whether to make the shipment depends only on the system parameters and the realized quota price. The decision is independent of the size of the realized demand.

If the realized demand is greater than the original procurement quantity, i.e., $x > y_0$, then the firm's profit function is

$$\pi_h(y_0, z, x) = \begin{cases} ry_0 - zy_0 - p(x - y_0), & \text{make shipment;} \\ -px + s_0 y_0, & \text{abandon shipment.} \end{cases} \tag{4.3}$$

When the realized demand is greater than the procurement quantity y_0, the firm ships all

procurement quantity if it is economical to do so. By (4.3), it is optimal to ship if and only if

$$ry_0 - zy_0 - p(x - y_0) \geq -px + s_0 y_0 \Rightarrow z \leq r + p - s_0.$$

Note that the optimality condition for the shipment decision is independent of realizations of x.

Let $G(\cdot)$ and $F(\cdot)$ denote the distribution function of $\tilde{z}$ and $\tilde{x}$, respectively. The firm's expected profit with respect to the random quota cost $\tilde{z}$ and stochastic demand $\tilde{x}$ is therefore

$$
\begin{aligned}
\pi(y_0) = -c_0 y_0 &+ \int_0^{s_2 - s_0} \int_0^{y_0} \left((r - s_2)x + (s_2 - z)y_0 \right) dF(x) dG(z) \\
&+ \int_{s_2 - s_0}^{r + p - s_0} \int_0^{y_0} \left((r - s_0 - z)x + s_0 y_0 \right) dF(x) dG(z) \\
&+ \int_0^{r + p - s_0} \int_{y_0}^{\infty} \left((r + p - z)y_0 - px \right) dF(x) dG(z) + \int_{r + p - s_0}^{\infty} \left(-pE[x] + s_0 y_0 \right) dG(z).
\end{aligned}
$$

$$(4.4)$$

Theorem 23. *With direct procurement strategy, the firm's profit function is concave in the procurement quantity. Furthermore, if $s_2 \leq s_0$, then the firm's optimal procurement quantity is given by*

$$y_0^* = F^{-1} \left(\frac{r + p - c_0 - A - (c_0 - s_0)B}{r + p - s_0 - A} \right),$$

$$(4.5)$$

where A is the expected unit quota cost, given that it is economical to ship the product, i.e.,

$$A = \frac{1}{G(r + p - s_0)} \int_0^{r + p - s_0} z \, dG(z),$$

and B is the odds that the quota cost is higher than the shipment threshold, i.e., $B = \frac{\overline{G}(r + p - s_0)}{G(r + p - s_0)}$.

Otherwise, if $s_2 > s_0$, then the firm's optimal procurement quantity is given by

$$y_0^* = F^{-1}\left(\frac{r + p - c_0 - A - (c_0 - s_0)B}{r + p - s_0 - A - C}\right), \tag{4.6}$$

where C is given by

$$C = \frac{1}{G(r + p - s_0)}\int_0^{s_2 - s_0}(s_2 - s_0 - z)dG(z).$$

Substituting the optimal procurement quantity y_0^* into (4.4), we have (for the case of $s_2 > s_0$)

$$\pi(y_0^*) = \int_0^{y_0^*}xdF(x) \cdot \left(\int_0^{r+p-s_0}(r + p - s_0 - z)dG(z) - \int_0^{s_2-s_0}(s_2 - s_0 - z)dG(z)\right) - pE[x]. \tag{4.7}$$

Note that if $s_2 \le s_0$, then the second integration term in the bracket should be dropped.

Corollary 4. *With direct procurement strategy, the firm's optimal procurement quantity is a) less than that without quota uncertainty and b) decreasing in the unit expected quota cost A.*

The above corollary is intuitive as a higher expected quota cost makes larger procurement quantity increasingly unattractive. From (4.5), it can be seen that the optimal procurement quantity y_0^* is also decreasing in the odds of the quota cost being higher than the shipment threshold. In other words, when the quota cost is more likely to exceed the shipment threshold, the optimal procurement quantity is reduced.

With respect to the firm's optimal profit, we can establish, by the envelope theorem, the following sensitivity result.

Proposition 1. *With direct procurement strategy, the firm's optimal profit is a) decreasing in the unit procurement cost c_0, i.e., $\partial_{c_0} \pi(y_0^*) = -y_0^*$, b) decreasing in the unit penalty cost p, c) increasing in the unit salvage value s_0 and s_2, and d) increasing in the unit revenue r.*

In what follows, we investigate how the characteristics of the unit quota price influence the firm's optimal profit. Here we adopt the concept of first order stochastic ordering, which ranks two random variables by the following relationship. A random variable X_1 is said to be stochastically dominated by the random variable X_2, if, for any given value of x, the probability of X_1 being less than x is greater than that of X_2 being less than x. In other words, $X_1 \leq_{st} X_2$ if $G_1(x) \geq G_2(x)$, where $G_i(\cdot)$ is the distribution function of random variable X_i, $i = 1, 2$. Given the above stochastic ordering concept, we have the following theorem.

Theorem 24. *With direct procurement strategy, the firm's optimal expected profit (weakly) decreases as the unit quota price stochastically increases .*

The above theorem establishes that a stochastically increasing quota price negatively impacts a firm's optimal profit. When the stochastic increase in the unit quota price becomes very large, however, such negative impact eventually diminishes because the firm may find it uneconomical to make the shipment. Therefore, the overall impact of the stochastically increasing quota price must be in a convex form, hence the word 'weakly' in the above theorem statement.

We note that, depending on the particular family of distribution, a stochastically increasing quota price does not necessarily correspond to a quota price with larger variance. However, a straightforward interpretation of the stochastically increasing quota price is simply a higher expected quota price for a given family of distribution functions. For the special case of the

normally distributed quota price, stochastic dominance can be ordered by the mean parameter μ (while keeping variance constant). In other words, a stochastically increasing quota price corresponds to the distribution with a higher μ. The following corollary, therefore, is a direct consequence of Theorem 24.

Corollary 5. *With direct procurement strategy, if the quota price is normally distributed with parameters $N(\mu, \sigma)$, then, everything else being equal, the firm's expected optimal profit is decreasing in the expected quota cost μ.*

Since a higher expected quota price μ increases the firm's overall cost, the optimal expected profit decreases. As noted earlier, the stochastic dominance concept does not imply the ordering of the variance. The following theorem investigates the how the variability of the quota price influences the firm's optimal expected profit.

Theorem 25. *With direct procurement strategy, if the quota price is normally distributed with parameters $N(\mu, \sigma)$, then, everything else being equal, the firm's expected optimal profit is increasing in the quota cost standard deviation σ.*

The above theorem tells us that, in the case of normally distributed quota price, an increase in the quota variance (while keeping the expected quota price constant) actually increases the firm's optimal expected profit. Note that this result holds true in general and is not parameter dependent. This surprising result can be explained as follows. The firm's downside exposure to the very high quota price is limited by the firm's option of not shipping the finished product and therefore the firm avoids incurring the high quota cost (at a loss). On the other hand, the upside of a very low quota price benefits the firm significantly. This asymmetry of the limited downside cost versus the upside benefit makes the firm better off when the quota variance is

higher.

We note that, in our comprehensive numeric study (see §4.4.2.1 for details), Theorem 25 holds (on average) even with Weibull distributed quota prices. We observed that the firm's optimal profit is, on average, convex increasing in the quota price coefficient of variation. For very low unit production cost c_0, however, we observed that the firm's optimal profit is not necessarily increasing in the quota price coefficient of variation. With Weibull distribution, an increase in the coefficient of variation entails a change in both the shape and the scale parameters. Such simultaneous change in general cannot be ranked by the stochastic ordering concept.

Furthermore, our numeric study confirmed that Theorem 24 holds in the case of the Weibull distributed quota price. For lower values of the expected quota price, the optimal expected profit declines approximately linearly with the expected quota price. The optimal expected profit eventually becomes independent of the expected quota price.

4.3.3 Split Procurement

In this case, the firm procures from two countries, LCC and MCC, where the LCC is subject to quota restrictions but the MCC is not subject to quota restrictions. Because both the LCC and the MCC are foreign countries, we assume they have the same leadtime. Consequently, orders have to be placed at the same time. To avoid trivial cases, we assume the unit cost from LCC is less than the unit cost from MCC, i.e., $c_0 < c_1$. Recall that we use s_0, s_1, and s_2 to denote the unit salvage value from LCC, MCC and DOM, respectively. In addition, y_0 and y_1 denote the firm's procurement quantity from the LCC and the MCC, respectively. In what follows,

we first describe the firm's profit function conditional on the realizations of the quota price z and the realized demand x. Then we derive the firm's expected profit function. Throughout the rest of this section, we assume that $s_2 > s_1 > s_0$. For the case of $s_2 < \min(s_0, s_1)$, see Appendix §A3.1 for details. Note the case of $s_1 > s_2 > s_0$ or $s_0 > s_2 > s_1$ can be similarly analyzed.

In the second stage, first consider the case when the realized demand is less than the total procurement quantity, i.e., $x \leq y_0 + y_1$. Following the analysis from §4.3.2, it is economical to ship product from LCC if and only if $z \leq r + p - s_0$. Therefore, if $z \leq r + p - s_0$, then the firm's revenue function is

$$\pi_l(\hat{y}_0, \hat{y}_1, y_0, y_1, x) = r \min\left(\sum_{i=0}^{1} \hat{y}_i, x\right) - z\hat{y}_0 - p\left(x - \sum_{i=0}^{1} \hat{y}_i\right)^+ + s_2 \left(\sum_{i=0}^{1} \hat{y}_i - x\right)^+ + \sum_{i=0}^{1} s_i(y_i - \hat{y}_i)$$

$$(4.8)$$

$$\text{s.t.} \quad \hat{y}_i \leq y_i, i = 0, 1,$$

where $\hat{y}_0$ and $\hat{y}_1$ are the procurement quantity shipped from LCC and MCC, respectively. Since π_l is increasing in $\hat{y}_0$ and $\hat{y}_1$ for $\sum_{i=0}^{1} \hat{y}_i < x$, we must have $\sum_{i=0}^{1} \hat{y}_i \geq x$. Therefore, (4.8) can be simplified as

$$\pi_l(\hat{y}_0, \hat{y}_1, y_0, y_1, x) = (r - s_2)x - z\hat{y}_0 + s_2 \sum_{i=0}^{1} \hat{y}_i + \sum_{i=0}^{1} s_i(y_i - \hat{y}_i). \qquad (4.9)$$

Note that $\partial_{\hat{y}_0}\pi_l(\hat{y}_0, \hat{y}_1, y_0, y_1, x) = s_2 - s_0 - z$, and $\partial_{\hat{y}_1}\pi_l(\hat{y}_0, \hat{y}_1, y_0, y_1, x) = s_2 - s_1 > 0$. Clearly, we must have $\hat{y}_1^* = y_1$, and $\hat{y}_0^* = y_0$ if $z \leq s_2 - s_0$ and $\hat{y}_0^* = \max(x - y_1, 0)$ otherwise.

Therefore, (4.9) can be further simplified to

$$
\pi_l(y_0, y_1, x) =
\begin{cases}
(r - s_2)x - zy_0 + s_2 \sum_{i=0}^{1} y_i, & z \le s_2 - s_0; \\
(r - s_2)x + (s_2 - s_0 - z)(x - y_1)^+ + s_0 y_0 + s_2 y_1, & \text{otherwise.}
\end{cases}
\tag{4.10}
$$

If $z > r + p - s_0$, then it is not economical to ship from LCC, but it remains profitable to ship from MCC. The firm's revenue function is

$$
\pi_l(y_0, y_1, x) = r \min(y_1, x) - p(x - y_1)^+ + s_2(y_1 - x)^+ + s_0 y_0.
\tag{4.11}
$$

Note that in this case the optimal amount shipped from MCC is given by $\hat{y}_1^* = y_1$ which is independent of the realized demand x or the quota price z.

We now turn our attention to the case where the realized demand is greater than the total procurement quantity, i.e, $x > y_0 + y_1$. Again, it is economical to ship from LCC if and only if $z \le r + p - s_0$. Therefore, if $z \le r + p - s_0$, then the firm's revenue function is

$$
\pi_h(y_0, y_1, x) = r \sum_{i=0}^{1} y_i - p\left(x - \sum_{i=0}^{1} y_i\right) - zy_0.
\tag{4.12}
$$

Otherwise, if $z > r + p - s_0$, then the firm's profit function is

$$
\pi_h(y_0, y_1, x) = ry_1 - p(x - y_1) + s_0 y_0.
\tag{4.13}
$$

Combining the above derivations, we can derive the firm's expected profit function. We have

$$
\pi(y_0, y_1) = \int_0^{s_2 - s_0} \int_0^{y_0 + y_1} \left((r - s_2)x - zy_0 + s_2(y_0 + y_1) \right) dF(x)dG(z)
$$
$$
+ \int_{s_2 - s_0}^{r+p-s_0} \left(\int_0^{y_1} \left((r - s_2)x + s_0y_0 + s_2y_1 \right) dF(x) \right.
$$
$$
\left. + \int_{y_1}^{y_0 + y_1} \left((r - s_0 - z)x + s_0y_0 + (s_0 + z)y_1 \right) dF(x) \right) dG(z)
$$
$$
+ \int_0^{r+p-s_0} \int_{y_0+y_1}^{\infty} \left((r+p)(y_0 + y_1) - px - zy_0 \right) dF(x)dG(z)
$$
$$
+ \overline{G}(r + p - s_0) \left(\int_0^{y_1} \left(rx + s_2(y_1 - x) + s_0y_0 \right) dF(x) \right.
$$
$$
\left. + \int_{y_1}^{\infty} \left(ry_1 - p(x - y_1) + s_0y_0 \right) dF(x) \right) - \sum_{i=0}^{1} c_i y_i. \qquad (4.14)
$$

Using (4.14), we have

$$
\partial_{y_0} \pi(y_0, y_1) = \overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z)
$$
$$
+ F(y_0 + y_1) \int_0^{s_2 - s_0} (s_2 - s_0 - z)dG(z) - (c_0 - s_0), \qquad (4.15)
$$

and

$$
\partial_{y_1} \pi(y_0, y_1) = (r + p)\overline{F}(y_1) + (\overline{F}(y_0 + y_1) - \overline{F}(y_1)) \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z)
$$
$$
+ (F(y_0 + y_1) - F(y_1)) \int_0^{s_2 - s_0} (s_2 - s_0 - z)dG(z) - (c_1 - s_2). \qquad (4.16)
$$

Lemma 10. *The hessian matrix associated with $\pi(y_0, y_1)$ is symmetric, negative, and diagonal dominant.*

It follows directly from Lemma 10 that the firm's objective function is jointly concave in the procurement quantity y_0 and y_1. To summarize the above analysis, we have the following

theorem.

Theorem 26. *With split procurement strategy, the firm's profit function is jointly concave in the procurement quantity $\vec{y} = (y_0, y_1)$. Furthermore, the firm's optimal procurement quantity can be obtained by setting (4.15) and (4.16) equal to zero (for interior solutions).*

Note that, in absence of foreign exchange rate risk, the direct procurement strategy can be viewed as a special case of the split procurement strategy, where it is optimal not to procure from MCC. The following theorem investigates the sensitivity of the firm's optimal expected profit on different parameter values.

Theorem 27. *With split procurement strategy, the firm's optimal expected profit is a) decreasing in the unit procurement cost c_0 and c_1, b) decreasing in the unit penalty cost p, c) increasing in the unit salvage value s_0 and s_2, and d) increasing in the unit revenue r.*

While the above theorem demonstrates that the sensitivity of the firm's optimal expected cost on system parameters is fairly intuitive, it is unclear how the variability of the quota cost influences the firm's optimal expected cost. In what follows, we consider the similar stochastic dominance concept of the quota price as discussed in §4.3.2.

Theorem 28. *With split procurement strategy, the firm's optimal expected profit (weakly) decreases as the quota price stochastically increases.*

The above theorem tells us that a stochastically increasing quota price decreases the firm's optimal expected cost. This result is consistent with the direct procurement strategy as discussed in §4.3.2. Again, it is important to realize that the stochastic dominance concept does not lend itself to the ranking of variances. Analogous to Theorem 25 and Corollary 5, we have the following theorem when the quota price is normally distributed.

Theorem 29. *With split procurement strategy, if the quota price is normally distributed with parameters $N(\mu, \sigma)$, then the firm's optimal expected profit is decreasing in the expected quota price μ and increasing in the quota standard deviation σ.*

Note that Theorems 28 and 29 collectively prove that an increase in the variance of the quota price does not necessarily hurt the firm, but a stochastically increasing quota price always hurts the firm's performance in terms of the optimal expected profit.

In a comprehensive numeric study (see §4.4.2.1 for details), we observed that the results of Theorem 29 hold true (on average) even with Weibull distributed quota prices. We observed that, on average, the optimal expected profit is convex increasing in the quota price coefficient of variation. However, for very low values of the LCC production cost c_0, we observed that the optimal expected profit is not necessarily increasing in the quota price coefficient of variation. These observations are also consistent with what we observed with the direct procurement strategy.

Our numeric study also confirms that the result of Theorem 28 holds true in the case of Weibull distributed quota price. In particular, we observed that the optimal expected profit is convex decreasing in the expected quota prices. As the expected quota price increases, the negative impact of the quota price diminishes as the firm shifts more and more production to the alternative MCC, which is not subject to quota restrictions.

4.3.4 Split Procurement versus Direct Procurement: Identical or Strict Dominance

It is intuitive that the split procurement strategy, in absence of fixed cost, dominates the direct procurement strategy. It is of interest, however, to understand when the split procurement strategy is identical to the direct procurement strategy and when it strictly dominates the direct procurement strategy. This helps a firm to select appropriate strategies if fixed costs come into consideration.

Whether the split procurement strategy strictly dominates the split procurement strategy is intimately linked with the procurement structure of the split procurement strategy, i.e., whether it is optimal to sole source from one country or dual source from both countries.

Remark: The split procurement strategy is identical to the direct procurement strategy if and only if, in the optimal solution, the firm procures only from LCC. Otherwise, the split procurement strategy strictly dominates the direct procurement strategy.

The above observation provides a crisp connection between the optimal structure of the split procurement strategy and whether the split procurement strategy strictly dominates the direct procurement strategy. Therefore, to determine whether the split procurement strategy strictly dominates the direct procurement strategy, we simply need to check whether the split procurement strategy actually procures from MCC at the optimal solution. The following theorem provides a sufficient condition for the split procurement strategy to strictly dominate the direct procurement strategy.

Theorem 30. *With split procurement strategy, it is optimal not to procure from LCC, i.e., $y_0^* = 0$, if $c_1 - s_2 \leq c_0 - s_0$.*

Consequently, the split procurement strategy strictly dominates the direct procurement strategy if $c_1 - s_2 \leq c_0 - s_0$. Note that the terms $c_0 - s_0$ and $c_1 - s_2$ are the firm's overage cost for LCC and MCC. Theorem 30 tells us that if the overage cost of procuring in MCC is less than that in LCC, then the firm is better off procuring from MCC only. In this case, the firm's decision not to procure from LCC is not influenced by the uncertain quota cost. In other words, the nature of the quota cost associated with LCC does not influence whether the split procurement strategy strictly dominates the direct procurement strategy. Note that if $s_0 = s_2$, then the stated condition never holds.

On the other hand, the following theorem provides a sufficient condition under which the split procurement strategy is identical to the direct procurement strategy.

Theorem 31. *With split procurement strategy, it is optimal not to procure from MCC, i.e., $y_1^* = 0$, if the unit production cost in MCC exceeds the following threshold*

$$c_1 > c_0 + s_2 + (r + p - s_0) - \int_0^{r+p-s_0} (r + p - s_0 - z) dG(z). \qquad (4.17)$$

Consequently, if (4.17) holds, then the split procurement strategy is identical to the direct procurement strategy. Therefore, with an incremental fixed cost, the direct procurement strategy strictly dominates the split procurement strategy if condition (4.17) is true. Note that the above condition is more likely to occur when the quota price is less likely to exceed the $r + p - s_0$ threshold.

We conducted a comprehensive numeric study to further investigate the relative dominance of the split procurement strategy over the direct procurement strategy. For our numeric study, we set the parameter values according to the base case scenario (see §4.4.2.1 for details), but

vary the mean quota cost from 0.1 to 0.5 with a step size of 0.1, the quota cv from 0.3 to 0.7 with a step size of .1, the unit cost c_1 from 0.1 to 0.9 with a step size of 0.1, the mean demand from 40 to 160 with a step size of 30, and the demand coefficient of variation from 0.1 to 0.5 with a step size of 0.1. This gives us in total 5,625 observations.

We observed that in 72.7% of the cases, the split procurement strategy uses only one country for total procurement quantity. Overall, the firm solely sources from the LCC (MCC) in 44.6% (28.2%) of cases. Consequently, the split procurement strategy is identical to the direct procurement strategy in 44.6% of the cases and it strictly dominates the direct procurement strategy in more than 55% of the cases. This indicates that, even with incremental fixed cost, the split procurement strategy can still be strictly preferred to the direct procurement strategy.

The above observations are influenced by a number of system parameters. On average, the firm is more likely to solely source from LCC and less likely to solely source from MCC as the demand coefficient of variation increases. In other words, the attractiveness of the split procurement strategy relative to the direct procurement strategy is reduced when the demand becomes more volatile. The advantage of the cheaper procurement cost from LCC seems to become more salient when the demand volatility becomes higher. We note that the propensity for the firm to use both countries in its optimal procurement can increase or decrease in the demand coefficient of variation. We observed that, on average, the firm is initially more likely to use both countries as the demand coefficient of variation increases, but then becomes less likely to do so when the demand coefficient of variation becomes very high.

On the other hand, the firm is significantly less inclined to use LCC and more inclined to use MCC as the expected quota cost increases. This is intuitive because a higher expected quota cost increases the effective production cost in LCC. Therefore, the split procurement strategy

strictly dominates the direct procurement strategy as the expected quota cost increases. We also observed, however, that the firm is also significantly more likely to use both countries as the expected quota cost increases. Table 4.1 lists the number of cases when the split procurement strategy is identical or strictly dominates the direct procurement strategy. In addition, the last column of the table lists, when the split procurement strategy strictly dominates the direct procurement strategy, the average percentage increase in the expected profit for the split procurement strategy over the direct procurement strategy.

Expected quota cost	Cases of Identical Profit	Cases of Strict Dominance	Profit Increase (%)
0.1	756	369	5.05
0.2	625	490	8.11
0.3	500	625	10.66
0.4	375	750	13.33
0.5	250	875	16.10
Total	2,506	3,119	10.65

Table 4.1: Comparison of the split procurement strategy versus the direct procurement strategy

The relative dominance of the split procurement strategy over the direct procurement strategy is also significantly influenced by the unit production cost in LCC. As expected, as the unit production cost c_0 increases, the split procurement strategy is more likely to strictly dominate the direct procurement strategy. In addition, when the split procurement strategy strictly dominates the direct procurement strategy, its relative advantage (in the expected profit) over the direct procurement strategy is convex increasing in the unit cost c_0. We note that, when the split procurement strategy strictly dominates the direct procurement strategy, the firm is more likely to procure from both countries when the unit production cost c_0 is moderate. When the

unit cost c_0 is high, the split procurement strategy is more likely to result in the firm solely sourcing strategy from the medium cost country.

4.3.5 Indirect Outward Processing Arrangement

In this case, the firm sources from MCC, which has an outward processing arrangement with LCC. The production from MCC is not subject to quota restrictions[5] and therefore the firm can take advantage of the lower production cost in LCC indirectly. Note that any firms in MCC have to maintain a certain level of domestic production activities in order to qualify for the outward processing arrangements with LCC. let γ represent the fraction of the value of the processing carried out in the foreign MCC. In the context of this chapter, γ is the fraction of the processing cost carried out in MCC, while the MCC and the LCC has certain OPA arrangements. Note that, in general, the parameter γ is product dependent, even within the same industry[6]. For the buying firm, therefore, the procurement cost can be represented by $(1-\gamma)c_0+\gamma c_1$, where c_0 and c_1 represent the production cost in LCC and MCC respectively. Note that, from the buying firm's perspective, its procurement problem is a fairly straightforward newsvendor problem, because the buying firm is not actively involved in the outward processing arrangement. In what follows, we characterize the firm's procurement problem.

---

[5]In certain product categories, MCC may not be entirely free of quota restrictions, but such restrictions can be significantly less severe than that in LCC. Therefore, from the buying firm's perspective, the quota risk from MCC is negligible.

[6]Specific examples can be found, for example, on http://www.tid.gov.hk/ under the trade circular section.

Let y denote the procurement quantity from MCC, then the firm's expected profit is

$$\pi(y_1) = -((1-\gamma)c_0 + \gamma c_1)y_1 + r \left(\int_0^{y_1} x dF(x) + \int_{y_1}^{\infty} y_1 dF(x) \right)$$
$$- p \int_{y_1}^{\infty} (x - y_1)dF(x) + s_2 \int_0^{y_1} (y_1 - x)dF(x). \qquad (4.18)$$

Note that in the above expression, for expositional clarity we implicitly assume that $s_2 > s_1$. The case of $s_2 < s_1$ can be similarly analyzed. It is a known result that the firm's optimal procurement quantity is of a newsvendor type, i.e.,

$$y_1^* = F^{-1} \left(\frac{r + p - (1-\gamma)c_0 - \gamma c_1}{r + p - s_2} \right). \qquad (4.19)$$

We note that, by substituting (4.19) into (4.18), we have the firm's optimal profit as

$$\pi(y_1^*) = (r + p - s_2) \int_0^{y_1^*} x dF(x) - pE[x]. \qquad (4.20)$$

Clearly, the firm's optimal procurement quantity is influenced by the fraction of the production activities arranged in LCC.

Lemma 11. *With the indirect OPA strategy, a) the optimal procurement quantity is decreasing in the local processing requirement γ, and b) the optimal expected profit is decreasing in the local processing requirement γ.*

The above lemma (proof omitted) tells us that the indirect OPA becomes less attractive as the fraction of the production activities in MCC increases. This is as expected because a higher level of γ represents a higher level of constraint on the system.

In closing, we note that the sensitivity analysis is straightforward and obvious for the

indirect OPA strategy and we therefore omit any further sensitivity results for this section.

4.3.6   Direct Outward Processing Arrangement

In this case, the firm procures from two countries, where a complete production line is maintained in the domestic country, while additional production capacity is sourced from LCC, which is subject to quota restriction. To avoid trivial cases, we assume the unit cost from LCC is less than that in DOM, i.e., $c_0 < c_2$. Recall that we use s_0 and s_2 to denote the unit salvage value from LCC and the home country, respectively. Note that in order for the firm to qualify for OPA, the firm has to maintain a certain level of domestic production. Towards this end, let α represent the fraction of the production quantity carried out in DOM. Note that, under the direct OPA, the policy may mandate the fraction of domestic production in terms of the processing value. Such a mandate can be converted into the quantity based requirement and therefore is mathematically equivalent to directly specifying the quantity requirement.

Recall that we let y_0 and y_2 denote the firm's procurement quantity from LCC and the domestic production quantity, respectively. The above domestic production requirement, therefore, can be expressed as follows. The firm's domestic procurement quantity has to be such that y_2 is no less than a certain fraction of the total procurement quantity, e.g., $\frac{y_2}{y_0+y_2} \geq \alpha$, where $0 \leq \alpha \leq 1$. Note that $\alpha = 0$ corresponds to the case of unrestricted domestic production level, whereas $\alpha = 1$ corresponds to pure domestic production case.

With all other strategies, we do not explicitly consider the shipment transit time, because all these strategies involve similar international shipping. It is therefore reasonable to assume that the transit times are identical with the earlier three strategies. With direct OPA, however,

domestic production does not involve international transit times and therefore the leadtime is shorter than all other strategies. We therefore introduce the notion of leadtime difference, denoted as $\Delta_L = L_A - L_B$, where L_A and L_B denote the transit time for LCC and DOM, respectively. The leadtime difference Δ_L may allow the firm to learn more information about its total demand. Let the starting time be 0; then Δ_L is the time at which the firm must decide its domestic production quantity y_2. We use $\tilde{\theta}$ to denote the possible information that may be obtained at time Δ_L. Furthermore, we use $F_{\Delta_L|\tilde{\theta}}(\cdot)$ to denote the updated demand distribution when the firm makes its domestic production decision at time Δ_L. See Figure 4.3.
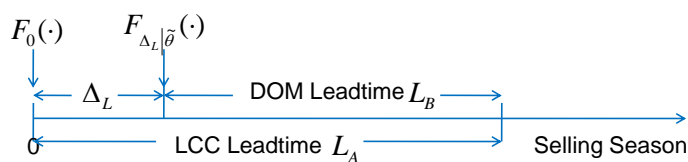
Figure 4.3: Time line for leadtime difference in the direct outward processing arrangement

In what follows, we first describe the firm's second stage decision problem for domestic production. We then derive the firm's first stage expected profit function.

4.3.6.1 Second Stage Problem

In the second stage, the firm's decision problem is to determine the domestic production quantity y_2. In addition, the firm may also adjust the amount of future shipment $w_0 \le y_0$, depending on the updated demand distribution. The firm's second stage problem is therefore a joint optimization problem: the firm needs to simultaneously determine w_0 and y_2. Note that in the case $w_0 < y_0$, the excess amount of $y_0 - w_0$ can be salvaged at s_0. Given the above description,

the firm's second stage expected profit function is

$$\pi_L(w_0, y_2 | y_0, \tilde{\theta}) = -c_2 y_2 + s_0(y_0 - w_0) + r\left(\int_0^{w_0 + y_2} x dF_{\Delta_L | \tilde{\theta}}(x) + \int_{w_0 + y_2}^\infty (w_0 + y_2) dF_{\Delta_L | \tilde{\theta}}(x)\right)$$

$$- p \int_{w_0 + y_2}^\infty (x - w_0 - y_2) dF_{\Delta_L | \tilde{\theta}}(x) + s_2 \int_0^{w_0 + y_2} (w_0 + y_2 - x) dF_{\Delta_L | \tilde{\theta}}(x),$$

$$\text{subject to:} \qquad w_0 \leq \min\left(y_0, \frac{1-\alpha}{\alpha} y_2\right). \tag{4.21}$$

We note that s_0 implicitly includes the savings in the transportation cost, which is assumed to be included in the unit cost c_0. For expositional clarity, we assume that $s_0 \leq s_2$.

For any given y_0 and y_2, $\partial_{w_0} \pi(w_0, y_2 | y_0) = (r + p - s_0) - (r + p - s_2) F_{\Delta_L | \theta}(w_0 + y_2)$, which is increasing in w_0 (recall that s_0 is assumed to be less than s_2). The revenue function $\pi(w_0, y_2 | y_0)$ is therefore always (weakly) increasing in w_0. Consequently, the constraint in (4.21) must be binding. In what follows, we consider the binding conditions in each region.

4.3.6.2 Shipment Bounded By OPA Procurement Quantity

If, at optimality, w_0 is bounded by y_0, then (4.21) can be rewritten (after some algebra) as

$$\pi_L(w_0, y_2 | y_0, \tilde{\theta}) = -c_2 y_2 + (r + p - s_2) \int_0^{y_0 + y_2} x dF_{\Delta_L | \tilde{\theta}}(x)$$

$$- (r + p - s_2)(y_0 + y_2) F_{\Delta_L | \tilde{\theta}}(y_0 + y_2) + (r + p)(y_0 + y_2) - p E_{\Delta_L | \tilde{\theta}}[x]$$

$$\text{subject to:} \qquad y_0 \leq \frac{1-\alpha}{\alpha} y_2.$$

For any given y_0 (and ignoring the constraint), the above expression is a standard newsvendor model, and the optimal unconstrained y_2^* is given by

$$y_2^* = \left(F_{\Delta_L|\theta}^{-1}\left(\frac{r+p-c_2}{r+p-s_2}\right) - y_0 \right)^+. \tag{4.22}$$

Therefore, if the optimal solution of w_0 with respect to (4.21) is bounded by y_0, then the optimal y_2^* must satisfy the following. Let $k_L = F_{\Delta_L|\theta}^{-1}\left(\frac{r+p-c_2}{r+p-s_2}\right)$.

$$y_2^* = \begin{cases} k_L - y_0, & y_0 \le (1-\alpha)k_L; \\[2mm] \frac{\alpha}{1-\alpha}y_0, & y_0 > (1-\alpha)k_L. \end{cases}$$

4.3.6.3 Shipment Bounded By Domestic Production Quantity

In the case where, at optimality, w_0 is bounded by $\frac{1-\alpha}{\alpha}y_2$, then (4.21) can be rewritten (after some algebra) as

$$\pi_L(w_0, y_2|y_0, \tilde{\theta}) = -c_2 y_2 + s_0\left(y_0 - \frac{1-\alpha}{\alpha}y_2\right) + (r+p-s_2)\int_0^{\frac{1}{\alpha}y_2} x\, dF_{\Delta_L|\tilde{\theta}}(x)$$

$$- (r+p-s_2)\frac{1}{\alpha}y_2 F_{\Delta_L|\tilde{\theta}}\left(\frac{1}{\alpha}y_2\right) + (r+p)\frac{1}{\alpha}y_2 - pE_{\Delta_L|\tilde{\theta}}[x]$$

subject to: $y_0 > \dfrac{1-\alpha}{\alpha}y_2.$

For any given y_0 (and ignoring the constraint), the above expression is a standard newsvendor model, and the optimal unconstrained y_2^* is given by

$$y_2^* = \alpha F_{\Delta_L|\theta}^{-1}\left(\frac{r+p-\alpha c_2 - (1-\alpha)s_0}{r+p-s_2}\right). \tag{4.23}$$

Therefore, if the optimal solution of w_0 with respect to (4.21) is bounded by $\frac{1-\alpha}{\alpha}y_2$, then the optimal solution to (4.21) must satisfy the following. Let $k_H = F_{\Delta_L|\theta}^{-1}\left(\frac{r+p-\alpha c_2-(1-\alpha)s_0}{r+p-s_2}\right)$.

$$w_0^* = (1-\alpha)k_H, \qquad y_2^* = \alpha k_H, \qquad \text{if } y_0 > (1-\alpha)k_H;$$

4.3.6.4   Comparison of the Bounds

For $y_0 > (1-\alpha)k_H$, in order to characterize the optimal solution to (4.21), we need to determine whether w_0 should be bounded by y_0 or $(1-\alpha)k_H$. Let π_1 denote the resulting revenue function with $w_0 = y_0$ substituted into (4.21). Analogously, let π_2 denote the resulting revenue function with $w_0 = (1-\alpha)k_H$ substituted into (4.21). One can show that

$$\pi_1 = -c_2\frac{\alpha}{1-\alpha}y_0 + (r+p-s_2)\int_0^{\frac{y_0}{1-\alpha}} x dF_{\Delta_L|\tilde{\theta}}(x) - (r+p-s_2)\frac{y_0}{1-\alpha}F_{\Delta_L|\tilde{\theta}}\left(\frac{y_0}{1-\alpha}\right)$$
$$+ (r+p)\frac{y_0}{1-\alpha} - pE_{\Delta_L|\tilde{\theta}}[x],$$

and

$$\pi_2 = (1-\alpha)k_H s_0 + (r+p-s_2)\int_0^{k_H} x dF_{\Delta_L|\tilde{\theta}}(x) - pE_{\Delta_L|\tilde{\theta}}[x].$$

Note that π_2 is a constant, independent of y_0 (for any $y_0 > (1-\alpha)k_H$). It can be easily verified that $\pi_1 = \pi_2$ for $y_0 = (1-\alpha)k_H$.

