Essays on Incentives in Rank-Order Tournaments

by
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Abstract

JUSTIN L. JOFFRION: Essays on Incentives in Rank-Order Tournaments.
(Under the direction of Sérgio O. Parreiras.)

This dissertation explores the effect of a dynamic tournament on strategic behavior. First, I examine the relevant theory and build a model of a two-player continuation contest. I find that there are two unique equilibria including one where an underdog can take the lead. Second, I analyze the effect that an educational signal from the Air Force Academy has on distant career outcomes. Using a regression discontinuity design, I show that the distinction of Distinguished Graduate has no effect on selection to In-residence Intermediate Developmental Education. Finally, I explore the impact that the tournament structure and the prize valuation have on strategic behavior of cadets at the Air Force Academy.

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Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

Contests are one of the essential tools used to motivate and screen agents. They have been used in a variety of settings. In sports, tournaments determine who is the best athlete. In politics, lobbyists vie to influence government. In business, employees compete for promotion. Additionally, in schools, students rival to be head of the class. In their seminal paper, Lazear and Rosen (1981) show that contests are an efficient means to induce worker effort. In the time since this finding was published, there has been effort to examine contest structure and the effect that the structure has on the outcome. This paper uses the all-pay framework to examine the competitive behavior of multiple agents in a two-period competition when there is a single prize and a cap on the observable output in each period. My dissertation will explore the complete equilibrium of this model. Furthermore, I will use data from the Air Force Academy to test the theoretical predictions of this model.

There are two segments of literature that examine dynamic or multi-stage contests. The first looks at elimination tournaments where a subset of the players compete in early rounds for a set of prizes that include the opportunity to compete against other early winners in later rounds for a grand prize. This form of tournament is frequently used in sporting competition where early heats narrow the field for the final competition or in bracket-style tournaments where losers are eliminated and the winners continue to compete until one is left standing. The other branch of this literature looks at multi-stage contests where players who lose continue to compete. In these continuation tournaments, either the principal, the competitors, or both receive information about the level of performance in early rounds. Several papers have examined the optimal design of contests so that this information is released to maximum effect. This paper contributes to this vein of research by examining a multiple-stage contest where agents learn their relative position after each stage and the prize is awarded to the agent with the highest cumulative measure of effort. In addition, I examine effort under linear and convex cost of effort.

It is appropriate to examine different functional forms of the cost of effort. In many circumstances, the marginal cost of effort in increasing. Furthermore when the contest is divided into stages, the cost of effort may "reset" between rounds. Examples include semesters for students, stages for the Tour de France, and other extended competitions. The convexity of the cost function can be such that an agent may determine it is no longer worthwhile to compete and drop-out of the competition after the information is revealed. For example, consider a five round competition where the marginal cost of effort in each round is increasing and the highest cumulative score wins. Specifically, the effort to score over five points exceeds every player's valuation. If after four rounds, a player is ahead by more than six points, no other player has an opportunity to win. Because the effort required to win is too costly, other players will "self-eliminate" from the contest until the final round. In this round, players will randomize the amount of effort they exert based on their standing in the competition.

In some competitions, there is no upper bound on the competitors. There are limits to the competition in terms of time or distance, but there is no limit on the measure of output. For instance, in a basketball game, the game ends after the clock expires. However while the clock is running, there is no limit to the number of points that a team can score. In other settings, there is a limit to the score. The motivating example is the academic setting where students compete for the prize of valedictorian. Setting aside all other prizes (high GPA for college, future value of human capital, etc.), students must allocate their effort towards being first among their peers based on their valuation of being first and the perceived likelihood that they will be first based on their effort.

Moldovanu and Sela (2006) examine a competition where agents are separated into sub-groups that compete for entry into later rounds of the tournament. They show that when individuals face convex cost functions, a contest designer can increase total effort by splitting the contestants into sub-groups and awarding prizes to finalists, as well as the overall winner. Unlike this paper, in their model, players do not observe opponents' effort levels in the first round. They only know that they are competing against other winners of the first round.

In examining carryovers, Baik and Lee (2000) find that in a two-stage elimination game, carryover rate affects the level of effort in the second period. When agents are allowed to carryover some of their effort from the first stage, they find that more effort is expended in the first stage and less in the second. When agents can carryover all of their effort from the first period, they find that no competition occurs in the second round. The winner is decided in the first round. In addition, the rent is completely dissipated so that the total effort outlay equals the value of the rent.

In a recent working paper, Petrenko (2008) examines continuation contests where agents face cost "carryover" so that resources spent in early periods result in higher costs in subsequent periods. More specifically, they use a "best of" model where the first player to reach a specified number of wins is the overall winner. They find that, in equilibrium, the leader after the first period expends more effort in subsequent periods. However, this effect is diminished when contestants face a budget constraint.

This dissertation explores the contest environment at the United States Air Force Academy. First, I develop a model that examines the behavior of two players who compete for a single prize in a multiple-stage game. The winner is the player who has the highest aggregate bid. Second, I use data from the graduating classes of 1982-1995 to examine the value of the top prize at the Academy, the designation of *Distinguished Graduate*. Finally, I explore the strategic behavior of cadets as predicted by the model of competition. This project will shed light on the role that contest design plays on strategic behavior and yield suggestions for how to improve design to extract more effort from participants.

Cadets are engaged in an economic tournament where they compete for prizes including the designation of Distinguished Graduate (DG), which is awarded to the top ten percent of each class. In terms of a tournament, this designation is one of the economic prizes. While there are other prizes available in the tournament, the environment at the Academy is useful to study because all cadets graduate in four years and are required to serve in the Air Force for five years following graduation. During these five years, they will receive the same monetary compensation and are promoted on a fixed schedule that does not vary regardless of job assignment or performance. However, despite the guarantee of job placement, they still have career preferences. Some would like to be rated and serve as pilots and aircrew. Others seek contracting, acquisitions, or other billets. Because assignments are based on class ranking, this is another prize in the tournament. However, this prize is more difficult to value due to the heterogeneous preferences over jobs described above. So, in this paper we focus our attention on the DG prize.

The Distinguished Graduate designation is important during the promotion to field grade officer, which does not happen until nine years after graduation. At that time, the promotion board meets again to rank-order all officers. The top twenty percent are identified as selectees for In-residence Intermediate Development Education (IDE). This is a strong signal for future promotion boards. It is possible to see if the officers who were Distinguished Graduates are now considered the top twenty percent of the officer corps. Knowing the value of the distinguished graduate signal will allow me to investigate the mechanism that the academy employs to incentivize cadets.

While the setting described above is a special case of tournament theory, this examination has general implications that are applicable to other tournament structures.

Background on the United States Air Force Academy

The United States Air Force Academy (USAFA) was established on April 1, 1954 and soon after the campus was sited near Colorado Springs, Colorado. The mission of the Academy is to "educate, train and inspire men and women to become officers of character, motivated to lead the United States Air Force in service to our nation." To accomplish this mission, the Academy has established a military, athletic, and academic environment to challenge officer trainees, commonly referred to as cadets, and develop their capabilities. Taxpayers spend approximately $250,000 to educate an Academy graduate. Thus, the Academy, to provide a sound return on this investment, must determine what cadets most need to know and incentivize cadets to learn as much as possible. USAFA is an accredited four-year university that currently offers thirty-two academic majors to just over four thousand men and women from all fifty states and numerous other countries and territories. Approximately ninety-five percent of the student body is from out-of-state. There are a small number of cadets who are citizens of foreign countries. These exchange cadets return to their home countries following graduation. There are over 500 officer and civilian faculty members, of whom forty percent hold terminal degrees. Athletes may participate in twenty-seven NCAA Division I sports. Unlike other universities, all cadets who graduate do so in four years with a

Bachelor of Science degree. In lieu of tuition, cadets agree to serve five years as commissioned officers in the United States Air Force. A small percentage cross-commission into other branches of the Armed Forces. Cadets receive academic, military, and athletic performance scores. These scores determine the overall order of merit at graduation. Cadets who graduate in the top ten percent of the order of merit are designated as Distinguished Graduates. This identifier becomes part of the officer's record that is presented to the promotion board. No other information about a cadet's performance is available to the board.

Officer advancement is based on the outcome of promotion boards held centrally at the Air Force Personnel Center at Randolph Air Force Base outside San Antonio, Texas. Promotion to service in a higher rank is not a reward for successful performance. Rather, it is a recognition that a service member is capable of additional duty and responsibility. While previous experience is an indicator of this capability, it is not the sole factor that determines promotion. The promotion board also considers recommendations from superiors, peer stratification, and training and education. Eligibility for promotion is based on time in grade. Graduates enter the Air Force with the rank of Second Lieutenant. After two years, officers become eligible for promotion to First Lieutenant. After two years as a First Lieutenant, officers are eligible to become Captains. The advancement to these ranks is fairly automatic. No formal promotion boards are held. Thus, the promotion to Major is the first time that an officer faces a promotion board. At this point, officers are selected for both promotion and In-residence Intermediate Development Education (IDE). The selection to IDE identifies the top twenty percent of the officers in that year group. Hence, it is possible to compare the top performing officers to the top performing cadets. Furthermore, In-residence selection is an important signal to future promotion boards. While all officers must complete IDE, either by correspondence or In-residence, to be eligible for promotions beyond Major, those selected for In-residence IDE are in effect "primed" for future promotion opportunities.

While at the Academy, cadets are competing in a rank-order tournament that extends over eight semesters. The outcome of this tournament determines job placement, graduate school opportunities, and the designation as a Distinguished Graduate (DG). After each semester, cadets receive grade reports that inform them of their position in the tournament. It is informative to examine the strategic behavior of cadets in the final semester at the academy when only the DG prize remains.

After the seventh semester, job placement, salary, and promotion schedule are all predetermined. Tournament theory suggests how agents should behave based on this scenario.

The remainder of this thesis is organized as follows: Chapter 2 presents a theoretical model of a multiple stage continuation contest and its solution. The third chapter explores the value of the educational signal that results from the USAFA tournament by examining its influence on a distance career outcome. Chapter 4 examines the strategic behavior of cadets at the Air Force Academy to determine if the theoretical predictions of the model are valid.

Chapter 2

Theoretical Model of a Two-Player Continuation Contest

2.1 Introduction

This chapter identifies the equilibrium of a contest where two players with known valuations compete for a single prize. Although each player values the prize, one player has a greater valuation. The player who at the end of all stages has exerted the highest cumulative effort receives the prize. Using an all-pay auction framework, I identify two Markov perfect equilibria. In the first, the player with the higher valuation wins the prize; and, in the second, the player with the lower valuation wins the prize. In addition, I identify the circumstances where either the first, the second, or both equilibria exist. Because there is an equilibrium where the lower valuation wins, the contest may fail to identify the player with the highest valuation. Furthermore, I find that as one player accumulates an advantage over the other, both players will reduce their exertion. Thus, this contest structure may not induce maximal output from the players.

There are several additional findings. In the last stage of the competition, each player uses a mixed bidding strategy. When agents have linear cost of effort, each player may exert zero effort up until the final stage where they will exert mixed effort. Under this circumstance, only the player with the higher valuation will expect to earn positive profits. However, when agents' cost is a convex function of effort, there are circumstances where either the high valuation player or the low valuation player will exert effort in every period and the other does not. This is true until the last period where both employ a mixed strategy.

In the basic theory of the firm, there is a principal, or owner, of the firm that relies upon agents, or employees, to provide effort that generates revenue for the firm. To maximize profit, the principal must elicit the maximum amount of effort from agents at the lowest wage possible. Hence, one of the central concerns of the principal is to find an optimal mechanism to incentivize agents.

Over time, many incentive structures have been implemented. Two of the most common mechanisms are the contract mechanism and the tournament (or contest) mechanism. With the contract mechanism, the principal and agent agree to compensation based on some observable measure of performance. Because this measure is often a function of output, this incentive mechanism is sometimes referred to as a piece-rate contract, where the piece-rate is the wage paid per measure of output. Under this mechanism, agents focus on their individual performance. This mechanism works well when agents are risk-neutral and the principle can measure performance cost-effectively. The tournament mechanism plays upon the notion that competition among agents will drive higher performance. With this mechanism, the principal and agent agree upon some assessment of performance. The principal ranks the agents according to this assessment and awards one or more agents with the incentive prize. The tournament serves two functions. It motivates the contestants to exert effort and it identifies the agents with high ability. In addition, because contestant performance is assessed relative to other competitors, common shocks are accounted for in the tournament.

It is instructive to think of the contest mechanism as an all-pay auction where there is a fixed number of "winners," but the auctioneer wants each participant to "bid" their maximum effort. In some circumstances, the auctioneer has information about each buyer's maximum willingness to pay. Like the auctioneer who designs the auction so that each bidder bids his maximum valuation, the principal's goal is to design the tournament so that each participant exerts maximal effort.

In both the tournament and contract mechanisms, the principal faces two constraints. The incentive structure must be such that the agent receives more than some minimal amount of utility. This is referred to as the individually rational or participation constraint. If the firm does not offer this level of compensation, the agent will either work for another principal or not work at all. Second, the incentive scheme must induce the agent to align his interests with the principal. If the agent can gain an advantage by shirking on effort, pretending to have lower ability, or otherwise misbehaving, then the incentive mechanism will not entice the agent properly. This constraint is the incentive compatibility constraint.

As there are multiple ways to incentivize agents, it is beneficial to investigate which mechanism is optimal. This is the fundamental question that motivated Lazear and Rosen's seminal paper (1981) on rank-order tournaments, in which they established that tournaments can be optimal under certain conditions. Following in their footsteps, other authors have attempted to refine the circumstances where the optimal contest mechanism outperforms optimal contracts. In addition, there are many ways to employ the tournament mechanism. Contests can be decided in a single round of competition or after multiple rounds. When there are multiple stages, the principle can affect behavior by choosing how to release information between stages, how to match competitors, and the timing of actions within rounds. In general, multi-stage contests can be separated into two categories: elimination contests and continuation contests. In elimination contests, contestants are divided into groups that compete in early rounds. From these initial stages, a subset of players from each division face each other in subsequent rounds of competition. Rosen (1986) completely characterizes the equilibrium of elimination contests. And, Moldovanu and Sela (2006) describe how this form compares to "winner-take-all" tournaments. Continuation contests allow contestants to compete in later rounds regardless of the outcome of earlier rounds. The principle must decide how earlier outcomes will affect the competition in later rounds.

Because of the many forms that a contest may take, there is a large number of papers pertaining to this research. In this space, I refer to a number of the more recent and pertinent studies. Baye, Kovenock, and de Vries (1996) detail the solution to the all-pay auction with complete information. I use their techniques to solve the model. With the all-pay auction, the uncertainty in the outcome is due to the simultaneity of bidding. Yildirim (2005) investigates a two-stage, two-player continuation contest with uneven players using a logit contest success function to provide uncertainty. He finds that a player who exerts lower effort in early rounds will exert additional effort in later rounds to make up the difference. Furthermore, his model rules out scenarios where the player with lower valuation leads. The results of my model suggest that an equilibrium exists where the underdog can lead, but only if the nature of the contest is such that the separation between players is diminished.

Additionally, as one player falls behind, both players will become less aggressive.

Ludwig and Lünser (2008) examine two identical agents in a two stage continuation contest. They identify a pure strategy equilibrium where both players exert the same effort as the other and equal effort in each period. In their study there is uncertainty in the outcome of the tournament due to measurement error in the observed effort. This uncertainty incites both agents to participate despite the advantage that one agent may have over another. Compounding this incentive is an upper bound that they place on the amount of effort that can occur in each round.

Finally, several works (Lazear and Rosen (1981), O'Keeffe, Viscusi, and Zeckhauser (1984), Shogren and Baik (1992), and Baik and Lee (2000)) recognize that contestants are not necessarily equally matched. Casas-Arce and Martínez-Jerez (2007) examine an all-pay auction where there are handicaps on the participants. In their model, each agent has the same valuation. However, there is a maximum bid amount that varies across participants. In a two-period contest, they find that agents will delay bidding until the final period. When I introduce convex cost of effort, my model eliminates this tendency for agents to delay. Konrad and Kovenock (2008) examine multi-battle contests where the winner is determined by the number of battles won. They find that players will exert effort in each stage if there are intermediate prizes.

This paper advances the literature by examining the case where two contestants with valuations, $v_1 > v_2 > 0$, compete in a multiple-stage continuation contest. In these contests, it is not the number of stages won that matters but the aggregate score. Depending on the outcome of previous stages, an agent may enter the current stage with an advantage. The magnitude of this advantage will affect behavior and outcome. The impact of the advantage depends on the difference between the players' valuations and the convexity of the cost function. In a two stage continuation contest, I find that in the case where agents have linear cost of effort, each player will exert zero effort in the first round and mixed effort in the second round. However, when agents' cost is a convex function of effort, there is a unique equilibrium where Player 1 exerts positive effort in the first round and Player 2 exerts zero effort. Then, in the second round, each player employs a mixed strategy. When this contest is played in multiple periods with convex costs, I find that in any stage prior to the last stage, only one player will compete. However, there are conditions where participation may come from either the player with higher valuation or lower valuation.

The rest of this chapter proceeds as follows. Section 2.2 describes the motivation behind the model. Section 2.3 provides the solution concept. Section 2.4 describes the model. Section 2.5 examines the equilibrium of a continuation contest limited to two stages. Section 2.6 extends the analysis to a multi-stage continuation contest. In section 2.7, I provide some notional examples of how the equilibrium will develop under different cost functions and valuation differences. Sections 2.8 provide remarks on the results and 2.9 offers the conclusion.

2.2 Motivation

One can evoke several examples of situations where agents engaged in competition obtain information about their relative positions before the end of the race. Perhaps the canonical example is from Aesop (translated by George Fyler Townsend[1]) in "The Hare and the Tortoise":

> A hare one day ridiculed the short feet and slow pace of the tortoise. The latter, laughing, said, "Though you be swift as the wind, I will beat you in a race." The hare, deeming her assertion to be simply impossible, assented to the proposal; and they agreed that the fox should choose the course, and fix the goal. On the day appointed for the race they started together. The tortoise never for a moment stopped, but went on with a slow but steady pace straight to the end of the course. The hare, trusting to his native swiftness, cared little about the race, and lying down by the wayside, fell fast asleep. At last waking up, and moving as fast as he could, he saw the tortoise had reached the goal, and was comfortably dozing after her fatigue.

In this example, the hare enters the race with a strong advantage. So strong, that the tortoise cannot overcome the hare unless circumstances beyond his control intervene. The tortoise, confident of the hubris of the hare, continues to exert effort despite the seeming impossibility of winning. Sure enough, happenstance allows for the hare to oversleep. Thus, by a fluke, the tortoise is able to take the prize. Of course, the moral invites those with less capability to participate in the contest. However, if the hare had simply run the race and napped afterward, the effort of the tortoise would be for nought. Thus, a contest where one agent is outmatched may not entice both agents to

[1] George Fyler Townsend, Three Hundred Aesop's Fables: Literally Translated from the Greek (London: George Routledge and Sons, 1867), pp. 9-10.

participate. In what follows, I shall show that this is not an equilibrium because had the hare acted prudently, he would have won the race. This is the irony of the fable.

Even if participants are equally matched at the beginning of the race, this cannot ensure full participation throughout the entire race. If a player amasses too much of a lead, the outcome of the contest may be determined before the final stage of the competition. Consider the example of the game show *Jeopardy*. In this game, players compete in three rounds. Each contestant enters the final round with the money they've earned in previous rounds. This final round consists of a single question. Before the players are asked this question, they must wager some or all of the money they've accrued in the previous rounds. Players who answer the question correctly, receive the amount of money they put up. And, those who answer incorrectly lose the amount they put forth. The maximum payoff occurs when a player bets all of his money and answers correctly. In most cases, the players have different amounts to risk in the final round. So, the player who enters the final round with the most money has an advantage. And, the player who has more money than the others has an advantage. There are times when the final round is not interesting because one player has an insurmountable lead over the other contestants.

Tournaments are frequently used to determine a sporting champion. The Tour de France is an example of a multi-stage continuation contest where the winner is the cyclist who has the lowest aggregate time. It is rare that a winner actually be first in many stages. And, when Greg LeMond won in 1990, he did so having never won an individual stage. This may be one of the reasons why the honor of wearing a yellow jersey is so important. In addition, the race recognizes the rider who wins the most individual stages. Also, riders are often members of teams. Thus, those who are out of the running for an overall win may continue to exert effort in order to assist a teammate by riding in front and giving him the benefit of slipstreaming.

Finally, consider an extended political race where two contestants face each other in polls. It is possible that one can gain such an advantage in the polls that the opponent may choose to exit. Or, supporters who no longer believe their candidate is viable reduce their contributions and their candidate can no longer sustain the race.

In what follows, I differentiate players based on their valuation. However, it is important to realize that this valuation can be interpreted in a variety of ways. The most straightforward interpretation is that the valuation refers to the amount of value that the individual assigns to the prize. It is also possible to consider the valuation a means to differentiate players based on their future value to an organization. For instance, the player with the higher valuation may be the type of player that a future principle would want to identify. As an example, a student who places a higher value on receiving a university honor such as "sigma cum laude" may have an inate drive to excel. Since most employers value motivated employees, a contest structure that successfully identifies such individuals would be beneficial. However, if the contest structure leads to outcomes where lower valuation employees take the prize, the selection effect of the contest is eliminated.

