Essays on Retail Operations

by
Olga Perdikaki

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Approved by:

Dr. Jayashankar M. Swaminathan, Advisor
Dr. Kyle D. Cattani, Committee Member
Dr. Saravanan Kesavan, Committee Member
Dr. Dimitris Kostamis, Committee Member
Dr. Ann Marucheck, Committee Member

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ABSTRACT

OLGA PERDIKAKI: Essays on Retail Operations.
(Under the direction of Dr. Jayashankar M. Swaminathan.)

The intensified competition that the retail industry faces with the increasing numbers of new players in both the local and global markets has forced retailers to critically examine and redesign their operations and marketing strategies. To remain competitive, many retailers have focused on the provision of enhanced customer experience and pursued practices of differentiation. In this dissertation comprising of three essays, we attempt to shed light on retail practices that enhance consumer valuation, on factors that affect store performance, and on temporal management of demand enhancing activities using both analytical and empirical methodologies. The aim of this research is to develop theoretical insights to help retailers understand their store performance and effectively manage strategies geared towards enhancing demand and consumer valuation about their product offerings.

In the first essay, we focus on technology investments that can affect consumer valuation. We examine the impact of such investments in a duopoly setting in which retailers compete in prices and consumers can search among the two retailers. In the second essay, we focus on store performance and examine the impact of labor and traffic characteristics on different store performance metrics using proprietary data of a retail chain. In the third essay, we focus on general services that retailers could provide to enhance demand and examine their temporal management under competition and demand uncertainty.

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CHAPTER 1

Introduction

1.1 Research Motivation

The retail industry is one of the most dynamic and influential industries in developed economies. In the U.S. the retail business represents about 40% of the Gross Domestic Product (GDP) and is the largest employer (Fisher and Raman (2001)). The intensified competition that the retail industry faces with the emergence of increasing numbers of new players in both the local and global markets has forced retailers to critically examine and redesign both their operations and marketing strategies. To remain competitive, many retailers have differentiated themselves by designing enhanced customer experiences and other strategies to distinguish themselves from competition. Such practices have provided a fertile ground and new contexts for research in the retail operations arena. This dissertation comprising of three essays aims to shed light to some important aspects of retail operations that have emerged due to the current practices. The aim of this research is to develop theoretical insights to help retailers understand their store performance and effectively manage strategies geared towards enhancing demand and consumer valuation about their product offerings. In Chapter 2 we examine investments in technologies that can affect consumer valuation and focus on analyzing the factors that affect retailers' decisions to undertake such investments. In

Chapter 3 we study the impact of labor and traffic characteristics on store performance using proprietary data of a retail chain. In Chapter 4 we focus on general services that retailers could provide to enhance demand and examine their temporal management under competition in the face of demand uncertainty. Chapter 5 concludes the dissertation. A brief overview of each chapter of the dissertation follows.

1.2 Dissertation Overview

1.2.1 Chapter 2

With increased competition in the retail industry many retailers such as Best Buy are investing in technology, employee training, and presentation in order to improve consumers' valuation of their product offerings. Such investments in pre-purchase activities to enhance consumer valuation are costly and are designed to increase the possibility of purchase, but they do not lead to "stickiness". In particular, increases in consumer valuation through pre-purchase services are prone to free-riding since consumers receive the benefits of such activities offered by a retailer, but may decide to purchase a product at another retailer. Gateway's Country Stores and their subsequent demise provide a characteristic example of the free riding problems (Frei (2006)).

Chapter 2 investigates the factors that retailers should consider before investing in pre-purchase activities in order to increase consumer valuation and examines their effect on retailers' pricing and profits. We develop a stylized model using first principles on the distribution of consumer valuation and study a duopoly in which retailers compete on the basis of price and consumer search is allowed between the two retailers. In such a setting, a retailer may make investments to increase consumer valuation for his product, but the final sale could be made at the other retailer, who did not invest in technologies, leading to free riding. We explore the Nash equilibria in terms of both investment and pricing through a computational study. Then we focus on the pricing game only and establish the pricing Nash equilibria. We characterize the competitive effects under different regimes related to market expansion, retailers' physical proximity, direction of consumer flow, and magnitude of change in consumer valuation for two asymmetric investment structures. Next, we focus on a special case in which the competing retailers are symmetric and characterize the possible Nash equilibria investment strategies depending on the investment cost. Finally, we present a model with an endogenous level of investment and analyze the symmetric equilibrium for a symmetric duopoly.

Our main results are as follows. When the investment decision is endogenous, we establish the surprising result that in the majority of instances both retailers will decide to invest in equilibrium but price the product in a manner to avoid consumer search between them. We also find that the proximity of retailers has an interesting non-monotonic impact on their decisions to invest. Retailers tend to invest in technology when they are either very close or very far away but refrain from investing in the intermediate range. When we further focus only on the pricing game, we find two major effects related to improvements in consumer valuation. First, consistent with popular belief, we find that there is a threshold effect whereby a retailer could overcome competitive effects by improving consumer valuation. However, there are situations where a greater improvement in consumer valuation by a retailer could lead to lower profits. Second, we find evidence for a free-riding effect where a retailer who does not invest in valuation enhancement practices could benefit from an innovative competitor who increases consumer valuation beyond a threshold. When we focus on symmetric retailers we find that as the investment cost increases the Nash equilibrium strategies shift from both retailers investing, to only one retailer investing (either retailer 1 or retailer 2), and finally to neither retailer investing. Finally, for the extension where the level of investment is endogenous, we show that a symmetric duopolist's optimal strategy to cover his whole local market or part of his market depends on the effectiveness of his investment cost and the optimal price may indeed decrease with the per unit cost of acquiring the product.

1.2.2 Chapter 3

The intensified competition in the retail industry has forced retailers to place enormous importance on store performance metrics. Several retailers nowadays track two metrics conversion rate, the percentage of incoming traffic who purchased, and basket value, the average dollar amount spent by customers. Both metrics are important indicators of store performance. Conversion rate, for example, has been found to be strongly correlated with customer loyalty while basket value, on the other hand, is typically linked to the profitability of the retailer. Both conversion rate and basket value can be correlated with traffic characteristics due to many factors including labor, consumer purchase behavior, economic conditions, product availability, and merchandise assortment.

In Chapter 3, we use proprietary data pertaining to a retail chain to conduct a descriptive study of conversion rate and basket value. Specifically, we consider the correlation between store performance and intra-day traffic variability and traffic uncertainty. We also measure traffic-labor mismatches and study if they explain the observed correlations in our sample.

The results of our study are as follows: First, we report the within-store results. We find that intra-day traffic variability is negatively correlated with both conversion rate and basket value. A 1% increase in traffic variability is associated with a 0.094% decrease in conversion rate in a store and 0.037% decrease in basket value. We also find that, for a given level of traffic, both conversion rate and basket value increase with an increase in store labor at a diminishing rate. A 1% increase in labor is associated with a 0.102% increase in conversion rate and 0.066% increase in basket value. In addition, we find that conversion rates are higher during holidays but basket values are lower suggesting that price promotions offered during the holiday season cause more customers to purchase but do not lead to higher levels of purchasing. Moreover, we find that both conversion rates and basket values exhibit significant seasonality.

Next, we report the across-store results. We find that stores with higher traffic uncertainty have lower conversion rates but similar basket values. We also find that stores that have higher traffic variability and higher traffic uncertainty have higher mismatches between required labor and actual labor. Furthermore, our tests reveal that stores that have lower foot-traffic have higher traffic uncertainty, resulting in mismatches between required labor and actual labor. A surprising result of our analysis is that competition as measured here does not affect conversion rates and basket values. Finally, we find that stores located in neighborhoods with higher per capita income have higher conversion rates but similar basket values.

1.2.3 Chapter 4

In Chapter 4 we focus on understanding the temporal management of investments in activities that can enhance demand under competition in the face of demand uncertainty. In many settings, retailer investments in experience activities are important in influencing demand for a product. For example, a retailer can stimulate demand through various ways such as provide training to its sales personnel to promote a given product, create special areas to show case a product or even invest in technology that offers a unique experience for a product. All the above activities which we will be referring to as “service” can affect the purchasing decision of customers. The planning of such costly activities can be very crucial especially when a new product is being launched in a market. In the case of new product introduction, the retailer needs not only to decide the optimal price and service investments for the product but also when to invest in such experience activities. Given that market demand could be highly uncertain, a retailer may choose to wait until he receives some information regarding the market state before investing in such activities or he may want to make all these investments upfront to take advantage of possible reduced investment costs.

In this chapter we develop a two-stage model in order to examine two alternatives that retailers typically have in terms of timing their investments under both monopoly and symmetric duopoly settings. The first alternative is to invest in service in advance of the selling season without knowing the market state (i.e., invest in the first stage) and the second alternative is to invest in service after the market state realizes (i.e., invest in the second stage). In both cases a retailer decides on pricing after observing the demand (i.e., in the second stage). For the monopoly we further examine a hybrid strategy in which a retailer can invest both before and after the demand state is known. Typically, investing after the demand state is known is associated with higher investment costs. We analyze these settings under both equal and different investment costs across stages. In addition, we investigate the deterministic demand case for the symmetric duopoly and contrast our results with the stochastic demand case. In the case of a deterministic demand these alternatives translate to making sequential decisions (i.e., first service and then price) as opposed to simultaneous decisions (both service and price).

Our major findings in Chapter 4 are as follows. For a monopolist who faces stochastic demand and incurs different investment costs across stages, we show that a hybrid strategy always dominates a strategy in which a retailer invests only before or only after the market state is known. In addition, we show that a monopolist would prefer to delay investments until demand is realized only when the market variability is high and the differential cost of investments across stages is low. In all other regimes, a monopolist would prefer to invest before the demand is realized. This result is in contrast to the case of equal investment costs in which a monopolist would always defer such investments until after the demand is realized.

For a symmetric duopolist who faces deterministic demand and incurs the same investments costs across stages we show that the dominant strategy is always to make service investments in the first stage. We find that this is not always the case when the duopolist incurs higher investment costs in the second stage. Interestingly, when the intensity of competition of service is high, a symmetric duopolist could be better off to invest in the second stage as the differential cost of investment in the two stages increases. We also find computationally that the equilibrium strategies for a symmetric retailer can shift in a non-monotonic fashion as the differential cost of investment in the two stages increases. In particular, a retailer could invest in the first stage for high and low differential costs and in the second stage for intermediate values of differential costs.

For a symmetric duopolist who faces stochastic demand and incurs same costs across stages, the dominant strategy is to invest in demand enhancing activities in the first stage in all regimes except for one characterized by high demand variability, low intensity of competition in service, and high investment cost. This result shows that the competitive dynamics could significantly diminish the value of delaying investments after demand is realized. We further characterize some of the investment strategies when a duopolist incurs higher investment costs in the second stage. Interestingly, we find that in the case of high intensity of service competition increase in demand variability could make investing more preferable in the first stage than in the second stage if the differential costs of investments across stages exceeds a given threshold.

CHAPTER 2

Improving Valuation Under Consumer Search:Implications for Pricing and Profits

2.1 Introduction

The retail sector is a vital sector in most modern economies. In the U.S. for example, the retail industry represents about 40% of the economy and is the largest employer (Fisher and Raman (2001)). As a result, several researchers have examined different retail operations issues, which include inventory management and store execution (see Eppen and Iyer (1997), Tsay and Agrawal (2000), Raman et al. (2001), Fisher et al. (2006), Nagarajan and Rajagopalan (2008) for representative work).

With increased competition, retailers are trying to identify ways to differentiate themselves. One strategy has been to invest in practices that increase consumer valuation of their product offerings. Our interactions with a former top executive at Magnolia Home theater stores, specialized stores at Best Buy geared towards high end electronic home theater items, gave us insights regarding the various practices that firms employ to improve consumer valuation of their product offerings (Freeland (2007)). These practices include improving the ambience of the store to enhance the presentation of the product category as well as hiring well-trained experts as salespeople to explain the product characteristics. Investments in strategies to increase consumer valuation entail some risk because they are usually costly and may not pay off in increased sales. Another risk is that a retailer may invest in an enhanced presentation to sell a product, but the customer decides to purchase it at another retailer, who hasn't made the investment - leading to the free-riding phenomenon. Gateway's country stores provides an example of free riding. As indicated by Frei (2006), "When Gateway's new stores opened in 1996, they were undeniably impressive. Employees were experienced, helpful, and abundant (the employee-to-customer ratio was unusually high). Excellent educational materials were on hand, and the stores were conveniently located to ensure heavy foot traffic. Unfortunately, Gateway hadn't guaranteed that the people receiving the benefits of all that pre-purchase accommodation would also bear the costs. Far too often, customers took their newly acquired understanding of what they needed and how the product worked and then placed an order with one of Gateway's low-price competitors".

These observations suggest that in addition to managing inventory in stores (which often is the focus of studies in operations management) there are other aspects of store operations that determine eventual sales. Our work studies retailer initiated increases in consumer valuation, which could affect consumers' purchasing decision and the resulting impact on retailers' profitability. In particular, we focus on increases in consumer valuations that are experiential i.e., remain with the consumer even when they go to another retailer. We do not consider retailer-specific ("sticky") increases in consumer valuations such as after-purchase services and coupon offerings which are not prone to free riding. Specifically, we are interested in addressing the following questions: How do market characteristics affect the retailer's decision to invest in such practices? How does the ability of a retailer to change consumer valuation for a product affect his retail price and profit under competition? These questions address issues that retailers need to be aware of when they engage in consumer valuation enhancement activities.

To answer these questions, we first develop a stylized model using first principles on the distribution of consumer valuation. We assume that consumer valuation is uniformly distributed and an improvement is captured by a right shift of the distribution. We consider a two-stage game in a duopoly setting, where consumers could search among two retailers who offer a single product. In the first stage, the retailers decide whether to invest in improvements in customer valuation. In the second stage, given the investment decisions in the first stage, the retailers engage in price competition. We explore the Nash equilibria in terms of both investment and pricing through a computational study. Then we focus on the pricing game only and establish the pricing Nash equilibria. We characterize the competitive effects under different regimes related to market expansion, retailers' physical proximity, direction of consumer flow, and magnitude of change in consumer valuation for two asymmetric investment structures. Next, we focus on a special case in which the competing retailers are symmetric and characterize the possible Nash equilibria investment strategies depending on the investment cost. Finally, we present a model with an endogenous level of investment and analyze the symmetric equilibrium for a symmetric duopoly.

Our main results are as follows. When the investment decision is endogenous, in the majority of instances both retailers decide to invest in equilibrium but they price the product in a manner to avoid consumer search between them. We also find that the proximity of retailers has an interesting non monotonic impact on their decisions to invest in that retailers tend to invest when they are very close or very far away but refrain from investing in the intermediate range. When we further focus on only the pricing game we find two major effects related to improvements in consumer valuation. First, consistent with popular belief, we find that a retailer could overcome competitive effects by improving consumer valuation beyond a certain threshold. However, there are situations where a greater improvement in consumer valuation by a retailer could lead to lower profits. Second, we find evidence of free-riding, where a retailer who does not invest could benefit from an innovative competitor who increases consumer valuation beyond a threshold. When we focus on symmetric retailers we find that as the investment cost increases, the Nash equilibrium strategies shift from both retailers investing, to only one retailer investing (either retailer 1 or retailer 2), and finally to neither retailer investing. Finally, for the extension where the level of investment is endogenous, we show that a symmetric duopolist's optimal strategy to cover his whole local market or part of his market depends on his investment cost effectiveness and the optimal price charged by him may indeed decrease with the per unit cost of acquiring the product.

The rest of the chapter is organized as follows. Section 2 briefly reviews the relevant literature. In Section 3, we describe the two-stage game for the duopoly and provide insights on the retailers' investment decisions. Section 4 focuses on the pricing game. Section 5 examines a special case in which both retailers are symmetric. In Section 6, we present an extension to the basic model. Finally, we conclude by summarizing our findings and provide future research directions in Section 7. We present a summary of our mathematical notation in Appendix A. Proofs of all the results are in Appendix.

2.2 Literature Review

Our work relates to the retail price competition literature that has been extensively studied by research in economics, marketing, as well as operations management. Initial works in this stream focused on competition among brick-and-mortar retailers. In addition to price competition among traditional retailers, there has recently been a number of papers that focus on pricing in different contexts such as multi-channel supply chains and e-commerce. Cattani et al. (2004) and Tsay and Agrawal (2004) provide extensive reviews of this literature.

There have also been works under retail competition that have incorporated a "service" component as a decision variable for retailers in addition to pricing. Two different types of service are found in the literature: (i) service experience which can be consumed by the customer without necessarily making a purchase at the retailer who offers it (e.g., informational type of service) and (ii) service experience whose benefits are derived by making a purchase at the retailer who offers it (e.g., generous warranties, free-delivery, and installation).

Our work is related to the stream of literature that considers the first type of service. In settings where the pre-sale activities are conducted independently from the actual sale of the product, the free-riding problem occurs. The main focus of the free-riding literature has been the negative impact of free riding on the retailers' incentive to provide costly pre-sale service and the way that retailers and manufacturers could prevent free riding (see Carlton and Perloff (2000), Carlton and Chevalier (2001)). From this literature the papers by Bernstein et al. (2006) and Shin (2007) are related to our work.

Bernstein et al. (2006) also explore the idea that consumer valuation could be increased by making appropriate investments. These authors consider manufacturer-retailer competition and are interested in how free-riding affects a manufacturer's decision to open a direct store. In contrast, we focus on retailer competition and are interested in understanding - (1) how the market characteristics, customer search behavior, and magnitude of change in consumer valuation could affect retailers' investment decisions; (2) how free-riding and profit-loss to competition are affected by the above factors. Bernstein et al. (2006) find that the direct store price is always higher than the retail price as long as visiting the direct store increases consumer valuation. Our computational study, on the other hand, shows cases where the retailer who invests to increase consumer valuation but has a smaller market share could offer a lower price relative to the retailer who does not make such an investment, particularly when the proportion of searching consumers is high.

Shin (2007) considers two retailers selling the same product competing for the same customers who are heterogenous in terms of their opportunity costs for shopping and can be of two types: informed and uninformed. The author focuses on sales assistance that resolves the matching uncertainty between the retailers' product specification and consumers' needs. In our work, we also consider two retailers selling the same product but competing for customers who are heterogenous in terms of their valuation for the retailers' product. We focus on retailers' investments which can increase customer valuation of their product and model explicitly the impact of such investments on the consumers' valuation distribution.

Our work contributes to the existing literature in the following aspects. First, we develop a model that allows us to study the impact of increasing consumer valuation. Second, we provide managerial insights on how the market characteristics and consumer search behavior could impact the retailers' investment strategies. Third, we provide insights on how the ability of retailers to influence consumer valuation impacts competitive and free riding effects under different regimes related to market expansion, retailers' physical proximity, consumer search behavior, and magnitude of changes in consumer valuation.

2.3 Model

This section is organized as follows. In §2.3.1 we present a model of how consumer valuation increases. Then, in §2.3.2 we present our duopoly setting and describe the consumer search behavior. In §2.3.3 we describe the two-stage investment and pricing model. Finally, we provide some computational insights in §2.3.4.

2.3.1 Modeling of Consumer Valuation Increase

We first consider a risk neutral firm (retailer 1) which offers one product in a market of population size μ. Consumers are heterogeneous in the valuation of the product. We denote the consumption value (alternatively called "willingness to pay") by V which is assumed to be uniformly distributed (across the population of consumers) between 0 and 1. The retailer offers the product at price p_1 and incurs cost per unit c_1 for acquiring the product. Consumers who visit the retailer incur a cost k_1. "This cost can include the opportunity cost of time, the real cost of travel, and the implicit cost of inconvenience" (Balasubramanian (1998)). We will be referring to k_1 as traveling cost. Therefore, consumers whose valuation is greater than or equal to the sum of retail price p_1 and cost k_1 ($p_1 + k_1$) buy the product from retailer 1. In that case, the retailer's expected demand and profit are $q_1^N = \mu P(V \geq p_1 + k_1) = \mu(1 - p_1 - k_1)$ and $\pi_1^N = (p_1 - c_1)q_1^N$ (superscript N refers to the scenario in which the retailer does not invest to increase consumer valuation). We provide a summary of all the scenarios in the Appendix (see Table A2). The retailer maximizes his expected profit with respect to his retail price. The resulting optimal price, demand, and profit are $p_1^{*N} = \frac{1+c_1-k_1}{2}$, $q_1^{*N} = \frac{(1-c_1-k_1)\mu}{2}$, and $\pi_1^{*N} = \frac{(1-c_1-k_1)^2\mu}{4}$, where $0 \leq c_1 + k_1 \leq 1$.

If the retailer engages in consumer-valuation-enhancing activities, a consumer whose valuation was v prior to visiting the retailer enjoys a valuation $\hat{v}$ after visiting the retailer. In the basic model, we assume that the investment leads to a fixed linear shift α (i.e., $\hat{V}$ will be uniformly distributed between α and $1+\alpha$ as illustrated in Figure 1). In order to accomplish these activities, we assume that the retailer incurs a fixed cost I_1. This assumption represents situations where fixed investments lead to a given increase in consumer valuation[1]. Note that a shift in the consumer valuation distribution leads to an increase in the average consumer willingness to pay for the product. As a result, for a price p_1 the demand before investment is $q_1^N = \mu(1-p_1-k_1)$ while the demand after investment is $q_1^I = \mu P(\hat{V} \geq p_1+k_1) = \mu(1+\alpha-p_1-k_1)$, which is identical to the demand expression obtained when offering a price discount of α per unit. Hence, a linear shift model of consumer valuation may seem similar to a price discount model. However, there are critical differences. First, a linear shift model of consumer valuation and a price discount model[2] lead to different profit functions which will result in different optimal pricing, demands, and profits. Second, and more importantly, a price discount creates loyalty to the retailer who runs the promotion whereas increasing consumer valuation for a product does not guarantee that the consumer would make the purchase from the retailer who exerts such efforts.

[1] We study an extension of this basic model in Section 6 in which α is endogenous and the investment cost is dependent on α.

[2] In a price discount model the profit function is $\pi_1 = (p_1 - \alpha - c_1)\mu(1+\alpha-p_1-k_1)$ whereas in our model the corresponding profit function is $\pi_1 = (p_1 - c_1)\mu(1+\alpha-p_1-k_1) - I_1$.

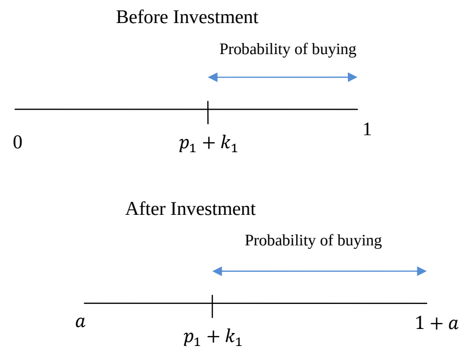


FIGURE 2.1: Consumer valuation before and after the investment.

The objective for the retailer is to maximize his expected profit $\pi_1^I = (p_1 - c_1)q_1^I$- I_1, by deciding on the price under the constraint that $\alpha \leq p_1 + k_1 \leq 1 + \alpha$. The optimal price, demand, and profit as functions of the mean shift α are $p_1^{*I} = \frac{1+\alpha+c_1-k_1}{2}$, $q_1^{*I} = \frac{(1+\alpha-c_1-k_1)\mu}{2}$, and $\pi_1^{*I} = \frac{(1+\alpha-c_1-k_1)^2\mu}{4} - I_1$, where $0 \leq \alpha \leq 1 + c_1 + k_1$.

2.3.2 Duopoly and Search

We now consider two retailers in the market (retailer 1 and retailer 2) who offer the same product at prices p_1 and p_2 and incur per unit costs of acquiring the product c_1 and c_2 respectively. Such differences in retailers' costs may be a reflection of buying power as well as their operational performance. Both retailers can invest to increase consumer valuation by incurring investment costs I_1 and I_2. We assume that the change in consumer valuation is identical for both retailers but the investment costs of the two retailers are different to allow for heterogeneity in the retailers' investment effectiveness. We further assume that the two retailers are located at some distance from each other and that the introduction of retailer 2 may bring additional people in to the market (Mahajan et al. (1993), Huang and Swaminathan (2009)). We consider an additive expansion in the market and denote by ϵ the additional people in the market (i.e., the new market size will be $\mu + \epsilon$, where $\epsilon \geq 0$).

We assume that γ proportion of the total population is located near retailer 1 and as a result this proportion visits retailer 1 first. The remaining (1-γ) visits retailer 2 first. Consumers who visit retailer 1 and retailer 2 incur traveling costs k_1 and k_2 respectively. The search occurs as follows (see Figure 2.2). A consumer visits her local retailer first. She buys the product from that retailer provided that she obtains non-negative consumer surplus i.e., $v \geq p_i + k_i$. If the consumer surplus is negative she does not buy the product at her local retailer. A fraction of such consumers δ $(0 \leq \delta \leq 1)$ who have visited their local retailer and have not obtained positive consumer surplus would be willing to search with updated consumer valuation the other retailer before leaving the system. The cost that a consumer incurs traveling from one retailer to the other is denoted by Δk. Similar to Lal and Sarvary (1999), when consumers visit their local retailer i first they incur a traveling cost k_i associated with the cost of undertaking the shopping trip, and when consumers go from one retailer to the other, they incur a traveling cost Δk of visiting an additional retailer. Note that we are not modeling the case where a consumer visits the other retailer after having visited her local retailer, seeking price-matching guarantee refunds. Though price matching is common in the retail industry, research shows that on average only about 5 to 7.4 percent of consumers seek price matching guarantee refunds (Moorthy and Winter (2004), Kukar-Kinney (2005)). We also do not consider the situation where, after visiting the second store a consumer returns to the original store to make the eventual purchase. This is a

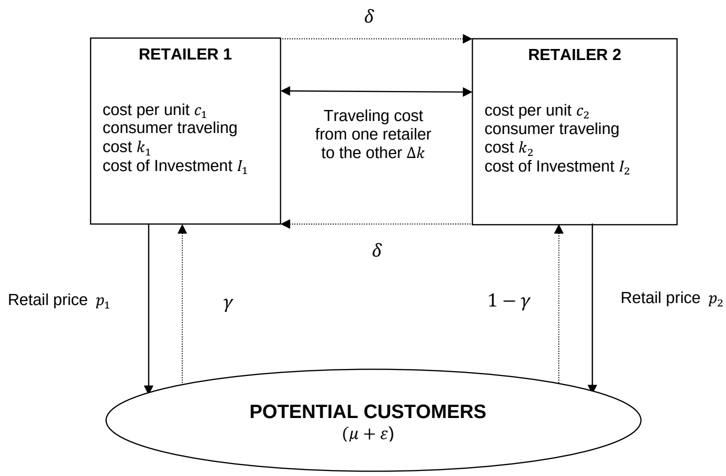


FIGURE 2.2: Duopoly.

reasonable assumption when the price differential between the stores is not significant. Our search scheme is close to that of Anupindi and Bassok (1999) and Ahn et al. (2002). In Anupindi and Bassok (1999) consumers buy the product from their local retailer. In the event of a stock-out a fraction of the unsatisfied customers looks for the product at another retailer. Since we assume that the demand is deterministic so there is no stock-out, consumers buy the product from their local retailer provided that they obtain positive consumer surplus. A fraction of consumers who have visited their local retailer and have not obtained positive consumer surplus will continue their search with updated consumer valuation. In Ahn et al. (2002) a manufacturer-owned store (outlet) and an independent retail store are located in different markets. Each consumer makes the initial attempt to purchase at the store that is closer. All consumers who have visited the independent retail store and did not obtain positive consumer surplus travel to the manufacturer-owned store (i.e., one way movement of consumers). Our search scheme generalizes the search scheme of Ahn et al. (2002) since: (i) consumers incur a cost when they visit their local retailer (in their case such cost is zero), (ii) consumers can search both retailers (in their case there is one way flow) and (iii) only a proportion of consumers who encounters non-positive consumer surplus searches both retailers, while the rest leave the system without making a purchase (in their case this proportion is one). In addition, our work is differentiated from the above search related literature in the incorporation of updated consumer valuations.

2.3.3 Investment Pricing Game

We consider two retailers engaging in an investment pricing game with the following sequence of events.

1. The retailers simultaneously decide whether to invest in improvements in customer valuation.

2. Based on their investment decision in event 1, the retailers simultaneously determine their prices.

The above events constitute a two-stage game. In the first stage the retailers make investment decisions to improve customer valuation. These decisions can lead to the following possible investment scenarios.

(i) Neither retailer invests (denoted as (NI,NI)).

(ii) Retailer 1 does not invest but retailer 2 invests (denoted as (NI,I)).

(iii) Retailer 1 invests but retailer 2 does not invest (denoted as (I,NI)).

(iv) Both retailers invest (denoted as (I,I)).

In the second stage given the investment decisions made at the first stage, the retailers decide on their prices. Let $x_i \in \{0,1\}$ denote retailer i's decision to invest or not in improving customer valuation. Then, retailer i's problem in stage 1 is given by

$$\max_{x_i \in \{0,1\}} \pi_i(x_i, x_j, p_i, p_j) = (p_i - c_i) q_i(x_i, x_j, p_i, p_j) - x_i I_i$$

where

$$\begin{aligned} q_i(x_i, x_j, p_i, p_j) &= (\mu+\epsilon)\gamma(x_i(1+\alpha-p_i-k_i) + (1-x_i)(1-p_i-k_i)) \\ &+ (1-\gamma)\delta(\mu+\epsilon)(1-x_i)(1-x_j)(p_j-p_i-\Delta k)^+ \\ &+ (1-\gamma)\delta(\mu+\epsilon)(1-x_i)x_j(p_j-p_i-\Delta k)^+ \\ &+ (1-\gamma)\delta(\mu+\epsilon)x_i(1-x_j)(p_j+\alpha-p_i-\Delta k)^+ \\ &+ (1-\gamma)\delta(\mu+\epsilon)x_i x_j(p_j-p_i-\Delta k)^+ \end{aligned}$$

The first term of the demand expression (i.e., $(\mu+\epsilon)\gamma x_i(1+\alpha-p_i-k_i)$) denotes the demand from local consumers if retailer i invests. The second term (i.e., $(\mu+\epsilon)\gamma(1-x_i)(1-p_i-k_i)$) denotes the demand from local consumers if retailer i does not invest. The remaining terms of the demand expression denote the demand from searching consumers for the above four possible investment strategies (i)-(iv) chosen by the retailers respectively.

We use backwards induction to solve this two stage-game. For each investment scenario ((i)-(iv)) we need to solve the second-stage problem and identify the pricing Nash equilibrium. In order to solve the second stage for a given investment scenario we distinguish four theoretically possible cases (because of the $(\cdot)^+$ operator) that correspond to the following four possible consumer search schemes depending on the retailers' prices:

(1) Consumers do not search (denoted as (NS,NS)).

(2) Consumers search from retailer 2 to retailer 1 (denoted as (NS,S)).

(3) Consumers search from retailer 1 to retailer 2 (denoted as (S,NS)).

(4) Consumers search both retailers (denoted as (S,S)).

Specifically, for investment scenarios (ii) and (iii) we need to consider all four cases (1)-(4) in the second stage whereas for investment scenarios (i) and (iv) we only need to consider cases (1)-(3) as shown in Proposition 1. All proofs are in Appendix.

Proposition 1 *In investment scenarios (ii) and (iii) both retailers can obtain sales from the consumers who search between retailers. In investment scenarios (i) and (iv) only one retailer can obtain sales from the consumers who search.*

Hence, there are 14 possible cases in the second stage of the game that need to be analyzed. However, we can indeed show that for a given combination of investment and consumer search scheme, there exists a unique equilibrium in the pricing game.

Proposition 2 *In the two-retailer pricing game a unique Nash equilibrium exists for every combination of investment scenario and consumer search scheme.*

Deriving the two-stage game Nash equilibrium $(x_i^*, x_j^*, p_i^*, p_j^*)$ for an arbitrary set of parameters is analytically intractable. Therefore, we first explore the Nash equilibria in terms of both investment and prices through a computational study and examine the effects of changing different parameters such as γ, δ, Δk, and different types of costs on the Nash equilibria. Then, in the following section, we focus on the pricing game only and theoretically characterize the pricing Nash equilibria for a given investment scenario and consumer search scheme.

2.3.4 Computational Insights

We now describe our computational study and some of the interesting insights that we obtained for the pricing and investment game. We used a full factorial design experiment summarized in Table A3 in Appendix. We identify the Nash equilibria in terms of the investment decisions of the two retailers in the two-stage game as follows: We start backwards to solve the second stage of the game first. For each of the four possible investment scenarios ((i)-(iv)), we compute the pricing Nash equilibria of the retailers assuming a given consumer search scheme ((1)-(4)). For each instance of a given investment scenario, we identify the consumer search scheme that is a Nash equilibrium (i.e., the consumer search scheme that is being induced by the Nash equilibrium pricing strategy of the retailers for a given investment scenario). We refer to this consumer search scheme as search Nash equilibrium. During our computational study, we found that there are instances that lead to two search Nash equilibria that cannot be ranked because one retailer is better off with one consumer search scheme and the other retailer is better off with the other search scheme. We did not consider such instances in the first stage of the game where the retailers make a decision on whether to invest or not. Since the payoffs for both retailers for each investment scenario are known from the second stage of the game, we can identify the Nash equilibrium investment strategy for the retailers.

Tables 2.1 and 2.2 provide a summary on the types of Nash equilibria investment strategies as well as the Nash equilibria search strategies which result from the retailers' pricing decisions. Table 2.3 summarizes the frequency of occurrence in our experimental setup of each consumer search scheme for a given investment equilibrium scenario.

TABLE 2.1: Characterization of investment Nash equilibria.

Type of strategies	(NI,NI)	(NI,I)	(I,NI)	(I,I)	Two Nash equilibria
%	16.143%	22.125%	27.697%	33.16%	0.874%

TABLE 2.2: Characterization of search Nash equilibria.

Type of strategies	(NS,NS)	(NS,S)	(S,NS)	(S,S)	Two Nash equilibria
%	30.948%	27.752%	25.239%	15.187%	0.874%

TABLE 2.3: Investment and Search Nash equilibria.

	(NS,NS)	(NS,S)	(S,NS)	(S,S)
(NI,NI)	57.699%	23.012%	19.289%	NA
(NI,I)	8.519%	40.37%	22.716%	28.395%
(I,NI)	13.609%	20.513%	33.728%	32.15%
(I,I)	48.188%	28.418%	23.394%	NA

Here are some interesting observations with respect to the impact of the retailers' investments on the consumer search scheme.

(1) We notice that the most prevalent Nash equilibrium investment strategy corresponds to both retailers investing. There is a very small proportion of instances (0.874%) that lead to two pure investment Nash equilibria (see Table 2.1). Interestingly, the majority of instances lead to retailers pricing in such a manner so that there is no search to either retailer (see Table 2.2).

(2) When both or none of the retailers invest, then they price in such a manner so as to discourage consumer search. Specifically, 48.188% and 57.699% of situations where the resulting Nash equilibria is such that both retailers invest or neither retailer invests, respectively, lead to no search between the retailers. This result is interesting particularly in the case of both investing, because it enables the retailers to avoid price competition due to search.

(3) In a number of cases, consumers may decide to search a retailer who invests. For example, Table 2.3 shows that 20.513% of the cases where retailer 1 invests lead to consumers searching that retailer and 22.716% of the cases where retailer 2 invests lead to consumers searching that retailer. In those cases, it is quite possible that the retailer who has made the investment decides to price lower than the other retailer, and this effect is more profound when the percentage of consumers willing to search increases. Such an example is illustrated in Table 2.4 with parameter values $\gamma = 0.2$, $\epsilon = 0.4$, $\mu = 1$, $c_1 = c_2 = 0.3$, $k_1 = k_2 = 0.1$, $\Delta k = 0.05$, $I_1 = 0.02$, $I_2 = 0.1$, and $\alpha = 0.2$. Since $I_1 << I_2$ only retailer 1 has an incentive to invest. Note that retailer 1 is at a disadvantage in terms of market share since $\gamma = 0.2$. At low values of δ, retailer 1's benefit from consumers who search is low, thus, his incentive to attract consumers from retailer 2 is low. Hence, retailer 1 prices higher than retailer 2. But as the proportion of consumers who are willing to search increases, retailer 1's benefit from the consumers who search is high. As a result, retailer 1 is willing to drop his price and prices lower than retailer 2 (even though retailer 1 is the one who has invested).

We further examined the impact of the different parameters on the investment

TABLE 2.4: An example of the impact of δ on the pricing Nash equilibrium.

δ	Investment NE	Search NE	p_1	p_2	q_1	q_2	π_1	π_2
0.2	(I,NI)	(NS,S)	0.622	0.6	0.162	0.336	0.032	0.101
0.5	(I,NI)	(NS,S)	0.583	0.6	0.238	0.336	0.047	0.101
0.8	(I,NI)	(NS,S)	0.567	0.6	0.314	0.336	0.064	0.101

decisions of the retailers. Here are some of our key insights.

Effect of Proximity: The proximity between the two retailers Δk has an interesting non-monotonic behavior on the retailers' investment decisions under certain conditions. Specifically, we find that at low and high values of Δk both retailers have an incentive to invest, yet for intermediate values of Δk, only one of the retailers has an incentive to invest. Table 2.5 illustrates such an example with parameter values $\gamma = 0.8$, $\delta = 0.8$, $\epsilon = 0.4$, $\mu = 1$, $c_1 = c_2 = 0.4$, $k_1 = k_2 = 0.3$, $I_1 = I_2 = 0.02$, and $\alpha = 0.5$.

TABLE 2.5: An example of the impact of Δk on the investment Nash equilibria.

Δk	Investment NE	Search NE	p_1	p_2	q_1	q_2	π_1	π_2
0.02	(I,I)	(S,NS)	0.8	0.64	0.448	0.282	0.159	0.048
0.1	(I,NI)	(S,S)	0.778	0.542	0.509	0.177	0.173	0.034
0.2	(I,I)	(NS,NS)	0.8	0.8	0.448	0.112	0.159	0.025

This can be explained because for low values of Δk, the retailers' competition is very intense which creates an incentive for both retailers to invest. For high values of Δk, each retailer is actually acting as a monopolist who can make independent decisions that do not have an impact on the other retailer. For the intermediate values of Δk, the competition is not so intense and does not justify the retailer who is at a disadvantage in terms of market share to commit to an investment decision. This insight suggests that a retailer should reciprocate investment decisions of a very close competitor but could ignore the investments made by a retailer who is further away.

Effect of Searching Consumers: In order to better understand the effects of the percentage of consumers willing to search δ, we considered three other values $\delta = 0.1, 0.5, 0.9$ (see example in Table 2.6 with the following parameter values: $\gamma = 0.2$, $\epsilon = 0.8$, $\mu = 1$, $c_1 = c_2 = 0.3$, $k_1 = k_2 = 0.1$, $\Delta k = 0.05$, $I_1 = I_2 = 0.02$, and $\alpha = 0.2$).

TABLE 2.6: An example of the impact of δ on the investment Nash equilibria.

δ	Investment NE	Search NE	p_1	p_2	q_1	q_2	π_1	π_2
0.1	(I,I)	(NS,S)	0.636	0.7	0.169	0.576	0.037	0.210
0.5	(NI,I)	(NS,S)	0.517	0.7	0.234	0.576	0.051	0.210
0.9	(NI,I)	(NS,S)	0.502	0.7	0.335	0.576	0.068	0.210

Note that in this case $\gamma = 0.2$, so retailer 1 is at a disadvantage with respect to the market share, and hence, needs to price lower in order to attract more consumers. As a result, consumers always search retailer 1 at equilibrium. Therefore, for low values of search (δ) both retailers invest at equilibrium. But as the proportion of search population (δ) increases, retailer 1 does not have an incentive to invest anymore and is better off free-riding from retailer 2. Therefore, only retailer 2 invests at equilibrium and consumers search from retailer 2 to retailer 1. We find a similar symmetric effect when $\gamma = 0.8$ since retailer 2 is at a disadvantage in that case with respect to the market share.