The relative attractiveness of π_1 versus π_2 therefore simply depends on the shape of π_1 for

$y_0 > (1 - \alpha)k_H$. Note that

$$\partial_{y_0}\pi_1 = \frac{1}{1 - \alpha}(r + p - \alpha c_2) - \frac{1}{1 - \alpha}(r + p - s_2)F_{\Delta_L|\tilde{\theta}}\left(\frac{y_0}{1 - \alpha}\right),$$

which is clearly a concave function of y_0. The revenue function π_1 reaches its maximum point

at

$$y_0 = (1 - \alpha)F^{-1}\left(\frac{r + p - \alpha c_2}{r + p - s_2}\right).$$

Let $k_G = F^{-1}\left(\frac{r+p-\alpha c_2}{r+p-s_2}\right)$. Therefore, there exists a unique $\hat{y}_0 > (1 - \alpha)k_G$ such that $\pi_1 \geq \pi_2$

for $y_0 \leq \hat{y}_0$ and $\pi_1 < \pi_2$ for $y_0 > \hat{y}_0$. Note that $\hat{y}_0$ must satisfy

$$\frac{y_0}{1 - \alpha}\left(r + p - \alpha c_2 - (r + p - s_2)F_{\Delta_L|\tilde{\theta}}\left(\frac{y_0}{1 - \alpha}\right)\right) + (r + p - s_2)\int_{k_H}^{\frac{y_0}{1-\alpha}} x dF_{\Delta_L|\tilde{\theta}}(x) = (1 - \alpha)k_H s_0.$$

Finally, we note that the derivative of the revenue function π_1 and π_2 with respect to y_0

must be bounded by s_0 for any given y_0, because in the worst case scenario, the firm can

simply salvage y_0 at unit value of s_0. Because the derivative of π_1 and π_2 at $y_0 = (1 - \alpha)k_H$ is

exactly s_0, we know the solution (w_0^*, y_2^*) at $y_0 = (1 - \alpha)k_H$ is optimal for any $y_0 > (1 - \alpha)k_H$.

Figure 4.4 depicts the firm's second stage revenue function as a function of the initial OPA

procurement quantity y_0. Note that the bolded line denotes the optimal revenue function.
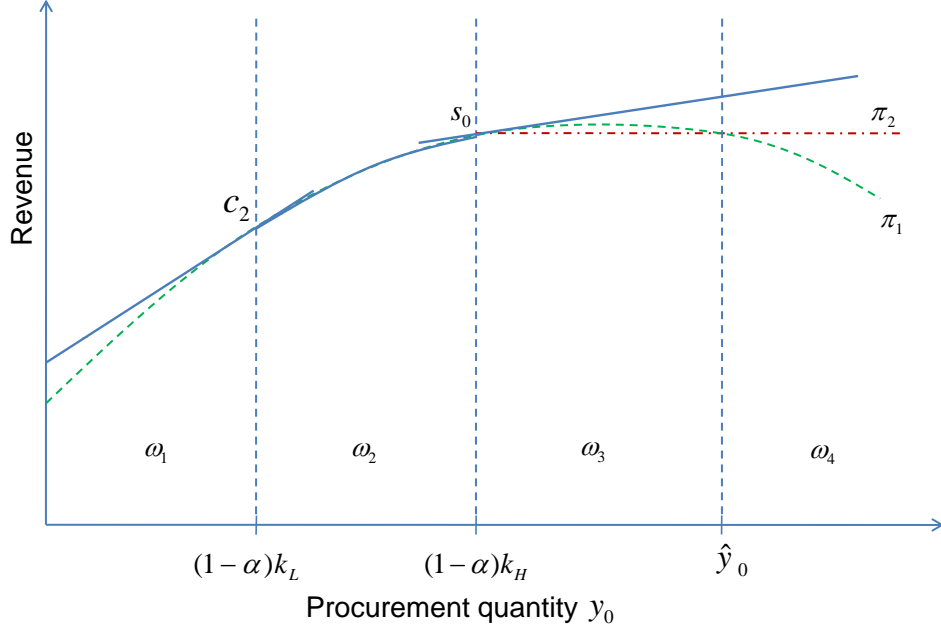
Figure 4.4: Second stage profit function as the LCC procurement quantity y_0 increases

To summarize the above analysis, we have the following theorem.

Theorem 32. *For any given OPA procurement quantity y_0, the firm's second stage optimal shipment, production, and revenue function is given by the following.*

$$
w_0^* = \begin{cases} y_0, & y_0 \leq (1-\alpha)k_H; \\[2mm] (1-\alpha)k_H, & y_0 > (1-\alpha)k_H. \end{cases}
$$

$$
y_2^* = \begin{cases} k_L - y_0, & y_0 \leq (1-\alpha)k_L; \\[2mm] \frac{\alpha}{1-\alpha}y_0, & (1-\alpha)k_L < y_0 \leq (1-\alpha)k_H; \\[2mm] \alpha k_H, & y_0 > (1-\alpha)k_H. \end{cases}
$$

$$\pi_L^*(w_0^*, y_2^* | y_0, \tilde{\theta}) = \begin{cases} c_2 y_0 + (r + p - s_2) \int_0^{k_L} x dF_{\Delta_L | \tilde{\theta}}(x) - pE_{\Delta_L | \tilde{\theta}}[x], & y_0 \leq (1 - \alpha)k_L; \\[3mm] (r + p - \alpha c_2) \frac{y_0}{1 - \alpha} + (r + p - s_2) \int_0^{\frac{y_0}{1-\alpha}} x dF_{\Delta_L | \tilde{\theta}}(x) \\[3mm] \quad -(r + p - s_2) \frac{y_0}{1 - \alpha} F_{\Delta_L | \tilde{\theta}} \left(\frac{y_0}{1 - \alpha} \right) - pE_{\Delta_L | \tilde{\theta}}[x], & (1 - \alpha)k_L < y_0 \leq (1 - \alpha)k_H; \\[3mm] s_0 y_0 + (r + p - s_2) \int_0^{k_H} x dF_{\Delta_L | \tilde{\theta}}(x) - pE_{\Delta_L | \tilde{\theta}}[x], & y_0 > (1 - \alpha)k_H. \end{cases}$$

$$(4.24)$$

The above theorem tells us that the firm will make the full shipment of OPA procurement quantity y_0 as long as y_0 is not too high as compared with the updated demand information. If y_0 is too high relative to the updated demand distribution, i.e., $y_0 > (1 - \alpha)k_H$, then the firm will only ship $(1 - \alpha)k_H$ and salvage the rest at a unit value of s_0.

In addition, the firm's second stage production quantity is increasing in the system parameter α, indicating that a more stringent OPA policy indeed leads to a higher level of domestic production. This higher level of domestic production, however, comes at the expense of the firm's expected profit. In the extreme case, when $\alpha = 1$, the firm does not have any LCC production y_0 and maintains full domestic production. On the other extreme, when $\alpha = 0$, the firm maintains, in expectation, no domestic production. Otherwise, the firm can simply increase its LCC production, with a lower production cost, to compensate for the domestic production. While the policy constraint α can be effective to maintain domestic production, it is important to realize that the firm may find the alternative of not using direct OPA more attractive than being subject to the domestic production requirement. We discuss the relative attractiveness of different strategies in the numeric studies section.

The following theorem proves that the firm's second stage problem is well behaved with respect to the first stage procurement quantity y_0.

Theorem 33. *The firm's second stage optimal revenue function is a concave function of y_0.*

Given the characterization of the firm's second stage problem, the firm's first stage problem can be expressed as

$$\pi(y_0) = -c_0 y_0 + E_{\tilde{\theta}} \left[\pi_L(w_0^*, y_2^* | y_0, \theta) \right].$$

(4.25)

Since the expectation operation preserves concavity, by Theorem 33, we have the following theorem.

Theorem 34. *With direct OPA, the firm's expected profit is a concave function of its initial production quantity y_0.*

Therefore, there exists a unique y_0^* that maximizes the firm's expected profit function in the first stage. We note that this characterization is very general with respect to the forecast evolution process. This result holds regardless of the particular type of demand updating process the firm may use.

It is of interest to understand how the leadtime difference Δ_L influences the firm's optimal expected profit. Because a longer leadtime difference allows the firm to gather more information about its total demand, one may suspect that a longer Δ_L increases the firm's expected optimal profit. The following theorem confirms that such intuition is indeed correct.

Theorem 35. *With the direct OPA strategy, the firm's optimal expected profit is non-decreasing in the leadtime difference Δ_L.*

The generality of the above analysis, however, prevents either implicit or closed form solutions for the optimal procurement level. To further investigate the properties of the optimal

solution, in what follows we impose a particular type of demand updating process and then further characterize the firm's optimal solution.

4.3.6.5 The Martingale Forecast Updating Process

Here we assume the firm has the martingale type of forecast evolution process. In particular, we adopt the additive martingale process as developed by Graves et al. (1986) and Heath and Jackson (1994). The martingale process is very flexible to model a variety of forecast updating processes. Since the martingale forecast updating process is intimately related with the length of the planning horizon, such a process dovetails well in our context, where the leadtime difference naturally leads to a valuable time window for the firm to update its demand forecast. We refer to Graves et al. (1986) for a detailed description of the additive martingale forecast updating process. For completeness, here we briefly describe the forecasting process and introduce relevant notations along the way.

Referring to Figure 4.3, the firm has an initial forecast of the total season demand at time 0, i.e., when the firm makes the decision on OPA production level y_0. We denote this initial forecast as μ_0, which is the firm's best estimate of the total season demand at time 0. From time 0 until the selling season, denoted as T, the firm makes multiple adjustments to its initial forecast of the total demand as more information becomes available as time elapses. We denote a forecast adjustment made at time $0 < t < T$ as δ_t, which is a random variable with mean zero and standard deviation σ_t. Furthermore, we assume these random adjustments are independent and normally distributed. With discrete adjustments, the firm's view of the total season demand at time 0 is given by $\mu_0 + \sum_t \delta_t$. Therefore, at time 0, the total season demand is normally distributed with parameter μ_0 and $\sigma_0^2 = \sum_t \sigma_t^2$, i.e., $F_0(\cdot) \sim N(\mu_0, \sigma_0)$.

For mathematical convenience, we assume these adjustments are continuous in time, i.e., $\delta(t)$ is a continuous function of t. Consequently, we have $\sigma_0^2 = \int_0^T \sigma^2(t)dt$.

At time Δ_L, the firm's forecast of total demand is the initial forecast of μ_0 plus the sum of forecast adjustments up to time Δ_L. We denote this forecast of demand at time Δ_L as $\tilde{\mu}_L$, which is a normally distributed random variable. For any given realization of μ_L, the firm's view of total season demand is normally distributed with parameter μ_L and σ_L, where $\sigma_L^2 = \int_L^T \sigma^2(t)dt$. Note that $E[\mu_L] = \mu_0$ and $\sigma_L^2 = \sigma_0^2 - \int_0^L \sigma^2(t)dt < \sigma_0^2$. This is consistent with an unbiased, gradually improving forecast updating process. As a final note, we use $G_L(\cdot)$ to denote the distribution function of μ_L, which is normally distributed with mean μ_0 and variance $\sigma_{\mu_L}^2 = \int_0^L \sigma^2(t)dt$.

To summarize, at time 0, the firm's view of the season demand is described by a normal distribution with an expected demand of μ_0 and variance of σ_0^2. At time Δ_L, for any realized forecast revision process, the firm's view of the season demand is described again by a normal distribution, but with an expected demand of μ_L and variance of σ_L^2. Note that the realization of μ_L can be higher or lower than the original forecast of μ_0, but the variance of the demand at time Δ_L is lower than the initial one, i.e., $\sigma_L^2 \leq \sigma_0^2$.

4.3.6.6 Optimal Procurement Problem with Martingale Forecast Updating

To further characterize the firm's optimal procurement problem, we substitute the martingale forecast updating process into (4.24). Recall that $k_L = F_{\Delta_L|\theta}^{-1}\left(\frac{r+p-c_2}{r+p-s_2}\right)$, which can also be expressed as $k_L = \mu_L + \Phi^{-1}\left(\frac{r+p-c_2}{r+p-s_2}\right)\sigma_L$, where $\Phi(\cdot)$ is the standard normal distribution function. Analogously, $k_H = F_{\Delta_L|\theta}^{-1}\left(\frac{r+p-\alpha c_2-(1-\alpha)s_0}{r+p-s_2}\right)$, which can also be expressed as $k_H =$

$$\mu_L + \Phi^{-1}\left(\frac{r+p-\alpha c_2-(1-\alpha)s_0}{r+p-s_2}\right)\sigma_L.$$

$$\pi_L(w_0^*, y_2^*|y_0, \mu_L) = \begin{cases} (r+p-s_2)\int_{-\infty}^{k_L} x dF_{\Delta_L|\mu_L}(x) + c_2 y_0 - p\mu_L, & y_0 \leq (1-\alpha)k_L; \\[2mm] (r+p-s_2)\int_{-\infty}^{\frac{y_0}{1-\alpha}} x dF_{\Delta_L|\mu_L}(x) + (r+p-\alpha c_2)\frac{y_0}{1-\alpha} \\[2mm] \quad -(r+p-s_2)\frac{y_0}{1-\alpha}F_{\Delta_L|\mu_L}\left(\frac{y_0}{1-\alpha}\right) - p\mu_L, & (1-\alpha)k_L < y_0 \leq (1-\alpha)k_H; \\[2mm] (r+p-s_2)\int_{-\infty}^{k_H} x dF_{\Delta_L|\mu_L}(x) + s_0 y_0 - p\mu_L, & y_0 > (1-\alpha)k_H. \end{cases}$$

$$(4.26)$$

We note that with normal distribution, it is possible for demand to go negative. However, here we may assume that the normal distribution parameter values are such that the probability of negative demand is negligible. By taking advantage of the special structure of the martingale forecast updating process, we can write out more explicitly the firm's second stage optimal profit function, which in turn will help us to characterize the firm's first stage problem. With martingale forecast updating, one can prove that (see Lemma A13)

$$\pi_L(w_0^*, y_2^*|y_0, \mu_L) = \begin{cases} -(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\frac{z_L^2}{2}} + (r-c_2)\mu_L + c_2 y_0, & y_0 \leq (1-\alpha)k_L; \\[3mm] -(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\left(\frac{\frac{y_0}{1-\alpha}-\mu_L}{\sqrt{2}\sigma_L}\right)^2} + (r+p-\alpha c_2)\frac{y_0}{1-\alpha} \\[3mm] \quad +(r+p-s_2)\left(\mu_L - \frac{y_0}{1-\alpha}\right)F_{\Delta_L|\mu_L}\left(\frac{y_0}{1-\alpha}\right) - p\mu_L, & (1-\alpha)k_L < y_0 \leq (1-\alpha)k_H; \\[3mm] -(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\frac{z_H^2}{2}} + (r-\alpha c_2 - (1-\alpha)s_0)\mu_L \\[3mm] \quad +s_0 y_0, & y_0 > (1-\alpha)k_H. \end{cases}$$

$$(4.27)$$

We note that the critical threshold k_H may not always exist, regardless of whether the firm's forecast updating process is a martingale. With $\alpha = 0$, i.e., there is no mandatory domestic production requirement, k_H never exists. In this case, the firm maintains, in expectation, no

domestic production. On the other hand, with $\alpha = 1$, then k_H always exists and the firm's LCC production is zero.

Substituting (4.27) into (4.25), we have

$$
\begin{aligned}
\pi(y_0) = -c_0 y_0 &+ \int_{\frac{y_0}{1-\alpha} - z_L \sigma_L}^{\infty} \left(-(r+p-s_2) \frac{\sigma_L}{\sqrt{2\pi}} e^{-\frac{z_L^2}{2}} + (r-c_2)\mu_L + c_2 y_0 \right) dG_L(\mu_L) \\
&+ \int_{\frac{y_0}{1-\alpha} - z_H \sigma_L}^{\frac{y_0}{1-\alpha} - z_L \sigma_L} \left(-(r+p-s_2) \frac{\sigma_L}{\sqrt{2\pi}} e^{-\left(\frac{\frac{y_0}{1-\alpha} - \mu_L}{\sqrt{2}\sigma_L}\right)^2} + (r+p-\alpha c_2) \frac{y_0}{1-\alpha} \right. \\
&\qquad\qquad \left. +(r+p-s_2)\left(\mu_L - \frac{y_0}{1-\alpha} \right) F_{\Delta_L|\mu_L}\left(\frac{y_0}{1-\alpha} \right) - p\mu_L \right) dG_L(\mu_L) \\
&+ \int_{-\infty}^{\frac{y_0}{1-\alpha} - z_H \sigma_L} \left(-(r+p-s_2) \frac{\sigma_L}{\sqrt{2\pi}} e^{-\frac{z_H^2}{2}} + (r-\alpha c_2 - (1-\alpha)s_0)\mu_L + s_0 y_0 \right) dG_L(\mu_L).
\end{aligned}
$$

$$(4.28)$$

In what follows, we explicitly derive the firm's first stage optimal procurement problem. One can show that (see Lemma A14)

$$
\begin{aligned}
\partial_{y_0} \pi(y_0) = -c_0 &+ c_2 \overline{G}_L \left(\frac{y_0}{1-\alpha} - z_L \sigma_L \right) + s_0 G_L \left(\frac{y_0}{1-\alpha} - z_H \sigma_L \right) \\
&+ \frac{1}{1-\alpha} \int_{\frac{y_0}{1-\alpha} - z_H \sigma_L}^{\frac{y_0}{1-\alpha} - z_L \sigma_L} \left[(r+p-\alpha c_2) - (r+p-s_2) F_{\Delta_L|\mu_L}\left(\frac{y_0}{1-\alpha} \right) \right] dG_L(\mu_L). \quad (4.29)
\end{aligned}
$$

Using (4.29), it is straightforward to show that

$$
\frac{\partial^2 \pi(y_0)}{\partial y_0^2} = -\frac{1}{(1-\alpha)^2} \int_{\frac{y_0}{1-\alpha} - z_H \sigma_L}^{\frac{y_0}{1-\alpha} - z_L \sigma_L} (r+p-s_2) F_{\Delta_L|\mu_L}\left(\frac{y_0}{1-\alpha} \right) dG_L(\mu_L) < 0,
$$

which confirms that the firm's first stage profit function is concave in y_0.

Theorem 36. *With additive martingale forecast updating, the firm's optimal OPA procurement*

quantity y_0^* satisfies

$$(r + p - c_2)G_L\left(\frac{y_0^*}{1-\alpha} - z_L\sigma_L\right) - (r + p - \alpha c_2 - (1-\alpha)s_0)G_L\left(\frac{y_0^*}{1-\alpha} - z_H\sigma_L\right)$$

$$- (r + p - s_2)\int_{\frac{y_0^*}{1-\alpha} - z_H\sigma_L}^{\frac{y_0^*}{1-\alpha} - z_L\sigma_L} F_{\Delta_L|\mu_L}\left(\frac{y_0^*}{1-\alpha}\right) dG_L(\mu_L) = (1-\alpha)(c_0 - c_2).$$

$$(4.30)$$

Note that (4.30) can be further simplified as

$$\int_{\frac{y_0^*}{1-\alpha} - z_H\sigma_L}^{\frac{y_0^*}{1-\alpha} - z_L\sigma_L} G_L(\mu_L)dF_{\Delta_L|\mu_L}\left(\frac{y_0^*}{1-\alpha}\right) = (1-\alpha)\frac{c_0 - c_2}{r + p - s_2}. \qquad (4.31)$$

The above theorem tells us that the firm's optimal OPA procurement quantity depends on the demand forecast of μ_L only through its distribution, not any particular realization of μ_L. However, the realization of μ_L significantly influences the firm's OPA shipment decision as well as the domestic production decision.

Corollary 6. *With additive forecast updating process as described in §4.3.6.5, the firm makes full shipment of its OPA production if the realized forecast level at time Δ_L is sufficiently high, i.e., $\mu_L \geq \frac{y_0}{1-\alpha} - \Phi^{-1}\left(\frac{r+p-\alpha c_2-(1-\alpha)s_0}{r+p-s_2}\right)\sigma_L$. Otherwise, the firm makes partial shipment of its OPA production.*

By Corollary 6, the firm is more likely to make full shipment of its OPA production if its domestic unit production cost c_2 is cheaper. This may seem counterintuitive, but it is reasonable because a lower domestic unit production cost will actually reduce the optimal OPA production quantity. Consequently, it is less likely for the firm to be in a position to reduce its OPA shipment quantity. It is straightforward to realize that the firm is also more

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likely to make the full shipment of its OPA production if the unit LCC cost c_0 is higher and/or the unit LCC salvage value s_0 is lower.

4.3.6.7   Leadtime Sensitivity

While we have proved in Theorem 35 that the firm's optimal expected profit is non-decreasing in the leadtime difference Δ_L for the direct OPA strategy, it is of interest to understand the magnitude of the influence induced by the leadtime difference Δ_L. Toward this end, we set up a numeric study according to the base case scenario parameter values (see §4.4.2.1 for details). We vary the leadtime Δ_L from 0 to 5 with a step size of 0.5. Because the impact of the leadtime difference Δ_L is likely to be linked with demand characteristics, we vary the demand mean from 40 to 160 with a step size of 30. We vary the demand coefficient of variation from 0.1 to 0.5 with a step size of 0.1. In addition, we vary the unit production cost c_0 from 0.1 to 0.9 with a step size of 0.1. Therefore, we have in total 2,475 observations.

We observed that, on average, the optimal expected profit is convex increasing in the leadtime difference Δ_L. Clearly, an increase in the leadtime difference Δ_L makes the direct OPA strategy become increasingly attractive to the firm. A key reason that a larger Δ_L is advantageous to the firm is the information learning effect associated with a longer leadtime difference. It is therefore reasonable to believe that the benefit associated with the leadtime difference will be sensitive to the inherent demand uncertainties. This intuition turns out to be correct. Figure 4.5 illustrates the firm's optimal expected profit as a function of the leadtime difference Δ_L as well as the demand coefficient of variation.
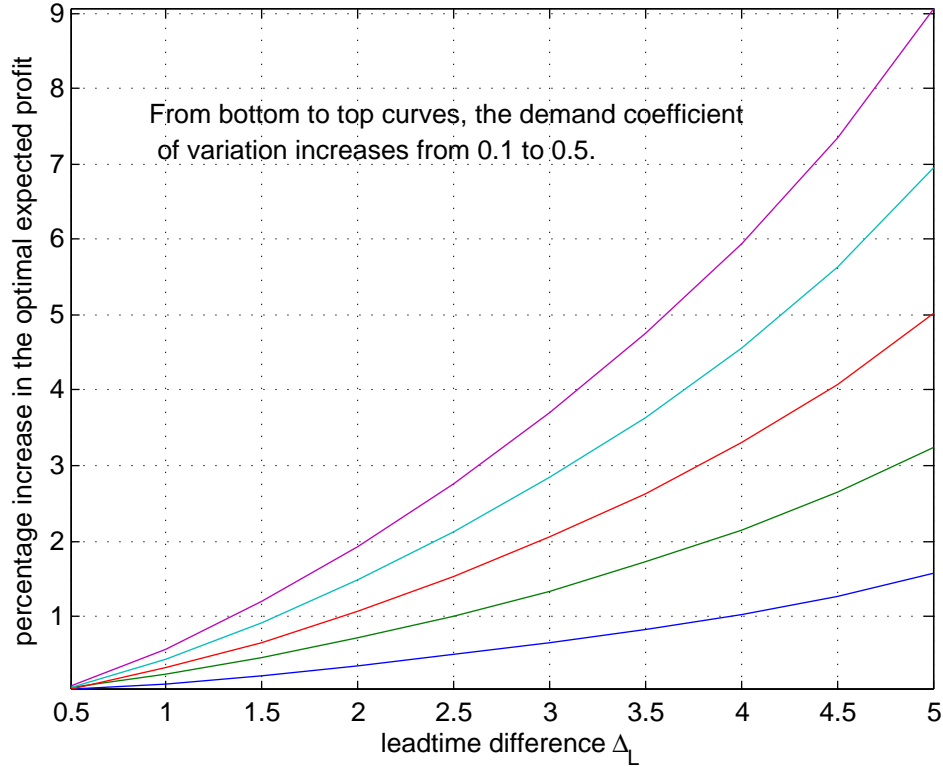
Figure 4.5: Direct OPA optimal expected profit as the leadtime difference increases

Note that an increase in the leadtime difference becomes increasingly more beneficial to the firm as the demand uncertainty becomes higher. When demand is highly uncertain, the inherent option value of having a postponed domestic production becomes much more valuable to the firm.

4.4 Comparison of Different Strategies

In this section, we investigate how the quota, demand, and cost characteristics affect the relative attractiveness of each strategy. We note that a firm's absolute preference for a particular strategy is influenced by a number of system parameters. Therefore, one can easily create cases

where one strategy is preferred to another strategy by changing different parameter values. The more interesting comparison, therefore, lies in the directional change of the relative attractiveness of different strategies. In what follows, we first describe the scenario when a parameter change influences only one strategy. We then investigate the scenario when a parameter change influences multiple strategies simultaneously.

4.4.1 Parameters Affecting A Single Strategy (Dual Strategies)

When a parameter affects only a single strategy, it is clear that if such a parameter change improves this single strategy, then, all else being equal, this parameter change must also increase the relative attractiveness of this single strategy over all other strategies. The following table summarizes the effect of an increase in the parameter values that affect only a single strategy (or, in the case of quota characteristics, that affect two strategies). Unless otherwise stated, all results in Table 4.2 are established analytically in the earlier sections (or when such results are straightforward to establish analytically).

Parameter	Direct Procurement	Split Procurement	Indirect OPA	Direct OPA
Expected Quota Price[a] μ	$\downarrow$	$\downarrow$	-	-
Quota Price Variance[b] σ	$\uparrow$	$\uparrow$	-	-
MCC Production Cost[c] c_1	-	$\downarrow$	$\downarrow$	-
Local Production Bound γ	-	-	$\downarrow$	-
DOM Production Bound α	-	-	-	$\downarrow$
Leadtime Difference Δ_L	-	-	-	$\uparrow$
DOM Production Cost c_2	-	-	-	$\downarrow$

[a]A more general concept of stochastic dominance also holds.

[b]For normally distributed quota price only.

[c]Note that the MCC production cost does not have to be identical for the split procurement and the indirect OPA strategies.

Table 4.2: Directional results on parameters affecting a single strategy or dual strategies

The above directional results are fairly intuitive, except that, for the case of normally distributed quota price, the firm's optimal expected profit increases in the quota price variance. This can be explained by the fact that the firm's downside risk of very high quota price is capped because the firm can simply salvage the product. On the other hand, the upside benefit of a very low quota price is very beneficial for the firm. This imbalance of the upside advantage versus the downside risk makes a more volatile quota price beneficial for the firm.

Based on Table 4.2, we now briefly discuss some straightforward intra-strategy comparisons. With respect to OPA strategies, because the OPA strategies are not influenced by the mean and the variance of the quota cost, an increase in the mean or decrease in the variance of the quota cost makes the OPA strategies more attractive. The comparison between the direct procurement strategy and the split procurement strategy is also straightforward, because the

split procurement strategy (weakly) dominates the direct procurement strategy. Thus, any incremental fixed cost toward the split procurement strategy simply makes the split procurement strategy less attractive. We also note that, even if the leadtime difference $\Delta_L = 0$, the direct OPA strategy is not necessarily dominated by the indirect OPA strategy. It depends on other system parameters, and in particular, on the relative restrictiveness of the domestic production bound α and γ.

4.4.2 Parameters Affecting Multiple Strategies

Some important parameters that influence multiple strategies include the LCC unit production cost and the unit revenue, as well as the demand characteristics. To investigate these parameter effects, we use a comprehensive numeric study to understand how the relative attractiveness of each different strategy is influenced by these parameter values. In what follows, we first describe the numeric study design and then present detailed results.

4.4.2.1 Numeric Study Design

We describe our numeric study in the sequence of cost/revenue parameters, quota parameters, demand parameters and policy specific parameters. In our description, we introduce parameter values for the base case scenario as well as the range values for any specific parameters.

For cost/revenue related parameters, we fix the domestic unit production cost at 1, i.e., $c_2 = 1$. The LCC unit production cost c_0 varies between 0.1 to 0.9, with a step size of 0.1. This gives us a total of 9 different cost ratios. Both the split procurement and the indirect OPA strategy involve a MCC, which we assume has a lower production cost as compared

with the domestic production cost. In addition, we assume the MCC production cost is more closely aligned with the domestic production cost. Therefore, we fix the MCC production cost $\hat{c}_1 = 0.8$, which is at the 80% of the domestic production cost. We note that the MCC involved in the split procurement strategy is not necessarily the same country involved in the indirect OPA strategy. For ease of exposition, we assume they have identical costs. Such an assumption merely shifts the absolute value of a given strategy and does not affect the change in the relative attractiveness of a particular strategy, which is our primary interest here.

We set the salvage value at 25% of the unit production cost. Therefore, s_2 is fixed at 0.25 and s_0 varies with c_0. Note that the salvage value for the MCC s_1 is set at 0.2, which is at 25% of the MCC production cost c_1. For the base case scenario, the unit revenue is set at $r = 3$. When the effect of the unit revenue is concerned, we vary the unit revenue from 1 to 5 with a step size of 1, which gives us a total of 5 values. We set the unit penalty cost at 20% of the unit revenue. Therefore, in the base case scenario, $p = 0.6$.

For the quota related parameter values, we chose the Weibull distribution for the quota cost distribution. The (two-parameter) Weibull distribution has non-negative support and allows very flexible distribution shapes. Note that the Weibull distribution can be completely specified by the mean and the coefficient of variation (cv). For the base case scenario, we set the mean and the cv of the quota price distribution as 0.3 and 0.5, respectively. Note that the base case scenario corresponds to the unimodal shape (resembles a skewed normal distribution). We vary the mean of the quota price from 0.1 to 0.5 with a step size of 0.1. Similarly, we vary the cv of the quota price from 0.3 to 0.7 with a step size of 0.1. Therefore, we have $5^2 = 25$ scenarios for the quota cost. In all these scenarios, the distribution is of the unimodal type as opposed to the exponential type. Therefore, the shape effect across these different scenarios is not very

significant.

For the demand related parameters, we use the normal distribution for the total demand. For the base case scenario, we set the mean demand equal to 100 with a coefficient of variation (cv) of .3. This implies that $\mu_0 = 100$ and $\sigma_0 = 30$ for the base case scenario. We vary the mean demand from 40 to 160 with a step size of 30. In addition, we vary the cv from 0.1 to 0.5 with a step size of 0.1. Thus, we have in total $5^2 = 25$ demand related scenarios.

There are some strategy-specific parameter values that need to be specified. For the direct OPA strategy, we adopt the linear martingale forecast updating process. In particular, we set the LCC production leadtime equal to 6, and we set the leadtime difference Δ_L equal to 1. This implies that the leadtime for domestic production is one period shorter than the LCC production. The standard deviation parameters σ_{μ_L} and σ_L can be derived from the demand and leadtime parameter. With the linear martingale process, we have $\sigma_{\mu_L} = \sqrt{L}\sigma$ and $\sigma_L = \sqrt{T - L}\sigma$, where $\sigma = \sigma_0/\sqrt{T}$. Therefore, for the base case scenario, we have $\sigma_{\mu_L} = 12.25$ and $\sigma_L = 27.39$. The value of σ_{μ_L} and σ_L will vary with the demand cv as well as the leadtime differences.

For the direct OPA policy parameter, we set the domestic production constraint $\alpha = 0.5$. (see DTI (2006)). For the indirect OPA approach, we set the "domestic" production constraint $\gamma = 0.4$. In this case, only the essential operation is required to be carried out in the host country (MCC in figure 4.1).

4.4.2.2   Demand and Information Characteristics

To investigate the demand characteristics on the firm's optimal expected profit under different strategies, we set all parameter values according to the base case scenario, but varying the mean demand from 40 to 160 with a step size of 30 and the demand cv from .1 to .5 with a step size of .1. In addition, we vary the LCC unit cost c_0 from .1 to .9 with a step size of .1. We also vary the leadtime difference Δ_L from 0 to 5 with a step size of 1. Thus, we have in total 1,225 observations.

We observed that the optimal expected profits for all four strategies increase approximately linearly in the expected demand. We also observed that the slope of the increase becomes steeper as the unit production cost c_0 increases. The slope of the increase in the expected profit, however, is different across different strategies. For a given set of parameter values, a strategy that starts off as the most preferred strategy will have the steepest slope, i.e., fastest growth in the expected profit as the expected demand increases. Similarly, a strategy that starts off as the least preferred strategy will have the shallowest slope, i.e., slowest growth in the expected profit as the expected demand increases. Consequently, an increase in the expected demand cannot change the rankings of the four mitigation strategies. An increase in the expected demand merely magnifies the absolute difference in the optimal expected cost among the different strategies.