2.3 Contest Modeling Approach and Solution Concept

There are two general approaches that researchers have used to study the use of tournaments as incentives. Both focus on the means by which the winner of the tournament is determined. In the first approach, the prize is awarded according to a set rule. There is no ambiguity in the determination of the winner. In the second approach, things are not as clear-cut as there is some uncertainty in the determination. Nevertheless, in both allocation methods, the contestants, by their actions, have some sway over the allocation of the prize. The first allocation method is typically modeled as an all-pay auction where the actions of the agents are modeled as bidding on a prize. These bids can be viewed as the level of effort put forth in the competition. The cost of effort can vary across agents. The second method introduces some ambiguity in the contest success function so that it is possible that an agent who exerts strictly lower effort may still have a positive probability of winning. Tullock pioneered this formulation, hence these competitions are often referred to as Tullock contests. Nitzan (1994) provides a summary of the "winner-take-all" contest literature that employs probabilistic contest success functions.

I use the all-pay auction framework to model the contest; whereas, others use a contest success function to analyze the contest. As I am using an auction framework, players in the contest are choosing how much effort to exert to win. I view this effort as their bid. So, the strategy they employ in the game is their bidding strategy. There is a cost to bidding that reduces an agent's payoff. The uncertainty in the model comes from not knowing what the other agent is bidding until after the bids are submitted. Therefore, the probability that an agent wins is the probability that after all rounds are complete, the total of the agent's bids is higher than the total of the other agent's bids.

The interpretation of this bidding behavior is flexible. The cost of bidding may be tied directly to the ability of the agent. Furthermore, the cost of bidding need not be linear. It is reasonable that the cost of effort is increasing in effort level. That is to say that the marginal cost of effort is increasing.

The solution that follows is a Markov equilibrium to the all-pay auction formulation. Hence, the strategies below provide for a Nash equilibrium in every subgame. However, these strategies do not rely on the complete history of the game. This rules out more complicated scenarios where a player may employ punishment to drive an outcome. Nevertheless, in a finite game, this restriction does not severely impede the implications of my findings. The state variable that sets each stage of the game is the advantage that players have accumulated over all prior stages.

2.4 Model

There are two players with commonly known valuations $v_1 > v_2 > 0$. Player i's bid in period t is denoted b_i^t. The timing of the game is as follows. Players enter the game evenly matched. At the beginning of each period, the players bid simultaneously. The cost of bidding occurs at the time of bidding, but no payoffs occur until the the end of the final stage. At the end of each stage the bids are revealed. Players enter the following period knowing the history of bids. In each stage that follows, players continue to bid simultaneously. At the end of the final stage, the player with the highest aggregate bid wins the prize. If there is a tie, then the prize will be divided between those who have the highest bid.

The gap between the stages allows each bidder to understand where he stands in relation to the other bidder. Thus, they design their bidding strategies in each period based on what they have observed in the past.

Definition 1 (Probability of winning)**.** The probability that Player 1 wins is:

$$P_1(\text{win}) = \begin{cases} 1 \text{ if } \sum_{n=1}^{N} b_1^n > \sum_{n=1}^{N} b_2^n \\ \frac{1}{2} \text{ if } \sum_{n=1}^{N} b_1^n = \sum_{n=1}^{N} b_2^n \\ 0 \text{ if } \sum_{n=1}^{N} b_1^n < \sum_{n=1}^{N} b_2^n \end{cases}$$

and the probablity that Player 2 wins is:

$$P_2(\text{win}) = 1 - P_1(\text{win})$$

Given the game described above, players seek to maximize expected profit. Payoff only occurs in the final round; however, bidding is costly in every round.

Definition 2 (Profit functions)**.** The profit in round n for player i is

$$\pi_i^n = -b_i^n$$

for all $n < N$, and

$$\pi_i^N = -b_i^N + v_i * P_i(\text{win})$$

in the final round, N. Therefore, the total profit, Π_i is

$$\Pi_i = \sum_{k=1}^{N} \pi_i^k$$

Because the players are accumulating an advantage over time, the model requires that we track the amount of this advantage. The following defines the variable that will track the advantage.

Definition 3. In any period n, Δ^n represents the advantage at the start of stage n.

$$\begin{aligned} \Delta^n &= \sum_{k=1}^{n-1} (b_1^k - b_2^k) \\ &= \sum_{k=1}^{n-1} b_1^k - \sum_{k=1}^{n-1} b_2^k \end{aligned}$$

2.4.1 Effective bid

Neither player will bid a positive amount if that bid yields a negative expected profit. From this statement, we know that $\pi_i^N \geq 0$. And this implies that $c(b_i^N) \leq v_i$ or $b_i^N \leq c^{-1}(v_i)$. Because players have different valuations, this results in different upper bounds on their bidding. Player 1 has a higher upper bound since $v_1 > v_2$ and c is strictly increasing.

If this was a one period game, that would be the end of the story. But due to the prior stages, the *effective*[2] bid for Player 1 is $b_1^N + \sum_{n=1}^{N-1} b_1^n$ and Player 2 is $b_2^N + \sum_{n=1}^{N-1} b_2^n$. That is, the $\sum_{n=1}^{N-1} b_i^n$ entering the round affects the outcome of the game, but the cost has already been paid.

Proposition 1. *Only the player with the higher limit on their effective bid in period N will earn positive expected profits.*

Proof. Suppose Player 1 has a higher limit on his effective bid in period N and does not earn positive profit. The effective bid is the bid place in the current period combined with the cumulative bids placed in early periods. The bids placed in early periods are a sunk cost. Therefore, the only constraint on the bidding in the current period is that the cost of the bid cannot exceed the player's valuation. So, $c(b_i) \leq v_i$. Therefore the maximum bid in the current period is $b_i \leq c^{-1}(v_i)$. This combined with previous bidding is the upper limit on Player i's effective bid:

$$c^{-1}(v_i) + \sum_{n=1}^{N-1} b_i^n$$

So, if Player 1 has the higher limit,

$$c^{-1}(v_1) + \sum_{n=1}^{N-1} b_1^n > c^{-1}(v_2) + \sum_{n=1}^{N-1} b_2^n$$

This implies that

$$c^{-1}(v_1) - c^{-1}(v_2) + \sum_{n=1}^{N-1} b_1^n - \sum_{n=1}^{N-1} b_2^n > 0$$
$$c^{-1}(v_1) - c^{-1}(v_2) + \Delta^N > 0$$

[2]Effective in terms of its impact on the probability of winning

In order for Player 1 to not earn positive profit

$$b_1^N + \sum_{n=1}^{N-1} b_1^n < b_2^N + \sum_{n=1}^{N-1} b_2^n \leq c^{-1}(v_2) + \sum_{n=1}^{N-1} b_2^n < c^{-1}(v_1) + \sum_{n=1}^{N-1} b_1^n$$

Which is to say,

$$\begin{aligned} b_1^N + \sum_{n=1}^{N-1} b_1^N &< c^{-1}(v_1) + \sum_{n=1}^{N-1} b_1^N \\ b_1^N &< c^{-1}(v_1) \end{aligned}$$

Yet this is suboptimal play for Player 1 who could increase b_1^N to $c^{-1}(v_2) - \Delta^N + \varepsilon$ without exceeding $c^{-1}(v_1)$. Hence as long as $c^{-1}(v_1) - c^{-1}(v_2) + \Delta^N > 0$ Player 1 can guarantee himself positive profit by bidding just over Player 2's maximum profitable bid. The same reasoning holds for Player 2, if Player 2 has the higher effective bid. □

This proposition has implications for how the players behave in the game. In all previous periods, players, aware of where they stand in the game, can look forward and determine the impact of their bids on the effective bids in the final round. They will only bid in the current period if they believe that bidding will result in having the highest effective bid in period N.

To illustrate the equilibrium, I start by examining a two stage continuation contest. Using the results of this examination, I will expand the solution to the multiple stage continuation contest.

2.5 The Two-Stage Continuation Contest

The contest is played in two stages, and players have valuations $v_1 > v_2 > 0$. At the beginning of stage 1, the players bid simultaneously. The bids are then revealed but no payoffs occur in the first period. In the second stage, players again bid simultaneously. At the end of the second stage, the player with the highest aggregate bid wins the prize.

Based on definition 1, the probability that Player 1 wins is:

$$P_1(\text{win}) = \begin{cases} 1 \text{ if } b_1^1 + b_1^2 > b_2^1 + b_2^2 \\ \frac{1}{2} \text{ if } b_1^1 + b_1^2 = b_2^1 + b_2^2 \\ 0 \text{ if } b_1^1 + b_1^2 < b_2^1 + b_2^2 \end{cases}$$

The probablity that Player 2 wins, according to definition 2, is:

$$P_2(\text{win}) = 1 - P_1(\text{win})$$

In the two-stage game, the profit for Player i is:

$$\begin{aligned} \Pi_i &= \pi_i^1 + \pi_i^2 \\ &= -b_i^1 - b_i^2 + v_i * P_i(\text{win}) \\ &= -b_i^1 - b_i^2 + v_i * P_i(b_i^1 + b_i^2 > b_{-i}^1 + b_{-i}^2) \end{aligned}$$

Finally, definition 3 allows us to calculate the advantage players hold when they enter the final period:

$$\Delta^N = b_1^1 - b_2^1$$

and this advantage is known to both players at the start of the final stage. To simplify notation, I will refer to Δ^N as Δ because in the two-stage game there can only be an advantage in the final round.

In a two-stage continuation contest, after the first round, there are three possible scenarios:

1. If $\Delta = 0$, neither player has an advantage
2. If $\Delta > 0$, Player 1 has an advantage
3. If $\Delta < 0$, Player 2 has an advantage

After the first stage, the bidding that has transpired is a sunk cost. Hence, the players will not factor that sunk cost into their subsequent bidding decisions. However, because the outcome of the game is based upon the cumulative bidding, the bidding in earlier rounds does affect the game. But only through the effect on the probability of winning the contest. Hence, in the second round, a players' maximum bid is only bound by their valuation of the prize. But, in the first round, players are aware of what it will take to win in the second round. Therefore, they will bid according to their expectation of how the game will progress in the second round.

In the first scenario, where neither player has an advantage, the players enter stage 2 perfectly matched. This is the case described by Che and Gale (1998). They find that without caps on bids, expected revenue is strictly below v_2. However, when bids are capped, there could be an increase in competition that could raise expected revenue.

In the second and third scenarios, where one of the players has an advantage, Szymanski (2003) explains the two concerns. First, if the players are far enough apart, the weak player may choose not to participate. And, second, even if the both agents compete, overall effort may be dampened as the advantaged player can exert less than his or her potential but still win.

The magnitude of the advantage will affect the behavior in the second stage. For example, if $b_2^1 - b_1^1 = v_1 - v_2$, then the second stage of the competition can be modeled as a symmetric all-pay auction because the upper limit of each players bid is equal. The bottom line is that players will behave strategically in the first round to shape the scenario in the second round.

2.5.1 Solution with linear cost of bidding

Consider the case where bidders face a linear cost of bidding (e.g, $c(b_i) = b_i$). The cost function is common to both players. Again, $v_1 > v_2 > 0$ and players have complete information.

I will be more formal later, but consider the following. Player 1 has a higher valuation than Player 2. Neither player will commit to a strategy that yields negative expected profit. Therefore, Player 1 starts the contest with an advantage in that he has a higher upper bound on his willingness to bid. In order for Player 2 to overcome this advantage, he must enter the final stage with Δ such that $v_2 - \Delta > v_1$. This gives Player 2 the "ability" to outbid Player 1 and earn positive expected profits. This is important as we will see because whomever has the ability to outbid will have positive expected profits. Because the creation of Δ is costly, Player 2 must find it profitable to expend the effort in round 1 necessary to create the situation where $v_2 - \Delta > v_1$. And, Player 1 must find is unprofitable to block such a move.

Lemma 1. *If $\Delta > v_2 - v_1$, then Player 1's expected payoff is positive and Player 2's expected payoff is zero. If $\Delta < v_2 - v_1$, Player 2 receives a positive expected payoff and Player 1 receives zero. And, if $\Delta = v_2 - v_1$, each player has an expected payoff of zero. The full solution to the second stage problem is presented in Table 2.1.*

Proof. The proof is in the Appendix. □

This outcome is due to the effective bid proposition above. If $\Delta > v_2 - v_1$, Player 1 has the higher limit on the effective bid. And if, $\Delta < v_2 - v_1$ then Player 2 has the higher limit. The solution of the second stage problem has three important results. First, one player will receive a positive payout and the other will receive nothing. Second, both players will play mixed strategies in the final round. Third, the magnitude of the advantage determines both who will receive a positive payout, the amount of the payoff, and the mixing strategy for both players.

Having explored the possible outcomes in stage two, we turn our attention to stage one to find the overall equilibrium of the model.

Proposition 2 (Equilibrium of the two stage continuation contest with linear cost of bidding)**.** *In a two stage contest where two bidders with unequal valuations compete with linear cost of bidding, the player with the lower valuation will bid zero in the first period. The player with the higher valuation will bid any amount between zero and player two's valuation. Then, in the second round, players will bid using a mixed strategy according the following CDFs in Table 2.1.*

Proof. The complete proof is provided below; however, this a brief sketch. Starting with round 2 and working backwards, we note that players will not employ a pure strategy in the final period unless the Δ is such that one player has no possibility of winning with positive profit. In this event, the outcome of the contest is essentially predetermined and both players will bid zero. More interesting is the case where there is competition in the final round.

At the beginning of stage one, both players are aware of v_1 and v_2. Players chose b_1^1 and b_2^1, respectively, to maximize individual profit. Figure 2.1 shows the relationship between bid combinations in the first round and payoffs in the second round. In the region shaded with green, Player 1 earns positive profit. In the red region, Player 2 earns positive profit. Note that it is not enough

Table 2.1: Summary of profit, CDF, and support of players in second stage with linear cost function

	Advantage				
	$\Delta > 0$	$\Delta < 0$			$\Delta = 0$
		Case 1 $v_1 = v_2 - \Delta$	Case 2 $v_1 > v_2 - \Delta$	Case 3 $v_1 < v_2 - \Delta$	
Profit	$\pi_1 = v_1 - v_2 + \Delta$ $\pi_2 = 0$	$\pi_1 = 0$ $\pi_2 = 0$	$\pi_1 = v_1 - v_2 + \Delta$ $\pi_2 = 0$	$\pi_1 = 0$ $\pi_2 = v_2 - v_1 - \Delta$	$\pi_1 = v_1 - v_2$ $\pi_2 = 0$
CDF	$G_1(x) = \frac{x+\Delta}{v_2}$ $G_2(x) = \frac{v_1 - v_2 + x}{v_1}$	$G_1(x) = \frac{x+\Delta}{v_2}$ $G_2(x) = \frac{x-\Delta}{v_1}$	$G_1(x) = \frac{x+\Delta}{v_2}$ $G_2(x) = \frac{v_1 - v_2 + x}{v_1}$	$G_1(x) = \frac{v_2 - v_1 + x}{v_2}$ $G_2(x) = \frac{x-\Delta}{v_1}$	$G_1(x) = \frac{x}{v_2}$ $G_2(x) = \frac{v_1 - v_2 + x}{v_1}$
Support	$b_1^2 \in [0, v_2 - \Delta]$ $b_2^2 \in \{0, [\Delta, v_2]\}$	$b_1^2 \in [0, v_1]$ $b_2^2 \in [0, v_2]$	$b_1^2 \in [0, v_2 - \Delta]$ $b_2^2 \in [0, v_2]$	$b_1^2 \in [0, v_1]$ $b_2^2 \in [0, v_1 + \Delta]$	$b_1^2 \in [0, v_2]$ $b_2^2 \in [0, v_2]$

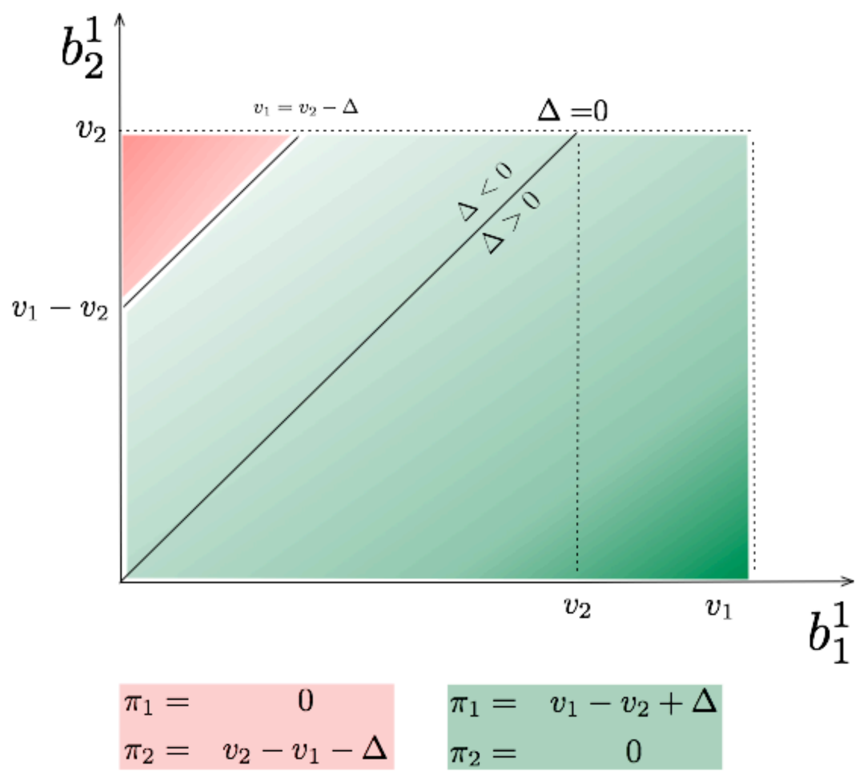

Figure 2.1: Payoffs from first round bids based on second round choices

for Player 2 to have $\Delta < 0$. It must be that $\Delta < v_2 - v_1 - \varepsilon$. Otherwise, in round 2 he earns zero profits. Therefore, he must bid at least $b_2^1 \geq v_1 - v_2 + \varepsilon$. Otherwise, he would prefer to bid zero. Also, Player 2 cannot bid greater than v_2 and earn positive profits. Hence, $b_2^1 \in \{0, [v_1 - v_2 + \varepsilon, v_2]\}$. Player 1 has the opportunity to earn nonnegative profits over the range $[0, v_1]$. However, because Player 2 will not bid higher than v_2, $b_1^1 \in [0, v_2]$.

I begin by checking if bidding a positive amount is dominated by bidding zero. The maximum profit for Player 2 occurs if Player 1 bids zero in the first round and Player 2 bids $v_1 - v_2 + \varepsilon$ so that he enters round 2 under case 3 at the lowest possible cost. In this case, $\Delta = b_1^1 - b_2^1 = -v_1 + v_2 - \varepsilon$. This yields profit in the second round of $\pi_2^2 = v_2 - v_1 - \Delta = v_2 - v_1 - (-v_1 + v_2 - \varepsilon) = \varepsilon$. But, since $\pi_2^1 = -(v_1 - v_2 + \varepsilon)$, overall profit for Player 2 is.

$$\begin{aligned}
\Pi_2 &= \pi_2^1 + \pi_2^2 \\
&= -(v_1 - v_2 + \varepsilon) + \varepsilon \\
&= -(v_1 - v_2)
\end{aligned}$$

Therefore, Player 2 earns negative profit for bidding any amount other than zero. Hence, in equilibrium, Player 2 bids zero in the first round.

Knowing that in stage 1 $b_2^1 = 0$, Player 1 may bid any amount in the first stage[3]. And, $\Delta = b_1^1$. Given expected profit in the second round is $v_1 - v_2 + \Delta$, overall profit is

$$\begin{aligned}
\Pi_1 &= -b_1^1 + v_1 - v_2 + \Delta \\
&= -b_1^1 + v_1 - v_2 + b_1^1 \\
&= v_1 - v_2
\end{aligned}$$

□

To summarize, we have that $b_1^1 = [0, v_2]$ and $b_1^2 = G_1(x) = \frac{x + b_1^1}{v_2}$. And, $b_2^1 = 0$ and $b_2^2 = G_2(x) = \frac{v_1 - v_2 + x}{v_1}$. These strategies result in expected profit of $\Pi_1 = v_1 - v_2$ and $\Pi_2 = 0$.

[3]Note that if there is discounting in the profit function, then Player 1 will delay bidding until the second period.

2.5.2 Solution with convex cost of bidding

Until now, the cost of bidding has been a linear function of effort. However, in many circumstances, the marginal cost of additional effort is increasing[4]. For example, in a sporting contest, it becomes increasingly difficult to reduce race times. And, in academics, it may not require much additional effort to raise a mark to a C from an F. But, to achieve an A+ versus an A may be highly costly. If bidding cost is convex, there is a unique equilibrium.

Definition 4. Let $c(b)$ be the convex cost of bidding in each period, where $c'(b) > 0$, $c''(b) > 0$ and $c(0) = 0$.