Effect of Market Share, Total Market, and Costs: As expected, market share γ plays a critical role in the retailer's decision to invest. As γ increases, retailer 1 is more likely to invest and retailer 2 is less likely to make such an investment. The impact of ϵ and μ on the retailers' investment decisions is very intuitive. As ϵ and/or μ

increase, the retailers have a higher incentive to invest because the market size increases. An increase in the marginal cost c_i results in retailer i having less incentive to invest. Similarly, an increase in search cost k_i forces the retailer to drop his price to make up for such an increase which subsequently creates disincentives for him to invest. Finally, as the investment cost increases, the retailers have less incentive to make an investment in increasing consumer valuation.

2.4 Special Case: Pricing Game

In the previous section, we computationally explored the Nash equilibria in terms of investment and pricing. We now focus on the pricing game between the retailers for a given consumer search scheme assuming that the investment game has been played and an equilibrium has been reached. Although we have analyzed the competitive effects for most of the investment scenarios we focus our discussion on two asymmetric investment scenarios (ii) and (iii) (described in subsection 3.3) which create free-riding opportunities for one of the retailers. Note that in our computational study in the previous section, we find that almost 50% of the situations lead to investment scenarios (ii) and (iii) as the resulting Nash equilibria in terms of investment and pricing. In addition, we focus our discussion on consumer search scheme (4) as it is the most general search scheme. We defer the presentation of consumer search schemes (1)-(3) in Appendix.

2.4.1 Benefiting from Innovative Competition

In this subsection, we study how the profits and prices of a retailer (who does not invest to improve consumer valuation) under monopoly compare to the case where there is another retailer in the market (retailer 2) who has decided to invest to improve consumer valuation. We denote this duopoly scenario as $NISS$ where the first two letters in the scenario acronym refer to the retailers investment decisions (N for "not invest", I for "invest") and the remaining letters refer to the consumers search scheme (N for "not search", S for "search"). The expressions of expected demands and profits for the retailers are:

$$q_1^{NISS} = (\mu+\epsilon)\gamma(1-p_1-k_1) + (1-\gamma)\delta(\mu+\epsilon)(p_2-p_1-\Delta k) \tag{2.1}$$

$$q_2^{NISS} = (\mu+\epsilon)(1-\gamma)(1+\alpha-p_2-k_2) + \gamma\delta(\mu+\epsilon)(p_1-p_2-\Delta k+\alpha) \tag{2.2}$$

$$\pi_1^{NISS} = (p_1-c_1)q_1^{NISS} \tag{2.3}$$

$$\pi_2^{NISS} = (p_2-c_2)q_2^{NISS} - I_2 \tag{2.4}$$

where p_1 and p_2 need to satisfy the following constraints which ensure nonnegative demands: $0 \leq p_1 + k_1 \leq 1$, $\alpha \leq p_2 + k_2 \leq 1+\alpha$, $\alpha \leq p_2 + k_1 + \Delta k \leq 1+\alpha$, $\alpha \leq p_1 + k_2 + \Delta k \leq 1 + \alpha$, $p_2 - p_1 - \Delta k > 0$, and $p_1 - p_2 - \Delta k + \alpha > 0$. The impact of competition on retailer 1's price, demand, and profit is illustrated in Table A6 in Appendix.

Proposition 3 *Let π_1^{*NISS} be the optimal profit of retailer 1 under duopoly, π_1^{*N} be the* *optimal profit under monopoly, and $\bar{\alpha}^{(1)}$ be a threshold in the consumer valuation mean shift (defined in Table A5 in Appendix). Then,*

a) if $\alpha < \bar{\alpha}^{(1)}$ then $\pi_1^{*N} > \pi_1^{*NISS}$ and

b) if $\alpha > \bar{\alpha}^{(1)}$ then $\pi_1^{*N} < \pi_1^{*NISS}$.

Proposition 3 summarizes the competitive effects on retailer 1's profit. Specifically, there exists a threshold in the consumer valuation mean shift $\bar{\alpha}^{(1)}$ such that if the mean shift is low ($\alpha < \bar{\alpha}^{(1)}$), the introduction of retailer 2 leads to retailer 1 losing profit to competition ($\pi_1^{*N} > \pi_1^{*NISS}$); if the mean shift is high ($\alpha > \bar{\alpha}^{(1)}$), retailer 2 creates a positive externality and retailer 1 free rides ($\pi_1^{*N} < \pi_1^{*NISS}$). The impact on retailer 1's profit due to retailer 2's investments depends on how many consumers search and what is their valuation level. When retailer 2 invests in a high level of improvement in consumer valuation ($\alpha > \bar{\alpha}^{(1)}$), he is likely to price higher which leads to a higher flow of searching consumers with higher valuations for the product at retailer 1. Therefore, the profits of retailer 1 under duopoly are higher than those under monopoly.

Note that retailer 1 can still benefit under competition even when retailer 2 does not bring any market expansion, provided that retailer 2 has made a sufficient investment to increase consumer valuation. Figure 3 illustrates such an example with parameter values $\gamma = 0.5$, $\delta = 0.5$, $\epsilon = 0$, $\mu = 3$, $c_1 = c_2 = 0.5$, $k_1 = k_2 = 0.3$, $\Delta k = 0.1$.

Proposition 4 *Let $\bar{\alpha}_1$, $\bar{\alpha}_2$, and $\bar{\alpha}^{(1)}$ be thresholds in the consumer valuation mean shift (i.e., the values of α defined in Table A5 in Appendix that equate retailer 1's prices, demands, and profits respectively under duopoly and monopoly regimes). Then,*

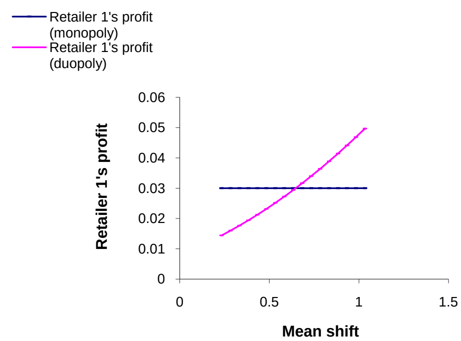


FIGURE 2.3: Retailer 1's profit versus mean shift in scenario NISS.

(i) $\frac{\partial \bar{\alpha}_1}{\partial \Delta k} > 0,$

(ii) $\frac{\partial \bar{\alpha}_2}{\partial \Delta k} > 0,$ *and*

(iii) $\frac{\partial \bar{\alpha}^{(1)}}{\partial \Delta k} > 0.$

Proposition 4 shows that as Δk increases (retailers are located further away), it will require higher levels of improvements in consumer valuation so that retailer 1 could have at least the same price, demand, and profit under duopoly as under monopoly.

If it is indeed true that a competitor could benefit from an innovative retailer's investment, then how could the innovative retailer protect himself? In this context it is interesting what Magnolia Home theater stores are doing. Magnolia Home theater stores are a successful "arm" of Best Buy and contributed total sales of $46 million during the quarter that ended in November 27, 2006 (Stock (2006)). Magnolia has placed tremendous emphasis on practices that increase consumer valuation of their product offerings. Such practices include (1) offering premium brands in a demonstration environment where consumers can actually try out the equipment and (2) employing knowledgeable sales professionals who can interact one to one with the home theater enthusiasts to guide them through the home theater experience. Magnolia investments to increase consumer valuation are significant which could allow its competitors to benefit/free-ride based on the results of this section. To mitigate this effect, Magnolia offers many unique assortments that are not available at other stores (Freeland (2007)). For example, in the television section, they are currently offering a special series of Samsung products like Samsung PN58A760 58" Class 1080p Flat-Panel Plasma HDTV that are unique to Magnolia stores. As a result, a customer with an increased valuation cannot purchase the same product at another store. Hence, one strategy for an innovative retailer to mitigate free riding could be to offer different assortments which is not captured in our single product model.

2.4.2 Implications of Competition for an Innovative Retailer

In this subsection, we study how the profits and prices for an innovative retailer who has decided to invest to improve consumer valuation under monopoly compare to the case where there is another retailer in the market (retailer 2) who does not invest. In that case the expressions of expected demands and profits for the two retailers are

$$q_1^{INSS} = (\mu+\epsilon)\gamma(1+\alpha-p_1-k_1)+(1-\gamma)\delta(\mu+\epsilon)(p_2+\alpha-p_1-\Delta k) \quad (2.5)$$

$$q_2^{INSS} = (\mu+\epsilon)(1-\gamma)(1-p_2-k_2)+\gamma\delta(\mu+\epsilon)(p_1-p_2-\Delta k) \quad (2.6)$$

$$\pi_1^{INSS} = (p_1-c_1)q_1^{INSS} - I_1 \quad (2.7)$$

$$\pi_2^{INSS} = (p_2-c_2)q_2^{INSS} \quad (2.8)$$

where p_1 and p_2 need to satisfy the following constraints: $\alpha \leq p_1 + k_1 \leq 1 + \alpha$, $0 \leq p_2 + k_2 \leq 1$, $\alpha \leq p_2 + k_1 + \Delta k \leq 1 + \alpha$, $\alpha \leq p_1 + k_2 + \Delta k \leq 1 + \alpha$, $p_2 + \alpha - p_1 - \Delta k > 0$, and $p_1 - p_2 - \Delta k > 0$. Table A8 in Appendix summarizes the impact of competition on retailer 1's price, demand, and profit. Figure 2.4 summarizes the impact of competition on retailer 1's profit according to different regimes related to market expansion, retailers' physical proximity, and magnitude of change in consumer valuation.

Proposition 5 *Let π_1^{*INSS} be the optimal profit of retailer 1 under duopoly, π_1^{*I} be the optimal profit of retailer 1 under monopoly, $\bar{\Delta k}$ be a threshold of the retailers' physical proximity, $\bar{\alpha}^{(2)}$, $\bar{\alpha_3}$, and $\bar{\alpha}$ be thresholds of the consumer valuation mean shifts and $\bar{\epsilon}$ be a threshold of the market expansion (defined in Table A7 in Appendix). Then,*

i a) if $\epsilon > \bar{\epsilon}$, $\Delta k > \bar{\Delta k}$, and $\alpha < \bar{\alpha}^{(2)}$, then $\pi_1^{*I} > \pi_1^{*INSS}$.

i b) if $\epsilon > \bar{\epsilon}$, $\Delta k > \bar{\Delta k}$, and $\alpha > \bar{\alpha}^{(2)}$, then $\pi_1^{*I} < \pi_1^{*INSS}$.

ii) if $\Delta k > \bar{\Delta k}$ and $\alpha < \bar{\alpha} = \min\{\bar{\alpha}^{(2)}, \bar{\alpha_3}\}$, then $\pi_1^{*I} > \pi_1^{*INSS}$.

Proposition 5 provides the following insights:

i) When the market expansion is high and the retailers' proximity is low, there exists a threshold in the consumer valuation mean shift $\bar{\alpha}^{(2)}$ such that if the mean shift is low ($\alpha < \bar{\alpha}^{(2)}$) the introduction of retailer 2 leads to retailer 1 obtaining lower profit ($\pi_1^{*I} > \pi_1^{*INSS}$). Otherwise, if the mean shift is high ($\alpha > \bar{\alpha}^{(2)}$) retailer 1's profit is higher under duopoly ($\pi_1^{*I} < \pi_1^{*INSS}$). Hence, retailer 1 can avoid profit loss to competition by making a sufficient improvement in consumer valuation.

ii) In a low proximity regime, a low improvement in consumer valuation ($\alpha < \bar{\alpha}$) leads to retailer 1 obtaining lower profit.

The remaining regions in Figure 2.4 cannot be characterized as clearly. We performed a computational study in order to gain more insights about the remaining regimes. The computational study leads to an interesting observation. For example, for the same regime we can obtain qualitatively different outcomes. Figure 2.5 demon-

High Market Expansion $(\varepsilon > \bar{\varepsilon})$

High Proximity		Low Proximity	
$\alpha < \bar{a}_3$	$\alpha > \bar{a}_3$	$\alpha < \bar{a}^{(2)}$	$\alpha > \bar{a}^{(2)}$
Not Determined	Higher Profit	Lower Profit	Higher Profit

Low Market Expansion $(\varepsilon < \bar{\varepsilon})$

High Proximity			Low Proximity	
$\alpha < min$	$min < \alpha < max$	$\alpha > max$	$\alpha < \bar{a}_3$	$\alpha > \bar{a}_3$
Not Fully Determined			Lower Profit	Not Determined

FIGURE 2.4: The impact on retailer 1's profit based on the analytical results in scenario INSS.

Note $\min \equiv \min\{\bar{\alpha}_3, \bar{\alpha}_4\}$ and $\max \equiv \max\{\bar{\alpha}_3, \bar{\alpha}_4\}$

strates an example of a low market and low proximity regime in which competition can impact retailer 1 in different ways. The parameter values for the left frame are: $\gamma = 0.45$, $\delta = 0.8$, $\epsilon = 0.02$, $\mu = 3$, $c_1 = 0.5$, $c_2 = 0.7$, $k_1 = 0.3$, $k_2 = 0.1$, $\Delta k = 0.01$, $I_1 = 0.05$ and for the right frame are: $\gamma = 0.45$, $\delta = 0.8$, $\epsilon = 0.02$, $\mu = 1$, $c_1 = 0.4$, $c_2 = 0.6$, $k_1 = 0.4$, $k_2 = 0.2$, $\Delta k = 0.02$, $I_1 = 0.05$. Note that for this example the value of $\bar{\alpha}_3$ (i.e., the value of α at which the price of retailer 1 under monopoly and under duopoly are equal) happens to be negative. In the right frame, higher levels of improvement in consumer valuation by retailer 1 lead to higher profits for that retailer.

Interestingly, in the left frame lower levels of improvement in consumer valuation by retailer 1 lead to higher profits for that retailer. The differential role of the magnitude of improvement in consumer valuation in these two instances can be explained by looking at the rate of change of profits with respect to the magnitude of improvements under monopoly and duopoly regimes. Specifically, the following expression $\frac{\partial \pi_1^{*INSS}}{\partial \alpha} - \frac{\partial \pi_1^{*I}}{\partial \alpha}$ can give some insights regarding this difference in behavior. The above difference can be expanded as follows:

$\frac{\partial \pi_1^{*INSS}}{\partial \alpha} - \frac{\partial \pi_1^{*I}}{\partial \alpha} = (\frac{\partial p_1^{*INSS}}{\partial \alpha} q_1^{*INSS} - \frac{\partial p_1^{*I}}{\partial \alpha} q_1^{*I}) + (\frac{\partial q_1^{*INSS}}{\partial \alpha} p_1^{*INSS} - \frac{\partial q_1^{*I}}{\partial \alpha} p_1^{*I}) - (c_1(\frac{\partial q_1^{*INSS}}{\partial \alpha} - \frac{\partial q_1^{*I}}{\partial \alpha}))$.

Note that the second term $(\frac{\partial q_1^{*INSS}}{\partial \alpha} p_1^{*INSS} - \frac{\partial q_1^{*I}}{\partial \alpha} p_1^{*I})$ and third term $(c_1(\frac{\partial q_1^{*INSS}}{\partial \alpha} - \frac{\partial q_1^{*I}}{\partial \alpha}))$ of this expression are respectively negative and positive for both instances whereas the first term $(\frac{\partial p_1^{*INSS}}{\partial \alpha} q_1^{*INSS} - \frac{\partial p_1^{*I}}{\partial \alpha} q_1^{*I})$ is positive in the right frame and negative in the left frame. Also note that the price of retailer 1 under duopoly increases at a higher rate than his corresponding price under monopoly $(\frac{\partial p_1^{*INSS}}{\partial \alpha} > \frac{\partial p_1^{*I}}{\partial \alpha})$. Since the market size in the left frame is higher than the market size in the right frame, retailer 1 gets much higher demand in the left frame than in the right frame which leads to the first term being negative in the left frame and positive in the right frame. Subsequently, this difference in the signs of the first term leads to the difference in the signs of the following expression $\frac{\partial \pi_1^{*INSS}}{\partial \alpha} - \frac{\partial \pi_1^{*I}}{\partial \alpha}$. The managerial implication of this result is that a higher market size can enable a firm to extract significantly more from consumer valuation increases under monopoly than under competition. In such a situation if $\alpha > \bar{\alpha}^{(2)}$ the firm gets hurt from competition.

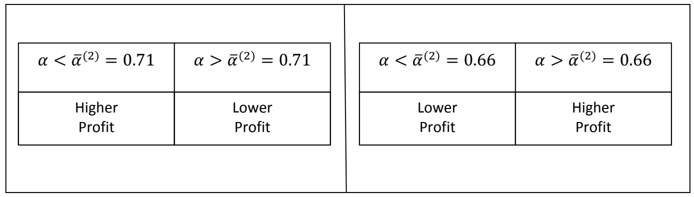


FIGURE 2.5: Impact of competition in a low proximity and low market regime.

2.5 Special Case: Symmetric Retailers

In this section, we focus on symmetric retailers and examine the investment Nash equilibrium. As we discussed in §2.3.3 asymmetry between retailers greatly confounds the comparison of the profit functions of the retailers across different investment scenarios. Hence, in this section we seek to obtain further insights by considering symmetric retailers. Note that for the two symmetric investment scenarios ((i) and (iv)) discussed in §2.3.3 since retailers are symmetric in all aspects including market share they will price identically. It is easy to show that for these two investment scenarios there is no demand from consumer search between the retailers. As a result, the retailers obtain demand only through their local consumers. For the two asymmetric investment scenarios ((ii) and (iii)) we will consider the most general case where both retailers obtain demand from searching consumers. Let π_i^{*j} denote the optimal payoff of retailer i for investment scenario j, where $i = \{1, 2\}$ and $j = \{(NNNN), (NISS), (INSS), (IINN)\}$. Recall that the first two letters in the scenario acronym refer to the retailers investment decisions (N for "not invest", I for "invest") and the remaining letters refer to the consumers search scheme (N for "not search", S for "search"). We provide the expressions of the optimal profits for each retailer for all the investment scenarios as well as the feasible range of the parameters in Appendix.

Note that since the retailers are symmetric they engage in a symmetric game where the payoffs for playing a particular strategy depend only on the other strategies employed, not on who is playing them. This fact simplifies the analysis to identify the Nash equilibrium strategies. We next characterize the investment Nash equilibrium strategies for different ranges of investment cost I.

Proposition 6 *Let A_1 and A_2 be two thresholds for investment cost I defined in Appendix. Then,*

a) if $0 < I < A_1$, the unique pure investment Nash equilibrium is both retailers to invest.

b) if $\max\{0, A_1\} < I < A_2$, the investment Nash equilibrium strategies are either retailer 1 or retailer 2 to invest

c) if $I > \max\{0, A_2\}$, the unique pure investment Nash equilibrium is neither retailer to invest.

Proposition 6 characterizes the two retailers investment strategies as a function of their investment cost. If the investment cost is very low (i.e., $0 < I < A_1$) then the optimal strategy for both symmetric retailers is to invest. If the investment cost is very high (i.e., $I > \max\{0, A_2\}$) then the optimal strategy for the two retailers is not to invest. For the intermediate values of investment cost retailer 1 investing but retailer 2 not investing or retailer 1 not investing but retailer 2 investing are two pure Nash equilibrium strategies.

The above holds provided that $A_2 > A_1 > 0$. Finding an ordering between A_1 and

A_2 is analytically challenging. As a result, we resorted to a computational study. We performed a full factorial analysis with parameters values described in Appendix (see Table A4) and found that for all instances considered (1728 in total) $A_2 > A_1 > 0$. This suggests that as the investment cost increases, the Nash equilibrium strategies shift from both retailers investing, to only one retailer investing (either 1 or 2), and finally to neither retailer investing.

2.6 A Variant With Endogenous Mean Shift

The analysis so far has assumed that α is exogenously given. We now explore a variant of the basic model in which each retailer is in a position to decide on the optimal mean shift and subsequent investment level. Our attempts to incorporate endogenous investment levels in the asymmetric duopoly led to very cumbersome expressions that were not amenable to analysis. Hence, we only analyzed the symmetric duopoly and focused on the equilibrium where both retailers invest.

We consider two symmetric retailers in the market who compete on investments and prices. We assume that when each retailer invests to increase consumer valuation, he incurs an investment cost $\lambda\alpha^2$, where λ corresponds to the investment cost factor of that retailer. The parameter λ captures each retailer's cost effectiveness in operational deployment of consumer-valuation-enhancing activities (i.e., a high value of λ denotes a low-cost effective retailer). In addition, we consider that the investment cost increases quadratically in α to capture diminishing returns of increases in consumer valuation.

The intuition behind the quadratic investment cost representation is as follows: In order for a retailer to achieve a small translation of the mean of consumer valuation to the right, some minimum services can be sufficient, and the resulting investment cost is low. But when a retailer targets higher shifts of the mean valuation, he needs to provide a greater level of services that will increase the cost significantly. This behavior suggests a convex type of investment cost. Similar approaches to modeling service effort, which can be broadly defined as any demand-enhancing effort activity, have been used in the literature (e.g., Tsay and Agrawal (2000)).

We only consider symmetric equilibria (i.e., the optimal mean shifts and prices for the two retailers are identical). As a result, there will be no demand from searching consumers to either retailer, and each retailer decides on prices and investment levels to serve his local consumers. Hence, each retailer acts as a monopolist in his own local market. Note that for this section we will be using for simplicity the scenario acronym (II) to refer to $(IINN)$.

The expected demand for the product is given by $q^{II} = \hat{\mu} P(\hat{V} \geq p + k) = \hat{\mu}(1 + \alpha - p - k)$, where $\hat{\mu} = \frac{\mu+\epsilon}{2}$. The objective for each retailer is to maximize his expected profit $\pi^{II} = (p-c)q^{II}$-$\lambda\alpha^2$, by deciding on the price under the constraint $\alpha \leq p + k \leq 1 + \alpha$. The optimal price, demand, and profit as functions of the mean shift α are $p^{*II}(\alpha) = \frac{1+\alpha+c-k}{2}$, $q^{*II}(\alpha) = \frac{(1+\alpha-c-k)\hat{\mu}}{2}$, and $\pi^{*II}(\alpha) = \frac{(1+\alpha-c-k)^2\hat{\mu}}{4} - \lambda\alpha^2$, where $0 \leq \alpha \leq 1 + c + k$. For the case in which $p + k < \alpha$, each retailer can increase his price such that $p+k = \alpha$, and he will be able to serve his whole local market ($\hat{\mu} = \frac{\mu+\epsilon}{2}$). Thus, when $p + k < \alpha$ the expected demand for the product $q^{II} = \hat{\mu}$, and the optimal price, demand, and profit as functions of the mean shift α are $p^{*II}(\alpha) = \alpha - k, \quad q^{*II} = \hat{\mu}$, and $\pi^{*II}(\alpha) = (\alpha - c - k)\hat{\mu} - \lambda\alpha^2$, where $\alpha \geq 1 + c + k$.

The following proposition details how this strategy depends on the value of each retailer's investment cost factor λ. Let α^{*IEIE} be the optimal value of the mean shift. The superscript $IEIE$ refers to a duopoly scenario in which the level of investment is endogenous (I for "invest", E for "endogenous").

Proposition 7 *The optimal value of the mean shift α^{*IEIE} depends on the value of each retailer's investment cost factor λ as follows:*

a) if $\lambda < \frac{\hat{\mu}}{2(1+c+k)}, \quad \alpha^{*IEIE} = \frac{\hat{\mu}}{2\lambda}$

b) if $\lambda > \frac{\hat{\mu}}{2(1+c+k)}, \quad \alpha^{*IEIE} = \frac{(1-c-k)\hat{\mu}}{4\lambda - \hat{\mu}}$.

Proposition 7 illustrates the existence of a threshold in the investment cost factor that determines the optimal mean shift of the consumer valuation distribution. If the investment cost factor is low ($\lambda < \frac{\hat{\mu}}{2(1+c+k)}$), each retailer can price low, which allows him to serve his whole local market $\hat{\mu}$, whereas if the investment cost factor is high ($\lambda > \frac{\hat{\mu}}{2(1+c+k)}$), each retailer serves only a part of his local market.

Table 2.7 presents a complete characterization of the optimal decisions of each retailer depending on the value of the investment cost factor and comparative statics. As anticipated, when the market is partially covered, an increase in the per unit cost of acquiring the product, traveling cost, and investment cost factor leads to a decrease in the optimal mean shift, demand, and profit; whereas an increase in the market size leads to an increase in the optimal mean shift, price, demand, and profit. When a retailer serves the whole market, several of the above parameters have no impact on the corresponding optimal mean shift, price, and demand.

TABLE 2.7: Impact of λ on a retailer's strategy and comparative statics.

Best retailer strategy	Serve part of his local market				Serve his whole local market			
	$\lambda \geq \frac{\hat{\mu}}{2(1+c+k)}$				$0 < \lambda \leq \frac{\hat{\mu}}{2(1+c+k)}$			
mean shift α^{*IEIE}	$\frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}}$				$\frac{\hat{\mu}}{2\lambda}$			
price p^{*IEIE}	$c+\frac{2(1-c-k)\lambda}{4\lambda-\hat{\mu}}$				$\frac{\hat{\mu}}{2\lambda}-k$			
demand q^{*IEIE}	$\frac{2(1-c-k)\lambda\hat{\mu}}{4\lambda-\hat{\mu}}$				$\hat{\mu}$			
profit π^{*IEIE}	$\frac{(1-c-k)^2\lambda\hat{\mu}}{4\lambda-\hat{\mu}}$				$\frac{\hat{\mu}(\hat{\mu}-4\lambda(c+k))}{4\lambda}$			
comparative statics	c	k	$\hat{\mu}$	λ	c	k	$\hat{\mu}$	λ
mean shift α^{*IEIE}	-	-	+	-	n/a	n/a	+	-
price p^{*IEIE}	+ /-	-	+	-	n/a	-	+	-
demand q^{*IEIE}	-	-	+	-	n/a	n/a	+	n/a
profit π^{*IEIE}	-	-	+	-	-	-	+	-

One of the important counterintuitive insights in Table 2.7 relates to how the optimal price changes with the per unit cost of acquiring the product when each retailer serves part of his local market. We find that when $\lambda > \max\{\frac{\mu}{2(1+c+k)}, \frac{\hat{\mu}}{2}\} = \frac{\hat{\mu}}{2}$, then the per unit cost of acquiring the product has an intuitive impact on the price. The price increases when the per unit cost of acquiring the product increases. However, when $\frac{\hat{\mu}}{2(1+c+k)} < \lambda < \frac{\hat{\mu}}{2}$, the price surprisingly *decreases* when the per unit cost of acquiring the product increases. We can explain this counterintuitive result from the expression of the optimal price $p^{*IEIE} = \frac{1+\alpha^{*IE}+c-k}{2}$, where $\alpha^{*IEIE} = \frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}}$. The impact of the per unit cost of acquiring the product on the optimal price is as follows: $\frac{\partial p^{*IEIE}}{\partial c} = \frac{\partial \alpha^{*IEIE}}{\partial c}\frac{1}{2}+\frac{1}{2}$, where $\frac{\partial \alpha^{*IEIE}}{\partial c} = -\frac{\hat{\mu}}{4\lambda_1-\hat{\mu}} < 0$. Note that while the per unit cost of acquiring the product increases price directly, it decreases the optimal mean shift as well. This decrease in the optimal mean shift leads to a decrease in price. Hence, the per unit cost of acquiring the product affects the optimal price directly and also indirectly through the optimal mean shift. As a result, the net impact of the per unit cost of acquiring the product on the optimal pricing depends on the relative magnitude of these two effects. When $\lambda = \frac{\hat{\mu}}{2}$ the two effects cancel out (i.e., the rate of decrease of the optimal mean shift is equal to the rate of increase of the per unit cost of acquiring the product) resulting in $\frac{\partial p^{*IEIE}}{\partial c} = 0$. When $\lambda > \frac{\hat{\mu}}{2}$, each retailer's cost effectiveness decreases, which results in a decrease in the optimal mean shift. As the per unit cost of acquiring the product increases, the rate of decrease of the optimal mean shift is lower than the rate of increase of the per unit cost of acquiring the product, which leads to an eventual price increase ($\frac{\partial p^{*IEIE}}{\partial c} > 0$). But when $\lambda < \frac{\hat{\mu}}{2}$, each retailer's cost effectiveness increases, which results in an increase in the optimal mean shift. As the per unit cost of acquiring the product increases, the rate of decrease of the optimal mean shift is now higher than the rate of increase of the per unit cost of acquiring the product, which leads to a price decrease ($\frac{\partial p^{*IEIE}}{\partial c} < 0$).

2.7 Conclusion

As retail competition becomes more intense with numerous entrants in the market, several retailers have engaged in practices that aim to increase consumer valuation about their product offerings. Our primary objective has been to explore the effect of such practices. To enable detailed analysis, we have developed a stylized model for changes in consumer valuation and how it affects the eventual purchase decision.

Our analysis has yielded a number of findings. Under duopoly, when the investment decision is endogenous, surprisingly in the majority of instances both retailers decide to invest in equilibrium but price the product in a manner to avoid consumer search between them. We also find that the proximity of retailers has an interesting non monotonic impact on their decisions to invest in that retailers tend to invest when they are very close or very far away but refrain from investing in the intermediate range. When we further focus on the pricing game only we find two major effects related to improvements in consumer valuation. First, consistent with popular belief, we find that a retailer could overcome competitive effects by improving consumer valuation beyond a certain threshold. However, there are situations where a greater improvement in consumer valuation by a retailer could lead to lower profits. Second, we find that a retailer who does not invest could benefit from an innovative competitor who increases consumer valuation beyond a threshold. When we focus on symmetric retailers we find that as the investment cost increases the Nash equilibrium strategies shift from both retailers investing, to only one retailer investing, and finally to neither retailer investing. Finally, for the extension where the level of investment is endogenous, we show that a symmetric duopolist's optimal strategy to cover his whole local market or part of his market depends on his investment cost effectiveness and the optimal price charged by him may indeed decrease with the per unit cost of acquiring the product.

Our study has two implications for practitioners. First, our model suggests that an equilibrium strategy could be both firms to invest in valuation enhancing practices but price in such a way to prevent focal customer base buying from the competitor. The above strategy could work as a protective mechanism against free-riding. Such strategy complements practices that are currently observed in the retail industry including the offering of unique assortments. Second, retailers need to take into account the physical proximity to their competitor and the proportion of consumers who are willing to search for better deals in the market in addition to the costs associated with such investments prior to deciding whether to engage or not in such practices.

Our model is highly stylized however it provides insights into the success that Magnolia Home theater stores have achieved by offering customer valuation enhancement activities for high-margin products targeting an affluent customer base. Our analysis also provides some insights into Gateway's Country Stores demise. Gateway started as one of the first widely successful direct sales PC companies targeting initially price-sensitive consumers. Hoping to grow in a different market and attract top management and engineers, Gateway opened a chain of retail stores called Gateway Country Stores and made significant investments in customer valuation enhancement activities without having established first the customer base who would be willing to bear the costs associated with these activities.

Although our model and its analysis has provided several interesting insights related to increasing consumer valuations, it has some limitations. To focus our attention on the effects of competition we use a deterministic demand that is more amenable to analysis of competition. Embedding stochastic demand in a duopoly setting has been found to dramatically complicate the analysis of even models that do not include increased consumer valuation (cf. Tsay and Agrawal (2000)).

In the asymmetric duopoly, we did not consider the retailers' decision of how much to invest to increase consumer valuation but explored such a decision only in the symmetric duopoly focusing on the symmetric equilibrium. Our preliminary attempts to incorporate endogenous investment levels in the asymmetric duopoly led to very cumbersome expressions that were not amenable to analysis or deeper managerial insights.

There are several research avenues that could be explored in the future. A possible extension is to consider the mixed impact of the practices employed by the retailers to increase consumer valuation on heterogeneous populations. For instance, providing enhanced informational services could reveal some positive and unique aspects of the product and subsequently could increase their willingness to pay for some consumers. On the other hand, enhanced informational services may also help some consumers identify that the product is not a good fit for them.

In our current model, we consider that consumers visit their local retailers first. If consumers do not obtain positive consumer surplus at their local retailer, they visit the competing retailer. Our model captures one of the possible consumer search behaviors but does not consider price sensitive consumers (i.e., consumers who search among retailers and buy from the retailer who maximizes their utility). Embedding price sensitive consumers in our model could be another direction to pursue in the future.

Another possible extension of our model relates to studying the supply chain implications of increased consumer valuation. In this area one could explore how increased consumer valuation provided by manufacturer-owned stores could impact supply chain performance, as well as each party's performance and study coordination mechanisms.

Although the above extensions are all interesting and relevant, the associated analysis is sufficiently complicated and different from the models included in the current chapter.

CHAPTER 3

The Impact of Labor and Traffic on Store Performance

3.1 Introduction

In a bricks-and-mortar channel the last mile of a customer's purchase occurs in the retail store. In the store, customers may decide whether to purchase or not and also how much to spend. Retailers track conversion rate, the percentage of incoming traffic who purchased, and basket value, the average dollar amount spent by customers, to measure their store performance and place enormous importance on these metrics. Conversion rate, for example, has been found to be strongly correlated with customer loyalty[1]. Basket value, on the other hand, would be linked to the profitability of the retailer.

While the difference in conversion rates and basket values across retailers are to be expected, we find that both these metrics exhibit considerable heterogeneity across stores as well as time. For example, in our proprietary data that pertains to an apparel

[1]Pat Conroy, vice chairman and national principal, consumer business practice at Deloitte and Touche states that "Customer conversion–it's the single most important thing that a retailer can do to sustain long-term growth" (Zaino (2007))

retailer, we find that conversion rate varies between 2% and 45.8% longitudinally and basket value varies between \$2.78 and \$365.83. Also, average conversion rate across stores vary between 9% and 18.9% and basket values vary between \$72.4 and \$127.9. Such a wide variation in these performance metrics is surprising given that most retailers tend to have uniform policies across their stores. Further, as Figures 3.3 and 3.4 show conversion rate and basket value are independent measures of store performance both within and across stores. Hence a study of the systematic factors that explain the longitudinal and cross-sectional variation in these variables would be useful to explain store performance.

Both conversion rate and basket value can be correlated with traffic characteristics. Figure 3.5 plots conversion rate intra-day traffic variability for one of the stores in the sample. Figure 3.6 plots average basket value across stores against traffic uncertainty. We see strong correlations between the pairs of variables in each Figure. Such strong correlations could be the result of many factors including labor, consumer purchase behavior, economy, product availability, and merchandise assortment.

In this chapter, we conduct a descriptive study of conversion rate and basket value for a retailer. Specifically, we consider the correlation between store performance and intra-day traffic variability and traffic uncertainty. We also measure traffic-labor mismatches and study if they explain the observed correlations in our sample.

There are two main reasons why we focus on conversion rate and basket value instead of profitability. First, the above two metrics provide additional information that a manager could use to better utilize his current resources. Information on average spending and customer conversion could allow the store manager to direct his resources as needed in order not to leave money on the table and increase the customer's average spending. Second, conversion rate could be a more appropriate metric of bench-marking store performance as opposed to profitability or sales. For instance, sales and profitability could be higher in store A than store B of the same chain simply because store A is located in an area that attracts more traffic than store B. But store B could have a higher conversion rate which indicates that it is a better performing store since it utilizes its existing resources and potential in a more efficient manner.

We report the following results in this chapter. First, we report the within-store results. We find that intra-day traffic variability is negatively correlated with both conversion rate and basket value. A 1% increase in traffic variability is associated with a 0.094% decrease in conversion rate in a store and 0.037% decrease in basket value. We also find that, for a given level of traffic, both conversion rate and basket value increase with an increase in store labor at a diminishing rate. A 1% increase in labor is associated with a 0.102% increase in conversion rate and 0.066% increase in basket value. In addition, we find that conversion rates are higher during holidays but basket values are lower suggesting that price promotions offered during the holiday season cause more customers to purchase but do not increase the size of the average customer purchase. Moreover, we find that both conversion rates and basket values exhibit significant seasonality.

Next, we report the across-store results. We find that stores with higher traffic uncertainty have lower conversion rates but similar basket values. We also find that stores that have higher traffic variability and higher traffic uncertainty have higher mismatches between required labor and actual labor. Furthermore, our tests reveal that stores that have lower foot-traffic have higher traffic uncertainty resulting in mismatches between required labor and actual labor. A surprising result of our analysis is that competition does not affect conversion rates and basket values. This suggests that the consumers' decision on whether or not to purchase and how much to purchase is unaffected by the presence of other competitors once they are in the store. Finally, we find that stores located in neighborhoods with higher per capita income have higher conversion rates but similar basket values.

Our research is closest to Fisher et al. (2009) who conduct a cross-sectional study to show that labor scheduling and execution significantly explain differences in basket value for a retailer. Our results are consistent with Fisher et al. (2009) in the sense that we find that mismatches between labor and traffic are correlated with lower basket values across stores. Furthermore, our use of data on hourly traffic, sales, and labor is novel and allows us to differentiate variability in traffic within a day and variability across days and show their association with the mismatches between traffic and labor. In addition, our research setting allows us to study within-store analysis of basket value, as well as conversion rates, a metric that has not been studied in operations management so far. Our study is also the first to study aggregate volatility and show its association with negative store performance.

Our study has two implications for practitioners. First, many retailers plan labor based on sales and traffic. Hence, stores that have higher sales and traffic tend to have more labor. However, our study shows that stores with lower sales and traffic tend to have higher volatility. This suggests that planning labor based on average traffic may be misleading and traffic volatility needs to be taken into account. Even though some part of traffic volatility is uncontrollable retailers need to handle volatilities that are under their control either using increased labor or through more flexible labor. Second, retailers need to analyze their actions to determine if they are resulting in increased traffic volatility and seek ways to address them. While several studies have considered the impact of retail actions such as advertising, price promotions etc., we do not know of any study that considered the impact of such actions on traffic volatility. For example, several retailers have "early bird specials" that would cause traffic variability to increase substantially. Hence, our study shows that it is important to coordinate actions that drive store traffic with those that help manage the potential increase in volatility.

The remaining of the chapter is organized as follows: Section 2 reviews relevant literature. Section 3 presents the store performance framework that we use. In section 4 we present our hypotheses. Sections 5 and 6 describe the data and the econometric model respectively. In Section 7 we discuss our results. Section 8 concludes the chapter.

3.2 Literature Review

Our work falls into the stream of literature that examines factors influencing store performance. Several factors have been studied in the literature including store-, market-, and consumer characteristics. We focus on the role of labor and traffic characteristics and review only papers that examine those factors. An interested reader could refer to Reinartz and Kumar (1999) for a brief literature review on different store performance studies.

Few papers in the retail operations literature empirically examine the role of labor capacity in retail store performance. Hise et al. (1983) focus on explaining cross-sectional variations in store-performance of a retail chain using survey data and find that among other factors the number of employees is statistically significant in explaining the financial performance of a store. Ton and Huckman (2005) examine the impact of employee turnover on store performance and show that the negative effect of turnover on performance is most pronounced in stores that have low level of process conformance. Fisher et al. (2006) analyze the drivers of a retail store's financial performance and find that store staffing levels have a significant impact on customer satisfaction and sales. Another study by Fisher et al. (2009) focuses on the impact of labor planning and execution practices on the financial performance of a retail location. They show that mismatches between the planned employee staffing level and the actual employee staffing level have negative impact on the financial performance of a store. Ton (2008) studies the effect of labor levels on retail store profitability through its impact on quality defined in terms of service and conformance. In the setting that she studies she finds that increasing labor has a significant effect on profitability through its impact on conformance quality but not its impact on service quality.

Our work is different from the above papers in the following aspect. To the best of our knowledge, our study is the first one to examine empirically the impact of labor levels on store conversion effects. The only empirical research to date that has studied conversion effects at the store level is the work by Lam et al. (2001) who study the effectiveness of marketing activities (i.e., promotions) on store performance including attraction, conversion, and spending effects. In contrast, our focus is to examine the impact of labor capacity as measured by labor hours as well as traffic on store performance as measured by conversion rate, basket value, and sales.

Our work contributes to existing literature that focuses on retail sales force planning based on store traffic forecasting. A representative work of this stream of research is the paper by Lam et al. (1998) which proposes a model that links store sales potential with store traffic volume, customer type, and customer response to sales force availability. Our research demonstrates that traffic variability and uncertainty in addition to average store traffic volume have an effect on store performance. This result has important operational implications in planning and scheduling sales force. According to our findings, a retailer while making his store labor force planning may need to take into account not only factors identified by previous literature (e.g., Lam et al. (1998)) such as average store traffic volume, customer type, and customer response to sales force availability but also traffic variability and uncertainty. To the best of our knowledge, our research is the first attempt in the operations literature that uses traffic data to empirically examine the effect of traffic variability and uncertainty on store performance and its potential implications on labor planning and scheduling.