For the demand coefficient of variation, we observed that the optimal expected profits for all four strategies decrease in the demand cv. The slope of the decline in the optimal expected profit for different strategies, however, is different. On average, the direct OPA strategy saw steepest decline in the optimal expected profit as the demand cv increases. On

the other extreme, the direct procurement strategy saw least decline in the optimal expected profit as the demand cv increases. On average, the rankings from the steepest decline to the shallowest decline by strategy is direct OPA, indirect OPA, split procurement, and lastly direct procurement. Consequently, in contrast to the effect of the expected demand, we observed that an increase in the demand cv can in fact shift the rankings of the four mitigation strategies. Figure 4.6 illustrates the average decline in the optimal expected profit for the four strategies as the demand cv increases.
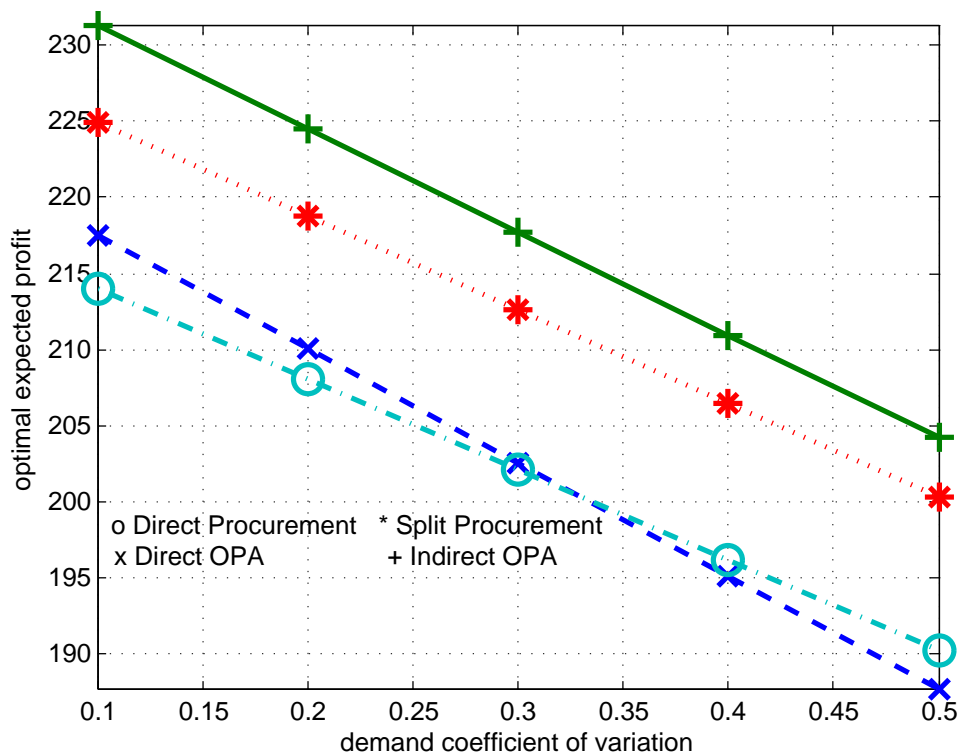
Figure 4.6: Optimal expected profits as the demand coefficient of variation increases

It is clear that, on average, as the demand cv increases, the direct procurement strategy becomes relatively more attractive. In particular, the direct procurement strategy overtakes the direct OPA strategy as the demand cv becomes large. Note that Figure 4.6 is an illustration of the average decline in the optimal expected profit as the demand cv increases. It does not

imply that the indirect OPA strategy is always preferred to all other strategies. In fact, the split procurement (direct procurement) strategy, for example, can be strictly preferred to all other strategies when the LCC unit production cost is relatively low. A detailed examination of the data reveals the following observations.

- The indirect OPA strategy declines faster than the split procurement strategy, except when the unit quota price is high and the unit production cost from LCC is moderate. With moderate LCC cost, the split procurement strategy is more likely to procure from both countries. In this case, the split procurement strategy seems to be more sensitive to the demand volatility, especially when the quota price is relatively high.

- The indirect OPA strategy always declines faster than the direct procurement strategy.

- The split procurement strategy declines faster than the direct procurement strategy, except when the LCC cost is extremely high. With extremely high LCC cost, the optimal procurement quantity for the direct procurement strategy becomes relatively more sensitive to changes in the demand volatility.

- The direct OPA strategy can decline faster or slower than any of the other strategies, depending on the leadtime difference Δ_L. With a lower leadtime difference, the direct OPA strategy quickly loses its appeal as the demand cv increases. With a higher leadtime difference, however, the direct OPA strategy can surpass all other strategies and become the most appealing strategy as the demand cv increases. Therefore, the leadtime difference Δ_L can significantly influence the attractiveness of the direct OPA strategy as the demand cv increases.

4.4.2.3   Cost Characteristics

The LCC unit production cost c_0 has a profound impact on the attractiveness of different strategies. We observed that the optimal expected profits for all four strategies decrease as the unit production cost c_0 increases. All but the split procurement strategy declines approximately linearly with respect to the unit cost c_0. As the LCC unit production cost increases, the relative attractiveness of different strategies shifts from direct procurement/split procurement strategies to the indirect OPA strategy and then to the split procurement strategy. See Figure 4.7. Note it is not the absolute values of the different strategies that makes such a shift interesting, but rather the relative speed of the change. In fact, the direct OPA strategy becomes steadily more attractive (except for the split procurement case) as the unit production cost c_0 increases, although with our base case scenario, the direct OPA strategy is never preferred. As discussed earlier, the appeal of the direct OPA strategy is strongest when the leadtime difference Δ_L is large and the demand volatility is high.
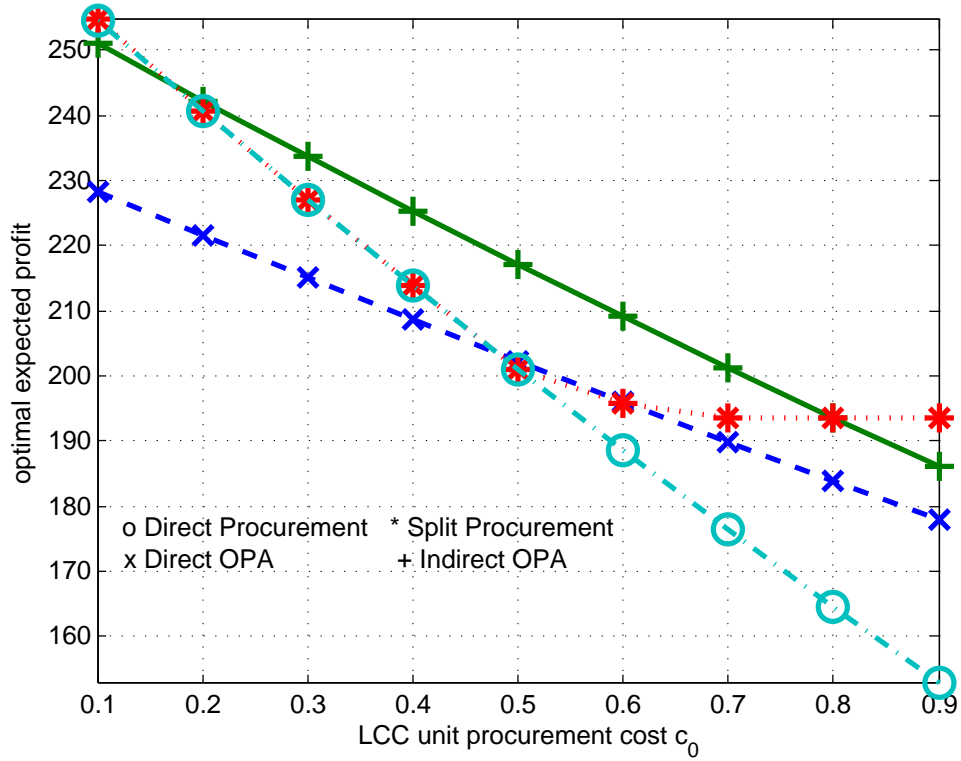
Figure 4.7: Optimal expected profits as the LCC unit procurement cost increases

On average, the direct procurement strategy declines the fastest with respect to the unit production cost c_0. This is due to the fact that the direct procurement strategy does not have a MCC production option as the unit cost c_0 increases. The direct OPA strategy is less sensitive to the unit cost c_0 as compared with the indirect OPA strategy. The reason is that, for the direct OPA strategy, the firm can adjust its LCC procurement quantity and its domestic production quantity (within the constraint α) to cope with the shift in the relative production cost. Finally, the split procurement strategy becomes independent of the LCC unit production cost when c_0 is very large, since the split procurement strategy simply degenerates to the single sourcing strategy from the MCC. Note that, if a fixed cost (however small) is charged to the split procurement strategy, then the direct procurement strategy is preferred to

the split procurement strategy for a large range of c_0 values.

In general, when the unit production cost c_0 is low, the sheer cost advantage makes the direct procurement and the split procurement strategy very attractive. However, as c_0 increases to a moderate level, the indirect OPA strategy can be more appealing since it balances the product cost without incurring uncertain quota cost. Finally, when the unit production cost c_0 is fairly large, the direct OPA strategy may become more attractive because of the leadtime advantage, especially when the demand is very volatile.

4.4.3 The Influence of Quota Cost on OPA Policy Bounds

Both the direct and the indirect OPA strategies remove the quota uncertainty, but at the expense of maintaining domestic (or MCC) production. This disadvantage is mainly reflected through the policy bound parameters α and γ for the direct and indirect OPA strategies, respectively. Therefore, it is of interest to gauge the upper bound of these two policies' parameters, at which both OPA strategies can still achieve at least as much expected profit as the direct procurement strategy.

To investigate this question, we set up a numeric study using base case scenario parameter values as described in §4.4.2.1, but varying the expected quota cost from 0.1 to 0.5 with a step size of 0.1, the unit cost c_0 from 0.1 to 0.9 with a step size of 0.1, and the demand cv from 0.1 to 0.5 with a step size of 0.1. We therefore have in total 225 scenarios for each OPA strategy. For each scenario, we search over the policy parameter values α and γ such that the optimal expected profit is at least as much as that of the direct procurement strategy.

We observed that adjusting the policy parameter always enables both OPA strategies to

achieve at least the expected profit achieved by the direct procurement policy. Furthermore, we observed that in 42% of the cases, the indirect OPA strategy achieved a higher expected profit than that achieved by the direct procurement strategy, even when the policy parameter γ is set to 1. On average, these cases achieved 7.12% higher expected profit than that by the direct procurement strategy. For the direct OPA strategy, we observed that in 33% of the cases it achieved a higher expected profit than that by the direct procurement strategy, even when the policy parameter α is set to 1. On average, these cases achieved 4.99% higher expected profit than that by the direct procurement strategy.

We observed that the upper bound of these policy parameters increases as the expected quota cost increases, indicating that removing the quota cost becomes more valuable. This also implies that the policy maker has a wider range of freedom to implement the OPA policies. Figure 4.8 illustrates the indifference curve for the direct OPA policy bound α and the optimal expected domestic production quantity. We note that both policy bounds α and γ (not plotted) is concave increasing in the expected quota cost, indicating more latitudes for the domestic (MCC) production requirements. This implies that, as the expected quota cost increases, the policy bounds for the OPA strategies increase in a concave fashion.
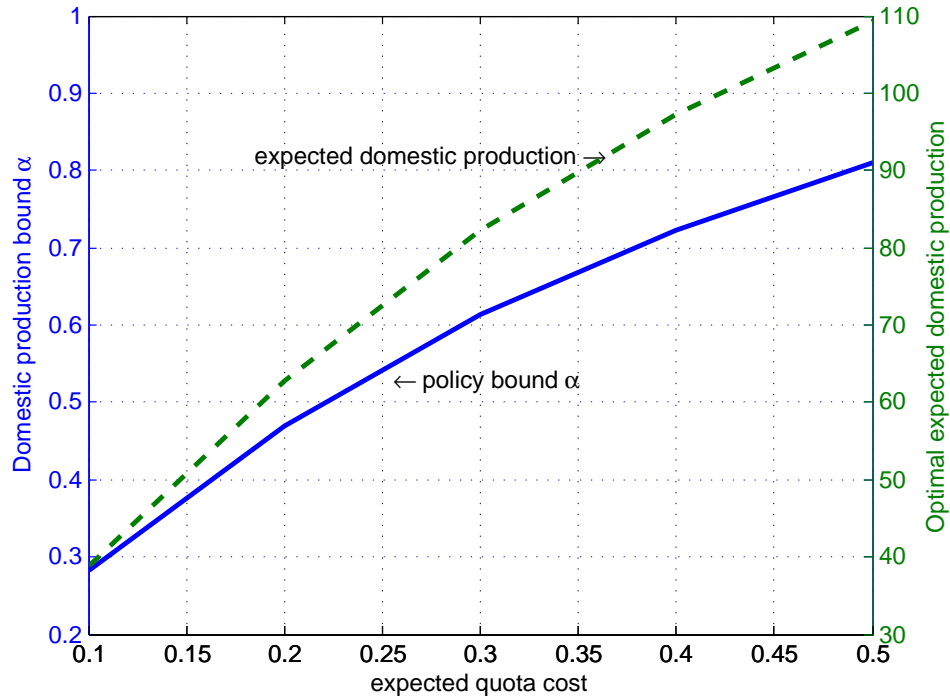
Figure 4.8: Policy bounds as the expected quota cost increases

Figure 4.8 indicates that an increase in the policy bound α indeed achieves the policy's purpose, that is, to maintain a higher level of domestic production quantity. However, we note that the policy bound α is an unstable indifference point, because a slight increase in α will make the direct procurement strategy more attractive. Consequently, if the policy bound exceeds these indifference points, then the firm may maintain zero domestic production quantities.

For the indirect OPA strategy, we note that an increase in the policy bound γ is accompanied by a decrease in the optimal procurement quantity. For the direct OPA strategy, an increase in the policy bound γ directly increases the costs for every unit procured. Consequently, the firm finds it optimal to reduce the procurement quantity to mitigate the significant increase in the procurement cost.

We now turn attention to the effect of demand volatility on the policy bounds α and γ. We observed that both policy bounds are decreasing in the demand coefficient of variation. This implies that the appropriate policy bounds for the OPA strategies should be differentiated by the demand characteristics. In particular, a higher policy bound can be implemented with a low demand volatility, but these policy bounds must be reduced when the demand becomes more volatile. Otherwise the OPA strategies will not be as competitive as the direct procurement (split procurement) strategy. See Figure 4.9.
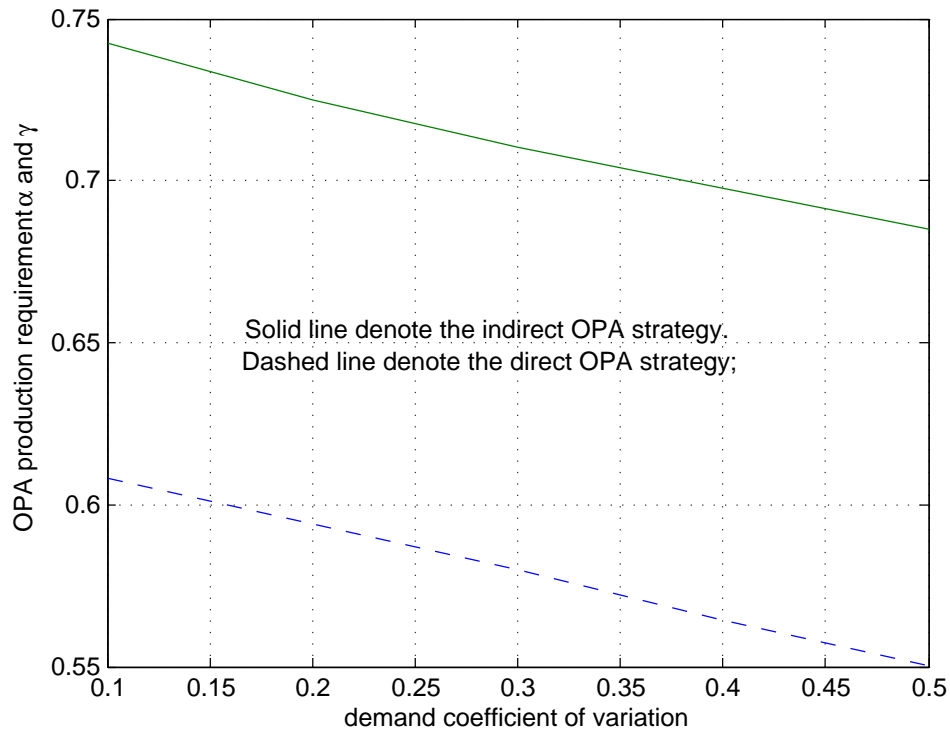
Figure 4.9: Policy bounds as the demand coefficient of variation increases

We also note that the policy bounds for α and γ increase in the LCC unit production cost c_0. This indicates that a higher unit production cost at the LCC makes the direct procurement strategy less attractive and therefore the OPA strategies more attractive, and this in turn relaxes the policy bound for α and γ. In fact, with very high LCC production cost, both α and

γ become 1, implying that complete domestic production can be preferred.

4.5   Conclusion

In this chapter, we contrast four different strategies for mitigating the supply risk induced by trade barriers. We characterize the optimal solutions for these strategies and provide managerial insights on the relative attractiveness of different strategies.

For the direct procurement and split procurement strategy, we prove that the optimal expected profit is decreasing in the quota cost in terms of the first order stochastic dominance. In other words, a stochastically increasing quota cost leads to a decrease in the optimal expected cost. In the special case of the normally distributed quota price, we prove that the firm's optimal expected profit is decreasing in the expected quota cost but increasing in the variance of the quota cost. For the split procurement strategy, we also provide sufficient conditions when the split procurement strategy strictly dominates the direct procurement strategy. In our comprehensive numeric study, we observed that the split procurement strategy strictly dominates the direct procurement strategy in more than 50% of the cases, indicating that the split procurement strategy can be strictly preferred even if there is an incremental fixed cost associated with the split procurement strategy.

For the direct OPA strategy, we prove that the firm's optimal expected profit is non-decreasing in the leadtime difference Δ_L. Furthermore, we investigate the special case of the martingale forecast updating process for the direct OPA strategy. We characterize the firm's optimal procurement decisions under the general martingale process. In our comprehensive numeric study, we observed that the firm's optimal expected profit is convex increasing in

the leadtime difference Δ_L. We also observed that the direct OPA strategy can become more attractive (relative to other strategies) as the demand coefficient of variation increases. A key advantage of the direct OPA strategy is its leadtime advantage. Our numeric study indicates that an increase in the leadtime advantage allows more flexible OPA policy bounds, i.e., the fraction of the domestic production requirement α.

When comparing the four strategies, we observed in our numeric study that an increase in the expected quota cost shifts the firm's preference from the direct procurement and split procurement strategies to the OPA strategies. We also observed that an increase in the mean demand does not shift the rankings of different strategies, whereas an increase in the demand coefficient of variation can alter the rankings of different strategies. In particular, we found that if the leadtime difference Δ_L is small, then an increase in the demand cv can make the direct procurement strategy more attractive than the direct OPA strategy. On the other hand, when Δ_L is large, then an increase in the demand cv can make the direct OPA strategy much more attractive and can surpass any other strategies.

Figure 4.10 is a schematic summarization of the firm's preference for the four different strategies, categorized by the demand coefficient of variation and the relative ratio of the LCC procurement cost versus the domestic procurement cost. We note that Figure 4.10 does not imply that, for any system parameters, the optimal strategy will transit as depicted. Instead, it illustrates possible progressions from one strategy to another. Depending on system parameters, a transition may not occur, or a particular strategy may be skipped. For example, as the demand coefficient of variation increases, the indirect OPA strategy may not necessarily transit to the direct OPA strategy, depending on the leadtime difference Δ_L for the direct OPA strategy.
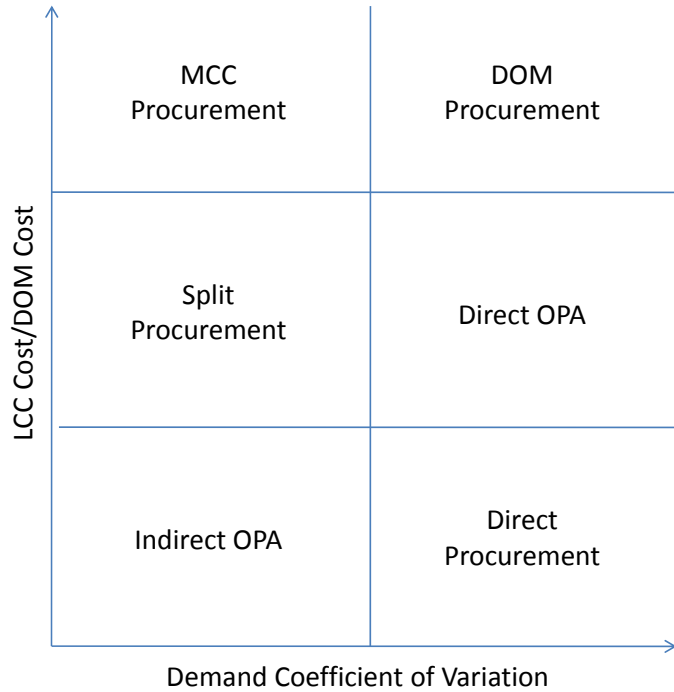
Figure 4.10: Summary of the mitigation strategy preferences by the unit procurement cost and demand volatility

Referring to Figure 4.10, we first focus on the relative LCC procurement cost. Note that when the LCC procurement cost is very low, the direct procurement strategy is more likely to be preferred. As the LCC procurement cost increases, the firm's preference transits from the direct procurement to the split procurement strategy, where the firm utilizes both the LCC and the MCC. As the LCC procurement cost continues to increase, the firm's preference will eventually reach the MCC procurement strategy.

When the indirect OPA strategy is preferred and when the LCC procurement cost is very low, an increase in the LCC cost can shift the firm's preference to the direct OPA strategy, because the direct OPA strategy is more flexible in adjusting its procurement quantity from LCC and from DOM. As the LCC procurement cost continues to increase, the firm can even-

tually prefer domestic procurement. We note that the firm will always procure from LCC as long as the LCC cost is lower than the domestic procurement cost. However, as the LCC cost approaches the domestic procurement cost, the procurement quantity from LCC diminishes.

We now turn attention to the demand volatility. When the firm adopts the split procurement and/or direct procurement strategy, everything else being equal, an increase in the demand volatility makes the firm more likely to prefer the direct procurement strategy. Given our base case scenario parameters, the newsvendor type of critical fractile exceeds 0.5. In this case, an increase in the demand volatility makes the strategy with lower procurement cost more attractive. This explains the transition from split procurement to direct procurement strategy. On the other hand, if the firm adopts the OPA strategies, then an increase in the demand volatility makes the direct OPA strategy more attractive relative to the indirect OPA strategy. The leadtime advantage associated with the direct OPA strategy becomes more valuable as the demand volatility increases, enabling the direct OPA strategy to overtake the indirect OPA strategy. Note that if the leadtime difference Δ_L is relatively small, then no transition will occur as the demand volatility increases.

We note that in Figure 4.10, a stochastically increasing quota cost will lead to a contraction in the split procurement and the direct procurement region, and an expansion of the direct OPA and the indirect OPA strategies. A stochastically increasing quota cost makes the direct procurement and the split procurement strategy less attractive, because they do not or only partially mitigate the quota risk.

A further extension of this chapter's work can be pursued in several directions. The four different mitigation strategies can also be categorized into one-country versus two-country strategies. This distinction can shift the firm's preference of different strategies when the exchange

rate risk is considered. Whether the exchange rate risk will influence the firm's preference, however, depends to a large degree on how the exchange rate risk is shared between the supplier and the buyer. Another direction would be to explore the strategic incentives between the supplier and the buying firm under different mitigation strategies. For example, with the direct procurement and the split procurement strategies, a firm can make different arrangements with its suppliers to jointly share the quota risk. Such an arrangement may significantly influence a firm's preference towards different strategies.

The VER risk is typically correlated with the market demand in the importing country for the specific controlled product. The higher the market demand, the less likely it is that a supplier will be able to obtain the necessary VER allocation. It is therefore of interest to investigate how the possible correlations between the quota risk and the market demand influence a firm's preference for different mitigation strategies.

Chapter 5

Summary

This dissertation research contributes, both academically and managerially, to several important aspects of the operational risks associated with global operations. In particular, the collection of chapters in this research studies three important challenges facing many firms operating in a global environment: leadtime uncertainty, capacity uncertainty, and trade barrier uncertainty. The insights obtained from this dissertation research are especially valuable and timely because, among the multitude of operational challenges posed by the proliferation of global trade, these three are some of the most frequently encountered challenges facing firms across many industries. A deeper understanding of, and more effective mitigation strategies toward, these challenges not only helps to advance the operations management field but also helps our field to be more relevant to practicing managers.

This dissertation also contributes, from a methodological point of view, to operational risk management in general. In contrast to the classic operations management approach, this dissertation research systematically takes a risk mitigation approach toward operational risk management. While many operational risks, such as demand risk and inventory risk, have

been studied since the early days of operations management (see Wagner (1975) for a detailed list of topics), it is only recently that these operational risks have been systematically studied from a risk mitigation perspective. A key difference between the risk mitigation perspective and the classic perspective is the following. With the classic approach, the objective is to find an optimal policy that minimizes, for example, the long run average system cost with a given set of system constraints. In contrast, with the risk mitigation perspective, the objective is to find an optimal strategy that does not necessarily treat the given set of system constraints as exogenous. Therefore, the risk mitigation perspective gives a richer set of strategic spaces that both academia and industry may find interesting and relevant. In what follows, we discuss the specific operational risks addressed in this dissertation and how this work has advanced the risk mitigation research. For specific operational risks studied in each chapter, we discuss the contributions from the theoretical, managerial, and technical aspects.

5.1 Leadtime Risk

Leadtime risk is not a new concept, as the leadtime uncertainty has been extensively studied both in the operations research and the operations management area. What distinguishes this research from the rest, however, is the unique treatment of leadtime risk. Typically, a leadtime uncertainty is treated either as a normal variation in the supply system, e.g., Zipkin (2000), or as a disruptive event, e.g., Kouvelis and Li (2006), but not at the same time. In many practical situations, however, the nature of the leadtime risk is often linked to the normal variation and the disruptive event. For example, a shipment might get delayed, and if it is delayed, then the length of the delay can be uncertain. Such mixed leadtime uncertainty has not, to the best of our knowledge, been studied before. A surprising consequence is that, everything else being

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equal, the firm may not necessarily prefer a supply leadtime with a lower variance.

Another distinguishing contribution of this chapter is the concept of using demand learning techniques to mitigate the supply leadtime risk. When the supply leadtime uncertainty is beyond the buying firm's control, the firm can use alternative techniques to mitigate such uncertainty. This indirect mitigation approach opens new opportunities for theoretical investigations beyond the context of this specific chapter. For example, when a risk factor is beyond a firm's control, the firm can develop alternative strategies to indirectly minimize its exposure to the particular risk factor.

This chapter develops several important managerial implications for firms facing supply leadtime risk. Among other significant managerial insights, we proved that a firm may not necessarily prefer a leadtime with a lower variance. The firm's preference depends on the leadtime distribution. This has important implications for firms that have multiple suppliers with different leadtime distributions.

In addition, we found that the firm's optimal timing decision is independent of actual revisions of the demand forecast (although the firm's optimal quantity decision does depend on the actual updates of the demand forecast process). This result has an important managerial implication: the firm can decide its procurement time in advance of the demand updating process, but it has to wait until that time to decide the procurement quantity. The very fact that the firm can decide its procurement time in advance is very beneficial from a planning perspective. The firm can arrange the future procurement schedule with its suppliers so that the suppliers can prepare beforehand.

To date, the supply leadtime risk and the demand risk have been studied separately. This

chapter is the first to integrate the firm's joint quantity and timing decision under the supply leadtime risk with the demand updating process. Even the basic, continuous time newsvendor model with supply leadtime uncertainty only (without demand forecast updating) has not been previously studied in the literature. In this chapter, we completely characterize the firm's optimal procurement quantity and timing decision for both the demand updating case and the no demand updating case.

In this chapter, we also introduce a continuous time, auto-correlated, unbiased demand forecast updating process. The multiplicative martingale model of forecast evolution can be seen as a special case of our model. In addition, the two-period, discrete time forecast updating process (e.g., Fisher and Raman (1996)) can also be seen as a special case[1] of our model.

5.2 Capacity Risk

Random capacity is a frequently encountered risk in global operations. A commonly adopted approach is to diversify across different suppliers to mitigate the capacity risk. Motivated by recent industry evidence, this chapter studies an alternative approach of investing in suppliers to improve their reliability. While process improvement itself has been extensively studied, e.g., Carrillo and Gaimon (2000), this research has focused on internal or intra-firm process improvement efforts with an emphasis on the particular approach of the process improvement techniques. In contrast, this chapter focus on the relative strength of the diversification strategy versus the improvement strategy. This chapter therefore advances potential mitigation strategies that firms may not have realized before. We found that the mere existence of the

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[1]Note that the demand in Fisher and Raman (1996) is normally distributed, whereas the demand in our model is lognormally distributed.

supplier improvement opportunity can significantly change a firm's allocation of its suppliers.

A distinguishing approach of this chapter is that we do not take supplier reliability as exogenous. Instead, consistent with the risk mitigation approach as discussed at the beginning of this chapter, we treat supplier reliability as a benchmark that can be improved upon. This risk mitigation approach allows a broader consideration of the firm's potential risk mitigation strategies. Furthermore, the risk mitigation approach can be applied to a far broader area of risk issues beyond the capacity uncertainty.

When the firm has the option to improve its suppliers' reliability, we found that the firm's supplier allocation strategy can be profoundly changed. In particular, a firm may choose to improve a supplier and source from that supplier, which would have never been chosen if there were no improvement opportunity. This surprising result also has significant implications from the supplier's perspective. The chance of a particular supplier being used can be significantly influenced by the buying firm's opportunity and willingness to improve supplier reliability.

In addition, we found that the firm's preference for a particular strategy, either improvement or diversification, cannot be consistently ranked as the industry environment changes. For example, as demand volatility increases, the firm may initially prefer the diversification strategy and then switch to the improvement strategy. This implies that the firm's strategic decisions with regard to supplier selection should depend on industry characteristics.

In this chapter, we completely characterize the optimal procurement policies under the diversification strategy. In fact, to the best of our knowledge, we are the first[2] to rigorously characterize the joint quantity decision problem under a general random capacity setting.

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[2]Note that Erdem (1999) studied the joint quantity setting problem. The result in that research is correct, however, no rigorous proof on the joint unimodal property was given.

In addition, we use the general stochastic dominance concept to characterize the relative attractiveness of the diversification versus the improvement strategy. By an innovative approach of modeling the reliability in a state space fashion, this research lays the foundation for future extensions in the direction of the multi-period problem.

5.3 Trade Barriers Risk

Regulatory trade barriers are among the most frequently encountered supply risks in global operations. Surprisingly, little research has been done in the operations management area to investigate the implications of regulatory trade barriers. This chapter is the first attempt at investigating the operational implications of a particular type of regulatory trade barriers, i.e., voluntary export restraints. While much of the analysis in this chapter is devoted to voluntary export restraints, many other forms of trade barriers, such as antidumping duties, can be easily incorporated into our framework.

A significant contribution of this chapter is its consideration of regulatory trade barriers in the operations management realm. While regulatory trade barriers have been under extensive investigation from the economics field, the focus of the economics literature is on the policy issue on the national level. Because regulatory trade barriers can have a profound impact on a firm's supply chain, it is essential for the operations management field to study and provide guidance for firms engaged in global operations. The mitigation strategies discussed in this chapter serve as the groundwork to inspire future research in this important and relevant area.

By contrasting several different risk mitigation strategies, we provide managerial implications on the firm's optimal strategy to reduce or eliminate voluntary export restraints. Without

such managerial guidance, it is difficult for firms to evaluate, let alone develop, consistent procurement strategies when facing different regulatory trade barriers across different countries. Our research indicates that the firm's risk mitigation strategy depends significantly on several important industry characteristics, such as demand volatility, unit procurement cost, and voluntary export restraints volatility.

Our research also has significant policy implications. We found that OPAs are indeed effective ways for a government to encourage domestic production. However, the attractiveness of OPAs can be significantly influenced by industry characteristics, such as demand volatility and the efficiency (in terms of production leadtime) of domestic production. In addition, we find that the outward processing policy is an unstable one; a slight increase in the outward processing policy requirement may lead to a complete loss of domestic production.

To the best of our knowledge, this chapter is the first to model regulatory trade barriers from the operations management perspective. In addition, we completely characterize a continuous time, additive martingale model of forecast evolution process in a two-stage, joint optimization problem with decision dependent constraints. Furthermore, we model the voluntary quota risk from the uncertain cost perspective, which allows a broader class of regulatory trade barriers to be treated in a mathematically equivalent form.

Appendices

A1 Appendices for Chapter 2

A1.1 Derivation of the Forecast Evolution

In what follows, we derive the time-dependent demand forecast given in (2.1). We begin with a discrete-time model of pre-season and in-season order evolution in which T is divided into $n \geq 1$ periods.

A1.1.1 Pre-Season Orders

Let X_i denote the amount of pre-season orders received up to and including period i. Let Δ_i denote the ratio of pre-season orders in period i. Define

$$X_i = \Delta_i X_{i-1}, \quad 1 \leq i \leq n, \tag{A-1}$$

$$\Delta_i = \Delta_{i-1}^{\phi} \xi_i^{1-\phi}, \quad 1 \leq i \leq n, \tag{A-2}$$

where ϕ is the autocorrelation parameter, and ξ_i are independent, identically distributed log-normal random variables with parameters μ and σ. We initialize $X_0 = x_0$ and $\Delta_0 = \delta_0$. Using

(A-1) and (A-2), it can be shown that

$$\ln X_i = \left(\sum_{j=1}^{i} \phi^j\right) \ln \delta_0 + (1-\phi)\left(\left(\sum_{j=1}^{i} \phi^{j-1}\right) \ln \xi_1 + \left(\sum_{j=1}^{i-1} \phi^{j-1}\right) \ln \xi_2 + \cdots \right.$$

$$\left. + \left(\sum_{j=1}^{1} \phi^{j-1}\right) \ln \xi_i\right) + \ln x_0. \tag{A-3}$$

Note that

$$\sum_{j=1}^{k} \phi^{j-1} = \frac{1-\phi^k}{1-\phi} \tag{A-4}$$

for any $k \geq 1$. Therefore, substitute (A-4) into (A-3),

$$\ln X_i = \frac{\phi}{1-\phi}\left(1-\phi^i\right) \ln \delta_0 + \left(1-\phi^i\right) \ln \xi_1 + \left(1-\phi^{i-1}\right) \ln \xi_2 + \cdots + (1-\phi) \ln \xi_i + \ln x_0$$

$$= \frac{\phi}{1-\phi}\left(1-\phi^i\right) \ln \delta_0 + \sum_{j=1}^{i}\left(1-\phi^{i-j+1}\right) \ln \xi_j + \ln x_0. \tag{A-5}$$

Note that (A-5) characterizes the cumulative pre-season orders i periods from now, given the current cumulative pre-season order is x_0 and pre-season order ratio is δ_0. By (A-5), the mean and variance of $\ln X_i$ is given by

$$E[\ln X_i] = \frac{\phi}{1-\phi}\left(1-\phi^i\right) \ln \delta_0 + \left(i - \frac{\phi}{1-\phi}\left(1-\phi^i\right)\right) \mu + \ln x_0. \tag{A-6}$$

$$Var[\ln X_i] = \sum_{j=1}^{i}\left(1-\phi^{i-j+1}\right)^2 \sigma^2 = \sum_{j=1}^{i}\left(1-2\phi^j+\phi^{2j}\right) \sigma^2$$

$$= \left(i - 2\frac{\phi}{1-\phi}\left(1-\phi^i\right) + \frac{\phi^2}{1-\phi^2}\left(1-\phi^{2i}\right)\right) \sigma^2$$

$$= \left(i - \frac{\phi}{1-\phi}\left(1-\phi^i\right) - \frac{\phi}{1-\phi^2}\left(1-\phi^i\right)\left(1-\phi^{i+1}\right)\right) \sigma^2 \tag{A-7}$$

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A1.1.2 In-Season Orders

Let X_D represent pre-season orders plus in-season orders. Define

$$X_D = \Delta_D X_n, \tag{A-8}$$

$$\Delta_D = \Delta_n^{\phi_D} \xi_D^{1-\phi_D}, \tag{A-9}$$

where ϕ_D is the between-season correlation parameter, and ξ_D is an independent, lognormal random variable with parameters μ_D and σ_D. Using (A-8) and (A-9), it can be shown that

$$
\begin{aligned}
\ln X_D &= \left(\phi_D \phi^n + \frac{\phi}{1-\phi} \left(1 - \phi^n\right) \right) \ln \delta_0 + \phi_D (1-\phi) \sum_{j=1}^{n} \phi^{n-j} \ln \xi_j \\
&\quad + \sum_{j=1}^{n} \left(1 - \phi^{n-j+1}\right) \ln \xi_j + (1 - \phi_D) \ln \xi_D + \ln x_0 \\
&= \left(\phi_D \phi^n + \frac{\phi}{1-\phi} \left(1 - \phi^n\right) \right) \ln \delta_0 + \sum_{j=1}^{n} \left(1 - \phi^{n-j} \left(\phi - (1-\phi)\phi_D\right)\right) \ln \xi_j \\
&\quad + (1 - \phi_D) \ln \xi_D + \ln x_0
\end{aligned} \tag{A-10}
$$

Note that by setting $n = 1$, we obtain a two-period demand model, where X_1 and X_D represent pre-season orders and pre-season plus in-season orders, respectively. In particular, the in-season orders $X_D - X_1$ is correlated with the pre-season orders X_1 through parameter ϕ_D.