The bidding strategy of the players affects profit through both the cost function and the probability of winning. Despite the non-linearity in the cost function, the cost enters the payoff function linearly. Therefore much of what has been determined with the linear cost functions carries over to the non-linear cost function. The bid strategy can be expressed as the inverse of the cost function. To determine the strategies, we again use backward induction.

In the second round, payoff functions for players are:

$$\begin{aligned}
\pi_1^2 &= v_1 G_2(b_1^2 + \Delta) - c(b_1^2) \\
\pi_2^2 &= v_2 G_1(b_2^2 - \Delta) - c(b_2^2)
\end{aligned}$$

This is because Player 2 is using strategy G_2 and the probability that Player 1 wins is:

$$\begin{aligned}
P(b_1^1 + b_1^2 > b_2^1 + b_2^2) &= P(b_2^2 < b_1^2 + b_1^1 - b_2^1) \\
&= P(b_2^1 < b_1^2 + \Delta) \\
&= G_2(b_1^2 + \Delta)
\end{aligned}$$

[4] In some specialized cases, such as when there is learning, there may be concave costs. However, I do not consider concave costs in this paper.

Similarly, the probability that Player 2 wins under Player 1's strategy of G_1 is:

$$\begin{aligned} P(b_2^1 + b_2^2 > b_1^1 + b_1^2) &= P(b_1^2 < b_2^2 + b_2^1 - b_1^1) \\ &= P(b_1^1 < b_2^2 - \Delta) \\ &= G_1(b_2^2 - \Delta) \end{aligned}$$

Lemma 2. *If $\Delta > c^{-1}(v_2) - c^{-1}(v_1)$, then Player 1's expected payoff is positive and Player 2's expected payoff is zero. If $\Delta < c^{-1}(v_2) - c^{-1}(v_1)$, Player 2 receives a positive expected payoff and Player 1 receives zero. And, if $\Delta = c^{-1}(v_2) - c^{-1}(v_1)$, each player has an expected payoff of zero. The full solution to the second stage problem is presented in Table 2.2.*

Proof. Using the same technique detailed in the appendix for linear costs, one can determine the payoffs, CDFs, and supports. □

This cost structure will make minor changes to the profit and strategies of the players. However, the substance of the game is unchanged. The most significant impact is on the effective bid. In the linear case, the difference between v_1 and v_2 determined the bid. Now, it is important to consider not only this difference, but also the possibility that there could be a high marginal cost of outbidding one's opponent. Although the difference in valuation may be large, the difference in maximum effective bid could be small.

Proposition 3. *In a two stage contest where two bidders with unequal valuations compete with convex cost of bidding, the player with the lower valuation will bid zero in the first period. The player with the higher valuation will bid $b_1^1 = \frac{c^{-1}(v_2)}{2}$. Then, in the second round, players will bid using a mixed strategy according the CDFs in Table 2.2.*

Proof. Using the same reasoning as before, in the first round, Player 2 will select $b_2^1 = 0$. Otherwise, Player 2 receives negative profits. However, now, to ensure minimal costs, Player 1 prefers to spread costs over the two rounds. Thus in round 1, Player 1 recognizes that expected profit in the second round is

$$\pi_1^2 = v_1 - c(c^{-1}(v_2) - b_1^1)$$

Table 2.2: Summary of profit, CDF, and support of players in second stage with convex cost function

	Advantage				
	$\Delta > 0$	$\Delta < 0$			$\Delta = 0$
		Case 1 $c^{-1}(v_1) = c^{-1}(v_2) - \Delta$	Case 2 $c^{-1}(v_1) > c^{-1}(v_2) - \Delta$	Case 3 $c^{-1}(v_1) < c^{-1}(v_2) - \Delta$	
Profit	$\pi_1 = v_1 - c(c^{-1}(v_2) - \Delta)$ $\pi_2 = 0$	$\pi_1 = 0$ $\pi_2 = 0$	$\pi_1 = v_1 - c(c^{-1}(v_2) - \Delta)$ $\pi_2 = 0$	$\pi_1 = 0$ $\pi_2 = v_2 - c(c^{-1}(v_1) + \Delta)$	$\pi_1 = v_1 - v_2$ $\pi_2 = 0$
CDF	$G_1(x) = \frac{c(x+\Delta)}{v_2}$ $G_2(x) = \frac{v_1 - c(c^{-1}(v_2)-\Delta) + c(x-\Delta)}{v_1}$	$G_1(x) = \frac{c(x+\Delta)}{v_2}$ $G_2(x) = \frac{c(x-\Delta)}{v_1}$	$G_1(x) = \frac{c(x+\Delta)}{v_2}$ $G_2(x) = \frac{v_1 - c(c^{-1}(v_2)-\Delta) + c(x-\Delta)}{v_1}$	$G_1(x) = \frac{v_2 - c(c^{-1}(v_1)+\Delta) + c(x+\Delta)}{v_2}$ $G_2(x) = \frac{c(x-\Delta)}{v_1}$	$G_1(x) = \frac{c(x)}{v_2}$ $G_2(x) = \frac{v_1 - v_2 + c(x)}{v_1}$
Support	$b_1^2 \in [0, c^{-1}(v_2) - \Delta]$ $b_2^2 \in \{0, [\Delta, c^{-1}(v_2)]\}$	$b_1^2 \in [0, c^{-1}(v_2) - \Delta]$ $b_2^2 \in [0, c^{-1}(v_1) + \Delta]$	$b_1^2 \in [0, c^{-1}(v_2) - \Delta]$ $b_2^2 \in [0, c^{-1}(v_2)]$	$b_1^2 \in [0, c^{-1}(v_1)]$ $b_2^2 \in [0, c^{-1}(v_1) + \Delta]$	$b_1^2 \in [0, c^{-1}(v_2)]$ $b_2^2 \in [0, c^{-1}(v_2)]$

And overall profit is

$$\Pi_1 = v_1 - c(c^{-1}(v_2) - b_1^1) - c(b_1^1)$$

So, Player 1's problem is to choose b_1^1 to maximize Π_1.

$$\begin{aligned}
& \arg\max_{\mathbf{b}_1^1} \Pi_1 \\
& \frac{\partial \Pi_1(b_1^1)}{\partial b_1^1} = 0 \\
0 &= c'(c^{-1}(v_2) - b_1^1) - c'(b_1^1) \\
c'(c^{-1}(v_2) - b_1^1) &= c'(b_1^1) \\
c^{-1}(v_2) - b_1^1 &= b_1^1 \\
c^{-1}(v_2) &= 2b_1^1 \\
b_1^1 &= \frac{c^{-1}(v_2)}{2}
\end{aligned}$$

The solution to Player 1's problem is to choose $b_1^1 = \frac{c^{-1}(v_2)}{2}$. Note that this is exactly one-half of the maximum that Player 1 will bid up to in the second round. If Player 1 does not make this commitment in the first round, Player 2 will realize that Player 1 may not intend to bid up to $c^{-1}(v_2)$ in the second round. This will lead Player 2 to bid more aggressively in round 2. □

Hence, the strategy for Player 1 is $b_1^1 = \frac{c^{-1}(v_2)}{2}$ and $b_1^2 = G_1(b_1^2) = \frac{c(x+\Delta)}{v_2}$). Player 2's strategy is $(b_2^1 = 0, G_2(b_2^2) = \frac{v_1 - c(c^{-1}(v_2)-\Delta)+c(x-\Delta)}{v_1})$. Expected profits are

$$\begin{aligned}
\Pi_1 &= v_1 - c(c^{-1}(v_2) - \frac{c^{-1}(v_2)}{2}) - c(\frac{c^{-1}(v_2)}{2}) \\
&= v_1 - 2c(\frac{c^{-1}(v_2)}{2})
\end{aligned}$$

and $\Pi_2 = 0$.

Now that we have established what happens in a two stage game, we investigate the multiple-stage game.

2.6 The Multiple-Stage Continuation Contest

This section solves the case where there are N rounds in the contest. As above, two players with valuations $v_1 > v_2 > 0$ compete and the prize is awarded to the player with the highest aggregate bid. The cost of bidding in every period, $c(b)$ is convex, with $c'(b) > 0$ and $c(0) = 0$, and the same for each player.

Definition 5. In any period n, Δ^n represents the advantage at the start of stage n.

$$\begin{aligned}\Delta^n &= \sum_{k=1}^{n-1}(b_1^k - b_2^k) \\ &= \sum_{k=1}^{n-1} b_1^k - \sum_{k=1}^{n-1} b_2^k\end{aligned}$$

At the beginning of the first stage the advantage is zero (i.e., $\Delta^1 = 0$). After each stage, contestants learn b_i^{n-1} and update Δ^n after each stage. If Δ^n is positive, Player 1 hold the advantage. Whereas, Player 2 has the advantage if Δ^n is negative.

The method of finding the solution is identical to the two-stage problem. Therefore, the solution for the last stage is unchanged. Thus, the tables above are still valid. However, when $N > 2$, there is more opportunity for players to accumulate an advantage.

Many of the proofs require comparison of inequalities. The following lemma is useful in this exercise.

Lemma 3. *If c is an increasing convex function and* $c(0) = 0$, *then* $c^{-1}(\lambda v_1) > \lambda c^{-1}(v_2)\ \forall \lambda \in [0, 1]$.

Proof. Proof: If c is a increasing convex function, then c^{-1} is an increasing concave function. And, by the definition of concavity, $\forall \lambda \in [0, 1]$ and $x = 0$:

$$\begin{aligned}c^{-1}(\lambda v_2 + (1-\lambda)x) &\geq \lambda c^{-1}(v_2) + (1-\lambda)c^{-1}(x) \\ c^{-1}(\lambda v_2) &\geq \lambda c^{-1}(v_2)\end{aligned}$$

And, because $v_1 > v_2$, this means that $\lambda v_1 > \lambda v_2$. And because c^{-1} is an increasing function, we have the result that:

$$\begin{aligned} c^{-1}(\lambda v_1) &> c^{-1}(\lambda v_2) \geq \lambda c^{-1}(v_2) \\ c^{-1}(\lambda v_1) &> \lambda c^{-1}(v_2) \end{aligned}$$

□

2.6.1 Period N

In this period, Player 1's objective is to maximize:

$$\pi_1^N = v_1 G_2(b_1^N + \Delta^N) - c(b_1^N)$$

and Player 2's objective is to maximize:

$$\pi_2^N = v_2 G_1(b_2^N - \Delta^N) - c(b_2^N)$$

This is the same subgame as the final period of the 2 stage game described above. Therefore, the solution is the same. Each player will play a mixed strategy represented by CDFs G_1 and G_2. And, each player's strategy is a function of the advantage, Δ^N.

Over the course of the contest, each player attempts to posture themselves so that they have the higher limit on their effective bid in the final round. The following sections will examine the bidding in prior periods. We will start with the $N-1$ stage and then generalize to the $N-k$ stage.

2.6.2 Period N-1

I now examine the case where players enter the second to last round without an advantage.

Proposition 4 (No mixed strategies). *Neither Player 1 nor Player 2 will employ a mixed strategy in his bidding.*

The formal proof is in the appendix. However, it is constructive to examine why this is true.

This is a winner take all contest. As a result, the players can work backwards from the end to know where they stand in the competition. They know at any stage in the game what their effective bid will be and the cost of raising their effective bid. Ultimately, one player will have the higher maximum effective bid and earn positive expected profits. As a result, the other player would prefer not to have bid at all.

Having established that players do not use mixed strategies, we turn our attention to pure strategy equilibria. To simplify notation, we will refer to bids without superscripts. So, in stage $N-1$, Player 1 bids b_1 and Player 2 bids b_2.

Proposition 5 (Pure strategy equilibria). *For stage $N-1$, the following describes the Markov perfect equilibria:*

If $\Delta^{N-1} \geq c^{-1}(v_2) - 2c^{-1}(\frac{v_1}{2})$ and $\Delta^{N-1} > 4c^{-1}\left(\frac{v_2}{2}\right) - 2c^{-1}(v_1) - c^{-1}(v_2)$, then an equilibrium exists where:

$$\begin{aligned} b_1^* &= \frac{c^{-1}(v_2) - \Delta^{N-1}}{2} \\ b_2^* &= 0 \end{aligned}$$

and the value functions are:

$$\begin{aligned} V_1 &= v_1 - 2c\left(\frac{c^{-1}(v_2) - \Delta^{N-1}}{2}\right) \\ V_2 &= 0 \end{aligned}$$

If $\Delta^{N-1} \leq 2c^{-1}\left(\frac{v_2}{2}\right) - c^{-1}(v_1)$ and $\Delta^{N-1} < 2c^{-1}(v_2) + c^{-1}(v_1) - 4c^{-1}(\frac{v_1}{2})$:

$$\begin{aligned} b_1^* &= 0 \\ b_2^* &= \frac{c^{-1}(v_1) + \Delta^{N-1}}{2} \end{aligned}$$

and the value functions are:

$$
\begin{aligned}
V_1 &= 0 \\
V_2 &= v_2 - 2c\left(\frac{c^{-1}(v_1) + \Delta^{N-1}}{2}\right)
\end{aligned}
$$

The proof is given in the Appendix, but the rationale is as follows. With the advantage, Δ^{N-1}, players must first decide if they wish to bid at all. There are four possible scenarios. The first scenario has both Player 1 and Player 2 bidding a positive amount. The second scenario is where only Player 1 bids a positive amount. In the third scenario, Player 2 bids a positive amount and Player 1 bids zero. Finally, in the fourth scenario, both players bid zero. First, I show that due to the threshold, it cannot be that both players will find it profitable to bid. Therefore, only one player will bid a positive amount. To determine which player, I must first determine the conditions where it is profitable for a player to bid. And, having established that condition, I must specify the condition where the other player would not find it profitable to deviate from bidding zero. Hence, either the advantage or the amount of the bid must be sufficient to discourage entry. Otherwise, the other player may wish to intercede. It may be that there is a region where both players are willing to bid. In this region, there are two possible equilbria: one where the high valuation player bids positively and wins. And, the other where the lower valuation bids profitably and wins. Finally, I show that there is no situation where both players bid zero. Because of the convexity of the cost function, the player who has the advantage in terms of valuation or delta, will want to bid in order to avoid the high marginal cost of bidding later in the game.

Having established the two possible equilibria and their necessary conditions, I can generalize these results to any period prior to the final period.

2.6.3 Period N-k

A player will only bid in period $N-k$ if the bid will lead to a positive expected profit in the final round. The analysis for period $N-1$ shows that there are four conditions that define the equilibria of this model.

Lemma 4 (Conditions for equilibria)**.** *There are four cases*[5] *to consider. In period $N-k$:*

1. *It is profitable for Player 1 to bid if Player 2 bids zero.*
2. *It is not profitable for Player 2 to outbid Player 1 if Player 1 bids.*
3. *It is profitable for Player 2 to bid if Player 1 bids zero.*
4. *It is not profitable for Player 1 to outbid Player 2 if Player 2 bids.*

If both conditions 1 and 2 are satisfied *and* if either condition 3 or 4 is not, then there is only one equilibrium where Player 1 bids and Player 2 does not. On the opposite side, if conditions 3 and 4 are satisfied *and* either 1 or 2 is not, then the only equilibrium is one in which Player 2 bids and Player 1 does not. But, if all four conditions are satisfied, then there are multiple equilibria. Either Player 1 will bid and Player 2 will not *or* Player 2 will bid and Player 1 will not.

I can now generalize the value functions and bid function for any period.

Proposition 6. *When Player 1 bids and Player 2 does not, the value functions for Player 1 and 2 are:*

$$\begin{aligned} V_1^{N-k} &= v_1 - (k+1)c\left(\frac{c^{-1}(v_2) - \Delta^{N-k}}{k+1}\right) \\ V_2^{N-k} &= 0 \end{aligned}$$

and Player 1 bids:

$$b_1^{N-k} = \frac{c^{-1}(v_2) - \Delta^{N-k}}{k+1}$$

And, when Player 2 bids and Player 1 does not, the value functions are:

$$\begin{aligned} V_1^{N-k} &= 0 \\ V_2^{N-k} &= v_2 - (k+1)c\left(\frac{c^{-1}(v_1) + \Delta^{N-k}}{k+1}\right) \end{aligned}$$

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[5] The cases presented here are endogeneous. However, as I have shown in the period $N-1$ analysis above, it is possible to develop these conditions from the primitives.

and Player 2 bids:

$$b_2^{N-k} = \frac{c^{-1}(v_1) + \Delta^{N-k}}{k+1}$$

Proof. This proof is by induction and is in the appendix. □

One important implication is that as a player's advantage increases, b_i^{N-k} decreases. Therefore, a contest where one player amasses a large advantage results in decreased effort later in the contest.

2.7 Examples

This section will explore two example situations to provide context and show the existence of the region where multiple equilibria exist.

First consider the case where $n = N - 1$, $c(x) = x^2$, $v_1 = 10$, and $v_2 = 9$. In this scenario, we are in the second to last round of the competition. If $\Delta^n > 0.218$, Player 2 will not find it profitable to challenge Player 1. The unique equilibrium will be one where Player 1 bids and Player 2 does not. If $\Delta^n < -0.839$, Player 1 will not challenge Player 2. Therefore, Player 2 will bid and Player 1 will not. However, if Δ^n is greater than -0.839 and less than 0.218, either Player 1 or Player 2 may bid a positive amount. But, the other player will bid zero. This is because there is no profitable deviation for either player if the other bids positively. For example, if $\Delta^n = 0$, there are two equilibrium.

1. $b_1 = 1.5$ and $b_2 = 0$ resulting in $V_1 = 5.5$ and $V_2 = 0$

2. $b_1 = 0$ and $b_2 = 1.58$ resulting in $V_1 = 0$ and $V_2 = 4$

In the first equilibrium, Player 2's most profitable deviation is to bid 2.33. However this still does not breach the threshold to win the contest. Therefore, the outcome from bidding such an amount is -2.33. In the second equilibrium, Player 1 can cross the threshold by bidding 2.29. However, the expected profit from doing so is -0.4934. Therefore, Player 1 would prefer not to deviate.

The first equilibrium is not efficient because 37.2% of the time, Player 1 does not receive the prize. And, the second equilibrium is even worse from an efficiency standpoint with Player 2 winning 63.1% of the time. However, if the principle is solely interested in inducing effort, the second equilibrium provides a higher expected revenue. The total expected revenue in the first equilibrium is 2.69; whereas, in the second equilibrium, it is 2.82.

As a second example, we will use the same scenario as before, except that the cost function will now be $c(x) = x^4$. Before the range for multiple equilibrium was $-0.839 < \Delta^n < 0.218$. With the same valuations, the difference in these bounds shrinks to ε. Hence, there is essentially no opportunity for multiple equilibria.

2.8 Remarks

If the cost function is highly convex and the players have close valuations, then there is a region where either player bidding is an equilibrium. The analysis above shows that the multi-stage all-pay auction may cause an agent to reduce overall output compared to the single-stage competition. This result is driven by the high level of information that agents have about valuations and their position in the competition. If there were more uncertainty in the model, there may be a higher level of total effort (Petrenko, 2008).

If an agent were to face a shock, especially early in the competition, it could allow an opening for a competitor with lower prize valuation to gain an upperhand and eventually win the competition. Because of this possibility, it is difficult to make the argument that the contest results in an outcome where the highest valuation contestant is selected. Hence, if the prize of the contest is meant to convey information to future employers about the quality of the contestant, this structure may not have the intended consequences.

As the distance between v_1 and v_2 increases, the range where there are multiple equilibria is reduced. The distance required to eliminate the possibility of multiple equilibria depends on the convexity of the cost function. As the cost function becomes more convex, less separation between the valuations is required to eliminate the possibility of multiple equilibria. This is because though the player with the higher valuation can place the higher effective bid, the marginal cost of increasing the bid makes it unprofitable to do so.

There are several empirical implications that arise from this model. Of course, there will be more ambiguity from an observed continuation contest. Nevertheless, we should see that a player who does not have the highest valuation winning the competition. This is perhaps the most striking result of the model. Furthermore, we should observe that players' level of effort changes based on where they are in relation to other players. A player at the top of the order in any period should remain in the top at the next period. And a player lower in the order should remain lower in the following periods. This is due to the dynamic nature of the contest. As the advantage a player holds over his opponent increases, the amount of effort required to hold that advantage decreases. The disadvantaged player observes that he is no longer "in the running" for the prize and therefore reduces effort. The advantaged player, aware of this, can reduce effort accordingly and still maintain the advantage necessary to win.

2.9 Conclusion

It is important when devising incentive structures that their implementation be robust. Though this model contains many simplifying assumptions, the underlying premise suggests that there is a likelihood that multiple equilibria may exist in competition. If the structure of the contest is meant to select the player with the highest valuation, there is the possibility that a player with a lower valuation may win. And, if the prize of the contest is meant to convey information to future employers, this possibility undermines the efficacy of the signal. Additionally, if the contest is meant to extract the maximum possible effort from participants, a large advantage may reduce effort from both players.