3.3 Store Performance Framework and Factors Influencing Store Performance

Figure 3.1 presents a modification of the framework suggested by Lam et al. (2001) for analyzing store performance. This framework breaks down store sales into three components: store traffic, conversion rate, and basket value. Such a partitioning of sales can be very useful and can lead to a better understanding of the different factors that affect sales. Our focus will be on three store performance measures (sales, conversion rate, and basket value). Conversion rate and basket value are related to the effectiveness of store related activities in converting potential customers into buyers and encouraging them to spend more. In addition to store related factors that are under the control of a retailer, there are several other factors beyond his control that can affect store performance.

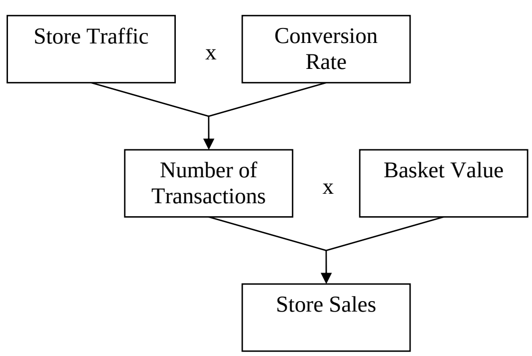


FIGURE 3.1: A Framework for Analyzing Store Performance

Lam et al. (2001) have suggested an organizing framework for studying the impact of different controllable and uncontrollable factors on store performance. We borrow his framework and classify the different factors based on time dimension into time-variant and time-invariant. Figure 3.2 illustrates the framework we use for our analysis. Note that consumer demographics and competitive conditions can actually vary with time. We assume that the above factors are time invariant for the period of our study.

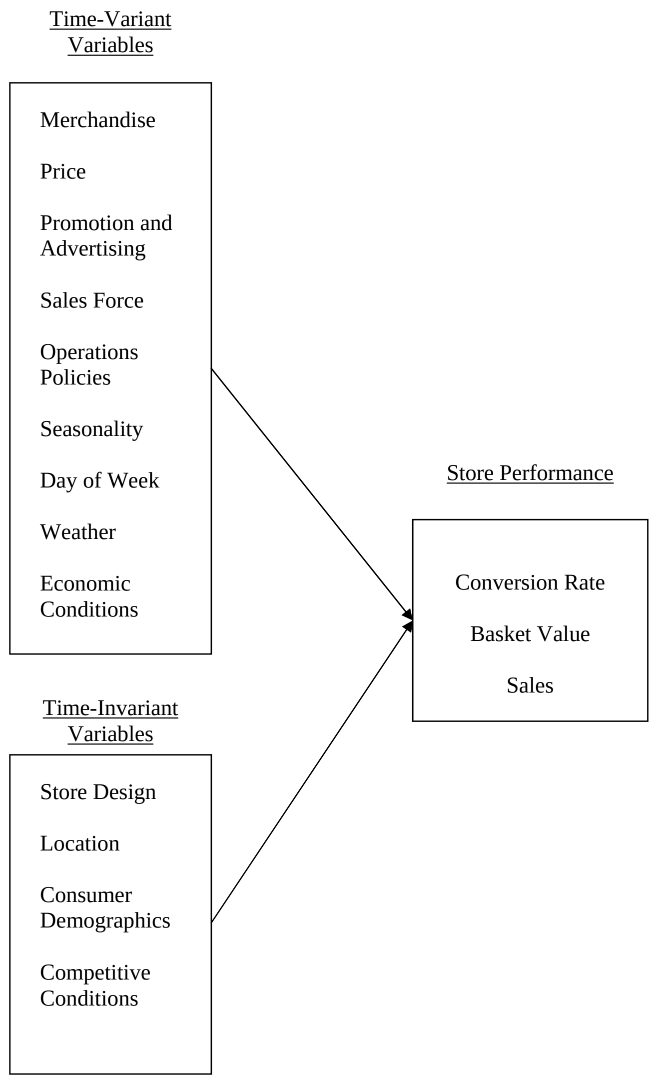


FIGURE 3.2: Factors Influencing Store Performance

Lam et al. (2001) examine two additional store performance measures (i.e., front traffic and store entry ratio) since they are interested in the impact of marketing activities on attraction in addition to conversion and spending effects. We do not treat traffic as a performance measure but focus on understanding the impact of traffic variability and traffic uncertainty on store performance. Traffic variability corresponds to intra-day traffic variability and traffic uncertainty is proxied by inter-day traffic variability. More information regarding the above traffic characteristics is provided in the data description.

In our analysis, we control for all the factors that affect store performance presented in Figure 3.2 besides merchandise and price for which we unfortunately have no data. In addition, since traffic variability could be correlated with days of the week we do not use days of the week as controls in our analysis.

3.4 Hypotheses

In this section, we develop hypotheses regarding the impact of labor and traffic on store performance. Our hypotheses are motivated based on practice as well as literature.

Impact of Staffing Levels. The linkage between staffing levels and store performance has been highlighted by several researchers who have identified mechanisms through which the former affects the latter. For example, Fisher et al. (2006) show that more labor at retail stores is associated with higher customer satisfaction and higher sales. Ton (2008) examines the effect of labor on profitability through its impact on quality. She finds that increasing labor is associated with an increase in profitability through its impact on conformance quality. Both studies provide empirical support that higher labor is associated with higher service quality.

The relationship between service quality and performance has been established by the literature on customer satisfaction. They find that higher service quality leads to higher customer satisfaction which then increases customer loyalty resulting in better performance (Heskett et al. (1994)). For instance, Babakus et al. (2004) find that both service and merchandise quality exert significant impact on store performance–measured by sales growth and customer growth–and this impact is mediated by customer satisfaction. Sulek et al. (1995) find empirical support that customer satisfaction has a positive impact on the sales per labor hour for a retail chain of 46 stores.

In the context of retailing, the research stream on customer satisfaction usually considers sales force as a vital driver of the sales process (and subsequently customer conversion and spending) by providing information to the customers about the product characteristics, features, prices, and brands, by helping customers identify the right product for their needs and locate the product in the store as well as by making the sales transaction process more efficient. Hence, as the number of sales force increases each employee will have more time to interact with the customer and assist him/her in identifying his/her needs with the offerings of the store (Wernerfelt (1994)). As a result, the customer is more likely to buy at the store. Moreover, customers who are interested in multiple items can make more purchases at the store if they find sufficient assistance. In addition, more time spent with customers can provide more opportunities to the sales force to engage in cross-selling and up-selling, and subsequently increasing the amount that customers eventually spend. Building on the above arguments we hypothesize the following:

Hypothesis 1 *Increase in labor is associated with higher store performance.*

Impact of Traffic Characteristics.

Traffic uncertainty affects store performance in two ways. First, increase in traffic uncertainty would lead to increase in demand uncertainty for individual items. Hence, for a given level of inventory, as traffic uncertainty increases we would expect more stockouts when the demand is very high and we would expect unused inventory that needs to be discounted to move when the demand is low.

Second, increase in traffic uncertainty would lead to greater difficulty in planning store labor. Retailers often use sales and traffic as input to derive their staffing levels. Based on the staffing levels and available labor, they schedule employees in their stores. For a given staffing level, increase in traffic uncertainty would lead to greater mismatches between required store labor (to manage in-store customers) and actual store labor. When the required store labor exceeds actual store labor, the customer service within the store would decline resulting in fewer customer purchases as well as decline in customer's basket value. This leads to the following hypothesis.

Hypothesis 2 *Increase in traffic uncertainty is associated with lower store performance.*

For a given level of daily traffic, we expect store performance to decline as the intra-day traffic variability increases due to the following reason. Labor scheduling is a complex function that requires matching the supply of available store labor with the traffic demand. Store labor usually comprises of full-time employees, part-time employees, and temporary workers. These employees may be available at different times of the day for different durations as opposed to a standard eight-hour work schedule. Furthermore, there are additional complications such as different skill sets of the employees, minimum staffing requirements, overtime and wages, budget constraints, vacations, leaves, etc. that need to be taken into account when scheduling employees. Therefore, as the variability in intra-day traffic increases, it will become more difficult for the retailer to schedule daily labor for different hours of the day resulting in over- and under-staffing at different hours of the day. Finally, Fisher et al. (2009) find that greater mismatches between store traffic and planned staffing levels result in lower financial performance across stores. Therefore, we hypothesize the following:

Hypothesis 3 *Increase in intra-day traffic variability is associated with lower store performance.*

Based on the above discussion, mismatches between labor and traffic seem to be one of the mechanisms through which traffic uncertainty and variability could affect store performance. We are interested in testing out that mechanism and see whether it indeed holds. To this end, we propose the following hypothesis:

Hypothesis 4 *Increase in traffic uncertainty and traffic variability are associated with* *higher mismatches between labor and traffic.*

Control Variables

In this section, we have presented hypotheses regarding the effect of labor as well as the effect of traffic variability and uncertainty on store performance. We have also hypothesized a mechanism called labor mismatches through which traffic variability and uncertainty affect store performance. We define labor mismatches as the deviation between required staffing levels to manage in store operations based on actual traffic and actual staffing levels. Note that the company could enforce certain labor rules regarding the staffing levels that need to be maintained which could be different from the staffing levels that are required to serve in store customers. In addition to the above explanatory variables, we also control for multiple variables that may impact store performance according to the framework presented in Figure 3.2. These include weather, seasonality, consumer demographics, competitive conditions, economic conditions, location, store design, and promotions.

3.5 Data Description

Our study requires store-level data which is not publicly available. Our store-level data pertains to a large retail chain provided under conditions of nondisclosure and anonymity. We will be referring to this retail chain as "Alpha" in this chapter.

"Alpha" is a women's apparel retail chain. Retailer "Alpha" as of July 2008 operated 212 stores in 35 states of the United States, Puerto Rico, the United States Virgin

Islands, and Canada. The stores are located primarily in regional shopping centers and in freestanding street locations. The study period was from January 1, 2007 to December 31, 2007. The retail chain used counters to record store traffic. As of 2007 the retailer had installed traffic counters in only 60 stores located in the United States. We obtained three types of data for retailer “Alpha”: financial data (i.e., retail transactions in units and store sales in dollars), labor data (i.e., employee hours), and traffic data for the year of 2007. The above data were provided to us in both hourly level as well as daily level. We also obtained regular business hours for each store by calling the stores and requesting this information from a sales associate.

In addition to the above data, we hand-collected data by accessing online the website of the mall where each store is located and recording stores in the mall directory that sold women’s apparel and targeted the same age group as retailer “Alpha”. These stores served as a proxy of competition. Out of 60 stores, 5 stores were located in freestanding street locations and 4 stores were located in malls that did not have a working website. Moreover, there were stores for which we did not have any observations for the whole year of 2007. Those store either opened during that year or they did not install traffic counters at the beginning of the year. To overcome this problem, we focused only on those stores for which we could obtain complete information with respect to the above variables. After we removed outliers in our data set our sample size dropped to 41 stores.

We also collected data on the daily average temperature of each store location. This data was obtained from the United States National Climatic Data Center in

Asheville, NC. The center archives data from the National Oceanic and Atmospheric Administration, a scientific agency within the United States Department of Commerce focused on the conditions of the oceans and the atmosphere. Climate-Data Inventories are accessible online and the website provides different search capabilities for locating weather stations including city, zip-code, state, and county. Each weather station has archived data on certain aspects of weather covering a specific time period. We identified weather stations searching by zip-code. There were 5 zip-codes for which we were not able to identify a weather station and had to use the closest station within 20 miles to that zip-code.

To control for economic conditions, we collected data on the Dow Jones Industrial Average (DJI) using the Wharton Research Data Services (WRDS) as well as data on the Standard & Poor Retail Index (RLX) using Thomson Datastream. We used the Dow Jones Industrial Average to do our analysis and the Standard & Poor Retail Index to validate the robustness of our results. For the days for which the stock market is closed and there are no data on the DJI or RLX, we used a five-day moving average to obtain estimates. In addition, for the RLX we used the previous quarter's data.

We also obtained demographic data for the population in each store location using U.S. Census data. We collected information on median household income and per capita income. The above variables were highly correlated and hence, we only used per capita income in our analysis.

Among other factors that affect store performance (see Figure 3.2) are time-invariant store characteristics such as location and store design. We control for such factors using store fixed effects.

Store performance is also affected by store promotions. We could not obtain any information regarding retailer "Alpha's" promotional activities. However, we control for major holidays in which retailers typically run promotions. We provide more information about the holidays we consider when we give a brief description of each variable.

Although we possessed data on the hourly level we performed our core analysis on the daily level. We used the hourly level to obtain one of the variables for the analysis as explained later in the section. The reason why we chose to work with the daily level was that we found that the financial and labor data for the daily and aggregated hourly level did not always match. However, this was not the case with the traffic information. We believe that the reason for this discrepancy is that the hourly level data is collected on the basis of hours the store is open. Sometimes store managers will get in early and run returns and employee sales transactions prior to the store opening hour. This would lead to lower sales and transactions in the hourly aggregated data when compared to the daily data.

To test our hypotheses it is necessary to estimate traffic variability and traffic uncertainty. We used as measures of traffic variability and traffic uncertainty the intra-day traffic variability within a store and the inter-day traffic variability across stores respectively.

To calculate the intra-day traffic variability within a store we used the hourly level data to obtain the coefficient of variation of traffic for each store for every day of the year. To calculate inter-day traffic variability across stores, we used the daily level data to obtain the coefficient of variation of traffic for each store for each week of the year. We then obtained the overall mean of the coefficient of variation of traffic across weeks for each store. To test the robustness of our results, we measured inter-day traffic variability using three alternate models. We describe these models in the sensitivity analysis section of the chapter.

To test our fourth hypothesis we also need to estimate labor mismatches. If we possessed information on the planned staffing levels we could obtain deviations between the actual labor schedule and the planned labor schedule. As in many empirical studies, our variable choices and definitions are driven by the data availability. Since we did not have any information on the planned labor and the retailer's labor planning is based on traffic we calculated labor mismatches for each store as follows: We regressed labor hours of each store for a given day on the previous week's traffic controlling for days of the week, months as well as holidays. We then obtained the residual of each regression for each store and calculated the mean residual for each store. In addition, we estimated labor mismatches using three other models that we present in the sensitivity analysis section.

Table 3.1 summarizes the variables that we obtained and Table 3.2 summarizes descriptive statistics for all variables. We use subscript i ranging from 1 to 41 to denote each store and we use subscript t ranging from 1 to 365 to denote each time period. Below we give a brief description of each variable.

$TRAFFIC(TRAFFIC_{it})$ is the number of customers that entered a store per day.

The average arrivals were 729.541 and ranged from 114 to 2719.

$SALES(SALES_{it})$ is the revenue in U.S. dollars at a store during the day. While sales averaged about \$9083.46 per day per store there was considerable variation both within a store and across stores.

$TRANSACTIONS(TRANS_{it})$ corresponds to the number of customer transactions recorded at the checkout counters at a store. On average, a store in a given day had 97.72 transactions.

$CONVERSION\ RATE(CR_{it})$ is the proportion of customers who made a transaction per day per store and is calculated as follows: $CR_{it} = \frac{TRANS_{it}}{TRAFFIC_{it}}$. The average conversion rate was about .14 and there was considerable variation both within a store and across stores.

$BASKET\ VALUE(BV_{it})$ is the value of customers' shopping basket and is calculated as follows: $BV_{it} = \frac{SALES_{it}}{TRANS_{it}}$. Basket value averaged about \$90.49 per day per store.

$LABOR\ HOURS(LBRHRS_{it})$ is the total number of employee hours reported per store in a given day. On average, a total number of 56.432 employee hours were reported in a store per day.

Because the store business hours varied across the study period (i.e., during holiday season stores had extended business hours) we divided the daily total employees hours, traffic, transactions, and sales with the regular business hours of each store to obtain an average staffing level, traffic level, average transactions, and average sales that we used for our analysis (see Section 6). We denote these variables as $ADJUSTLBRHRS_{it}$,

$ADJUSTTRAFFIC_{it}$, $ADJUSTTRANS_{it}$, and $ADJUSTSALES_{it}$.

$COMPETITION(COMP_i)$ is measured by the total number of women's apparel stores which target the same age group as retailer "Alpha" and are located in the same mall/shopping center with retailer's store i. On average, there were 32 such competitors in each mall.

$TEMPERATURE$ ($TEMP_{it}$) corresponds to the daily temperature for each store location. Lam et al. (2001) treated daily temperature as a categorical variable and we adopted his approach. We used the following temperature ranges as Lam et al. (2001): (i) below 15 °F, (ii) 15 − 40 °F, (iii) 40 − 60 °F, (iv) 60 − 85 °F, (v) above 85 °F.

$DOW\ JONES\ INDEX$ (DJI_t) denotes the Dow Jones Industrial Average that varied between 12127.81 and 14086.09 during the period of the study.

$PER\ CAPITA\ INCOME$ ($PERCAPINC_i$) denotes the per capita income that was on average $35946.53.

$INTER-DAY\ TRAFFIC\ VARIABILITY$ ($TRAFFICUNC_i$) denotes the inter-day traffic variability for each store that was on average .365.

$INTRA-DAY\ TRAFFIC\ VARIABILITY$ ($TRAFFICVAR_{it}$) denotes the intra-day traffic variability per store in a given day that was on average .643.

$HOLIDAYS$ (δ_h) corresponds to major holidays such as Christmas season (Dec 23-31), Easter, Memorial Day, Independence Day, Labor Day, Martin Luther King Day, Mother's Day, Veterans Day, and Thanksgiving Day. We also control for 3 days before the occurrence of these holidays since retailers typically run different types of promotions before these holidays.

Table 3.3 shows the correlations among longitudinal variables. Note that traffic variability is negatively correlated with sales, basket value, and conversion rate. In addition, labor hours are positively correlated with store performance. Table 3.7 shows correlations among cross-sectional variables. Note that traffic variability and traffic uncertainty are positively correlated with labor mismatches.

Tables 3.5 and 3.6 present summary statistics for store related variables for different time slots within a day. Note that average traffic is lowest at the beginning of the store operating hours (i.e., 10:00am-12:00pm) and reaches its peak between 4:00pm-6:00pm on a weekday and between 2:00pm-4:00pm on a weekend. Sales and number of transactions are on average lowest at the beginning of the day and are highest between 6:00pm-8:00pm on a weekday and between 4:00pm-6:00pm on a weekend. On a weekday conversion rate reaches its peak at the beginning of the day (i.e., 10:00am-12:00pm) which is also the time period that we observe the lowest number of customer arrivals and is lowest between 4:00pm-6:00pm which is the time that we have the highest traffic. On a weekend conversion rate reaches its peak at the end of the day (6:00pm-8:00pm) and is lowest between 2:00pm-4:00pm which is the period with the highest traffic. On a weekday basket value is lowest around noon time (12:00pm-2:00pm) and is highest between 4:00pm-6:00pm whereas on a weekend basket value is lowest at the beginning of the day (10:00am-12:00pm) and reaches its peak closer to the end of the store operating hours (6:00pm-8:00pm). On a weekday labor hours are lowest at the opening hours of the store which is also the time with the lowest customer traffic and are highest between 2:00pm-4:00pm. On a weekend labor hours are lowest at the opening hours of the store and highest between 2:00pm-4:00pm which are the periods with the lowest and highest customer traffic respectively.

Table 3.7 summarizes descriptive statistics for store related variables across different days of the week. Traffic, sales, and number of transactions are highest on Saturday and lowest on Tuesday. The day with the highest intra-day traffic variability is Sunday and the days with the lowest are Tuesday and Wednesday. Conversion rate is lowest on Sunday, which is also the day with the highest traffic variability, and reaches its peak on Thursday. Basket value is lowest on Monday and highest on Thursday and labor hours are lowest on Sunday and reach their peak on Saturday the day with the highest traffic.

Table 3.8 presents summary statistics for store related variables across months. Traffic, sales, and number of transactions are highest in December and while traffic and transactions are lowest in October, sales are lowest in the beginning of the year (i.e., January). Intra-day traffic variability is lowest in December and is highest in September. May and June are the months that we observe the highest conversion rate and October is the month in which we observe the lowest average conversion rate. Basket value is lowest in January and highest in October and labor hours are lowest in February and reach their peak in December the month with the highest traffic.

We also conducted independent samples t-tests to compare the average traffic, labor hours, sales, transactions, basket values, and conversion rates of the holiday season (that corresponds to all major holidays described in this section) and non-holiday season. There is significant difference between the average traffic, labor hours, sales, transactions, basket value, and conversion rate of the holiday season and non-holiday season as illustrated in Table 3.11. Note that the average conversion rate and basket value are significantly higher during non-holiday season as opposed to average traffic, labor hours, sales, and transactions which are significantly higher during holiday season.

We computed the logarithm of each variable in order to construct a multiplicative model that we present in the next section. The variables obtained after taking logarithm are denoted by lower-case letters, i.e., $adjusttraffic_{it}$, $adjusttrans_{it}$, bv_{it}, $adjustlbrhrs_{it}$, etc.

3.6 Econometric Model

Since we have in our possession both longitudinal variables in addition to cross-sectional variables, we formulate a two-stage econometric model for each store performance measure following Figure 3.2 to test the proposed hypotheses. As in Lam et al. (2001) we use a multiplicative model. The first stage of the model focuses on explaining within-store variations using fixed effects while the second stage focuses on explaining across-store variations. In the first stage, the dependent variables are the adjusted number of transactions, the basket value, and the adjusted sales. The independent variables are adjusted store traffic, adjusted labor hours, traffic variability, the Dow Jones Industrial Average Index, dummy variables for daily temperature ranges, as well as dummy variables for months and holidays. We also control for differences across stores by using time-invariant store fixed effects.

Based on Figure 3.2, we specify the first stage equations for transactions, basket value, and sales as:

$$
\begin{aligned}
adjusttrans_{it} &= F_i + \alpha_{11} adjusttraffic_{it} + \alpha_{12} adjustlbrhrs_{it} + \alpha_{13} trafficvar_{it} \\
&+ \alpha_{14} dji_t + \alpha_{15}\delta_h + \boldsymbol{\alpha}'_{16t}\boldsymbol{\delta}_t + \boldsymbol{\alpha}'_{17m}\boldsymbol{\delta}_m + \epsilon_{it} \qquad (3.1) \\
bv_{it} &= J_i + \alpha_{21} adjusttraffic_{it} + \alpha_{22} adjustlbrhrs_{it} + \alpha_{23} trafficvar_{it} \\
&+ \alpha_{24} dji_t + \alpha_{25}\delta_h + \boldsymbol{\alpha}'_{26t}\boldsymbol{\delta}_t + \boldsymbol{\alpha}'_{27m}\boldsymbol{\delta}_m + \zeta_{it} \qquad (3.2) \\
adjustsales_{it} &= H_i + \alpha_{31} adjusttraffic_{it} + \alpha_{32} adjustlbrhrs_{it} + \alpha_{33} trafficvar_{it} \\
&+ \alpha_{34} dji_t + \alpha_{35}\delta_h + \boldsymbol{\alpha}'_{36t}\boldsymbol{\delta}_t + \boldsymbol{\alpha}'_{37m}\boldsymbol{\delta}_m + \phi_{it} \qquad (3.3)
\end{aligned}
$$

Each equation consists of store fixed effects (F_i, J_i, H_i), coefficients of the independent variables, and an error term (ϵ_{it}, ζ_{it}, ϕ_{it}). Note that α_{11}, α_{12}, α_{13}, α_{14}, α_{15}, α_{21}, α_{22}, α_{23}, α_{24}, α_{25}, α_{31}, α_{32}, α_{33}, α_{34}, and α_{35} are scalars. Let δ_h be a dummy variable used to control for holidays. We denote by $\boldsymbol{\alpha}'$ the transpose of a vector and by $\boldsymbol{\delta}$ a vector of dummy variables for each temperature range $\boldsymbol{\delta}_t$ and for months $\boldsymbol{\delta}_m$.

In equation (3.1) we use as dependent variable the number of transactions instead of conversion rate for the following reason: Retailer "Alpha" makes his staffing decisions based on forecasted traffic. Hence, since conversion rate is the ratio of the number of transactions to traffic having conversion rate as a dependent variable would create endogeneity issues because traffic would be affecting labor hours and consequently adjusted labor hours that is one of the explanatory variables in the right-hand side. To avoid endogeneity of our regressors, we used the number of transactions as a dependent variable controlling for traffic.

In the second stage, the dependent variables are the store fixed effects (F_i, J_i, H_i) obtained from the equations of the first-stage. The independent variables are competition, per capita income, and the average traffic uncertainty of a store. These are the variables that we treat as time-invariant. Hence, the second stage equations are as follows:

$$F_i = \beta_{11} comp_i + \beta_{12} percapinc_i + \beta_{13} trafficunc_i + \omega_i \quad (3.4)$$

$$J_i = \beta_{21} comp_i + \beta_{22} percapinc_i + \beta_{23} trafficunc_i + \chi_i \quad (3.5)$$

$$H_i = \beta_{31} comp_i + \beta_{32} percapinc_i + \beta_{33} trafficunc_i + \psi_i \quad (3.6)$$

In order to test the labor mismatch hypothesis we specify the following equation:

$$mismatches_i = \gamma_{11} trafficunc_i + \gamma_{12} \overline{trafficvar_i} + \eta_i \quad (3.7)$$

In the above equation we denote by $\overline{trafficvar_i}$ the mean of the longitudinal variable $trafficvar_{it}$.

3.7 Results and Discussion

Tables 3.11, 3.12, and 3.13 present the estimates of the parameters for equations (3.1)-(3.7). Below we present the effects of labor and traffic characteristics on store performance.

Staffing levels. The estimates of the parameters for equations (3.1)-(3.3) are presented in Table 3.11. Note that since we used a log-linear model the coefficient estimates need to be interpreted as elasticities. Labor is positively correlated with conversion rate and basket value. An increase of 1% in labor is associated with a 0.102% increase in conversion rate and with a 0.066% increase in basket value. We also find that labor is positively correlated with sales. An increase of 1% in labor is associated with a 0.173% increase in sales. Note that the elasticity of labor on store performance is less than 1 which suggests that store performance increases with an increase in store labor at a diminishing rate. We conclude that Hypothesis 1 is supported in our data.

The significant positive effect of labor on store performance is consistent with previous literature (Fisher et al. (2006), Ton (2008)). Moreover, our results supplement Fisher et al. (2006) findings by confirming that labor and other performance metrics such as conversion rate and basket value in addition to sales are related through a concave increasing function.

Traffic Characteristics. The parameter estimates for traffic variability and traffic uncertainty on store performance are presented in Tables 3.11 and 3.12 respectively. The corresponding parameter estimates for average traffic variability and uncertainty on labor mismatches are presented in Table 3.13.

First and foremost, we see that stores with higher traffic uncertainty have lower conversion rates and sales but similar basket values. Hence, higher traffic uncertainty is associated with lower conversion rates and sales. Our results show support for Hypothesis 2 for conversion rate and sales but not basket value which requires further examination. Second, we see that traffic variability is negatively correlated with both conversion rate and basket value. A 1% increase in traffic variability is associated with a 0.094% decrease in conversion rate at a store and 0.037% decrease in basket value. Hence, our results support Hypothesis 3. Moreover, we find that both traffic uncertainty and average traffic variability are positively correlated with mismatches between required labor and actual labor supporting Hypothesis 4. Hence, our results confirm that customer traffic uncertainty and variability can have implications in the labor planning and scheduling process by resulting in labor mismatches. Furthermore, our tests reveal that stores that have lower foot-traffic have higher traffic uncertainty (see Figure 3.7) resulting in mismatches between required labor and actual labor.

Covariates. The parameter estimates of the covariates provide support for the framework in Figure 3.2 regarding factors that affect store performance. For example, the significant positive estimates of the Dow Jones Index support the idea that the economy affects consumers' ability and their confidence of making purchases. Our results show that consumer demographics and specifically the per capita income has a significant and positive impact on store performance. We find that stores located in neighborhoods with higher per capita income have higher conversion rates but similar basket values. We also find that both conversion rate and basket values exhibit significant seasonality. A surprising result of our analysis is that competition as measured here does not affect conversion rates and basket values.

Holidays have a significant effect on store performance. Our results show that holiday season has a positive conversion effect but a negative spending effect which results in a negative overall effect on sales. A possible explanation could be that price promotions offered during the holiday season affect consumers' probability of making a purchase at a store. The direction of the effect of price promotions on basket value depends on the tradeoff between the sales gain resulting from an increase in the quantity of the goods sold and the sales loss due to the price discount. Our results indicate that the latter effect was dominant which is consistent with some of the findings in the marketing literature (e.g., Lam et al. (2001)).

Summary. Both the hypotheses and the results obtained from our analysis are summarized in Table 3.14. We find full support for Hypotheses 1, 3, and 4. Regarding Hypothesis 2 we find support for conversion rate and sales but not basket value which merits further investigation.

3.8 Sensitivity Analysis

In this section, we perform some sensitivity analysis to test the robustness of our results. During this analysis we use different model specifications for hypothesis testing and alternate models to define some of our variables.

First, we checked the robustness of our results testing different traffic uncertainty models. Recall that we used as measure of traffic uncertainty the inter-day traffic variability across stores. For our core analysis we obtained this measure as follows: we calculated the coefficient of variation of traffic for each store over a week and then obtained the overall mean of the coefficient of variation across weeks for each store.

We calculated the inter-day traffic variability using three alternate approaches. In the first approach which we refer to as Model 1 we calculate the coefficient of variation of traffic for each store over a month and then obtain the overall mean across months for each store. In the second approach (Model 2) we calculate the coefficient of variation for each store over two weeks and obtain the overall mean for each store across the two-week periods. In the third approach (Model 3) we calculate the coefficient of variation of traffic for each store over the whole year.

Tables 3.15 and 3.16 present the results of regression equations (3.4) and (3.7). Table 3.15 shows that the results obtained from the three alternate model specifications for traffic uncertainty are robust. We find that traffic uncertainty is negatively correlated with conversion rate and sales under all model specifications. Similarly, we find that the coefficient estimates of Models 1-3 in Table 3.16 confirm support for Hypothesis 4.

Second, we checked the robustness of our results using different labor mismatch models. In the main analysis we had used the following model to obtain mismatches:

$$\begin{aligned} LBRHRS_{it} &= b_{11}TRAFFIC_{it-1} + b_{12}TRAFFIC_{it-2} + b_{13}TRAFFIC_{it-3} \\ &+ b_{14}TRAFFIC_{it-4} + b_{15}TRAFFIC_{it-5} + b_{16}TRAFFIC_{it-6} \\ &+ b_{17}TRAFFIC_{it-7} + b_{18}\delta_h + \boldsymbol{b}'_{19d}\boldsymbol{\delta}_d + \boldsymbol{b}'_{20m}\boldsymbol{\delta}_m + \eta_{it} \end{aligned} \tag{3.8}$$

We defined as a mismatch the residuals of the above regression which we convert into a single number for each store by obtaining the mean residual for each store in order to enable cross-sectional analysis to test our fourth hypothesis. The advantage of this definition is that we do utilize longitudinal data to create the mismatch variable.

To test the robustness we tried three alternative model specifications for mismatches. We first removed the dummy variable for holidays from the regression equation (3.8). We refer to this specification as Model 4. Models 5 and Model 6 were constructed sequentially from Model 4 by removing days of the week and months from the covariates respectively. Table 3.17 present the results of regression equation (3.7) for the three alternate model specifications for mismatches. Table 3.17 shows that our results support fully Hypothesis 4.

Third, we also tested the robustness of the results from the first stage model in which we removed one of the covariates at a time and rerun the model. First, we removed from regression equations (3.1)-(3.3) the dummy variable for holidays. This is refered to as Model 7. Second, we removed from regression equations (3.1)-(3.3) the month covariates (i.e., Model 8). The results of the estimation of the above models are presented in Table 3.17. We find under all alternate model specifications that traffic variability is negatively associated with store performance which is consistent with our previous findings.

3.9 Conclusion

Motivated by the increasing efforts that retailers make to track conversion rate and basket value and the importance that they place on such store performance metrics we conduct a descriptive study of these metrics for a retailer. Specifically, we consider the correlation between store performance and intra-day traffic variability and traffic uncertainty. We also measure traffic-labor mismatches and study if they explain the observed correlations in our sample.

We report the following results in this chapter. First, we present the within-store results. We find that intra-day traffic variability is negatively correlated with both conversion rate and basket value. A 1% increase in traffic variability is associated with a 0.094% decrease in conversion rate in a store and 0.037% decrease in basket value. We also find that, for a given level of traffic, both conversion rate and basket value increase with an increase in store labor at a diminishing rate. A 1% increase in labor is associated with a 0.102% increase in conversion rate and 0.066% increase in basket value. In addition, we find that conversion rates are higher during holidays but basket values are lower suggesting that price promotions offered during the holiday season cause more customers to purchase but do not make the average customer purchase more. Moreover, we find that both conversion rates and basket values exhibit significant seasonality.

Next, we report the across-store results. We find that stores with higher traffic uncertainty have lower conversion rates but similar basket values. We also find that stores that have higher traffic variability and higher traffic uncertainty have higher mismatches between required labor and actual labor. Furthermore, our tests reveal that stores that have lower foot-traffic have higher traffic uncertainty resulting in mismatches between required labor and actual labor. A surprising result of our analysis is that competition does not affect conversion rates and basket values. This suggests that consumers decision to whether or not to purchase and how much to purchase is unaffected by the presence of other competitors once they are in the store. Finally, we find that stores located in neighborhoods with higher per capita income have higher conversion rates but similar basket values.

Although our analysis has provided several interesting insights related to the linkages between staffing levels, traffic variability, traffic uncertainty, and store performance we need to acknowledge its limitations. One of the drivers of store performance is the availability of inventory at a store. Our stores are under the same ownership so one would expect that the target inventory levels should be similar in all stores. However, there could be large variations in the actual inventory levels across stores as well as within a store over time. Unfortunately, we could not obtain any information with respect to the inventory levels for the retailer we studied and as a result, we could not control for actual inventory in our analysis. Another information that we could not obtain pertains to the type and size of assortment. Even though the stores that we considered in our analysis are of the same type there could be large variations in the size as well as type of assortment that they offer.

Summarizing our study has the following implications for practitioners. First, according to our study retailers need to plan labor not only based on sales and traffic which has been the traditional approach but also take traffic volatility into account. Even though some part of traffic volatility is uncontrollable retailers need to handle volatilities that are under their control either using increased labor or through more flexible labor. Second, retailers often can induce traffic volatility through the actions that they take. For example, several retailers have "early bird specials" that would cause traffic variability to increase substantially. Hence, our study shows that it is important to coordinate actions that drive store traffic with those that help manage the potential increase in volatility.

3.10 Supplement with Tables and Figures

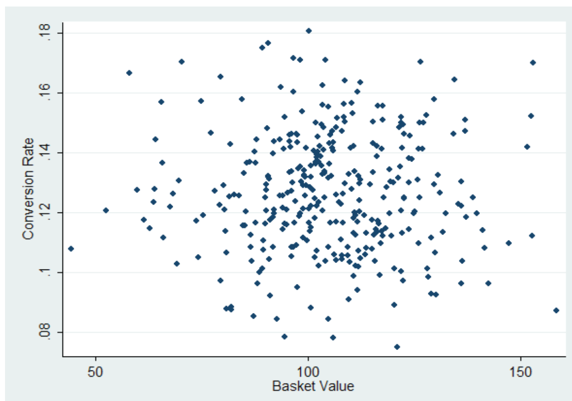


FIGURE 3.3: Correlation between conversion rate and basket value for one store

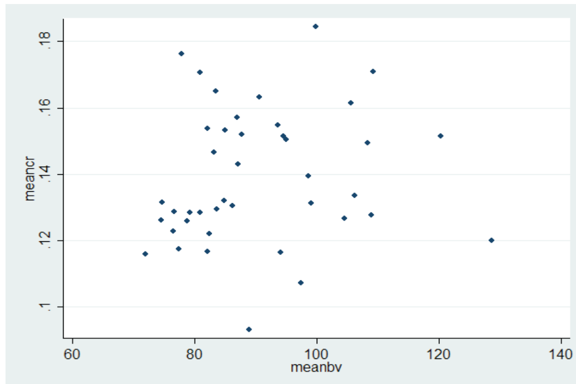


FIGURE 3.4: Correlation between average conversion rate and average basket value across stores

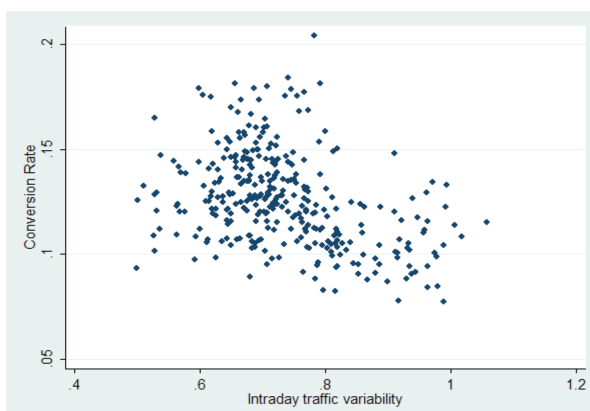


FIGURE 3.5: Correlation between conversion rate and intra-day traffic variability for one of the stores

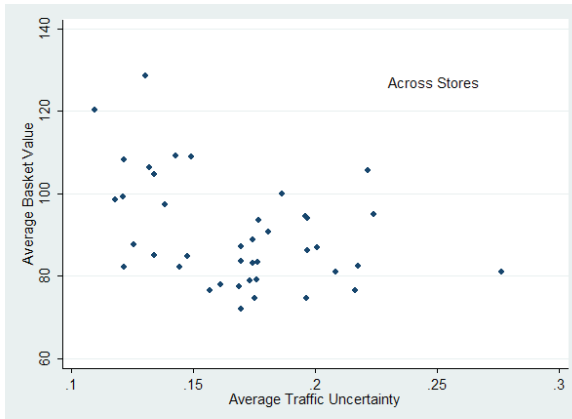


FIGURE 3.6: Correlation between basket value and traffic uncertainty across stores

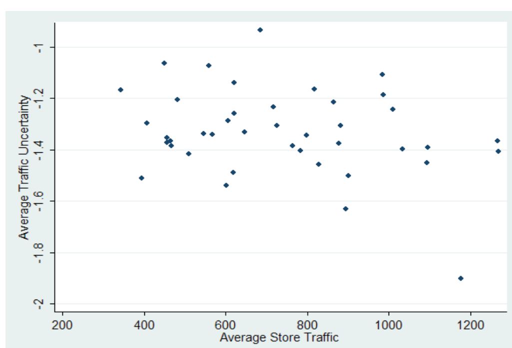


FIGURE 3.7: Correlation between average traffic uncertainty and average store traffic across stores

TABLE 3.1: Description of Variables

Variable	Description
$TRAFFIC_{it}$	Number of customers who entered store i on day t
$ADJUSTTRAFFIC_{it}$	Total number of customers who entered store i on day t divided by the regular business hours of store i on day t
$SALES_{it}$	Revenue in U.S. dollars for store i on day t
$ADJUSTSALES_{it}$	Revenue in U.S. dollars for store i on day t divided by the regular business hours of store i on day t
$TRANS_{it}$	Number of customer transactions at store i on day t
$ADJUSTTRANS_{it}$	Number of customer transactions at store i on day t divided by the regular business hours of store i on day t
CR_{it}	Proportion of customers who made a transaction at store i on day t
BV_{it}	Value in U.S. dollars of customers' shopping basket at store i on day t
$LBRHRS_{it}$	Total number of employee hours reported at store i on day t
$ADJUSTLBRHRS_{it}$	Total number of employee hours reported at store i on day t divided by the regular business hours of store i on day t
$COMP_i$	Total number of stores that are in the mall where store i is located which sell similar assortment as "Alpha"
$TEMP_{it}$	Daily temperature for store location i
DJI_t	Dow Jones Industrial Average on day t
$PERCAPINC_i$	Per capita income for store location i
$TRAFFICUNC_i$	Average inter-day traffic variability for store location i
$TRAFFICVAR_{it}$	Intra-day traffic variability for store location i on day t

TABLE 3.2: Summary Statistics

Variable	Mean	Std.Dev.	Min.	Max.
LONGITUDINAL VAR				
TRAFFIC	729.541	379.506	114	2719
TRAFFICVAR	.643	.161	.269	1.065
SALES	9083.464	5618.848	140.21	56666.14
TRANS	97.716	47.314	17	371
CR	.139	.035	.020	.458
BV	90.494	24.256	2.780	365.83
LBRHRS	56.432	19.508	16.63	140.76
OPERATING HRS	10.500	1.400	6	14
TEMP	64.171	15.299	18	93
DJI	13177.24	507.689	12127.81	14086.09
CROSS – SECTIONAL VAR				
COMP	31.523	11.994	15	71
PERCAPINC	35946.530	19417.730	12763	92940
TRAFFICUNC	.365	.062	.216	.524

TABLE 3.3: Correlations among longitudinal variables

Variable Name	1	2	3	4	5	6	7	8	9	10	11	12	13
1. $ADJUSTSALES$	1.00												
2. $ADJUSTTRANS$	0.88*	1.00											
3. $ADJUSTTRAFFIC$	0.77*	0.87*	1.00										
4. $TRAFFICVAR$	-0.01*	0.03*	0.12*	1.00									
5. $ADJUSTLBRHRS$	0.67*	0.68*	0.56*	-0.05*	1.00								
6. CR	0.10*	0.09*	-0.34*	-0.15*	0.12*	1.00							
7. BV	0.55*	0.17*	0.13*	-0.11*	0.27*	0.06*	1.00						
8. DJI	0.03*	0.01	0.00	-0.01	0.05*	0.01	0.07*	1.00					
9. $Temp\ 15-40\,°F$	-0.05*	-0.01	-0.01	-0.03*	-0.05*	0.00	-0.08*	-0.21*	1.00				
10. $Temp\ 40-60\,°F$	0.03*	0.05*	0.05*	-0.04*	0.04*	0.02*	-0.02*	-0.30*	-0.17	1.00			
11. $Temp\ 60-85\,°F$	-0.01*	-0.04*	-0.05*	0.03*	-0.01	0.01	0.04*	0.33*	-0.37*	-0.72*	1.00		
12. $Temp > 85\,°F$	0.03*	0.00	0.02*	0.04*	0.02	-0.06	0.06*	0.10*	-0.07	-0.14*	-0.31	1.00	
13. $HOLIDAY$	0.10*	0.20*	0.22*	0.11*	0.07*	-0.05*	-0.07*	0.01*	0.05*	0.02	-0.03*	-0.02	1.00

Note:* denotes statistical significance at the 5% level and higher.