By (A-10), the mean and variance of $\ln X_D$ is given by

$$E[\ln X_D] = \left(\phi_D \phi^n + \frac{\phi}{1-\phi} (1 - \phi^n) \right) \ln \delta_0 +$$

$$\left(n - (1 - \phi^n) \left(\frac{\phi}{1-\phi} - \phi_D \right) \right) \mu + (1 - \phi_D) \mu_D + \ln x_0. \qquad \text{(A-11)}$$

$$Var[\ln X_D] = \sum_{j=1}^{n} \left(1 - \phi^{n-j} (\phi - (1-\phi) \phi_D) \right)^2 \sigma^2 + (1 - \phi_D)^2 \sigma_D^2. \qquad \text{(A-12)}$$

Let $A = \phi - (1 - \phi) \phi_D$, we have

$$\sum_{j=1}^{n} \left(1 - A\phi^{n-j} \right)^2 = \sum_{j=1}^{n} \left(1 - 2A\phi^{n-j} + A^2 \phi^{(n-j)^2} \right)$$

$$= n - 2A \frac{1 - \phi^n}{1 - \phi} + A^2 \frac{1 - (\phi^2)^n}{1 - \phi^2}$$

$$= n - (1 - \phi^n) \left(\frac{\phi}{1-\phi} - \phi_D \right) \left(2 - (\phi - (1-\phi)\phi_D) \frac{1 + \phi^n}{1 + \phi} \right). \qquad \text{(A-13)}$$

Substitute (A-13) into (A-12),

$$Var[\ln X_D] = \left[n - (1 - \phi^n) \left(\frac{\phi}{1-\phi} - \phi_D \right) \left(2 - \left(\frac{\phi}{1-\phi} - \phi_D \right) (1 + \phi^n) \right) \right] \sigma^2 + (1 - \phi_D)^2 \sigma_D^2.$$

$$\text{(A-14)}$$

With the continuous time analog of the above analysis, the total pre-season and in-season orders X_D is lognormally distributed, with n replaced by $(T - t)$ in (A-11) and (A-14). This leads to the forecast evolution given by (2.1). Without loss of generality, setting $\delta_0 = 1$ and $x_0 = 1$ leads to the initial forecast given by (2.2).

A1.2 Heuristics

While we can efficiently solve the time-and-quantity optimization problem, it is of interest to explore the performance of some simple heuristics. In this section we investigate the performance of three heuristics: a) the mean demand heuristic; b) the mean leadtime heuristic; and c) the zero planned leadtime heuristic. Recall that for a given procurement time, the optimal procurement quantity is easily found using (2.7). Therefore, the heuristics determine a procurement time and then use the optimal procurement quantity given the chosen procurement time.

In the case of deterministic demand, the expected cost is convex in the procurement time, and one can show that the optimal procurement time is given by

$$t^* \;=\; T - L - G^{-1}\left(\frac{p}{p+h} - \frac{\theta}{1-\theta}\frac{h}{p+h}\right),$$

i.e., a newsvendor-type result is obtained. In the mean demand heuristic, we set the optimal procurement time using this equation. This mean demand heuristic captures the trade-off between the holding costs incurred by an early arrival and the tardiness costs incurred by a late arrival. The mean leadtime heuristic sets the procurement time as $t = T - L - (1 - \theta) E[\omega]$, i.e., the order is placed such that the expected arrival time coincides with the selling time. The zero planned leadtime heuristic sets the procurement time as $t = T - L$. We note that our mean demand and mean leadtime heuristics generalize the mean demand and mean leadtime heuristics in Song et al. (2000) to the case of $\theta > 0$.

We explore the performance of the heuristics using the same factorial design described in §2.6.3. For each problem instance, we calculate the relative increase in the expected cost

(for a given heuristic) as compared to the optimal cost and we also determine which heuristic performs best. In Table 5.1, we report the average and maximum relative increases as well as the percentage of cases in which a particular heuristic performed best. Other than the mean demand heuristic in the case of no pre-season orders, we see from Table 5.1 that none of the heuristics perform particularly well on average. In all cases, the worst case performances are very poor.

Scenario		Mean demand	Mean leadtime	Zero planned leadtime
	Average	0.05%	1.54%	7.32%
$\lambda = 0$	Max	0.61%	10.11%	101.78%
	Cases preferred	96.69%	3.31%	0.00%
	Average	0.84%	1.90%	4.97%
$\lambda = 0.5$	Max	14.75%	21.86%	94.85%
	Cases preferred	63.96%	26.06%	9.97%
	Average	5.61%	6.55%	3.88%
$\lambda = 1$	Max	51.06%	63.36%	86.43%
	Cases preferred	41.78%	24.76%	33.46%

Table 5.1: Average and maximum percentage increase in the expected cost with heuristics.

It is worth qualitatively comparing our no pre-season order results with those of Song et al. (2000). Recall that Song et al. (2000) did not consider pre-season orders. We note that Song et al. (2000) use Poisson demand and a negative binomial leadtime distribution in their numeric study, but we don't consider this to be a material difference. They observed that "the mean demand heuristic, although suboptimal, never produced poor results" (p. 294). This conclusion is not supported in our study. A detailed review of the data uncovered a reason for this seeming anomaly. The study in Song et al. (2000) considered two possible values of r/c; 1.5 and 2.5. Our study encompassed a much wider range of r/c values. When the revenue is much higher than the purchasing cost, i.e., at high r/c values, then the firm's procurement quantity is large. A large procurement quantity results in a large inventory cost if the order arrives early, and

so high r/c values tend to be associated with later procurement times. We observed that the mean demand heuristic, while capturing some of the earliness effect through the h term, did not compensate sufficiently for large procurement quantities, and so performed quite poorly for high r/c values.

More often than not, the optimal procurement time with pre-season orders was later than that without pre-season orders. Because the zero planned leadtime heuristic results in the latest procurement time among the three heuristics, its performance was better in the case of pre-season orders than in the case of no pre-season orders. For high pre-season order volatilities and high r/c values, the zero planned leadtime heuristic performed well because it is beneficial for the firm to delay its procurement as long as possible in such settings.

Generally speaking, we advise caution before using any of these heuristics. While one can find settings where a particular heuristic performs quite well, none of the heuristics are robust.

A1.3   Technical Lemmas

Lemma A1. *For $\theta < 1$, define*

$$H(z) = y^*(z) \frac{\theta + (1-\theta)G(z)}{(1-\theta)(1-G(z))}.$$

(a) $H(0) \geq H(\bar{z}) = 0$. (b) $H'(z) = 0 \Rightarrow H''(z) < 0$. (c) $H(z)$ is unimodal in z if $\sup(H(z)) > H(0)$, and $H(z)$ is non-increasing in z otherwise.

The $H(z)$ function is closely related to $v'(z)$, the derivative of the objective function with respect to z. Therefore, characterizing $H(z)$ helps us to identify the stationary points of the

objective function. Part (b) of Lemma A1 states that whenever $H'(z) = 0$, then $H''(z) < 0$. Combining this with part (a) guarantees that $H(z)$ has at most a single peak over $[0, \bar{z}]$. Part (c) recognizes the possibility of a corner point.

Proof of Lemma A1. Note that

$$H'(z) = \frac{y'(z)(\theta + (1-\theta)G(z))}{(1-\theta)(1-G(z))} + \frac{y(z)g(z)}{(1-\theta)(1-G(z))^2}. \tag{A-15}$$

(a) Follows from the fact that $H(0) = y^*(0)\frac{\theta}{1-\theta} > 0$ and $H(\bar{z}) = 0$. (b) Note

$$H''(z) = \frac{y''(z)(\theta + (1-\theta)G(z))}{(1-\theta)(1-G(z))} + \frac{2y'(z)g(z)}{(1-\theta)(1-G(z))^2} + \frac{y(z)g'(z)}{(1-\theta)(1-G(z))^2} + \frac{2y(z)g^2(z)}{(1-\theta)(1-G(z))^3}.$$

At $H'(z) = 0$, $H''(z)$ can be further simplified to

$$H''(z) = \frac{y''(z)(\theta + (1-\theta)G(z)) + 2(1-\theta)y'(z)g(z)}{(1-\theta)(1-G(z))} + \frac{y(z)g'(z)}{(1-\theta)(1-G(z))^2}. \tag{A-16}$$

To determine the sign of $H''(z)$ at $H'(z) = 0$, we first obtain the expression for $y'(z)$ and $y''(z)$. From (2.7), we have

$$y'(z) = \frac{1}{F'\left[F^{-1}\left[\frac{r-c-hA(z)}{r-s}\right]\right]} \left(-\frac{h}{r-s}A'(z)\right) = -\frac{1}{f(y(z))}\frac{h}{r-s}(\theta + (1-\theta)G(z)) = -\frac{h(\theta + (1-\theta)G(z))}{(r-s)f(y(z))}$$

$$\tag{A-17}$$

and

$$y''(z) = -\frac{h(1-\theta)g(z)}{(r-s)f(y(z))} + \frac{h(\theta + (1-\theta)G(z))}{(r-s)[f(y(z))]^2}f'(y(z))y'(z)$$

$$= \frac{h}{(r-s)f(y(z))}\left[-(1-\theta)g(z) + (\theta + (1-\theta)G(z))\frac{f'(y(z))}{f(y(z))}y'(z)\right].\qquad\text{(A-18)}$$

From (A-15), we have

$$y'(z) = -\frac{y(z)g(z)}{(1-G(z))(\theta + (1-\theta)G(z))}\qquad\text{(A-19)}$$

at $H'(z) = 0$. Combining (A-17) and (A-19), we have

$$f(y(z)) = \frac{h(1-G(z))[\theta + (1-\theta)G(z)]^2}{(r-s)y(z)g(z)}.$$

Therefore,

$$f'(y(z)) = \frac{-hg(z)[\theta + (1-\theta)G(z)]^2 + h(1-G(z))2[\theta + (1-\theta)G(z)](1-\theta)g(z)}{(r-s)y(z)g(z)}$$

$$-\frac{h(1-G(z))[\theta + (1-\theta)G(z)]^2}{(r-s)[y(z)g(z)]^2}[y'(z)g(z) + y(z)g'(z)].$$

Hence,

$$\frac{f'(y(z))}{f(y(z))} = \left\{\frac{-hg(z)[\theta + (1-\theta)G(z)]^2 + h(1-G(z))2[\theta + (1-\theta)G(z)](1-\theta)g(z)}{(r-s)y(z)g(z)}\right.$$

$$\left.-\frac{h(1-G(z))[\theta + (1-\theta)G(z)]^2}{(r-s)[y(z)g(z)]^2}[y'(z)g(z) + y(z)g'(z)]\right\}$$

$$\times\frac{(r-s)y(z)g(z)}{h(1-G(z))[\theta + (1-\theta)G(z)]^2}$$

$$= -\frac{g(z)}{1-G(z)} + \frac{2(1-\theta)g(z)}{\theta + (1-\theta)G(z)} - \frac{y'(z)g(z) + y(z)g'(z)}{y(z)g(z)}.$$

Now, consider the terms in squared bracket in (A-18), we have

$$-(1-\theta)g(z) + [\theta + (1-\theta)]G(z)\frac{f'(y(z))}{f(y(z))}y'(z)$$

$$= -(1-\theta)g(z) + \{-\frac{g(z)[\theta + (1-\theta)G(z)]}{1-G(z)} + 2(1-\theta)g(z)$$

$$- \frac{y'(z)}{y(z)}[\theta + (1-\theta)G(z)] - \frac{g'(z)}{g(z)}[\theta + (1-\theta)G(z)]\}y'(z)$$

$$= -(1-\theta)g(z) + \left\{3(1-\theta)g(z) - \frac{g'(z)}{g(z)}[\theta + (1-\theta)G(z)]\right\}y'(z). \qquad \text{(A-20)}$$

From (A-19) we know $y'(z) < 0$. Combining with the assumption that $g'(z) \leq 0$, it follows that (A-20) is negative. Consequently, (A-18) is negative, i.e., $y''(z) < 0$. Therefore, (A-16) is negative, i.e., $H''(z) < 0$. (c) We show that there exists a unique local maximum if $\sup(H(z)) > H(0)$. Because $\sup(H(z)) > H(0)$, there exists a point z^* such that $H(z^*) > H(0) > H(\bar{z})$. Since $H(\cdot)$ is continuous in z, there exists two points $z_a < z^* < z_b$ such that $\partial H(z)/\partial z|_{z=z_a} > 0$ and $\partial H(z)/\partial z|_{z=z_b} < 0$. Because $H'(z)$ is also continuous in z, by Intermediate Value Theorem (Royden, 1988) there exists a point $z_a < z_c < z_b$ such that $\frac{\partial H(z)}{\partial z}|_{z=z_c} = 0$. It follows from part (b) that z_c is a local maximum. We prove the uniqueness through contradiction. Suppose $H(\cdot)$ achieves another local maximum at z_d. Without loss of generality, assume $z_c < z_d < \bar{z}$. From part (b), $H'(z_c + \epsilon) < 0$ and $H'(z_d - \epsilon) > 0$. By continuity of $H'(\cdot)$, there exists a point $z_c < z_e < z_d$ such that $H'(z_e) = 0$ and $H''(z_e) > 0$. But this cannot happen given the results in part (b). The case of $\sup(H(z)) \leq H(0)$ can be analogously proved by realizing that $H(0)$ is both a maximum and a corner point of $H(z)$. □

Lemma A2. $\mathcal{Z} = \{z : v'(z) = 0\}$, *i.e.,* $\mathcal{Z}$ *is the set of stationary points.* $(a)\mathcal{Z} = \varnothing$ *if* $\sup(H(z)) < \frac{p}{h}E[X_D]$. *(b)* $\mathcal{Z} = \{z_1\}$ *and* $0 < z_1 < \bar{z}$ *if* $H(0) > \frac{p}{h}E[X_D]$. *(c)* $\mathcal{Z} = \{z_1, z_2\}$ *and* $0 \leq z_1 \leq z_2 < \bar{z}$ *if* $H(0) \leq \frac{p}{h}E[X_D]$ *and* $\sup(H(z)) \geq \frac{p}{h}E[X_D]$.

Lemma A2 tells us that $v(z)$ has zero, one, or two stationary points, depending on how the $H(\cdot)$ function compares to the ratio of the tardiness penalty cost and the unit holding cost.

Proof of Lemma A2. Substitute $y^*(z)$ into (2.9), we have

$$
\begin{aligned}
\frac{\partial v(z)}{\partial z} &= h\left(\theta + (1-\theta)\int_0^z g(\omega)d\omega\right)y^*(z) + \left(h\left(\theta z + (1-\theta)\int_0^z (z-\omega)g(\omega)d\omega\right) - (r-c)\right)\frac{\partial y^*(z)}{\partial z} \\
&\quad + (r-s)\int_0^{y^*(z)}\frac{\partial y^*(z)}{\partial z}f(\xi)d\xi + p(1-\theta)E[X_D]\int_z^\infty -g(\omega)d\omega \\
&= hy^*(z)\left(\theta + (1-\theta)\int_0^z g(\omega)d\omega\right) + \left(hA(z) - (r-c) + (r-s)\int_0^{y^*(z)}f(\xi)d\xi\right)\frac{\partial y^*(z)}{\partial z} \\
&\quad - p(1-\theta)E[X_D]\int_z^\infty g(\omega)d\omega \\
&= hy^*(z)\left(\theta + (1-\theta)G(z)\right) - p(1-\theta)E[X_D](1-G(z)). \tag{A-21}
\end{aligned}
$$

Therefore, $\frac{\partial v(z)}{\partial z} = 0 \Leftrightarrow H(z) = \frac{p}{h}E[X_D]$. (a) Follows directly from the fact that $\sup(H(z)) < \frac{p}{h}E[X_D]$. (b) Follows from part (c) of Lemma A1 and the fact that $H(0) > \frac{p}{h}E[X_D]$. (c) Follows analogously from part (b). $\qquad\square$

Lemma A3. $v(t, x_t, \delta_t)$ *(see (2.6)) is separable in (x_t, δ_t) and t, i.e., there exists two real valued functions $v_1(\cdot)$ and $v_2(\cdot)$ such that $v(t, x_t, \delta_t) = v_1(x_t, \delta_t) \cdot v_2(t)$.*

Proof of Lemma A3. First we write out the explicit expression for $v(t, x_t, \delta_t)$

$$
\begin{aligned}
v(t, x_t, \delta_t) =& (hA(T-L-t) - (r-c))y^*(t) + (r + pB(T-L-t))\, x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right) \\
&+ (r-s)\int_0^{y^*(t)}(y^*(t) - \xi)f_t(\xi|x_t, \delta_t)d\xi. \tag{A-22}
\end{aligned}
$$

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Because $F_t(y^*(t)|x_t, \delta_t) = \frac{r-c-hA(T-L-t)}{r-s}$, (A-22) can be simplified to

$$v(t, x_t, \delta_t) = (hA(T - L - t) - (r - c)) y^*(t) + (r + pB(T - L - t)) x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right)$$

$$+ (r - s)y^*(t) \int_0^{y^*(t)} f_t(\xi|x_t, \delta_t)d\xi - (r - s) \int_0^{y^*(t)} \xi f_t(\xi|x_t, \delta_t)d\xi$$

$$= (hA(T - L - t) - (r - c)) y^*(t) + (r + pB(T - L - t)) x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right)$$

$$+ (r - s)y^*(t)\frac{r - c - hA(T - L - t)}{r - s} - (r - s) \int_0^{y^*(t)} \xi f_t(\xi|x_t, \delta_t)d\xi$$

$$= (r + pB(T - L - t)) x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right) - (r - s) \int_0^{y^*(t)} \xi f_t(\xi|x_t, \delta_t)d\xi.$$

$$(A\text{-}23)$$

Note

$$\int_0^{y^*(t)} \xi f_t(\xi|x_t, \delta_t)d\xi$$

$$= \frac{1}{\sqrt{2\pi\psi_\sigma(t)}} \int_0^{y^*(t)} \exp\left(-\frac{(\ln\xi - \psi_\mu(t) - \psi_\delta(t) - \ln x_t)^2}{2\psi_\sigma(t)}\right) d\xi$$

$$= \frac{\exp\left(\psi_\mu(t) + \psi_\delta(t) + \ln x_t + \frac{1}{2}(\psi_\sigma(t))\right)}{\sqrt{2\pi\psi_\sigma(t)}} \sqrt{\frac{\pi}{2}} \sqrt{\psi_\sigma(t)} \cdot \operatorname{erfc}\left(\frac{\psi_\mu(t) + \psi_\delta(t) + \ln x_t + \psi_\sigma(t) - \ln y^*(t)}{\sqrt{2(\psi_\sigma(t))}}\right)$$

$$= \frac{1}{2}x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right)\left(1 - \operatorname{erf}\left(\frac{\psi_\mu(t) + \psi_\delta(t) + \ln x_t + \psi_\sigma(t) - \ln y^*(t)}{\sqrt{2(\psi_\sigma(t))}}\right)\right)$$

$$= \frac{1}{2}x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right)\left(1 + \operatorname{erf}\left(\frac{\ln y^*(t) - \psi_\mu(t) - \psi_\delta(t) - \ln x_t}{\sqrt{2(\psi_\sigma(t))}} - \frac{\sqrt{\psi_\sigma(t)}}{\sqrt{2}}\right)\right),$$

$$(A\text{-}24)$$

where we have used the fact

$$\int_0^y \exp\left(-\frac{(\ln\xi - M)^2}{2S}\right) d\xi = \exp(M + \frac{S}{2})\sqrt{\frac{\pi}{2}}\sqrt{S} \cdot \operatorname{erfc}\left(\frac{M + S - \ln y}{\sqrt{2S}}\right).$$

Combining (2.7) and (2.11), we have

$$\text{erf}\left(\frac{\ln y^*(t) - \psi_\mu(t) - \psi_\delta(t) - \ln x_t}{\sqrt{2\psi_\sigma(t)}}\right) \equiv 2\frac{r - c - hA(T - L - t)}{r - s} - 1.$$

Define

$$k(t) = \frac{\ln y^*(t) - \psi_\mu(t) - \psi_\delta(t) - \ln x_t}{\sqrt{2\psi_\sigma(t)}} = \text{erf}^{-1}\left(2\frac{r - c - hA(T - L - t)}{r - s} - 1\right), \qquad \text{(A-25)}$$

and substitute (A-25) into (A-24), we have

$$\int_0^{y^*(t)} \xi f_t(\xi|x_t, \delta_t)d\xi = \frac{1}{2}x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right)\left(1 + \text{erf}\left(k(t) - \frac{\sqrt{\psi_\sigma(t)}}{\sqrt{2}}\right)\right).$$

$$\text{(A-26)}$$

Substitute (A-26) into (A-23), we have

$$v(t, x_t, \delta_t) = x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right) \cdot$$
$$\left(r + pB(T - L - t) - \frac{1}{2}(r - s)\left(1 + \text{erf}\left(k(t) - \frac{\sqrt{\psi_\sigma(t)}}{\sqrt{2}}\right)\right)\right). \qquad \text{(A-27)}$$

By (A-27), $v(t, x_t, \delta_t)$ is separable in (x_t, δ_t) and t, i.e., $v(t, x_t, \delta_t) = v_1(x_t, \delta_t) \cdot v_2(t)$, where

$v_1(\cdot, \cdot)$ is just the first two terms and $v_2(\cdot)$ is the rest of terms in (A-27) respectively. $\qquad \square$

Lemma A4. $E\left[\chi(t, X_t, \Delta_t)|x_0, \delta_0\right]$ *is independent of* t, *where*

$$\chi(t, X_t, \Delta_t) = X_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right).$$

Proof of Lemma A4. First consider $t = 0$. Recall without loss of generality, we scale $x_0 = 1$

and $\delta_0 = 1$.

$$E\left[\chi(t, X_t, \Delta_t)|x_0, \delta_0\right] = x_0 \exp\left(\psi_\mu(0) + \psi_\delta(0) + \frac{1}{2}\psi_\sigma(0)\right) = \exp\left(\psi_\mu(0) + \frac{1}{2}\psi_\sigma(0)\right),$$

which is a constant. Now, consider an arbitrary $t > 0$.

$$E\left[\chi(t, X_t, \Delta_t)|x_0, \delta_0\right] = E\left[X_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right)\middle|x_0, \delta_0\right]$$

$$= \exp\left(\psi_\mu(t) + \frac{1}{2}\psi_\sigma(t)\right) E\left[X_t \exp\left(\psi_\delta(t)\right)|x_0, \delta_0\right].$$

Note that $X_t \exp\left(\psi_\delta(t)\right)$ is lognormally distributed and therefore $\ln(X_t \exp\left(\psi_\delta(t)\right))$ is normally distributed. In what follows, we first derive the mean and variance parameter for $\ln(X_t \exp\left(\psi_\delta(t)\right))$ and then derive the expectation of $X_t \exp\left(\psi_\delta(t)\right)$. Using (A-2), we have

$$\ln \Delta_n = \phi^n \ln \delta_0 + (1 - \phi)\left[\sum_{j=1}^{n} \phi^{j-1} \ln \xi_{n-j+1}\right].$$

Therefore,

$$E[\ln \Delta_n] = \phi^n \ln \delta_0 + (1 - \phi)\left[\sum_{j=1}^{n} \phi^{j-1}\mu\right] = \phi^n \ln \delta_0 + (1 - \phi^n)\mu.$$

The continuous time analog is

$$E[\ln \Delta_t] = \phi^t \ln \delta_0 + (1 - \phi^t)\mu. \tag{A-28}$$

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Using (A-28), we have

$$E[\ln(X_t \exp(\psi_\delta(t)))] = E[\ln X_t] + E[\psi_\delta(t)]$$

$$= \left(t - \frac{\phi}{1-\phi}\left(1-\phi^t\right)\right)\mu + \left[\phi_D\phi^{T-t} + \frac{\phi}{1-\phi}\left(1-\phi^{T-t}\right)\right]\left(1-\phi^t\right)\mu$$

$$= \left(\phi_D\left[\phi^{T-t} - \phi^T\right] + t - \frac{\phi}{1-\phi}\left[\phi^{T-t} - \phi^T\right]\right)\mu. \qquad \text{(A-29)}$$

Now consider the following discrete case,

$$\ln X_i + \psi_\delta(i) = \sum_{j=1}^{i}(1-\phi^{i-j+1})\ln\xi_j + \left(\phi_D\phi^{n-i} + \frac{\phi}{1-\phi}(1-\phi^{n-i})\right)\ln\Delta_i$$

$$= \sum_{j=1}^{i}(1-\phi^{i-j+1})\ln\xi_j + \left(\phi_D\phi^{n-i} + \frac{\phi}{1-\phi}(1-\phi^{n-i})\right)(1-\phi)\sum_{j=1}^{i}\phi^{i-j}\ln\xi_j$$

$$= \sum_{j=1}^{i}\left(1-\phi^{i-j}\left[\phi-(1-\phi)\phi_D\phi^{n-i} - \phi(1-\phi^{n-i})\right]\right)\ln\xi_j.$$

Therefore,

$$Var[\ln X_i + \psi_\delta(i)] = \left(i - (1-\phi^i)\phi^{n-i}\left[\frac{\phi}{1-\phi} - \phi_D\right]\left[2 - \phi^{n-i}\left(\phi-(1-\phi)\phi_D\right)\frac{1+\phi^i}{1+\phi}\right]\right)\sigma^2.$$

The continuous time analog is

$$Var[\ln X_t + \psi_\delta(t)] = \left(t - (1-\phi^t)\phi^{T-t}\left[\frac{\phi}{1-\phi} - \phi_D\right]\left[2 - \phi^{T-t}\left(\phi-(1-\phi)\phi_D\right)\frac{1+\phi^t}{1+\phi}\right]\right)\sigma^2.$$

Therefore,

$$E\left[\chi(t, X_t, \Delta_t)|x_0, \delta_0\right] = \exp\left(\psi_\mu(t) + \frac{1}{2}\psi_\sigma(t)\right.$$

$$+ \left(\phi_D\left[\phi^{T-t} - \phi^T\right] + t - \frac{\phi}{1-\phi}\left[\phi^{T-t} - \phi^T\right]\right)\mu$$

$$+ \frac{1}{2}\left(t - (1-\phi^t)\phi^{T-t}\left[\frac{\phi}{1-\phi} - \phi_D\right]\left[2 - \phi^{T-t}\left(\phi - (1-\phi)\phi_D\right)\frac{1+\phi^t}{1+\phi}\right]\right)\sigma^2\right)$$

$$= \exp\left(\psi_\mu(0) + \frac{1}{2}\psi_\sigma(0)\right),$$

which is independent of t. $\qquad\square$

Lemma A5. *Define* $\eta(t) = \text{erf}(\gamma(t))$, *where* $\gamma(t) = k(t) - \rho(t)$ *and* $\rho(t) = \sqrt{\frac{1}{2}\psi_\sigma(t)}$. *If* $\phi = \phi_D = 0$, *then*

(i) *A sufficient set of condition for* $\eta'''(t) \le 0$ *is* $C1 \cap ((C2 \cap C3) \cup (C4 \cap C5))$,

(ii) *A sufficient set of condition for* $\eta'''(t) \ge 0$ *is* $C1 \cap ((C2 \cap C6) \cup (C4 \cap C7))$,

where

$$C1 \quad : \quad r + s - 2c \le 0$$

$$C2 \quad : \quad \gamma''(T - L) \le 0$$

$$C3 \quad : \quad 2\gamma(0)^2 - 1 - \frac{\rho'''(T-L) - k'''(0)}{2\gamma'(T-L)^3} - 3\frac{\gamma(T-L)\gamma''(T-L)}{\gamma'(0)^2} \le 0$$

$$C4 \quad : \quad \gamma''(0) \ge 0$$

$$C5 \quad : \quad 2\gamma(0)^2 - 1 - \frac{\rho'''(T-L) - k'''(0)}{2\gamma'(0)^3} - 3\frac{\gamma(0)\gamma''(T-L)}{\gamma'(0)^2} \le 0$$

$$C6 \quad : \quad 2\gamma(T-L)^2 - 1 - \frac{\rho'''(0) - k'''(T-L)}{2\gamma'(0)^3} - 3\frac{\gamma(0)\gamma''(0)}{\gamma'(T-L)^2} \ge 0$$

$$C7 \quad : \quad 2\gamma(T-L)^2 - 1 - \frac{\rho'''(0) - k'''(T-L)}{2\gamma'(T-L)^3} - 3\frac{\gamma(T-L)\gamma''(0)}{\gamma'(T-L)^2} \ge 0$$

Proof of Lemma A5. Applying $\phi = \phi_D = 0$, we have

$$\rho'(t) = -\frac{\sigma^2}{4}\left(\frac{(T-t)\sigma^2 + \sigma_D^2}{2}\right)^{-\frac{1}{2}} < 0,$$

$$\rho''(t) = -\frac{\sigma^4}{16}\left(\frac{(T-t)\sigma^2 + \sigma_D^2}{2}\right)^{-\frac{3}{2}} < 0,$$

$$\rho'''(t) = -\frac{3\sigma^6}{64}\left(\frac{(T-t)\sigma^2 + \sigma_D^2}{2}\right)^{-\frac{5}{2}} < 0,$$

$$\rho''''(t) = -\frac{15\sigma^8}{256}\left(\frac{(T-t)\sigma^2 + \sigma_D^2}{2}\right)^{-\frac{7}{2}} < 0.$$

In addition, $r \le 2c - s \Rightarrow k(t) \le 0$. Therefore,

$$k'(t) = \frac{1}{\mathrm{erf}'(k(t))}\frac{2h}{r-s}A'(T-L-t) = \frac{1}{\mathrm{erf}'(k(t))}\frac{2h}{r-s}\left(\theta + (1-\theta)G(T-L-t)\right) > 0,$$

$$k''(t) = -\frac{1}{\left(\mathrm{erf}'(k(t))\right)^2}\mathrm{erf}''(k(t))k'(t)\frac{2h}{r-s}A'(T-L-t) - \frac{1}{\mathrm{erf}'(k(t))}\frac{2h}{r-s}(1-\theta)g(T-L_t)$$

$$= 2k(t)\left(k'(t)\right)^2 - \frac{1}{\mathrm{erf}'(k(t))}\frac{2h}{r-s}(1-\theta)g(T-L-t) < 0,$$

$$k'''(t) = 2\left(k'(t)\right)^3 + 4k(t)k'(t)k''(t) + \frac{1}{\left(\mathrm{erf}'(k(t))\right)^2}\mathrm{erf}''(k(t))k'(t)\frac{2h}{r-s}(1-\theta)g(T-L-t)$$

$$= 2\left(k'(t)\right)^3 + 4k(t)k'(t)k''(t) - \frac{2k(t)k'(t)}{\mathrm{erf}'(k(t))}\frac{2h}{r-s}(1-\theta)g(T-L-t) > 0,$$

$$k''''(t) = 6\left(k'(t)\right)^2 k''(t) + 4(k'(t))^2 k''(t) + 4k(t)(k''(t))^2 + 4k(t)k'(t)k''(t)$$

$$- \left(\frac{2(k'(t))^2 + k(t)k''(t)}{\mathrm{erf}'(k(t))} + \frac{4(k(t)k'(t))^2}{\mathrm{erf}'(k(t))}\right)\frac{2h}{r-s}(1-\theta)g(T-L-t)$$

$$= 10\left(k'(t)\right)^2 k''(t) + 4k(t)(k''(t))^2 + 4k(t)k'(t)k'''(t)$$

$$- \frac{1}{\mathrm{erf}'(k(t))}\left(2(k'(t))^2 + k(t)k''(t) + 4(k(t)k'(t))^2\right)\frac{2h}{r-s}(1-\theta)g(T-L-t) < 0.$$

Note

$$\eta'''(t) = \text{erf}'\left(k(t) - \rho(t)\right)\left(k'''(t) - \rho'''(t)\right) \tag{$n1$}$$

$$- 6\left(k(t) - \rho(t)\right)\left(k'(t) - \rho'(t)\right)\left(k''(t) - \rho''(t)\right) \tag{$n2$}$$

$$+ 4(k(t) - \rho(t))^2(k'(t) - \rho'(t))^3 \tag{$n3$}$$

$$- 2(k'(t) - \rho'(t))^3\Big). \tag{$n4$}$$

The lemma statements follow by appropriately bounding $(n1) - (n4)$ terms. The derivation is rather tedious and therefore is omitted here. The details are available upon request. $\qquad \square$

Lemma A6. *(a)* $\iota'(t) \le 0$, *where* $\iota(t) = T - t - \left(1 - \phi^{T-t}\right)\left(\frac{\phi}{1-\phi} - \phi_D\right)\left(2 - (\phi - (1-\phi)\phi_D)\frac{1+\phi^{T-t}}{1+\phi}\right)$.

(b) $\kappa'(t) \le 0$, *where* $\kappa(t) = T - t - \left(1 - \phi^{T-t}\right)\left(\frac{\phi}{1-\phi} - \phi_D\right)$.

Proof of Lemma A6. Here we provide proof for part (a). The simpler case of part (b) can be analogously proved. Note that

$$
\begin{aligned}
\iota'(t) &= -1 - \left(\frac{\phi}{1-\phi} - \phi_D\right)\left[\phi^{T-t}\ln\phi\left(2 - (\phi - (1-\phi)\phi_D)\frac{1+\phi^{T-t}}{1+\phi}\right)\right. \\
&\qquad\qquad\qquad\qquad \left. + \left(1 - \phi^{T-t}\right)(\phi - (1-\phi)\phi_D)\frac{\phi^{T-t}\ln\phi}{1+\phi}\right] \\
&= -1 - \left(\frac{\phi}{1-\phi} - \phi_D\right)\phi^{T-t}\ln\phi\left[2 - 2(\phi - (1-\phi)\phi_D)\frac{\phi^{T-t}}{1+\phi}\right]. \tag{A-30}
\end{aligned}
$$

Clearly, if $\frac{\phi}{1-\phi} - \phi_D \le 0$ then $\iota'(t) < 0$. Now consider the case of $\frac{\phi}{1-\phi} - \phi_D > 0$. By (A-30),

$$\iota''(t) = 2\left(\frac{\phi}{1-\phi} - \phi_D\right)\phi^{T-t}\ln^2\phi\left[1 - 2\left(\frac{\phi}{1-\phi} - \phi_D\right)\phi^{T-t}\frac{1-\phi}{1+\phi}\right]. \tag{A-31}$$

Therefore, $\iota''(t) \ge 0$ if and only if the terms in the squared bracket in (A-31) is non-negative.