Chapter 3

Value of an Educational Signal on a Distant Career Outcomes

This chapter investigates the effect that an educational signal has on a distant career outcome. The purpose behind this investigation is to identify if a signal, which is bestowed on top graduates, has value from a career standpoint. Using data from the United States Air Force Academy (USAFA), I measure the effect that the Distinguished Graduate (DG) designation has on subsequent officer performance. One empirical challenge in measuring the effect of the DG designation is that we cannot observe how an individual would have performed had they not achieved this distinction. However, because the designation of DG is based on an assignment rule, it is possible to use a Regression Discontinuity Design (RDD) to identify the causal effect of the DG signal on the likelihood that graduates will be in the top twenty percent of their officer cohort, as revealed through Intermediate Developmental Education (IDE) "In-Residence" selection. As there are limited opportunities for development, the Air Force struggles to identify those with great potential early in their careers.[1]

Whenever a designation is bestowed based on performance, one may ask if later outcomes are due to the characteristics associated with the performance or if the designation in and of itself is the causal factor. The latter has become known as the "sheepskin" effect because possession of a degree may act as a signal to employers. Spence (1973) and Arrow (1973) were the pioneers of the educational signaling literature. And, Weiss (1995) argues that any attempt to examine the economic

[1] *Panel Prods Academies to Groom Elite Officers,* Air Force Times, 20 July 2009.

returns to education must consider the impact of signaling effects. Furthermore, there is a rich body of research to determine the returns to education. In this regard, the following analysis explores the signal of DG designation and its impact on distant career outcomes. Though this analysis capitalizes on the unique properties of the USAFA environment, the findings are applicable in a broader setting. The DG designation is comparable to the *summa cum laude* designation bestowed by many civilian universities. Furthermore, in an environment where there is increasing grade inflation (Chan, Hao, and Suen, 2007), a rank-based distinction may help identify which students are truly at the head of the class.

While it is fruitful in itself to explore this signaling value, determining the effect of DG on IDE is part of my broader research agenda where I explore the incentive structure at the United States Air Force Academy. The situation at USAFA is unique in that it allows empirical analysis, particularly of the final semester at the Academy. Cadets compete in a multi-stage tournament for DG recognition. A central facet of the tournament is the prize for which agents compete. In addition to the DG prize, cadets vie for higher class rank so they can have a better choice of assignment, opportunities for graduate school, and extra privileges while at USAFA. Because these other prizes have been awarded by the last semester, the analysis of the competition is greatly simplified. Therefore, this analysis focuses on the valuation of the DG prize. The valuation includes the intrinsic value of the prize held by the cadet and the extrinsic value that the prize may have on future career outcomes. The prize's intrinsic value may be the pride that a winner feels from having done well. This intrinsic value is heterogeneous across competitors. The extrinsic value of the prize is the impact that the DG status has on an officer's career progression. As the intrinsic valuation varies greatly across individuals, I focus on the extrinsic valuation.

Agents who have higher valuation for a prize will exert higher effort. In addition, average effort increases as the number of prizes available increases (Harbring and Irlenbusch, 2003). Early research of tournaments focused on agents drawn from an identical distribution. Hence, the principal was concerned with effort levels rather than selection. Clark and Riis (2001) consider the case where principals have preferences over agents. They show that a two-tiered prize structure can ensure that selection is perfect. Before I examine the strategic behavior of cadets, I must first have a better understanding of the prize for which they compete.

This chapter is organized into five sections. The next section describes the scenario at USAFA and the data. The second section explains the identification strategy. Section three explains the various methodologies employed to examine the effect of DG on IDE selection. Section four contains remarks on the results of the empirical analysis. The final section explains the ramifications of these results on the incentives at USAFA.

3.1 Background and Data

Many authors have attempted to characterize the relationship between educational signals and career outcomes. Card (1999) provides a survey of the empirical issues and some attempts to overcome these challenges. Since that article was written, the search for a connection between education and earnings has continued. Psacharopoulos and Patrinos (2004) provide a compendium of returns to education studies using data from various countries. Finding data that are robust enough to address the central question is challenging. The Current Population Survey (CPS) and the National Longitudinal Surveys (NLS) are popular sources for empirical work. My research relies upon the unique properties of the United States Air Force Academy and the United States Air Force promotion system to address the impact that an educational signal has on career outcomes. While most studies in this vein address the impact of an additional year of schooling, this contribution is unique because there are no studies that examine how graduation honors impact earnings. As grade inflation becomes more of an issue, employers may begin to view honors as a distinguishing characteristic. Yale has taken steps to ensure that its honors designations are limited to those students at the top of the class rather than those above a GPA threshold.[2]

Among colleges and universities in the United States, the Academy is unique in that it serves a dual role: provide a liberal education with a technical focus as well as prepare graduates for military service. Graduates of the Academy have been successful in both military and academic settings. Since its establishment in 1959, it has produced 35 Rhodes Scholars, 10 Marshall Scholars, 14 Truman Scholars, and 36 Fulbright Scholars. In addition, over 470 graduates have served as general

[2] *Yale Moves to Make Cum Laude Mean More,* New York Times, 22 May 1988.

officers.[3] At the same time, cadets who attend the Academy are not unlike those who attend other prestigious universities. Previous studies that have used data on Air Force Academy cadets have found that the demographics at the Academy are similar to civilian schools that focus on technical degrees.[4]

Upon graduation, cadets fulfill their commitment to serve for five years as active-duty Air Force officers. Based on their performance at the Academy, cadets choose a job classification according to their preferences from a menu of career openings. The Air Force refers to these job classifications by Air Force Speciality Codes (AFSC). There are nine top-level speciality codes: operations, logistics, support, medical, legal, acquisitions, office of special investigations (OSI), special, and student. Furthermore, there are two broad classes of officers in the Air Force. Those who are medically qualified may choose to attend pilot or navigator school. Officers who complete this training are known as rated officers. Those who do not are considered non-rated officers. Typically, rated officers are assigned an operations speciality code.[5] Non-rated officers serve in logistics, communications, maintenance, special investigations, and acquisitions.[6] Rated officers enter pilot training following graduation. Based on their performance in pilot training, they are assigned to specific aircraft. This aircraft type will dictate the type of mission and location of assignments. Hence, non-rated officers compete for their job and basing location while at the Academy. Rated officers do not compete for these details until they enter pilot training.

At graduation, cadets are recognized as Distinguished Graduates if they are in the top ten percent of their class according to their Officer Performance Average (OPA). The OPA consists of measures of performance from academic, military, and athletic endeavors. While the weighting of these three measures has changed slightly over time, the largest component of the OPA is the academic grade point average (GPA). Typically, 80 percent of OPA is GPA. Upon graduation, cadet records are sealed. Only the DG designation follows a cadet into the officer corps. Hence, one of the benefits

[3] USAF Academy Institutional Self-Study Report (2009)

[4] See Carrell, Page, and West (2009), Table 1 for comparison to peer institutions

[5] In the data used in this study, 85 percent of rated officers have an operations AFSC.

[6] A small subset of cadets enter the legal and medical branches. Because these career fields are promoted in separate boards, graduates in these fields are not included in this analysis.

of achieving this distinction is that it is reflected in the officer selection brief when graduates come before their promotion board. Part of the value of the prize is the impact that the designation has on promotion.

Officer advancement is based on the outcome of promotion boards held centrally at Air Force Personnel Center at Randolph Air Force Base outside San Antonio, Texas. Promotion to service in a higher rank is not a reward for successful performance. Rather, it is a recognition that a service member is capable of additional duty and responsibility. While previous experience is an indicator of this capability, it is not the sole factor that determines promotion. The promotion board also considers recommendations from superiors, peer stratification, and training and education. Again, records from the Academy are sealed. However, the board is informed of the officer's commissioning source (USAFA, ROTC, or OTS) and whether he graduated with distinction.

Eligibility for promotion is based on time in grade. Graduates enter the Air Force with the rank of Second Lieutenant. After two years, officers become eligible for promotion to First Lieutenant. After two years as a First Lieutenant, officers are eligible to become Captains. The advancement to these ranks is fairly automatic. No formal promotion boards are held. Thus, the promotion to Major is the first time that an officer faces a promotion board. At this point, officers are selected for both promotion and In-residence Intermediate Development Education (IDE). This selection identifies the top twenty percent of the officers in that year group. Hence, it is possible to compare the top performing officers, those selected for In-residence IDE, to the top performing cadets, those designated as Distinguished Graduates. Furthermore, In-residence selection is an important signal to future promotion boards. While all officers must complete IDE, either by correspondence or in-residence, to be eligible for promotions beyond Major, those selected for In-residence IDE are in effect "primed" for future promotion opportunities. If Air Force promotion boards use the DG information as a signal of capability, then there should be a positive correlation between DG and IDE.

The data used in this thesis was supplied by the USAF Academy's Office of Plans and Programs under the supervision of the Academy's Institutional Review Board (IRB). The data contain observations from the class of 1982 to 1995, the most recent class to complete IDE. For each cadet, there is demographic information, admissions data, cadet performance measures, and officer performance measures. The variables and summary statistics are listed in Appendix A.

3.2 Identification Strategy

The aim of this paper is to measure the impact of achieving the designation of "Distinguished Graduate" (DG) on the selection to attend Intermediate Developmental Education (IDE) "In-Residence". There are significant empirical difficulties in identifying the impact that an honors designation has on future career outcomes. One inherent problem is that it is impossible to know what the in-residence determination would have been had an individual, who graduated as a DG, not received this designation. Similarly, the data cannot identify what a non-DG's outcome would have been if she or he had been a DG. Econometric techniques can mitigate this problem.

Due to the nature of these data, there are no observations with identical covariates who progress through the system with and without the signal. Without such a counterfactual there is no possibility of using matching as an identification model. In my analysis, I have explored using LPM, probit, and bivariate probit models to identify the causal effect of the DG signal. However, due to the unobserved heterogeneity between the DG signal and the IDE selection outcome variable, these models produce a biased effect of DG on IDE. To correct for this issue and give weight to the results, I use a discontinuity design based on the assignment rule for DG to isolate the effect of the DG designation. As long as the covariates associated with DG are continuous over the cutoff, we can infer that those in a small neighborhood on opposite sides of the cutoff are equivalent except for the possession of the DG designation. Given this equivalence, we can determine the unconfounded influence of DG on IDE selection.

3.3 Empirical Analysis

3.3.1 LPM and Probit

It is constructive to run a simple linear probability model (LPM) to determine if DG has a statistically significant effect on the likelihood that an officer is selected for IDE. In this most basic model, DG has a significant effect on IDE selection. This is true in the five specifications detailed in Table

3.1. The first, second, and third specifications contain all observations from the classes of 1982-1995 who are commissioned as Air Force officers and achieve the rank of Major. The fourth and fifth specification divide the observations into those who are rated and those who are non-rated.[7] The LPM model is:

$$IDE_i = \beta_0 + \beta_1 DG_i + \beta_2' \mathbf{S}_i + \varepsilon_i$$

The covariates included in $\mathbf{S}_i$ vary across specifications. All specifications include controls for overall performance average,[8] state of residence, and class year. The first specification includes demographic variables for gender,[9] race, and academic composite score. The second specification adds recruited athlete, intercollegiate athlete,[10] and age at commissioning to the first specification. The third, fourth, and fifth specifications include the officer job classification, whether the officer ever married, the number of dependents the officer has, and whether the officer has earned an advanced academic degree. The excluded group is the white male who was not a recruited athlete, did not participate in intercollegiate sports, and serves under the operator AFSC.

Because selection to In-residence IDE is a binary variable and the linear probability model may yield predictions outside this range, Table 3.2 contains the same specifications described above using a probit model. Now, the results are mixed. The first two specifications indicate that DG has a positive effect on In-residence IDE selection. But, once post-graduation controls are added to the model, this effect is no longer statistically significant.

While the probit model does restrict predicted probabilities to between zero and one, serious issues remain. Almost assuredly, there is endogenity with regards to the DG variable and the unobserved characteristics. Consider an individual who is highly motivated. This motivation may not

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[7]Rated officers receive their assignments in a different manner than non-rated officers. Therefore, I have included tables in the Appendix that focus solely on rated officers. Also, only three rated officers who attain the rank of Major serve under the OSI job classification. None of these officers are selected for In-Residence IDE. Hence, the coefficient on specification (4) for AFSC_osi is zero in the linear probability model and these observations are dropped from the probit model.

[8]Because DG is determined by OPA, I have also run the same specifications without controlling for OPA. These results, presented in the Appendix, do not differ substantially.

[9]Because women comprise less than twenty percent of a graduating class, I've included addition tables in the appendix that focus on males only. Again, the results are not substantially different.

[10]There may be concern that recruited athlete and intercollegiate athlete are highly correlated. In the sample, the correlation coefficient between these variables is only 0.4754.

Table 3.1: Marginal Effect on Selection to In-Residence IDE – by LPM

VARIABLES	(1) All	(2) All	(3) All	(4) Rated	(5) Non-rated
Distinguished Graduate	0.0716*** (0.0234)	0.0703*** (0.0233)	0.0557** (0.0229)	0.0464* (0.0280)	0.0724* (0.0425)
OPA	0.154*** (0.0169)	0.152*** (0.0166)	0.126*** (0.0159)	0.130*** (0.0194)	0.124*** (0.0291)
female	-0.00890 (0.0144)	-0.0145 (0.0149)	-0.0153 (0.0156)	-0.0460* (0.0243)	-0.00876 (0.0202)
minority (non-white)	-0.0155 (0.0123)	-0.0103 (0.0124)	-0.00826 (0.0116)	-0.0264* (0.0147)	0.0173 (0.0198)
intercollegiate athlete		0.00474 (0.0129)	0.0112 (0.0124)	0.0123 (0.0137)	0.00724 (0.0211)
recruited athlete		0.0343** (0.0144)	0.0282** (0.0139)	0.0139 (0.0154)	0.0593** (0.0235)
academic composite score	-8.38e-05*** (1.95e-05)	-7.06e-05*** (2.03e-05)	-7.92e-05*** (1.92e-05)	-9.05e-05*** (2.33e-05)	-6.61e-05* (3.52e-05)
AFSC - logistics			0.0220 (0.0226)	-0.0162 (0.0592)	-0.0181 (0.0278)
AFSC - support			0.0118 (0.0175)	-0.0316 (0.0437)	-0.0173 (0.0242)
AFSC - acquisitions			-0.0328** (0.0144)	-0.0451 (0.0440)	-0.0561*** (0.0204)
AFSC - osi			-0.0332 (0.0524)	0 (0)	-0.0566 (0.0555)
AFSC - special			-0.0841*** (0.0250)	-0.0509 (0.0311)	-0.146*** (0.0406)
AFSC - student			-0.0284 (0.0200)	-0.0143 (0.0223)	-0.0863** (0.0435)
ever married – binary			0.0714*** (0.0159)	0.0609*** (0.0202)	0.0961*** (0.0288)
advanced academic degree			0.259*** (0.00778)	0.257*** (0.00920)	0.247*** (0.0135)
Age at Graduation		-0.0141** (0.00565)	-0.0128** (0.00530)	-0.0148** (0.00611)	-0.0138 (0.00989)
maximum number of dependents			0.00671** (0.00309)	0.00992** (0.00385)	-0.000349 (0.00562)
Constant	0.461** (0.202)	0.743*** (0.227)	0.385* (0.228)	0.256* (0.154)	0.424 (0.363)
Observations	8476	8476	8476	5673	2803
R^2	0.069	0.070	0.153	0.175	0.138

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, AFSC - operator

Table 3.2: Marginal Effect on Selection to In-Residence IDE – by probit

VARIABLES	(1) PR1	(2) PR2	(3) PR3	(4) PR4	(5) PR5
Distinguished Graduate	0.0486**	0.0473**	0.0247	0.00829	0.0580
	(0.0217)	(0.0215)	(0.0184)	(0.0186)	(0.0420)
OPA	0.165***	0.163***	0.127***	0.122***	0.130***
	(0.0175)	(0.0173)	(0.0153)	(0.0170)	(0.0305)
female	-0.00918	-0.0144	-0.00585	-0.0347*	-0.00378
	(0.0144)	(0.0148)	(0.0139)	(0.0183)	(0.0206)
minority (non-white)	-0.0168	-0.0115	-0.00908	-0.0275**	0.0206
	(0.0126)	(0.0129)	(0.0108)	(0.0116)	(0.0217)
intercollegiate athlete		0.00498	0.00764	0.00844	0.00596
		(0.0132)	(0.0113)	(0.0114)	(0.0206)
recruited athlete		0.0342**	0.0262**	0.0104	0.0618**
		(0.0148)	(0.0130)	(0.0127)	(0.0245)
academic composite score	-8.86e-05***	-7.67e-05***	-7.58e-05***	-8.39e-05***	-5.93e-05*
	(1.96e-05)	(2.05e-05)	(1.75e-05)	(1.92e-05)	(3.50e-05)
AFSC - logistics			0.0206	-0.0221	-0.0163
			(0.0185)	(0.0446)	(0.0249)
AFSC - support			0.0148	-0.0481	-0.0119
			(0.0154)	(0.0478)	(0.0231)
AFSC - acquisitions			-0.0216**	-0.0184	-0.0532***
			(0.0109)	(0.0276)	(0.0190)
AFSC - osi			-0.0231		-0.0543
			(0.0398)		(0.0479)
AFSC - special			-0.0811***	-0.0608**	-0.133***
			(0.0183)	(0.0248)	(0.0267)
AFSC - student			-0.0219	-0.00927	-0.0696*
			(0.0216)	(0.0227)	(0.0418)
Age at Graduation		-0.0157**	-0.0108**	-0.0117**	-0.0148
		(0.00613)	(0.00520)	(0.00551)	(0.0106)
maximum number of dependents			0.00772***	0.00938***	0.00105
			(0.00292)	(0.00331)	(0.00557)
Observations	8476	8476	8476	5654	2800

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, AFSC - operator

be captured in the covariates. Hence, it appears in the error term. At the same time, that motivation will affect both the likelihood that an individual becomes a DG and the likelihood that he is selected for IDE. Therefore, the results of the LPM and probit specifications detailed in Table 3.1 and 3.2 are biased.[11] To determine if there is correlation between the two binary outcomes, DG and IDE, we can employ a bivariate probit model.

3.3.2 Bivariate Probit

There may be unobserved heterogeneity that affects both selection to In-residence IDE and selection as a DG. To test for this, I used a bivariate probit specification as described in Cameron and Trivedi (2005):

$$IDE_i = \beta_0 + \beta_1 DG_i + \beta_2' \mathbf{S}_i + \varepsilon_i \tag{3.1}$$

$$DG_i = \alpha_0 + \alpha_1' \mathbf{T}_i + \mu_i \tag{3.2}$$

where $\mathbf{S}_i$ and $\mathbf{T}_i$ are observable characteristics affecting IDE and DG selection for individual i. If ρ, the correlation between ε_i and μ_i, is zero, then (3.1) and (3.2) can be considered separately.

The results of the bivariate probit model are presented in Table 3.3. Because ρ is not significant, we have that there is no correlation between the error terms. Specification (1) in Table 3.3, which corresponds to equation (3.1) above, shows that DG does not affect IDE selection. However OPA, from which the DG distinction is based, does have a significant effect on selection for IDE. I also considered a model that omits OPA from equation (3.1). The effect of DG on IDE remains insignificant.[12] The insignificance of ρ is another indication that IDE and DG selection are unrelated by unobservables. However, the results are unstable, and the model is probably not identified. There may be endogeneity in the covariates which could be marring these results.

[11] Blackburn and Neumark (1995) find that omitted ability measures may lead to upward bias in OLS estimation of economic returns to schooling. This suggests that the impact of DG may be overstated.

[12] Appendix C, Table C.3.

Table 3.3: Bivariate Probit

VARIABLES	(1) IDE	(2) DG	(3) ρ
Distinguished Graduate	0.300		
	(0.404)		
OPA	0.596***		
	(0.0666)		
female	-0.0130	-0.390***	
	(0.0699)	(0.0889)	
minority (non-white)	-0.0294	-0.131	
	(0.0683)	(0.146)	
intercollegiate athlete	0.0481	-0.0964*	
	(0.0441)	(0.0573)	
recruited athlete	0.102**	-0.0616	
	(0.0422)	(0.0721)	
academic composite score	-0.000398***	0.00263***	
	(0.000140)	(0.000164)	
advanced academic degree	1.667***	0.415***	
	(0.169)	(0.0614)	
attended prep school	0.112**	-0.871***	
	(0.0561)	(0.155)	
AFSC - logistics	0.0801		
	(0.113)		
AFSC - support	0.0689		
	(0.0657)		
AFSC - acquisitions	-0.106**		
	(0.0464)		
AFSC - osi	-0.102		
	(0.228)		
AFSC - special	-0.513***		
	(0.125)		
AFSC - student	-0.0925		
	(0.196)		
Age at Graduation	-0.0769***	0.0715**	
	(0.0170)	(0.0340)	
maximum number of dependents	0.0364**		
	(0.0166)		
parent attended an academy		-0.115	
		(0.116)	
sibling attended an academy		0.0276	
		(0.0827)	
Constant	-0.169	-16.61	-0.112
	(0.706)	(.)	(0.218)
Observations	8476	8476	8476

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, afsc_operator

3.3.3 Regression Discontinuity Design

There are several approaches to dealing with the potential endogeneity of explanatory variables. Panel methods, matching, and instrumental variables are traditional means to obtain unbiased estimates. Because the DG signal is bestowed according to an assignment rule, I can use the regression-discontinuity design approach to study the outcome in a quasi-experimental manner. In a pure experiment, I would be able to randomly assign a treatment, in this case the DG designation, to individuals and observe the impact on the outcome of interest, selection to In-residence IDE. With observational data, the treatment is not assigned by either the researcher or a defined process; therefore, the analysis is more difficult. However, with an assignment rule and if certain assumptions hold, it is possible to treat the observation data as if it were experimental. This is the purpose of the regression discontinuity design (RDD).