TABLE 3.4: Correlations among cross-sectional variables

Variable Name	1	2	3	4	5
1. $MISMATCH$	1.00				
2. $\overline{TRAFFICVAR}$	0.04*	1.00			
3. $TRAFFICUNC$	0.15*	0.15*	1.00		
4. $COMP$	-0.12*	-0.42*	-0.11*	1.00	
5. $PERCAPINC$	-0.14*	-0.35*	-0.07*	0.29*	1.00

Note:* denotes statistical significance at the 5% level and higher.

TABLE 3.5: Summary Statistics for Store Related Variables for Different Time Slots Within a Weekday

Time Slots	Variable	Mean	Std.Dev.	Min.	Max.
	$TRAFFIC$	29.479	21.831	1	235
	$SALES$	425.580	441.719	19	3772.670
$10:00AM - 12:00PM$	$TRANS$	4.767	3.658	1	36
	CR	.164	.099	.030	1
	BV	89.671	55.239	5.825	317
	$LBRHRS$	3.401	1.491	2	11
	$TRAFFIC$	56.441	30.596	4	237
	$SALES$	747.063	594.365	19	3812.190
$12:00PM - 2:00PM$	$TRANS$	8.332	4.776	1	40
	CR	.156	.071	.030	.705
	BV	89.696	48.564	5.7	316.426
	$LBRHRS$	4.226	1.709	2	11
	$TRAFFIC$	65.174	35.724	1	238
	$SALES$	840.986	634.361	19.8	3796.250
$2:00PM - 4:00PM$	$TRANS$	9.031	5.027	1	40
	CR	.148	.066	.030	1
	BV	93.224	48.243	5.72	316.150
	$LBRHRS$	5.018	1.899	2	11
	$TRAFFIC$	66.009	35.875	1	238
	$SALES$	849.663	645.999	19.980	3812.940
$4:00PM - 6:00PM$	$TRANS$	9.029	5.134	1	40
	CR	.145	.067	.030	1
	BV	93.909	47.776	5.802	317
	$LBRHRS$	4.888	1.896	2	11
	$TRAFFIC$	65.997	35.595	1	238
	$SALES$	857.168	633.986	19	3798.600
$6:00PM - 8:00PM$	$TRANS$	9.184	5.057	1	35
	CR	.150	.077	.030	1
	BV	93.312	46.710	5.8	316.863
	$LBRHRS$	4.465	1.689	2	11

TABLE 3.6: Summary Statistics for Store Related Variables for Different Time Slots Within a Weekend

Time Slots	Variable	Mean	Std.Dev.	Min.	Max.
	$TRAFFIC$	38.815	30.3680	1	205
	$SALES$	572.516	598.870	19	3809.610
$10:00AM - 12:00PM$	$TRANS$	6.427	4.970	1	38
	CR	.140	.091	.030	1
	BV	86.539	48.532	5.691	315.990
	$LBRHRS$	3.388	1.471	2	11
	$TRAFFIC$	93.579	42.027	4	237
	$SALES$	999.621	723.544	21	3813.250
$12:00PM - 2:00PM$	$TRANS$	11.092	5.963	1	46
	CR	.121	.049	.030	.464
	BV	88.787	42.111	5.871	314.618
	$LBRHRS$	4.764	1.880	2	11
	$TRAFFIC$	131.650	45.771	8	238
	$SALES$	1335.082	778.738	19.980	3812.120
$2:00PM - 4:00PM$	$TRANS$	14.853	6.408	2	46
	CR	.115	.039	.030	.342
	BV	89.665	37.591	5.870	314.128
	$LBRHRS$	5.819	1.951	2	11
	$TRAFFIC$	122.563	48.438	1	238
	$SALES$	1381.591	784.754	29	3813.500
$4:00PM - 6:00PM$	$TRANS$	14.959	6.556	1	42
	CR	.128	.051	.030	.833
	BV	92.301	37.228	6.117	309
	$LBRHRS$	5.602	1.863	2	11
	$TRAFFIC$	79.831	57.536	1	238
	$SALES$	1097.237	768.951	19.990	3801.210
$6:00PM - 8:00PM$	$TRANS$	11.310	6.649	1	50
	CR	.190	.176	.030	1
	BV	99.266	46.486	5.792	316
	$LBRHRS$	4.967	1.776	2	11

TABLE 3.7: Summary Statistics for Store Related Variables for Different Days (Monday-Sunday)

Day	Variable	Mean	Std.Dev.	Min.	Max.
	TRAFFIC	599.476	306.532	209	2464
	TRAFFICVAR	.627	.142	.269	1.064
	SALES	7065.308	4382.507	716.630	33281.200
MONDAY	*TRANS*	82.291	39.169	26	313
	CR	.143	.035	.025	.338
	BV	83.740	24.379	9.146	236.926
	LBRHRS	55.599	18.600	22.600	131.05
	TRAFFIC	542.024	239.48	187	2264
	TRAFFICVAR	.606	.138	.269	.898
	SALES	6919.516	4002.317	269.7	28311.43
TUESDAY	*TRANS*	76.975	33.134	28	262
	CR	.146	.035	.029	.458
	BV	88.021	26.792	2.780	347.547
	LBRHRS	61.773	19.899	26.69	136.91
	TRAFFIC	591.691	285.951	214	2462
	TRAFFICVAR	.606	.135	.272	1.064
	SALES	7743.733	4231.748	140.21	35399.85
WEDNESDAY	*TRANS*	84.030	37.829	29	299
	CR	.146	.035	.027	.344
	BV	91.309	26.024	3.260	365.83
	LBRHRS	54.107	18.536	27.02	135.59
	TRAFFIC	630.881	296.698	217	2378
	TRAFFICVAR	.609	.135	.270	1.044
	SALES	8984.044	5526.249	935.64	48252.4
THURSDAY	*TRANS*	92.312	41.750	29	281
	CR	.150	.037	.020	.392
	BV	94.550	25.493	15.411	236.585
	LBRHRS	54.232	18.900	26.65	140.76
	TRAFFIC	815.497	341.595	228	2467
	TRAFFICVAR	.607	.134	.269	.900
	SALES	10922.740	5621.024	1876.290	46115.310
FRIDAY	*TRANS*	114.969	45.029	32	312
	CR	.144	.031	.033	.313
	BV	93.045	21.623	22.562	193.533
	LBRHRS	59.883	19.311	27.310	129.330
	TRAFFIC	1175.169	418.113	323	2605
	TRAFFICVAR	.674	.135	.278	1.037
	SALES	13781.840	6718.914	2691.190	56666.140
SATURDAY	*TRANS*	144.261	52.403	40	371
	CR	.125	.028	.046	.322
	BV	93.611	20.171	29.199	270.075
	LBRHRS	62.769	19.232	27.260	138.970
	TRAFFIC	730.317	306.934	114	2719
	TRAFFICVAR	.778	.222	.274	1.065
	SALES	7814.846	4530.942	574.220	36815.990
SUNDAY	*TRANS*	85.832	37.736	17	324
	CR	.119	.028	.040	.291
	BV	88.757	22.853	17.944	190.576
	LBRHRS	45.854	16.699	16.630	127.810

TABLE 3.8: Summary Statistics for Store Related Variables for Different Months (January-June)

Months	Variable	Mean	Std.Dev.	Min.	Max.
	TRAFFIC	666.119	323.674	114	2355
	TRAFFICVAR	.649	.161	.273	1.064
	SALES	6950.871	4477.252	140.210	33281.200
JANUARY	*TRANS*	89.629	42.895	17	313
	CR	.140	.039	.025	.3
	BV	76.164	25.113	2.780	186.375
	LBRHRS	53.923	18.290	17.210	131.610
	TRAFFIC	704.794	375.571	197	2544
	TRAFFICVAR	.642	.160	.2690	1.061
	SALES	8675.499	5217.262	1230.470	39091.450
FEBRUARY	*TRANS*	91.956	43.898	26	324
	CR	.138	.038	.020	.3
	BV	91.726	23.786	37.342	347.547
	LBRHRS	51.134	16.533	18.700	111.230
	TRAFFIC	733.200	369.313	219	2338
	TRAFFICVAR	.646	.162	.279	1.061
	SALES	9489.285	6073.535	716.630	44070.110
MARCH	*TRANS*	97.213	46.858	26	289
	CR	.139	.042	.027	.360
	BV	93.821	24.430	18.375	190.779
	LBRHRS	55.004	19.534	18.750	129.470
	TRAFFIC	711.324	359.870	202	2394
	TRAFFICVAR	.646	.161	.269	1.060
	SALES	9153.486	5566.904	1639.340	35412.020
APRIL	*TRANS*	96.983	46.553	23	297
	CR	.141	.038	.059	.458
	BV	91.212	21.699	29.628	250.492
	LBRHRS	56.042	19.322	18.710	121.300
	TRAFFIC	732.220	365.294	233	2605
	TRAFFICVAR	.637	.158	.272	1.065
	SALES	9802.685	5868.811	1582.080	49676.780
MAY	*TRANS*	104.065	48.214	31	371
	CR	.146	.032	.076	.344
	BV	91.323	20.724	36.258	177.238
	LBRHRS	56.804	19.765	23.260	134.280
	TRAFFIC	693.245	331.949	183	2413
	TRAFFICVAR	.638	.162	.271	1.062
	SALES	8593.011	5207.741	870.580	38705.410
JUNE	*TRANS*	99.191	46.781	23	291
	CR	.146	.031	.062	.305
	BV	84.082	20.450	15.010	228.025
	LBRHRS	56.954	18.582	18.240	136.910

TABLE 3.9: Summary Statistics for Store Related Variables for Different Months (July-December)

Months	Variable	Mean	Std.Dev.	Min.	Max.
	TRAFFIC	703.181	314.899	190	2199
	TRAFFICVAR	.651	.161	.269	1.063
	SALES	7967.695	4303.862	907.760	35416.820
JULY	*TRANS*	92.816	40.685	24	266
	CR	.135	.030	.040	.260
	BV	84.426	21.135	14.641	190.269
	LBRHRS	53.828	17.973	18.440	126.940
	TRAFFIC	701.352	323.768	209	2273
	TRAFFICVAR	.643	.156	.270	1.051
	SALES	8858.745	5464.121	574.220	48252.400
AUGUST	*TRANS*	92.836	42.077	30	275
	CR	.135	.031	.051	.295
	BV	92.122	24.565	17.944	200.217
	LBRHRS	56.546	20.003	18.720	140.760
	TRAFFIC	692.778	367.163	212	2336
	TRAFFICVAR	.657	.166	.276	1.065
	SALES	8957.168	5632.290	996.710	56666.140
SEPTEMBER	*TRANS*	91.849	43.513	27	337
	CR	.139	.035	.054	.288
	BV	94.904	25.481	26.229	365.830
	LBRHRS	56.001	19.702	18.560	136.610
	TRAFFIC	648.295	348.222	215	2401
	TRAFFICVAR	.640	.161	.280	1.065
	SALES	8560.075	5253.109	1636.450	38890.890
OCTOBER	*TRANS*	83.576	37.699	27	291
	CR	.13680	.033	.064	.273
	BV	99.010	26.682	41.875	270.075
	LBRHRS	56.726	19.270	16.630	132.220
	TRAFFIC	727.054	396.239	217	2449
	TRAFFICVAR	.645	.164	.273	1.061
	SALES	9371.947	5743.932	1302.240	39794.560
NOVEMBER	*TRANS*	93.044	44.960	26	312
	CR	.134	.031	.045	.269
	BV	97.513	25.144	27.442	304.886
	LBRHRS	60.665	21.628	18.540	138.970
	TRAFFIC	1065.840	501.390	204	2719
	TRAFFICVAR	.616	.158	.270	1.063
	SALES	12901.610	6481.820	1783.480	41693.870
DECEMBER	*TRANS*	141.259	57.844	31	299
	CR	.139	.032	.072	.279
	BV	90.960	22.306	15.411	165.988
	LBRHRS	63.728	20.246	21.440	131.050

TABLE 3.10: Independent-Samples T-Tests for Comparison of Non-Holidays (G1) versus Holidays (G2)

Variables	$Mean(G1)$	$Std.Dev.(G1)$	$Mean(G2)$	$Std.Dev.(G2)$	$t_{satterthwaite}$	p
$TRAFFIC$	703.165	359.084	988.221	467.310	-20.819	< 0.001
$SALES$	8908.919	5551.430	10795.280	5979.661	-10.629	< 0.001
$TRANS$	94.789	45.489	126.419	54.710	-19.636	< 0.001
CR	.140	.035	.133	.033	6.806	< 0.001
BV	91.111	24.305	84.447	22.921	9.676	< 0.001
$LBHRS$	56.098	19.345	59.711	20.771	-5.860	< 0.001

TABLE 3.11: Regression Results of First Stage Equations

	Transactions	Basket Value	Sales
$adjusttraffic$	.753***	.046***	.804***
	(.004)	(.005)	(.006)
$adjustlbrhrs$	.102***	.066***	.173***
	(.006)	(.008)	(.010)
$trafficvar$	-.094***	-.037***	-.122***
	(.008)	(.011)	(.014)
dji	.522***	.486***	.961***
	(.107)	(.130)	(.173)
$40-60\,°$F	.003	-.005	.004
	(.006)	(.008)	(.010)
$60-85\,°$F	.015*	-.010	.014
	(.007)	(.009)	(.012)
$> 85\,°$F	.004	-.021*	-.000
	(.010)	(.012)	(.017)
Jan	-.082***	-.092***	-.172***
	(.011)	(.013)	(.017)
Feb	-.073***	.038***	-.028**
	(.010)	(.012)	(.017)
Mar	-.053***	.073***	.018
	(.012)	(.015)	(.020)
Apr	-.060***	.043***	-.014
	(.010)	(.012)	(.016)
May	-.042***	.031***	-.006
	(.008)	(.010)	(.013)
$June$	-.047***	-.062***	-.106***
	(.009)	(.010)	(.014)
$July$	-.124***	-.046***	-.168***
	(.009)	(.010)	(.014)
Aug	-.116***	.032***	-.084***
	(.009)	(.011)	(.014)
Sep	-.106***	.056***	-.044***
	(.009)	(.010)	(.014)
Oct	-.158***	.067***	-.089***
	(.009)	(.011)	(.015)
Nov	-.126***	.095***	-.027**
	(.008)	(.009)	(.013)
$Holidays$	.025***	-.082***	-.052***
	(.005)	(.006)	(.008)
$Constant$	-6.220***	-.577	-6.374***
	(1.021)	(1.239)	(1.652)
$Wald\ \chi^2$	88560.01***	8156.73***	45121.47***

Note:*,**,*** denote statistical significance at the 10%, 5% and 1% levels, respectively. Fixed store effects are included in the regressions but not shown in the table. The numbers below the parameter estimates are the respective standard errors. Wald test statistic compares the fit of the model including explanatory variables to fit of model with only the intercept. GLS estimators are used.

TABLE 3.12: Regression Results of Second Stage Equation

	Transactions	Basket Value	Sales
comp	.010	.071	.083
	(.052)	(.070)	(.095)
percapinc	.081**	-.007	.076
	(.038)	(.050)	(.067)
trafficunc	-.342***	-.119	-.459**
	(.095)	(.107)	(.178)
Constant	-7.313***	-.746**	-7.666***
	(.372)	(.415)	(.564)

Note:*,**,*** denote statistical significance at the 10%, 5% and 1% levels, respectively. The numbers below the parameter estimates are the respective robust standard errors.

TABLE 3.13: Regression Results for the Labor Mismatches Equation

	Labor Mismatches Equation
trafficunc	.014***
	(.005)
$\overline{trafficvar}$	.014**
	(.005)
Constant	3.804***
	(.004)

Note:*,**,*** denote statistical significance at the 10%, 5% and 1% levels, respectively. The numbers below the parameter estimates are the respective robust standard errors.

TABLE 3.14: Summary of Hypotheses and Results

Indep. Variable	Hypothesis	Dep. Variable	Anticipated Sign	Results
adjustlbrhrs	H_1	*adjusttrans*	+	Supported
adjustlbrhrs	H_1	*bv*	+	Supported
adjustlbrhrs	H_1	*adjustsales*	+	Supported
trafficunc	H_2	*F*	-	Supported
trafficunc	H_2	*J*	-	Not Supported
trafficunc	H_2	*H*	-	Supported
trafficvar	H_3	*adjusttrans*	-	Supported
trafficvar	H_3	*bv*	-	Supported
trafficvar	H_3	*adjustsales*	-	Supported
$\overline{trafficvar}$	H_4	*mismatch*	+	Supported
trafficunc	H_4	*mismatch*	+	Supported

TABLE 3.15: Sensitivity Analysis for H2 for different Traffic Uncertainty Models

	Transactions			Basket Value			Sales		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
comp	.013	.011	.035	.074	.072	.079	.090	.086	.116
	(.052)	(.053)	(.060)	(.070)	(.070)	(.068)	(.098)	(.096)	(.102)
percapinc	.077*	.079*	.066	-.008	-.008	-.0147	.0709	.072	.053
	(.039)	(.039)	(.044)	(.051)	(.050)	(.048)	(.069)	(.069)	(.070)
trafficunc	-.372***	-.350**	-.344***	-.100	-.113	-.176	-.467**	-.459**	-.513***
	(.099)	(.097)	(.091)	(.110)	(.108)	(.117)	(.180)	(.180)	(.167)
Constant	-7.311***	-7.301***	-7.190***	-.722*	-.735*	-.733*	-7.638***	-7.642***	-7.529***
	(.368)	(.378)	(.382)	(.415)	(.414)	(.381)	(.561)	(.570)	(.527)

Note:*,**,*** denote statistical significance at the 10%, 5% and 1% levels, respectively. The numbers below the parameter estimates are the respective robust standard errors. Traffic uncertainty is computed as follows: (i) In Model 1 we calculate the coefficient of variation of traffic for each store over a month's period and then take the average. (ii) In Model 2 we calculate the coefficient of variation of traffic for each store over a two week's period and then take the average. (iii) In Model 3 we calculate the coefficient of variation of traffic for each store over the whole year.

TABLE 3.16: Sensitivity Analysis for H4 for different Traffic Uncertainty Models

	Model 1	Model 2	Model 3
$trafficunc$	.017***	.014***	.027***
	(.005)	(.005)	(.007)
$\overline{trafficvar}$	.013**	.013**	.010**
	(.005)	(.005)	(.005)
$Constant$	3.807***	3.805***	3.812***
	(.004)	(.004)	(.005)

Note:*,**,*** denote statistical significance at the 10%, 5% and 1% levels, respectively. The numbers below the parameter estimates are the respective robust standard errors. Traffic uncertainty is computed as follows: (i) In Model 1 we calculate the coefficient of variation of traffic for each store over a month's period and then take the average. (ii) In Model 2 we calculate the coefficient of variation of traffic for each store over a two week's period and then take the average. (iii) In Model 3 we calculate the coefficient of variation of traffic for each store over the whole year.

TABLE 3.17: Sensitivity Analysis for H4 for different Mismatch Models

	Model 4	Model 5	Model 6
$trafficunc$	.014***	.019***	.021***
	(.005)	(.006)	(.007)
$\overline{trafficvar}$	.013**	.0203***	.021**
	(.005)	(.006)	(.008)
$Constant$	3.804***	3.809***	3.805***
	(.004)	(.005)	(.007)

Note:*,**,*** denote statistical significance at the 10%, 5% and 1% levels, respectively. The numbers below the parameter estimates are the respective robust standard errors. Mismatches are computed as the residuals of the following regressions: In Model 4 we regressed labor hours of a given day and a given store on the previous week's traffic controlling for days of the week and months. In Model 5 we regressed labor hours of a given day and a given store on the previous week's traffic controlling for days of the week. In Model 6 we regressed labor hours of a given day and a given store on the previous week's traffic.

TABLE 3.18: Sensitivity Analysis for H1 and H2

	Transactions		Basket Value		Sales	
	Model 7	Model 8	Model 7	Model 8	Model 7	Model 8
adjusttraffic	.757***	.771***	.035***	.043***	.796***	.818***
	(.004)	(.004)	(.005)	(.005)	(.006)	(.006)
adjustlbrhrs	.101***	.102***	.068***	.075***	.175***	.184***
	(.006)	(.006)	(.008)	(.008)	(.010)	(.010)
trafficvar	-.089***	-.120***	-.051***	-.027**	-.131***	-.143***
	(.008)	(.008)	(.011)	(.011)	(.014)	(.014)
dji	.476***	.238***	.658***	.430***	.512***	.623***
	(.107)	(.049)	(.131)	(.058)	(.162)	(.077)
$40-60\,^{\circ}$F	-.005	.021**	.020	.018	.000	.030*
	(.011)	(.010)	(.012)	(.011)	(.017)	(.015)
$60-85\,^{\circ}$F	-.002***	.017**	.017*	.025***	.006	.037***
	(.008)	(.007)	(.010)	(.009)	(.013)	(.012)
$>85\,^{\circ}$F	.009	.020*	.011	.027***	.016	.042***
	(.007)	(.007)	(.008)	(.008)	(.011)	(.010)
Jan	-.086***		-.077***		-.164***	
	(.011)		(.013)		(.017)	
Feb	-.080***		.062***		-.0137	
	(.010)		(.012)		(.016)	
Mar	-.062***		.103***		.036*	
	(.012)		(.015)		.020	
Apr	-.066***		.063***		-.002	
	(.010)		(.012)		(.016)	
May	-.041***		.027***		-.009	
	(.008)		(.010)		(.013)	
June	-.052***		-.049***		-.098***	
	(.009)		(.010)		(.014)	
July	-.125***		-.044***		-.167***	
	(.009)		(.011)		(.0147)	
Aug	-.121***		.045***		-.075**	
	(.009)		(.011)		(.014)	
Sep	-.108***		.062***		-.041***	
	(.009)		(.010)		(.014)	
Oct	-.160***		.0744***		-.085***	
	(.009)		(.011)		(.015)	
Nov	-.126***		.094***		-.027**	
	(.008)		(.010)		(.013)	
Holidays		.027***		-.082***		-.049***
		(.005)		(.006)		(.008)
Constant	-5.783***	-3.689***	-2.219*	-2.219*	-7.288***	-3.323***
	(1.021)	(.472)	(.5594)	(1.249)	(1.650)	(.740)
Wald χ^2	87847.69***	79732.15***	7689.97***	5807.91***	44765.86***	40508.45***

Note:*,**,*** denote statistical significance at the 10%, 5% and 1% levels, respectively. Fixed store effects are included in the regressions but not shown in the table. The numbers below the parameter estimates are the respective standard errors. Wald test statistic compares the fit of the model including explanatory variables to fit of model with only the intercept. GLS estimators are used. In Model 7 we have removed holidays from the covariates and in Model 8 we have removed months.

CHAPTER 4

Temporal Management of Service Investments under Demand Uncertainty and Competition

4.1 Introduction

In many settings, retailer investments in customer experience activities are important in influencing demand for a product. For example, a retailer can stimulate demand through various ways such as provide training to its sales personnel to promote a given product, create special areas to show case a product or even invest in technology that offers a unique experience for a product. In addition to price, all the above activities which we will be referring to as "service" can affect the purchasing decision of customers. The planning of such costly activities can be very crucial especially when a new product is being launched in a market. In the case of new product introduction, the retailer needs not only to decide the optimal price and service investments for the product but also when to invest in such experience activities. Given that market demand could be highly uncertain, a retailer may choose to wait until he receives some information regarding the market state before investing in such activities or he may want to make all these investments upfront to take advantage of possible reduced investment costs.

A retailer typically has three alternatives regarding when to make such investments.

(i) A retailer can make investments in experience activities in advance of the selling season without knowing the market state to take advantage of possible reduced investment costs.

(ii) A retailer can make investments in experience activities close to the selling season but typically incur a higher investment cost.

(iii) A retailer may follow a hybrid strategy by making some investments before and some after the market state realizes.

In this chapter we focus on understanding the temporal management of service investments under demand uncertainty and competition. We develop a two-stage model in order to examine two alternatives that retailers typically have in terms of timing their investments under both monopoly and symmetric duopoly settings. The first alternative is to invest in service in advance of the selling season without knowing the market state (i.e., invest in the first stage) and the second alternative is to invest in service after the market state realizes (i.e., invest in the second stage). In both cases a retailer decides on pricing after observing the demand (i.e., in the second stage). For the monopoly we further examine a hybrid strategy in which a retailer can invest both before and after the demand state is known. Typically, investing after the demand state is known is associated with higher investment costs. We analyze these settings under both equal and different investment costs across stages. In addition, we investigate the deterministic demand case for the symmetric duopoly and contrast our results with the stochastic demand case. In the case of a deterministic demand these alternatives translate to making sequential decisions (i.e., first service and then price) as opposed to simultaneous decisions (both service and price).

Our major findings are as follows. For a monopolist who faces stochastic demand and incurs different investment costs across stages we show that a hybrid strategy always dominates a strategy in which a retailer invests only before or only after the market state is known. In addition, we show that a monopolist would prefer to delay investments until demand realizes only when the market variability is high and the differential cost of investments across stages is low. In all other regimes a monopolist would prefer to invest before the demand realizes. This result is in contrast to the case of equal investment costs in which a monopolist would always defer investments in service after the demand realizes.

For symmetric duopolists who face deterministic demand and incur same investments costs across stages we show that the dominant strategy is always to invest in service in the first stage. We find that this is not always the case when the duopolists incur higher investment costs in the second stage. Interestingly, when the intensity of service competition exceeds a given threshold, a symmetric duopolist could be better off to invest in the second stage as the differential cost of investment in the two stages increases. We also find computationally that the equilibrium strategies for a symmetric retailer can shift in a non-monotonic fashion as the differential cost of investment in the two stages increases. In particular, a retailer could invest in the first stage for high and low differential costs and in the second stage for intermediate values of differential costs.

For symmetric duopolists who face stochastic demand and incur the same costs across stages the dominant strategy is to invest in service in the first stage in all regimes except for one characterized by high demand variability, low intensity of competition in service, and high investment cost. This result shows that the competitive dynamics could diminish significantly the value of delaying investments after demand realizes. We further characterize some of the investment strategies when a duopolist incurs higher investment costs in the second stage. Interestingly, we find that in the case of high intensity of service competition an increase in demand variability could make investing in the first stage more preferable than investing in the second stage provided that the differential costs of investments across stages exceeds a given threshold.

The remaining of the chapter proceeds as follows. In Section 4.2, we briefly discuss related literature. In Section 4.3, we present and analyze the monopoly. In Sections 4.4 and 4.5 we discuss the symmetric duopoly under deterministic and stochastic demands and characterize the dominant strategies. Finally in Section 4.6 we conclude this chapter.

4.2 Literature Review

Our research focuses on temporal management of demand-enhancing activities. The operations literature has extensively studied timing issues regarding capacity and inventory/production (see Van Mieghem and Dada (1999) for representative work) but the timing of investments in service activities that stimulate demand has escaped its attention. Regarding service provision, most papers in the literature have focused on designing contractual mechanisms to improve channel coordination. Winter (1993) studies a manufacturer with competing retailers who specify both price and service. The author finds that vertical restraints could achieve the first-best solution. Desiraju and Moorthy (1997) find that when the retailer has private demand information, the manufacturer could achieve higher profit by enforcing retail price and service performance requirements. Perry and Porter (1990) show that resale price maintenance and franchise fees could correct the sub-optimal level of retail service although resale price maintenance alone is not enough. Iyer (1998) studies how a manufacturer should respond to the consumer's location difference and the difference in the willingness to pay for the retail service when two retailers specify both the retail price and the quality of service for the manufacturer's product. Tsay and Agrawal (2000) study a setting in which a single manufacturer sells its product through two different retailers. They find that the retailers are better off when service plays a role in their competition than when they compete only based on price. Our demand model is a modification of Tsay and Agrawal (2000) for a stochastic demand environment but our focus is quite different. First, we are interested in understanding under what circumstances a monopolist should postpone or not the service investments until demand realizes. Second, we aim to understand the equilibrium investment strategies in a symmetric duopoly where each retailer can invest in service either before or after the demand realizes. Since there is no study in the literature studying the timing of service investments in an uncertain demand environment under competition, we aim to fill this gap with this study.

4.3 Model-Monopoly Benchmark

In the following, we first introduce the demand model, the sequence of events, and the firm's decisions.

We consider one retailer in the market introducing a new product. The retailer experiences a stochastic linear demand that is decreasing in retailer's price (p_k) and increasing in the retailer's service (s_k). Service captures all the activities mentioned in the introduction that could stimulate demand.

$$D_k(p_k, s_k) = \alpha_k - b_p p_k + b_s s_k \tag{4.1}$$

where $\alpha_k > 0$, $b_p > 0$ and $b_s \geq 0$.

α_k denotes the market base for the retailer at demand state k, $k = \{H, L\}$ taking values $\alpha_L = m - u$ and $\alpha_H = m + u$ with equal probabilities. Hence, $P(\alpha = m + u) = P(\alpha = m - u) = \frac{1}{2}$ where m is a measure of the mean demand and u is a measure of demand variability. We restrict our analysis to $0 < u < m$. We refer to α_L as the "low" market and similarly α_H as the "high" market. Mathematically, α_k is the demand faced by the retailer at demand state k when the retailer prices at 0 and offers no accompanying service. b_p and b_s measure the responsiveness of the retailer's market demand to his price and service respectively.

The retailer's cost of providing service level s_k is $\eta\frac{s_k^2}{2}$, where the quadratic form suggests diminishing returns of such expenditures and η (a strictly positive term which we refer to as "investment cost factor") denotes the effectiveness of the retailer in operational deployment of service. The above demand model is a modification of Tsay and Agrawal's model (cf. Tsay and Agrawal (2000)) for a monopoly setting with stochastic demand.

The retailer needs to decide the timing of service investments. We examine three alternatives regarding when to make such investments.

(i) A retailer can make investments in experience activities in advance of the selling season without knowing the market state to take advantage of possible reduced investment costs.

(ii) A retailer can make investments in experience activities close to the selling season but typically incur a higher investment cost.

(iii) A retailer may follow a hybrid strategy by making some investments before and some after the market state realizes.

We denote these three scenarios as F (investment only in first stage), S (investment only in second stage) and B (investment in both stages). In particular, in Stage 1 the retailer knows the distribution of demand and in Stage 2 he knows the realization of the demand. Stage 2 corresponds roughly to the beginning of the sales season and at this stage the retailer decides on pricing. At this time, the firm has observed interest in and demand for samples and most of the market uncertainty has been resolved. In the following subsection, we present each scenario and assume that making an investment at the second stage (i.e, after the demand is realized) is more expensive than at the first stage ($\eta_2 > \eta_1$). A reason behind this is that the retailer may have to incur expedited costs after the demand has realized in order to make the service investments.

4.3.1 Scenario F (Investment Only in First Stage)

In this scenario, the retailer decides on price after the demand has realized and on service before the demand realization. The demand at state k will be as follows: $D_k(p_k, s) = \alpha_k - b_p p_k + b_s s$. We solve this two-stage problem by backwards induction.

Second stage solution

In the second stage the demand is revealed to the retailer. The retailer decides on the optimal price to maximize his profit given in (4.2).

$$\pi_k(p_k, s) = p_k(\alpha_k - b_p p_k + b_s s), \quad k = \{H, L\} \tag{4.2}$$

The optimal price can be obtained by the first-order conditions since the profit function given the service level is concave in price (i.e., $\frac{\partial^2 \pi_k}{\partial p_k^2} = -2b_p < 0$).

$$p_i^*(s) = \frac{\alpha_k + b_s s}{2b_p}, \qquad k = \{H, L\}$$

First stage solution

In the first stage the retailer maximizes his expected profit ($\hat{\pi}$) provided in (4.3) by choosing the service level s.

$$\max_s \hat{\pi}(s) = \frac{1}{2}(p_L D_L(s) - \eta_1 \frac{s^2}{2}) + \frac{1}{2}(p_H D_H(s) - \eta_1 \frac{s^2}{2}) \tag{4.3}$$

The profit function is strictly concave in the service level s if $2\eta_1 b_p - b_s^2 > 0$. The optimal service level is found by applying the first order conditions and substituting the optimal service in the price, demand, and profit expressions, thus obtaining the following:

$$\begin{aligned}
p_L^F &= \frac{2\alpha_L(2\eta_1 b_p - b_s^2) + b_s^2(\alpha_H + \alpha_L)}{4b_p(2\eta_1 b_p - b_s^2)} \\
p_H^F &= \frac{2\alpha_H(2\eta_1 b_p - b_s^2) + b_s^2(\alpha_H + \alpha_L)}{4b_p(2\eta_1 b_p - b_s^2)} \\
s^F &= \frac{b_s(\alpha_H + \alpha_L)}{2(2\eta_1 b_p - b_s^2)} \\
D_L^F &= \frac{b_s^2(\alpha_H - \alpha_L) + 4b_p \alpha_L \eta_1}{2(4\eta_1 b_p - 2b_s^2)} \\
D_H^F &= \frac{4b_p \alpha_H \eta_1 - b_s^2(\alpha_H - \alpha_L)}{2(4\eta_1 b_p - 2b_s^2)} \\
\hat{\pi}^F &= \frac{4b_p(\alpha_H^2 + \alpha_L^2)\eta_1 - b_s^2(\alpha_H - \alpha_L)^2}{16b_p(2\eta_1 b_p - b_s^2)}
\end{aligned}$$

4.3.2 Scenario S (Investment Only in Second Stage)

In this scenario, the retailer decides on the service level and price simultaneously after the demand has realized, which reduces the problem to a single stage. The retailer solves

$$\max_{p_k,s_k} \pi_k(p_k s_k) = p_k D_k(p_k, s_k) - \eta_2 \frac{s_k^2}{2} \tag{4.4}$$

The first-order conditions are:

$$\frac{\partial \pi_k}{\partial p_k} = \alpha_k - 2b_p p_k + b_s s_k = 0 \tag{4.5}$$

$$\frac{\partial \pi_k}{\partial s_k} = -\eta_2 s_k + b_s p_k = 0 \tag{4.6}$$

The Hessian is

$$\begin{pmatrix} \frac{\partial^2 \pi_k}{\partial p_k^2} & \frac{\partial^2 \pi_k}{\partial p_k s_k} \\ \frac{\partial^2 \pi_k}{\partial s_k p_k} & \frac{\partial^2 \pi_k}{\partial s_k^2} \end{pmatrix} = \begin{pmatrix} -2b_p & b_s \\ b_s & -\eta_2 \end{pmatrix}$$

Second order conditions for profit maximization by the retailer will be satisfied if $2\eta_2 b_p - b_s^2 > 0$. The optimal price and service are obtained solving a system of equations from the first order conditions. Substituting the optimal price and service expressions on the demand function and profit we obtain the following expressions.

$$
\begin{aligned}
p_k^S &= \frac{\alpha_k \eta_2}{2\eta_2 b_p - b_s^2}, \quad k = \{H, L\} \\
s_k^S &= \frac{\alpha_k b_s}{2\eta_2 b_p - b_s^2}, \quad k = \{H, L\} \\
D_k^S &= \frac{\alpha_k \eta_2 b_p}{2\eta_2 b_p - b_s^2}, \quad k = \{H, L\} \\
\pi_k^S &= \frac{\alpha_k^2 \eta_2}{2(2\eta_2 b_p - b_s^2)}, \quad k = \{H, L\} \\
\hat{\pi}^S &= \frac{1}{2}\pi_L^S + \frac{1}{2}\pi_H^S
\end{aligned}
$$

4.3.3 Scenario B (Investment in Both Stages)

In this scenario, we assume that the retailer can invest s_1 in the first stage and s_2 in the second stage. We further make the assumption that the two service levels have an additive impact on demand i.e., $D_k(p_k, s_1, s_{2k}) = \alpha_k - b_p p_k + b_s(s_1 + s_{2k})$.

Second stage solution

In the second stage, the retailer decides on the service level s_2 and price after the demand has realized. The retailer solves the following problem

$$\max_{p_k, s_{2k}} \pi_k(p_k, s_1, s_{2k}) = p_k D_k(p_k, s_1, s_{2k}) - \eta_2 \frac{s_{2k}^2}{2} \tag{4.7}$$

Second order conditions for profit maximization by the retailer are satisfied if $2b_p\eta_2 - b_s^2 > 0$. From first order conditions we obtain:

$$\begin{aligned} p_k(s_1) &= \frac{(\alpha_k + b_s s_1)\eta_2}{2b_p\eta_2 - b_s^2}, \quad k = \{H, L\} \\ s_{2k}(s_1) &= \frac{(\alpha_k + b_s s_1)b_s}{2b_p\eta_2 - b_s^2}, \quad k = \{H, L\} \end{aligned}$$

First stage solution

In the first stage, the retailer maximizes his expected profit ($\hat{\pi}$) by choosing the service level s_1.

$$\max_{s_1} \hat{\pi}(s_1) = \frac{1}{2}(p_L D_L(s_1) - \eta_1\frac{s_1^2}{2} - \eta_2\frac{s_{2L}^2}{2}) + \frac{1}{2}(p_H D_H(s_1) - \eta_1\frac{s_1^2}{2} - \eta_2\frac{s_{2H}^2}{2}) \tag{4.8}$$

The profit function is strictly concave in the service level s_1 if $2b_p\eta_1\eta_2 - b_s^2(\eta_1 + \eta_2) > 0$. The optimal service ($s_1$) is obtained from first order conditions. From substitution we obtain the following expressions:

$$\begin{aligned}
p_L^B &= \frac{\eta_2(2\alpha_L(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2)) + b_s^2\eta_2(\alpha_H+\alpha_L))}{2(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))(2b_p\eta_2 - b_s^2)} \\
p_H^B &= \frac{\eta_2(2\alpha_H(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2)) + b_s^2\eta_2(\alpha_H+\alpha_L))}{2(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))(2b_p\eta_2 - b_s^2)} \\
s_1^B &= \frac{b_s(\alpha_H+\alpha_L)\eta_2}{2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2)} \\
s_{2L}^B &= \frac{b_s(2\alpha_L(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2)) + \eta_2 b_s^2(\alpha_H+\alpha_L))}{2(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))(2b_p\eta_2 - b_s^2)} \\
s_{2H}^B &= \frac{b_s(2\alpha_H(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2)) + \eta_2 b_s^2(\alpha_H+\alpha_L))}{2(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))(2b_p\eta_2 - b_s^2)} \\
D_L^B &= \frac{b_p\eta_2(4b_p\alpha_L\eta_1\eta_2 + b_s^2((\alpha_H-\alpha_L)\eta_2 - \alpha_L\eta_1))}{2(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))(2b_p\eta_2 - b_s^2)} \\
D_H^B &= \frac{b_p\eta_2(4b_p\alpha_H\eta_1\eta_2 - b_s^2(\alpha_H\eta_1 + (\alpha_H-\alpha_L)\eta_2))}{2(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))(2b_p\eta_2 - b_s^2)} \\
\hat{\pi}^B &= \frac{\eta_2(4b_p(\alpha_H^2+\alpha_L^2)\eta_1\eta_2 - b_s^2(2(\alpha_H^2+\alpha_L^2)\eta_1 + (\alpha_H-\alpha_L)^2\eta_2))}{8(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))(2b_p\eta_2 - b_s^2)}
\end{aligned}$$

4.3.4 Comparison of the scenarios in Monopoly

In this subsection, we compare the three scenarios in monopoly. The following proposition summarizes the effect of timing of service investments on the retailer's price, service level, demand, and profit.