Let $v(t) = 1 - 2\left(\frac{\phi}{1-\phi} - \phi_D\right)\phi^{T-t}\frac{1-\phi}{1+\phi}$. Then $v'(t) = 2\left(\frac{\phi}{1-\phi} - \phi_D\right)\phi^{T-t}\ln\phi\frac{1-\phi}{1+\phi} < 0$. Thus, if

$v(T) > 0$ then we know $v(t) > 0$ for any t. But $v(T) = 1 - 2\left(\frac{\phi}{1-\phi} - \phi_D\right)\frac{1-\phi}{1+\phi} > 0$ because

$$1 - \phi > -2(1-\phi)\phi_D \Rightarrow \phi - 2(1-\phi)\phi_D < 1 \Rightarrow 2(\phi - (1-\phi)\phi_D) < 1 + \phi$$

$$\Rightarrow 2\frac{\phi - (1-\phi)\phi_D}{1+\phi} < 1 \Rightarrow 2\left(\frac{\phi}{1-\phi} - \phi_D\right)\frac{1-\phi}{1+\phi} < 1 \Rightarrow v(T) > 0.$$

Therefore, we have proved that $\iota''(t) \geq 0$, which indicates that $\iota'(t)$ is increasing in t. Hence, if

we prove that $\iota'(T) < 0$ then it follows $\iota'(t) < 0$ for any t. Note that

$$\iota'(T) = -1 - \left(\frac{\phi}{1-\phi} - \phi_D\right)\ln\phi\left[2 - 2\left(\phi - (1-\phi)\phi_D\right)\frac{1}{1+\phi}\right].$$

It follows that

$$\frac{\partial\iota'(T)}{\partial\phi_D} = 2\ln\phi\left[1 - 2\left(\phi - (1-\phi)\phi_D\right)\frac{1}{1+\phi}\right] = 2\ln\phi\ v(T) < 0.$$

Therefore, if $\iota'(T) < 0$ at $\phi_D = 0$ then $\iota'(T) < 0$ for any ϕ_D. Substitute $\phi_D = 0$ into $\iota'(T)$, we

have

$$\iota'(T) = -1 - \frac{\phi}{1-\phi}\ln\phi\left[2 - 2\frac{\phi}{1+\phi}\right] = -1 - 2\frac{\phi}{1-\phi^2}\ln\phi.$$

The lemma statement follows if $v(\phi) = \frac{\phi}{1-\phi^2}\ln\phi > -\frac{1}{2}$. Note

$$v'(\phi) = \frac{1+\phi^2}{(1-\phi^2)^2}\left[\ln\phi + \frac{1-\phi^2}{1+\phi^2}\right].$$

Note the terms in the squared bracket in the above expression, denote as $g(\phi)$, is increasing in

ϕ because

$$g'(\phi) = \frac{1}{\phi} - 2\phi\frac{1}{1+\phi^2} - 2\phi\frac{1-\phi^2}{(1+\phi^2)^2} = \frac{1}{\phi}\left(\frac{1-\phi^2}{1+\phi^2}\right)^2 \geq 0.$$

Therefore, if $g(\phi) \leq 0$ at $\phi = 1$ then $g(\phi) \leq 0$ at for any ϕ. Clearly, $g(\phi|\phi = 1) = 0$. Thus, we have $v'(\phi) \leq 0$. Therefore, $v(\phi)$ is lower bounded at $\phi = 1$. Note

$$\lim_{\phi\to1} v(\phi) = \frac{\ln 1}{1-1} = 0 \cdot \infty \Rightarrow \lim_{\phi\to1} v(\phi) = \lim_{\phi\to1} \frac{\frac{d\ln\phi}{d\phi}}{\frac{d(1-\phi^2)}{d\phi}} = \lim_{\phi\to1} -\frac{1}{2\phi^2} = -\frac{1}{2}.$$

$\square$

Lemma A7. *Define*

$$\eta(t) = \mathrm{erf}(k(t) - \rho(t)),$$

where $\rho(t) = \sqrt{\frac{1}{2}\psi_\sigma(t)}$. *If* $\eta'''(t) < 0$, *then (a) If* $\eta''(T-L) \geq \frac{2p}{r-s}(1-\theta)g(0)$, *then* $M(t)$ *is a concave function of* t *for* $0 \leq t \leq T - L$. *(b) If* $\eta''(0) \leq \frac{2p}{r-s}(1-\theta)g(T-L)$, *then* $M(t)$ *is a convex function of* t *for* $0 \leq t \leq T - L$. *(c) Otherwise, let* $0 < t_s < T - L$ *denote the unique solution to* $\eta''(t) = \frac{2p}{r-s}(1-\theta)g(T-L-t)$. *Then,* $M(t)$ *is a concave function of* t *for* $0 \leq t \leq t_s$ *and a convex function of* t *for* $t_s < t \leq T - L$.

Lemma A8. *Define*

$$\eta(t) = \mathrm{erf}(k(t) - \rho(t)),$$

where $\rho(t) = \sqrt{\frac{1}{2}\psi_\sigma(t)}$. *If* $\eta'''(t) \geq 0$, *then (a) If* $\eta''(0) \geq \frac{2p}{r-s}(1-\theta)g(T-L)$, *then* $M(t)$ *is a concave function of* t *for* $0 \leq t \leq T - L$. *(b) If* $\eta''(T-L) \leq \frac{2p}{r-s}(1-\theta)g(0)$, *then* $M(t)$ *is a convex function of* t *for* $0 \leq t \leq T - L$. *(c) Otherwise, let* $0 < t_s < T - L$ *denote the unique solution to* $\eta''(t) = \frac{2p}{r-s}(1-\theta)g(T-L-t)$. *Then,* $M(t)$ *is a convex function of* t *for* $0 \leq t \leq t_s$

and a concave function of t for $t_s < t \leq T - L$.

Lemma A7 and A8 tells us that $M(t)$ can be concave and/or convex in different regions of t, depending on relative parameter values.

Proof of Lemma A7. Recall

$$\eta(t) = \mathrm{erf}(k(t) - \rho(t)),$$

where $\rho(t) = \sqrt{\frac{1}{2}\psi_\sigma(t)}$. We have

$$\eta'(t) = \left[\mathrm{erf}'(k(t) - \rho(t))\right]\left[k'(t) - \rho'(t)\right],$$

$$\eta''(t) = \left[\mathrm{erf}'(k(t) - \rho(t))\right]\left[-2(k(t) - \rho(t))(k'(t) - \rho'(t))^2 + k''(t) - \rho''(t)\right],$$

$$\eta'''(t) = \left[\mathrm{erf}'(k(t) - \rho(t))\right]\left[k'''(t) - \rho'''(t) - 6(k(t) - \rho(t))(k'(t) - \rho'(t))(k''(t) - \rho''(t))\right.$$

$$\left. + 2(k'(t) - \rho'(t))^3 \left(2(k(t) - \rho(t))^2 - 1\right)\right].$$

Note that, for $t \leq T - L$,

$$B''(T - L - t) = (1 - \theta)g(T - L - t) \geq 0,$$

$$B'''(T - L - t) = -(1 - \theta)g'(T - L - t) \geq 0.$$

Therefore, $(1 - \theta)g(T - L - t)$ is convex increasing in $[0, T - L]$. By assumption $\eta''(t)$ is monotonically decreasing in $[0, T - L]$. The lemma statements follow by realizing that $B''(T - L - t)$ and $\eta''(t)$ cross at most once. $\square$

Proof of Lemma A8. Analogous to the proof of Lemma A7. $\square$

A1.4 Proofs

Proof of Theorem 1. Let

$$c(y) = (hA(T-L-t)-(r-c))y + (r-s)E[(y-X_D)^+|x_t,\delta_t] + (r+pB(T-L-t))\,E[X_D|x_t,\delta_t].$$

One can show that $c(y)$ is convex in y, and

$$c'(y) = hA(T-L-t) - (r-c) + (r-s)F_t(y|x_t,\delta_t). \tag{A-32}$$

Setting (A-32) $= 0$ obtains the desired result. $\qquad\square$

Proof of Theorem 2. When $h = 0$, $v'(z) \leq 0$. The result then follows directly by realizing that $v(z)$ is constant for $z \geq \hat{\omega}$. $\qquad\square$

Proof of Theorem 3. First we show that $z^* \geq 0$. It is necessary and sufficient to show that $v(0) \leq v(-\epsilon)$ for any $\epsilon > 0$. From (2.9),

$$v(0) = (hA(0) - (r-c))y^*(0) + (r-s)E[(y^*(0) - X_D)^+] + pB(0)E[X_D]$$

$$= -(r-c)y^*(0) + (r-s)E[(y^*(0) - X_D)^+] + pB(0)E[X_D]$$

$$\leq -(r-c)y^*(0) + (r-s)E[(y^*(0) - X_D)^+] + pB(-\epsilon)E[X_D]$$

$$= -(r-c)y^*(-\epsilon) + (r-s)E[(y^*(-\epsilon) - X_D)^+] + pB(-\epsilon)E[X_D]$$

$$= v(-\epsilon).$$

Next we show that $z^* \leq \bar{z}$. By assumption, $\hat{\omega} \geq \bar{z}$. By definition, $y^*(\bar{z}) = 0$. Because $y^*(z)$ is

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non-increasing in z, $y^*(z) = 0$ for any $z \geq \bar{z}$. It follows directly from (2.9) that $v(\bar{z}) \leq v(\bar{z} + \epsilon)$ for any $\epsilon > 0$. $\qquad\square$

Proof of Theorem 4. Please see the proof of Theorem 10 which proves a more general case. $\qquad\square$

Proof of Theorem 5. (i)For $\theta = 1$, $A(z) = z^+$ and $B(z) = z^-$. By Theorem 3, $z^* \geq 0$. By (2.9), $v'(z) > 0 \Rightarrow z^* = 0$. (ii)(a) Note that $\mathcal{Z} = \varnothing \Leftrightarrow v'(z) \leq 0$. Hence, $v(z) \leq v(z - \epsilon)$ for any $\epsilon > 0$. (b) In this case $v(\cdot)$ is maximized at z_1. Because $H(z)$ is unimodal or non-increasing, the optimal $z^* \in \{0, \bar{z}\}$. Let $v(\varnothing)$ represent the non-participation cost, i.e., the maximum cost when the firm decides not to procure/sell. But $v(\bar{z}) = pB(\bar{z})E[X_D] > 0 = v(x_\varnothing)$. Therefore $v(\bar{z})$ cannot minimize $E[v(\cdot)]$. It follows directly that $v(0) < v(\bar{z})$, i.e., $z^* = 0$. (c) Note that $v(\cdot)$ is minimized at z_1 and maximized at z_2. It follows that $v(z_1) < v(\bar{z}) < v(z_2)$, i.e., $z^* = z_1$. $\qquad\square$

Proof of Theorem 6. Let $D(z^*)$ denote (A-21) being set to 0, i.e.,

$$D(z^*) = hy^*(z^*)\left(\theta + (1 - \theta)G(z^*)\right) - p(1 - \theta)E[X_D](1 - G(z^*)). \qquad \text{(A-33)}$$

Because $0 < z^* < \bar{z}$, $\mathcal{Z} = \{z_1, z_2\}$. From part (c) of Lemma A1, $z^* = z_1$ and $\frac{\partial D(z^*)}{\partial z^*} > 0$.
(a) By the Implicit Function Theorem, $\frac{\partial z^*}{\partial p} = -\left(\frac{\partial D(z^*)}{\partial z^*}\right) \big/ \left(\frac{\partial D(z^*)}{\partial p}\right)$. But it follows directly from (A-33) that $\frac{\partial D(z^*)}{\partial p} \leq 0$. Hence $\frac{\partial z^*}{\partial p} \geq 0$. (b) Following similar logic, $\frac{\partial D(z^*)}{\partial c} = h\left(\theta + (1 - \theta)G(z^*)\right)\left(\frac{\partial y^*(z^*)}{\partial c}\right)$. But $\frac{\partial y^*(z^*)}{\partial c} = -\frac{1}{f(y^*(z^*))(r-s)} < 0$. Hence, $\frac{\partial z^*}{\partial c} \geq 0$. (c) Analogous to part (b), we have $\frac{\partial y^*(z^*)}{\partial s} = \frac{r-c-hA(z)}{f(y^*(z^*))(r-s)^2} > 0$. Hence, $\frac{\partial z^*}{\partial s} \leq 0$. (d) Following similar logic, $\frac{\partial y^*(z^*)}{\partial r} = \frac{c-s+hA(z)}{f(y^*(z^*))(r-s)^2} > 0$. Hence, $\frac{\partial z^*}{\partial r} \leq 0$. (e) The result follows from the fact that $\frac{\partial z^*}{\partial L} = 0$. $\qquad\square$

Proof of Theorem 7. All the desired results can be proved through application of the envelope theorem on (2.9). □

Proof of Theorem 8. Note that for $\theta = 1$, the optimal planned leadtime $z^* = 0$. From (2.9), we have

$$v(z^*|\theta = 1) = \min_y \left\{ (hA(0) - (r - c))y + (r - s)E[(y - X_D)^+] + pB(0)E[X_D] \right\}$$

$$= \min_y \left(-(r - c)y + (r - s)E[(y - X_D)^+] \right).$$

Suppose that the optimal planned leadtime for an given $\theta < 1$ is z'. By Theorem 3, $z' \geq 0$. Therefore, $A(z') \geq 0$ and $B(z') \geq 0$. It follows that $v(z^*|\theta = 1) \leq v(z'|\theta < 1)$. □

Proof of Theorem 9. Because $X_D = 1$ with probability 1, (2.9) can be simplified to

$$v(z) = \min_{z \geq 0} \left\{ hA(z) - (r - c) + pB(z) \right\}. \tag{A-34}$$

Note

$$\frac{\partial v(z)}{\partial z} = h\theta + h(1 - \theta)G(z) - p(1 - \theta)(1 - G(z)), \tag{A-35}$$

$$\frac{\partial^2 v(z)}{\partial z^2} = h(1 - \theta)g(z) + p(1 - \theta)g(z).$$

By (A-35), we have

$$z^* = G^{-1} \left[\frac{p - h\frac{\theta}{1-\theta}}{p + h} \right]. \tag{A-36}$$

Note z^* is decreasing in θ and z^* is interior if $\theta \leq \bar{\theta} = \frac{p}{p+h}$. Substitute (A-36) into (A-34) and

drop the term $(r - c)$, we have

$$\tilde{v}(z^*) = h\theta z^* + h(1-\theta)z^* \int_0^{z^*} g(\omega)d\omega - h(1-\theta) \int_0^{z^*} \omega g(\omega)d\omega$$

$$+ p(1-\theta) \int_{z^*}^{\infty} \omega g(\omega)d\omega - p(1-\theta)z^* \int_{z^*}^{\infty} g(\omega)d\omega$$

$$= z^* \left[h\theta + h(1-\theta)G(z^*) - p(1-\theta)(1 - G(z^*)) \right]$$

$$+ p(1-\theta) \int_{z^*}^{\infty} \omega g(\omega)d\omega - h(1-\theta) \int_0^{z^*} \omega g(\omega)d\omega$$

$$= (1-\theta) \left[pE[\omega] - (p+h) \int_0^{z^*} \omega g(\omega)d\omega \right]. \tag{A-37}$$

Note

$$\frac{\partial \tilde{v}(z^*)}{\partial \theta} = - \left[pE[\omega] - (p+h) \int_0^{z^*} \omega g(\omega)d\omega \right] - (1-\theta)(p+h)z^* g(z^*) \frac{\partial z^*}{\partial \theta}$$

$$= - \left[pE[\omega] - (p+h) \int_0^{z^*} \omega g(\omega)d\omega \right] + \frac{hz^*}{1-\theta} \tag{A-38}$$

because $\frac{\partial z^*}{\partial \theta} = -\frac{h}{(p+h)g(z^*)(1-\theta)^2}$. Furthermore,

$$\frac{\partial^2 \tilde{v}(z^*)}{\partial \theta^2} = (p+h)z^* g(z^*) \frac{\partial z^*}{\partial \theta} + \frac{h}{1-\theta} \frac{\partial z^*}{\partial \theta} + \frac{hz^*}{(1-\theta)^2} = -\frac{h^2}{(p+h)(1-\theta)^3 g(z^*)} < 0.$$

Therefore, the optimal expected cost is a concave function of θ. From (A-38), we have

$$- \left[pE[\omega] - (p+h) \int_0^{z^*} \omega g(\omega)d\omega \right] + \frac{hz^*}{1-\theta^*} = 0$$

$$\Rightarrow \theta^* = 1 - \frac{hz^*}{pE[\omega] - (p+h) \int_0^{z^*} \omega g(\omega)d\omega}. \tag{A-39}$$

If the optimal θ^* satisfying (A-39) is less than or equal to zero, then the cost function would

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be strictly decreasing in θ. This implies that if the leadtime distribution function satisfies

$$pE[\omega] - hz^* \mid_{\theta=0} \leq (p+h) \int_0^{z^* \mid_{\theta=0}} \omega g(\omega) d\omega$$

then the optimal expected cost is strictly decreasing in θ. Otherwise the optimal expected cost is a concave function of θ, initially increasing in θ and then decreasing in θ. $\qquad \square$

Proof of Theorem 10. By Lemma A3, $v(t, x_t, \delta_t)$ can be written as $v(t, x_t, \delta_t) = \chi(t, x_t, \delta_t) \cdot \widetilde{v}(t)$, where

$$\chi(t, x_t, \delta_t) = x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right),$$

$$\widetilde{v}(t) = \left(r + pB(T - L - t) - \frac{1}{2}(r-s)\left(1 + \mathrm{erf}\left(k(t) - \frac{\sqrt{\psi_\sigma(t)}}{\sqrt{2}}\right)\right)\right).$$

In taking the expectation of $v(t, x_t, \delta_t)$ over x_t and δ_t, we have $J(x_0, \delta_0) = E[v(t, X_t, \Delta_t)|x_0, \delta_0] = E[\chi(t, X_t, \Delta_t) \cdot \widetilde{v}(t)|x_0, \delta_0] = E[\chi(t, X_t, \Delta_t)] \cdot \widetilde{v}(t)$ because $\widetilde{v}(t)$ is independent of (x_t, δ_t). By Lemma A4, $E[\chi(t, X_t, \Delta_t)|x_0, \delta_0]$ is independent of t. The theorem statement then follows. $\qquad \square$

Proof of Theorem 11. (a) By (A-27), for any given time t,

$$\frac{\partial v(t, x_t, \delta_t)}{\partial \sigma} = x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right) \cdot$$

$$\left((T-t) - (1-\phi^{T-t})\left(\frac{\phi}{1-\phi} - \phi_D\right)\left(2 - (\phi - (1-\phi)\phi_D)\frac{1+\phi^{T-t}}{1+\phi}\right)\right)\sigma \cdot$$

$$\left(r + pB(T-L-t) - \frac{1}{2}(r-s)\left(1 + \mathrm{erf}\left(k(t) - \frac{\sqrt{\psi_\sigma(t)}}{\sqrt{2}}\right)\right)\right)$$

$$- \frac{1}{2}x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right)(r-s)\frac{\partial \mathrm{erf}\,(I)}{\partial \sigma}$$

$$\geq x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right) \cdot$$

$$\left((T-t) - (1-\phi^{T-t})\left(\frac{\phi}{1-\phi} - \phi_D\right)\left(2 - (\phi - (1-\phi)\phi_D)\frac{1+\phi^{T-t}}{1+\phi}\right)\right)\sigma \cdot$$

$$(r + pB(T-L-t) - (r-s)) > 0,$$

where $I = k(t) - \frac{\sqrt{\psi_\sigma(t)}}{\sqrt{2}}$ and we have used the fact that $\mathrm{erf}(\cdot) \leq 1$ and

$$\frac{\partial \mathrm{erf}\,(I)}{\partial \sigma} = -\mathrm{erf}'(I)\frac{\left((T-t) - (1-\phi^{T-t})\left(\frac{\phi}{1-\phi} - \phi_D\right)\left(2 - (\phi - (1-\phi)\phi_D)\frac{1+\phi^{T-t}}{1+\phi}\right)\right)\sigma}{2\sqrt{2(\psi_\sigma(t))}}.$$

Therefore, applying envelope theorem at t^* obtains the desired result. (b) Can be analogously

proved as in (a). □

Proof of Theorem 12. Note that $\frac{\partial \psi_\mu(t)}{\partial \lambda} = (F-1)\mu_N < 0$, where

$$F = \frac{T - t - (1-\phi^{T-t})\left(\frac{\phi}{1-\phi} - \phi_D\right)}{T - (1-\phi^T)\left(\frac{\phi}{1-\phi} - \phi_D\right)}.$$

Note that by Lemma A6, $F \leq 1$. Similarly, $\frac{\partial \psi_\sigma(t)}{\partial \lambda} = (G-1)\sigma_N^2 < 0$, where

$$G = \frac{T - t - \left(1 - \phi^{T-t}\right)\left(\frac{\phi}{1-\phi} - \phi_D\right)\left(2 - (\phi - (1-\phi)\phi_D)\frac{1+\phi^{T-t}}{1+\phi}\right)}{T - \left(1 - \phi^T\right)\left(\frac{\phi}{1-\phi} - \phi_D\right)\left(2 - (\phi - (1-\phi)\phi_D)\frac{1+\phi^T}{1+\phi}\right)}.$$

By (A-27), for any given time t,

$$\frac{\partial v(t, x_t, \delta_t)}{\partial \lambda} = x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right)\left(\frac{\partial \psi_\mu(t)}{\partial \lambda} + \frac{1}{2}\frac{\partial \psi_\sigma(t)}{\partial \lambda}\right).$$
$$\left(r + pB(T - L - t) - \frac{1}{2}(r - s)\left(1 + \mathrm{erf}\left(k(t) - \frac{\sqrt{\psi_\sigma(t)}}{\sqrt{2}}\right)\right)\right)$$
$$- \frac{1}{2}x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right)(r - s)\frac{\partial \mathrm{erf}\,(I)}{\partial \lambda} \leq 0,$$

where $I = k(t) - \frac{\sqrt{\psi_\sigma(t)}}{\sqrt{2}}$ and we have used the fact that $\mathrm{erf}(\cdot) \leq 1$ and

$$\frac{\partial \mathrm{erf}\,(I)}{\partial \lambda} = -\mathrm{erf}'(I)\frac{\frac{\partial \psi_\sigma(t)}{\partial \lambda}}{2\sqrt{2(\psi_\sigma(t))}} \geq 0.$$

Therefore, applying envelope theorem at t^* obtains the desired result. $\square$

Proof of Theorem 13. Rearranging terms in (A-27), we have

$$\begin{aligned} v(t, x_t, \delta_t) &= \frac{1}{2}(r - s)x_t \exp\left(\psi_\mu(t) + \psi_\delta(t) + \frac{1}{2}\psi_\sigma(t)\right)\left(\frac{r + s}{r - s} + M(t)\right) \\ &= \frac{1}{2}(r - s)\chi(t, x_t, \delta_t)\left(\frac{r + s}{r - s} + M(t)\right). \end{aligned}$$

By Lemma A4, $E[\chi(t, X_t, \Delta_t)|x_0, \delta_0]$ is independent of t. Therefore, to determine the optimal

order time t^*, it is sufficient to investigate the properties of $M(t)$. $\square$

Proof of Theorem 14. (i) Note that $\mathrm{erf}(\cdot)$ is bounded between $(-1, 1)$. By (2.14), it pays to

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order earlier if the accrued tardiness cost from time 0 to $T-L$ exceeds 2. The theorem statement then follows directly. (ii) For $t > T - L$, we prove (2.14) is concave in t for $T - L \leq t \leq T$ if the respective conditions are met. Because $\frac{\partial^2 B(T-L-t)}{\partial t^2} = 0$ for $t > T - L$, we only need to consider erf$(\cdot)$ in (2.14). Note

$$
\frac{\partial \, \mathrm{erf}(\cdot)}{\partial t} = \frac{1}{4} \, \mathrm{erf}'(\cdot) \frac{\sigma^2}{\sqrt{\frac{1}{2}\psi_\sigma(t)}},
$$

$$
\begin{aligned}
\frac{\partial^2 \, \mathrm{erf}(\cdot)}{\partial t^2} &= \frac{\sigma^4}{8\psi_\sigma(t)} \left[\mathrm{erf}''(\cdot) + \mathrm{erf}'(\cdot) \frac{1}{\sqrt{\frac{1}{2}\psi_\sigma(t)}} \right] \\
&= \frac{\sigma^4}{8\psi_\sigma(t)} \, \mathrm{erf}'(\cdot) \left[-2\,\mathrm{erf}^{-1}\left(2\frac{r-c}{r-s} - 1 \right) + \frac{\sqrt{2}\,(\psi_\sigma(t) + 1)}{\sqrt{\psi_\sigma(t)}} \right].
\end{aligned}
\tag{A-40}
$$

Thus, (2.14) is concave in t if (A-40) is non-negative for all $T - L \leq t \leq T$. Consider the terms in squared bracket in (A-40), we have

$$
\frac{\partial[\cdot]}{\partial t} = \frac{\sigma^2}{\sqrt{2\psi_\sigma(t)}} \left(-1 + \frac{1}{\psi_\sigma(t)} \right),
\tag{A-41}
$$

and

$$
\frac{\partial^2[\cdot]}{\partial t^2} = \frac{\sigma^4}{(2\psi_\sigma(t))^{3/2}} \left(-1 + \frac{3}{\psi_\sigma(t)} \right),
\tag{A-42}
$$

By (A-41) and (A-42), $\frac{\partial^2[\cdot]}{\partial t^2} > 0$ whenever $\frac{\partial[\cdot]}{\partial t} = 0$. Thus, the terms in squared bracket in (A-40) is unimodal in t, where is minimum point is obtained by setting (A-41) equal to zero. This minimum point exists if and only if $\sigma_D^2 \leq 1$ and $(T - L)\sigma^2 + \sigma_D^2 > 1$, which corresponds to case (b) of the theorem statement. If at this minimum point (A-40) is positive, then $M(t)$ is concave in t. Theorem statement follows by substituting $\psi_\sigma(t) = 1$ into (A-40) and set (A-40) > 0, recognizing that $\Phi(x) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{x}{\sqrt{2}} \right) \right)$. Part (a) and (c) can be analogously proved. $\qquad \square$

Proof of Corollary 1. Part (a) follows directly from Theorem 14. (b) As a special case, by setting $\theta = 1$ one can easily construct examples where the optimal procurement time $t > T - L$. Details of example are available upon request. □

Proof of Theorem 15. Note that the functional form of $M(t)$ is characterized by Lemma A8. (i) In this case $M(t)$ is a concave function of t for $0 \le t \le T - L$. The theorem statement follows. (ii) In this case $M(t)$ is a convex function of t for $0 \le t \le T - L$. The theorem statement follows by recognizing that $M(t)$ can be convex increasing or decreasing. (iii) $M(t)$ is convex in t for $0 \le t \le t_s$ and concave in t for $t_s \le t \le T - L$. □

Proof of Theorem 16. Analogous to Theorem 15. We note, however, this theorem does not require assumption that $G(\cdot)$ is uniform, because in this case we only need $G''(\cdot) \le 0$ for Lemma A7 to hold. □

Proof of Corollary 2. (i) and (ii) follows directly from Theorem 15 and 16. □

Proof of Theorem 17. (a) When $\theta = 1$, the supply leadtime is constant and equals to L. Consequently, we have $t_N^* \ge T - L$ and $t_P^* \ge T - L$. By Theorem 3, however, $t_N^* \le T - L$. Therefore, we have $t_N^* = T - L$. In addition, by Theorem 15 and 16, t_P^* can be T. Combining the above result, we have $t_P^* \ge t_N^*$. (b) When $h = 0$, we have $t_N^* \le T - L - \hat{z}$ by Theorem 2. Recall $\hat{z} = \min\{\hat{\omega}, \bar{z}\}$. But one can show that $t_P^* \ge T - L - \hat{z}$. Hence, $t_P^* \ge t_N^*$. (c) We prove this by an example. Assume the delay ω follows a Weibull distribution with the shape parameter $\alpha = 0.85$ and expectation $E[\omega] = 2$. Set unit cost $c = 2.1$. In addition, we set $r = 7$, $p = .7$, $s = 2$, $h = .14$, $L = 2$, $T = 6$, $\theta = .5$, $\phi = \phi_D = .05$. We set expected demand equal to 100 and the demand CV equal to .8. Note that $\alpha = 0.85 \Rightarrow G''(\cdot) \le 0$. For the no pre-season order

case, by definition in Lemma A1, $H(0) = y^*(0)\frac{\theta}{1-\theta} = 331.07$. Since $\frac{p}{h}E[X_D] = 500 > H(0)$ and

$\sup H(z) = 2,387.56 > \frac{p}{h}E[X_D]$, by part (c) of Lemma A2, $z^* = z_1$. By evaluating equation

(A-21), we obtain $z_1 = 0.41$ and hence the optimal order time $t_N^* = T - L - z_1 = 3.59$. For

the pre-season order case, consider equation (2.14). For $0 \le t < L$, $k(t) - \sqrt{\frac{1}{2}\psi_\sigma(t)}$ is strictly

decreasing in t. Combining with the fact that $B(T - L - t)$ is linear in t, one can verify that

$B(-L) = 3$, $B(0) = 1$, so that $t > T - L$ cannot be optimal. For $t \le T - L$, note $B(\cdot)$ is convex

increasing in t and $\lim_{t \to -\infty} B(\cdot) = 0$, and $\mathrm{erf}(k(t) - \sqrt{\frac{1}{2}\psi_\sigma(t)})$ is monotonically increasing in

t and $\lim_{t \to -\infty} \mathrm{erf}(\cdot) = -1$. As both functions are continuous and bounded, by an exhaustive

numeric search we obtain $t_P^* = 3.55 < t_N^*$. $\qquad\square$

A2   Appendices for Chapter 3

A2.1   Technical Lemmas

Lemma A9. *The profit function (3.4) can be simplified into a more compact form as (3.5).*

Proof of Lemma A9. Using (3.4), we have

$$
\begin{aligned}
\tilde{\pi}(\vec{y}, \vec{u}) &= r\left(\sum_k \tilde{y}_k - \left(\sum_k \tilde{y}_k - \tilde{D}\right)^+\right) + v\left(\sum_k \tilde{y}_k - \tilde{D}\right)^+ - p\left(\tilde{D} - \sum_k \tilde{y}_k\right)^+ - \sum_k \tilde{C}(y_k) \\
&= r\sum_k \tilde{y}_k - (r - v)\left(\sum_k \tilde{y}_k - \tilde{D}\right)^+ - p\left(\tilde{D} - \sum_k \tilde{y}_k\right)^+ - \sum_k \tilde{C}(y_k) \\
&= r\sum_k \tilde{y}_k - (r + p - v)\left(\sum_k \tilde{y}_k - \tilde{D}\right)^+ + p\left(\left(\sum_k \tilde{y}_k - \tilde{D}\right)^+ - \left(\tilde{D} - \sum_k \tilde{y}_k\right)^+\right) \\
&\quad - \sum_k \tilde{C}(y_k) \\
&= (r + p)\sum_k \tilde{y}_k - \sum_k c_k(\eta_k y_k + (1 - \eta_k)\tilde{y}_k) - (r + p - v)\left(\sum_k \tilde{y}_k - \tilde{D}\right)^+ - p\tilde{D}.
\end{aligned}
$$

$$\text{(A-43)}$$

Substitute ψ_k and ϕ_k into (A-43), we have (3.5). $\qquad\square$

Lemma A10. *(a) For a given i, $\nabla_{y_i}\pi(\vec{y})|_{y_i=0} > 0$ if and only if $\phi_i + \frac{\psi_i}{G_i(K_i)} > E_{\vec{S}}\left[F\left(\sum_{k\neq i}\tilde{y}_k\right)\right]$.*
(b) $\nabla_{y_i}\pi(\vec{y})|_{y_i=K_i} \leq 0$.