The discontinuity design procedure is set forth by Imbens and Lemieux (2008) and van der Klaauw (2008). Though this technique was first used by Thistlethwaite and Campbell in 1960, it has recently grown in popularity, particularly in the research on education interventions. This has led to empirical advances and the codification of best practices. There are two types of discontinuities exploited by RDD: sharp and fuzzy. A sharp discontinuity is one where there is a firm cut-off where all observations that meet this threshold receive the treatment. In a fuzzy discontinuity, there is strong evidence of an assignment rule, but it is not deterministic. Rather, there is some variability around the cut-off as to whether the treatment is observed.

Data suited for RDD do not admit observations where an individual both receives treatment in one instance and then does not receive the treatment in another. In addition, an important assumption required for identification is that the regression functions, conditioned on the covariates, are smooth and continuous. One drawback of the discontinuity design is that it identifies only the treatment effect for the covariates close to the cut-off. While these effects may be useful for the question at hand, the results are not applicable in a more general setting.

Key Assumptions

The discontinuity design exploits the rank-ordering inherent in the DG signal. Because there is a cut-off that determines DG, it is reasonable to assume that those who are just short of the mark are similar to those who just make it. Based on this reasoning, I can consider individuals who have OPAs close to the cut-off as the same for econometric purposes. Thus, if members of this group who have the DG distinction are more likely to be selected for In-residence IDE than those who are not DGs, then we can conclude that the DG designation has an impact. Close is a relative term. Adjustments to the bandwidth around the cutoff will impact the results. The results presented use a bandwidth of ± 0.1 from the OPA cutoff for DG. However the results hold for a range of bandwidths. Table 3.4 shows how the choice of bandwidth changes the effect of DG on IDE. The table investigates four cutoffs: all observations with OPA > 1, observations with OPA within 0.1 of the cutoff, within 0.05 of the cutoff, and within 0.01 of the cutoff.

Having established the appropriate bandwidth with which to examine the discontinuity, it is important to test the assumption that those who border the cut-off are the same except for the treatment. This is accomplished by examining covariates both graphically and analytically. Graphical results are presented in Figure 3.1. These variables, which are detailed in Table C.10 of Appendix C, allow me to note if any observable characteristics are responsible for differences in in-residence selection. If, for example, gender plays a role, then I cannot isolate the effect of the DG signal. So, to conclude that it is the DG distinction that has an impact and no other, I must examine other factors that may play a role in the selection for IDE. The figures represent a local linear regression on each side of the cut-off for DG. Gender, race, age at commissioning, high school GPA, academic composite, candidate fitness test score, military performance average, and rated status are depicted. The graphs of these covariates all appear smooth and continuous across the cut-off. Detailed examination of the coefficient on DG for each covariate confirms that the characteristics are smooth across all except military performance average (MPA). This is not unreasonable given that MPA accounts for between fifteen and twenty percent of OPA depending on the class year. However, the coefficient on treatment variable is insignificant for all other covariates.

Table 3.4: Effect of Distinguished Graduate on Selection to In-Residence IDE – by bandwidth

VARIABLES	(1) All	(2) ±0.1	(3) ±0.05	(4) ±0.01
Distinguished Graduate	-0.0107	0.0729	0.205	-0.208
	(0.0636)	(0.0865)	(0.122)	(0.183)
female	-0.0151	-0.106	-0.115	-0.709
	(0.0157)	(0.0846)	(0.101)	(0.348)
minority (non-white)	-0.00883	-0.0469	-0.153	-0.232
	(0.0116)	(0.102)	(0.123)	(0.382)
intercollegiate athlete	0.0112	-0.00936	-0.0251	-0.277
	(0.0124)	(0.0462)	(0.0444)	(0.152)
recruited athlete	0.0277**	0.00761	0.0203	-0.348
	(0.0140)	(0.0944)	(0.186)	(0.148)
academic composite score	-7.99e-05***	-4.25e-05	-0.000159	0.000373
	(1.93e-05)	(0.000106)	(0.000134)	(0.000230)
AFSC - logistics	0.0214	0.185	0.0238	0.723
	(0.0226)	(0.168)	(0.298)	(0.479)
AFSC - support	0.0114	-0.0772	-0.0706	-0.566
	(0.0175)	(0.0987)	(0.147)	(0.484)
AFSC - acquisitions	-0.0336**	-0.0734	-0.0269	-0.113
	(0.0143)	(0.0586)	(0.0759)	(0.245)
AFSC - osi	-0.0334	0.186	-0.180	0.453
	(0.0527)	(0.220)	(0.264)	(0.825)
AFSC - special	-0.0831***	-0.0638	0.0433	0
	(0.0250)	(0.105)	(0.154)	(0)
AFSC - student	-0.0286	-0.0302	0.0871	1.168**
	(0.0201)	(0.117)	(0.179)	(0.274)
advanced academic degree	0.260***	0.331***	0.399***	0.611**
	(0.00779)	(0.0403)	(0.0569)	(0.133)
Age at Graduation	-0.0129**	-0.0540*	-0.102***	-0.0594
	(0.00530)	(0.0272)	(0.0277)	(0.100)
maximum number of dependents	0.00656**	0.0292**	0.0468**	0.0657
	(0.00308)	(0.0133)	(0.0178)	(0.0389)
Constant	0.396*	2.067**	2.431**	0.246
	(0.229)	(0.791)	(0.961)	(2.676)
Observations	8476	596	335	93
R^2	0.153	0.239	0.370	0.731

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, afsc_operator

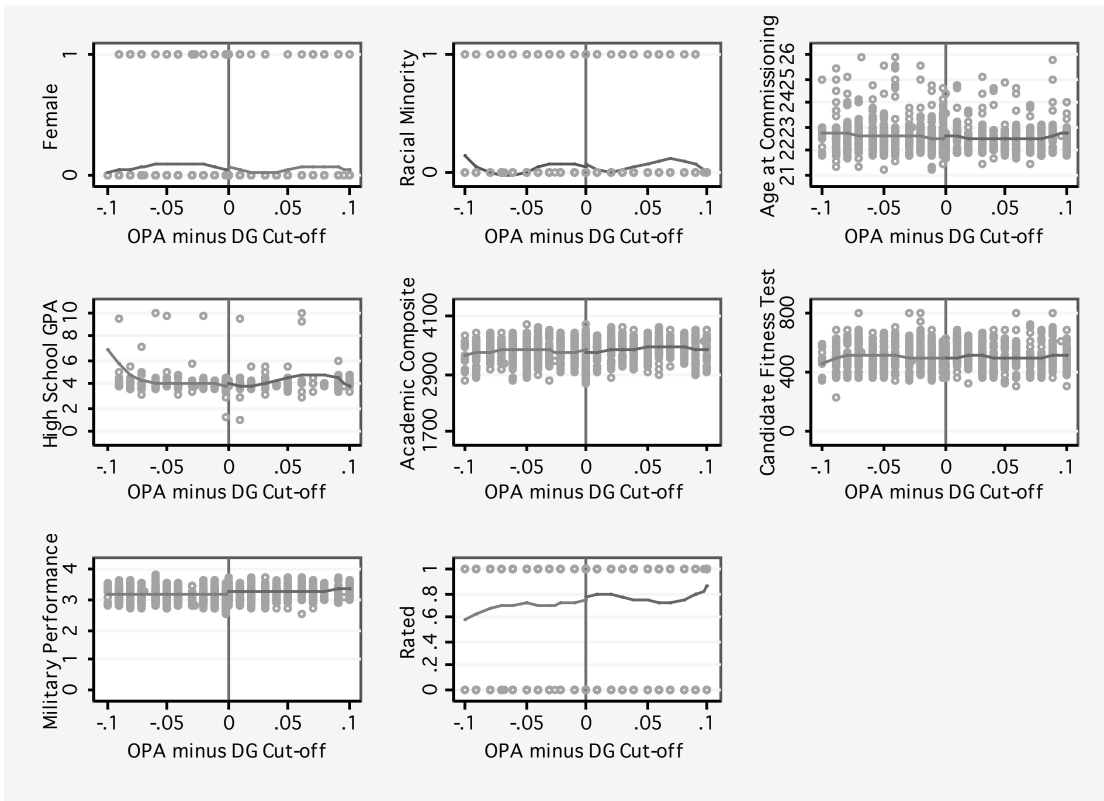

Figure 3.1: Estimates of discontinuity in observable characteristics

Model Specification

The discontinuity approach is the straightforward "sharp" discontinuity whereby individuals are assigned a treatment based on achieving a threshold over a continuous measure. Specifically, cadets who are in the ninetieth percentile of their class according to their officer performance average (OPA). This treatment, $DG = 1$ occurs according to the following assignment rule:

$$DG_{ic} = \mathbb{1}_{\left[OPA_{ic} \geq \overline{OPA_c}\right]}$$

where $\overline{OPA_c}$ is the threshold for each graduating class c above which individual i is in the top tenth percentile of their peers. Appendix B lists the cutoffs for each graduating class. The final cutoff for DG is determined just prior to graduation because the cutoff depends on the number of graduates. Because cadets do not know the cutoff in advance, they cannot adjust their behavior to self-select into DG.

Since it is true that the covariates are smooth over the cutoff, the DG signal can be examined independently from the covariates within a bandwidth of the cutoff. This allows us to examine the effect of DG on the selection to IDE using the following model:

$$IDE_i = \beta_0 + \beta_1 DG_i + \beta_2' \mathbf{S_i} + \varepsilon_i$$

where **S** is a vector of observable characteristics.

Assumptions:

1. Cadets do not have advanced knowledge of where the DG cutoff will be. This is plausible because the assignment rule is based on the top ten percent of cadets. The cutoff point is not established until all scores are final.

2. Also, $\mathrm{E}[\varepsilon|DG, \mathbf{S}] = \mathrm{E}[\varepsilon|\mathbf{S}]$. This is the foundation of the discontinuity approach. This assumption implies that the unobserved heterogeneity is not conditional on the treatment Cameron and Trivedi (2005). Therefore, the effect of the treatment is the difference in the

outcomes of the individuals just below the threshold and those just above:

$$\lim_{S \downarrow \bar{S}} E[y|\mathbf{S}] - \lim_{S \uparrow \bar{S}} \mathbf{E}[\mathbf{y}|\mathbf{S}] = \alpha + \lim_{S \downarrow \bar{S}} \mathbf{E}[\varepsilon|\mathbf{S}] - \lim_{S \uparrow \bar{S}} \mathbf{E}[\varepsilon|\mathbf{S}]$$

Following Lee, Moretti, and Butler (2004) and Hoekstra (2008), I use a cubic polynomial specification for the local linear regressions on each side of the DG discontinuity. All specifications are a cubic polynomial of officer performance average with interactions with a dummy variable for DG designation. Table 3.5 shows that regardless of the choice of polynomial the effect of DG on IDE is insignificant.

Results

Treatment variable

Figure 3.2 shows that this is indeed a sharp discontinuity where the treatment is present only for those observations that are above the cut-off.

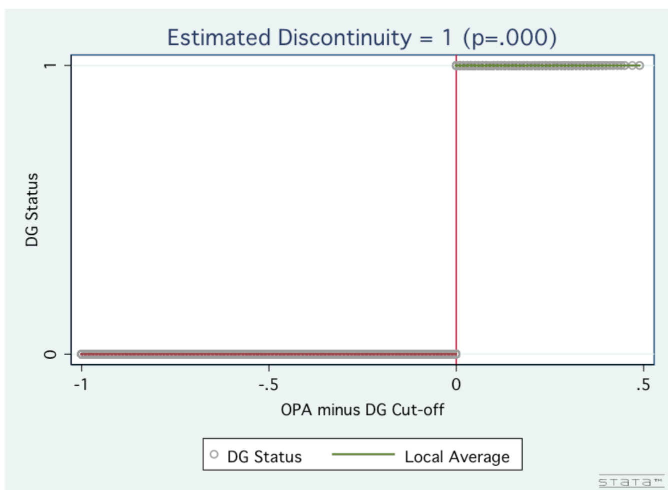

Figure 3.2: Sharp discontinuity in treatment

Outcome variable

Conditional upon achieving the rank of Major[13], I find no evidence that DG is a factor in IDE

[13] I also performed the analysis including those who did not make Major with an IDE value of zero. There were no substantive changes to the results.

Table 3.5: Polynomial Specifications - within 0.1 of cutoff

VARIABLES	(1) linear	(2) quadratic	(3) cubic	(4) quartic
Distinguished Graduate	0.00373	0.0481	0.0248	0.0105
	(0.0681)	(0.0705)	(0.0765)	(0.0718)
OPA	0.563	1.421	-0.505	5.542
	(0.610)	(3.926)	(7.378)	(13.37)
OPA^2		8.785	-40.75	238.1
		(41.72)	(163.4)	(689.2)
OPA^3			-335.2	4059
			(1030)	(11367)
OPA^4				22181
				(58096)
$DG \cdot OPA$	-0.373	-4.683	2.738	-7.417
	(1.060)	(4.600)	(8.763)	(17.28)
$DG \cdot OPA^2$		24.78	-67.36	-144.7
		(41.80)	(185.8)	(818.5)
$DG \cdot OPA^3$			1271	-6331
			(1179)	(13975)
$DG \cdot OPA^4$				-6174
				(65072)
Constant	0.133*	0.144**	0.129**	0.154**
	(0.0679)	(0.0534)	(0.0597)	(0.0587)
Observations	596	596	596	596
R^2	0.062	0.064	0.065	0.066

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

In-residence Selection. Table 3.6 shows the outcome of several specifications based upon the local linear regressions.[14] All specifications include gender, race, and high school zip code dummy variables. Specification (1) focuses on the academic ability at admission by including academic composite score. Specification (2) looks at the role that age and athletics play in IDE selection. There is a general perception at the Academy that athletic participation builds leadership and character. In addition, older cadets often have prior college or military experience which may affect performance. The third specification, (3), includes the previous variables and adds officer characteristics such as job classification, Air Force Specialty Code (AFSC), marital status, and number of dependents. These variables shape the type of job assignments that officers can fill. Assignments lead to experiences which are assessed by the promotion boards. Finally, the Air Force is divided between those who have aeronautical ratings and those without. Specifications (4) and (5) look at these separately.

3.4 Remarks on the Prize Valuation

As there is no discontinuity in selection to In-residence IDE across the treatment, RDD indicates that DG has no effect on this distant career outcome. In all specifications there is no evidence that DG status affects the likelihood of selection to In-Residence IDE. While it appears that some other covariates such as gender may play a role based on the results of specifications (1) and (2), the more thorough specification, (3), which includes AFSC, does not show that gender is significant. Because much of the information available to the promotion board concerns job performance, the AFSC is an important factor. Note, the board does not consider AFSC directly when determining promotion and IDE selection. However, they do consider the range of experiences and level of responsibility held. This is directly related to AFSC. The high level of significance in the AFSC dummy variables shows the linkage between AFSC and the unobserved factors.

Specifications (4) and (5) compare rated and non-rated officers. For rated officers, there is evidence that race has a negative effect on IDE selection. Also, rated officers benefit more in terms of IDE selection from having an advanced academic degree. This is likely because flying duties rarely

[14] To validate this result, I also used the software package developed by Nichols (2007). His program employs a more flexible kernel across several bandwidths. The results from this exploration, though not presented in this paper, yield the same conclusion which is that the DG treatment has no impact on selection to In-residence IDE.

Table 3.6: Effect of Distinguished Graduate on Selection to In-Residence IDE

VARIABLES	(1) All ±0.1 OPA	(2) All ±0.1 OPA	(3) All ±0.1 OPA	(4) Rated ±0.1 OPA	(5) Non-rated ±0.1 OPA
Distinguished Graduate	0.0298	0.0393	0.0729	0.0892	-0.350
	(0.0917)	(0.0928)	(0.0865)	(0.104)	(0.244)
female	-0.139	-0.142	-0.106	-0.0461	-0.120
	(0.0854)	(0.0847)	(0.0846)	(0.135)	(0.123)
minority (non-white)	-0.0569	-0.0756	-0.0469	-0.149**	0.0578
	(0.111)	(0.109)	(0.102)	(0.0676)	(0.253)
intercollegiate athlete		-0.0363	-0.00936	-0.0326	0.0195
		(0.0489)	(0.0462)	(0.0546)	(0.127)
recruited athlete		0.0766	0.00761	-0.00419	0.0171
		(0.103)	(0.0944)	(0.132)	(0.163)
academic composite score	-3.51e-05	-4.87e-05	-4.25e-05	-6.56e-05	-2.18e-05
	(0.000103)	(0.000105)	(0.000106)	(0.000151)	(0.000277)
AFSC - logistics			0.185	-0.244	0.237
			(0.168)	(0.220)	(0.250)
AFSC - support			-0.0772	0	-0.0643
			(0.0987)	(0)	(0.146)
AFSC - acquisitions			-0.0734	-0.0366	-0.0443
			(0.0586)	(0.130)	(0.113)
AFSC - osi			0.186	0	0.408
			(0.220)	(0)	(0.388)
AFSC - special			-0.0638	-0.211**	0.225
			(0.105)	(0.0942)	(0.352)
AFSC - student			-0.0302	-0.00123	0.0706
			(0.117)	(0.129)	(0.268)
ever married – binary			0.0602	-0.0275	0.191
			(0.0847)	(0.110)	(0.180)
advanced academic degree			0.331***	0.350***	0.274
			(0.0403)	(0.0444)	(0.231)
Age at Graduation		-0.0590**	-0.0540*	-0.0440	-0.0593
		(0.0247)	(0.0272)	(0.0410)	(0.0766)
maximum number of dependents			0.0292**	0.0440***	-0.0228
			(0.0133)	(0.0158)	(0.0240)
Constant	-0.102	1.249*	0.704	0.689	0.227
	(0.370)	(0.729)	(0.807)	(1.093)	(3.056)
Observations	596	596	596	436	160
R^2	0.164	0.173	0.239	0.306	0.514

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Excluded group: male, white, non-recruit, non-athlete, afsc_operator

Controls	gender race academic composite high school residence	gender race, age academic composite high school residence intercollegiate status recruit status	gender race, age academic composite high school residence intercollegiate status recruit status	gender race, age academic composite high school residence intercollegiate status recruit status	gender race, age academic composite high school residence intercollegiate status recruit status

allow time for rated members to attend school on weekends and evenings. Also, once the Air Force has invested in pilot training, it is reluctant to release members from flying billets to attend graduate school full time. In many non-rated career fields, officers are expected to have a master's degree to be eligible for promotion.

Also, it is important to keep in mind that the results from covariates in this analysis are not broadly applicable. As discussed when I introduced the RDD procedure, the validity of the coefficients on the covariates are only applicable to the those cadets close to the cutoff. The purpose of the discontinuity design is not to determine the effect of any particular covariate, but to measure the impact of the treatment on an outcome. Hence, this analysis finds no effect of DG on IDE.

Finally, the results of this analysis of the DG prize have implications about the theoretical parameters from Chapter 2. Specifically, the lack of impact on IDE selection may indicate that DG is not distinguishing between those who have high valuation and low valuation. Or, it may be that those who have high valuation for the DG prize are not the individuals who become the best officers. Because I do not have a measure of an individuals valuation for the prize or the marginal cost of effort, I cannot use the theoretical model to predict which type of cadet is more likely graduate with DG distinction.

3.5 Conclusion

The absence of a DG effect suggests that the promotion boards place more emphasis on the officer's career performance rather than education signals. Hence, as a prize, DG status may lack extrinsic appeal. If the tournament structure at the Academy fails to offer an appealing prize, the effort and selection outcomes will be inefficient. Yet, it could also be that cadets place a strong intrinsic value on the prize of DG. If this is the case, competition would be strong in the group that does place a high value on the prize. Whereas those who do not value the prize would not expend costly effort. If the Academy makes it a priority to award those who strive for *excellence* in all they do regardless of extrinsic value, then this structure may identify and reward those individuals. However, the Air Force is no likelier to select these individuals for In-residence IDE.

Chapter 4

Analysis of Strategic Behavior by US Air Force Academy Cadets

In the final chapter of my dissertation, I examine the strategic behavior of Academy cadets in the multi-stage tournament for distinguished graduate. To isolate the main prize most effectively, I focus on cadet behavior in the final semester at the Academy. At this point, cadets have already made their job selection. Hence, the only prize available is the designation of distinguished graduate. As the analysis in Chapter 3 shows, the DG designation has no discernible impact on later career outcomes. Additionally many cadets enter the Academy due to the abundance of pilot slots as compared to the Reserve Officer Training Corps (ROTC) program. The assignment of those slots is based on the overall class ranking. However, it is typical that there are over 500 pilot slots. Therefore, most cadets who are medically qualified to fly stand a good chance of obtaining a slot. In 1992, there was a shortage of pilot slots due to the draw down after the Cold War. This led to the first class who had more cadets who were medically qualified to fly than there were slots available. However, it is also important to note that cadets have little foresight into the number of slots that will be allocated to the Academy. For example, in 1988 the Congressional Budget Office[1] predicted a shortage of 2,000 pilots by 1992.