Proposition 8 *For a monopolist we have the following ordering of prices, service levels, demands, and profits.*

(1) $\hat{p}^B > \hat{p}^F > \hat{p}^S$

(2) $s^F > \hat{s}^S$

(3) $s_1^B > s^F$

(4) $s_2^B > \hat{s}^S$

(5) $\hat{D}^B > \hat{D}^F > \hat{D}^S$

(6) $\hat{\pi}^B > \hat{\pi}^F$

(7) $\hat{\pi}^B > \hat{\pi}^S$

(8a) $\hat{\pi}^F \geq \hat{\pi}^S$ *if* $\eta_2 \geq \tilde{\eta_2}$

(8b) $\hat{\pi}^F \geq \hat{\pi}^S$ *if* $\eta_2 < \tilde{\eta_2}$ *and* $u \leq \hat{u}$ *and* $\hat{\pi}^F < \hat{\pi}^S$ *if* $\eta_2 < \tilde{\eta_2}$ *and* $u > \hat{u}$

where $\tilde{\eta_2} = \frac{4b_p\eta_1 - b_s^2}{2b_p}$ *and* $\hat{u} = m\sqrt{\frac{2b_p(\eta_2-\eta_1)}{2b_p\eta_1 - b_s^2}}$

Proposition 8 shows that a monopolist always benefits from having the opportunity to make service investments in both stages. Interestingly, the service level that a monopolist invests in the first stage in scenario B is higher than the corresponding service level in scenario F. This result is driven by the assumption that the service levels in both stages have an additive impact on demand, which creates demand enhancement and leads to service levels s_1 and s_2 being complements as opposed to substitutes. If this opportunity of investing in both stages does not exist and the monopolist has to decide between investing at the first stage only or at the second stage only, then the scenario that dominates depends on the relative magnitude of the investment cost factors in the two stages as well as the demand variability. If the investment cost at the second stage is high enough ($\eta_2 > \tilde{\eta_2}$), then scenario F dominates scenario S. Even if the investment cost at the second stage is low enough ($\eta_2 < \tilde{\eta_2}$), scenario F can still dominate scenario S provided that the demand variability is low ($u < \hat{u}$). In the case of high demand variability and low investment cost at the second stage scenario S dominates scenario F. Note that when the investment costs in the two stages are the same ($\eta_1 = \eta_2$), then a monopolist is always better off to postpone investments in service to the second stage (i.e., scenario S dominates scenario F). Figure 4.1 is a pictorial representation of the

η_2 \ u	L	H
L	F	S
H	F	F

FIGURE 4.1: Scenario Dominance for a Monopolist Under Stochastic Demand and Different Investment Costs

regimes under which a monopolist would prefer to invest before or after the demand realization. Note that low investment cost in the second stage corresponds to $\eta_2 < \tilde{\eta_2}$ and low demand variability corresponds to $u < \hat{u}$.

Lemma 1 *The impact of different parameters on thresholds $\tilde{\eta_2}$ and $\hat{u}$ is as follows:*

i) $\frac{\partial \tilde{\eta_2}}{\partial b_p} > 0$, $\frac{\partial \tilde{\eta_2}}{\partial b_s} < 0$, $\frac{\partial \tilde{\eta_2}}{\partial \eta_1} > 0$

ii) $\frac{\partial \hat{u}}{\partial b_p} < 0$, $\frac{\partial \hat{u}}{\partial b_s} > 0$, $\frac{\partial \hat{u}}{\partial m} > 0$, $\frac{\partial \hat{u}}{\partial \eta_1} < 0$, $\frac{\partial \hat{u}}{\partial \eta_2} > 0$

Lemma 1 presents comparative statics on thresholds $\tilde{\eta_2}$ and $\hat{u}$. Let us consider the case where the investment cost at the second stage is low (i.e., $\eta_2 < \tilde{\eta_2}$). Note that as the elasticity of price (b_p) increases the region under which scenario S dominates F becomes larger. This is because as the elasticity of price increases (i.e., consumers are very price sensitive) a retailer could be better off offering low price (which is the case under regime S) even for low demand variability. On the other hand, as the elasticity of service (b_s) increases the region under which scenario S dominates F becomes smaller. As the elasticity of service increases (i.e., consumers are very service sensitive) a retailer would only be better off offering low service (which is the case under regime S) only if demand variability is very high.

In the following section we describe the duopoly model. We first discuss the deterministic demand in order to gain some insights regarding the dynamics of price and service competition and then analyze the stochastic demand.

4.4 Duopoly - Deterministic Demand

We now consider two symmetric retailers in the market indexed by $i \in \{1, 2\}$ and $j = 3-i$ selling the same product. The retailers engage in price and service competition. The customer demand faced by retailer i is: $D_i(p_i, p_j, s_i, s_j) = \alpha - b_p p_i + \theta_p(p_j - p_i) + b_s s_i - \theta_s(s_j - s_i)$, where $\alpha, b_p > 0$, b_s, θ_p, $\theta_s \geq 0$. θ_p, θ_s denote intensity of competition between the two retailers with regards to pricing and service behavior. This functional form of demand has the desirable property that for a fixed set of retailers actions, the total market size $(D_1 + D_2)$ is invariant to changes in θ_p or θ_s.

We assume that both retailers have the ability to make service investments. We focus on the following symmetric outcomes:

(1) Both retailers invest in the first stage (scenario FF).

(2) Both retailers invest in the second stage (scenario SS).

In the following subsection we analyze the two scenarios (FF and SS) under the assumption that investment costs of the two stages are the same (i.e., $\eta_1 = \eta_2 = \eta$) in order to isolate the effect of competition on the decision of the retailers to time such investments. We will later waive this assumption and consider the case where the investment at the second stage is more expensive than that at the first stage ($\eta_2 > \eta_1$).

4.4.1 Scenario FF (Two Competing Retailers Investing in First Stage)

In this scenario, both retailers decide on prices in the second stage and on service levels in the first stage.

Second stage solution

In the second stage, the retailers engage only in price competition. The demand for retailer i is given by $D_i(p_i, p_j, s_i, s_j) = \alpha - b_p p_i + \theta_p(p_j - p_i) + b_s s_i - \theta_s(s_j - s_i)$.

The profit function for retailer i is equal to his revenue since he does not incur any investment cost at the second stage.

$$\pi_i(p_i, p_j, s_i, s_j) = p_i(\alpha - b_p p_i + \theta_p(p_j - p_i) + b_s s_i - \theta_s(s_j - s_i)) \quad i \in \{1, 2\},\ j = 3 - i,$$

The first-order conditions are:

$$\frac{\partial \pi_i}{\partial p_i} = \alpha - b_p p_i + b_s s_i - p_i(b_p + \theta_p) + \theta_p(p_j - p_i) - (s_j - s_i)\theta_s = 0 \qquad (4.9)$$

Since $\frac{\partial^2 \pi_i}{\partial p_i^2} = -2(b_p + \theta_p) < 0$, the profit function given service levels is strictly concave in prices. Solving for p_1 and p_2 simultaneously from the above two equations, we obtain the equilibrium prices:

$$p_i(s_i, s_j) = \frac{R + Ss_i - Ts_j}{W}, \quad i \in \{1, 2\},\ j = 3 - i, \tag{4.10}$$

where

$R = \alpha(2b_p + 3\theta_p)$

$S = 2b_p(\theta_s + b_s) + \theta_p(2b_s + \theta_s)$

$T = \theta_s(2b_p + \theta_p) - \theta_p b_s$

$W = (2b_p + \theta_p)(2b_p + 3\theta_p)$

The corresponding demand quantities at the equilibrium prices are:

$$D_i(s_i, s_j) = \frac{(R + Ss_i - Ts_j)(\theta_p + b_p)}{W}, \quad i \in \{1, 2\},\ j = 3 - i. \tag{4.11}$$

From (4.10) and (4.11) note that when $T < 0$, for a fixed choice of retailer i's service level, retailer i will see its equilibrium price as well as the demand at this equilibrium price increase as competing retailer j increases its service level. Hence, we assume that $T > 0$, i.e.,

$$T > 0 \Leftrightarrow \theta_s(2b_p + \theta_p) - \theta_p b_s > 0 \tag{4.12}$$

First stage solution

In the first stage, retailer i maximizes his profit (π_i) by choosing his service level s_i.

$$\max \pi_i(s_i) = p_i D_i(s_i) - \eta \frac{s_i^2}{2}. \tag{4.13}$$

The profit function is strictly concave in the service level s_i if $W^2\eta - 2(\theta_p + b_p)S^2 > 0$. Further, for a pure-strategy Nash equilibrium to exist, we require that the reaction functions for the two retailers intersect once. A sufficient condition for this to happen is the following (Tirole (1990)):

$$\left|\frac{\partial^2 \pi_i}{\partial s_i^2}\right| > \left|\frac{\partial^2 \pi_i}{\partial s_i s_j}\right| \Leftrightarrow W^2\eta - 2(\theta_p + b_p)S^2 - 2ST(\theta_p + b_p) > 0 \tag{4.14}$$

The optimal service levels, prices, demands, and profits are provided below:

$$\begin{aligned}
p_i^{FF} &= \frac{RW\eta}{W^2\eta - 2S(S-T)(\theta_p + b_p)} \quad i \in \{1,2\} \\
s_i^{FF} &= \frac{2RS(\theta_p + b_p)}{W^2\eta - 2S(S-T)(\theta_p + b_p)} \quad i \in \{1,2\} \\
D_i^{FF} &= \frac{RW\eta(\theta_p + b_p)}{W^2\eta - 2S(S-T)(\theta_p + b_p)} \quad i \in \{1,2\} \\
\pi_i^{FF} &= \frac{R^2\eta(\theta_p + b_p)(W^2\eta - 2S^2(\theta_p + b_p))}{(W^2\eta - 2S(S-T)(\theta_p + b_p))^2} \quad i \in \{1,2\}
\end{aligned}$$

Note that the diagonal dominance condition (4.14) is sufficient to ensure concavity of the profit function in the service levels as well as positive service levels, prices, and demands.

4.4.2 Scenario SS (Two Competing Retailers Investing in Second Stage)

In this scenario, both retailers decide on service levels and prices in the second stage. As a result, the retailers engage in simultaneous price and service competition.

Retailer i solves $\max \pi_i(p_i, p_j, s_i, s_j) = p_i(\alpha - b_p p_i + \theta_p(p_j - p_i) + b_s s_i - \theta_s(s_j - s_i)) - \eta \frac{s_i^2}{2}$, $i \in \{1, 2\}$ and $j = 3 - i$. The first-order conditions are:

$$\frac{\partial \pi_i}{\partial p_i} = \alpha - b_p p_i + b_s s_i - p_i(b_p + \theta_p) + \theta_p(p_j - p_i) - (s_j - s_i)\theta_s = 0 \quad (4.15)$$

$$\frac{\partial \pi_i}{\partial s_i} = -\eta s_i + p_i(b_s + \theta_s) = 0 \quad (4.16)$$

The Hessian is

$$\begin{pmatrix} \frac{\partial^2 \pi_i}{\partial p_i^2} & \frac{\partial^2 \pi_i}{\partial p_i s_i} \\ \frac{\partial^2 \pi_i}{\partial s_i p_i} & \frac{\partial^2 \pi_i}{\partial s_i^2} \end{pmatrix} = \begin{pmatrix} -2(b_p + \theta_p) & b_s + \theta_s \\ b_s + \theta_s & -\eta \end{pmatrix}$$

Second order conditions for profit maximization by retailer i will be satisfied if $2\eta(b_p + \theta_p) - (b_s + \theta_s)^2 > 0$. Joint concavity of the profit function in (p, s) ensures existence of a pure equilibrium strategy. Uniqueness is not too difficult to demonstrate in this case since the equilibrium is symmetric. Since the players have two-dimensional strategies, finding a symmetric equilibrium reduces to determining whether a system of 2 equations has a unique solution (Cachon and Netessine (2004)). The conditions that ensure joint concavity are sufficient in our case to ensure the existence of a unique solution. The optimal price, service level, demand and profit expressions for retailer i

are given below:

$$\begin{aligned} p_i^{SS} &= \frac{\alpha\eta}{\eta(2b_p+\theta_p)-b_s(b_s+\theta_s)} \quad i\in\{1,2\} \\ s_i^{SS} &= \frac{\alpha(\theta_s+b_s)}{\eta(2b_p+\theta_p)-b_s(b_s+\theta_s)} \quad i\in\{1,2\} \\ D_i^{SS} &= \frac{\alpha\eta(\theta_p+b_p)}{\eta(2b_p+\theta_p)-b_s(b_s+\theta_s)} \quad i\in\{1,2\} \\ \pi_i^{SS} &= \frac{\alpha^2\eta(2\eta(\theta_p+b_p)-(b_s+\theta_s)^2)}{2(\eta(2b_p+\theta_p)-b_s(b_s+\theta_s))^2} \quad i\in\{1,2\} \end{aligned}$$

We impose $2\eta(b_p+\theta_p)-(b_s+\theta_s)^2>0$ to ensure that the second order conditions are satisfied. In addition, we impose $\eta(2b_p+\theta_p)-b_s(b_s+\theta_s)>0$ to ensure positive service and price.

4.4.3 Comparison of scenarios FF and SS (same investment costs)

In this subsection, we compare scenarios FF and SS in duopoly under deterministic demand and same investment costs across stages. The following proposition summarizes the effect of timing of service investments on the duopolists' prices, service levels, demands, and profits.

Proposition 9 *For duopolist i we have the following ordering of prices, service levels, demands, and profits.*

(1) $p_i^{FF} < p_i^{SS}$

(2) $s_i^{FF} < s_i^{SS}$

(3) $D_i^{FF} < D_i^{SS}$

(4) $\pi_i^{FF} > \pi_i^{SS}$

Proposition 9 shows that a symmetric duopolist always charges higher price and invests in a higher level of service when he does not commit to service (i.e., invests in the second stage). In addition, he enjoys higher demand if he invests in service provision in the second stage. Interestingly, a duopolist will have higher profit if he makes an investment in the first stage. The reason behind this is that the investment cost is quadratic and the investment cost per unit of service η needs to be large enough to ensure existence and uniqueness of the NE in both scenarios. As a result, a duopolist will prefer to invest less in service which is the case if he makes a service investment upfront. Hence, a duopolist will always be better off in Scenario FF.

We next consider the case where the investment costs in the two stages are different and specifically when $\eta_2 > \eta_1$. Since the derivation of the expressions for scenarios FF and SS is done in a similar fashion as in the case with the same investment costs it can be omitted. We next present the comparison of the scenarios FF and SS under different investment costs.

4.4.4 Comparison of scenarios FF and SS (different investment costs)

The following proposition summarizes the effect of timing of service investments on the duopolist's price, service level, and demand in the case where the investment cost in the second stage is higher than the corresponding cost in the first stage.

Proposition 10 *For duopolist i we have the following ordering of prices, service levels, and demands.*

(1) If $\frac{\eta_2}{\eta_1} \geq \bar{\eta}$*, then* $p_i^{FF} \geq p_i^{SS}$ *and if* $\frac{\eta_2}{\eta_1} < \bar{\eta}$*, then* $p_i^{FF} < p_i^{SS}$.

(2) If $\frac{\eta_2}{\eta_1} \geq \bar{\eta}$*, then* $s_i^{FF} \geq s_i^{SS}$ *and if* $\frac{\eta_2}{\eta_1} < \bar{\eta}$*, then* $s_i^{FF} < s_i^{SS}$.

(3) If $\frac{\eta_2}{\eta_1} \geq \bar{\eta}$*, then* $D_i^{FF} \geq D_i^{SS}$ *and if* $\frac{\eta_2}{\eta_1} < \bar{\eta}$*, then* $D_i^{FF} < D_i^{SS}$,

where $\bar{\eta} = \frac{(2b_p+\theta_p)(2b_p+3\theta_p)(b_s+\theta_s)}{2(b_p+\theta_p)(2b_s(b_p+\theta_p)+\theta_s(2b_p+\theta_p))}$.

Proposition 10 shows that if the investment cost in the second stage is high enough ($\frac{\eta_2}{\eta_1} > \bar{\eta}$), a symmetric duopolist will charge higher price, invest in higher service, and enjoy higher demand in the first stage. Otherwise, he will invest in higher service in the second stage and hence charge higher price and enjoy higher demand.

For the remaining of the analysis we let for simplicity of exposition $\eta_1 = \eta$ and $\eta_2 = \eta + \epsilon$, where $\epsilon > 0$. We first illustrate how the profit function of a symmetric duopolist in scenario SS changes with respect to ϵ. This allow us to derive some of the conditions under which investment in the second stage dominates investment in the first stage and vice versa.

Proposition 11 *Let* $\epsilon_1 = \frac{b_s(b_s+\theta_s)^2+\eta(2b_p+\theta_p)\theta_s-b_s(2b_p+3\theta_p)}{b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s}$*, then we have the following cases:*

1. *If* $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ *and* $\epsilon \geq \epsilon_1$*, then* $\frac{\partial \pi_i^{SS}}{\partial \epsilon} \leq 0$.
2. *If* $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ *and* $\epsilon < \epsilon_1$*, then* $\frac{\partial \pi_i^{SS}}{\partial \epsilon} > 0$.
3. *If* $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$*, then* $\frac{\partial \pi_i^{SS}}{\partial \epsilon} > 0$.

Note that when there is competition across two dimensions (i.e., price and service), then an increase in the differential investment cost of the two stages (ϵ) could result in profit increase for a symmetric duopolist.

The following proposition presents two of the regimes that we are able to fully characterize analytically.

Proposition 12 *a) If* $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ *and* $\eta > \max\{\frac{b_s(b_s+\theta_s)^2}{b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s}, \eta_{max}\}$, *then* $\pi_i^{SS} < \pi_i^{FF}$ $\forall\ \epsilon > 0$.

b) If $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$, *then there exists a threshold* $\bar{\epsilon_3}$ *such that if* $\epsilon < \bar{\epsilon_3}$, *then* $\pi_i^{SS} < \pi_i^{FF}$ *and if* $\epsilon > \bar{\epsilon_3}$, *then* $\pi_i^{SS} > \pi_i^{FF}$. *The expressions for* η_{max} *and* $\bar{\epsilon_3}$ *are provided in the Appendix.*

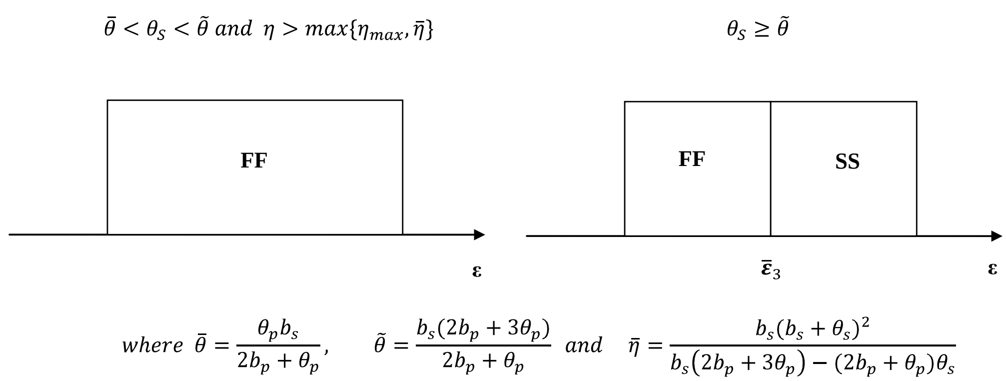


FIGURE 4.2: Scenario Dominance (obtained analytically) for a Symmetric Duopolist Under Deterministic Demand and Different Investment Costs

Figure 4.2 summarizes the regimes that we obtain analytically.

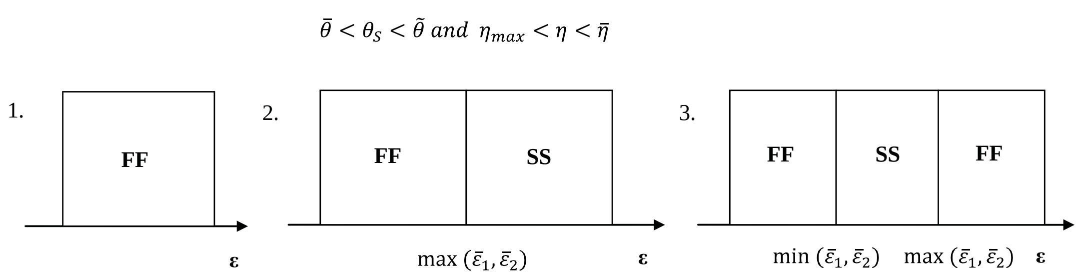
The regime that we are not able to fully characterize is the following: $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\eta_{max} < \eta < \frac{b_s(b_s+\theta_s)^2}{b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s}$. For the above regime a numerical study shows that there could be several different cases depending on the parameter values.

1. We could have $\pi_i^{SS} < \pi_i^{FF}\ \forall\ \epsilon > 0$.

2. There could exist two thresholds of ϵ (i.e., $\bar{\epsilon_1}$, $\bar{\epsilon_2} > 0$) such that if $\epsilon < \min\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$ or $\epsilon > \max\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$ then $\pi_i^{SS} < \pi_i^{FF}$ else if $\min\{\bar{\epsilon_1}, \bar{\epsilon_2}\} \leq \epsilon \leq \max\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$ then $\pi_i^{SS} \geq \pi_i^{FF}$.

3. There could exist two thresholds of ϵ (i.e., $\bar{\epsilon_1}$, $\bar{\epsilon_2}$) such that $\min\{\bar{\epsilon_1}, \bar{\epsilon_2}\} < 0$ and $\max\{\bar{\epsilon_1}, \bar{\epsilon_2}\} > 0$. If $0 < \epsilon < \max\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$ then $\pi_i^{SS} < \pi_i^{FF}$ and if $\epsilon > \max\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$ then $\pi_i^{SS} > \pi_i^{FF}$.

Figure 4.3 summarizes the regimes that we obtain numerically. Note that unlike the



$$where\ \bar{\theta} = \frac{\theta_p b_s}{2b_p + \theta_p}, \quad \tilde{\theta} = \frac{b_s(2b_p + 3\theta_p)}{2b_p + \theta_p}\ and\ \bar{\eta} = \frac{b_s(b_s + \theta_s)^2}{b_s(2b_p + 3\theta_p) - (2b_p + \theta_p)\theta_s}$$

FIGURE 4.3: Scenario Dominance (obtained numerically) for a Symmetric Duopolist Under Deterministic Demand and Different Investment Costs

deterministic case with equal costs, where the dominant strategy is always to invest in the first stage, in the case of higher investment costs in the second stage we can have a richer set of investment strategies depending on the parametric setting.

First, when the intensity of competition with regard to service (θ_s) is low (i.e., $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$) and the investment cost is high (i.e., $\eta > \max\{\bar{\eta}, \eta_{max}\}$), then the dominant strategy would be for a symmetric retailer to invest in the first stage.

Second, when the intensity of competition with regard to service (θ_s) is low (i.e., $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$) and the investment cost is low (i.e., $\eta_{max} < \eta < \bar{\eta}$), then we could observe a non-monotonic behavior on the Nash equilibrium strategies in the sense that for low (i.e., $\epsilon < \min\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$) or high values of ϵ (i.e., $\epsilon > \max\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$), a symmetric duopolist would be better of investing in the first stage whereas for intermediate values he would be better off postponing investments to the second stage.

Third, when the intensity of service competition is high (i.e., $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$), then ϵ has an interesting impact on the dominant outcome. We show that as the cost differential of the two stages (i.e., ϵ) increases, a symmetric duopolist would be better off making the investment in the second stage. We also observe such instances numerically even when the intensity of service competition is low for some parameter values.

4.5 Duopoly - Stochastic Demand

In this section, we describe the duopoly model and discuss the timing of the service investment under demand uncertainty. The customer demand faced by retailer i at demand state k is: $D_{ik}(p_{ik}, p_{jk}, s_{ik}, s_{jk}) = \alpha_k - b_p p_{ik} + \theta_p(p_{jk} - p_{ik}) + b_s s_{ik} - \theta_s(s_{jk} - s_{ik})$ where $\alpha_k, b_p > 0$ and $b_s, \theta_p, \theta_s \geq 0$.

As in the monopoly case, α_k takes values $\alpha_L = m - u$ and $\alpha_H = m + u$ for low demand and high demand state with equal probabilities. We focus our attention on the following scenarios:

(1) Both retailers invest before the demand realizes (scenario FF).

(2) Both retailers invest after the demand realizes (scenario SS).

As in the case of deterministic demand, we first analyze the two scenarios (FF and SS) under the assumption that investment costs of the two stages are the same (i.e., $\eta_1 = \eta_2 = \eta$) and later examine the case where the investment cost at the second stage is more expensive than that at the first stage ($\eta_2 > \eta_1$).

4.5.1 Scenario FF (Two Competing Retailers Investing in First Stage)

In this case both retailers decide on price after the demand has realized and on service before the demand realization. The demand for retailer i at demand state k is given by $D_{ik}(p_{ik}, p_{jk}, s_i, s_j) = \alpha_k - b_p p_{ik} + \theta_p(p_{jk} - p_{ik}) + b_s s_i - \theta_s(s_j - s_i)$.

Second stage solution

The profit function for retailer i is equal to his revenue since he does not incur any investment cost at the second stage.

$$\pi_{ik}(p_{ik}, p_{jk}, s_i, s_j) = p_{ik}(\alpha_k - b_p p_{ik} + \theta_p(p_{jk} - p_{ik}) + b_s s_i - \theta_s(s_j - s_i)), \qquad (4.17)$$

$$i \in \{1, 2\},\ j = 3 - i,\ k \in \{H, L\}$$

The first-order conditions are:

$$\frac{\partial \pi_{ik}}{\partial p_{ik}} = \alpha_k - 2(b_p + \theta_p)p_{ik} + \theta_p p_{jk} + (b_s + \theta_s)s_{ik} - \theta_s s_{jk} = 0, \quad i \in \{1,2\},\ j = 3-i,\ k \in \{H, L\} \tag{4.18}$$

Since $\frac{\partial^2 \pi_{ik}}{\partial p_{ik}^2} = -2(b_p + \theta_p) < 0$, the profit function given the service levels is strictly concave in prices. Solving for p_{1k} and p_{2k} simultaneously from the above two equations, we obtain the equilibrium price for each retailer:

$$p_{ik}(s_i, s_j) = \frac{R_k + Ss_i - Ts_j}{W} \quad i \in \{1,2\},\ j = 3-i,\ k \in \{H, L\} \tag{4.19}$$

where

$R_k = \alpha_k(2b_p + 3\theta_p),\ k \in \{H, L\}$

$S = 2b_p(\theta_s + b_s) + \theta_p(2b_s + \theta_s)$

$T = \theta_s(2b_p + \theta_p) - \theta_p b_s$

$W = (2b_p + \theta_p)(2b_p + 3\theta_p)$

The corresponding demand quantities at the equilibrium prices are:

$$D_{ik}(s_i, s_j) = \frac{(R_k + Ss_i - Ts_j)(\theta_p + b_p)}{W} \quad i \in \{1,2\},\ j = 3-i,\ k \in \{H, L\} \tag{4.20}$$

From (4.19) and (4.20) note that when $T < 0$ for a fixed choice of retailer i's service level, retailer i will see its equilibrium price as well as the demand at this equilibrium price increase as competing retailer j increases its service level. Hence, we assume that

$T > 0$, i.e.,

$$T > 0 \Leftrightarrow \theta_s(2b_p + \theta_p) - \theta_p b_s > 0 \tag{4.21}$$

First stage solution

In this stage retailer i maximizes his expected profit ($\hat{\pi}_i$) by choosing his service level s_i.

$$\max \hat{\pi}_i(s_i) = \frac{1}{2}(p_{iL}D_{iL}(s_i) - \eta\frac{s_i^2}{2}) + \frac{1}{2}(p_{iH}D_{iH}(s_i) - \eta\frac{s_i^2}{2}) \tag{4.22}$$

The profit function is strictly concave in the service level s_i if $W^2\eta - 2S^2(b_p + \theta_p) > 0$. Further, for a pure-strategy Nash equilibrium to exist, we require that the reaction functions for the two retailers intersect once. A sufficient condition for this to happen is the following (Tirole (1990)):

$$\left|\frac{\partial^2\hat{\pi}_i}{\partial s_i^2}\right| > \left|\frac{\partial^2\hat{\pi}_i}{\partial s_i s_j}\right| \Leftrightarrow W^2\eta - 2S(S+T)(\theta_p + b_p) > 0 \tag{4.23}$$

The optimal service, price, demand, and profit expressions are as follows:

$$
\begin{aligned}
p_{iL}^{FF} &= \frac{2W^2\eta R_L + 2S(S-T)(b_p+\theta_p)(R_H-R_L)}{2W(W^2\eta - 2S(S-T)(b_p+\theta_p))} \quad i \in \{1,2\} \\
p_{iH}^{FF} &= \frac{2W^2\eta R_H - 2S(S-T)(b_p+\theta_p)(R_H-R_L)}{2W(W^2\eta - 2S(S-T)(b_p+\theta_p))} \quad i \in \{1,2\} \\
s_i^{FF} &= \frac{2S(R_H+R_L)(b_p+\theta_p)}{2(W^2\eta - 2S(S-T)(b_p+\theta_p))} \quad i \in \{1,2\} \\
D_{iL}^{FF} &= \frac{(b_p+\theta_p)(2W^2\eta R_L + 2S(S-T)(b_p+\theta_p)(R_H-R_L))}{2W(W^2\eta - 2S(S-T)(b_p+\theta_p))} \quad i \in \{1,2\} \\
D_{iH}^{FF} &= \frac{(b_p+\theta_p)(2W^2\eta R_H - 2S(S-T)(b_p+\theta_p)(R_H-R_L))}{2W(W^2\eta - 2S(S-T)(b_p+\theta_p))} \quad i \in \{1,2\} \\
\hat{\pi}_i^{FF} &= \frac{(b_p+\theta_p)(R_H-R_L)^2(W^2\eta - 2S(S-T)(b_p+\theta_p))^2}{4W^2(W^2\eta - 2S(S-T)(b_p+\theta_p))^2} \\
&+ \frac{(b_p+\theta_p)(R_H+R_L)^2W^2\eta(W^2\eta - 2S^2(b_p+\theta_p))}{2W^2(W^2\eta - 2S(S-T)(b_p+\theta_p))^2} \quad i \in \{1,2\}
\end{aligned}
$$

Note that the diagonal dominance condition (4.23) is sufficient to ensure concavity of the profit function in the service level as well as positive service, price, and demand.

4.5.2 Scenario SS (Two Competing Retailers Investing in Second Stage)

In this case both retailers decide on both the service level and price after the demand has realized. Retailer i solves $\max \pi_{ik}(p_{ik}, p_{jk}, s_{ik}, s_{jk}) = p_{ik}D_{ik}(p_{ik}, p_{jk}, s_{ik}, s_{jk}) - \eta\frac{s_{ik}^2}{2}$, $i \in \{1,2\}$, $j = 3-i$, and $k \in \{H, L\}$.

The first-order conditions are:

$$\frac{\partial \pi_{ik}}{\partial p_{ik}} = \alpha_k - 2(b_p + \theta_p)p_{ik} + \theta_p p_{jk} + (b_s + \theta_s)s_{ik} - \theta_s s_{jk} = 0 \tag{4.24}$$

$$\frac{\partial \pi_{ik}}{\partial s_{ik}} = -\eta s_{ik} + p_{ik}(b_s + \theta_s) = 0 \tag{4.25}$$

The Hessian is

$$\begin{pmatrix} \frac{\partial^2 \pi_{ik}}{\partial p_{ik}^2} & \frac{\partial^2 \pi_{ik}}{\partial p_{ik} s_{ik}} \\ \frac{\partial^2 \pi_{ik}}{\partial s_{ik} p_{ik}} & \frac{\partial^2 \pi_{ik}}{\partial s_{ik}^2} \end{pmatrix} = \begin{pmatrix} -2(b_p + \theta_p) & b_s + \theta_s \\ b_s + \theta_s & -\eta \end{pmatrix}$$

Second order conditions for profit maximization by retailer i will be satisfied if $2\eta(b_p + \theta_p) - (b_s + \theta_s)^2 > 0$ which is also a sufficient condition to ensure existence and uniqueness of the Nash equilibrium in terms of price and service (see previous discussion in the deterministic demand). The optimal expressions for retailer i are given below:

$$p_{ik}^{SS} = \frac{\alpha_k \eta}{\eta(2b_p + \theta_p) - b_s(b_s + \theta_s)} \quad i \in \{1,2\},\ k \in \{H, L\}$$

$$s_{ik}^{SS} = \frac{\alpha_k (b_s + \theta_s)}{\eta(2b_p + \theta_p) - b_s(b_s + \theta_s)} \quad i \in \{1,2\},\ k \in \{H, L\}$$

$$D_{ik}^{SS} = \frac{\alpha_k \eta (b_p + \theta_p)}{\eta(2b_p + \theta_p) - b_s(b_s + \theta_s)} \quad i \in \{1,2\},\ k \in \{H, L\}$$

$$\pi_{ik}^{SS} = \frac{\alpha_k^2 \eta (2\eta(b_p + \theta_p) - (b_s + \theta_s)^2)}{2(\eta(2b_p + \theta_p) - b_s(b_s + \theta_s))^2} \quad i \in \{1,2\},\ k \in \{H, L\}$$

$$\hat{\pi}_i^{SS} = \frac{1}{2}\pi_{iL}^{SS} + \frac{1}{2}\pi_{iH}^{SS} \quad i \in \{1,2\}$$

We impose $2\eta(b_p + \theta_p) - (b_s + \theta_s)^2 > 0$ to ensure that the second order conditions are satisfied. In addition, we impose $(2b_p + \theta_p)\eta - b_s(b_s + \theta_s) > 0$ to ensure positive service and price.

4.5.3 Comparison of scenarios FF and SS (same investment costs)

In this subsection, we compare scenarios FF and SS in duopoly under stochastic demand and same investment costs across stages. The following proposition summarizes the effect of timing of service investments on the duopolist's price, service level, demand, and profits.

Proposition 13 *For duopolist i we have the following ordering of prices, service levels, demands, and profits.*

(1) $\hat{p_i}^{FF} < \hat{p_i}^{SS}$

(2) $s_i^{FF} < \hat{s_i}^{SS}$

(3) $\hat{D_i}^{FF} < \hat{D_i}^{SS}$

(4) If $\eta_{max} < \eta < \hat{\eta}$ *then* $\hat{\pi_i}^{FF} > \hat{\pi_i}^{SS}$.

(5) If $\eta > max\{\hat{\eta}, \eta_{max}\}$, $\frac{b_s\theta_p}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$, *and* $u \leq \bar{u}$ *then* $\hat{\pi_i}^{FF} \geq \hat{\pi_i}^{SS}$ *whereas if* $u > \bar{u}$ *then* $\hat{\pi_i}^{FF} < \hat{\pi_i}^{SS}$.

(6) If $\eta > max\{\hat{\eta}, \eta_{max}\}$ *and* $\theta_s > \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ *then* $\hat{\pi_i}^{FF} > \hat{\pi_i}^{SS}$.

The expressions for $\hat{\eta}$, η_{max}, *and* $\bar{u}$ *are provided in the Appendix.*

Proposition 13 shows that a duopolist always prices lower, offers lower service, and has lower demand if he invests in the first stage. The strategy that a duopolist should follow to maximize his profits depends on his investment cost, the intensity of competition in

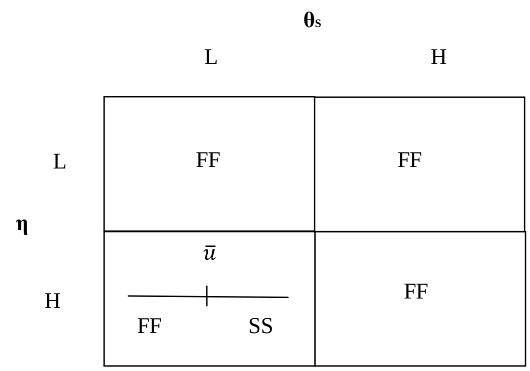


FIGURE 4.4: Scenario Dominance for a Symmetric Duopolist Under Stochastic Demand and Equal Investment Costs

service as well as the degree of demand variability. Figure 4.4 is a pictorial representation of the regimes under which a duopolist would prefer to invest before or after the demand realization. Note that low investment cost corresponds to $\eta_{max} < \eta < \hat{\eta}$, low intensity of competition in service corresponds to $\frac{b_s\theta_p}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and low demand variability corresponds to $u < \bar{u}$. There are four possible quadrants depending on the value of the investment cost and the intensity of competition in service. Quadrant 1 corresponds to the case of low investment cost and low intensity of competition in service. Quadrant 2 corresponds to the case of low investment cost and high intensity of competition in service. Quadrant 3 corresponds to the case of high investment cost and low intensity of competition in service. Quadrant 4 corresponds to the case of high investment cost and high intensity of competition in service. Note that in contrast to a monopolist whose dominant strategy is always to postpone service provision under same investment costs, a duopolist would consider commitment to service as the dominant strategy in most regimes. The only regime under which a duopolist would delay service investments after demand realization is the case of high investment cost, low intensity of competition in service, and high demand variability. Hence, we find that in the face of competition the value of delaying service investments can be significantly diminished. The following proposition presents an ordering of the effect of demand variability on profits for scenarios F, S (under monopoly) and FF, SS (under duopoly).

Proposition 14 *i) if η and θ_s are in quadrants 1,2, or 4 (see Figure 4.4), then* $\frac{\partial \hat{\pi}_i{}^S}{\partial u} > \frac{\partial \hat{\pi}_i{}^F}{\partial u} > \frac{\partial \hat{\pi}_i{}^{FF}}{\partial u} > \frac{\partial \hat{\pi}_i{}^{SS}}{\partial u}$.

ii) if η and θ_s are in quadrant 3 (see Figure 4.4), then $\frac{\partial \hat{\pi}_i{}^S}{\partial u} > \frac{\partial \hat{\pi}_i{}^F}{\partial u}$ *and* $\frac{\partial \hat{\pi}_i{}^{SS}}{\partial u} > \frac{\partial \hat{\pi}_i{}^{FF}}{\partial u}$.

In all quadrants other than 3 (where the investment cost is high and the intensity of competition in service is low) demand variability has a greater impact on profits in the monopoly as opposed to the duopoly. Moreover, in those quadrants demand variability has the most impact on a monopolist's profits who postpones investment to the second stage and the least impact on a symmetric duopolist's profits who postpones investment to the second stage.