Proof of Lemma A10. Omitted. $\qquad\square$

Lemma A11. *Let δ represent the capacity differential, such that supplier 1's capacity is $K + \delta$*

and supplier 2's cost is $K - \delta$, where K is the base capacity level. Then,

$$\frac{\partial \pi^*(\cdot)}{\partial \delta} = \phi_1 G_1(K + \delta, u_1) + \psi_1 - \overline{G}_2(K - \delta, u_2)E[F(\tilde{y}_1^*)]$$

$$- \phi_2 G_2(K - \delta, u_2) - \psi_2 + \overline{G}_1(K + \delta, u_1)E[F(\tilde{y}_2^*)]. \tag{A-44}$$

Proof of Lemma A11. Setting $n = 2$ and applying envelope theorem to (3.6), we have

$$\frac{\partial \pi(\vec{y}, \vec{u})}{\partial \delta} = \sum_{k=1}^{2} \phi_k \frac{\partial E_{\vec{S}}[\tilde{y}_k]}{\partial \delta} - \frac{\partial E_{\vec{S}, D}\left[\left(\sum_{k=1}^{2} \tilde{y}_k - \tilde{D}\right)^+\right]}{\partial \delta}. \tag{A-45}$$

First note that for supplier 1, we have

$$\frac{\partial E_{\vec{S}}[\tilde{y}_1]}{\partial \delta} = \nabla_\delta \left(\int_0^{K+\delta-y_1^*} y_1^* dG_1(r_1, u_1) + \int_{K+\delta-y_1^*}^{K+\delta} (K + \delta - r_1)dG_1(r_1, u_1) \right)$$

$$= y_1^* g_1(K + \delta - y_1^*) + \int_{K+\delta-y_1^*}^{K+\delta} dG_1(r_1, u_1) - y_1^* g_1(K + \delta - y_1^*)$$

$$= G_1(K + \delta, u_1) - G_1(K + \delta - y_1^*, u_1). \tag{A-46}$$

Analogously, for supplier 2, we have

$$\frac{\partial E_{\vec{S}}[\tilde{y}_2]}{\partial \delta} = - \left(G_2(K - \delta, u_2) - G_2(K - \delta - y_2^*, u_2) \right). \tag{A-47}$$

Furthermore, $\dfrac{\partial E_{\vec{S},D}\left[\left(\sum_{k=1}^{2}\tilde{y}_k-\tilde{D}\right)^{+}\right]}{\partial\delta}=$

$$
\begin{aligned}
\nabla_\delta\Bigg[& \int_0^{K+\delta-y_1^*}\int_0^{K-\delta-y_2^*}\int_0^{y_1^*+y_2^*}(y_1^*+y_2^*-D)dF(D)dG_2(r_2,u_2)dG_1(r_1,u_1) \\[4pt]
& +\int_0^{K+\delta-y_1^*}\int_{K-\delta-y_2^*}^{K-\delta}\int_0^{y_1^*+K-\delta-r_2}(y_1^*+K-\delta-r_2-D)dF(D)dG_2(r_2,u_2)dG_1(r_1,u_1) \\[4pt]
& +\int_0^{K+\delta-y_1^*}\int_{K-\delta}^{\infty}\int_0^{y_1^*}(y_1^*-D)dF(D)dG_2(r_2,u_2)dG_1(r_1,u_1) \\[4pt]
& +\int_{K+\delta-y_1^*}^{K+\delta}\int_0^{K-\delta-y_2^*}\int_0^{K+\delta-r_1+y_2^*}(K+\delta-r_1+y_2^*-D)dF(D)dG_2(r_2,u_2)dG_1(r_1,u_1) \\[4pt]
& +\int_{K+\delta-y_1^*}^{K+\delta}\int_{K-\delta-y_2^*}^{K-\delta}\int_0^{K-r_1+K-r_2}(K-r_1+K-r_2-D)dF(D)dG_2(r_2,u_2)dG_1(r_1,u_1) \\[4pt]
& +\int_{K+\delta-y_1^*}^{K+\delta}\int_{K-\delta}^{\infty}\int_0^{K+\delta-r_1}(K+\delta-r_1-D)dF(D)dG_2(r_2,u_2)dG_1(r_1,u_1) \\[4pt]
& +\int_{K+\delta}^{\infty}\int_0^{K-\delta-y_2^*}\int_0^{y_2^*}(y_2^*-D)dF(D)dG_2(r_2,u_2)dG_1(r_1,u_1) \\[4pt]
& +\int_{K+\delta}^{\infty}\int_{K-\delta-y_2^*}^{K-\delta}\int_0^{K-\delta-r_2}(K-\delta-r_2-D)dF(D)dG_2(r_2,u_2)dG_1(r_1,u_1)\Bigg]. \qquad\text{(A-48)}
\end{aligned}
$$

The above can be written out term by term (label as $T1-T8$) as

$$
\begin{aligned}
T1=-&\int_0^{K+\delta-y_1^*}\left[\int_0^{y_1^*+y_2^*}(y_1^*+y_2^*-D)dF(D)g_2(K-\delta-y_2^*,u_2)\right]dG_1(r_1,u_1) \\[4pt]
+&\int_0^{K-\delta-y_2^*}\int_0^{y_1^*+y_2^*}(y_1^*+y_2^*-D)dF(D)dG_2(r_2,u_2)g_1(K+\delta-y_1^*,u_1).
\end{aligned}
$$

$$
\begin{aligned}
T2=\int_0^{K+\delta-y_1^*}\Bigg[&-\int_{K-\delta-y_2^*}^{K-\delta}\int_0^{y_1^*+K-\delta-r_2}dF(D)dG_2(r_2,u_2)-\int_0^{y_1^*}(y_1^*-D)dF(D)g_2(k-\delta,u_2) \\[4pt]
&+\int_0^{y_1^*+y_2^*}(y_1^*+y_2^*-D)dF(D)g_2(K-\delta-y_2^*,u_2)\Bigg]dG_1(r_1,u_1) \\[4pt]
+\int_{K-\delta-y_2^*}^{K-\delta}&\int_0^{y_1^*+K-\delta-r_2}(y_1^*+K-\delta-r_2-D)dF(D)dG_2(r_2,u_2)g_1(K+\delta-y_1^*,u_1).
\end{aligned}
$$

$$T3 = \int_0^{K+\delta-y_1^*} \left[\int_0^{y_1^*} (y_1^* - D)dF(D)g_2(K-\delta, u_2) \right] dG_1(r_1, u_1)$$

$$+ \int_{K-\delta}^{\infty} \int_0^{y_1^*} (y_1^* - D)dF(D)dG_2(r_2, u_2)g_1(K+\delta-y_1^*, u_1).$$

$$T4 = \int_{K+\delta-y_1^*}^{K+\delta} \left[\int_0^{K-\delta-y_2^*} \int_0^{K+\delta-r_1+y_2^*} dF(D)dG_2(r_2, u_2) \right.$$

$$\left. - \int_0^{K+\delta-r_1+y_2^*} (K+\delta-r_1+y_2^*-D)dF(D)g_2(k-\delta-y_2^*, u_2) \right] dG_1(r_1, u_1)$$

$$+ \int_0^{K-\delta-y_2^*} \int_0^{y_2^*} (y_2^* - D)dF(D)dG_2(r_2, u_2)g_1(K+\delta, u_1)$$

$$- \int_0^{K-\delta-y_2^*} \int_0^{y_1^*+y_2^*} (y_1^* + y_2^* - D)dF(D)dG_2(r_2, u_2)g_1(K+\delta-y_1^*, u_1).$$

$$T5 = \int_{K+\delta-y_1^*}^{K+\delta} \left[- \int_0^{K+\delta-r_1} (K+\delta-r_1-D)dF(D)g_2(K-\delta, u_2) \right.$$

$$\left. + \int_0^{K+\delta-r_1+y_2^*} (K+\delta-r_1+y_2^*-D)dF(D)g_2(k-\delta-y_2^*, u_2) \right] dG_1(r_1, u_1)$$

$$+ \int_{K-\delta-y_2^*}^{K-\delta} \int_0^{K-\delta-r_2} (K-\delta-r_2-D)dF(D)dG_2(r_2, u_2)g_1(K+\delta, u_1)$$

$$- \int_{K-\delta-y_2^*}^{K-\delta} \int_0^{y_1^*+K-\delta-r_2} (y_1^* + K-\delta-r_2-D)dF(D)dG_2(r_2, u_2)g_1(K+\delta-y_1^*, u_1).$$

$$T6 = \int_{K+\delta-y_1^*}^{K+\delta} \left[\int_{K-\delta}^{\infty} \int_0^{K+\delta-r_1} dF(D)dG_2(r_2, u_2) \right.$$

$$\left. + \int_0^{K+\delta-r_1} (K+\delta-r_1-D)dF(D)g_2(k-\delta, u_2) \right] dG_1(r_1, u_1)$$

$$- \int_{K-\delta}^{\infty} \int_0^{y_1^*} (y_1^* - D)dF(D)dG_2(r_2, u_2)g_1(K+\delta-y_1^*, u_1).$$

$$T7 = -\int_{K+\delta}^{\infty}\int_{0}^{y_2^*}(y_2^* - D)dF(D)g_2(K - \delta - y_2^*, u_2)dG_1(r_1, u_1)$$

$$-\int_{0}^{K-\delta-y_2^*}\int_{0}^{y_2^*}(y_2^* - D)dF(D)dG_2(r_2, u_2)g_1(K + \delta, u_1).$$

$$T8 = \int_{K+\delta}^{\infty}\left[-\int_{K-\delta-y_2^*}^{K-\delta}\int_{0}^{K-\delta-r_2}dF(D)dG_2(r_2, u_2)\right.$$

$$\left.+\int_{0}^{y_2^*}(y_2^* - D)dF(D)g_2(K - \delta - y_2^*, u_2)\right]dG_1(r_1, u_1)$$

$$-\int_{K-\delta-y_2^*}^{K-\delta}\int_{0}^{K-\delta-r_2}(K - \delta - r_2 - D)dF(D)dG_2(r_2, u_2)g_1(K + \delta, u_1).$$

Combining all the above eight terms $T1 - T8$ together, we have $\dfrac{\partial E_{\vec{S},D}\left[(\tilde{y}_1^* + \tilde{y}_2^* - \tilde{D})^+\right]}{\partial\delta} =$

$$-\int_{0}^{K+\delta-y_1^*}\int_{K-\delta-y_2^*}^{K-\delta}F(y_1^* + K - \delta - r_2)dG_2(r_2, u_2)dG_1(r_1, u_1)$$

$$+\int_{K+\delta-y_1^*}^{K+\delta}\int_{0}^{K-\delta-y_2^*}F(K + \delta - r_1 + y_2^*)dG_2(r_2, u_2)dG_1(r_1, u_1)$$

$$+\int_{K+\delta-y_1^*}^{K+\delta}\int_{K-\delta}^{\infty}F(K + \delta - r_1)dG_2(r_2, u_2)dG_1(r_1, u_1)$$

$$-\int_{K+\delta}^{\infty}\int_{K-\delta-y_2^*}^{K-\delta}F(K - \delta - r_2)dG_2(r_2, u_2)dG_1(r_1, u_1)$$

$$= -G_1(K + \delta - y_1^*, u_1)\int_{K-\delta-y_2^*}^{K-\delta}F(y_1^* + K - \delta - r_2)dG_2(r_2, u_2)$$

$$+G_2(K - \delta - y_2^*, u_2)\int_{K+\delta-y_1^*}^{K+\delta}F(K + \delta - r_1 + y_2^*)dG_1(r_1, u_1)$$

$$+\overline{G}_2(K - \delta)\int_{K+\delta-y_1^*}^{K+\delta}F(K + \delta - r_1)dG_1(r_1, u_1)$$

$$-\overline{G}_1(K + \delta)\int_{K-\delta-y_2^*}^{K-\delta}F(K - \delta - r_2)dG_2(r_2, u_2). \tag{A-49}$$

Substitute (A-46), (A-47) and (A-49) into (A-45), we have

$$
\frac{\partial \pi(\vec{y}, \vec{u})}{\partial \delta} = \phi_1(G_1(K + \delta, u_1) - G_1(K + \delta - y_1^*, u_1)) - \phi_2\left(G_2(K - \delta, u_2) - G_2(K - \delta - y_2^*, u_2)\right)
$$

$$
- G_1(K + \delta - y_1^*, u_1) \int_{K-\delta-y_2^*}^{K-\delta} F(y_1^* + K - \delta - r_2)dG_2(r_2, u_2)
$$

$$
+ G_2(K - \delta - y_2^*, u_2) \int_{K+\delta-y_1^*}^{K+\delta} F(K + \delta - r_1 + y_2^*)dG_1(r_1, u_1)
$$

$$
+ \overline{G}_2(K - \delta) \int_{K+\delta-y_1^*}^{K+\delta} F(K + \delta - r_1)dG_1(r_1, u_1)
$$

$$
- \overline{G}_1(K + \delta) \int_{K-\delta-y_2^*}^{K-\delta} F(K - \delta - r_2)dG_2(r_2, u_2). \tag{A-50}
$$

Note by (3.9), we have

$$
\phi_1 G_1(K + \delta - y_1^*, u_1) = -\psi_1 + G_1(K + \delta - y_1^*, u_1) \int_0^{K-\delta-y_2^*} F(y_1^* + y_2^*)dG_2(r_2, u_2)
$$

$$
+ G_1(K + \delta - y_1^*, u_1) \int_{K-\delta-y_2^*}^{K-\delta} F(y_1^* + K - \delta - r_2)dG_2(r_2, u_2)
$$

$$
+ G_1(K + \delta - y_1^*, u_1) \int_{K-\delta}^{\infty} F(y_1^*)dG_2(r_2, u_2), \tag{A-51}
$$

and

$$
\phi_2 G_2(K - \delta - y_2^*, u_2) = -\psi_2 + G_2(K - \delta - y_2^*, u_2) \int_0^{K+\delta-y_1^*} F(y_1^* + y_2^*)dG_1(r_1, u_1)
$$

$$
+ G_2(K + \delta - y_1^*, u_2) \int_{K+\delta-y_1^*}^{K+\delta} F(y_2^* + K + \delta - r_1)dG_1(r_1, u_1)
$$

$$
+ G_2(K + \delta - y_1^*, u_2) \int_{K+\delta}^{\infty} F(y_2^*)dG_1(r_1, u_1). \tag{A-52}
$$

Substitute (A-51) and (A-52) into (A-50), we have

$$\frac{\partial \pi(\vec{y}, \vec{u})}{\partial \delta} = \phi_1 G_1(K + \delta, u_1) + \psi_1 - G_1(K + \delta - y_1^*)\overline{G}_2(K - \delta)F(y_1^*)$$

$$- \phi_2 G_2(K - \delta, u_2) - \psi_2 + G_2(K - \delta - y_2^*)\overline{G}_1(K + \delta)F(y_2^*)$$

$$- \overline{G}_2(K - \delta, u_2) \int_{K+\delta-y_1^*}^{K+\delta} F(K + \delta - r_1)dG_1(r_1, u_1)$$

$$+ \overline{G}_1(K + \delta, u_1) \int_{K-\delta-y_2^*}^{K-\delta} F(K - \delta - r_2)dG_2(r_2, u_2)$$

$$= \phi_1 G_1(K + \delta, u_1) + \psi_1 - \overline{G}_2(K - \delta)E\left[\tilde{y}_1^*\right]$$

$$- \phi_2 G_2(K - \delta, u_2) - \psi_2 + \overline{G}_1(K + \delta)E\left[\tilde{y}_2^*\right].$$

$\square$

A2.2   Proofs

Proof of Lemma 1. Note that the first part of the lemma statement is true if and only if $\nabla_{y_i y_i}\pi(\vec{y}, \vec{u}) - \nabla_{y_i y_j}\pi(\vec{y}, \vec{u}) \le 0$. Using (3.12) and (3.13), and realizing that

$$G_i(K_i - y_i)G_j(K_j - y_j)E_{\vec{S}}\left[f\left(y_i + y_j + \sum_{k \ne i,j} \tilde{y}_k\right)\right]$$

$$= G_i(K_i - y_i)E_{\vec{S}}\left[G_j(K_j - y_j)f\left(y_i + y_j + \sum_{k \ne i,j} \tilde{y}_k\right)\right]$$

$$\le G_i(K_i - y_i)E_{\vec{S}}\left[f\left(y_i + \sum_{k \ne i} \tilde{y}_k\right)\right],$$

we have that a sufficient condition for $\nabla_{y_i y_i}\pi(\vec{y}, \vec{u}) - \nabla_{y_i y_j}\pi(\vec{y}, \vec{u}) \le 0$ is $\phi_i - E_{\vec{S}}\left[F\left(y_i + \sum_{k \ne i} \tilde{y}_k\right)\right] \ge 0$. This condition is guaranteed as long as $\vec{y}$ is an interior solution, which implies that $\nabla_i \pi(\vec{y}) \ge 0$. The second part of the lemma statement can be analogously proved. $\square$

Proof of Lemma 2. Setting (3.11) equal to zero, we have $\phi_i - E_{\vec{S}}\left[F\left(y_i + \sum_{k \neq i} \tilde{y}_k\right)\right] > 0$. Substitute this condition into (3.12), we have $\nabla_{y_i y_i} \pi(\vec{y}) < 0$. □

Proof of Theorem 19. We prove the theorem by induction on the number of suppliers. Note that the proof here is in the same spirit to the proof of Proposition 2 in Aydin and Porteus (2005).

First note that if there is a single supplier, (a) follows directly from Lemma A10. (b) is true by Lemma A10 and Lemma 2, which proves that $\pi(\cdot, \vec{u})$ is unimodal in y_1.

Now suppose there are two suppliers such that both y_1 and y_2 are decision variables. At a given y_1, $\pi(\vec{y}, \vec{u})$ reduces to a function of one variable. By the induction assumption, at any given y_1, there exists a unique $y_2^*(y_1)$ such that $\nabla_{y_i} \pi(y_1, y_2^*(y_1), \vec{u}) \leq 0$ and $y_i \nabla_{y_i} \pi(y_1, y_2^*(y_1), \vec{u}) = 0$. Note that $y_2^*(y_1)$ maximizes $\pi(\cdot, \vec{u})$ at y_1. Instead of maximizing the $\pi(\cdot, \vec{u})$ function directly, we can maximize the induced function $\pi(y_1, y_2^*(y_1))$. By envelope theorem,

$$\frac{\partial \pi(y_1, y_2^*(y_1))}{\partial y_1} = \nabla_{y_1} \pi(y_1, y_2^*(y_1), \vec{u}).$$

Therefore,

$$\frac{\partial^2 \pi(y_1, y_2^*(y_1), \vec{u})}{\partial y_1^2} = \nabla_{y_1 y_1} \pi(y_1, y_2^*(y_1), \vec{u}) + \nabla_{y_1 y_2} \pi(y_1, y_2^*(y_1)) \frac{\partial y_2^*(y_1)}{\partial y_1}. \tag{A-53}$$

Setting (3.11) equal to zero and applying implicit function theorem, we have

$$\frac{\partial y_2^*(y_1)}{\partial y_1} = -\frac{C_2}{L_2}, \tag{A-54}$$

where

$$C_2 = G_2(K_2 - y_2)G_1(K_1 - y_1)f(y_1 + y_2),$$

$$L_2 = g_2(K_2 - y_2)\left(\phi_2 - E_{\vec{S}}[F(\tilde{y}_1 + y_2)]\right) + G_2(K_2 - y_2)E_{\vec{S}}[f(\tilde{y}_1 + y_2)].$$

Substitute (3.12), (3.13) and (A-54) into (A-53), we have

$$\frac{\partial^2 \pi(y_1, y_2^*(y_1), \vec{u})}{\partial y_1^2} = -L_1 + \frac{C_2^2}{L_2}. \tag{A-55}$$

Note that $\pi(\cdot, \vec{u})$ is unimodal if and only if $-L_1 L_2 + C_2^2 < 0$. If $y_2^* = 0$ then the above reduces to a single variable problem. If $y_2^* > 0$, then by (3.11) and (3.12) $L_2 > C_2$. Analogously, we have $L_1 > C_2$. The theorem statement then follows directly. $\qquad\square$

Proof of Lemma 3. Setting (3.11) equal to zero and applying implicit function theorem, we have (note that we have suppressed the dependence of $u(\cdot)$ on the improvement efforts and simply write u_i)

$$\frac{\partial y_i^*(u_i)}{\partial u_i} = -\frac{\frac{\partial G_i(K_i - y_i^*, u_i)}{\partial u_i}\left(\phi_i - E_{\vec{S}}\left[F\left(y_i^* + \sum_{k \neq i}\tilde{y}_k\right)\right]\right)}{-g_i(K_i - y_i^*)\left(\phi_i - E_{\vec{S}}\left[F\left(y_i^* + \sum_{k \neq j}\tilde{y}_k\right)\right]\right) - G_i(K_i - y_i^*)E_{\vec{S}}\left[f\left(y_i^* + \sum_{k \neq j}\tilde{y}_k\right)\right]}$$

Note at optimality if $y_i^* > 0$ then $\phi_i - E_{\vec{S}}\left[F\left(y_i^* + \sum_{k \neq j}\tilde{y}_k\right)\right] > 0$. In addition, $\frac{\partial G_i(K_i - y_i^*, u_i)}{\partial u_i} > 0$ by definition. Combine these together, we must have $\frac{\partial y_i^*(u_i)}{\partial u_i} \geq 0$. $\qquad\square$

Proof of Lemma 4. Using (3.6), we have

$$\frac{\partial \pi^*(\vec{u})}{\partial u_i} = \phi_i \frac{\partial E_{\vec{S}}[\tilde{y}_i]}{\partial u_i} - \frac{\partial E_{\vec{S},D}\left[\left(\sum_{k=1}^n \tilde{y}_k - \tilde{D}\right)^+\right]}{\partial u_i}. \tag{A-56}$$

Note that

$$
\begin{aligned}
\frac{\partial E_{\vec{S}}\left[\tilde{y}_i\right]}{\partial u_i} &= y_i^* \frac{\partial G_i(K_i - y_i^*, u_i)}{\partial u_i} + \int_{K_i - y_i^*}^{K_i} (K_i - r_i) d\frac{\partial G_i(r_i, u_i)}{\partial u_i} \\
&= y_i^* \frac{\partial G_i(K_i - y_i^*, u_i)}{\partial u_i} + (K_i - r_i)\frac{\partial G_i(r_i, u_i)}{\partial u_i}\Big|_{K_i - y_i^*}^{K_i} - \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} d(K_i - r_i) \\
&= \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} dr_i.
\end{aligned}
\tag{A-57}
$$

In addition, we have $\dfrac{\partial E_{\vec{S},D}\left[\left(\sum_{k=1}^n \tilde{y}_k - \tilde{D}\right)^+\right]}{\partial u_i} =$

$$
\begin{aligned}
E_{\vec{S}}&\Bigg[\int_0^{K_i - y_i^*} \int_0^{y_i^* + \sum_{k \neq i} \tilde{y}_k} \left(y_i + \sum_{k \neq i} \tilde{y}_k - D \right) dF(D) d\frac{\partial G_i(r_i, u_i)}{\partial u_i} \\
&+ \int_{K_i - y_i^*}^{K_i} \int_0^{K_i - r_i + \sum_{k \neq i} \tilde{y}_k} \left(K_i - r_i + \sum_{k \neq i} \tilde{y}_k - D \right) dF(D) d\frac{\partial G_i(r_i, u_i)}{\partial u_i} \\
&+ \int_{K_i}^{\infty} \int_0^{\sum_{k \neq i} \tilde{y}_k} \left(\sum_{k \neq i} \tilde{y}_k - D \right) dF(D) d\frac{\partial G_i(r_i, u_i)}{\partial u_i} \Bigg] \\
= E_{\vec{S}}&\Bigg[\frac{\partial G_i(K_i - y_i^*, u_i)}{\partial u_i} \int_0^{y_i^* + \sum_{k \neq i} \tilde{y}_k} \left(y_i + \sum_{k \neq i} \tilde{y}_k - D \right) dF(D) \\
&+ \frac{\partial G_i(r_i, u_i)}{\partial u_i} \int_0^{K_i - r_i + \sum_{k \neq i} \tilde{y}_k} \left(K_i - r_i + \sum_{k \neq i} \tilde{y}_k - D \right) dF(D)\Big|_{K_i - y_i^*}^{K_i} \\
&- \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} d\left[\int_0^{K_i - r_i + \sum_{k \neq i} \tilde{y}_k} \left(K_i - r_i + \sum_{k \neq i} \tilde{y}_k - D \right) dF(D) \right] \\
&- \frac{\partial G_i(K_i, u_i)}{\partial u_i} \int_0^{\sum_{k \neq i} \tilde{y}_k} \left(\sum_{k \neq i} \tilde{y}_k - D \right) dF(D) \Bigg] \\
= E_{\vec{S}}&\left[\int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + \sum_{k \neq i} \tilde{y}_k \right) dr_i \right].
\end{aligned}
\tag{A-58}
$$

Substitute (A-57) and (A-58) into (A-56), we have

$$
\frac{\partial \pi^*(\vec{u})}{\partial u_i} = \phi_i \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} dr_i - E_{\vec{S}} \left[\int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + \sum_{k \neq i} \tilde{y}_k \right) dr_i \right]
$$

$$
\geq \phi_i \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} dr_i - E_{\vec{S}} \left[\int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(y_i^* + \sum_{k \neq i} \tilde{y}_k \right) dr_i \right]
$$

$$
= \phi_i \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} dr_i - E_{\vec{S}} \left[F\left(y_i^* + \sum_{k \neq i} \tilde{y}_k \right) \right] \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} dr_i
$$

$$
= \left(\phi_i - E_{\vec{S}} \left[F\left(y_i^* + \sum_{k \neq i} \tilde{y}_k \right) \right] \right) \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} dr_i \geq 0. \tag{A-59}
$$

Note that (A-59) follows from the fact that $\frac{\partial G_i(r_i, u_i)}{\partial u_i} \geq 0$, and $\phi_i - E_{\vec{S}} \left[F\left(y_i^* + \sum_{k \neq i} \tilde{y}_k \right) \right] \geq 0$

by (3.11). $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Proof of Theorem 20. When $\eta_1 = \eta_2 = 0$, (3.14) can be simplified to

$$
G_1(K_1 - y_1, u_1) \left(\phi_1 - E_{\tilde{S}_2} \left[F\left(y_1 + \tilde{y}_2 \right) \right] \right) = 0,
$$

$$
G_2(K_2 - y_2, u_2) \left(\phi_2 - E_{\tilde{S}_1} \left[F\left(y_2 + \tilde{y}_1 \right) \right] \right) = 0. \tag{A-60}
$$

(a) For any interior solution, it is clear that $y_1^* + y_2^* \geq D$, because otherwise (A-60) is strictly positive. The theorem statement follows since $y_1^* + y_2^*$ is upper bounded by $K_1 + K_2$. In addition, both y_1^* and y_2^* are strictly less than D because otherwise one or both conditions in (A-60) is (are) strictly negative. (b) We prove the theorem statement by considering different regions of the demand. First note that when $y_1 + y_2 = D$, both equations in (A-60) are strictly positive. We must have $\phi_1 G_1(K_1 - y_1) = \phi_2 G_2(K_2 - y_2)$, unless $\phi_1 G_1(K_1 - D) \geq \phi_2 G_2(K_2) \Rightarrow D \leq K_1 - G_1^{-1}\left(\frac{r - c_2}{r - c_1} G_2(K_2) \right)$. The theorem statements then follow directly. Note that if $\phi_1 G_1(K_1) < \phi_2 G_2(K_2)$, the above condition can be similarly derived as $D \leq$

$K_2 - G_2^{-1}\left(\frac{r-c_1}{r-c_2}G_1(K_1)\right)$. Next we consider the case when $y_1 + y_2 > D$. Using (3.14), the optimal procurement quantity is given by

$$y_1^* = D - (K_2 - G_2^{-1}(r - c_1)), \quad y_2^* = D - (K_1 - G_1^{-1}(r - c_2)). \qquad \text{(A-61)}$$

For $y_1 + y_2 > D$ to hold, therefore, we must have $D > K_1 + K_2 - G_1^{-1}(r - c_2) - G_2^{-1}(r - c_1)$. However, for (A-61) to be interior solution, we must have

$$\phi_1 - G_2(K_2 - (D - K_1)) \leq 0 \Rightarrow D \leq K_1 + K_2 - G_2^{-1}(r - c_1),$$

$$\phi_2 - G_1(K_1 - (D - K_2)) \leq 0 \Rightarrow D \leq K_1 + K_2 - G_1^{-1}(r - c_2).$$

Therefore, we must have $D \leq K_1 + K_2 - \min(G_1^{-1}(r - c_2), G_2^{-1}(r - c_1))$ for firm to dual source without hitting the capacity limit from one of the suppliers. Note that when the firm utilize the fully capacity of one supplier, the total procurement quantity $y_1^* + y_2^* > D$. Otherwise, suppose $y_1^* < K_1$ and $y_2^* = K_2$, then we have $y_1^* + K_2 = (D - K_2 + G_2^{-1}(r - c_1)) + K_2 = D + G_2^{-1}(r - c_1) > D$. The case of both suppliers' capacities are fully utilized can be similarly proved. The last region in the theorem statement follows directly from the above analysis. □

Proof of Theorem 21. Using (3.19), we have

$$\frac{\partial g(u(z), u(0))}{\partial u(z)} = -m\frac{\partial h^{-1}(u(z), u(0))}{\partial u(z)} + \theta\frac{\partial \pi^*(u(z))}{\partial u(z)}, \quad \text{and}$$

$$\frac{\partial^2 g(u(z), u(0))}{\partial u(z)^2} = -m\frac{\partial^2 h^{-1}(u(z), u(0))}{\partial u(z)^2} + \theta\frac{\partial^2 \pi^*(u(z))}{\partial u(z)^2}. \qquad \text{(A-62)}$$

Let $h'(u(0), z) = \frac{\partial h(u(0), z)}{\partial z}$ and $h''(u(0), z) = \frac{\partial^2 h(u(0), z)}{\partial z^2}$, we have

$$\frac{\partial^2 h^{-1}(u(z), u(0))}{\partial u(z)^2} = -\frac{1}{(h'(u(0), z))^2} \frac{h''(u(0), z)}{h'(u(0), z)}. \tag{A-63}$$

Substitute (A-63) and (A-59) into (A-62), and applying appropriate assumptions (T1-T3), we have $\frac{\partial^2 g(u(z), u(0))}{\partial u(z)^2} \leq 0$. $\square$

Proof of Lemma 9. It is necessary and sufficient to show that $\frac{\partial^2 g(\vec{u}(z), \vec{u}(0))}{\partial u_i(z) \partial u_j(z)} \leq 0$. Using (A-59), we have (suppressing the dependence of $u(\cdot)$ on the improvement efforts)

$$
\begin{aligned}
\frac{\partial^2 \pi^*(\vec{u})}{\partial u_i \partial u_j} &= -\partial_{u_j} E_{\vec{\mathcal{S}}} \left[\int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + \sum_{k \neq i} \tilde{y}_k\right) dr_i \right] \\
&= -E_{\vec{\mathcal{S}}} \Bigg[\frac{\partial G_j(K_j - y_j^*, u_j)}{\partial u_j} \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + y_j^* + \sum_{k \neq i,j} \tilde{y}_k\right) dr_i \\
&\quad + \frac{\partial G_j(r_j, u_j)}{\partial u_j} \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + K_j - r_j + \sum_{k \neq i,j} \tilde{y}_k\right) dr_i \Bigg|_{K_j - y_j^*}^{K_j} \\
&\quad - \int_{K_j - y_j^*}^{K_j} \frac{\partial G_j(r_j, u_j)}{\partial u_j} d\left(\int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + K_j - r_j + \sum_{k \neq i,j} \tilde{y}_k\right) dr_i \right) \\
&\quad - \frac{\partial G_j(K_j, u_j)}{\partial u_j} \int_{K_i - y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + \sum_{k \neq i,j} \tilde{y}_k\right) dr_i \Bigg] \\
&= -E_{\vec{\mathcal{S}}} \left[\int_{K_j - y_j^*}^{K_j} \int_{K_i - y_i^*}^{K_i} \frac{\partial G_j(r_j, u_j)}{\partial u_j} \frac{\partial G_i(r_i, u_i)}{\partial u_i} f\left(K_i - r_i + K_j - r_j + \sum_{k \neq i,j} \tilde{y}_k\right) dr_i dr_j \right].
\end{aligned}
\tag{A-64}
$$

Note that (A-64) is non-positive because both $\frac{\partial G_i(\cdot, u_j)}{\partial u_j} \geq 0$ and $\frac{\partial G_i(\cdot, u_i)}{\partial u_i} \geq 0$. The lemma statement then follows because $\frac{\partial^2 g(\vec{u}^1, \vec{u}^0)}{\partial u_i \partial u_j}$ is a positive, linear combination of $\frac{\partial^2 \pi^*(\vec{u})}{\partial u_i \partial u_j}$. $\square$

Proof of Theorem 22. We will rewrite (A-59) in a more amenable form. First note that with

two suppliers, we have

$$E_{\vec{S}}\left[\int_{K_i-y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + \sum_{k \neq i} \tilde{y}_k\right) dr_i\right]$$

$$= G_j(K_j - y_j^*, u_j) \int_{K_i-y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + y_j^*\right) dr_i$$

$$+ \overline{G}_j(K_j, u_j) \int_{K_i-y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i\right) dr_i$$

$$+ G_j(r_j, u_j) \int_{K_i-y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + K_j - r_j\right) dr_i \Big|_{K_j-y_j^*}^{K_j}$$

$$- \int_{K_j-y_j^*}^{K_j} G_j(r_j, u_j) d\left[\int_{K_i-y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i + K_j - r_j\right) dr_i\right]$$

$$= \int_{K_i-y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} F\left(K_i - r_i\right) dr_i$$

$$+ \int_{K_j-y_j^*}^{K_j} G_j(r_j, u_j) \int_{K_i-y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} f\left(K_i - r_i + K_j - r_j\right) dr_i dr_j. \qquad \text{(A-65)}$$

Substitute (A-65) into (A-59), we have

$$\frac{\partial \pi^*(\vec{u})}{\partial u_i} = \int_{K_i-y_i^*}^{K_i} [\phi_i - F(K_i - r_i)] \frac{\partial G_i(r_i, u_i)}{\partial u_i} dr_i$$

$$- \int_{K_j-y_j^*}^{K_j} G_j(r_j, u_j) \int_{K_i-y_i^*}^{K_i} \frac{\partial G_i(r_i, u_i)}{\partial u_i} f\left(K_i - r_i + K_j - r_j\right) dr_i dr_j. \qquad \text{(A-66)}$$

Using (3.22) and the fact that $\frac{\partial^2 \pi^*(u_1(z_1), u_2(z_2))}{\partial u_i(z_i)^2} \leq 0$ for $i = 1, 2$, we have

$$\frac{\partial^2 g(\vec{u}(z), \vec{u}(0))}{\partial u_i(z_i)^2} = m\frac{h''(u_i(0), z_i)}{(h'(u_i(0), z_i))^3} + \theta_i \theta_j \frac{\partial^2 \pi^*(u_i(z_i), u_j(z_j))}{\partial u_i(z_i)^2} + \theta_i(1 - \theta_j)\frac{\partial^2 \pi^*(u_i(z_i), u_j(0))}{\partial u_i(z_i)^2}$$

$$\leq m\frac{h''(u_i(0), z_i)}{(h'(u_i(0), z_i))^3}, \quad i = 1, 2; \quad j = 3 - i, \qquad \text{(A-67)}$$

and

$$\frac{\partial^2 g(\vec{u}(z), \vec{u}(0))}{\partial u_i(z_i)\partial u_j(z_j)} = \theta_i\theta_j\frac{\partial^2 \pi^* \left(u_1(z_1), u_2(z_2)\right)}{\partial u_i(z_i)\partial u_j(z_j)}$$

$$= -\theta_1\theta_2\int_{K_j-y_j^*}^{K_j}\frac{\partial G_j(r_j, u_j)}{\partial u_j}\int_{K_i-y_i^*}^{K_i}\frac{\partial G_i(r_i, u_i)}{\partial u_i}f\left(K_i - r_i + K_j - r_j\right)dr_idr_j$$

$$\geq -\theta_1\theta_2\int_{K_j-y_j^*}^{K_j}G_j(r_j, u_j)\int_{K_i-y_i^*}^{K_i}\frac{\partial G_i(r_i, u_i)}{\partial u_i}f\left(K_i - r_i + K_j - r_j\right)dr_idr_j$$

$$\geq -\theta_1\theta_2\int_{K_i-y_i^*}^{K_i}\left[\phi_i - F(K_i - r_i)\right]\frac{\partial G_i(r_i, u_i)}{\partial u_i}dr_i, \quad i = 1, 2; \quad j = 3 - i. \quad \text{(A-68)}$$

Subtract (A-68) from (A-67), we have

$$\frac{\partial^2 g(\vec{u}(z), \vec{u}(0))}{\partial u_i(z_i)^2} - \frac{\partial^2 g(\vec{u}(z), \vec{u}(0))}{\partial u_i(z_i)\partial u_j(z_j)} \leq m\frac{h''\left(u_i(0), z_i\right)}{\left(h'\left(u_i(0), z_i\right)\right)^3} + \theta_1\theta_2\int_{K_i-y_i^*}^{K_i}\left[\phi_i - F(K_i - r_i)\right]\frac{\partial G_i(r_i, u_i)}{\partial u_i}dr_i$$

$$\leq m\frac{h''\left(u_i(0), z_i\right)}{\left(h'\left(u_i(0), z_i\right)\right)^3} + \theta_1\theta_2\phi_i\int_{K_i-y_i^*}^{K_i}G_i(r_i, u_i)dr_i$$

$$\leq m\frac{h''\left(u_i(0), z_i\right)}{\left(h'\left(u_i(0), z_i\right)\right)^3} + \theta_1\theta_2\phi_iK_i, \quad i = 1, 2; \quad j = 3 - i.$$

Therefore, a sufficient condition for $g(\vec{u}(z), \vec{u}(0))$ to be jointly unimodal in $\vec{u}(z)$ is given by (3.23). $\qquad\square$

Proof of Corollary 3. Let

$$L = \theta\int_{K-y^*}^{K}\left(\left(\phi - F(K - s)\right)\frac{\partial G(s, u(z^*))}{\partial u(z^*)}\right)ds - \frac{m}{h'(u(0), z^*)}.$$

We have

$$\frac{\partial L}{\partial z} = \theta\int_{K-y^*}^{K}\left(\left(\phi - F(K - s)\right)\frac{\partial^2 G(s, u(z^*))}{\partial u(z^*)^2}\right)\frac{\partial u(z^*)}{\partial z^*}ds + \frac{m}{h'(u(0), z^*)^2}h''(u(0), z^*) < 0.$$

The corollary statements (a) through (d) follow directly by applying implicit function theorem on (3.20). For part (e), note that

$$
\begin{aligned}
\frac{\partial L}{\partial K} &= \theta \left[-\int_{K-y^*}^{K} \left((f(K-s)) \frac{\partial G(s, u(z^*))}{\partial u(z^*)} \right) ds + \phi - \left(\phi - F(y^*) \frac{\partial G(K-y^*, u(z^*))}{\partial u(z^*)} \right) \right] \\
&= \theta \left[\int_{K-y^*}^{K} \frac{\partial G(s, u(z^*))}{\partial u(z^*)} dF(K-s) + F(y^*) \frac{\partial G(K-y^*, u(z^*))}{\partial u(z^*)} \right] \\
&= -\theta \int_{K-y^*}^{K} F(K-s) \frac{\partial g(s, u(z^*))}{\partial u(z^*)} ds.
\end{aligned}
$$

The corollary statements then follows. $\square$

Proof of Lemma 6. We prove the lemma statement by constructing a particular example. Suppose there are two suppliers which differ in cost and reliability, such that $c_1 < c_2$ and $u_1 < u_2$. As a special case, let $u_2 = \infty$, i.e., the supplier 2 is perfectly reliable. Furthermore, we assume the demand is deterministic, denote as D. For simplicity, we assume $\eta_1 = \eta_2 = 0$ and $\theta = 1$. All other parameters are assumed to be identical.