The second chapter of this thesis shows that there is a threshold that distinguishes the level of participation in the tournament for DG. Since there are other motivations at the Academy, one

[1]http://www.cbo.gov/ftpdocs/49xx/doc4953/doc09.pdf

must be careful to disentangle the various incentives that are in play. However, in the last semester, cadets have already received their assignments. Hence, the DG prize is left as the foremost motivating factor. In addition, the actions of cadets have limited impact on the rank-ordering.

Because of this perceived futility, cadets may become cynical, morose, or, perhaps worst of all, indifferent. This is despite receiving what many consider to be a first-class education. Recently, for the third year running, the Academy was named the best baccalaureate college in the west by the U.S. News & World Report's "America's Best Colleges" rankings. Forbes magazine consistently ranks USAFA as a best value in education. And, the Princeton Review rates the Academy highly for its easily accessible professors. Furthermore, the entire cost of tuition, supplies, meals, housing, and health care is provided for in exchange for five years of service in the Air Force. Such accolades and value suggest that cadets would be pleased with their situation. Yet, according to a GAO study[2], twenty-four percent of USAFA cadets were overall dissatisfied with the Academy. And, while ninety-two percent view the overall academic program as good or excellent, sixty-nine percent perceive that the workload is too high. I am inclined to believe that cadets are frustrated with some of the incentive structures in place at the Academy. Specifically, those who fall behind in this extended competition are left to "go through the motions" knowing that they are nor longer competitive.

This chapter provides a brief examination of the behavior of cadets. First, I show that cadets respond to a lack of incentive by reducing their effort. Next, I provide a brief look at officer promotion data. The third section examines how cadets alter their effort from one semester to the next. Finally, I look for a threshold that differentiates cadets who exert effort and those who relax. The purpose of this chapter is to provide a descriptive analysis of these topics and preview future avenues for research.

4.1 Cadet Motivation

The incentive structure at the Academy has measurable effects on cadet behavior. Consider, for example, the class of 2007. This class year was informed of their duty assignments in the second

[2] http://www.gao.gov/new.items/d031001.pdf

semester of their junior year. This is a full year earlier than previous classes. While the reasons for this revelation are unclear, the ramifications are easily seen in the data. Once the cadets were aware of their job assignment, the incentive to compete in the rank-order tournament was dramatically reduced. Hence, the two types of cadets revealed themselves earlier and more strongly. There were those who were still in competition for DG and graduate school opportunities. And, those who no longer felt that it was necessary to compete.

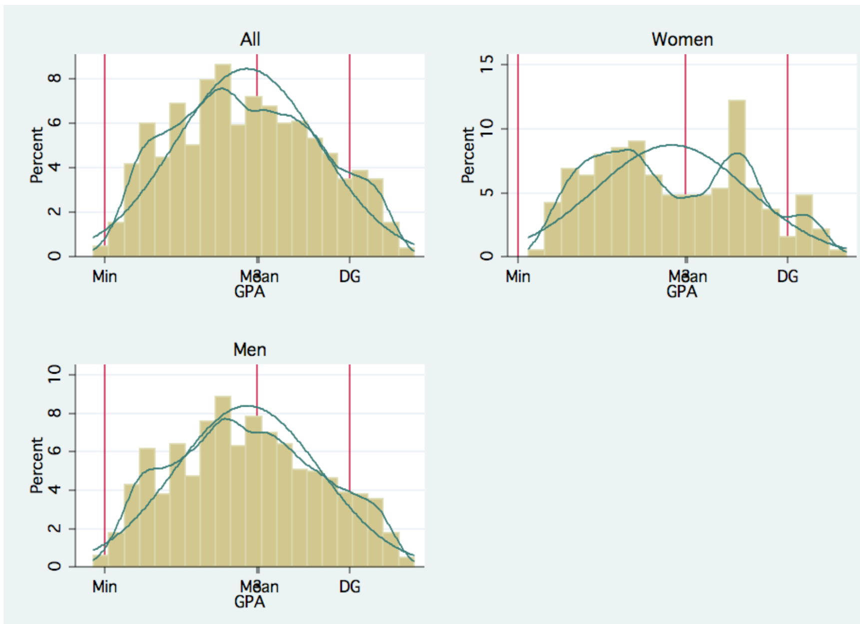

Figure 4.1: Histogram of Class of 2007 Final Semester GPA

If the cadets are indeed behaving strategically, theory predicts that those cadets who have little probability of success should exert little effort. Because I have observations of each cadet's performance by semester, I am able to examine the behavior of individual cadets as information is revealed over the four-year Academy tournament. This will allow me to observe if cadets who no longer believe they are able to succeed are indeed reducing their effort.

Figure 4.1 shows a histogram of final semester GPA from the class of 2007. This is the GPA from the final semester only and not the cumulative GPA. In the upper left-hand chart, all cadets are shown. The other charts display the cadets according to gender. In each chart, there are three vertical red lines. The line on the left is at 2.0. This is the minimum GPA required of cadets. If the cumulative GPA is lower than 2.0, cadets may not graduate. The center line is at the mean GPA. And the line on the right represents the cutoff for DG distinction. In addition to the bars of the histogram, each graph shows two fitted distributions. One is the normal distribution, and the other is the kernel density.

The charts show that bifurcation occurs. This is seen most clearly from the graph of the women. Here we see a mass that is well below the mean and two masses above the mean. While there is a distinct mass above the cutoff for DG, there is also a mass below the DG cutoff. One possible reason for this is that some cadets may be trying to maintain a GPA high enough for consideration to attend graduate school. In the men, we also see some bifurcation, but the results are not as pronounced. There are significantly more men than women in the sample, and this could be causing more smoothing. Nevertheless, there is strong indication that cadets behave as theory predicts.

4.2 Officer Promotions

In the Chapter 3, the data show that the DG signal has little effect on an officer's selection to IDE In-Residence. Given the weight that In-Residence status has on future promotion opportunities, this result suggests that the DG designation has little bearing on an officer's career path. Furthermore, this lack of impact suggests that other factors such as job assignment and performance have a larger effect on promotion boards. If the extrinsic value of the DG signal is low and uncorrelated with officer performance, then USAFA may be relying too heavily on a tournament prize that is not highly valued by those who are most likely to become successful officers as measured by IDE selection. The weakness of this prize may be cause for concern with regard to the efficiency of the tournament incentive structure.

Figure 4.2 shows that up until Captain, DG designation plays little role. However, of those who become Field Grade Officers,[3] a higher percentage of DGs attained the rank of Colonel than do those who do not graduate as DGs. This fact seems to contradict the previous notion that DG plays little role in distant career outcomes. Additional measures are required to determine if DG impacts advancement to Colonel.

[3]Field Grade Officers include those holding the rank of Major, Lieutenant Colonel, or Colonel.

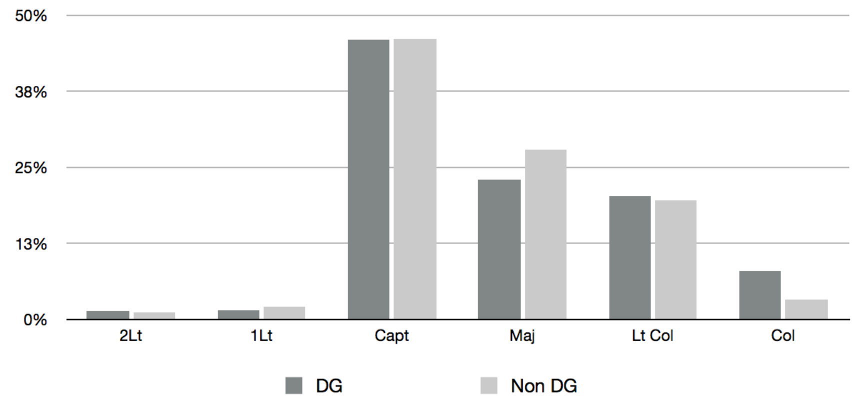

Figure 4.2: Highest Rank Attained as Percentage of DG Status

4.3 Semester Transition Matrices

As cadets progress through the four year program, they have the opportunity after each semester to assess their performances relative to their peers. Based on their class standings, they may choose to alter their effort levels. Figure 4.3 shows the movement between deciles. This is an aggregation across all semesters for the graduates from the classes of 1982-2004. The y-axis indicates the decile in which the cadet resided in period t. As an example, the bars above the number four represent the cadets who are in the fortieth percentile in period t. The bars clustered over each decile depict the percentage of cadets who entered the decile that corresponds to the bar in period $t+1$. This figure shows that cadets who start in the lowest decile tend to remain in a lower decile in the next period. Similarly, cadets who exit period t in the highest decile have a tendency to remain in the highest decile in $t+1$.

Table 4.1 provides the information in Figure 4.3 as a transition matrix for all transitions. In other words, based on aggregating the movements across all 8 semesters, 35.89% of cadets in the bottom 10% will appear in the bottom 10% in the next period. This quantity is represented by the leftmost bar in Figure 4.3 and the upper lefthand cell in Table 4.1.

The empirical implications of the theoretical model from Chapter 2 are difficult to predict without information about how each cadet values the prizes in the contest. However, one implication that is observable is that players who amass a strong advantage will maintain that advantage as those who lag behind recognize that they have a lower probability of winning and reduce their effort

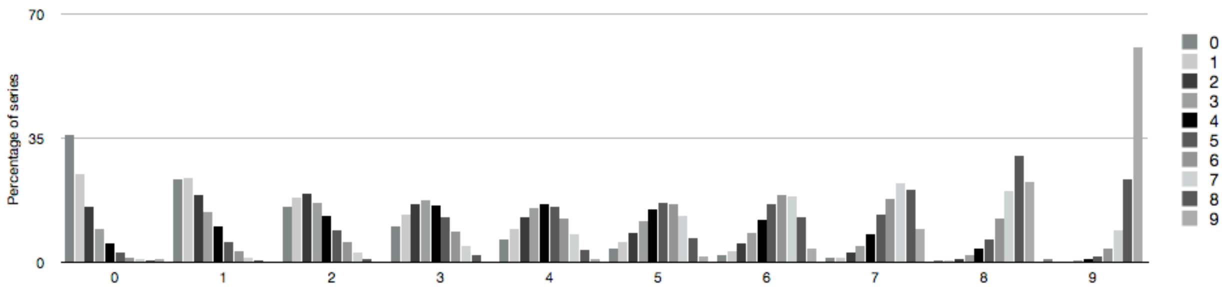

Figure 4.3: Movement between deciles

Table 4.1: Observed percentage of movement between deciles

	Decile in period t+1									
Decile in period t	0	1	2	3	4	5	6	7	8	9
0	35.89	23.45	15.60	10.19	6.35	3.82	1.97	1.15	0.68	0.90
1	24.91	23.57	18.12	13.52	9.27	5.68	3.17	1.23	0.42	0.10
2	15.57	18.95	19.15	16.47	12.65	8.17	5.27	2.68	0.91	0.18
3	9.43	14.08	16.70	17.34	15.34	11.63	8.30	4.73	1.99	0.45
4	5.14	9.94	13.01	16.03	16.40	14.99	12.10	7.95	3.66	0.79
5	2.74	5.84	8.84	12.58	15.68	16.78	16.22	13.21	6.36	1.74
6	1.38	2.96	5.53	8.43	12.44	16.25	19.06	17.81	12.31	3.83
7	0.83	1.24	2.69	4.44	7.92	12.93	18.73	22.41	19.92	8.88
8	0.63	0.35	0.69	1.80	3.29	6.60	12.49	20.57	30.11	23.48
9	0.79	0.12	0.14	0.31	0.69	1.59	3.93	9.17	22.56	60.69

level.

Using an ordered logit specification, I can estimate the transition probabilities rather than rely solely on observation. This analysis is performed on a semester by semester basis. Table 4.2 shows the observed transitions between between semesters 1 and 2. The estimated percentage of cadets to transition into the semester 2 deciles based on semester 1 decile is shown in Table 4.3.

Table 4.2: Observed percentage of movement between deciles from semester 1 to semester 2

		Decile in period t+1									
		0	1	2	3	4	5	6	7	8	9
Decile in period t	0	38.55	23.39	15.20	10.24	6.49	3.61	1.7	0.61	0.13	0.09
	1	25.51	23.34	18.90	13.57	9.09	5.12	2.94	1.15	0.38	0.00
	2	15.95	18.31	18.31	17.34	13.04	7.85	5.57	2.45	1.05	0.13
	3	9.88	14.45	16.11	18.63	15.28	10.84	8.18	4.44	1.92	0.26
	4	5.05	9.47	12.72	15.83	16.42	15.41	11.58	8.04	4.63	0.84
	5	2.51	5.91	8.99	13.98	15.21	16.27	16.36	12.52	6.57	1.68
	6	1.14	3.02	6.21	8.52	12.80	16.06	17.91	17.05	13.20	4.08
	7	0.59	1.22	3.08	4.52	7.47	14.10	19.85	21.03	18.71	9.42
	8	0.29	0.37	0.71	1.37	3.82	6.86	12.92	20.73	29.04	23.89
	9	0.08	0.08	0.12	0.44	0.80	2.03	4.38	9.92	23.86	58.30

Table 4.3: Estimated percentage of movement between deciles from semester 1 to semester 2

		Decile in period t+1									
		0	1	2	3	4	5	6	7	8	9
Decile in period t	0	42.87	25.93	14.57	8.14	4.17	2.16	1.15	0.57	0.29	0.13
	1	27.22	25.14	19.07	12.90	7.40	4.07	2.25	1.13	0.58	0.27
	2	15.70	19.68	20.08	17.36	11.83	7.23	4.24	2.20	1.14	0.53
	3	8.49	12.94	16.86	18.88	16.15	11.62	7.57	4.18	2.24	1.06
	4	4.42	7.54	11.65	16.33	17.85	15.96	12.27	7.55	4.31	2.11
	5	2.25	4.09	7.00	11.54	15.67	17.78	17.08	12.47	7.96	4.15
	6	1.13	2.13	3.86	7.04	11.20	15.73	19.35	17.88	13.67	8.00
	7	0.57	1.08	2.03	3.92	6.88	11.31	17.43	21.07	20.85	14.86
	8	0.28	0.55	1.04	2.07	3.84	6.98	12.74	19.79	26.77	25.94
	9	0.14	0.27	0.52	1.06	2.03	3.91	7.95	15.00	27.83	41.28

Table 4.4 looks at the transition between deciles between semester 7 and 8. From this table, we can see that of the 2,618 cadets who exit the seventh semester in the top decile, 56.5% remain at the top after semester 8. This number is consistent with the 58.3% that remain in the top decile from semester 1 to semester 2 as shown in Table 4.2. And, Table 4.5 provides a "zoomed-in" look at the

top percentiles as cadets transition from semester 7 to semester 8. In this table, we see that 19.3% of cadets remain in the 99th percentile between the seventh and eighth semesters. Each of these tables indicates that cadets tend to remain in the same decile that they were in before. In other words, you don't see a surge of effort at the end of the competition where those at the bottom pick up the pace and those at the top react to maintain their lead. Rather, most cadets tend to hold their current position.

Table 4.4: Transition between percentiles – Semester 7 to 8

		Decile in semester 8									
		0	1	2	3	4	5	6	7	8	9
Decile in semester 7	0	876	518	339	217	130	80	41	16	6	3
	1	591	547	471	346	232	141	108	34	16	6
	2	377	422	416	386	301	192	157	71	28	12
	3	243	329	416	418	336	291	218	98	66	18
	4	137	245	297	352	401	342	321	201	113	30
	5	62	147	235	302	339	370	351	269	163	53
	6	38	93	157	190	319	369	492	438	322	106
	7	11	41	80	118	230	308	470	519	505	236
	8	4	16	29	66	104	174	309	492	657	579
	9	2	8	8	14	29	56	161	275	587	1478

4.4 Switching Regression

Based on the theory set forth in Chapter 1, there should be some point where cadets go from exerting effort to coasting. The purpose of this analysis is to examine the data from the last semester at the Academy to determine if such a threshold exists. To do this, I propose a simple switching regression.

$$GPA_{i8} = \beta_0 + \beta_1 GPA_{i7} + \beta_2' \mathbf{S}_i + \beta_3' h(\mathbf{S}_i) + \varepsilon_i \tag{4.1}$$

where GPA_{ij} is the GPA of a cadet, i, in semester, j. $\mathbf{S}_i$ is a vector of covariates that affect GPA. There are issues with endogeneity in that most variables that affect a cadet's GPA in semester 8 will also affect a cadets GPA in semester 7. However, putting this issue aside, we can test for a threshold where cadets above a certain GPA behave differently than those below. $h(\mathbf{S}_i)$ is a function of the covariates interacted with the GPA threshold. To determine the optimal threshold, we run the

Table 4.5: Transition between percentiles – Semester 7 to 8

		Percentile in semester 8																				
		0	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
Percentile in semester 7	0	17602	166	120	157	108	142	123	113	108	97	85	69	83	58	53	62	44	36	23	21	15
	80	153	3	4	10	7	3	2	1	9	9	12	3	4	4	7	4	6	3	1	2	3
	81	149	6	4	4	4	5	4	3	4	10	8	3	7	6	8	5	2	4	7	3	1
	82	115	6	4	9	4	8	5	10	6	8	3	8	8	5	4	8	1	5	1	2	3
	83	104	2	5	2	2	2	2	3	4	5	5	10	4	2	5	6	6	3	3	4	1
	84	155	12	9	11	6	13	5	7	3	14	9	7	9	14	11	10	4	7	6	5	4
	85	102	4	7	8	5	4	5	5	5	7	9	5	10	5	4	5	4	4	5	1	4
	86	133	12	8	8	5	10	7	15	2	11	12	8	5	13	10	10	9	11	3	4	4
	87	92	7	2	6	8	5	3	11	7	7	5	9	7	4	14	5	10	8	3	2	8
	88	74	2	7	7	4	6	1	9	2	12	6	9	3	11	3	11	8	2	8	1	1
	89	117	6	6	10	4	8	11	6	9	17	18	10	8	11	7	12	8	7	4	2	10
	90	87	6	2	6	11	10	3	14	5	12	4	9	12	3	10	13	14	7	4	8	10
	91	85	7	4	7	3	9	4	5	6	5	12	7	13	9	8	17	4	7	16	9	13
	92	68	3	6	5	7	4	6	13	9	9	10	11	19	6	12	10	15	14	14	14	7
	93	63	6	6	8	5	3	4	6	9	4	8	10	11	9	15	11	14	13	15	12	14
	94	51	2	5	7	4	5	8	7	4	9	8	10	13	7	9	12	14	20	14	15	12
	95	45	2	4	4	5	12	8	10	6	7	6	9	6	17	10	12	11	18	12	16	14
	96	35	4	5	6	6	6	7	4	6	5	10	8	11	15	15	11	12	25	22	25	24
	97	28	5	3	3	3	5	6	8	5	8	7	5	11	8	7	15	15	21	25	21	25
	98	13	2	2	2	2	1	3	5	0	7	4	1	6	9	7	10	5	13	24	15	30
	99	78	0	3	6	4	5	6	7	8	10	9	17	18	15	19	21	23	31	51	59	93

model repeated while incrementing the threshold. After each calculation, we note the fit of the model by its adjusted R-squared. The model with the highest adjusted R-squared value results from the optimal threshold.

Figure 4.4 shows the adjusted R-squared over thresholds ranging from 2.0 to 4.0. The best fit occurs when the threshold is 3.09. This is a very reasonable result given that the cutoff for DG is only slightly higher.