4.5.4 Comparison of scenarios FF and SS (different investment costs)

We now consider the stochastic demand case where the cost of investing in the second stage is higher than that in the first stage. We first illustrate how demand variability affects the profits of a symmetric duopolist in scenarios FF and SS. This allows us to derive some of the conditions under which investment in the second stage would dominate investment in the first stage and vice versa.

Proposition 15 *Let* $\epsilon_2 = \frac{2b_s^2(b_s+\theta_s)(b_p+\theta_p)+\eta((2b_p+\theta_p)\theta_s-b_s(2b_p+3\theta_p))(2b_p+\theta_p)}{(b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s)(2b_p+\theta_p)}$*, then we have the following cases:*

1. *If* $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ *and* $\epsilon \leq \epsilon_2$*, then* $\frac{\partial \hat{\pi}_i{}^{FF}}{\partial u} \geq \frac{\partial \hat{\pi}_i{}^{SS}}{\partial u}$.
2. *If* $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ *and* $\epsilon > \epsilon_2$*, then* $\frac{\partial \hat{\pi}_i{}^{FF}}{\partial u} < \frac{\partial \hat{\pi}_i{}^{SS}}{\partial u}$.
3. *If* $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$*, then* $\frac{\partial \hat{\pi}_i{}^{FF}}{\partial u} > \frac{\partial \hat{\pi}_i{}^{SS}}{\partial u}$.

Note that as the variability of demand increases there are regimes where the expected profit of a symmetric duopolist in scenario FF increases at a higher rate than the corresponding expected profit at scenario SS. As a result there could be regimes in which the dominant strategy could shift from investing in the second stage to investing in the first stage as the variability of demand increases.

Proposition 16 *a) If* $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ *and* $\epsilon < \bar{\epsilon_3}$*, then* $\hat{\pi}_i{}^{SS} < \hat{\pi}_i{}^{FF}$*. If* $\epsilon > \bar{\epsilon_3}$ *then there exists a threshold* $\bar{u_1}$ *such that if* $u < \bar{u_1}$*, then* $\hat{\pi}_i{}^{SS} > \hat{\pi}_i{}^{FF}$ *and if* $u \geq \bar{u_1}$*, then* $\pi_i^{SS} \leq \pi_i^{FF}$.

b) If $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ *and* $\eta > \max\{\frac{b_s(b_s+\theta_s)^2}{b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s}, n_{max}\}$*, then there exists a threshold* $\bar{u_2}$ *such that if* $u < \bar{u_2}$*, then* $\hat{\pi}_i{}^{SS} < \hat{\pi}_i{}^{FF}$ *and if* $u \geq \bar{u_2}$*, then* $\hat{\pi}_i{}^{SS} \geq \hat{\pi}_i{}^{FF}$.

The expressions for η_{max}, $\bar{\epsilon_3}$, $\bar{u_1}$ *and* $\bar{u_2}$ *are provided in the Appendix.*

Proposition 16 presents the characterization of some of the possible regimes and the dominant strategies which are illustrated in Figure 4.5. Recall that in the deterministic demand case with different costs, when the intensity of competition of service is high (i.e., $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$), the dominant investment strategy for a symmetric duopolist

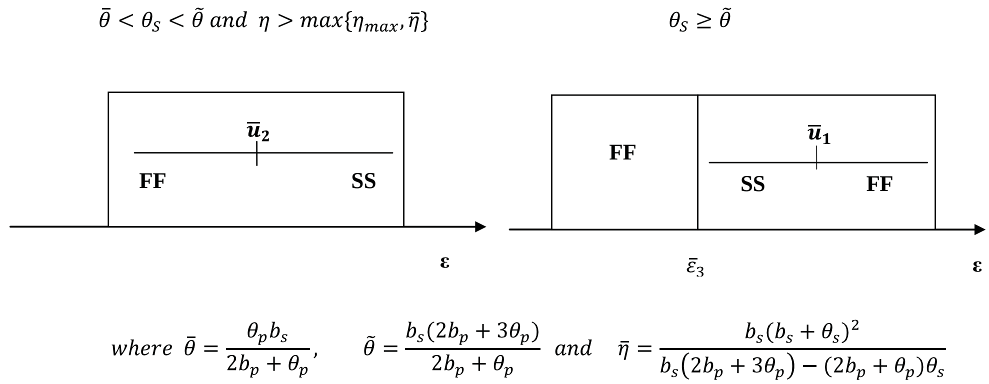


FIGURE 4.5: Scenario Dominance (obtained analytically) for a Symmetric Duopolist Under Stochastic Demand and Different Investment Costs

would shift from investing in the first stage to investing in the second stage as the differential costs of investments across stages (ϵ) increases (Proposition 12(b)). In the case of stochastic demand we could have a non-monotonic behavior in the investment strategies. Specifically, when the differential cost of investments (ϵ) is low, the dominant strategy for a retailer would be to make investments in the first stage. Beyond a given value of ϵ and for low levels of demand variability a symmetric duopolist would be better off to invest in the second stage but interestingly, as demand variability increases, the dominant strategy would be to make service investments in the first stage.

When the intensity of competition of service is low ($\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$) and the investment cost is high ($\eta > \max\{\frac{b_s(b_s+\theta_s)^2}{b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s}, n_{max}\}$), then a symmetric duopolist would always make an investment in the first stage under deterministic demand (Proposition 12(a)). In the case of stochastic demand a symmetric duopolist would still prefer to invest in the first stage for low demand uncertainty. But as demand uncertainty increases there will be value in postponing investment after the demand realizes which could lead to investing in the second stage. Note that we have not been

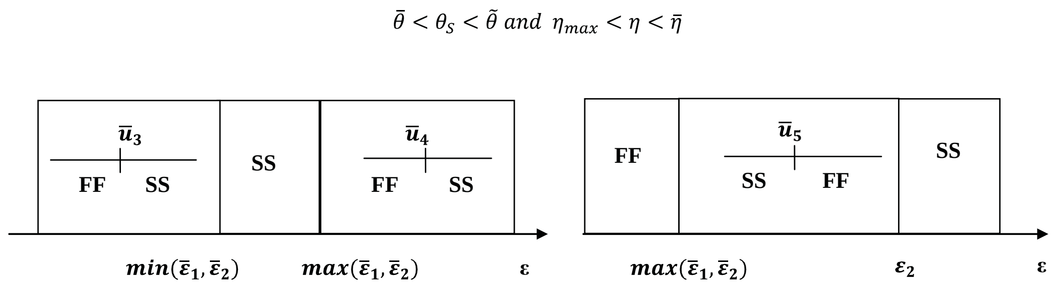


FIGURE 4.6: Scenario Dominance (obtained numerically) for a Symmetric Duopolist Under Stochastic Demand and Different Investment Costs

able to fully analytically characterize the regime in which $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $n_{max} < \eta < \frac{b_s(b_s+\theta_s)^2}{b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s}$. A computational study demonstrated that we could have a rich number of different strategies in the above regime. Some of the possible cases are described below and illustrated in Figure 4.6

1. There could exist two thresholds of ϵ (i.e., $\bar{\epsilon_1}$, $\bar{\epsilon_2} > 0$). If $\epsilon < \min\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$ there could exist a threshold $\bar{u_3}$ such that if $u < \bar{u_3}$ then $\hat{\pi}_i^{SS} < \hat{\pi}_i^{FF}$ and if $u \geq \bar{u_3}$ then $\hat{\pi}_i^{SS} \geq \hat{\pi}_i^{FF}$. Similarly if $\epsilon > \max\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$ then there could exist a threshold $\bar{u_4}$ such that if $u < \bar{u_4}$ then $\hat{\pi}_i^{SS} < \hat{\pi}_i^{FF}$ and if $u \geq \bar{u_4}$ then $\hat{\pi}_i^{SS} \geq \hat{\pi}_i^{FF}$. If $\min\{\bar{\epsilon_1}, \bar{\epsilon_2}\} \leq \epsilon \leq \max\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$ then $\hat{\pi}_i^{SS} \geq \hat{\pi}_i^{FF}$.

2. There could exist two thresholds of ϵ (i.e., $\bar{\epsilon_1}$, $\bar{\epsilon_2}$) such that $\min\{\bar{\epsilon_1}, \bar{\epsilon_2}\} < 0$ and $\max\{\bar{\epsilon_1}, \bar{\epsilon_2}\} > 0$. If $0 < \epsilon < \max\{\bar{\epsilon_1}, \bar{\epsilon_2}\}$ then $\hat{\pi}_i^{SS} < \hat{\pi}_i^{FF}$. If $\epsilon > \epsilon_2$ then $\hat{\pi}_i^{SS} > \hat{\pi}_i^{FF}$. If $\max\{\bar{\epsilon_1}, \bar{\epsilon_2}\} < \epsilon < \epsilon_2$ then there exists a threshold $\bar{u_5}$ such that if $u < \bar{u_5}$ then $\hat{\pi}_i^{SS} > \hat{\pi}_i^{FF}$ and if $u \geq \bar{u_5}$ then $\hat{\pi}_i^{SS} \leq \hat{\pi}_i^{FF}$.

4.6 Conclusion

Motivated by the increasing importance that retailers place on activities that enhance demand we examine the temporal management of investments in service in the face of demand uncertainty and competition.

We develop a two-stage model in order to understand the trade-offs that retailers need to take into consideration while deciding on the timing of service investments. We focus mainly on two alternatives that retailers typically have in terms of timing their investments and examine them under both monopoly and symmetric duopoly settings. The first alternative is to make service investments in advance of the selling season without knowing the market state (i.e., invest in the first stage) and the second alternative is to make service investments after the market state realizes (i.e., invest in the second stage). In both cases a retailer decides on pricing after observing the demand (i.e., in the second stage). For the monopoly case we further examine a hybrid strategy in which a retailer can invest both before and after the demand state is known. Typically, investing after the demand state is known is associated with higher investment costs. We analyze these settings under both equal and different investment costs across stages. In addition, we investigate the deterministic demand case for the symmetric duopoly and contrast our results with the stochastic demand case. In the case of a deterministic demand these alternatives translate to making sequential decisions (i.e., first service and then price) as opposed to simultaneous decisions (both service and price).

Our major findings are as follows. For a monopolist who faces stochastic demand and incurs different investment costs across stages we show that a hybrid strategy always dominates a strategy in which a retailer invests only before or only after the market state is known. In addition, we show that a monopolist would prefer to delay investments until demand realizes only when the market variability is high and the differential cost of investments across stages is low. In all other regimes a monopolist would prefer to invest before the demand realizes. This result is in contrast to the case of equal investment costs in which a monopolist would always defer service investments after the demand realizes.

For a symmetric duopolist who faces deterministic demand and incurs same investments costs across stages we show that the dominant strategy is always to make service investments in the first stage. We find that this is not always the case when the duopolist incurs higher investment costs in the second stage. Interestingly, when the intensity of service competition is high, a symmetric duopolist could be better off to invest in the second stage as the differential cost of investment in the two stages increases. We also find computationally that the equilibrium strategies for a symmetric retailer can shift in a non-monotonic fashion as the differential cost of investment in the two stages increases. In particular, a retailer could invest in the first stage for high and low differential costs and in the second stage for intermediate values of differential costs.

For a symmetric duopolist who faces stochastic demand and incurs same costs across stages the dominant strategy is to invest in service in the first stage in all regimes except for one characterized by high demand variability, low intensity of competition in service, and high investment cost. This result shows that the competitive dynamics could diminish significantly the value of delaying investments after demand realizes. We further characterize some of the investment strategies when a duopolist incurs higher investment costs in the second stage. Interestingly, we find that in the case of high intensity of competition in service increase in demand variability could make investing in the first stage more preferable than investing in the second stage provided that the differential costs of investments across stages exceeds a given threshold.

One of the key messages of this research, apart from offering several insights with regards to temporal management of investments in demand enhancing activities is that competition could actually diminish the value of postponing such investments after demand realization.

Our results, of course, reflect our assumptions. This immediately suggests a number of opportunities to build upon this work. While our linear demand relationships are analytically tractable, more general demand relationships could be considered. These could include different functional forms and different type of service costs to reflect economies of scale.

We have consider symmetric players to keep the analysis tractable but one could introduce different types of asymmetries that can potentially exist between retail competitors. For example, the cross-retailer demand effects could be nonsymmetric, as could the functional forms of the service cost terms.

The game structure we have considered is only a subset of the possibilities that can occur. One could introduce a manufacturer who could be a Stackelberg leader and examine the implications of service investments on the manufacturer's profitability as well as the supply chain as a whole. Similarly, we could also consider competition between distinct channels, each of which consists of a manufacturer and a retailer.

CHAPTER 5

Conclusion

Retail Operations has emerged as an important area of research in recent years. There are several reasons that have led to this advent. First, retailing is a large, growing, and dynamic sector in most countries both developed and developing. Second, retail operations has some unique aspects that are different from other issues faced by other players in a supply chain providing a fertile ground for operations management research. Third, the advanced computer technology, the advent of the Internet, and the emergence of new players in both the local and global markets has create tremendous opportunities to study new applications, contexts, and theory in retail operations. In this dissertation comprising of three essays, we attempt to shed light on retail practices that enhance consumer valuation, on factors that affect store performance, and on temporal management of investments in demand enhancing activities using both analytical and empirical methodologies.

The second chapter of the dissertation titled "Improving Valuation Under Consumer Search:Implications for Pricing and Profits" investigates technology type of investments that can affect consumer valuation. In this chapter, we employ analytical methodology in order to investigate first how consumer valuation practices followed by retailers could affect their prices and profits under competition and second how market characteristics could affect retailers' decision to invest in the above practices. We study a duopoly in which retailers compete in prices and consumers can search among the two retailers. In such a setting, a retailer may incur investments to increase consumer valuation for his product, but the final sale could be made at the other retailer, who may not incur such investments, leading to free riding. We explore the Nash equilibria in terms of both investment and pricing through a computational study. Then we focus on the pricing game only and establish the pricing Nash equilibria. Next, we focus on a special case in which the competing retailers are symmetric and characterize the possible Nash equilibria investment strategies depending on the investment cost. Finally, we present a model with an endogenous level of investment and analyze the symmetric equilibrium for a symmetric duopoly.

Among other results our key findings in Chapter 2 are as follows. When the investment decision is endogenous, surprisingly in the majority of instances both retailers decide to invest in equilibrium but price the product in a manner to avoid consumer search between them. We also find that the proximity of retailers has an interesting non monotonic impact on their decisions to invest in that retailers tend to invest when they are very close or very far away but refrain from investing in the intermediate range. When we further focus on the pricing game only we find two major effects related to improvements in consumer valuation. First, consistent with popular belief, we find that a retailer could overcome competitive effects by improving consumer valuation beyond a certain threshold. However, there are situations where a greater improvement in consumer valuation by a retailer could lead to lower profits. Second, we find that a retailer who does not invest could benefit from an innovative competitor who increases consumer valuation beyond a threshold. When we focus on symmetric retailers we find that as the investment cost increases the Nash equilibrium strategies shift from both retailers investing, to only one retailer investing (either retailer 1 or retailer 2), and finally to neither retailer investing. Finally, for the extension where the level of investment is endogenous, we show that a symmetric duopolist's optimal strategy to cover his whole local market or part of his market depends on his investment cost effectiveness and the optimal price charged by him may indeed decrease with the per unit cost of acquiring the product.

In the third chapter of the dissertation titled "The Impact of Labor and Traffic on Store Performance" we investigate the effect of store labor and customer traffic characteristics on store performance. We conduct a descriptive study of two store performance metrics, conversion rate and basket value using proprietary data pertaining to a retailer. Specifically, we consider the correlation between store performance and intra-day traffic variability and traffic uncertainty. We also measure traffic-labor mismatches and study if they explain the observed correlations in our sample.

We report the following results in Chapter 3. First, we report the within-store results. We find that intra-day traffic variability is negatively correlated with both conversion rate and basket value. We also find that, for a given level of traffic, both conversion rate and basket value increase with an increase in store labor at a diminishing rate. In addition, we find that conversion rates are higher during holidays but basket values are lower suggesting that price promotions offered during the holiday season cause more customers to purchase but do not make the average customer purchase more. Moreover, we find that both conversion rates and basket values exhibit significant seasonality.

Next, we report the across-store results. We find that stores with higher traffic uncertainty have lower conversion rates but similar basket values. We also find that stores that have higher traffic variability and higher traffic uncertainty have higher mismatches between required labor and actual labor. Furthermore, our tests reveal that stores that have lower foot-traffic have higher traffic uncertainty resulting in mismatches between required labor and actual labor. A surprising result of our analysis is that competition does not affect conversion rates and basket values. This suggests that consumers decision to whether or not to purchase and how much to purchase is unaffected by the presence of other competitors once they are in the store. Finally, we find that stores located in neighborhoods with higher per capita income have higher conversion rates but similar basket values.

Finally, in the fourth chapter of the dissertation, titled "Temporal Management of Service Investments under Demand Uncertainty and Competition" we employ analytical methodology to investigate the timing of investments in activities that enhance demand under competition in the face of demand uncertainty. We develop a two-stage model in order to examine two alternatives that retailers typically have in terms of timing their investments under both monopoly and symmetric duopoly settings. The first alternative is to invest in advance of the selling season without knowing the market state (i.e., invest in the first stage) and the second alternative is to invest after the market state realizes (i.e., invest in the second stage). In both cases a retailer decides on pricing after observing the demand (i.e., in the second stage). For the monopoly we further examine a hybrid strategy in which a retailer can invest both before and after the demand state is known. Typically, investing after the demand state is known is associated with higher costs of investments. We analyze these settings under both equal and different costs of investment across stages. In addition, we investigate the deterministic demand case for the symmetric duopoly and contrast our results with the stochastic demand case. In the case of a deterministic demand these alternatives translate to making sequential decisions (i.e., first service and then price) as opposed to simultaneous decisions (both service and price).

Our major findings in Chapter 4 are as follows. For a monopolist who faces stochastic demand and incurs different investment costs across stages we show that a hybrid strategy always dominates a strategy in which a retailer invests only before or only after the market state is known. In addition, we show that a monopolist would prefer to delay investments until demand realizes only when the market variability is high and the differential cost of investments across stages is low. In all other regimes a monopolist would prefer to invest before the demand realizes. This result is in contrast to the case of equal investment costs in which a monopolist would always defer service investments after the demand realizes.

For a symmetric duopolist who faces deterministic demand and incurs same investments costs across stages we show that the dominant strategy is always to make investments in service in the first stage. We find that this is not always the case when the duopolist incurs higher investment costs in the second stage. Interestingly, when the intensity of service competition is high a symmetric duopolist could be better off to invest in the second stage as the differential cost of investment in the two stages increases. We also find computationally that the equilibrium strategies for a symmetric retailer can shift in a non-monotonic fashion as the differential cost of investment in the two stages increases. In particular, a retailer could invest in the first stage for high and low differential costs and in the second stage for intermediate values of differential costs.

For a symmetric duopolist who faces stochastic demand and incurs same costs across stages the dominant strategy is to invest in service in the first stage in all regimes except for one characterized by high demand variability, low intensity of competition in service, and high investment cost. This result shows that the competitive dynamics could diminish significantly the value of delaying investments after demand realizes. We further characterize some of the investment strategies when a duopolist incurs higher investment costs in the second stage. Interestingly, we find that in the case of high intensity of competition in service increase in demand variability could make investing in the first stage more preferable than investing in the second stage provided that the differential costs of investments across stages exceeds a given threshold.

Together these three essays can contribute to the better management of retail practices and activities that could enhance demand and consumers' valuation as well as enable retailers to improve store performance. Summarizing these essays provide the following key messages for practitioners. First, a retailer's decision to engage in consumer valuation enhancing practices needs to be made taking into consideration the following important factors in addition to the costs associated with such investments: the physical proximity to their competitor as well as the demographics of their local customers such as willingness to pay and search for better deals in the market. Second, the timing of activities that enhance demand depends on the existence of competitors in the market. Retailers need to be aware that strategies regarding the timing of such activities which could be dominant in a monopolistic environment may have diminishing value in the face of competition. Moreover, the nature of competition such as the relative competitive intensity in price and service plays an important role in the adoption of the most suitable strategy. Third, with respect to labor planning and its impact on store performance retailers need to refrain from traditional labor planning approaches that are based solely on sales and average customer traffic and acknowledge the importance of taking traffic volatilities into account as well. Finally, even though traffic volatilities can be uncontrollable to a certain extent, retailers need to understand that their own actions could often induce them and seek ways to address such volatilities.

This dissertation can be extended on several fronts. For example, an interesting extension to the first essay, which has focused solely on retailer induced increases in customer valuations, would be to consider the supply chain implications of increased consumer valuation. Intensified retail competition can also affect the manufacturers' market share and profitability which results in manufacturers adopting different strategies of enhancing customer valuation. Specifically, several manufacturers, especially in the electronics industry, have engaged in activities such as improving product design that may differentiate their products, increase consumers valuation about their product line, and command higher prices (Guth et al., 2008)[1]. In settings where a manufacturer sells directly to consumers, stocking decisions and investment decisions could be planned together to improve the firm's profitability. In supply chains where a manufacturer sells his product through an independent retailer, who stocks competing products from multiple manufacturers, the manufacturer may need to make his investment decisions to enhance product valuation before the retailer commits to his stocking levels. An interesting question that arises in such a setting is how the presence of substitutable products in a stochastic demand environment affects the incentive of a manufacturer to invest in activities that increase consumer valuation for his product.

Another possible extension relates to understanding how the opportunities to invest in demand enhancing services for a product line affect the interactions between a manufacturer and her retailer. In the third essay, we focused on the timing of demand enhancing services that are typically undertaken by a retailer. Many demand enhancing services, e.g. information about how to install or use the product, after sales support, warranty repair etc. can be provided either by the manufacturer or they

[1]Guth R.A., J. Scheck, Clark D. (2008). "Window Dressing: PC Makers Take a Stylish Turn to Tackle Apple-Pink, Spotted Laptops Aimed at New Buyers; Designers Rule at Dell." The Wall Street Journal, January 4 2008.

can be delegated to the retailer. Such services can increase the consumer's perceived value of the product. An interesting question in such setting would be to explore how the competitive environment and the retailer's relative efficiency to the manufacturer's relative efficiency interacts with the manufacturer's decision about whether to deliver the services directly to a consumer or to outsource these services to the retailer.

A1 Appendix for Chapter 2

TABLE A1: Notation.

Notation	Description
V	Consumer willingness to pay prior to the investment
f	Pdf of V
F	Cdf of V
$\hat{V}$	Consumer willingness to pay after the investment
$\hat{f}$	Pdf of $\hat{V}$
$\hat{F}$	Cdf of $\hat{V}$
γ	Fraction of consumers who visits retailer 1 first
δ	Fraction of consumers who is willing to search
μ	Market size
ϵ	Market expansion
c_i	Retailer i's per unit cost of acquiring the product
k_i	Consumer traveling cost at retailer i
Δk	Consumer traveling cost from one retailer to the other
I_i	Retailer i's investment cost
λ_i	Retailer i's investment cost factor
α	Consumer valuation distribution mean shift
p_i^j	Retailer i's price in scenario j
d_i^j	Retailer i's demand in scenario j
π_i^j	Retailer i's profit in scenario j

Before we present the proofs, we review a useful definition on supermodularity (see Topkis (1979) for details on submodular games).

Definition 1 *Suppose $f(x_1, x_2)$ is twice differentiable, then $f(x_1, x_2)$ is supermodular in (x_1, x_2) if and only if* $\frac{\partial^2 f(x_1, x_2)}{\partial x_1 \partial x_2} \geq 0$.

Proof of Proposition 1. Let V and $\hat{V}$ be the consumer valuation before and after the investment, with corresponding domains [0,1] and [α,1+α]. We could theoretically have four possible investment scenarios: (i) Neither retailer invests, (ii) Retailer 1 does not invest but retailer 2 invests, (iii) Retailer 1 invests but retailer 2 does not invest,

TABLE A2: Scenarios examined.

Scenario	Description
N	Retailer 1 does not invest
I	Retailer 1 invests
IEIE	Both retailers invest and α is endogenous
IINN	Both retailers invest and there is no consumer search
NNNN	Neither retailer invests and there is no consumer search
NISS	Retailer 1 does not invest, retailer 2 invests, and consumers search both retailers
NISN	Retailer 1 does not invest, retailer 2 invests, and consumers search from retailer 1 to 2
NINS	Retailer 1 does not invest, retailer 2 invests, and consumers search from retailer 2 to 1
INSS	Retailer 1 invests, retailer 2 does not invest, and consumers search both retailers
INSN	Retailer 1 invests, retailer 2 does not invest, and consumers search from retailer 1 to 2
INNS	Retailer 1 invests, retailer 2 does not invest, and consumers search from retailer 2 to 1

(iv) Both retailers invest.

(i) Consumers who do not obtain positive consumer surplus at retailer 1 buy from retailer 2 if $p_2+k_1+\Delta k \leq u \leq p_1+k_1$, which implies that $p_1 > p_2+\Delta k$. Consumers who do not obtain positive consumer surplus at retailer 2 buy from retailer 1 if $p_1+k_2+\Delta k \leq u \leq p_2 + k_2$, which implies that $p_2 > p_1 + \Delta k$. But $p_1 > p_2 + \Delta k$ and $p_2 > p_1 + \Delta k$ cannot be satisfied at the same time hence, only one retailer can obtain sales from the consumers who search (i.e., consumers who did not buy at their local retailer and visit the competing retailer).

(ii) Consumers who do not obtain positive consumer surplus at retailer 1 buy from retailer 2 if $p_2 + k_1 + \Delta k \leq \hat{u} \leq p_1 + k_1 + \alpha$, which implies that $p_1 > p_2 + \Delta k - \alpha$. Consumers who do not obtain positive consumer surplus at retailer 2 buy from retailer

TABLE A3: Full Factorial Design 1.

Parameters	Values
γ	0.2, 0.5, 0.8
δ	0.2, 0.8
ϵ	0, 0.4, 0.8
μ	1
c_1	0.3, 0.4
c_2	0.3, 0.4
k_1	0.1, 0.3
k_2	0.1, 0.3
Δk	0.05, 0.2
I_1	0.02, 0.1
I_2	0.02, 0.1
α	0.2, 0.5

TABLE A4: Full Factorial Design 2.

Parameters	Values
δ	0.1, 0.3, 0.5, 0.7
$\hat{\mu}$	5, 10, 15, 20
c	0.1, 0.3, 0.5
k	0.1, 0.2, 0.3
Δk	0.5, 1, 1.5, 2
α	T_3, T_4, T_5

Note:The feasible range for the parameter α is $\alpha \in (T_1, T_2)$ where $T_1 = \frac{\delta\Delta k(2+3\delta)}{\delta(1+\delta)}$ and $T_2 = \frac{((1+\delta)(1-c-k)+\delta\Delta k)(2+3\delta)}{\delta(1+\delta)}$. For our computational study, we considered three values of α, T_3, T_4 and T_5 ($T_3 = \frac{T_1+T_2}{2}$, $T_4 = \frac{T_1+T_3}{2}$, and $T_5 = \frac{T_2+T_3}{2}$) that divide the interval (T_1, T_2) in three equally-spaced intervals

1 if $p_1 + k_2 + \Delta k \leq \hat{u} \leq p_2 + k_2$, which implies that $p_2 > p_1 + \Delta k$. Both retailers can obtain sales from the consumers who search under the following conditions: $\Delta k < \alpha/2$, $p_1 > p_2 + \Delta k - \alpha$, and $p_2 > p_1 + \Delta k$.

(iii) Consumers who do not obtain positive consumer surplus at retailer 1 buy from retailer 2 if $p_2 + k_1 + \Delta k \leq \hat{u} \leq p_1 + k_1$, which implies that $p_1 > p_2 + \Delta k$. Consumers who do not obtain positive consumer surplus at retailer 2 buy from retailer

1 if $p_1+k_2+\Delta k \le \hat{u} \le p_2+k_2+\alpha$, which implies that $p_2 > p_1+\Delta k-\alpha$. Both retailers can obtain sales from the consumers who search under the following conditions: $\Delta k < \alpha/2$, $p_1 > p_2 + \Delta k$, and $p_2 > p_1 + \Delta k - \alpha$.

(iv) Consumers who do not obtain positive consumer surplus at retailer 1 buy from retailer 2 if $p_2 + k_1 + \Delta k \le \hat{u} \le p_1 + k_1$, which implies that $p_1 > p_2 + \Delta k$. Consumers who do not obtain positive consumer surplus at retailer 2 buy from retailer 1 if $p_1 + k_2 + \Delta k \le \hat{u} \le p_2 + k_2$, which implies that $p_2 > p_1 + \Delta k$. But $p_1 > p_2 + \Delta k$ and $p_2 > p_1 + \Delta k$ cannot be satisfied at the same time hence, only one retailer can obtain sales from the consumers who search (i.e., consumers who did not buy at their local retailer and visit the competing retailer).

Concluding, only in investment scenarios (ii) and (iii) both retailers can obtain sales from the consumers who search. ■

Proof of Proposition 2. Using Definition 1, we can prove the supermodularity of the expected profit function of each retailer. We illustrate the supermodularity for the case in which retailer 1 does not invest, retailer 2 invests, and consumers search in both directions (i.e., scenario $NISS$). The same logic applies for the rest of the scenarios and we omit the proof to conserve space. Retailer 1's profit in scenario $NISS$ is:

$\pi_1^{NISS} = (p_1 - c_1)((\mu + \epsilon)\gamma(1 - p_1 - k_1) + \delta(\mu + \epsilon)(1 - \gamma)(p_2 - p_1 - \Delta k))$

$\frac{\partial^2 \pi_1^{NISS}}{\partial p_1 \partial p_2} = \delta(\mu+\epsilon)(1-\gamma) \ge 0$, which proves the supermodularity of π_1^{NISS}. Similarly, we can prove that π_2^{NISS} is supermodular in (p_1, p_2). Because we have a supermodular game there exists at least one Nash equilibrium. ■

Conditions of uniqueness of NE

The literature has considered several methods for proving NE uniqueness (Cachon and Netessine (2004)). One of the sufficient conditions for the uniqueness of a NE is the "diagonal dominance" condition, which in a two-player game is stated as follows: If a Nash equilibrium exists and $|\frac{\partial^2 \pi_i(p_1,p_2)}{\partial p_i \partial p_j}| < |\frac{\partial^2 \pi_i(p_1,p_2)}{\partial p_i^2}|$, $i,j = 1,2$, $i \neq j$, then the Nash equilibrium is unique. Using "diagonal dominance" the conditions we obtain for the uniqueness of the NE for the scenarios in Table A2 hold. Hence, the NE is unique.

Proof of Proposition 3.

The retailers' profit functions π_1^{NISS} and π_2^{NISS} are concave in their corresponding prices p_1 and p_2. The optimal prices p_1^{*NISS} and p_2^{*NISS} for the two retailers are the intersection of the best response functions obtained from the first order conditions of retailers' profits (see equations (3) and (4)). Evaluating the expressions for demands q_1^{NISS} and q_2^{NISS} and profits π_1^{NISS} and π_2^{NISS} at their optimal prices provides us with the corresponding optimal demand and profit expressions. We find the expressions for $\bar{\alpha}_1$, $\bar{\alpha}_2$, and $\bar{\alpha}^{(1)}$ respectively by solving the following equations: $p_1^{*NISS} = p_1^{*N}$, $q_1^{*NISS} = q_1^{*N}$, and $\pi_1^{*NISS} = \pi_1^{*N}$. We present all the expressions in Table A5.

Comparing retailer 1's optimal price and demand under monopoly and duopoly regimes we obtain the results summarized in Table A6. Note that Table A6 presents sufficient conditions (but not necessary). In Table A6 retailer 1's profit is lower under duopoly than under monopoly for low levels of α and it is higher under duopoly than under monopoly for high levels of α. Hence, there exists a threshold in the consumer valuation mean shift $\bar{\alpha}^{(1)}$ such that if $\alpha < \bar{\alpha}^{(1)}$ then $\pi_1^{*N} > \pi_1^{*NISS}$ and if $\alpha > \bar{\alpha}^{(1)}$ then $\pi_1^{*N} < \pi_1^{*NISS}$.

TABLE A5: Expressions for scenario $NISS$.

Notation	Expression
p_1^{*NISS}	$(\delta+\gamma(2(1-\delta)-\gamma(2-3\delta))+\alpha(1-\gamma)(1-\gamma(1-\delta))\delta-(1-\gamma)\delta(2-\gamma(2-3\delta))\Delta k$ $+2(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)c_1+(1-\gamma)(1-\gamma(1-\delta))\delta c_2$ $-2\gamma(1-\gamma(1-\delta))k_1-(1-\gamma)^2\delta k_2)/(4\delta+(1-\gamma)\gamma(2-\delta)(2-3\delta))$
q_1^{*NISS}	$((\gamma(1-\delta)+\delta)(\epsilon+\mu)(\delta+\gamma(2(1-\delta)-\gamma(2-3\delta))+\alpha(1-\gamma)(1-\gamma(1-\delta))\delta$ $-(1-\gamma)\delta(2-\gamma(2-3\delta))\Delta k-(1-\gamma)^2\delta k_2-(2\delta+(1-\gamma)\gamma(2-(4-\delta)\delta))c_1$ $-2\gamma(1-\gamma(1-\delta))k_1+(1-\gamma)(1-\gamma(1-\delta))\delta c_2)/(4\delta+(1-\gamma)\gamma(2-\delta)(2-3\delta))$
π_1^{*NISS}	$((\gamma(1-\delta)+\delta)(\epsilon+\mu)(\delta+\gamma(2(1-\delta)-\gamma(2-3\delta))+\alpha(1-\gamma)(1-\gamma(1-\delta))\delta$ $-(1-\gamma)\delta(2-\gamma(2-3\delta))\Delta k-(1-\gamma)^2\delta k_2-(2\delta+(1-\gamma)\gamma(2-(4-\delta)\delta))c_1$ $-2\gamma(1-\gamma(1-\delta))k_1+(1-\gamma)(1-\gamma(1-\delta))\delta c_2)^2)/(4\delta+(1-\gamma)\gamma(2-\delta)(2-3\delta))^2$
$\bar{\alpha_1}$	$(2(1-\gamma)+3\gamma\delta+2(2(1-\gamma+3\gamma\delta))\Delta k-\gamma\delta c_1-2(1-\gamma(1-\delta))c_2$ $-(4-4\gamma+3\gamma\delta)k_1+2(1-\gamma)k_2)/(2-2\gamma(1-\delta))$
ζ	$-2(1-\gamma)(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\delta(\delta+\gamma(2-2\delta-\gamma(2-3\delta))$ $-(1-\gamma)\delta(2-\gamma(2-3\delta))\delta k)(\epsilon+\mu)$
η	$(1-\gamma)^2(1-\gamma(1-\delta))^2(\gamma(1-\delta)+\delta)\delta^2(4\delta+\gamma(1-\gamma)(2-\delta)(2-3\delta))^2\mu(\epsilon+\mu)(1-c_1-k_1)^2$
θ	$2(1-\gamma)^3(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\delta^2(\epsilon+\mu)k_2$ $+4(1-\gamma)\gamma(1-\gamma(1-\delta))^2(\gamma(1-\delta)+\delta)\delta(\epsilon+\mu)k_1$ $-2(1-\gamma)^2(1+\gamma(1-\delta))(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\delta^2(\epsilon+\mu)c_2$ $+2(1-\gamma)(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\delta(2\delta+(1-\gamma)\gamma(2-(4-\delta)\delta))(\epsilon+\mu)c_1$
τ	$-2(\gamma(1-\delta)+\delta)(\delta+\gamma(2-2\delta-\gamma(2-3\delta))-(1-\gamma)\delta(2-\gamma(2-3\delta))\Delta k)$
υ	$(1-\gamma)(2\delta(2-\delta+2\delta\Delta k)-2\gamma^2(1-\delta)(2-3\delta)(1+\delta\Delta k)$ $+\gamma(4-\delta(10-4\Delta k-\delta(7-8\Delta k+6\delta\Delta k))))$
ϕ	$2(\gamma(1-\delta)+\delta)(2\delta+(1-\gamma)\gamma(2-(4-\delta)\delta))\epsilon-(1-\gamma)(4(1-\delta)\delta$ $-2\gamma^2(1-\delta)(2-(4-\delta)\delta)+\gamma(4-(4-\delta)\delta(3-2\delta)))\mu$
χ	$2(1-\gamma)(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\delta(\epsilon+\mu)$
ψ	$-4\gamma(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\epsilon+(1-\gamma)(4\delta+\gamma(4-4\gamma(1-\delta)^2-(8-3\delta)\delta))\mu$
ω	$-2(1-\gamma)^2(\gamma(1-\delta)+\delta)\delta(\epsilon+\mu)$
$\bar{\alpha_2}$	$(\tau\epsilon+\upsilon\mu+\phi c_1-\chi c_2-\psi k_1-\omega k_2)/(2(1-\gamma)(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\delta(\epsilon+\mu))$
$\bar{\alpha}^{(1)}$	$(\zeta\pm\sqrt{\eta}+\theta)/(2(1-\gamma)^2(1-\gamma(1-\delta))^2(\gamma(1-\delta)+\delta)\delta^2(\epsilon+\mu))$

■

Proof of Proposition 4.

Taking the partial derivatives of $\bar{\alpha_1}$, $\bar{\alpha_2}$, and $\bar{\alpha}^{(1)}$ with respect to Δk we obtain that:

$\frac{\partial\bar{\alpha_1}}{\partial\Delta k}=\frac{\partial\bar{\alpha_2}}{\partial\Delta k}=\frac{\partial\bar{\alpha}^{(1)}}{\partial\Delta k}=\frac{2-2\gamma+3\gamma\delta}{1-\gamma+\gamma\delta}>0.$ ■

Proof of Proposition 5. The retailers' profit functions π_1^{INSS} and π_2^{INSS} are concave in their corresponding prices p_1 and p_2. The optimal prices p_1^{*INSS} and p_2^{*INSS} for the two retailers are the intersection of the best response functions obtained from the first order conditions of retailers' profits (see equations (7) and (8)). Evaluating the

TABLE A6: Impact of competition on retailer 1 in scenario $NISS$.

Sufficient Conditions	Impact
$\mu < (\epsilon+\mu)(\gamma+\delta(1-\gamma))$ and $\alpha < \bar{\alpha_2}$	$p_1^{*N} > p_1^{*NISS}$, $q_1^{*N} > q_1^{*NISS}$, and $\pi_1^{*N} > \pi_1^{*NISS}$
$\mu < (\epsilon+\mu)(\gamma+\delta(1-\gamma))$ and $\bar{\alpha_2} < \alpha < \bar{\alpha_1}$	$p_1^{*N} > p_1^{*NISS}$ and $q_1^{*N} < q_1^{*NISS}$
$\mu < (\epsilon+\mu)(\gamma+\delta(1-\gamma))$ and $\alpha > \bar{\alpha_1}$	$p_1^{*N} < p_1^{*NISS}$, $q_1^{*N} < q_1^{*NISS}$, and $\pi_1^{*N} < \pi_1^{*NISS}$
$\mu > (\epsilon+\mu)(\gamma+\delta(1-\gamma))$ and $\alpha < \bar{\alpha_1}$	$p_1^{*N} > p_1^{*NISS}$, $q_1^{*N} > q_1^{*NISS}$, and $\pi_1^{*N} > \pi_1^{*NISS}$
$\mu > (\epsilon+\mu)(\gamma+\delta(1-\gamma))$ and $\bar{\alpha_1} < \alpha < \bar{\alpha_2}$	$p_1^{*N} < p_1^{*NISS}$ and $q_1^{*N} > q_1^{*NISS}$
$\mu > (\epsilon+\mu)(\gamma+\delta(1-\gamma))$ and $\alpha > \bar{\alpha_2}$	$p_1^{*N} < p_1^{*NISS}$, $q_1^{*N} < q_1^{*NISS}$, and $\pi_1^{*N} < \pi_1^{*NISS}$

expressions for demands q_1^{INSS} and q_2^{INSS} and profits π_1^{INSS} and π_2^{INSS} at their optimal prices provides us with the corresponding optimal demand and profit expressions. We obtain the expressions for $\bar{\alpha_3}$, $\bar{\alpha_4}$, and $\bar{\alpha}^{(2)}$ respectively by solving the following equations: $p_1^{*INSS} = p_1^{*I}$, $q_1^{*INSS} = q_1^{*I}$, and $\pi_1^{*INSS} = \pi_1^{*I}$. The equation $q_1^{*INSS} = q_1^{*I}$ is a first degree polynomial of the form $\alpha x = y$. We derive the expression for $\bar{\epsilon}$ and $\bar{\Delta k}$ by setting $x = 0$ and $y = 0$ and solving for ϵ and Δk respectively. We present all these expressions in Table A7 other than the expression for $\bar{\alpha}^{(2)}$, which is very long and hence, we omit it. Comparing retailer 1's optimal price and demand under monopoly and duopoly regimes we obtain the results summarized in Table A8.