Given the above assumptions, the firm's expected profit function, given the firm sources from supplier i, $i = 1, 2$, is $\phi_i E_{S_i}[\tilde{y}_i] - E_{S_i}[(\tilde{y}_i - D)^+]$. It is straightforward to show that $y_i^* = \min\{D, K\}$, $i = 1, 2$. Therefore, the firm's optimal expected profit function is simply $\phi_i E_{S_i}[\min\{D, K\}]$ if the firm procures from supplier i, $i = 1, 2$. In what follows, we consider the case where $D \le K$. The analysis of $D > K$ can be similarly carried out. Using (A-76), we have

$$
\phi_i E_{S_i}[D] = (r - c_i) \int_{K-D}^{K} G_i(s, u_i(0)) ds. \tag{A-69}
$$

If there is no improvement opportunity, then the firm will choose to procure from supplier 2 if

and only if

$$(r - c_2) \int_{K-D}^{K} G_2(s, u_2(0))ds > (r - c_1) \int_{K-D}^{K} G_1(s, u_1(0))ds. \tag{A-70}$$

Substitute the assumption that $u_2(0) = \infty$ into (A-70), we have

$$(r - c_2)D > (r - c_1) \int_{K-D}^{K} G_1(s, u_1(0))ds. \tag{A-71}$$

Since $D > \int_{K-D}^{K} G_1(s, u_1(0))ds$, it is easy to find parameter values such that $c_1 < c_2$ but (A-71) holds. In other words, the firm prefers to procure from supplier 2 as opposed to supplier 1, because the reliability gain of supplier 2 out weights the cost savings from supplier 1. However, if the firm has opportunity to make improvement, it will either make no improvement and continue to procure from supplier 2, or it will make improvement from supplier 1 and procure from supplier 1 afterwards. In particular, using (3.20), the optimal process capability after improvement is given by

$$u_1(z^*) = u_1(0) + \frac{r - c_1}{2m} \int_{K-D}^{K} \frac{\partial G_1(s, u_1(z^*))}{\partial u_1(z^*)} ds. \tag{A-72}$$

To facilitate intuition, we further assume that $G(\cdot, u)$ is uniformly distributed with support greater than or equal to K. We can then obtain closed form solution as

$$u_1(z^*) = u_1(0) + \frac{1}{2m}\frac{1}{2}(r - c_1)(K^2 - (K - D)^2). \tag{A-73}$$

Let Δ denote the LHS - RHS of (A-71), then if the optimal expected revenue after improvement on supplier 1 is greater than Δ then the firm will procure from supplier 1 after improvement

effort. Substitute (A-73) into (3.19), we have

$$g^* = -\frac{1}{4mu_1^2(0)} \left[\frac{1}{2}(r-c_1)(K^2-(K-D)^2)\right]^2 + \left[u_1(0) + \frac{1}{2m}\frac{1}{2}(r-c_1)(K^2-(K-D)^2)\right] \cdot$$
$$\frac{1}{2m}\frac{1}{2}(r-c_1)(K^2-(K-D)^2).$$

$$(\text{A-74})$$

If g^* minus the right hand side of (A-71) is greater than Δ, then the lemma statement is true.

Note this can be simplified to

$$\frac{1}{2m}\left(1 - \frac{1}{2u_1^2(0)}\right)\left[\frac{1}{2}(r-c_1)(K^2-(K-D)^2)\right]^2 > \Delta,$$

which can be easily satisfied if $u_1(0) > \frac{\sqrt{2}}{2}$ and m is small.   □

Proof of Lemma 7. By envelope theorem,

$$\frac{\partial(\pi_D^* - \pi_I^*)}{\partial c} = -2(\eta y_D^* + (1-\eta)E[y_D^*]) + (\eta y_I^* + (1-\eta)E[y_I^*]) = -2E[y_D^*] + E[y_I^*]. \quad (\text{A-75})$$

Note that

$$E[y_D^*] = \int_0^{K-y_D^*} y_D^* dG(r, u_D) + \int_{K-y_D^*}^K (K-r)dG(r, u_D)$$

$$= y_D^* G(K-y_D^*, u_D) + (K-r)G(r, u_D)|_{K-y_D^*}^K - \int_{K-y_D^*}^K G(r, u_D)d(K-r)$$

$$= \int_{K-y_D^*}^K G(r, u_D)dr. \quad (\text{A-76})$$

Substitute (A-76) into (A-75), and realizing $y_I^* \geq y_D^*$ (submodular property),

$$\frac{\partial(\pi_D^* - \pi_I^*)}{\partial c} = -2 \int_{K-y_D^*}^{K} G(r, u_D)dr + \int_{K-y_I^*}^{K} G(r, u_I)dr$$

$$= \int_{K-y_D^*}^{K} (-2G(r, u_D) + G(r, u_I))dr + \int_{K-y_I^*}^{K-y_D^*} G(r, u_I)dr.$$

Now, using (A-76),

$$\frac{\partial^2(\pi_D^* - \pi_I^*)}{\partial c^2} = -2\frac{\partial E[y_D^*]}{\partial c} + \frac{\partial E[y_I^*]}{\partial c} = -2G(K - y_D^*, u_D)\frac{\partial y_D^*}{\partial c} + G(K - y_I^*, u_I)\frac{\partial y_I^*}{\partial c}. \quad (A\text{-}77)$$

Applying implicit function theorem to (3.14), we have

$$\frac{\partial y_D^*}{\partial c} = -\frac{\eta + (1-\eta)G(K - y_D^*, u_D)}{g(K - y_D^*, u_D)[\phi - E[F(y_D^* + \tilde{y}_D^*)]] + G(K - y_D^*, u_D)E[f(y_D^* + \tilde{y}_D^*)]}.$$

Setting $\eta = 0$, the above equation can be simplified to

$$\frac{\partial y_D^*}{\partial c} = -\frac{1}{E[f(y_D^* + \tilde{y}_D^*)]} = -\frac{1}{f(\cdot)}.$$

Substitute the above function into (A-77), we have

$$\frac{\partial^2(\pi_D^* - \pi_I^*)}{\partial c^2} = -\frac{1}{f(\cdot)}\left(-2G(K - y_D^*, u_D) + G(K - y_I^*, u_I)\right). \quad (A\text{-}78)$$

Both claims 1 and 2 follow directly. □

Proof of Lemma 8. We first prove that the optimal expected revenue is increasing in cost re-

lated δ. Using (3.7), and substitute $c_1 = c - \delta$ and $c_c = c + \delta$, we have

$$\frac{\pi^*(\vec{u})}{\partial \delta} = \nabla_\delta \left(\sum_{i=1}^{2} \psi_i y_i^* + \sum_{i=1}^{2} \phi_i E[\tilde{y}_i^*] \right)$$

$$= \eta(y_1^* - y_2^*) + (1 - \eta) \left(E[\tilde{y}_1^*] - E[\tilde{y}_2^*] \right). \tag{A-79}$$

Using (3.9), we have, at optimality, that

$$G(K - y_1^*)(\phi_1 - E[F(y_1^* + \tilde{y}_2^*)]) + \psi_1 = 0,$$

$$G(K - y_2^*)(\phi_2 - E[F(y_2^* + \tilde{y}_1^*)]) + \psi_2 = 0.$$

By assumption, $\phi_1 \geq \phi_2$ and $\psi_1 \leq \psi_2$, it follows that $y_1^* \geq y_2^*$. Further more, because the two suppliers are identical in reliability, $y_1^* \geq y_2^* \Rightarrow E[\tilde{y}_1^*] \geq E[\tilde{y}_2^*]$. Substitute into (A-79), we have $\frac{\pi^*(\vec{u})}{\partial \delta} \geq 0$.

We next prove that $\frac{\pi^*(\vec{u})}{\partial \delta}$ is increasing in u_1. By Lemma 3, $\frac{\partial y_1^*}{\partial u_1} \geq 0$. Because $\pi^*(\cdot)$ is a submodular function in (y_1^*, y_2^*) by Lemma 1, it follows that $\frac{\partial y_2^*}{\partial u_1} \leq 0$. (Note this can also be directly verified by applying implicit function theorem to (3.9).) Combine the above together, we have $\frac{\pi^*(\vec{u})}{\partial \delta}$ is increasing in u_1. Setting $n = 1$ in (3.22), we have

$$\frac{\partial \pi_I^*}{\partial \delta} = \nabla_\delta g^*(u^1, u^0) = \theta_1 \frac{\partial \pi^* \left(u^{1*} \right)}{\partial \delta} + (1 - \theta_1) \frac{\partial \pi^* \left(u^{0*} \right)}{\partial \delta}$$

$$\geq \frac{\partial \pi^* \left(u^{0*} \right)}{\partial \delta} = \eta y_1^* + (1 - \eta) E[\tilde{y}_1^*]$$

$$\geq \eta(y_1^* - y_2^*) + (1 - \eta) \left(E[\tilde{y}_1^*] - E[\tilde{y}_2^*] \right)$$

$$= \frac{\partial \pi_D^*}{\partial \delta}.$$

Note the y_1^* differs in the above equation for the pure investment and pure diversification case

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because the capability level is different and the number of suppliers are also different. Because y^* is increasing in capability and the diversification function is a submodular function, we must have $y_I^* > y_D^*$.

We now prove that the optimal expected revenue is increasing in η related δ. Using (3.7), and substitute $\eta_1 = \eta - \delta$ and $\eta_2 = \eta + \delta$, we have

$$
\begin{aligned}
\frac{\pi^*(\vec{u})}{\partial \delta} &= \nabla_\delta \left(\sum_{i=1}^{2} \psi_i y_i^* + \sum_{i=1}^{2} \phi_i E[\tilde{y}_i^*] \right) \\
&= c\left(y_1^* - y_2^* - E[\tilde{y}_1^*] + E[\tilde{y}_2^*] \right) \\
&= c\left(y_1^* - y_2^* - \left(\int_0^{K-y_1^*} y_1^* dG(r,u) + \int_{K-y_1^*}^{K} (K-r)dG(r,u) \right) \right. \\
&\qquad \left. + \left(\int_0^{K-y_2^*} y_2^* dG(r,u) + \int_{K-y_2^*}^{K} (K-r)dG(r,u) \right) \right) \\
&= c\left(y_1^* - y_2^* - \left(G(K-y_1^*,u)y_1^* + (K-r)G(r,u)\Big|_{K-y_1^*}^{K} + \int_{K-y_1^*}^{K} G(r,u)dr \right) \right. \\
&\qquad \left. + \left(G(K-y_2^*,u)y_2^* + (K-r)G(r,u)\Big|_{K-y_2^*}^{K} + \int_{K-y_2^*}^{K} G(r,u)dr \right) \right) \\
&= c\left(y_1^* - y_2^* - \int_{K-y_1^*}^{K} G(r,u)dr + \int_{K-y_2^*}^{K} G(r,u)dr \right). \tag{A-80}
\end{aligned}
$$

Using (3.9), one can easily prove that $y_1^* \geq y_2^*$. We can therefore rewrite (A-80) as

$$
\begin{aligned}
\frac{\pi^*(\vec{u})}{\partial \delta} &= c\left(y_1^* - y_2^* - \int_{K-y_1^*}^{K-y_2^*} G(r,u)dr \right) \\
&\geq c\left(y_1^* - y_2^* - \int_{K-y_1^*}^{K-y_2^*} dr \right) = 0. \tag{A-81}
\end{aligned}
$$

Next we show that $\nu - \int_{K-\nu}^{K} G(r,u)dr$ is an increasing function of ν. Note that

$$
\nabla_\nu \left(\nu - \int_{K-\nu}^{K} G(r,u)dr \right) = 1 - G(K-\nu,u) \geq 0. \tag{A-82}
$$

Intuitively, as δ increases, y_1^* increases and y_2^* decreases (for diversification case). Therefore, the marginal benefit of δ is increasingly beneficial. (this can be easily proved). However, for the investment case, the increase in δ may also cause a change in optimal investment level u'. So how does optimal u change with η? By first order condition, this should be negative $= -c$

We now prove the capacity related δ. Setting all parameters identical and by Lemma A11, we have

$$\frac{\partial \pi^*(\cdot)}{\partial \delta} = \phi \left(G(K + \delta, u) - G(K - \delta, u) \right) - \overline{G}(K - \delta, u)E[F(\tilde{y}_1^*)] + \overline{G}(K + \delta, u)E[F(\tilde{y}_2^*)].$$

We prove that the above expression is always non-negative by showing (1) the above expression is equal to zero when $\delta = 0$, and (2) the derivative of the above expression is non-negative.

For part (1), it is easy to see that at $\delta = 0$ the above expression equals to zero. For part (2), note

$$\frac{\partial^2 \pi^*(\cdot)}{\partial \delta^2} = \phi \left(g(K + \delta, u) + g(K - \delta, u) \right) - g(K + \delta, u)E[F(\tilde{y}_2^*)] + \overline{G}(K + \delta)\frac{\partial E[F(\tilde{y}_2^*)]}{\partial \delta}$$
$$- g(K - \delta, u)E[F(\tilde{y}_1^*)] - \overline{G}(K - \delta)\frac{\partial E[F(\tilde{y}_1^*)]}{\partial \delta}. \tag{A-83}$$

Note that

$$\frac{\partial E[F(\tilde{y}_2^*)]}{\partial \delta} = -\int_{K-\delta-y_2^*}^{K-\delta} f(K - \delta - r_2)dG(r_2, u), \tag{A-84}$$

and

$$\frac{\partial E[F(\tilde{y}_1^*)]}{\partial \delta} = -\int_{K+\delta-y_1^*}^{K+\delta} f(K + \delta - r_1)dG(r_1, u), \tag{A-85}$$

Substitute (A-84) and (A-85) into (A-83), we have

$$
\begin{aligned}
\frac{\partial^2 \pi^*(\cdot)}{\partial \delta^2} = {} & g(K+\delta, u)(\phi - E[F(\tilde{y}_2^*)]) + g(K-\delta, u)(\phi - E[F(\tilde{y}_1^*)]) \\
& - \overline{G}(K+\delta) \int_{K-\delta-y_2^*}^{K-\delta} f(K-\delta-r_2) dG(r_2, u) \\
& + \overline{G}(K-\delta) \int_{K+\delta-y_1^*}^{K+\delta} f(K+\delta-r_1) dG(r_1, u).
\end{aligned}
\tag{A-86}
$$

Let $x_1 = K + \delta - r_1$ and $x_2 = K - \delta - r_2$, and substitute into (A-86), we have

$$
\begin{aligned}
\frac{\partial^2 \pi^*(\cdot)}{\partial \delta^2} = {} & g(K+\delta, u)(\phi - E[F(\tilde{y}_2^*)]) + g(K-\delta, u)(\phi - E[F(\tilde{y}_1^*)]) \\
& - \overline{G}(K+\delta) \int_0^{y_2^*} f(x_2) g(K-\delta-x_2, u) dx_2 \\
& + \overline{G}(K-\delta) \int_0^{y_1^*} f(x_1) g(K+\delta-x_1, u) dx_1.
\end{aligned}
\tag{A-87}
$$

If $g(\cdot)$ is uniformly distributed, then (A-87) is non-negative.

We now prove that π_D^* is increasing in revenue related δ. Using (A-59), we have

$$
\begin{aligned}
\frac{\partial \pi^*(\vec{u})}{\partial \delta} = {} & \phi \int_{K-y_1^*}^{K} \frac{\partial G(r_1, u_1)}{\partial u_1} dr_1 - E_{\vec{S}} \left[\int_{K-y_1^*}^{K} \frac{\partial G(r_1, u_1)}{\partial u_1} F(K-r_1+\tilde{y}_2^*) dr_1 \right] \\
& - \phi \int_{K-y_2^*}^{K} \frac{\partial G(r_2, u_2)}{\partial u_2} dr_2 + E_{\vec{S}} \left[\int_{K-y_2^*}^{K} \frac{\partial G(r_2, u_2)}{\partial u_2} F(K-r_2+\tilde{y}_1^*) dr_1 \right] \\
= {} & \int_{K-y_1^*}^{K} \frac{\partial G(r_1, u_1)}{\partial u_1} \left(\phi - E_{\vec{S}}[F(K-r_1+\tilde{y}_2^*)] \right) dr_1 \\
& - \int_{K-y_2^*}^{K} \frac{\partial G(r_2, u_2)}{\partial u_2} \left(\phi - E_{\vec{S}}[F(K-r_2+\tilde{y}_1^*)] \right) dr_2.
\end{aligned}
$$

Let $x = K - r$, we have

$$
\begin{aligned}
\frac{\partial \pi^*(\vec{u})}{\partial \delta} &= \int_0^{y_1^*} \frac{\partial G(K - x, u_1)}{\partial u_1} \left(\phi - E_{\vec{S}}\left[F\left(x + \tilde{y}_2^*\right)\right] \right) dx \\
&\quad - \int_0^{y_2^*} \frac{\partial G(K - x, u_2)}{\partial u_2} \left(\phi - E_{\vec{S}}\left[F\left(x + \tilde{y}_1^*\right)\right] \right) dx \\
&\geq \int_0^{y_2^*} \frac{\partial G(K - x, u_1)}{\partial u_1} \left(\phi - E_{\vec{S}}\left[F\left(x + \tilde{y}_2^*\right)\right] \right) dx \\
&\quad - \int_0^{y_2^*} \frac{\partial G(K - x, u_1)}{\partial u_1} \left(\phi - E_{\vec{S}}\left[F\left(x + \tilde{y}_1^*\right)\right] \right) dx \\
&= \int_0^{y_2^*} \frac{\partial G(K - x, u_1)}{\partial u_1} \left(E_{\vec{S}}\left[F\left(x + \tilde{y}_1^*\right)\right] - E_{\vec{S}}\left[F\left(x + \tilde{y}_2^*\right)\right] \right) dx \geq 0. \qquad \text{(A-88)}
\end{aligned}
$$

Note the inequalities follow from the fact that $u_1 > u_2 \Rightarrow y_1^* \geq y_2^*$.

Now, for the pure investment strategy, we must have

$$
\begin{aligned}
\frac{\partial \pi^*(\vec{u})}{\partial \delta} &= \int_0^{\hat{y}_1^*} \frac{\partial G(K - x, \hat{u}_1)}{\partial \hat{u}_1} \left(\phi - F(x) \right) dx \\
&\geq \int_0^{y_1^*} \frac{\partial G(K - x, \hat{u}_1)}{\partial \hat{u}_1} \left(\phi - E_{\vec{S}}\left[F\left(x + \tilde{y}_2^*\right)\right] \right) dx. \qquad \text{(A-89)}
\end{aligned}
$$

Therefore, as δ increases, the pure investment strategy is increasingly preferred to diversification case. $\qquad \square$

A3   Appendices for Chapter 4

A3.1   Split Procurement Strategy with $s_2 \leq \min(s_0, s_1)$

In the second stage, first consider the case when the realized demand is less than the total procurement quantity, i.e., $x \leq y_0 + y_1$. Following the analysis from §4.3.2, it is economical to ship product from LCC if and only if $z \leq r + p - s_0$. Therefore, if $z \leq r + p - s_0$, then the firm's revenue function is

$$\pi_l(\hat{y}_0, \hat{y}_1, y_0, y_1, x) = r \min\left(\sum_{i=0}^{1} \hat{y}_i, x\right) - z\hat{y}_0 - p\left(x - \sum_{i=0}^{1} \hat{y}_i\right)^+ + s_2\left(\sum_{i=0}^{1} \hat{y}_i - x\right)^+ + \sum_{i=0}^{1} s_i(y_i - \hat{y}_i)$$

(A-90)

$$\text{s.t.} \quad \hat{y}_i \leq y_i, i = 0, 1,$$

where $\hat{y}_0$ and $\hat{y}_1$ are the procurement quantity shipped from LCC and MCC, respectively.

For the case of $s_2 < \min(s_0, s_1)$, it is optimal for the firm to ship just as much as the realized demand x, because the firm can realize higher salvage value for any left over quantities. Since π_l is increasing in $\hat{y}_0$ and $\hat{y}_1$ for $\sum_{i=0}^{1} \hat{y}_i < x$, we must have $\sum_{i=0}^{1} \hat{y}_i = x$. Substitute $\hat{y}_1 = x - \hat{y}_0$ into (A-90),

$$\pi_l(\hat{y}_0, \hat{y}_1, y_0, y_1, x) = (r - s_1)x + (s_1 - s_0 - z)\hat{y}_0 + \sum_{i=0}^{1} s_i y_i. \qquad (A-91)$$

Note that $\pi_l(\hat{y}_0, x)$ is linear in $\hat{y}_0$, therefore, $\hat{y}_0^* = \min(y_0, x)$ if $z \leq s_1 - s_0$ and $\hat{y}_0^* = \max(x - y_1, 0)$

otherwise. Therefore, (A-91) can be simplified to

$$
\pi_l(y_0, y_1, x) =
\begin{cases}
(r - s_1)x + (s_1 - s_0 - z)\min(y_0, x) + \sum_{i=0}^{1} s_i y_i, & z \le s_1 - s_0; \\
(r - s_1)x + (s_1 - s_0 - z)(x - y_1)^+ + \sum_{i=0}^{1} s_i y_i, & \text{otherwise.}
\end{cases}
\tag{A-92}
$$

If $z > r + p - s_0$, then it is not economical to ship from LCC, but it remains profitable to ship from MCC. The firm's revenue function is

$$
\pi_l(y_0, y_1, x) = r\min(y_1, x) - p(x - y_1)^+ + s_1(y_1 - x)^+ + s_0 y_0.
\tag{A-93}
$$

Note that in this case the amount shipped from MCC is given by $\hat{y}_1 = \min(y_1, x)$.

We now turn our attention to the case where the realized demand is greater than the total procurement quantity, i.e, $x > y_0 + y_1$. Again, it is economical to ship from LCC if and only if $z \le r + p - s_0$. Therefore, if $z \le r + p - s_0$, then the firm's revenue function is

$$
\pi_h(y_0, y_1, x) = r\sum_{i=0}^{1} y_i - p\left(x - \sum_{i=0}^{1} y_i\right) - z y_0.
\tag{A-94}
$$

Otherwise, if $z > r + p - s_0$, then the firm's profit function is

$$
\pi_h(y_0, y_1, x) = r y_1 - p(x - y_1) + s_0 y_0.
\tag{A-95}
$$

Combining the above derivations, we can derive the firm's expected profit function for any given s_0, s_1. Note that if $s_0 \ge s_1$, then at the second stage, it is optimal for the firm to try to ship as much of product from MCC as possible. Otherwise if $s_0 < s_1$ then at the second stage, as long as the unit quota price is not too high, it is optimal for the firm to try to ship as much

of product from LCC as possible. First consider the case of $s_0 < s_1$, we have

$$
\begin{aligned}
\pi(y_0, y_1) = & \left(\int_0^{s_1-s_0} (s_1 - s_0 - z)dG(z) \right) \left(\int_0^{y_0} x dF(x) + \int_{y_0}^{y_0+y_1} y_0 dF(x) \right) \\
& + \left(\int_{s_1-s_0}^{r+p-s_0} (s_1 - s_0 - z)dG(z) \right) \int_{y_1}^{y_0+y_1} (x - y_1)dF(x) \\
& - (r + p - s_1)\overline{G}(r + p - s_0) \int_{y_1}^{y_0+y_1} (x - y_1)dF(x) \\
& + y_0 \overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r + p - z)dG(z) + s_0 y_0 \overline{F}(y_0 + y_1)\overline{G}(r + p - s_0) \\
& + \int_0^{y_0+y_1} \left((r - s_1)x + \sum_{i=0}^{1} s_i y_i \right) dF(x) + \int_{y_0+y_1}^{\infty} (ry_1 - p(x - y_1)) \, dF(x) - \sum_{i=0}^{1} c_i y_i.
\end{aligned}
$$

$$(A\text{-}96)$$

For the case of $s_0 \geq s_1$, we have

$$
\begin{aligned}
\pi(y_0, y_1) = & \left(\int_0^{r+p-s_0} (s_1 - s_0 - z)dG(z) \right) \int_{y_1}^{y_0+y_1} (x - y_1)dF(x) \\
& - (r + p - s_1)\overline{G}(r + p - s_0) \int_{y_1}^{y_0+y_1} (x - y_1)dF(x) \\
& + y_0 \overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r + p - z)dG(z) + s_0 y_0 \overline{F}(y_0 + y_1)\overline{G}(r + p - s_0) \\
& + \int_0^{y_0+y_1} \left((r - s_1)x + \sum_{i=0}^{1} s_i y_i \right) dF(x) + \int_{y_0+y_1}^{\infty} (ry_1 - p(x - y_1)) \, dF(x) - \sum_{i=0}^{1} c_i y_i.
\end{aligned}
$$

Note that the $s_0 \geq s_1$ case is similar to that of the $s_0 < s_1$ case. In what follows, we focus on

the more general case, i.e., the $s_0 < s_1$ case. Using (A-96), we have

$$
\begin{aligned}
\partial_{y_0} \pi(y_0, y_1) = & \overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z) \\
& + (F(y_0 + y_1) - F(y_0)) \int_0^{s_1-s_0} (s_1 - s_0 - z)dG(z) - (c_0 - s_0),
\end{aligned}
\qquad (A\text{-}97)
$$

and

$$\partial_{y_1}\pi(y_0, y_1) = (r + p - s_1)\overline{F}(y_1) - (F(y_0 + y_1) - F(y_1)) \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z)$$

$$+ (F(y_0 + y_1) - F(y_1)) \int_0^{s_1-s_0} (s_1 - s_0 - z)dG(z) - (c_1 - s_1). \qquad \text{(A-98)}$$

Lemma A12. *The hessian matrix associated with $\pi(y_0, y_1)$ is symmetric, negative, and diagonal dominant.*

Proof of Lemma A12. The second order derivative with respect to y_0 and y_1 follows from (A-97) and (A-98).

$$\frac{\partial^2 \pi(y_0, y_1)}{\partial y_0^2} = -f(y_0 + y_1) \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z)$$

$$+ (f(y_0 + y_1) - f(y_0)) \int_0^{s_1-s_0} (s_1 - s_0 - z)dG(z), \qquad \text{(A-99)}$$

and

$$\frac{\partial^2 \pi(y_0, y_1)}{\partial y_1^2} = -(r + p - s_1)f(y_1) - (f(y_0 + y_1) - f(y_1)) \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z)$$

$$+ (f(y_0 + y_1) - f(y_1)) \int_0^{s_1-s_0} (s_1 - s_0 - z)dG(z). \quad \text{(A-100)}$$

In addition, we have

$$\frac{\partial^2 \pi(y_0, y_1)}{\partial y_0 y_1} = -f(y_0 + y_1) \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z)$$

$$+ f(y_0 + y_1) \int_0^{s_1-s_0} (s_1 - s_0 - z)dG(z). \qquad \text{(A-101)}$$

It is straightforward to verify that $\frac{\partial^2 \pi(y_0, y_1)}{\partial y_0^2} < 0$, $\frac{\partial^2 \pi(y_0, y_1)}{\partial y_1^2} < 0$, and $\frac{\partial^2 \pi(y_0, y_1)}{\partial y_0 y_1} < 0$. Therefore,

the Hessian matrix is negative and symmetric. In addition, it is easy to verify that the Hessian

matrix is also diagonal dominant. This entails showing that

$$-(r + p - s_1) + \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z) - \int_0^{s_1-s_0} (s_1 - s_0 - z)dG(z) \leq 0.$$

Note that the above equation can be simplified as follows.

$$
\begin{aligned}
&- \int_0^{r+p-s_0} (r + p - s_1)dG(z) - \int_{r+p-s_0}^{\infty} (r + p - s_1)dG(z) \\
&+ \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z) - \int_0^{s_1-s_0} (s_1 - s_0 - z)dG(z) \\
&= \int_0^{r+p-s_0} (s_1 - s_0 - z)dG(z) - \int_{r+p-s_0}^{\infty} (r + p - s_1)dG(z) - \int_0^{s_1-s_0} (s_1 - s_0 - z)dG(z) \\
&= \int_{s_1-s_0}^{r+p-s_0} (s_1 - s_0 - z)dG(z) - \int_{r+p-s_0}^{\infty} (r + p - s_1)dG(z) < 0.
\end{aligned}
$$

$\square$

It follows directly from Lemma A12 that the firm's objective function is jointly concave in

the procurement quantity y_0 and y_1. To summarize the above analysis, we have the following

theorem.

Theorem A1. *With split procurement strategy, the firm's profit function is jointly concave in*

the procurement quantity $\vec{y} = (y_0, y_1)$. Furthermore, the firm's optimal procurement quantity

can be obtained by setting (A-97) and (A-98) equal to zero (for interior solutions).

Theorem A2. *With split procurement strategy, the firm's optimal expected profit is a) de-*

creasing in the unit procurement cost c_0 and c_1, b) decreasing in the unit penalty cost p, c)

increasing in the unit salvage value s_0 and s_1, and d) increasing in the unit revenue r.

Proof of Theorem A2. The theorem statements can be obtained by applying envelope theorem to (A-96). Part a) follows directly. For part b), note that

$$\partial_p \pi(y_0, y_1) = -\overline{G}(r + p - s_0) \int_{y_1}^{y_0+y_1} (x - y_1) dF(x) - \int_{y_0+y_1}^{\infty} (x - y_1) dF(x) < 0.$$

For part c), note that

$$\partial_{s_0} \pi(y_0, y_1) = -G(s_1 - s_0) \left(\int_0^{y_0} x dF(x) + \int_{y_0}^{y_0+y_1} y_0 dF(x) \right) - \int_{s_1-s_0}^{r+p-s_0} dG(z) \int_{y_1}^{y_0+y_1} (x - y_1) dF(x)$$

$$+ y_0 \overline{F}(y_0 + y_1) \overline{G}(r + p - s_0) + \int_0^{y_0+y_1} y_0 dF(x)$$

$$> \overline{G}(s_1 - s_0) \int_0^{y_0+y_1} y_0 dF(x) - \int_{s_1-s_0}^{r+p-s_0} dG(z) \int_{y_1}^{y_0+y_1} (x - y_1) dF(x)$$

$$+ y_0 \overline{F}(y_0 + y_1) \overline{G}(r + p - s_0)$$

$$> \overline{G}(s_1 - s_0) \int_0^{y_0+y_1} y_0 dF(x) - \overline{G}(s_1 - s_0) \int_{y_1}^{y_0+y_1} (x - y_1) dF(x)$$

$$+ y_0 \overline{F}(y_0 + y_1) \overline{G}(r + p - s_0)$$

$$> y_0 \overline{F}(y_0 + y_1) \overline{G}(r + p - s_0) > 0.$$

In addition, we have

$$\partial_{s_1} \pi(y_0, y_1) = G(s_1 - s_0) \left(\int_0^{y_0} x dF(x) + \int_{y_0}^{y_0+y_1} y_0 dF(x) \right) + \int_{s_1-s_0}^{r+p-s_0} dG(z) \int_{y_1}^{y_0+y_1} (x - y_1) dF(x)$$

$$+ \overline{G}(r + p - s_0) \int_{y_1}^{y_0+y_1} (x - y_1) dF(x) + \int_0^{y_0+y_1} (-x + y_1) dF(x)$$

$$= G(s_1 - s_0) \left(\int_0^{y_0} x dF(x) + \int_{y_0}^{y_0+y_1} y_0 dF(x) \right) + \overline{G}(s_1 - s_0) \int_{y_1}^{y_0+y_1} (x - y_1) dF(x)$$

$$+ \int_0^{y_1} (y_1 - x) dF(x) - \int_{y_1}^{y_0+y_1} (x - y_1) dF(x)$$

$$= G(s_1 - s_0) \left(\int_0^{y_0} x dF(x) + \int_{y_0}^{y_0+y_1} y_0 dF(x) - \int_{y_1}^{y_0+y_1} (x - y_1) dF(x) \right) + \int_0^{y_1} (y_1 - x) dF(x).$$

But one can easily verify that, regardless of whether $y_0 \geq y_1$ or $y_0 < y_1$,

$$\int_0^{y_0} x dF(x) + \int_{y_0}^{y_0+y_1} y_0 dF(x) - \int_{y_1}^{y_0+y_1} (x - y_1) dF(x) > 0,$$

which proves the part c) of the theorem statement. As for part d), note that

$$\partial_r \pi(y_0, y_1) = \overline{G}(r + p - s_0) \int_{y_1}^{y_0+y_1} (x - y_1) dF(x) + y_0 \overline{F}(y_0 + y_1) G(r + p - s_0)$$
$$+ \int_0^{y_0+y_1} x dF(x) + \int_{y_0+y_1}^{\infty} y_1 dF(x) > 0.$$

This proves the theorem statements. □

Theorem A3. *With split procurement strategy, the firm's optimal expected profit (weakly) decreases as the quota price stochastically increases.*

Proof of Theorem A3. From the proof of Theorem 24, we know that, for any given two random variables $Z_1 \leq_{st} Z_2$,

$$\int_0^A (A - z_1) dG_1(z_1) \geq \int_0^A (A - z_2) dG_1(z_2). \tag{A-102}$$

Therefore, to prove the theorem statement, it is sufficient to prove that

$$y_0 \overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r + p - z) dG(z) + s_0 y_0 \overline{F}(y_0 + y_1) \overline{G}(r + p - s_0) \tag{A-103}$$

satisfies (A-102). But (A-103) can be simplified as

$$y_0 \overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r+p-z)dG(z) + s_0 y_0 \overline{F}(y_0 + y_1) - s_0 y_0 \overline{F}(y_0 + y_1)G(r+p-s_0)$$

$$= y_0 \overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r+p-s_0-z)dG(z) + s_0 y_0 \overline{F}(y_0 + y_1),$$

which clearly satisfies condition (A-102). The theorem statement follows directly. $\square$

Theorem A4. *With split procurement strategy, if the quota price is normally distributed with parameters $N(\mu, \sigma)$, then the firm's optimal expected profit is decreasing in the expected quota price μ and increasing in the quota standard deviation σ.*

Proof of Theorem A4. Follows similar approach to the proof of Theorem 25 and Corollary 5. $\square$

A3.2 The Exchange Curve Between Leadtime Difference Δ_L and Domestic Production Bound α

One distinctive advantage of the direct OPA strategy is its ability to postpone the domestic production to better match demand and supply. This leadtime advantage, however, is balanced by the policy constraint of the domestic production quantity fraction α. It is of interest to understand the exchange curves between the leadtime difference Δ_L and the domestic production requirement α. Toward this end, we designed a numeric study to compute the exchange curve. We use the base case parameter values for our numeric study, but varying the LCC unit cost c_0 from 0.1 to 0.9 with a step size of 0.1, the demand cv from 0.1 to 0.5 with a step size of 0.1. We then vary the leadtime difference Δ_L from 1 to 5. For each Δ_L, we search the value of α

that gives a target profit of 200 (the average profit in our base case scenario). We therefore have 225 observations in total.