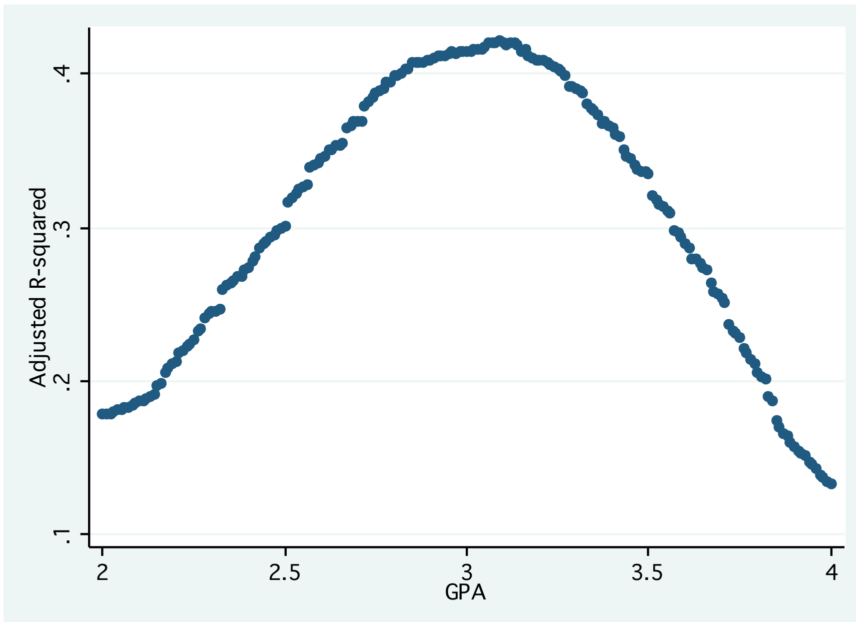

Figure 4.4: Adjusted R-squared vs GPA threshold

Again, setting aside issues with endogeneity for now, I employ a basic switching model using the 3.09 GPA threshold established above:

$$GPA_{i8} = \beta_0 + \beta_1 T + \beta_2 GPA_{i7} + \beta_3 (T \cdot GPA_{i7}) + \beta_4' \mathbf{S}_i + \varepsilon_i \tag{4.2}$$

The results of this model are presented in Table 4.6. Though this is a rough approximation, this result suggests that cadets who have a GPA above 3.09 behave differently from those below. Specifically, at the threshold, there is a 0.181 increase in predicted semester 8 GPA. And the marginal impact of an increase in seventh semester on predicted eighth semester GPA rises by 0.569.

Based on the theoretical model developed in the first chapter, I expect cadets above a certain

Table 4.6: Estimated Switching Model with Threshold at $GPA_7 = 3.09$

VARIABLES	(1) GPA_8
T	-1.577***
	(0.0723)
$T \cdot GPA_7$	0.569***
	(0.0241)
GPA_7	0.210***
	(0.0208)
female	0.0675***
	(0.0115)
minority (non-white)	-0.0823***
	(0.00781)
intercollegiate athlete	0.0131*
	(0.00755)
recruited athlete	-0.0358***
	(0.00741)
attended prep school	-0.0697***
	(0.00717)
Age at Graduation	-0.00815*
	(0.00454)
parent attended an academy	0.0166
	(0.00979)
sibling attended an academy	0.0159*
	(0.00786)
Constant	2.341***
	(0.0985)
Observations	21088
R^2	0.498

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

threshold to complete more rigorously than those below. These results seem to indicate that this is the case.

4.5 Comments

This chapter provides a brief examination of some of the theoretical predictions of the model of a continuation contest. Cadets indeed have a tendency to behave differently depending upon which side of the threshold they fall. If this behavior is not optimal from the Air Force's viewpoint, then the contest should be adjusted to change cadet behavior. Since the release of information is what allows cadets to adjust their exertion over time, it may be better to mask the cadets' relative performances. If this is not plausible, then there are other options available to adjust behavior. The Academy could restructure the nature of the tournament by allowing for elimination. Or, the Academy could provide intermediate prizes after each semester. Given the caliber of cadets and the level of effort necessary to rank highly, these prizes would have to be substantial.

Although this dissertation examines cadet behavior from an economic standpoint, it could be that the Academy has other designs. One of the core values of the Academy is *Excellence in all we do*. Perhaps USAFA is interested in cadets who behave in a certain way despite the strategically "correct" thing to do. In other words, they are testing cadets to determine if they will strive to excel regardless of the prize at stake. While this may be a nobel ambition, the incentive structure leads to a situation where cadets who are no longer in the running for Distinguished Graduate may not feel compelled to perform to their potential. And, it is difficult to properly test for striving without regard to the extrinsic value of the prize when there are no repercussions for failure – especially when introducing such consequences invalidates the purpose of the test.

Appendix A

Data and Summary Statistics

The following variables are used in Chapter 2. These data were provided under IRB supervision by USAFA Plans and Programs (USAFA/XP).

Table A.1: Summary Statistics – Variables Used in Chapter 2

Variable	Mean	Std. Dev.	Min.	Max.
class year	1988.721	3.92	1982	1995
Distinguished Graduate	0.07	0.256	0	1
IDE attendance	0.209	0.406	0	1
overall performance average	2.873	0.368	2	4.24
academic composite score	3179.645	270.891	2259	4120
female	0.087	0.282	0	1
minority (non-white)	0.135	0.342	0	1
ever married – binary	0.940	0.238	0	1
maximum number of dependents	2.775	1.523	0	19
Age at Graduation	22.761	0.764	21.225	27.039
intercollegiate athlete	0.256	0.437	0	1
recruited athlete	0.183	0.386	0	1
advanced academic degree	0.742	0.438	0	1
Rated Officer	0.669	0.47	0	1
parent attended an academy	0.027	0.164	0	1
sibling attended an academy	0.093	0.291	0	1
N		8476		

	IDE attendance					
	N		Y		Total	
	No.	%	No.	%	No.	%
class year						
1982	439	6.5%	33	1.9%	472	5.6%
1983	532	7.9%	46	2.6%	578	6.8%
1984	484	7.2%	63	3.6%	547	6.5%
1985	472	7.0%	67	3.8%	539	6.4%
1986	524	7.8%	97	5.5%	621	7.3%
1987	532	7.9%	123	7.0%	655	7.7%
1988	478	7.1%	136	7.7%	614	7.2%
1989	481	7.2%	139	7.9%	620	7.3%
1990	512	7.6%	172	9.7%	684	8.1%
1991	457	6.8%	171	9.7%	628	7.4%
1992	557	8.3%	198	11.2%	755	8.9%
1993	377	5.6%	160	9.0%	537	6.3%
1994	404	6.0%	222	12.6%	626	7.4%
1995	459	6.8%	141	8.0%	600	7.1%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%
Distinguished Graduate						
N	6,327	94.3%	1,552	87.8%	7,879	93.0%
Y	381	5.7%	216	12.2%	597	7.0%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%

Source: USAFA/XP

	IDE attendance					
	N		Y		Total	
	No.	%	No.	%	No.	%
female						
M	6,114	91.1%	1,624	91.9%	7,738	91.3%
F	594	8.9%	144	8.1%	738	8.7%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%
minority (non-white)						
N	5,767	86.0%	1,561	88.3%	7,328	86.5%
Y	941	14.0%	207	11.7%	1,148	13.5%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%
AFSC (single digit level)						
1	4,748	70.8%	1,238	70.0%	5,986	70.6%
2	261	3.9%	93	5.3%	354	4.2%
3	533	7.9%	168	9.5%	701	8.3%
6	745	11.1%	208	11.8%	953	11.2%
7	37	0.6%	10	0.6%	47	0.6%
8	121	1.8%	13	0.7%	134	1.6%
9	263	3.9%	38	2.1%	301	3.6%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%
attended prep school						
0	5,531	82.5%	1,498	84.7%	7,029	82.9%
1	1,177	17.5%	270	15.3%	1,447	17.1%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%
intercollegiate athlete						
N	5,006	74.6%	1,299	73.5%	6,305	74.4%
Y	1,702	25.4%	469	26.5%	2,171	25.6%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%

Source: USAFA/XP

	IDE attendance					
	N		Y		Total	
	No.	%	No.	%	No.	%
Rated Officer						
N	2,123	31.6%	680	38.5%	2,803	33.1%
Y	4,585	68.4%	1,088	61.5%	5,673	66.9%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%
parent attended an academy						
N	6,535	97.4%	1,708	96.6%	8,243	97.3%
Y	173	2.6%	60	3.4%	233	2.7%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%
sibling attended an academy						
N	6,101	91.0%	1,583	89.5%	7,684	90.7%
Y	607	9.0%	185	10.5%	792	9.3%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%
recruited athlete						
N	5,529	82.4%	1,399	79.1%	6,928	81.7%
Y	1,179	17.6%	369	20.9%	1,548	18.3%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%

Source: USAFA/XP

	Distinguished Graduate					
	N		Y		Total	
	No.	%	No.	%	No.	%
female						
M	7,170	91.0%	568	95.1%	7,738	91.3%
F	709	9.0%	29	4.9%	738	8.7%
Total	7,879	100.0%	597	100.0%	8,476	100.0%
minority (non-white)						
N	6,764	85.8%	564	94.5%	7,328	86.5%
Y	1,115	14.2%	33	5.5%	1,148	13.5%
Total	7,879	100.0%	597	100.0%	8,476	100.0%
recruited athlete						
N	6,381	81.0%	547	91.6%	6,928	81.7%
Y	1,498	19.0%	50	8.4%	1,548	18.3%
Total	7,879	100.0%	597	100.0%	8,476	100.0%
Commissioned						
Y	7,879	100.0%	597	100.0%	8,476	100.0%
Total	7,879	100.0%	597	100.0%	8,476	100.0%
attended prep school						
0	6,449	81.9%	580	97.2%	7,029	82.9%
1	1,430	18.1%	17	2.8%	1,447	17.1%
Total	7,879	100.0%	597	100.0%	8,476	100.0%
intercollegiate athlete						
N	5,806	73.7%	499	83.6%	6,305	74.4%
Y	2,073	26.3%	98	16.4%	2,171	25.6%
Total	7,879	100.0%	597	100.0%	8,476	100.0%

Source: USAFA/XP

	Distinguished Graduate					
	N		Y		Total	
	No.	%	No.	%	No.	%
parent attended an academy						
N	7,658	97.2%	585	98.0%	8,243	97.3%
Y	221	2.8%	12	2.0%	233	2.7%
Total	7,879	100.0%	597	100.0%	8,476	100.0%
sibling attended an academy						
N	7,142	90.6%	542	90.8%	7,684	90.7%
Y	737	9.4%	55	9.2%	792	9.3%
Total	7,879	100.0%	597	100.0%	8,476	100.0%

Source: USAFA/XP

Academic Major	IDE attendance					
	N		Y		Total	
	No.	%	No.	%	No.	%
Aeronautical Engineering	441	6.6%	133	7.5%	574	6.8%
Astronautical Engineering	307	4.6%	95	5.4%	402	4.7%
Aviation Science	81	1.2%	15	0.8%	96	1.1%
Basic Sciences	933	13.9%	152	8.6%	1,085	12.8%
Behavioral Sciences	587	8.8%	143	8.1%	730	8.6%
Biology	211	3.1%	74	4.2%	285	3.4%
Chemistry	51	0.8%	19	1.1%	70	0.8%
Civil Engineering	356	5.3%	87	4.9%	443	5.2%
Computer Science	201	3.0%	46	2.6%	247	2.9%
Economics	151	2.3%	36	2.0%	187	2.2%
Electrical Engineering	308	4.6%	68	3.8%	376	4.4%
Engineering Sciences	321	4.8%	84	4.8%	405	4.8%
English	45	0.7%	27	1.5%	72	0.8%
General Engineering	182	2.7%	53	3.0%	235	2.8%
Geography	311	4.6%	74	4.2%	385	4.5%
Meteorology	1	0.0%	1	0.1%	2	0.0%
History	485	7.2%	138	7.8%	623	7.4%
Humanities	94	1.4%	29	1.6%	123	1.5%
International Affairs	318	4.7%	69	3.9%	387	4.6%
Legal Studies	45	0.7%	22	1.2%	67	0.8%
Management	512	7.6%	137	7.7%	649	7.7%
Mathematics	101	1.5%	42	2.4%	143	1.7%
Operations Research	204	3.0%	42	2.4%	246	2.9%
Physics	139	2.1%	49	2.8%	188	2.2%
Political Science	250	3.7%	116	6.6%	366	4.3%
Social Sciences	73	1.1%	17	1.0%	90	1.1%
Total	6,708	100.0%	1,768	100.0%	8,476	100.0%

Source: USAFA/XP

Academic Major	Distinguished Graduate					
	N		Y		Total	
	No.	%	No.	%	No.	%
Aeronautical Engineering	485	6.2%	89	14.9%	574	6.8%
Astronautical Engineering	331	4.2%	71	11.9%	402	4.7%
Aviation Science	92	1.2%	4	0.7%	96	1.1%
Basic Sciences	1,083	13.7%	2	0.3%	1,085	12.8%
Behavioral Sciences	697	8.8%	33	5.5%	730	8.6%
Biology	272	3.5%	13	2.2%	285	3.4%
Chemistry	59	0.7%	11	1.8%	70	0.8%
Civil Engineering	423	5.4%	20	3.4%	443	5.2%
Computer Science	227	2.9%	20	3.4%	247	2.9%
Economics	172	2.2%	15	2.5%	187	2.2%
Electrical Engineering	328	4.2%	48	8.0%	376	4.4%
Engineering Sciences	369	4.7%	36	6.0%	405	4.8%
English	70	0.9%	2	0.3%	72	0.8%
General Engineering	231	2.9%	4	0.7%	235	2.8%
Geography	352	4.5%	33	5.5%	385	4.5%
Meteorology	2	0.0%	0	0.0%	2	0.0%
History	589	7.5%	34	5.7%	623	7.4%
Humanities	119	1.5%	4	0.7%	123	1.5%
International Affairs	353	4.5%	34	5.7%	387	4.6%
Legal Studies	67	0.9%	0	0.0%	67	0.8%
Management	638	8.1%	11	1.8%	649	7.7%
Mathematics	122	1.5%	21	3.5%	143	1.7%
Operations Research	227	2.9%	19	3.2%	246	2.9%
Physics	148	1.9%	40	6.7%	188	2.2%
Political Science	333	4.2%	33	5.5%	366	4.3%
Social Sciences	90	1.1%	0	0.0%	90	1.1%
Total	7,879	100.0%	597	100.0%	8,476	100.0%

Source: USAFA/XP

Academic Major	female					
	M		F		Total	
	No.	%	No.	%	No.	%
Aeronautical Engineering	557	7.2%	17	2.3%	574	6.8%
Astronautical Engineering	378	4.9%	24	3.3%	402	4.7%
Aviation Science	83	1.1%	13	1.8%	96	1.1%
Basic Sciences	1,023	13.2%	62	8.4%	1,085	12.8%
Behavioral Sciences	630	8.1%	100	13.6%	730	8.6%
Biology	227	2.9%	58	7.9%	285	3.4%
Chemistry	60	0.8%	10	1.4%	70	0.8%
Civil Engineering	429	5.5%	14	1.9%	443	5.2%
Computer Science	233	3.0%	14	1.9%	247	2.9%
Economics	173	2.2%	14	1.9%	187	2.2%
Electrical Engineering	365	4.7%	11	1.5%	376	4.4%
Engineering Sciences	380	4.9%	25	3.4%	405	4.8%
English	49	0.6%	23	3.1%	72	0.8%
General Engineering	226	2.9%	9	1.2%	235	2.8%
Geography	347	4.5%	38	5.1%	385	4.5%
Meteorology	2	0.0%	0	0.0%	2	0.0%
History	571	7.4%	52	7.0%	623	7.4%
Humanities	89	1.2%	34	4.6%	123	1.5%
International Affairs	335	4.3%	52	7.0%	387	4.6%
Legal Studies	58	0.7%	9	1.2%	67	0.8%
Management	587	7.6%	62	8.4%	649	7.7%
Mathematics	126	1.6%	17	2.3%	143	1.7%
Operations Research	225	2.9%	21	2.8%	246	2.9%
Physics	181	2.3%	7	0.9%	188	2.2%
Political Science	324	4.2%	42	5.7%	366	4.3%
Social Sciences	80	1.0%	10	1.4%	90	1.1%
Total	7,738	100.0%	738	100.0%	8,476	100.0%

Source: USAFA/XP

Table A.2: Variables used in Analysis

Variable	Definition
personal id	Unique identifier provided by USAFA/XP
gender	Dummy variable = 1 if individual is female
race	Dummy variable = 1 if individual is not caucasian
IDE	Dummy variable = 1 if individual attended Intermediate Developmental Education in-residence
Distinguished Graduate	Dummy variable = 1 if individual is a distinguished graduate
Officer Performance Average	Composite of Academy grade point average, military performance average, and physical education average
Military Performance Average	Assessment of the cadets performance on military roles and responsibilities
Academic Composite	Aggregate measure of individuals academic performance as an applicant; based on high school GPA and standardized test scores
Recruit Status	Dummy variable = 1 if individual is a recruited athlete
Intercollegiate Athlete	Dummy variable = 1 if individual participated in intercollegiate sports
Air Force Specialty Code	Air Force job classification (1 = Operations, 2 = Logistics, 3 = Support, 4 = Medical, 5 = Law/Chaplain, 6 = Aquisitions/Financial Management, 7 = Special Investigations, 8 = Special Duty, 9 = Student/Trainee/Patient*
Zip Code	High school zip code aggregated to the one-digit level with 11 = overseas, 12 = APO/FPO Europe, 13 = APO/FPO Asia
Age	Age at graduation from Academy
Marital Status	Dummy variable = 1 if individual was ever married
Dependents	Number of household dependents
Advanced Academic Degree	Dummy variable = 1 if individual has an advanced academic degree
Rated	Dummy variable = 1 if individual is rated for aircrew duty (e.g. Pilot or navigator)

Appendix B

Thresholds for Distinguished Graduate Designation

Table B.1: OPA Thresholds for Distinguished Graduate Designation

Class year	Num of cadets	OPA cutoff
1982	815	3.50
1983	925	3.48
1984	994	3.49
1985	889	3.45
1986	935	3.47
1987	953	3.46
1988	1,036	3.48
1989	973	3.47
1990	940	3.44
1991	920	3.50
1992	1,030	3.47
1993	895	3.46
1994	965	3.40
1995	938	3.46
1996	868	3.41
1997	748	3.38
1998	900	3.40

Appendix C

Additional Specifications Regarding the Valuation of an Educational Signal

This appendix contains the following additional specifications in support of Chapter 2:

Table C.1 Marginal Effect on Selection to In-Residence IDE – by LPM without OPA

Table C.2 Marginal Effect on Selection to In-Residence IDE – by probit without OPA

Table C.3 Bivariate Probit – without OPA

Table C.4 Marginal Effect on Selection to In-Residence IDE – by LPM – Males only

Table C.5 Marginal Effect on Selection to In-Residence IDE – by probit – Males only

Table C.6 Bivariate Probit – Males only

Table C.7 Marginal Effect on Selection to In-Residence IDE – by LPM – Rated only

Table C.8 Marginal Effect on Selection to In-Residence IDE – by probit – Rated only

Table C.9 Bivariate Probit – Rated only

Table C.10 Covariates

Table C.1: Marginal Effect on Selection to In-Residence IDE – by LPM without OPA

VARIABLES	(1) All	(2) All	(3) All	(4) Rated	(5) Non-rated
Distinguished Graduate	0.163***	0.161***	0.129***	0.120***	0.147***
	(0.0214)	(0.0213)	(0.0210)	(0.0258)	(0.0388)
female	-0.0118	-0.0174	-0.0150	-0.0438*	-0.00686
	(0.0146)	(0.0152)	(0.0158)	(0.0243)	(0.0205)
minority (non-white)	-0.0331***	-0.0282**	-0.0218*	-0.0396***	0.00505
	(0.0123)	(0.0124)	(0.0116)	(0.0147)	(0.0196)
intercollegiate athlete		0.00340	0.0101	0.00976	0.00839
		(0.0131)	(0.0125)	(0.0138)	(0.0212)
recruited athlete		0.0320**	0.0264*	0.0120	0.0578**
		(0.0145)	(0.0140)	(0.0155)	(0.0235)
academic composite score	1.08e-06	1.02e-05	-1.32e-05	-2.26e-05	-1.74e-06
	(1.77e-05)	(1.81e-05)	(1.68e-05)	(2.00e-05)	(3.10e-05)
AFSC - logistics			0.00315	-0.0174	-0.0297
			(0.0230)	(0.0586)	(0.0278)
AFSC - support			0.000289	-0.0428	-0.0211
			(0.0177)	(0.0440)	(0.0243)
AFSC - acquisitions			-0.0320**	-0.0396	-0.0480**
			(0.0144)	(0.0447)	(0.0205)
AFSC - osi			-0.0302	0	-0.0454
			(0.0527)	(0)	(0.0550)
AFSC - special			-0.0842***	-0.0530*	-0.138***
			(0.0248)	(0.0313)	(0.0408)
AFSC - student			-0.0229	-0.00611	-0.0823*
			(0.0199)	(0.0222)	(0.0433)
ever married – binary			0.0710***	0.0606***	0.0961***
			(0.0161)	(0.0204)	(0.0288)
advanced academic degree			0.265***	0.263***	0.256***
			(0.00787)	(0.00930)	(0.0134)
Age at Graduation		-0.0184***	-0.0159***	-0.0181***	-0.0162
		(0.00579)	(0.00542)	(0.00612)	(0.0100)
maximum number of dependents			0.00763**	0.0110***	-0.000124
			(0.00310)	(0.00384)	(0.00561)
Constant	0.104	0.498**	0.167	0.0318	0.183
	(0.202)	(0.228)	(0.228)	(0.148)	(0.360)
Observations	8476	8476	8476	5673	2803
R^2	0.059	0.061	0.146	0.168	0.133

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, AFSC - operator

Table C.2: Marginal Effect on Selection to In-Residence IDE – by probit without OPA