From Table A8 we can make the following observations: (i) When the retailers' proximity is low ($\Delta k > \bar{\Delta k}$) and market expansion is high ($\epsilon > \bar{\epsilon}$) retailer 1's profit for low levels of α is lower under duopoly than under monopoly and for high levels of α is higher under duopoly than under monopoly. Hence, there exists a threshold in the consumer valuation mean shift $\bar{\alpha}^{(2)}$ such that if $\alpha < \bar{\alpha}^{(2)}$ then $\pi_1^{*I} > \pi_1^{*INSS}$ and if $\alpha > \bar{\alpha}^{(2)}$ then $\pi_1^{*I} < \pi_1^{*INSS}$. (ii) When the retailers' proximity is low ($\Delta k > \bar{\Delta k}$) and market expansion is low ($\epsilon < \bar{\epsilon}$), then $\pi_1^{*I} > \pi_1^{*INSS}$ for $\alpha < \bar{\alpha_3}$. Hence, from (i) if the retailers' proximity is low there exists a threshold $\bar{\alpha} = \min\{\bar{\alpha}^{(2)}, \bar{\alpha_3}\}$ such that if $\alpha < \bar{\alpha}$

TABLE A7: Expressions for scenario INSS.

Notation	Expression
p_1^{*INSS}	$(\delta+\gamma(2(1-\delta)+\gamma(3\delta-2))+2\alpha((1-\gamma)\gamma(1-\delta)^2+\delta)-(1-\gamma)\delta(2+\gamma(3\delta-2))\Delta k$ $+2(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)c_1+(1-\gamma)(1-\gamma(1-\delta))\delta c_2$ $-2\gamma(1-\gamma+\gamma\delta)k_1-(1-\gamma)^2\delta k_2)/(4\delta+(1-\gamma)\gamma(2-\delta)(2-3\delta))$
q_1^{*INSS}	$(\delta+\gamma(2(1-\delta)+\gamma(3\delta-2))+2\alpha((1-\gamma)\gamma(1-\delta)^2+\delta)-(1-\gamma)\delta(2+\gamma(3\delta-2))\Delta k$ $-(2\delta+\gamma(1-\gamma)(2-(4-\delta)\delta))c_1+(1-\gamma)(1-\gamma(1-\delta))\delta c_2$ $-2\gamma(1-\gamma+\gamma\delta)k_1-(1-\gamma)^2\delta k_2)(\gamma(1-\delta)+\delta)(\epsilon+\mu)/(4\delta+(1-\gamma)\gamma(2-\delta)(2-3\delta))$
π_1^{*INSS}	$(\delta+\gamma(2(1-\delta)+\gamma(3\delta-2))+2\alpha((1-\gamma)\gamma(1-\delta)^2+\delta)-(1-\gamma)\delta(2+\gamma(3\delta-2))\Delta k$ $-(2\delta+\gamma(1-\gamma)(2-(4-\delta)\delta))c_1+(1-\gamma)(1-\gamma(1-\delta))\delta c_2-2\gamma(1-\gamma+\gamma\delta)k_1$ $-(1-\gamma)^2\delta k_2)^2(\gamma(1-\delta)+\delta)(\epsilon+\mu)/(4\delta+(1-\gamma)\gamma(2-\delta)(2-3\delta))^2-I_1$
$\bar{\alpha_3}$	$(2(1-\gamma)+3\gamma\delta+2(2(1-\gamma)+3\gamma\delta)\Delta k-\gamma\delta c_1-2(1-\gamma(1-\delta))c_2$ $-(4(1-\gamma)+3\gamma\delta)k_1+2(1-\gamma)k_2)/\gamma\delta$
τ_1	$-2(\gamma(1-\delta)+\delta)(\delta+\gamma(2-2\delta-\gamma(2-3\delta))-(1-\gamma)\delta(2-\gamma(2-3\delta))\Delta k)$
υ_1	$(1-\gamma)(2\delta(2-\delta+2\delta\Delta k)-2\gamma^2(1-\delta)(2-3\delta)(1+\delta\Delta k)$ $+\gamma(4+\delta(-10+4\Delta k+\delta(7-8\Delta k+6\delta\Delta k))))$
ϕ_1	$2(\gamma(1-\delta)+\delta)(2\delta+(1-\gamma)\gamma(2-(4-\delta)\delta))\epsilon-(1-\gamma)(4(1-\delta)\delta$ $-2\gamma^2(1-\delta)(2-(4-\delta)\delta)+\gamma(4-(4-\delta)\delta(3-2\delta)))\mu$
χ_1	$2(1-\gamma)(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\delta(\epsilon+\mu)$
ψ_1	$4\gamma(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\epsilon+(1-\gamma)(4\delta+\gamma(4-4\gamma(1-\delta)^2+(8-3\delta)\delta))\mu$
ω_1	$-2(1-\gamma)^2(\gamma(1-\delta)+\delta)\delta(\epsilon+\mu)$
τ_2	$2(\gamma(1-\delta)+\delta)(\delta+\gamma(2-2\delta-\gamma(2-3\delta)))$
υ_2	$-(1-\gamma)(-4(1-\gamma)\gamma-2(2-5(1-\gamma)\gamma)\delta+(2-\gamma(7-6\gamma))\delta^2)$
ϕ_2	$2(\gamma(1-\delta)+\delta)(2\delta+(1-\gamma)\gamma(2-(4-\delta)\delta))\epsilon-(1-\gamma)(4(1-\delta)\delta$ $-2\gamma^2(1-\delta)(2-(4-\delta)\delta)+\gamma(4-(4-\delta)\delta(3-2\delta)))\mu$
χ_2	$2(1-\gamma)(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\delta(\epsilon+\mu)$
ψ_2	$-4\gamma(1-\gamma(1-\delta))(\gamma(1-\delta)+\delta)\epsilon-(1-\gamma)(-4\delta-\gamma(4-4\gamma(1-\delta)^2-(8-3\delta)\delta))\mu$
ω_2	$-2(1-\gamma)^2(\gamma(1-\delta)+\delta)\delta(\epsilon+\mu)$
η_1	$4(1-\gamma(1-\delta))(\gamma+\delta-\gamma\delta)^2\epsilon-(1-\gamma)(-4\gamma^2(1-\delta)^3+4(1-\delta)\delta$ $+\gamma(4-\delta(12-(11-4\delta)\delta)))\mu$
$\bar{\alpha_4}$	$(\tau_1\epsilon+\upsilon_1\mu+\phi_1c_1-\chi_1c_2-\psi_1k_1-\omega_1k_2)/\eta_1$
$\bar{\epsilon}$	$(1-\gamma)\mu(4\gamma^2(\delta-1)^3+4(1-\delta)\delta)+\gamma(4+\delta((11-4\delta)\delta-12))/(4(1-\gamma(1-\delta))(\gamma+\delta-\gamma\delta)^2)$
Δk	$(\tau_2\epsilon-\upsilon_2\mu-\phi_2c_1+\chi_2c_2+\psi_2k_1+\omega_2k_2)/(2(1-\gamma)(\gamma(1-\delta)+\delta)\delta(2-\gamma(2-3\delta))(\epsilon+\mu))$

then $\pi_1^{*I} > \pi_1^{*INSS}$.

■

Proof of Proposition 6.

When the retailers are symmetric the expressions of profit for the four investment scenarios examined (IINN), (NNNN), (INSS), (NISS) are as follows:

$$\pi_i^{IINN} = (p-c)\hat{\mu}(1+\alpha-p-k)-I, \quad i=1,2$$

$$\pi_i^{NNNN} = (p-c)\hat{\mu}(1-p-k), \quad i=1,2$$

TABLE A8: Impact of competition on retailer 1 for scenario INSS.

Sufficient Conditions	Impact
$\epsilon > \bar{\epsilon}$, $\Delta k < \bar{\Delta k}$ and $\alpha < \bar{\alpha_3}$	$p_1^{*I} > p_1^{*INSS}$ and $q_1^{*I} < q_1^{*INSS}$
$\epsilon > \bar{\epsilon}$, $\Delta k < \bar{\Delta k}$, and $\alpha > \bar{\alpha_3}$	$p_1^{*I} < p_1^{*INSS}$, $q_1^{*I} < q_1^{*INSS}$, and $\pi_1^{*I} < \pi_1^{*INSS}$
$\epsilon > \bar{\epsilon}$, $\Delta k > \bar{\Delta k}$, and $\alpha < \min\{\bar{\alpha_3}, \bar{\alpha_4}\}$	$p_1^{*I} > p_1^{*INSS}$, $q_1^{*I} > q_1^{*INSS}$, and $\pi_1^{*I} > \pi_1^{*INSS}$
$\epsilon > \bar{\epsilon}$, $\Delta k > \bar{\Delta k}$, and $\bar{\alpha_3} < \alpha < \bar{\alpha_4}$	$p_1^{*I} < p_1^{*INSS}$ and $q_1^{*I} > q_1^{*INSS}$
$\epsilon > \bar{\epsilon}$, $\Delta k > \bar{\Delta k}$, and $\bar{\alpha_4} < \alpha < \bar{\alpha_3}$	$p_1^{*I} > p_1^{*INSS}$ and $q_1^{*I} < q_1^{*INSS}$
$\epsilon > \bar{\epsilon}$, $\Delta k > \bar{\Delta k}$, and $\alpha > \max\{\bar{\alpha_3}, \bar{\alpha_4}\}$	$p_1^{*I} < p_1^{*INSS}$, $q_1^{*I} < q_1^{*INSS}$, and $\pi_1^{*I} < \pi_1^{*INSS}$
$\epsilon < \bar{\epsilon}$, $\Delta k < \bar{\Delta k}$, and $\alpha < \min\{\bar{\alpha_3}, \bar{\alpha_4}\}$	$p_1^{*I} > p_1^{*INSS}$ and $q_1^{*I} < q_1^{*INSS}$
$\epsilon < \bar{\epsilon}$, $\Delta k < \bar{\Delta k}$, and $\bar{\alpha_3} < \alpha < \bar{\alpha_4}$	$p_1^{*I} < p_1^{*INSS}$, $q_1^{*I} < q_1^{*INSS}$, and $\pi_1^{*I} < \pi_1^{*INSS}$
$\epsilon < \bar{\epsilon}$, $\Delta k < \bar{\Delta k}$, and $\bar{\alpha_4} < \alpha < \bar{\alpha_3}$	$p_1^{*I} > p_1^{*INSS}$, $q_1^{*I} > q_1^{*INSS}$, and $\pi_1^{*I} > \pi_1^{*INSS}$
$\epsilon < \bar{\epsilon}$, $\Delta k < \bar{\Delta k}$ and $\alpha > \max\{\bar{\alpha_3}, \bar{\alpha_4}\}$	$p_1^{*I} < p_1^{*INSS}$ and $q_1^{*I} > q_1^{*INSS}$
$\epsilon < \bar{\epsilon}$, $\Delta k > \bar{\Delta k}$, and $\alpha < \bar{\alpha_3}$	$p_1^{*I} > p_1^{*INSS}$, $q_1^{*I} > q_1^{*INSS}$, and $\pi_1^{*I} > \pi_1^{*INSS}$
$\epsilon < \bar{\epsilon}$, $\Delta k > \bar{\Delta k}$, and $\alpha > \bar{\alpha_3}$	$p_1^{*I} < p_1^{*INSS}$ and $q_1^{*I} > q_1^{*INSS}$

$$\pi_1^{INSS} = (p_1 - c)(\hat{\mu}(1 + \alpha - p_1 - k) + \hat{\mu}\delta(p_2 + \alpha - p_1 - \Delta k)) - I$$

$$\pi_2^{INSS} = (p_2 - c)(\hat{\mu}(1 - p_2 - k) + \hat{\mu}\delta(p_1 - p_2 - \Delta k))$$

$$\pi_1^{NISS} = (p_1 - c)(\hat{\mu}(1 - p_1 - k) + \hat{\mu}\delta(p_2 - p_1 - \Delta k))$$

$$\pi_2^{NISS} = (p_2 - c)(\hat{\mu}(1 + \alpha - p_2 - k) + \hat{\mu}\delta(p_1 - p_2 - \Delta k + \alpha))$$

Note that in order to ensure positive demand from searching consumers and local consumers we need to constrain α in the following region:

$$\alpha \in (\tfrac{\delta\Delta k(2+3\delta)}{\delta(1+\delta)}, \tfrac{((1+\delta)(1-c-k)+\delta\Delta k)(2+3\delta)}{\delta(1+\delta)}).$$

Since we have symmetric retailers the following should hold:

$\pi_1^{*IINN} = \pi_2^{*IINN}$, $\pi_1^{*NNNN} = \pi_2^{*NNNN}$, $\pi_1^{*INSS} = \pi_2^{*NISS}$, and $\pi_1^{*NISS} = \pi_2^{*INSS}$, where the optimal profit expressions are given below:

$$\pi_i^{*IINN} = \frac{\hat{\mu}(1+\alpha-c-k)^2}{4} - I, \quad i = 1, 2$$

$$\pi_i^{*NNNN} = \frac{\hat{\mu}(1-c-k)^2}{4}, \quad i = 1, 2$$

$$\pi_1^{*INSS} = \frac{(1+\delta)\hat{\mu}((2+3\delta)(1-c-k-\delta\Delta k)+2\alpha(1+\delta)^2)^2}{(4+8\delta+3\delta^2)^2} - I$$

$$\pi_2^{*INSS} = \frac{(1+\delta)\hat{\mu}((2+3\delta)(1-c-k-\delta\Delta k)+\alpha\delta(1+\delta))^2}{(4+8\delta+3\delta^2)^2}$$

$\pi_1^{*NISS} = \frac{(1+\delta)\hat{\mu}((2+3\delta)(1-c-k-\delta\Delta k)+\alpha\delta(1+\delta))^2}{(4+8\delta+3\delta^2)^2}$

$\pi_2^{*NISS} = \frac{(1+\delta)\hat{\mu}((2+3\delta)(1-c-k-\delta\Delta k)+2\alpha(1+\delta)^2)^2}{(4+8\delta+3\delta^2)^2} - I$

We now explore the different possible Nash Equilibrium strategies.

Case 1: Both retailers invest

Both retailers invest is a unique Nash equilibrium strategy if the following holds: $\pi_1^{*IINN} > \pi_1^{*NISS}$.

Let A_1, A_2 denote the values of investment cost at which $\pi_1^{*IINN} = \pi_1^{*NISS}$ and $\pi_1^{*INSS} = \pi_1^{*NNNN}$ respectively.

$A_1 = \frac{\hat{\mu}(1+\alpha-c-k)^2}{4} - \frac{(1+\delta)\hat{\mu}((2+3\delta)(1-c-k-\delta\Delta k)+\alpha\delta(1+\delta))^2}{(4+8\delta+3\delta^2)^2}$ and

$A_2 = \frac{(1+\delta)\hat{\mu}((2+3\delta)(1-c-k-\delta\Delta k)+2\alpha(1+\delta)^2)^2}{(4+8\delta+3\delta^2)^2} - \frac{\hat{\mu}(1-c-k)^2}{4}$

One can easily show that $\pi_1^{*IINN} > \pi_1^{*NISS}$ if $0 < I < A_1$.

Hence, both retailers invest is a unique Nash equilibrium strategy if $0 < I < A_1$.

Case 2: Neither retailer invests

Neither retailer invests is a unique Nash equilibrium strategy if the following holds: $\pi_1^{*INSS} < \pi_1^{*NNNN}$.

One can easily show that $\pi_1^{*INSS} < \pi_1^{*NNNN}$ if $I > \max\{0, A_2\}$.

Hence, neither retailer invests is a unique Nash equilibrium strategy if $I > \max\{0, A_2\}$.

Case 3: Only one retailer invests (either retailer 1 or retailer 2)

Either retailer 1 invests and retailer 2 does not invest or retailer 1 does not invest and retailer 2 invests are two unique Nash equilibrium strategies if the following holds:

(i) $\pi_1^{*INSS} > \pi_1^{*NNNN}$ and (ii) $\pi_1^{*IINN} < \pi_1^{*NISS}$.

One can easily show that (i) $\pi_1^{*INSS} > \pi_1^{*NNNN}$ and (ii) $\pi_1^{*IINN} < \pi_1^{*NISS}$ if

$\max\{0, A_1\} < I < A_2$. Hence, either retailer 1 invests and retailer 2 does not invest or retailer 1 does not invest and retailer 2 invests are two unique Nash equilibrium strategies if $\max\{0, A_1\} < I < A_2$.

■

Proof of Proposition 7.

The expressions of profit for a retailer for a given α are : $\pi^{*II}(\alpha) = \frac{(1+\alpha-c-k)^2\hat{\mu}}{4} - \lambda\alpha^2$, where $0 \leq \alpha \leq 1+c+k$ and $\pi^{*II}(\alpha) = (\alpha - c - k)\hat{\mu} - \lambda\alpha^2$, where $\alpha \geq 1+c+k$. The corresponding local optima are $\alpha^{*IEIE} = \frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}}$, for $0 \leq \alpha \leq 1+c+k$ and $\alpha^{*IEIE} = \frac{\hat{\mu}}{2\lambda}$, for $\alpha \geq 1+c+k$. Thus, each retailer's profit function has two branches. The first branch $\pi^{*II}(\alpha) = \frac{(1+\alpha-c-k)^2\hat{\mu}}{4} - \lambda\alpha^2$ is concave when $\frac{\hat{\mu}}{\lambda} < 4$ and convex otherwise, while the second branch $\pi^{*II}(\alpha) = (\alpha - c - k)\hat{\mu} - \lambda\alpha^2$ is concave. In order to identify the global optimum we consider all possible cases in terms of relative magnitude of $\frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}}$, $\frac{\hat{\mu}}{2\lambda}$, and $1+c+k$.

Case A: First branch is concave ($\frac{\hat{\mu}}{\lambda} < 4$).

We have the following subcases: (i) $\frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}} < \frac{\hat{\mu}}{2\lambda} < 1+c+k$, (ii) $1+c+k < \frac{\hat{\mu}}{2\lambda} < \frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}}$, (iii) $\frac{\hat{\mu}}{2\lambda} < \frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}} < 1+c+k$, (iv) $\frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}} < 1+c+k < \frac{\hat{\mu}}{2\lambda}$, (v) $\frac{\hat{\mu}}{2\lambda} < 1+c+k < \frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}}$ and (vi) $1+c+k < \frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}} < \frac{\hat{\mu}}{2\lambda}$. The only feasible subcases are (i) and (ii), in which the global optima are $\alpha^{*IEIE} = \frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}}$, and $\alpha^{*IEIE} = \frac{\hat{\mu}}{2\lambda}$ respectively.

Case B: Second branch is convex ($\frac{\hat{\mu}}{\lambda} > 4$).

We have the following subcases: (i) $1+c+k < \frac{\hat{\mu}}{2\lambda}$ and (ii) $\frac{\hat{\mu}}{2\lambda} < 1+c+k$. The only feasible subcase is (i) and the global optimum is $\frac{\hat{\mu}}{2\lambda}$.

Combining cases A and B we obtain that a) if $\lambda < \frac{\hat{\mu}}{2(1+c+k)}$, then $\alpha^{*IEIE} = \frac{\hat{\mu}}{2\lambda}$ b) if $\lambda > \frac{\hat{\mu}}{2(1+c+k)}$, then $\alpha^{*IEIE} = \frac{(1-c-k)\hat{\mu}}{4\lambda-\hat{\mu}}$.

■

Consumers search from retailer 1 to retailer 2

The profit expressions for retailer 1 for investment scenarios (ii) and (iii) are as follows:

(ii) Retailer 1 does not invest but retailer 2 invests

$\pi_1^{NISN} = (p_1 - c_1)(\mu + \epsilon)\gamma(1 - p_1 - k_1)$.

(iii) Retailer 1 invests but retailer 2 does not invest

$\pi_1^{INSN} = (p_1 - c_1)(\mu + \epsilon)\gamma(1 + \alpha - p_1 - k_1) - I_1$.

Let π_1^{*k}, d_1^{*k}, and π_1^{*k} be the optimal prices, demands and profits in scenarios $k = \{NISN, INSN\}$.

Proposition A1 *Retailer's 1 optimal price, demand, and profit are impacted from competition as follows:*

(i) $p_1^{*N} = p_1^{*NISN}$ *and* $p_1^{*I} = p_1^{*INSN}$

(ii) If $\mu > (\mu + \epsilon)\gamma$ *then a)* $q_1^{*N} > q_1^{*NISN}$ *and* $q_1^{*I} > q_1^{*INSN}$ *and b)* $\pi_1^{*N} > \pi_1^{*NISN}$ *and* $\pi_1^{*I} > \pi_1^{*INSN}$.

(iii) If $\mu < (\mu + \epsilon)\gamma$ *then a)* $q_1^{*N} < q_1^{*NISN}$ *and* $q_1^{*I} < q_1^{*INSN}$ *and b)* $\pi_1^{*N} < \pi_1^{*NISN}$ *and* $\pi_1^{*I} < \pi_1^{*INSN}$.

Proof of Proposition A1.

(i) Since retailer 1's profits under monopoly and duopoly are concave in p_1 the first order conditions are both sufficient and necessary. Taking first order conditions of the profit expressions we have:

$$\frac{\partial \pi_1^{NISN}}{\partial p_1} = (1 - p_1 - k_1)(\mu + \epsilon)\gamma - (p_1 - c_1)(\mu + \epsilon)\gamma = 0 \quad \text{(A-1)}$$

$$\frac{\partial \pi_1^{INSN}}{\partial p_1} = (1 + \alpha - p_1 - k_1)(\mu + \epsilon)\gamma - (p_1 - c_1)(\mu + \epsilon)\gamma = 0 \quad \text{(A-2)}$$

Let $p_1^{*k} = arg\{\max_{p_1 \geq c_1} \pi_1^k\}$, where $k = \{NISN, INSN\}$. Hence, from (A-1) and (A-2) it follows that $p_1^{*N} = p_1^{*NISN} = \frac{1+c_1-k_1}{2}$ and $p_1^{*I} = p_1^{*INSN} = \frac{1+\alpha+c_1-k_1}{2}$.

(ii) a) The expressions of optimal demands are: $q_1^{*N} = \frac{(1-c_1-k_1)\mu}{2}$, $q_1^{*NISN} = \frac{(1-c_1-k_1)(\mu+\epsilon)\gamma}{2}$, $q_1^{*I} = \frac{(1+\alpha-c_1-k_1)\mu}{2}$, and $q_1^{*INSN} = \frac{(1+\alpha-c_1-k_1)(\mu+\epsilon)\gamma}{2}$. Hence, if $\mu > (\mu + \epsilon)\gamma$, then $q_1^{*N} > q_1^{*NISN}$ and $q_1^{*I} > q_1^{*INSN}$. If $\mu < (\mu+\epsilon)\gamma$, then $q_1^{*N} < q_1^{*NISN}$ and $q_1^{*I} < q_1^{*INSN}$.

(ii) b) The expressions of the optimal profits are: $\pi_1^{*N} = \frac{(1-c_1-k_1)^2\mu}{4}$, $\pi_1^{*NISN} = \frac{(1-c_1-k_1)^2(\mu+\epsilon)\gamma}{4}$, $\pi_1^{*I} = \frac{(1+\alpha-c_1-k_1)^2\mu}{4} - I_1$, and $\pi_1^{*INSN} = \frac{(1+\alpha-c_1-k_1)^2(\mu+\epsilon)\gamma}{4} - I_1$. Hence, if $\mu > (\mu+\epsilon)\gamma$, then $\pi_1^{*N} > \pi_1^{*NISN}$ and $\pi_1^{*I} > \pi_1^{*INSN}$. If $\mu < (\mu+\epsilon)\gamma$, then $\pi_1^{*N} < \pi_1^{*NISN}$ and $\pi_1^{*I} < \pi_1^{*INSN}$.

■

Proposition A1 shows that when consumers search from retailer 1 to retailer 2 (consumer search scheme (3)), retailer 1 prices identically under monopoly and duopoly regimes. The impact of competition on retailer 1's profit and demand depends solely on the relative magnitude of retailer 1's market potential under monopoly (μ) and duopoly regimes ($(\mu + \epsilon)\gamma$). Specifically, if the market potential of retailer 1 under monopoly is higher than the corresponding market potential under duopoly ($\mu > (\mu +$

$\epsilon)\gamma$), competition leads to retailer 1 obtaining lower demand and profit. If the market potential of retailer 1 under monopoly is lower than the corresponding market potential under duopoly ($\mu < (\mu + \epsilon)\gamma$), retailer 1 obtains higher demand and profit under competition. The impact of competition on retailer 1's price and profit is intuitive since there is no search of consumers from retailer 2 to retailer 1. As a result, the only factor that affects retailer 1 is the market expansion. It can be shown similarly that identical results hold for consumer search scheme (1).

Benefiting from Innovative Competition

Consumers search from retailer 2 to retailer 1

Figure A1 summarizes the impact of competition on retailer 1's profit according to different regimes related to market expansion, retailers' physical proximity, and magnitude of change in consumer valuation.

Proposition A2 *Let π_1^{*NINS} be the optimal profit of retailer 1 under duopoly, π_1^{*N} be the optimal profit of retailer 1 under monopoly, $\tilde{\Delta k_1}$ be a threshold of the retailers' physical proximity, $\tilde{\alpha}^{(1)}$, $\tilde{\alpha_2}$, and $\tilde{\alpha}$ be thresholds of the consumer valuation mean shifts (defined in Table A9). Then,*

i) if $\Delta k < \tilde{\Delta k_1}$ and $(\mu + \epsilon)(\gamma + \delta(1-\gamma)) > \mu$, then $\pi_1^{*N} < \pi_1^{*NINS}$,

ii) if $\Delta k < \tilde{\Delta k_1}$ and $\alpha > \tilde{\alpha} = \max\{0, \tilde{\alpha_2}\}$, then $\pi_1^{*N} < \pi_1^{*NINS}$,

iii a) if $\Delta k > \tilde{\Delta k_1}$ and $\alpha < \tilde{\alpha}^{(1)}$, then $\pi_1^{*N} > \pi_1^{*NINS}$,

iii b) if $\Delta k > \tilde{\Delta k_1}$ and $\alpha > \tilde{\alpha}^{(1)}$, then $\pi_1^{*N} < \pi_1^{*NINS}$.

Proposition A2 provides the following insights:

i) Retailer 1 has higher profits under competition in a high market expansion and high proximity regime. This result is intuitive. Since the retailers' proximity is high, consumers are more likely to buy from retailer 1 after having visited retailer 2, which combined with high market expansion will lead to higher profit for retailer 1 under duopoly.

ii) When the retailers are close (i.e., proximity is high)–irrespective of the market expansion–retailer 1 will obtain higher profit provided that the magnitude of change in consumer valuation exceeds a certain threshold (denoted as $\tilde{\alpha}$).

iii) When the retailers are far (i.e., proximity is low)–irrespective of the market expansion–retailer 1 has higher profit under competition, provided that the magnitude of change in consumer valuation exceeds a certain threshold (denoted as $\tilde{\alpha}^{(1)}$). The intuition behind this is that since the retailers' proximity is low, the consumer flow from retailer 2 to retailer 1 will be limited. As a result, retailer 1 benefits under competition only if his competitor makes a significant investment to increase consumer valuation. If the magnitude of change in consumer valuation is below a certain threshold then retailer 1's profit is lower under duopoly.

High Market Expansion

$\boldsymbol{\mu < (\mu + \varepsilon)(\gamma + \delta(1 - \gamma))}$

High Proximity	Low Proximity	
	$\alpha < \tilde{\alpha}^{(1)}$	$\alpha > \tilde{\alpha}^{(1)}$
Higher Profit	Lower Profit	Higher Profit

Low Market Expansion

$\boldsymbol{\mu > (\mu + \varepsilon)(\gamma + \delta(1 - \gamma))}$

High Proximity		Low Proximity	
$\alpha < \tilde{\alpha}_2$	$\alpha > \tilde{\alpha}_2$	$\alpha < \tilde{\alpha}^{(1)}$	$\alpha > \tilde{\alpha}^{(1)}$
Not Determined	Higher Profit	Lower Profit	Higher Profit

FIGURE A1: The impact on retailer 1's profit based on the analytical results in scenario NINS.

Note that in Figure A1 there is a regime (low market expansion and high proximity) in which the competitive effects are not fully determined. To gain more insights regarding the competitive outcomes under that regime, we performed a computational study, which showed the existence of a threshold in the consumer valuation mean shift $\tilde{\alpha}^{(1)}$. If the mean shift is low ($\alpha < \tilde{\alpha}^{(1)}$), the introduction of retailer 2 leads to retailer 1 obtaining lower profit and if the mean shift is high ($\alpha > \tilde{\alpha}^{(1)}$), retailer 2 creates a positive externality and retailer 1 free rides.

In summary, our analytical results along with the computational study demonstrate that competition may not always harm retailer 1. Even if retailer 2 brings in a small market expansion, retailer 1 can be more profitable under competition, provided that retailer 2 has made a sufficient improvement in consumer valuation. This result is consistent with the findings of the general case.

Proof of Proposition A2.

The retailers' profit functions π_1^{NINS} and π_2^{NINS} are concave in their corresponding prices p_1 and p_2. The optimal prices p_1^{*NINS} and p_2^{*NINS} for the two retailers are the intersection of the best response functions obtained from the first order conditions of retailers' profits. Evaluating the expressions for demands q_1^{NINS} and q_2^{NINS} and profits π_1^{NINS} and π_2^{NINS} at their optimal prices provides us with the corresponding optimal demand and profit expressions. We obtain expressions for $\tilde{\alpha_1}$, $\tilde{\alpha_2}$, and $\tilde{\alpha}^{(1)}$ respectively by solving the following equations: $p_1^{*NINS} = p_1^{*I}$, $q_1^{*NINS} = q_1^{*I}$, and $\pi_1^{*NINS} = \pi_1^{*I}$. We find the expression for $\tilde{\Delta k_1}$ by setting $\tilde{\alpha_1} = 0$ and solving for Δk. We provide these expressions in Table A9.

TABLE A9: Expressions for scenario $NINS$.

Notation	Expression
p_1^{*NINS}	$(2\gamma+\delta(1-\gamma)+\alpha\delta(1-\gamma)-2\delta(1-\gamma)\Delta k+2(\gamma(1-\delta)+\delta)c_1$ $+(1-\gamma)\delta c_2-2\gamma k_1-(1-\gamma)\delta k_2)/(4\gamma(1-\delta)+4\delta)$
q_1^{*NINS}	$(\epsilon+\mu)(2\gamma+\delta(1-\gamma)+\alpha\delta(1-\gamma)-2\delta(1-\gamma)\Delta k-2(\gamma(1-\delta)+\delta)c_1$ $+(1-\gamma)\delta c_2-2\gamma k_1-(1-\gamma)\delta k_2)/4$
π_1^{*NINS}	$((\epsilon+\mu)(2\gamma+\delta(1-\gamma)+\alpha\delta(1-\gamma)-2\delta(1-\gamma)\Delta k-2(\gamma(1-\delta)+\delta)c_1$ $+(1-\gamma)\delta c_2-2\gamma k_1-(1-\gamma)\delta k_2)^2)/(16(\gamma(1-\delta)+\delta))$
$\tilde{\alpha_1}$	$1+2\Delta k-c_2-2k_1+k_2$
$\tilde{\alpha_2}$	$(2\mu-(2\gamma+\delta(1-\gamma)-2(1-\gamma)\delta\Delta k)(\epsilon+\mu)-2(\mu-(\gamma(1-\delta)+\delta)(\epsilon+\mu))c_1-$ $(1-\gamma)\delta(\epsilon+\mu)c_2+2(\gamma\epsilon-(1-\gamma)\mu)k_1+(1-\gamma)\delta(\epsilon+\mu)k_2)/((1-\gamma)\delta(\epsilon+\mu))$
$\tilde{\alpha}^{(1)}$	$(-\frac{2\delta(1-c_1-k_1)}{1-\gamma}\pm\frac{2\sqrt{(1-\gamma)^2(\gamma(1-\delta)+\delta)\delta^2\mu(\epsilon+\mu)(1-c_1-k_1)^2}}{(1-\gamma)^2(\epsilon+\mu)}+$ $\delta(2-\delta+2\delta\Delta k-2(1-\delta)c_1-\delta c_2-2k_1+\delta k_2))/\delta^2$
$\Delta\tilde{k}_1$	$(c_2+2k_1-k_2-1)/2$

Comparing retailer 1's optimal price and demand under monopoly and duopoly regimes we obtain the results summarized in Table A10. From Table A10 we can make the following observations: (i) When the retailers' proximity is high ($\Delta k < \Delta\tilde{k}_1$) if the market expansion is high ($\mu < (\mu+\epsilon)(\gamma+\delta(1-\gamma))$), then $\pi_1^{*N} < \pi_1^{*NINS}$. (ii) When the retailers' proximity is high ($\Delta k < \Delta\tilde{k}_1$) if the market expansion is low ($\mu > (\mu+\epsilon)(\gamma+\delta(1-\gamma))$), then $\pi_1^{*N} < \pi_1^{*NINS}$, for $\alpha > \tilde{\alpha_2}$. Hence, from (i) if the retailers' proximity is high there exists a threshold $\tilde{\alpha} = \max\{0, \tilde{\alpha_2}\}$ such that if $\alpha > \tilde{\alpha}$ then $\pi_1^{*N} < \pi_1^{*NINS}$. (iii) If the retailers' proximity is low ($\Delta k > \Delta\tilde{k}_1$) retailer 1's profit is lower under duopoly than under monopoly for low levels of α and it is higher under duopoly than under monopoly for high levels of α. Hence, there exists a threshold in the consumer valuation mean shift $\tilde{\alpha}^{(1)}$ such that if $\alpha < \tilde{\alpha}^{(1)}$ then $\pi_1^{*N} > \pi_1^{*NINS}$ and if $\alpha > \tilde{\alpha}^{(1)}$ then $\pi_1^{*N} < \pi_1^{*NINS}$.

■

Implications of Competition for an Innovative Retailer

TABLE A10: Impact of competition on retailer 1 for scenario $NINS$.

Sufficient Conditions	Impact
$\mu < (\epsilon+\mu)(\gamma+\delta(1-\gamma))$ and $\Delta k < \tilde{\Delta k_1}$	$p_1^{*N} < p_1^{*NINS}$, $q_1^{*N} < q_1^{*NINS}$, and $\pi_1^{*N} < \pi_1^{*NINS}$
$\mu < (\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k > \tilde{\Delta k_1}$, and $\alpha < \tilde{\alpha_2}$	$p_1^{*N} > p_1^{*NINS}$, $q_1^{*N} > q_1^{*NINS}$, and $\pi_1^{*N} > \pi_1^{*NINS}$
$\mu < (\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k > \tilde{\Delta k_1}$, and $\tilde{\alpha_2} < \alpha < \tilde{\alpha_1}$	$p_1^{*N} > p_1^{*NINS}$ and $q_1^{*N} < q_1^{*NINS}$
$\mu < (\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k > \tilde{\Delta k_1}$, and $\alpha > \tilde{\alpha_1}$	$p_1^{*N} < p_1^{*NINS}$, $q_1^{*N} < q_1^{*NINS}$, and $\pi_1^{*N} < \pi_1^{*NINS}$
$\mu > (\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k < \tilde{\Delta k_1}$, and $\alpha < \tilde{\alpha_2}$	$p_1^{*N} < p_1^{*NINS}$ and $q_1^{*N} > q_1^{*NINS}$
$\mu > (\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k < \tilde{\Delta k_1}$, and $\alpha > \tilde{\alpha_2}$	$p_1^{*N} < p_1^{*NINS}$, $q_1^{*N} < q_1^{*NINS}$, and $\pi_1^{*N} < \pi_1^{*NINS}$
$\mu > (\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k > \tilde{\Delta k_1}$, and $\alpha < \tilde{\alpha_1}$	$p_1^{*N} > p_1^{*NINS}$, $q_1^{*N} > q_1^{*NINS}$, and $\pi_1^{*N} > \pi_1^{*NINS}$
$\mu > (\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k > \tilde{\Delta k_1}$, and $\tilde{\alpha_1} < \alpha < \tilde{\alpha_2}$	$p_1^{*N} < p_1^{*NINS}$ and $q_1^{*N} > q_1^{*NINS}$
$\mu > (\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k > \tilde{\Delta k_1}$, and $\alpha > \tilde{\alpha_2}$	$p_1^{*N} < p_1^{*NINS}$, $q_1^{*N} < q_1^{*NINS}$, and $\pi_1^{*N} < \pi_1^{*NINS}$

Consumers search from retailer 2 to retailer 1

Figure A2 presents the impact of competition on retailer 1's profit according to different regimes related to market expansion, retailers' physical proximity, and magnitude of change in consumer valuation. We find that retailer 1 always benefits under competition in a high market expansion and high proximity regime but retailer 1's profit is always lower under competition in a low market expansion and low proximity regime.

Proposition A3 *Let π_1^{*INNS} be the optimal profit of retailer 1 under duopoly, π_1^{*I} be the optimal profit of retailer 1 under monopoly, $\tilde{\Delta k_1}$, $\tilde{\Delta k_2}$ be thresholds of the retailers' physical proximity, and $\tilde{\alpha_3}$ be a threshold of the consumer valuation mean shift (defined in Table A11). Then,*

i) if $\Delta k > \max\{\tilde{\Delta k_1}, \tilde{\Delta k_2}\}$ and $\alpha < \tilde{\alpha_3}$, then $\pi_1^{*I} > \pi_1^{*INNS}$.

ii) if $\Delta k < \min\{\tilde{\Delta k_1}, \tilde{\Delta k_2}\}$ and $\alpha < \tilde{\alpha_3}$ then $\pi_1^{*I} < \pi_1^{*INNS}$.

Proposition A3 shows that in a low proximity regime ($\Delta k > \max\{\tilde{\Delta k_1}, \tilde{\Delta k_2}\}$), if retailer 1 invests in low improvement in consumer valuation his profit will be lower

High Market Expansion
$\boldsymbol{\mu < (\mu + \varepsilon)(\gamma + \delta(1 - \gamma))}$

High Proximity	Medium Proximity	Low Proximity	
		$\alpha < \tilde{\alpha}_3$	$\alpha > \tilde{\alpha}_3$
Higher Profit	Not Determined	Lower Profit	Not Determined

Low Market Expansion
$\boldsymbol{\mu > (\mu + \varepsilon)(\gamma + \delta(1 - \gamma))}$

High Proximity		Medium Proximity	Low Proximity
$\alpha < \tilde{\alpha}_3$	$\alpha > \tilde{\alpha}_3$		
Higher Profit	Not Determined	Not Determined	Lower Profit

FIGURE A2: The impact on retailer 1's profit based on the analytical results in scenario INNS.

under competition. Interestingly, in a high proximity regime ($\Delta k < \min\{\Delta\tilde{k}_1, \Delta\tilde{k}_2\}$) a low improvement in consumer valuation allows retailer 1 to obtain higher profit under duopoly.