Note that the exchange curve may not be always feasible. In fact, in 17.3% of cases, we observed that it is not possible to achieve the average profit even when the α value equals to zero. Of these 17.3% of cases, the average achievable expected profit is 6.5% less than the target profit level of 200.

Because an increase in the leadtime difference level Δ_L makes the direct OPA strategy more advantageous, it is intuitive that the exchange curve is increasing in the leadtime difference Δ_L. Figure 5.1 illustrates the exchange curve between the leadtime Δ_L and the policy parameter α, stratified by the demand coefficient of variation levels.
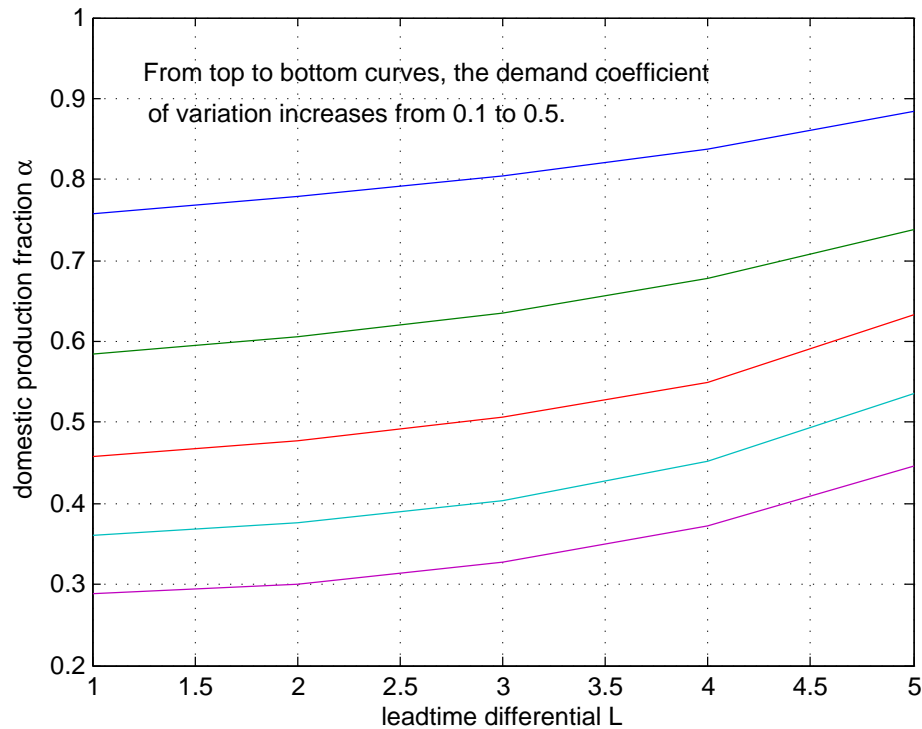

Figure 5.1: Exchange curve as the demand coefficient of variation increases

Figure 5.1 shows that the policy parameter value α is convex increasing in the leadtime difference. As the firm's domestic production has shorter leadtimes, the OPA policy has increasingly more latitude in setting the α value without compromise the competitiveness of the OPA strategy. However, this latitude is significantly influenced by the demand volatility. As the demand volatility increases, the policy parameter value α is significantly reduced, indicating that a more relaxed OPA policy is required to achieve similar competitiveness. We do note, however, that a shorter leadtime for the domestic production can mitigate the negative impact of the demand volatility on the policy parameter α. Figure 5.2 illustrates the exchange curve stratified by the unit cost c_0.
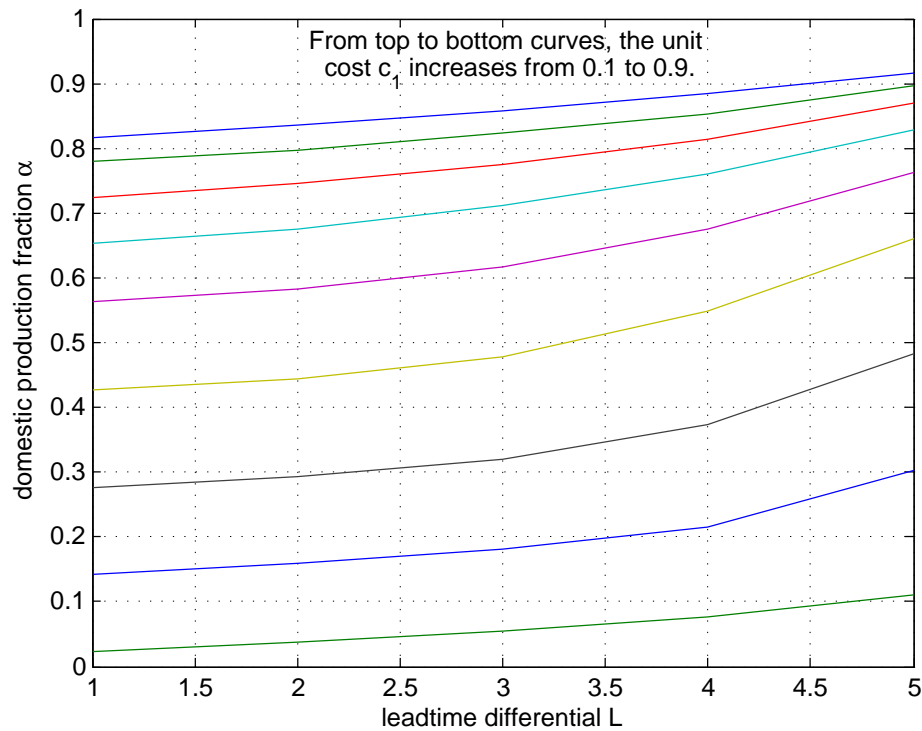

Figure 5.2: Exchange curve as the LCC unit procurement cost increases

Note that a higher level of c_0 makes the feasible α level significantly constrained. However, such observation should not be construed as a lack of competitiveness for the direct OPA

strategy as c_0 increases. These exchange curves are created at an average expected profit for the direct OPA strategy. As the unit cost c_0 increases, the non-OPA strategies may decline faster than that of the direct OPA strategy.

A3.3   Proofs

Proof of Theorem 23. It is straightforward to show that

$$\pi'(y_0) = -c_0 + F(y_0) \int_0^{s_2-s_0} (s_2 - z)dG(z) + s_0 F(y_0) \int_{s_2-s_0}^{r+p-s_0} dG(z)$$

$$+ \overline{F}(y_0) \int_0^{r+p-s_0} (r+p-z)dG(z) + s_0 \overline{G}(r+p-s_0), \qquad \text{(A-104)}$$

and

$$\pi''(y_0) = -f(y_0) \left(\int_0^{r+p-s_0} (r+p-s_0-z)dG(z) - \int_0^{s_2-s_0} (s_2 - s_0 - z)dG(z) \right) < 0.$$

The firm's revenue function is therefore concave in the procurement quantity and the optimal procurement quantity y_0^* can be obtained by setting (A-104) equal to zero (for the interior solution). $\qquad \square$

Proof of Proposition 1. The proposition statements can be obtained by applying the envelope theorem to (4.4). Part a) is straightforward. For part b), we have

$$\partial_p \pi(y_0^*) = G(r+p-s_0) \int_0^{y_0^*} x dF(x) - E[x] < 0.$$

For part c), note that (4.4) can be rewritten as

$$\pi(y_0) = y_0 \int_0^{r+p-s_0} (r+p-s_0-z)dG(z) - (c_0-s_0)y_0 - pE[x]$$

$$- \int_0^{y_0} (y_0-x)dF(x) \left(\int_0^{r+p-s_0} (r+p-s_0-z)dG(z) - \int_0^{s_2-s_0} (s_2-s_0-z)dG(z) \right)$$

It follows that

$$\partial_{s_0}\pi(y_0^*) = y_0^* \overline{G}(r+p-s_0) - \int_0^{y_0^*} (y_0^*-x)dF(x) \left(-G(r+p-s_0) + G(s_2-s_0) \right) \geq 0,$$

and

$$\partial_{s_2}\pi(y_0^*) = \int_0^{y_0^*} (y_0^*-x)dF(x)G(s_2-s_0) \geq 0.$$

Finally, for part d), we have

$$\partial_r \pi(y_0^*) = G(r+p-s_0) \left(y_0^* - \int_0^{y_0^*} (y_0^*-x)dF(x) \right) > 0.$$

$\square$

Proof of Theorem 24. Using (4.4), we can re-write the firm's revenue function as

$$\pi(y_0) = \int_0^{r+p-s_0} (r+p-s_0-z)dG(z) \left(\int_0^{y_0} xdF(x) + y_0\overline{F}(y_0) \right)$$

$$+ \int_0^{y_0} (y_0-x)dF(x) \int_0^{s_2-s_0} (s_2-s_0-z)dG(z) - (c_0-s_0)y_0 - pE[x]. \qquad \text{(A-105)}$$

To prove the theorem statement, it is suffice to prove that, given two distribution function

$G_1(\cdot) \geq G_2(\cdot)$, then $\pi(y|G_1(\cdot)) \geq \pi(y|G_2(\cdot))$ holds. Note that

$$\int_0^{r+p-s_0} (r+p-s_0-z)dG(z) = (r+p-s_0-z)G(z)|_0^{r+p-s_0} - \int_0^{r+p-s_0} -G(z)dz = \int_0^{r+p-s_0} G(z)dz.$$

It follows that, if $G_1(\cdot) \geq G_2(\cdot)$, then

$$\int_0^{r+p-s_0} (r+p-s_0-z)dG_1(z) \geq \int_0^{r+p-s_0} (r+p-s_0-z)dG_2(z) \Rightarrow \pi(y_0|G_1(\cdot)) \geq \pi(y_0|G_2(\cdot)).$$

Note that in the above derivation, the integrating term $\int_0^{s_2-s_0}(s_2-s_0-z)dG(z)$ can be similarly

incorporated when $s_2 > s_0$. □

Proof of Theorem 25. By (A-105) in the proof of theorem 24, the firm's expected profit function

depends on the variance of the quota price only through the following term.

$$V = \int_0^{r+p-s_0} (r+p-s_0-z)dG(z).$$

Substitute the normal distribution with parameters (μ, σ) into the above equation, we have

$$V = \int_{-\infty}^{r+p-s_0} (r+p-s_0-z) \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{z-\mu}{\sqrt{2}\sigma}\right)^2} dz.$$

A caveat with the normal distribution is that there are possibilities for negative quota prices,

which cannot happen in reality. Therefore, the parameter values of μ and σ have to be setup

in a way such that the probability of the quota prices becoming negative is negligible. Note

that the above equation can be simplified as

$$V = \frac{\sigma}{\sqrt{2\pi}} e^{-\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)^2} + \frac{1}{2}(r+p-s_0-\mu)\left(1+\text{erf}\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)\right).$$

Therefore, we have

$$\partial_\sigma V = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)^2} + \frac{1}{\sqrt{2\pi}} \left(\frac{r+p-s_0-\mu}{\sigma}\right)^2 e^{-\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)^2} - \frac{1}{2}\left(\frac{r+p-s_0-\mu}{\sigma}\right)^2 \frac{2}{\sqrt{2\pi}} e^{-\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)^2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)^2} > 0.$$

The theorem statement then follows directly by the envelope theorem. We note that the integration term $\int_0^{s_2-s_0}(s_2 - s_0 - z)dG(z)$ can be similarly incorporated when $s_2 > s_0$. □

Proof of Corollary 5. The theorem statement follows directly from Theorem 24. For concreteness, however, we provide an alternative proof. Analogous to the proof of Theorem 25, the firm's expected profit function depends on the mean of the quota price only through

$$V = \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z).$$

In the case of the normal distribution with parameters (μ, σ), we have

$$V = \frac{\sigma}{\sqrt{2\pi}} e^{-\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)^2} + \frac{1}{2}(r + p - s_0 - \mu)\left(1 + \mathrm{erf}\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)\right).$$

Therefore, we have

$$\partial_\mu V = \frac{r+p-s_0-\mu}{\sigma\sqrt{2\pi}} e^{-\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)^2} - \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)\right) - \frac{r+p-s_0-\mu}{\sigma\sqrt{2\pi}} e^{-\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)^2}$$

$$= -\frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{r+p-s_0-\mu}{\sqrt{2}\sigma}\right)\right) < 0.$$

The theorem statement then follows directly by the envelope theorem. We note that the integration term $\int_0^{s_2-s_0}(s_2 - s_0 - z)dG(z)$ can be similarly incorporated when $s_2 > s_0$. □

Proof of Lemma 10. The second order derivative with respect to y_0 and y_1 follows from (4.15) and (4.16).

$$\frac{\partial^2 \pi(y_0, y_1)}{\partial y_0^2} = -f(y_0 + y_1) \int_0^{r+p-s_0} (r+p-s_0-z)dG(z) + f(y_0+y_1) \int_0^{s_2-s_0} (s_2-s_0-z)dG(z),$$

$$\text{(A-106)}$$

and

$$\frac{\partial^2 \pi(y_0, y_1)}{\partial y_1^2} = -(r+p)f(y_1) - (f(y_0+y_1)-f(y_1)) \int_0^{r+p-s_0} (r+p-s_0-z)dG(z)$$

$$+ (f(y_0+y_1)-f(y_1)) \int_0^{s_2-s_0} (s_2-s_0-z)dG(z). \quad \text{(A-107)}$$

In addition, we have

$$\frac{\partial^2 \pi(y_0, y_1)}{\partial y_0 y_1} = -f(y_0+y_1) \int_0^{r+p-s_0} (r+p-s_0-z)dG(z)$$

$$+ f(y_0+y_1) \int_0^{s_2-s_0} (s_2-s_0-z)dG(z). \quad \text{(A-108)}$$

It is straightforward to verify that $\frac{\partial^2 \pi(y_0,y_1)}{\partial y_0^2} < 0$, $\frac{\partial^2 \pi(y_0,y_1)}{\partial y_1^2} < 0$, and $\frac{\partial^2 \pi(y_0,y_1)}{\partial y_0 y_1} < 0$. Therefore, the Hessian matrix is negative and symmetric. In addition, it is easy to verify that the Hessian matrix is also diagonal dominant. This entails showing that

$$-(r+p) + \int_0^{r+p-s_0} (r+p-s_0-z)dG(z) - \int_0^{s_2-s_0} (s_2-s_0-z)dG(z) \le 0,$$

which is obviously true. $\qquad\square$

Proof of Theorem 27. The theorem statements can be obtained by applying envelope theorem

to (4.14). Part a) follows directly. For part b), note that

$$\partial_p \pi(y_0, y_1) = -\overline{G}(r + p - s_0) \int_{y_1}^{\infty} (x - y_1) dF(x) - G(r + p - s_0) \int_{y_0 + y_1}^{\infty} (x - y_0 - y_1) dF(x) < 0.$$

For part c), note that

$$\partial_{s_0} \pi(y_0, y_1) = \int_{s_2 - s_0}^{r + p - s_0} \left(\int_0^{y_1} y_0 dF(x) + \int_{y_1}^{y_0 + y_1} (y_0 + y_1 - x) dF(x) \right) dG(z) + \overline{G}(r + p - s_0) y_0 > 0.$$

In addition, we have

$$\partial_{s_2} \pi(y_0, y_1) = \int_0^{s_2 - s_0} \int_0^{y_0 + y_1} (y_0 + y_1 - x) dF(x) dG(z) + \int_{s_2 - s_0}^{\infty} \int_0^{y_1} (y_1 - x) dF(x) dG(z) > 0.$$

As for part d), note that

$$\partial_r \pi(y_0, y_1) = G(r + p - s_0) \left(\int_0^{y_0 + y_1} x dF(x) + (y_0 + y_1) \overline{F}(y_0 + y_1) \right) > 0.$$

This proves the theorem statements. $\qquad \square$

Proof of Theorem 28. From the proof of Theorem 24, we know that, for any given two random variables $Z_1 \leq_{st} Z_2$,

$$\int_0^A (A - z_1) dG_1(z_1) \geq \int_0^A (A - z_2) dG_1(z_2). \tag{A-109}$$

Note that (4.14) can be simplified as

$$
\pi(y_0, y_1) = \int_0^{s_2-s_0} (s_2 - s_0 - z)dG(z)y_0F(y_0) + \left(\int_{s_2-s_0}^{r+p-s_0} (s_2 - s_0 - z)dG(z) \right) \int_{y_1}^{y_0+y_1} (x - y_1)dF(x)
$$

$$
- (r + p - s_2)\overline{G}(r + p - s_0) \int_{y_1}^{y_0+y_1} (x - y_1)dF(x) + y_0\overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r + p - z)dG(z)
$$

$$
+ s_0y_0\overline{F}(y_0 + y_1)\overline{G}(r + p - s_0) + \int_0^{y_0+y_1} ((r - s_2)x + s_0y_0 + s_2y_1)\, dF(x)
$$

$$
+ \int_{y_1}^{y_0+y_1} ((r - s_2)x + s_0y_0 + s_2y_1)\, dF(x) + \int_{y_0+y_1}^{\infty} (ry_1 - p(x - y_1))\, dF(x) - \sum_{i=0}^{1} c_iy_i.
$$

$$(\text{A-110})$$

Therefore, to prove the theorem statement, it is sufficient to prove that

$$
y_0\overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r + p - z)dG(z) + s_0y_0\overline{F}(y_0 + y_1)\overline{G}(r + p - s_0) \qquad (\text{A-111})
$$

satisfies (A-109). But (A-111) can be simplified as

$$
y_0\overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r + p - z)dG(z) + s_0y_0\overline{F}(y_0 + y_1) - s_0y_0\overline{F}(y_0 + y_1)G(r + p - s_0)
$$

$$
= y_0\overline{F}(y_0 + y_1) \int_0^{r+p-s_0} (r + p - s_0 - z)dG(z) + s_0y_0\overline{F}(y_0 + y_1),
$$

which clearly satisfies condition (A-109). The theorem statement follows directly. □

Proof of Theorem 29. Follows similar approach to the proof of Theorem 25 and Corollary 5. □

Proof of Theorem 30. The stated result can be obtained by substituting the condition of $c_1 - s_2 \le c_0 - s_0$ into the equation arrays of (4.15) and (4.16). □

Proof of Theorem 31. The proof can be obtained by setting (4.16) to be less than zero at $y_1 = 0$ (in which case the optimal y_0^* satisfies (4.15) for an interior solution) . $\qquad\square$

Proof of Theorem 33. From Theorem 32, $\pi_L^*(w_0^*, y_2^*|y_0, \tilde{\theta})$ is continuous in y_0. Furthermore, $\pi_L^*(w_0^*, y_2^*|y_0, \tilde{\theta})$ is linear in y_0 for $y_0 \leq (1-\alpha)k_L$ and $y_0 > (1-\alpha)k_H$. For $(1-\alpha)k_H < y_0 \leq (1-\alpha)k_H$, $\pi_L^*(w_0^*, y_2^*|y_0, \tilde{\theta})$ is concave in y_0. Because the upper and lower limit at $y_0 = (1-\alpha)k_L$ are identical and equal to c_2, and the upper and lower limit at $y_0 = (1-\alpha)k_H$ are also identical and equal to s_0, the theorem statement follows directly. $\qquad\square$

Proof of Theorem 35. Suppose the leadtime differences are such that $L_A < L_B$. For any given y_0 and any realizations of $\tilde{\theta}$, the optimal expected profit, $\pi_{L_A}^*(w_{0A}^*, y_{2A}^*|y_0, \tilde{\theta}) \leq \pi_{L_B}^*(w_{0A}^*, y_{2A}^*|y_0, \tilde{\theta}) \leq \pi_{L_B}^*(w_{1B}^*, y_{2B}^*|y_0, \tilde{\theta})$ because, for L_B, the firm can simply act on decisions made at time L_A and achieve at least the same optimal expected profit. Since for any given y_0, we have $\pi_{L_A}^*(\cdot|y_0) \leq \pi_{L_B}^*(\cdot|y_0)$, in expectation we have $\pi(y_{0A}^*|L_A) \leq \pi(y_{0B}^*|L_B)$. $\qquad\square$

Lemma A13. *With martingale forecast updating process, the firm's second stage optimal profit function is given by*

$$
\pi_L(w_0^*, y_2^*|y_0, \mu_L) =
\begin{cases}
-(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\frac{z_L^2}{2}} + (r-c_2)\mu_L + c_2 y_0, & y_0 \leq (1-\alpha)k_L; \\[2ex]
-(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\left(\frac{\frac{y_0}{1-\alpha}-\mu_L}{\sqrt{2}\sigma_L}\right)^2} + (r+p-\alpha c_2)\frac{y_0}{1-\alpha} \\[1ex]
\quad + (r+p-s_2)\left(\mu_L - \frac{y_0}{1-\alpha}\right)F_{\Delta_L|\mu_L}\left(\frac{y_0}{1-\alpha}\right) - p\mu_L, & (1-\alpha)k_L < y_0 \leq (1-\alpha)k_H; \\[2ex]
-(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\frac{z_H^2}{2}} + (r-\alpha c_2 - (1-\alpha)s_0)\mu_L \\[1ex]
\quad + s_0 y_0, & y_0 > (1-\alpha)k_H.
\end{cases}
$$

$$\text{(A-112)}$$

Proof of Lemma A13. Note that

$$\int_{-\infty}^{k_L} x dF_{\Delta_L|\mu_L}(x) = -\frac{\sigma_L}{\sqrt{2\pi}} \exp\left(-\left(\frac{k_L - \mu_L}{\sqrt{2}\sigma_L}\right)^2\right) + \mu_L \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{k_L - \mu_L}{\sqrt{2}\sigma_L}\right)\right), \quad \text{(A-113)}$$

where $\operatorname{erf}(\cdot)$ is the standard error function. But

$$F_{\Delta_L|\mu_L}(y) = \int_{-\infty}^{y} \frac{1}{\sigma_L\sqrt{2\pi}} e^{-\left(\frac{x-\mu_L}{\sqrt{2}\sigma_L}\right)^2} dx = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{y - \mu_L}{\sqrt{2}\sigma_L}\right)\right). \quad \text{(A-114)}$$

Substitute (4.23) into (A-114), we have

$$\frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{k_L - \mu_L}{\sqrt{2}\sigma_L}\right)\right) = \frac{r + p - c_2}{r + p - s_2}. \quad \text{(A-115)}$$

Substitute (A-115) into (A-113), we have

$$\int_{-\infty}^{k_L} x dF_{\Delta_L|\mu_L}(x) = -\frac{\sigma_L}{\sqrt{2\pi}} \exp\left(-\left(\frac{k_L - \mu_L}{\sqrt{2}\sigma_L}\right)^2\right) + \mu_L \frac{r + p - c_2}{r + p - s_2}. \quad \text{(A-116)}$$

Notice that

$$\frac{k_L - \mu_L}{\sigma_L} = \Phi^{-1}\left(\frac{r + p - c_2}{r + p - s_2}\right),$$

which is a constant, independent of μ_L and σ_L. In fact, this is the standard newsvendor service level, which is influenced only by the revenue and cost parameter values. Let $z_L = \Phi^{-1}\left(\frac{r+p-c_2}{r+p-s_2}\right)$ and substitute K into (A-116), we have

$$\int_{-\infty}^{k_L} x dF_{\Delta_L|\mu_L}(x) = -\frac{\sigma_L}{\sqrt{2\pi}} e^{-\frac{z_L^2}{2}} + \mu_L \frac{r + p - c_2}{r + p - s_2}. \quad \text{(A-117)}$$

Analogously, we have

$$\int_{-\infty}^{k_H} x dF_{\Delta_L|\mu_L}(x) = -\frac{\sigma_L}{\sqrt{2\pi}} e^{-\frac{z_H^2}{2}} + \mu_L \frac{r + p - \alpha c_2 - (1-\alpha)s_0}{r + p - s_2}, \qquad \text{(A-118)}$$

where $z_H = \Phi^{-1}\left(\frac{r+p-\alpha c_2-(1-\alpha)s_0}{r+p-s_2}\right)$.

Note that

$$\int_{-\infty}^{\frac{y_0}{1-\alpha}} x dF_{\Delta_L|\mu_L}(x) = -\frac{\sigma_L}{\sqrt{2\pi}} \exp\left(-\left(\frac{\frac{y_0}{1-\alpha} - \mu_L}{\sqrt{2}\sigma_L}\right)^2\right) + \mu_L \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{\frac{y_0}{1-\alpha} - \mu_L}{\sqrt{2}\sigma_L}\right)\right).$$

$$\text{(A-119)}$$

But

$$F_{\Delta_L|\mu_L}\left(\frac{y_0}{1-\alpha}\right) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{\frac{y_0}{1-\alpha} - \mu_L}{\sqrt{2}\sigma_L}\right)\right). \qquad \text{(A-120)}$$

Substitute (A-120) into (A-119), we have

$$\int_{-\infty}^{\frac{y_1}{1-\alpha}} x dF_{\Delta_L|\mu_L}(x) = -\frac{\sigma_L}{\sqrt{2\pi}} \exp\left(-\left(\frac{\frac{y_1}{1-\alpha} - \mu_L}{\sqrt{2}\sigma_L}\right)^2\right) + \mu_L F_{\Delta_L|\mu_L}\left(\frac{y_1}{1-\alpha}\right). \qquad \text{(A-121)}$$

The lemma statement follows by substituting (A-117), (A-118), and (A-121) into (4.26). $\square$

Lemma A14. *With martingale forecast updating under the direct OPA strategy, the derivative of the firm's revenue function with respect to OPA procurement quantity y_0, i.e., $\partial_{y_0}\pi(y_0)$, is*

given by

$$\partial_{y_0}\pi(y_0) = -c_1 + c_2\overline{G}_L\left(\frac{y_0}{1-\alpha} - z_L\sigma_L\right) + s_0 G_L\left(\frac{y_0}{1-\alpha} - z_H\sigma_L\right)$$

$$+ \frac{1}{1-\alpha}\int_{\frac{y_0}{1-\alpha}-z_H\sigma_L}^{\frac{y_0}{1-\alpha}-z_L\sigma_L}\left[(r+p-\alpha c_2) - (r+p-s_2)F_{\Delta_L|\mu_L}\left(\frac{y_0}{1-\alpha}\right)\right]dG_L(\mu_L).$$

$$\text{(A-122)}$$

Proof of Lemma A14. Using (4.28), we have

$$\partial_{y_0}\pi(y_0) = -c_1 + \int_{\frac{y_0}{1-\alpha}-z_L\sigma_L}^{\infty} c_2 dG_L(\mu_L) + \int_{-\infty}^{\frac{y_0}{1-\alpha}-z_H\sigma_L} s_0 dG_L(\mu_L)$$

$$- \left[-(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\frac{z_L^2}{2}} + (r-c_2)\left(\frac{y_0}{1-\alpha}-z_L\sigma_L\right) + c_2 y_0\right]g_L\left(\frac{y_0}{1-\alpha}-z_L\sigma_L\right)\frac{1}{1-\alpha}$$

$$+ \int_{\frac{y_0}{1-\alpha}-z_H\sigma_L}^{\frac{y_0}{1-\alpha}-z_L\sigma_L}\left[(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\left(\frac{\frac{y_0}{1-\alpha}-\mu_L}{\sqrt{2}\sigma_L}\right)^2}2\frac{\frac{y_0}{1-\alpha}-\mu_L}{\sqrt{2}\sigma_L}\frac{1}{\sqrt{2}\sigma_L}\frac{1}{1-\alpha}\right.$$

$$+ (r+p-\alpha c_2)\frac{1}{1-\alpha} - (r+p-s_2)\frac{1}{1-\alpha}F_{\Delta_L|\mu_L}\left(\frac{y_0}{1-\alpha}\right)$$

$$\left. + (r+p-s_2)\left(\mu_L - \frac{y_0}{1-\alpha}\right)F_{\Delta_L|\mu_L}\left(\frac{y_0}{1-\alpha}\right)\frac{1}{1-\alpha}\right]dG_L(\mu_L)$$

$$+ \left[-(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\frac{z_L^2}{2}} + (r+p-\alpha c_2)\frac{y_0}{1-\alpha} - (r+p-s_2)z_L\sigma_L F_{\Delta_L|\mu_L=\frac{y_0}{1-\alpha}-z_L\sigma_L}\left(\frac{y_0}{1-\alpha}\right)\right.$$

$$\left. - p\left(\frac{y_0}{1-\alpha}-z_L\sigma_L\right)\right]g_L\left(\frac{y_0}{1-\alpha}-z_L\sigma_L\right)\frac{1}{1-\alpha}$$

$$- \left[-(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\frac{z_H^2}{2}} + (r+p-\alpha c_2)\frac{y_0}{1-\alpha} - (r+p-s_2)z_H\sigma_L F_{\Delta_L|\mu_L=\frac{y_0}{1-\alpha}-z_H\sigma_L}\left(\frac{y_0}{1-\alpha}\right)\right.$$

$$\left. - p\left(\frac{y_0}{1-\alpha}-z_H\sigma_L\right)\right]g_L\left(\frac{y_0}{1-\alpha}-z_H\sigma_L\right)\frac{1}{1-\alpha}$$

$$+ \left[-(r+p-s_2)\frac{\sigma_L}{\sqrt{2\pi}}e^{-\frac{z_H^2}{2}} + (r-\alpha c_2 - (1-\alpha)s_0)\left(\frac{y_0}{1-\alpha}-z_H\sigma_L\right)\right.$$

$$\left. + s_0 y_0\right]g_L\left(\frac{y_0}{1-\alpha}-z_H\sigma_L\right)\frac{1}{1-\alpha}.$$

Simplifying the above expression, we have

$$
\partial_{y_0} \pi(y_0) = -c_1 + c_2 \overline{G}_L \left(\frac{y_0}{1-\alpha} - z_L \sigma_L \right) + s_0 G_L \left(\frac{y_0}{1-\alpha} - z_H \sigma_L \right)
$$

$$
+ \frac{1}{1-\alpha} g_L \left(\frac{y_0}{1-\alpha} - z_L \sigma_L \right) \left[(r+p-c_2) z_L \sigma_L - (r+p-s_2) z_L \sigma_L F_{\Delta_L | \mu_L = \frac{y_0}{1-\alpha} - z_L \sigma_L} \left(\frac{y_0}{1-\alpha} \right) \right]
$$

$$
+ \frac{1}{1-\alpha} g_L \left(\frac{y_0}{1-\alpha} - z_H \sigma_L \right) \left[- (r+p-\alpha c_2 - (1-\alpha) s_0) z_H \sigma_L \right.
$$

$$
\left. + (r+p-s_2) z_H \sigma_L F_{\Delta_L | \mu_L = \frac{y_0}{1-\alpha} - z_H \sigma_L} \left(\frac{y_0}{1-\alpha} \right) \right]
$$

$$
+ \frac{1}{1-\alpha} \int_{\frac{y_0}{1-\alpha} - z_H \sigma_L}^{\frac{y_0}{1-\alpha} - z_L \sigma_L} \left[(r+p-\alpha c_2) - (r+p-s_2) F_{\Delta_L | \mu_L} \left(\frac{y_0}{1-\alpha} \right) \right] dG_L(\mu_L).
$$

$$(\text{A-123})$$

Note that

$$
F_{\Delta_L | \mu_L}(k_L) = \int_{-\infty}^{k_L} \frac{1}{\sigma_L \sqrt{2\pi}} e^{-\left(\frac{x - \mu_L}{\sqrt{2} \sigma_L} \right)^2} dx = \int_{-\infty}^{\mu_L + z_L \sigma_L} \frac{1}{\sigma_L \sqrt{2\pi}} e^{-\left(\frac{x - \mu_L}{\sqrt{2} \sigma_L} \right)^2} dx \qquad (\text{A-124})
$$

Substitute $\mu_L = \frac{y_0}{1-\alpha} - z_L \sigma_L$ into (A-124), we have

$$
F_{\Delta_L | \mu_L = \frac{y_0}{1-\alpha} - z_L \sigma_L} \left(\frac{y_0}{1-\alpha} \right) = F_{\Delta_L | \mu_L}(k_L) = \frac{r+p-c_2}{r+p-s_2}. \qquad (\text{A-125})
$$

Following similar logic, we have

$$
F_{\Delta_L | \mu_L = \frac{y_0}{1-\alpha} - z_H \sigma_L} \left(\frac{y_0}{1-\alpha} \right) = F_{\Delta_L | \mu_L}(k_H) = \frac{r+p-\alpha c_2 - (1-\alpha) s_0}{r+p-s_2}. \qquad (\text{A-126})
$$

The lemma statement follows by substituting (A-125) and (A-126) into (A-123). $\qquad \square$

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