Because several of the regimes in Figure A2 cannot be fully characterized analytically, we performed some computational study to gain more insights on the undetermined regimes. The computational study showed the existence of a threshold in the consumer valuation mean shift ($\tilde{\alpha}^{(2)}$). When the market expansion is high, a low mean shift ($\alpha < \tilde{\alpha}^{(2)}$) leads to retailer 1 being worse off under duopoly and a high mean shift ($\alpha > \tilde{\alpha}^{(2)}$) allows retailer 1 to benefit under duopoly. Interestingly, when the market expansion is low, a low mean shift is beneficial for retailer 1 in terms of profits, whereas a high mean shift can hurt retailer 1's profits (see Figure A3 with parameter values $\gamma = 0.1$, $\delta = 0.3$, $\epsilon = 1.5$, $\mu = 3$, $c_1 = 0.2$, $c_2 = 0.7$, $k_1 = 0.7$, $k_2 = 0.1$, $\Delta k = 0.1$, $I_1 = 0.05$). The nature of search partially drives the counterintuitive re-

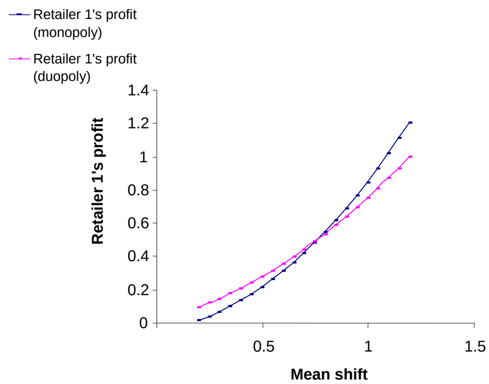


FIGURE A3: Retailer 1's profit versus mean shift in scenario INNS.

sult illustrated in Figure A3. Because the retailers are closely located to each other, consumers, who visit retailer 2 and do not obtain positive surplus, are likely to buy from retailer 1 if they search. As a result, retailer 1 is in an advantageous position which allows him to increase his price (i.e., $p_1^{*INNS} > p_1^{*I}$). Note that as the consumer valuation mean shift increases by $\Delta\alpha$, retailer 1's demands q_1^{*INNS} and q_1^{*I} increase by $\frac{\Delta\alpha}{2}(\mu + \epsilon)(\gamma\delta + (1 - \gamma)\delta)$ and $\frac{\Delta\alpha}{2}\mu$, respectively. Hence, q_1^{*I} increases at a faster rate than q_1^{*INNS} because the market expansion is low ($\mu > (\mu + \epsilon)(\gamma\delta + (1 - \gamma)\delta)$). Hence, even if originally the values of α are such that $q_1^{*INNS} > q_1^{*I}$ and consequently $\pi_1^{*INNS} > \pi_1^{*I}$, as α increases, q_1^{*I} becomes eventually higher than q_1^{*INNS}, which can reverse the relative magnitude of profits. In summary, our analytical results along with the computational study demonstrate that in a high market expansion regime retailer 1 should strive for a high improvement in consumer valuation in order to overcome profit loss to competition. This is not the case for a low expansion regime. When retailer 1 faces low market expansion, he should target low improvements in consumer valuation.

Finally, there are some regimes such as high proximity, high market expansion and low proximity, low market expansion where the outcome of competition does not depend on retailer 1's magnitude of improvement in consumer valuation. In such cases, the market characteristics (i.e., market expansion and retailers' physical proximity) determine the competitive effects on retailer 1's profits.

Proof of Proposition A3.

The retailers' profit functions π_1^{INNS} and π_2^{INNS} are concave in their corresponding prices p_1 and p_2. The optimal prices p_1^{*INNS} and p_2^{*INNS} for the two retailers are the intersection of the best response functions obtained from the first order conditions of retailers' profits. Evaluating the expressions for demands q_1^{INNS} and q_2^{INNS} and profits π_1^{INNS} and π_2^{INNS} at their optimal prices provides us with the corresponding optimal demand and profit expressions. We obtain the expressions for $\tilde{\Delta k_1}$, $\tilde{\alpha_3}$, and $\tilde{\alpha}^{(2)}$ respectively by solving the following equations: $p_1^{*INNS} = p_1^{*I}$, $q_1^{*INNS} = q_1^{*I}$, and $\pi_1^{*INNS} = \pi_1^{*I}$. The equation $q_1^{*INNS} = q_1^{*I}$ is a first degree polynomial of the form $\alpha x_1 = y_1$. We derive the expression for $\tilde{\Delta k_2}$ by setting $y_1 = 0$ and solving for Δk. We present all these expressions in Table A11.

Comparing retailer 1's optimal price and demand under monopoly and duopoly regimes we obtain the results summarized in Table A12. In Table A12 we can see that if the retailers' proximity is low ($\Delta k > \max\{\tilde{\Delta k_1}, \tilde{\Delta k_2}\}$), then $\pi_1^{*I} > \pi_1^{*INNS}$ for $\alpha < \tilde{\alpha_3}$. In the case that the retailers' proximity is high ($\Delta k < \max\{\tilde{\Delta k_1}, \tilde{\Delta k_2}\}$) then $\pi_1^{*I} < \pi_1^{*INNS}$ for $\alpha < \tilde{\alpha_3}$.

■

TABLE A11: Expressions for scenario $INNS$.

Notation	Expression
p_1^{*INNS}	$(2\gamma+\delta(1-\gamma)+2\alpha(\gamma(1-\delta)+\delta)-2\delta(1-\gamma)\Delta k+2(\gamma(1-\delta)+\delta)c_1$ $+(1-\gamma)\delta c_2-2\gamma k_1-(1-\gamma)\delta k_2)/(4\gamma(1-\delta)+4\delta)$
q_1^{*INNS}	$(\epsilon+\mu)(2\gamma+\delta(1-\gamma)+2\alpha(\gamma(1-\delta)+\delta)-2\delta(1-\gamma)\Delta k-2(\gamma(1-\delta)+\delta)c_1$ $+(1-\gamma)\delta c_2-2\gamma k_1-(1-\gamma)\delta k_2)/4$
π_1^{*INNS}	$((\epsilon+\mu)(2\gamma+\delta(1-\gamma)+2\alpha(\gamma(1-\delta)+\delta)-2\delta(1-\gamma)\Delta k-2(\gamma(1-\delta)+\delta)c_1$ $+(1-\gamma)\delta c_2-2\gamma k_1-(1-\gamma)\delta k_2)^2)/(16\gamma(1-\delta)+16\delta)-I_1$
$\tilde{\alpha_3}$	$(-2\mu+(2\gamma+\delta(1-\gamma)-2(1-\gamma)\delta\Delta k)(\epsilon+\mu)+2(\mu+(\gamma(1-\delta)+\delta)(\epsilon+\mu))c_1$ $+(1-\gamma)\delta(\epsilon+\mu)c_2-2(\gamma\epsilon-(1-\gamma)\mu)k_1-(1-\gamma)\delta(\epsilon+\mu)k_2)/(2(\mu-(\gamma(1-\delta)+\delta)(\epsilon+\mu)))$
ζ_1	$-(\gamma(1-\delta)+\delta)(-2\mu+(2\gamma+\delta(1-\gamma)-2(1-\gamma)\delta\Delta k)(\epsilon+\mu))$
η_1	$(1-\gamma)^2(\gamma(1-\delta)+\delta)\delta^2\mu(\epsilon+\mu)(1+2\Delta k-c_2-2k_1+k_2)^2$
θ_1	$(\gamma(1-\delta)+\delta)(2(\mu-(\gamma(1-\delta)+\delta)(\epsilon+\mu))c_1+(1-\gamma)\delta(\epsilon+\mu)c_2-2(\gamma\epsilon-(1-\gamma)\mu)k_1-(1-\gamma)\delta(\epsilon+\mu)k_2)$
$\tilde{\alpha}^{(2)}$	$(\zeta_1\pm\sqrt{\eta_1}-\theta_1)/(-2(\gamma(1-\delta)+\delta)(\mu-(\gamma(1-\delta)+\delta)(\epsilon+\mu)))$
$\Delta\tilde{k}_1$	$(c_2+2k_1-k_2-1)/2$
$\Delta\tilde{k}_2$	$(-2\mu+(\gamma(2-\delta)+\delta)(\epsilon+\mu)+2(\mu-(\gamma(1-\delta)+\delta)(\epsilon+\mu))c_1+$ $(1-\gamma)\delta(\epsilon+\mu)c_2+2(\mu-\gamma(\epsilon+\mu))k_1-(1-\gamma)\delta(\epsilon+\mu)k_2)/(2(1-\gamma)\delta(\epsilon+\mu))$

TABLE A12: Impact of competition on retailer 1 for scenario $INNS$.

Sufficient Conditions	Impact
$\mu<(\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k<\min\{\Delta\tilde{k}_1,\Delta\tilde{k}_2\}$	$p_1^{*I}<p_1^{*INNS}$, $q_1^{*I}<q_1^{*INNS}$, and $\pi_1^{*I}<\pi_1^{*INNS}$
$\mu<(\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta\tilde{k}_1<\Delta k<\Delta\tilde{k}_2$	$p_1^{*I}>p_1^{*INNS}$ and $q_1^{*I}<q_1^{*INNS}$
$\mu<(\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k>\max\{\Delta\tilde{k}_1,\Delta\tilde{k}_2\}$, and $\alpha<\tilde{\alpha_3}$	$p_1^{*I}>p_1^{*INNS}$, $q_1^{*I}>q_1^{*INNS}$, and $\pi_1^{*I}>\pi_1^{*INNS}$
$\mu<(\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k>\max\{\Delta\tilde{k}_1,\Delta\tilde{k}_2\}$ and $\alpha>\tilde{\alpha_3}$	$p_1^{*I}>p_1^{*INNS}$ and $q_1^{*I}<q_1^{*INNS}$
$\mu>(\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k<\min\{\Delta\tilde{k}_1,\Delta\tilde{k}_2\}$, and $\alpha<\tilde{\alpha_3}$	$p_1^{*I}<p_1^{*INNS}$, $q_1^{*I}<q_1^{*INNS}$, and $\pi_1^{*I}<\pi_1^{*INNS}$
$\mu>(\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k<\min\{\Delta\tilde{k}_1,\Delta\tilde{k}_2\}$, and $\alpha>\tilde{\alpha_3}$	$p_1^{*I}<p_1^{*INNS}$ and $q_1^{*I}>q_1^{*INNS}$
$\mu>(\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta\tilde{k}_2<\Delta k<\Delta\tilde{k}_1$	$p_1^{*I}<p_1^{*INNS}$ and $q_1^{*I}>q_1^{*INNS}$
$\mu>(\epsilon+\mu)(\gamma+\delta(1-\gamma))$, $\Delta k>\max\{\Delta\tilde{k}_1,\Delta\tilde{k}_2\}$	$p_1^{*I}>p_1^{*INNS}$, $q_1^{*I}>q_1^{*INNS}$, and $\pi_1^{*I}>\pi_1^{*INNS}$

B1 Appendix for Chapter 4

Proof of Proposition 8.

(1) The difference between the optimal expected prices in scenarios F and S is

$\hat{p}^F-\hat{p}^S=\frac{(\eta_2-\eta_1)b_s^2(\alpha_H+\alpha_L)}{2(2b_p\eta_1-b_s^2)(2b_p\eta_2-b_s^2)}$

Since, $\eta_2>\eta_1$, $2b_p\eta_1-b_s^2>0$, and $2b_p\eta_2-b_s^2>0$ we have $\hat{p}^F>\hat{p}^S$.

The difference between the optimal expected prices in scenarios B and F is

$\hat{p}^B - \hat{p}^F = \frac{b_s^2 \eta_1^2 (\alpha_H + \alpha_L)}{2(2b_p\eta_1 - b_s^2)(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))}$

Since, $2b_p\eta_1 - b_s^2 > 0$ and $2b_p\eta_1\eta_2 - b_s^2(\eta_1 + \eta_2) > 0$ we have $\hat{p}^B > \hat{p}^F$. Hence, $\hat{p}^B > \hat{p}^F > \hat{p}^S$.

(2) $s^F = arg\max_s\{E\pi(s)\}$ and $\hat{s}^S = E(arg\max_s \pi(s))$ where E is the expectation with respect to demand. Since $\pi(s)$ is strictly concave in s then applying Jensen's inequality we get $s^F = arg\max_s\{E\pi(s)\} > E(arg\max_s \pi(s)) = \hat{s}^S$. Hence, $s^F > \hat{s}^S$.

(3) The difference in s_1 in scenarios B and F is

$s_1^B - s_1^F = \frac{\eta_1 b_s^3 (\alpha_H + \alpha_L)}{2(2b_p\eta_1 - b_s^2)(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))}$

Since $2b_p\eta_1 - b_s^2 > 0$ and $2b_p\eta_1\eta_2 - b_s^2(\eta_1 + \eta_2) > 0$ we have $s_1^B > s_1^F$.

(4) The difference in s_2 in scenarios B and S is

$s_2^B - \hat{s}^S = \frac{b_s^3 (\alpha_H + \alpha_L)\eta_2}{2(2b_p\eta_2 - b_s^2)(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))}$

Since $2b_p\eta_2 - b_s^2 > 0$ and $2b_p\eta_1\eta_2 - b_s^2(\eta_1 + \eta_2) > 0$ we have $s_2^B > \hat{s}^S$.

(5) The difference in expected demands in scenarios F and S is

$\hat{D}^F - \hat{D}^S = \frac{(\eta_2 - \eta_1) b_p b_s^2 (\alpha_H + \alpha_L)}{2(2b_p\eta_1 - b_s^2)(2b_p\eta_2 - b_s^2)}$

Since, $2b_p\eta_1 - b_s^2 > 0$ and $2b_p\eta_2 - b_s^2 > 0$ we have $\hat{D}^F > \hat{D}^S$.

The difference in expected demands in scenarios B and F is

$\hat{D}^B - \hat{D}^F = \frac{\eta_1^2 b_s^2 b_p (\alpha_H + \alpha_L)}{2(2b_p\eta_1 - b_s^2)(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))}$

Since, $2b_p\eta_1 - b_s^2 > 0$ and $2b_p\eta_1\eta_2 - b_s^2(\eta_1 + \eta_2) > 0$ we have $\hat{D}^B > \hat{D}^F$.

Hence, $\hat{D}^B > \hat{D}^F > \hat{D}^S$.

(6) The difference in expected profits in scenarios B and F is

$\hat{\pi}^B - \hat{\pi}^F = \frac{2u^2 b_s^2 (2b_p\eta_1 - b_s^2)(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2)) + 4m^2 b_p b_s^2 \eta_1^2 (2b_p\eta_2 - b_s^2)}{8b_p(2b_p\eta_1 - b_s^2)(2b_p\eta_2 - b_s^2)(2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2))}$

Since, $2b_p\eta_1 - b_s^2 > 0$ and $2b_p\eta_2 - b_s^2 > 0$, and $2b_p\eta_1\eta_2 - b_s^2(\eta_1 + \eta_2) > 0$ we have

$\hat{\pi}^B > \hat{\pi}^F$.

(7) The difference in expected profits in scenarios B and S is

$\hat{\pi}^B - \hat{\pi}^S = \frac{b_s^2\eta_2^2(\alpha_H+\alpha_L)^2}{4(2b_p\eta_2-b_s^2)(2b_p\eta_1\eta_2-b_s^2(\eta_1+\eta_2))}$

Since, $2b_p\eta_1 - b_s^2 > 0$ and $2b_p\eta_1\eta_2 - b_s^2(\eta_1+\eta_2) > 0$ we have $\hat{\pi}^B > \hat{\pi}^S$.

(8) The difference in expected profits in scenarios F and S is

$\hat{\pi}^F - \hat{\pi}^S = \frac{-2u^2b_s^2(2b_p\eta_1-b_s^2)+4m^2b_pb_s^2(\eta_2-\eta_1)}{8b_p(2b_p\eta_1-b_s^2)(2b_p\eta_2-b_s^2)}$

Obviously the sign of $\hat{\pi}^F-\hat{\pi}^S$ depends on the sign of $-2u^2b_s^2(2b_p\eta_1-b_s^2)+4m^2b_pb_s^2(\eta_2-\eta_1)$ which is a second degree polynomial in terms of u. The above polynomial has two roots one of which is negative. Hence, only one root which we denote by $\hat{u} = m\sqrt{\frac{2b_p(\eta_2-\eta_1)}{2b_p\eta_1-b_s^2}}$ is positive. $\hat{u} < m$ if and only if $\eta_2 < \tilde{\eta_2} = \frac{4b_p\eta_1-b_s^2}{2b_p}$. Hence, if $\eta_2 \geq \tilde{\eta_2}$ then $\hat{\pi}^F \geq \hat{\pi}^S$. If $\eta_2 < \tilde{\eta_2}$ and $u < \hat{u}$ then $\hat{\pi}^F > \hat{\pi}^S$ else if $\eta_2 < \tilde{\eta_2}$ and $u > \hat{u}$ we have $\hat{\pi}^F < \hat{\pi}^S$.

■

Proof of Lemma 1.

i) $\frac{\partial\tilde{\eta_2}}{\partial b_p} = \frac{b_s^2}{2b_p^2} > 0$, $\frac{\partial\tilde{\eta_2}}{\partial b_s} = -\frac{b_s}{b_p} < 0$, $\frac{\partial\tilde{\eta_2}}{\partial \eta_1} = 2 > 0$

ii) $\frac{\partial\hat{u}}{\partial b_p} = -\frac{mb_s^2(\frac{b_p(\eta_2-\eta_1)}{2b_p\eta_1-b_s^2})^{3/2}}{\sqrt{2}b_p^2(\eta_2-\eta_1)} < 0$, $\frac{\partial\hat{u}}{\partial b_s} = \frac{\sqrt{2}mb_s\sqrt{\frac{b_p(\eta_2-\eta_1)}{2b_p\eta_1-b_s^2}}}{2b_p\eta_1-b_s^2} > 0$, $\frac{\partial\hat{u}}{\partial m} = \sqrt{\frac{2b_p(\eta_2\eta_1)}{2b_p\eta_1-b_s^2}} > 0$, $\frac{\partial\hat{u}}{\partial \eta_1} = -\frac{mb_p(2b_p\eta_2-b_s^2)}{\sqrt{2}(2b_p\eta_1-b_s^2)^2\sqrt{\frac{b_p(\eta_2-\eta_1)}{2b_p\eta_1-b_s^2}}} < 0$, $\frac{\partial\hat{u}}{\partial \eta_2} = \frac{m\sqrt{\frac{(\eta_2-\eta_1)b_p}{4b_p\eta_1-2b_s^2}}}{\eta_2-\eta_1} > 0$

■

Proof of Proposition 9.

(1) The difference between the optimal prices in scenarios FF and SS is

$p_i^{FF} - p_i^{SS} = -\frac{\alpha\eta b_s\theta_p(2b_p+3\theta_p)T}{(W^2\eta-2S(S-T)(\theta_p+b_p))(\eta(2b_p+\theta_p)-b_s(b_s+\theta_s))}$

Since $T > 0$, $(W^2\eta - 2S(S-T)(\theta_p+b_p)) > 0$, and $\eta(2b_p+\theta_p) - b_s(b_s+\theta_s) > 0$ we have $p_i^{FF} < p_i^{SS}$.

(2) The difference between the optimal services in scenarios FF and SS is

$s_i^{FF} - s_i^{SS} = -\frac{\alpha\eta\theta_p(2b_p+\theta_p)(2b_p+3\theta_p)T}{(W^2\eta-2S(S-T)(\theta_p+b_p))(\eta(2b_p+\theta_p)-b_s(b_s+\theta_s))}$

Since $T > 0$, $(W^2\eta - 2S(S-T)(\theta_p + b_p)) > 0$, and $\eta(2b_p + \theta_p) - b_s(b_s + \theta_s) > 0$ we have $p_i^{FF} < p_i^{SS}$.

(3) The difference between the optimal demands in scenarios FF and SS is

$D_i^{FF} - D_i^{SS} = -\frac{\alpha\eta\theta_p b_s(2b_p+3\theta_p)T(b_p+\theta_p)}{(W^2\eta-2S(S-T)(\theta_p+b_p))(\eta(2b_p+\theta_p)-b_s(b_s+\theta_s))}$

Since $T > 0$, $(W^2\eta - 2S(S-T)(\theta_p + b_p)) > 0$, and $\eta(2b_p + \theta_p) - b_s(b_s + \theta_s) > 0$ we have $D_i^{FF} - D_i^{SS}$.

(4) The difference between the optimal profits in scenarios FF and SS is

$\pi_i^{FF} - \pi_i^{SS} = \frac{\alpha^2\eta^2\theta_p(2b_p+3\theta_p)^2T^2Y}{2(W^2\eta-2S(S-T)(\theta_p+b_p))^2(\eta(2b_p+\theta_p)-b_s(b_s+\theta_s))^2}$

where $Y = (2b_p + \theta_p)^2(4b_p + 5\theta_p)\eta - 2b_s(\theta_p + b_p)(4b_p(b_s + \theta_s) + \theta_p(3b_s + 2\theta_s))$

Hence, if $Y > 0$ then $\pi_i^{FF} > \pi_i^{SS}$, else $\pi_i^{FF} \leq \pi_i^{SS}$.

$Y > 0 \Leftrightarrow \eta > \frac{2b_s(b_p+\theta_p)(4b_p(b_s+\theta_s)+\theta_p(3b_s+2\theta_s))}{(2b_p+\theta_p)^2(4b_p+5\theta_p)}$

In order to ensure existence and uniqueness of the NE in scenarios FF and SS we should impose the following condition on η, $\eta > max\{\frac{2(b_p+\theta_p)S(S+T)}{W^2}, \frac{(b_s+\theta_s)^2}{2(b_p+\theta_p)}\} = n_{max}$

It is easy to show that $n_{max} > \frac{2b_s(b_p+\theta_p)(4b_p(b_s+\theta_s)+\theta_p(3b_s+2\theta_s))}{(2b_p+\theta_p)^2(4b_p+5\theta_p)}$ as a result Y cannot be negative or zero. Thus, $Y > 0$. So we have $\pi_i^{FF} > \pi_i^{SS}$.

■

Proof of Proposition 10.

(1) The difference between the optimal prices in scenarios FF and SS is

$p_i^{FF} - p_i^{SS} = \frac{2\alpha b_s(2b_p+3\theta_p)(b_p+\theta_p)(2b_p(b_s+\theta_s)+\theta_p(2b_s+\theta_s))\eta_2-\alpha b_s(2b_p+\theta_p)(2b_p+3\theta_p)^2(b_s+\theta_s)\eta_1}{(W^2\eta_1-2S(S-T)(\theta_p+b_p))(\eta_2(2b_p+\theta_p)-b_s(b_s+\theta_s))}$

Hence if $\frac{\eta_2}{\eta_1} \geq \frac{(2b_p+\theta_p)(2b_p+3\theta_p)(b_s+\theta_s)}{2(b_p+\theta_p)(2b_p(b_s+\theta_s)+\theta_p(2b_s+\theta_s))} = \bar{\eta}$ then $p_i^{FF} \geq p_i^{SS}$ and if $\frac{\eta_2}{\eta_1} < \bar{\eta}$ then $p_i^{FF} < p_i^{SS}$.

(2) The difference between the optimal services in scenarios FF and SS is

$s_i^{FF} - s_i^{SS} = \frac{2\alpha b_s(2b_p+3\theta_p)(b_p+\theta_p)(2b_p+\theta_p)(2b_p(b_s+\theta_s)+\theta_p(2b_s+\theta_s))\eta_2 - \alpha b_s(2b_p+\theta_p)^2(2b_p+3\theta_p)^2(b_s+\theta_s)\eta_1}{(W^2\eta_1-2S(S-T)(\theta_p+b_p))(\eta_2(2b_p+\theta_p)-b_s(b_s+\theta_s))}$

Hence if $\frac{\eta_2}{\eta_1} \geq \frac{(2b_p+\theta_p)(2b_p+3\theta_p)(b_s+\theta_s)}{2(b_p+\theta_p)(2b_p(b_s+\theta_s)+\theta_p(2b_s+\theta_s))} = \bar{\eta}$ then $s_i^{FF} \geq s_i^{SS}$ and if $\frac{\eta_2}{\eta_1} < \bar{\eta}$ then $s_i^{FF} < s_i^{SS}$.

(3) The difference between the optimal demands in scenarios FF and SS is

$D_i^{FF} - D_i^{SS} = (b_p+\theta_p)\frac{2\alpha b_s(2b_p+3\theta_p)(b_p+\theta_p)(2b_p(b_s+\theta_s)+\theta_p(2b_s+\theta_s))\eta_2 - \alpha b_s(2b_p+\theta_p)(2b_p+3\theta_p)^2(b_s+theta_s)\eta_1}{(W^2\eta_1-2S(S-T)(\theta_p+b_p))(\eta_2(2b_p+\theta_p)-b_s(b_s+\theta_s))}$

Hence if $\frac{\eta_2}{\eta_1} \geq \frac{(2b_p+\theta_p)(2b_p+3\theta_p)(b_s+\theta_s)}{2(b_p+\theta_p)(2b_p(b_s+\theta_s)+\theta_p(2b_s+\theta_s))} = \bar{\eta}$ then $D_i^{FF} \geq D_i^{SS}$ and if $\frac{\eta_2}{\eta_1} < \bar{\eta}$ then $D_i^{FF} < D_i^{SS}$.

■

Proof of Proposition 11.

We consider the impact of ϵ on the optimal profits of a duopolist in scenario SS. We have $sign(\frac{\partial \pi_i^{SS}}{\partial \epsilon}) = sign((\epsilon+\eta)((2b_p+\theta_p)\theta_s - b_s(2b_p+3\theta_p)) + b_s(b_s+\theta_s)^2)$.

Recall that we have imposed the following condition on θ_s to exclude positive externalities of retailer i's service on retailer j's demand: $\theta_s > \frac{b_s\theta_p}{2b_p+\theta_p}$.

We let $\epsilon_1 = \frac{b_s(b_s+\theta_s)^2+\eta(2b_p+\theta_p)\theta_s - b_s(2b_p+3\theta_p)}{b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s}$ and examine the following cases:

Case 1. If $\frac{b_s\theta_p}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\epsilon \geq \epsilon_1$ we have $\frac{\partial \pi_i^{SS}}{\partial \epsilon} \leq 0$.

Case 2. If $\frac{b_s\theta_p}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\epsilon < \epsilon_1$ we have $\frac{\partial \pi_i^{SS}}{\partial \epsilon} > 0$.

Case 3. If $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ we have $\frac{\partial \pi_i^{SS}}{\partial \epsilon} > 0$.

■

Proof of Proposition 12.

Let $\eta_1 = \eta$ and $\eta_2 = \eta + \epsilon$. Recall that in the case of equal costs (i.e., $\eta_1 = \eta_2 = \eta$) $\pi_i^{FF} > \pi_i^{SS}$. Let $\epsilon_1 = \frac{b_s(b_s+\theta_s)^2+\eta(2b_p+\theta_p)\theta_s - b_s(2b_p+3\theta_p)}{b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s}$. In order to ensure existence and uniqueness of the NE in scenarios FF and SS we should impose the following condition on η, $\eta > max\{\frac{2(b_p+\theta_p)S(S+T)}{W^2}, \frac{(b_s+\theta_s)^2}{2(b_p+\theta_p)} - \epsilon\} = n_{max}$

Case a. If $\frac{b_s\theta_p}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\eta > \max\{\frac{b_s(b_s+\theta_s)^2}{b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s}, n_{max}\}$ then $\epsilon_1 < 0$ and hence from Proposition 11 we have $\frac{\partial \pi_i^{SS}}{\partial \epsilon} \leq 0$. As a result we have $\pi_i^{SS} < \pi_i^{FF}$ $\forall\ \epsilon > 0$

Case b. If $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ then from Proposition 11 we have $\frac{\partial \pi_i^{SS}}{\partial \epsilon} > 0$. Hence, then there exists a threshold $\bar{\epsilon_3}$ such that if $\epsilon < \bar{\epsilon_3}$ then $\pi_i^{SS} < \pi_i^{FF}$ and if $\epsilon > \bar{\epsilon_3}$ then $\pi_i^{SS} > \pi_i^{FF}$.

Note that $\pi_i^{FF} - \pi_i^{SS}$ could be expressed as a second degree polynomial in terms of ϵ. $\bar{\epsilon_3}$ is the positive root of that polynomial for the parameter setting under which $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$.

■

Proof of Proposition 13.

(1) The difference between the optimal expected prices in scenarios FF and SS is

$\hat{p_i}^{FF} - \hat{p_i}^{SS} = -\frac{\eta b_s(\alpha_H+\alpha_L)\theta_p(2b_p+3\theta_p)T}{2((2b_p+\theta_p)\eta - b_s(b_s+\theta_s))(W^2\eta - 2S(S-T)(b_p+\theta_p))} < 0$

Hence, we have $\hat{p_i}^{FF} < \hat{p_i}^{SS}$.

(2) The difference between the optimal expected service levels in scenarios FF and SS is

$s_i^{FF} - \hat{s_i}^{SS} = -\frac{\eta(\alpha_H+\alpha_L)\theta_p(2b_p+\theta_p)(2b_p+3\theta_p)T}{2(W^2\eta - 2S(S-T)(b_p+\theta_p))((2b_p+\theta_p)\eta - b_s(b_s+\theta_s))} < 0$

Hence, we have $s_i^{FF} < \hat{s_i}^{SS}$.

(3) The difference between the optimal expected demands in scenarios FF and SS is

$\hat{D}_i^{FF} - \hat{D}_i^{SS} = -\eta(b_p + \theta_p)\frac{b_s(\alpha_H+\alpha_L)\theta_p(2b_p+3\theta_p)T}{2((2b_p+\theta_p)\eta - b_s(b_s+\theta_s))(W^2\eta - 2S(S-T)(b_p+\theta_p))} < 0$

Hence, we have $\hat{D}_i^{FF} < \hat{D}_i^{SS}$.

(4) Recall, that in the deterministic demand case (i.e., $u = 0$) when $\eta_1 = \eta_2 = \eta$ the dominant strategy for a symmetric duopolist is to invest in the first stage. We now examine the impact of demand variability on profits in order to characterize the dominant strategy under stochastic demand.

$\frac{\partial \hat{\pi}_i^{FF}}{\partial u} = \frac{2u(b_p+\theta_p)}{(2b_p+\theta_p)^2} > 0$, $\frac{\partial \hat{\pi}_i^{SS}}{\partial u} = \frac{u\eta(2\eta(b_p+\theta_p)-(b_s+\theta_s)^2)}{(\eta(2b_p+\theta_p)-b_s(b_s+\theta_s))^2} > 0$.

Recall that $\eta > \eta_{max} = max\{\frac{2(b_p+\theta_p)S(S+T)}{W^2}, \frac{(b_s+\theta_s)^2}{2(b_p+\theta_p)}\}$ to ensure existence and uniqueness of NE for the scenarios SS and FF. In addition, we have $\theta_s > \frac{b_s\theta_p}{2b_p+\theta_p}$. Let $\hat{\eta} = \frac{2b_s^2(b_p+\theta_p)(b_s+\theta_s)}{(2b_p+\theta_p)(b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s)}$.

1. If $\frac{b_s\theta_p}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\eta_{max} < \eta < \hat{\eta}$ (i.e., quadrant 1) then $\frac{\partial \hat{\pi}_i^{FF}}{\partial u} > \frac{\partial \hat{\pi}_i^{SS}}{\partial u}$.

2. If $\theta_s > \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ (i.e., quadrant 2 and quadrant 4) then $\frac{\partial \hat{\pi}_i^{FF}}{\partial u} > \frac{\partial \hat{\pi}_i^{SS}}{\partial u}$.

3. If $\frac{b_s\theta_p}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\eta > max\{\eta_{max}, \hat{\eta}\}$ (i.e., quadrant 3) then $\frac{\partial \hat{\pi}_i^{FF}}{\partial u} < \frac{\partial \hat{\pi}_i^{SS}}{\partial u}$. Hence,

i) if η and θ_s are in quadrants 1, 2, or 4 then $\frac{\partial \hat{\pi}_i^{FF}}{\partial u} > \frac{\partial \hat{\pi}_i^{SS}}{\partial u}$ as a result FF will always be a dominant strategy for those cases.

ii) if η and θ_s are in quadrant 3 then $\frac{\partial \hat{\pi}_i^{SS}}{\partial u} > \frac{\partial \hat{\pi}_i^{FF}}{\partial u}$. Hence, there will exist a threshold $\bar{u}$ such that if $u > \bar{u}$ scenario SS is the dominant strategy and if $u < \bar{u}$ scenario FF is the dominant strategy.

The expression for $\bar{u}$ is $\bar{u} = \sqrt{\frac{C}{-A}}$, where $C = \eta^2 m^2 \theta_p(2b_p+\theta_p)^2(2b_p+3\theta_p)^4(b_s\theta_p -$

$(2b_p+\theta_p)\theta_s)^2(\eta(2b_p+\theta_p)^2(4b_p+5\theta_p)-2b_s(b_p+\theta_p)(4b_p(b_s+\theta_s)+\theta_p(3b_s+2\theta_s)))$ and $A=-\eta(2b_p+\theta_p)(b_s+\theta_s)(b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s)+2b_s^2(b_p+\theta_p)(b_s+\theta_s)^2$. We next prove that $C>0$. $C>0 \Leftrightarrow \eta > \frac{2b_s(b_p+\theta_p)(4b_p(b_s+\theta_s)+\theta_p(3b_s+2\theta_s))}{(2b_p+\theta_p)^2(4b_p+5\theta_p)}$. One can easily show that $\eta_{max} > \frac{2b_s(b_p+\theta_p)(4b_p(b_s+\theta_s)+\theta_p(3b_s+2\theta_s))}{(2b_p+\theta_p)^2(4b_p+5\theta_p)}$. Hence, $C>0$. In addition, $-A>0 \Leftrightarrow \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ which holds in quadrant 3. ■

Proof of Proposition 14. The impact of demand variability on profits under monopoly for scenarios S and F is given below:

$\frac{\partial \hat{\pi}_i{}^F}{\partial u} = \frac{u}{2b_p} > 0$, $\frac{\partial \hat{\pi}_i{}^S}{\partial u} = \frac{u\eta}{2b_p\eta - b_s^2} > 0$. It can be easily shown that $\frac{\partial \hat{\pi}_i{}^S}{\partial u} > \frac{\partial \hat{\pi}_i{}^F}{\partial u}$.

The impact of demand variability on profits under duopoly for scenarios SS and FF is given below:

$\frac{\partial \hat{\pi}_i{}^{FF}}{\partial u} = \frac{2u(b_p+\theta_p)}{(2b_p+\theta_p)^2} > 0$, $\frac{\partial \hat{\pi}_i{}^{SS}}{\partial u} = \frac{u\eta(2\eta(b_p+\theta_p)-(b_s+\theta_s)^2)}{(\eta(2b_p+\theta_p)-b_s(b_s+\theta_s))^2} > 0$. It can be easily shown that $\frac{\partial \hat{\pi}_i{}^F}{\partial u} > \frac{\partial \hat{\pi}_i{}^{FF}}{\partial u}$.

Hence, $\frac{\partial \hat{\pi}_i{}^S}{\partial u} > \frac{\partial \hat{\pi}_i{}^F}{\partial u} > \frac{\partial \hat{\pi}_i{}^{FF}}{\partial u}$. In Proposition 13 we showed the following:

1. If $\frac{b_s\theta_p}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\eta_{max} < \eta < \hat{\eta}$ (i.e., quadrant 1) then $\frac{\partial \hat{\pi}_i{}^{FF}}{\partial u} > \frac{\partial \hat{\pi}_i{}^{SS}}{\partial u}$.

2. If $\theta_s > \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ (i.e., quadrant 2 and quadrant 4) then $\frac{\partial \hat{\pi}_i{}^{FF}}{\partial u} > \frac{\partial \hat{\pi}_i{}^{SS}}{\partial u}$.

3. If $\frac{b_s\theta_p}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\eta > max\{\eta_{max}, \hat{\eta}\}$ (i.e., quadrant 3) then $\frac{\partial \hat{\pi}_i{}^{FF}}{\partial u} < \frac{\partial \hat{\pi}_i{}^{SS}}{\partial u}$. Hence,

i) If η and θ_s are in quadrants 1,2, or 4 then $\frac{\partial \hat{\pi}_i{}^S}{\partial u} > \frac{\partial \hat{\pi}_i{}^F}{\partial u} > \frac{\partial \hat{\pi}_i{}^{FF}}{\partial u} > \frac{\partial \hat{\pi}_i{}^{SS}}{\partial u}$.

ii) If η and θ_s are in quadrant 3 then $\frac{\partial \hat{\pi}_i{}^S}{\partial u} > \frac{\partial \hat{\pi}_i{}^F}{\partial u}$ and $\frac{\partial \hat{\pi}_i{}^{SS}}{\partial u} > \frac{\partial \hat{\pi}_i{}^{FF}}{\partial u}$.

■

Proof of Proposition 15.

Let $\epsilon_2 = \frac{2b_s^2(b_s+\theta_s)(b_p+\theta_p)+\eta((2b_p+\theta_p)\theta_s-b_s(2b_p+3\theta_p))(2b_p+\theta_p)}{(b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s)(2b_p+\theta_p)}$.

Next, we identify the impact of demand variability on the duopolist profits for scenario FF and scenario SS.

$\frac{\partial \hat{\pi_i}^{FF}}{\partial u} = \frac{2u(b_p+\theta_p)}{(2b_p+\theta_p)^2}$

$\frac{\partial \hat{\pi_i}^{SS}}{\partial u} = \frac{u(\eta+\epsilon)(2(\eta+\epsilon)(b_p+\theta_p)-(b_s+\theta_s)^2)}{(\eta+\epsilon)((2b_p+\theta_p)-b_s(b_s+\theta_s))^2}$

$sign(\frac{\partial \hat{\pi_i}^{FF}}{\partial u} - \frac{\partial \hat{\pi_i}^{SS}}{\partial u}) = sign((\eta+\epsilon)(2b_p+\theta_p)(b_s+\theta_s)((2b_p+\theta_p)\theta_s - b_s(2b_p+3\theta_p)) + 2b_s^2(b_p+\theta_p)(b_s+\theta_s)^2)$

Case 1. If $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\epsilon \leq \epsilon_2$ then $\frac{\partial \hat{\pi_i}^{FF}}{\partial u} \geq \frac{\partial \hat{\pi_i}^{SS}}{\partial u}$.

Case 2. If $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\epsilon > \epsilon_2$ then $\frac{\partial \hat{\pi_i}^{FF}}{\partial u} < \frac{\partial \hat{\pi_i}^{SS}}{\partial u}$.

Case 3. If $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ then $\frac{\partial \hat{\pi_i}^{FF}}{\partial u} > \frac{\partial \hat{\pi_i}^{SS}}{\partial u}$.

■

Proof of Proposition 16.

Since we have characterized some of the strategies for the deterministic demand case under different costs we will use this as a guidance for the stochastic demand case.

(a) If $\theta_s \geq \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ then from Proposition 15 we have $\frac{\partial \hat{\pi_i}^{FF}}{\partial u} > \frac{\partial \hat{\pi_i}^{SS}}{\partial u}$. Hence, if $\epsilon < \bar{\epsilon_3}$ then $\hat{\pi_i}^{SS} < \hat{\pi_i}^{FF}$. If $\epsilon > \bar{\epsilon_3}$ then there exists a threshold $\bar{u_1}$ such that if $u < \bar{u_1}$ then $\hat{\pi_i}^{SS} > \hat{\pi_i}^{FF}$ and if $u \geq \bar{u_1}$ then $\hat{\pi_i}^{SS} \leq \hat{\pi_i}^{FF}$.

b) If $\frac{\theta_p b_s}{2b_p+\theta_p} < \theta_s < \frac{b_s(2b_p+3\theta_p)}{2b_p+\theta_p}$ and $\eta > \max\{\frac{b_s(b_s+\theta_s)^2}{b_s(2b_p+3\theta_p)-(2b_p+\theta_p)\theta_s}, n_{max}\}$ then from Proposition 15 we have $\frac{\partial \hat{\pi_i}^{FF}}{\partial u} < \frac{\partial \hat{\pi_i}^{SS}}{\partial u}$. As a result, there exists a threshold $\bar{u_2}$ such that if $u < \bar{u_2}$ then $\hat{\pi_i}^{SS} < \hat{\pi_i}^{FF}$ and if $u \geq \bar{u_2}$ then $\hat{\pi_i}^{SS} \geq \hat{\pi_i}^{FF}$. Note that $\hat{\pi_i}^{FF} - \hat{\pi_i}^{SS}$ can be expressed as a second degree polynomial in terms of u. $\bar{u_1}$ and $\bar{u_2}$ are the positive roots of that polynomial for the respective parametric regimes. Recall, that $n_{max} = max\{\frac{2(b_p+\theta_p)S(S+T)}{W^2}, \frac{(b_s+\theta_s)^2}{2(b_p+\theta_p)} - \epsilon\}$ ■

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