VARIABLES	(1) All	(2) All	(3) All	(4) Rated	(5) Non-rated
Distinguished Graduate	0.167***	0.164***	0.116***	0.0943***	0.155***
	(0.0229)	(0.0227)	(0.0209)	(0.0236)	(0.0426)
female	-0.0127	-0.0180	-0.00603	-0.0336*	-0.00302
	(0.0145)	(0.0149)	(0.0141)	(0.0191)	(0.0208)
minority (non-white)	-0.0338***	-0.0289**	-0.0211**	-0.0361***	0.00740
	(0.0120)	(0.0123)	(0.0105)	(0.0114)	(0.0208)
intercollegiate athlete		0.00368	0.00696	0.00580	0.00855
		(0.0133)	(0.0114)	(0.0115)	(0.0206)
recruited athlete		0.0315**	0.0246*	0.0102	0.0585**
		(0.0148)	(0.0130)	(0.0129)	(0.0245)
academic composite score	1.14e-07	8.02e-06	-1.14e-05	-2.08e-05	5.20e-06
	(1.79e-05)	(1.85e-05)	(1.56e-05)	(1.67e-05)	(3.15e-05)
AFSC - logistics			-0.000240	-0.0251	-0.0275
			(0.0173)	(0.0443)	(0.0241)
AFSC - support			0.00164	-0.0547	-0.0165
			(0.0149)	(0.0464)	(0.0231)
AFSC - acquisitions			-0.0213*	-0.0140	-0.0445**
			(0.0110)	(0.0303)	(0.0193)
AFSC - osi			-0.0213		-0.0441
			(0.0408)		(0.0500)
AFSC - special			-0.0809***	-0.0631**	-0.128***
			(0.0190)	(0.0260)	(0.0289)
AFSC - student			-0.0204	-0.00513	-0.0687
			(0.0222)	(0.0241)	(0.0423)
Age at Graduation		-0.0200***	-0.0137**	-0.0144***	-0.0175
		(0.00633)	(0.00536)	(0.00555)	(0.0108)
maximum number of dependents			0.00861***	0.0105***	0.00116
			(0.00297)	(0.00338)	(0.00558)
Observations	8476	8476	8476	5654	2800

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, AFSC - operator

Table C.3: Bivariate Probit – without OPA

VARIABLES	(1) IDE	(2) DG	(3) ρ
Distinguished Graduate	0.454		
	(0.370)		
female	-0.0191	-0.391***	
	(0.0691)	(0.0886)	
minority (non-white)	-0.0771	-0.125	
	(0.0628)	(0.147)	
intercollegiate athlete	0.0447	-0.0968*	
	(0.0452)	(0.0571)	
recruited athlete	0.103**	-0.0605	
	(0.0400)	(0.0725)	
academic composite score	-3.93e-05	0.00263***	
	(0.000138)	(0.000164)	
advanced academic degree	1.680***	0.417***	
	(0.169)	(0.0620)	
attended prep school	0.0142	-0.870***	
	(0.0501)	(0.156)	
AFSC - logistics	-0.0117		
	(0.110)		
AFSC - support	0.00955		
	(0.0669)		
AFSC - acquisitions	-0.101**		
	(0.0478)		
AFSC - osi	-0.0788		
	(0.220)		
AFSC - special	-0.498***		
	(0.122)		
AFSC - student	-0.0865		
	(0.196)		
Age at Graduation	-0.0635***	0.0705**	
	(0.0157)	(0.0339)	
maximum number of dependents	0.0388**		
	(0.0160)		
parent attended an academy		-0.121	
		(0.117)	
sibling attended an academy		0.0198	
		(0.0847)	
Constant	-1.959***	-16.58	-0.0114
	(0.652)	(.)	(0.197)
Observations	8476	8476	8476

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, afsc_operator

Table C.4: Marginal Effect on Selection to In-Residence IDE – by LPM – Males only

VARIABLES	(1) All Males	(2) All Males	(3) All Males	(4) Rated Males	(5) Non-rated Males
Distinguished Graduate	0.0732***	0.0718***	0.0577**	0.0469*	0.0827*
	(0.0243)	(0.0242)	(0.0241)	(0.0283)	(0.0464)
minority (non-white)	-0.0210	-0.0159	-0.0135	-0.0281*	0.0102
	(0.0136)	(0.0137)	(0.0127)	(0.0156)	(0.0229)
intercollegiate athlete		0.0108	0.0149	0.00738	0.0304
		(0.0138)	(0.0130)	(0.0142)	(0.0241)
recruited athlete		0.0370**	0.0288**	0.0182	0.0486*
		(0.0148)	(0.0142)	(0.0160)	(0.0257)
academic composite score	-8.96e-05***	-7.39e-05***	-8.32e-05***	-9.32e-05***	-7.18e-05*
	(2.04e-05)	(2.12e-05)	(1.99e-05)	(2.38e-05)	(4.05e-05)
AFSC - logistics			0.0135	-0.0203	-0.0284
			(0.0239)	(0.0648)	(0.0318)
AFSC - support			-0.00434	-0.0656*	-0.0313
			(0.0184)	(0.0397)	(0.0271)
AFSC - acquisitions			-0.0335**	-0.0434	-0.0577**
			(0.0153)	(0.0468)	(0.0226)
AFSC - osi			-0.0305	0	-0.0575
			(0.0575)	(0)	(0.0624)
AFSC - special			-0.0556*	-0.0407	-0.106**
			(0.0288)	(0.0353)	(0.0464)
AFSC - student			-0.0127	-0.0133	-0.0328
			(0.0219)	(0.0232)	(0.0597)
Age at Graduation		-0.0156***	-0.0134**	-0.0151**	-0.0139
		(0.00595)	(0.00560)	(0.00636)	(0.0114)
maximum number of dependents			0.00670**	0.00996**	-0.00106
			(0.00304)	(0.00388)	(0.00598)
Constant	0.612**	0.917***	0.505	0.256	0.633
	(0.295)	(0.309)	(0.319)	(0.162)	(0.517)
Observations	7738	7738	7738	5451	2287
R^2	0.070	0.072	0.158	0.177	0.141

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, AFSC - operator

Table C.5: Marginal Effect on Selection to In-Residence IDE – by probit – Males only

VARIABLES	(1) All Males	(2) All Males	(3) All Males	(4) Rated Males	(5) Non-rated Males
Distinguished Graduate	0.0504**	0.0491**	0.0254	0.00950	0.0706
	(0.0225)	(0.0223)	(0.0191)	(0.0192)	(0.0470)
minority (non-white)	-0.0218	-0.0166	-0.0114	-0.0283**	0.0190
	(0.0138)	(0.0142)	(0.0117)	(0.0123)	(0.0250)
intercollegiate athlete		0.0111	0.00995	0.00379	0.0246
		(0.0143)	(0.0117)	(0.0117)	(0.0236)
recruited athlete		0.0374**	0.0277**	0.0143	0.0551**
		(0.0152)	(0.0131)	(0.0133)	(0.0257)
academic composite score	-9.56e-05***	-8.15e-05***	-8.02e-05***	-8.60e-05***	-6.96e-05*
	(2.04e-05)	(2.13e-05)	(1.76e-05)	(1.95e-05)	(3.95e-05)
AFSC - logistics			0.0133	-0.0203	-0.0260
			(0.0192)	(0.0462)	(0.0280)
AFSC - support			-0.000397	-0.0862***	-0.0269
			(0.0152)	(0.0224)	(0.0253)
AFSC - acquisitions			-0.0227**	-0.0172	-0.0575***
			(0.0110)	(0.0285)	(0.0208)
AFSC - osi			-0.0199		-0.0575
			(0.0427)		(0.0522)
AFSC - special			-0.0586**	-0.0532*	-0.104***
			(0.0234)	(0.0293)	(0.0353)
AFSC - student			-0.00377	-0.00555	-0.0165
			(0.0242)	(0.0242)	(0.0586)
Age at Graduation		-0.0178***	-0.0123**	-0.0132**	-0.0152
		(0.00652)	(0.00539)	(0.00566)	(0.0124)
maximum number of dependents			0.00768***	0.00945***	0.000788
			(0.00284)	(0.00332)	(0.00593)
Observations	7737	7737	7737	5432	2283

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, AFSC - operator

Table C.6: Bivariate Probit – Males only

VARIABLES	(1) IDE	(2) DG	(3) ρ
Distinguished Graduate	0.273		
	(0.488)		
minority (non-white)	-0.0479	-0.111	
	(0.0742)	(0.149)	
intercollegiate athlete	0.0561	-0.118**	
	(0.0376)	(0.0506)	
recruited athlete	0.110***	-0.113	
	(0.0342)	(0.0720)	
academic composite score	-0.000411**	0.00259***	
	(0.000174)	(0.000162)	
advanced academic degree	1.746***	0.418***	
	(0.187)	(0.0568)	
attended prep school	0.121*	-0.912***	
	(0.0654)	(0.155)	
AFSC - logistics	0.0440		
	(0.114)		
AFSC - support	0.00357		
	(0.0680)		
AFSC - acquisitions	-0.113**		
	(0.0517)		
AFSC - osi	-0.0868		
	(0.274)		
AFSC - special	-0.348**		
	(0.138)		
AFSC - student	-0.000528		
	(0.205)		
Age at Graduation	-0.0874***	0.0848**	
	(0.0200)	(0.0332)	
maximum number of dependents	0.0372**		
	(0.0145)		
parent attended an academy		-0.0930	
		(0.121)	
sibling attended an academy		0.0589	
		(0.0802)	
Constant	-0.0445	-16.87	-0.0944
	(0.796)	(.)	(0.266)
Observations	7738	7738	7738

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, afsc_operator

Table C.7: Marginal Effect on Selection to In-Residence IDE – by LPM – Rated only

VARIABLES	(1) Rated	(2) Rated	(3) Rated
Distinguished Graduate	0.0633**	0.0624**	0.0464*
	(0.0282)	(0.0282)	(0.0280)
female	-0.0582**	-0.0655***	-0.0460*
	(0.0227)	(0.0230)	(0.0243)
minority (non-white)	-0.0351**	-0.0338**	-0.0264*
	(0.0162)	(0.0164)	(0.0147)
intercollegiate athlete		0.00782	0.0123
		(0.0146)	(0.0137)
recruited athlete		0.0168	0.0139
		(0.0164)	(0.0154)
academic composite score	-9.54e-05***	-8.92e-05***	-9.05e-05***
	(2.33e-05)	(2.40e-05)	(2.33e-05)
AFSC - logistics			-0.0162
			(0.0592)
AFSC - support			-0.0316
			(0.0437)
AFSC - acquisitions			-0.0451
			(0.0440)
AFSC - special			-0.0509
			(0.0311)
AFSC - student			-0.0143
			(0.0223)
Age at Graduation		-0.0172**	-0.0148**
		(0.00675)	(0.00611)
maximum number of dependents			0.00992**
			(0.00385)
Constant	0.340***	0.699***	0.256*
	(0.0787)	(0.166)	(0.154)
Observations	5673	5673	5673
R^2	0.078	0.080	0.175

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, AFSC - operator

Table C.8: Marginal Effect on Selection to In-Residence IDE – by probit – Rated only

VARIABLES	(1) Rated	(2) Rated	(3) Rated
Distinguished Graduate	0.0319	0.0311	0.00829
	(0.0243)	(0.0241)	(0.0186)
female	-0.0602***	-0.0667***	-0.0347*
	(0.0222)	(0.0217)	(0.0183)
minority (non-white)	-0.0382**	-0.0371**	-0.0275**
	(0.0167)	(0.0170)	(0.0116)
intercollegiate athlete		0.00924	0.00844
		(0.0152)	(0.0114)
recruited athlete		0.0167	0.0104
		(0.0167)	(0.0127)
academic composite score	-0.000102***	-9.76e-05***	-8.39e-05***
	(2.34e-05)	(2.40e-05)	(1.92e-05)
AFSC - logistics			-0.0221
			(0.0446)
AFSC - support			-0.0481
			(0.0478)
AFSC - acquisitions			-0.0184
			(0.0276)
AFSC - special			-0.0608**
			(0.0248)
AFSC - student			-0.00927
			(0.0227)
Age at Graduation		-0.0202***	-0.0117**
		(0.00742)	(0.00551)
maximum number of dependents			0.00938***
			(0.00331)
Observations	5654	5654	5654

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, AFSC - operator

Table C.9: Bivariate Probit – Rated only

VARIABLES	(1) IDE	(2) DG	(3) athrho
Distinguished Graduate	-0.0392		
	(0.653)		
female	-0.199	-0.437***	
	(0.139)	(0.167)	
minority (non-white)	-0.147	-0.0693	
	(0.0958)	(0.146)	
intercollegiate athlete	0.0564	-0.188**	
	(0.0619)	(0.0750)	
recruited athlete	0.0596	-0.0164	
	(0.0541)	(0.0820)	
academic composite score	-0.000384	0.00266***	
	(0.000302)	(0.000138)	
advanced academic degree	1.829***	0.490***	
	(0.211)	(0.0748)	
attended prep school	0.0446	-0.861***	
	(0.0761)	(0.200)	
AFSC - logistics	-0.0886		
	(0.282)		
AFSC - support	-0.333		
	(0.584)		
AFSC - acquisitions	-0.105		
	(0.151)		
AFSC - special	-0.534*		
	(0.287)		
AFSC - student	-0.0361		
	(0.194)		
Age at Graduation	-0.0740***	0.0766**	
	(0.0227)	(0.0349)	
maximum number of dependents	0.0517***		
	(0.0161)		
parent attended an academy		-0.109	
		(0.148)	
sibling attended an academy		0.0833	
		(0.0877)	
Constant	-0.304	-15.87	0.0464
	(1.152)	(0)	(0.380)
Observations	5673	5673	5673

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Excluded group: male, white, non-recruit, non-athlete, afsc_operator

Table C.10: Covariates

VARIABLES	(1) female	(2) nonwhite	(3) hs_gpa	(4) aca_cmp	(5) rated_condensed	(6) cft_score	(7) age	(8) mpa
Distinguished Graduate	0.0365	0.0285	0.371	-35.44	0.0271	-10.03	0.115	0.0939**
	(0.0408)	(0.0433)	(0.576)	(52.56)	(0.0747)	(22.46)	(0.306)	(0.0421)
OPA	-4.236**	-5.826***	-39.31*	1953	3.046	1159	-3.167	-0.480
	(2.050)	(1.820)	(22.80)	(2298)	(3.245)	(807.7)	(10.92)	(1.822)
opasq	-73.12	-204.0***	-1290*	72952	80.77	47145**	-10.62	-14.97
	(50.15)	(37.99)	(693.0)	(48619)	(94.43)	(20362)	(245.2)	(51.69)
opacube	-304.4	-1559***	-12129**	644874**	667.3	400134***	19.44	-107.6
	(348.6)	(258.4)	(5235)	(308609)	(713.4)	(142063)	(1571)	(387.1)
Dopasq	188.9*	394.5***	2769*	-66184	-209.0	-75953**	-39.96	14.77
	(96.71)	(78.27)	(1380)	(102869)	(162.1)	(35781)	(450.4)	(98.52)
Dopacube	-459.3*	169.1	1069	-865609***	399.3	-201881*	922.4	265.6
	(244.8)	(195.6)	(6569)	(260746)	(484.2)	(100903)	(1006)	(337.5)
Constant	0.0303	0.0428	3.736***	3403***	0.746***	500.7***	22.52***	3.150***
	(0.0312)	(0.0277)	(0.125)	(34.93)	(0.0366)	(7.689)	(0.140)	(0.0353)
Observations	602	602	83	602	602	602	602	601
R^2	0.011	0.033	0.118	0.022	0.013	0.015	0.015	0.066

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Appendix D

Proofs

Proof of Lemma 1: Stage N outcome with linear cost of bidding. To find the Markov perfect equilibrium, I rely on backward induction. Thus, the first step is to determine the solution to the final stage. The solution is determined as follows: conjecture support, analyze payoffs, solve for distributions, confirm supports.

Entering the last stage, both players are aware of v_1, v_2, b_1^1, and b_2^1. Hence, they are aware of Δ. Players choose b_1^2 and b_2^2, independently, to maximize individual profit. The following examines the three possible scenarios players face with regards to Δ.

SCENARIO 1: $\Delta > 0$

Conjecture Support: The purpose here is to determine the range of bids that players my place with positive probability. Player 2 will never bid above v_2. Otherwise, $\max \pi_2 = v_2 - b_2^2 < 0$. Player 1, knowing that $b_2^2 \leq v_2$, will never bid above $v_2 - \Delta + \varepsilon$.[1] Otherwise, Player 1 could lower his bid and increase π_1. Furthermore, Player 2 will never bid in the range $(0, \Delta)$. Any bid in this range is not enough to overcome Player 1's advantage, Δ. Therefore, a bid in this range will result in $\pi_2 < 0$. The support of Player 2's strategy is $b_2^2 \in \{0, [\Delta, v_2]\}$.

Let G_1 and G_2 be the CDFs that describe the bidding strategies of Player 1 and Player 2. Based on the supports above, $G_1(v_2 - \Delta) = 1$ and $G_2(v_2) = 1$. And, $G_1(0) = 0$. Player 2 may put a mass at 0. To see this, assume $\Delta = v_2$. In this scenario, Player 2 would set $b_2^2 = 0$ with probability 1. Let, $G_2(0) = \alpha$ where α is the mass at 0. Given that no bids occur in the range $(0, \Delta)$, $G_2(b_2^2) = \alpha$ for all b_2^2 in this range.

[1] To simplify, I will not carry the ε through the rest of the analysis.

Analyze Payoffs: Having established the range over which players bid, I wish to determine the bidding strategy that maximizes the players' payoffs, which are:

$$\Pi_1 = \pi_1^N = v_1 G_2(b_1^2 + \Delta) - b_1^2 \tag{D.1}$$

$$\Pi_2 = \pi_2^N = v_2 G_1(b_2^2 - \Delta) - b_2^2 \tag{D.2}$$

Note that for each Player, the bids placed in the first round are unrecoverable sunk costs. Hence, they do not enter the payoff functions in the second stage other than through their impact on the advantage, Δ. What remains is to solve for the distributions G_1 and G_2. Player 1 has the higher limit on his effective bid. According to Proposition 1, he will earn positive expected profit and Player 2 will earn zero expected profit. Player 1 has the opportunity to win assuredly by bidding $v_2 - \Delta$. Therefore he will employ a mixed strategy that gives him this profit in expectation. Similarly, Player 2 expects zero profit and mixes accordingly. Any mixed strategy equilibrium requires that expected profits equal:

$$\pi_1 = v_1 - v_2 + \Delta \tag{D.3}$$

$$\pi_2 = 0 \tag{D.4}$$

Solving equations (D.1), (D.2), (D.3), (D.4) simultaneously yields the distributions:

$$G_1(x) = \frac{x+\Delta}{v_2} \text{ for } x \in [0, v_2 - \Delta]$$

$$G_2(x) = \frac{v_1 - v_2 + x}{v_1} \text{ for } x \in \{0, [\Delta, v_2]\}$$

Finally, note that Player 2 places a mass point at zero. When $x = 0$, $\alpha = \frac{v_1 - v_2}{v_1}$. Figure D.1 plots the CDF describing Player 2's mixed bidding strategy.

SCENARIO 2: $\Delta < 0$

In the second round, Player 2 enters with an advantage. For exposition, define $\Lambda = -\Delta = b_2^1 - b_1^1$. So, Λ is the advantage that Player 2 holds. There are three cases, defined by the magnitude of Λ, to

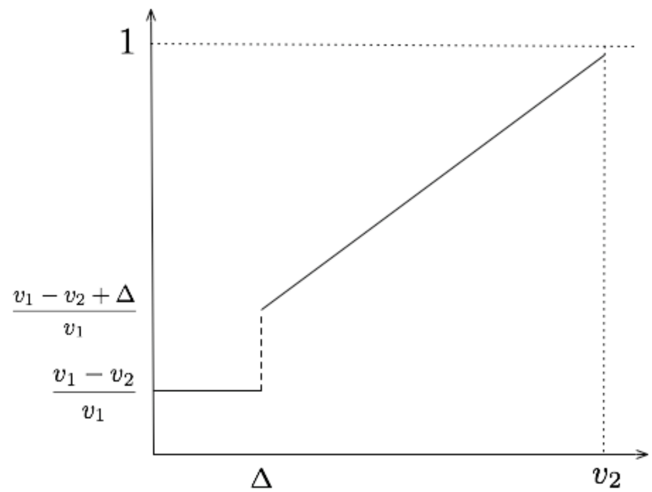

Figure D.1: Player 2's second round mixed strategy CDF

consider. If $\Lambda = v_1 - v_2$, then players enter the second round "even" in terms of their maximum effective bids. Therefore, each will receive expected profits of zero. If Λ is smaller than this amount, that is even with the advantage $v_1 > v_2 + \Lambda$, Player 1 will receive a positive expected payoff though it will be lower than when he held the advantage. Finally, if $\Lambda > v_1 - v_2$, then Player 2 can effectively outbid Player 1 in the second round and receive positive profits. The following will consider each of these three cases using the solution methodology outlined above.

Case 1: $v_1 = v_2 + \Lambda$

Conjecture Support: Player 2 will never bid above v_2. Otherwise, max $\pi_2 = v_2 - b_2^2 < 0$. For the same reasons, Player 1 will not bid above v_1. For Player 1 to win, he must bid above $b_2^2 + \Lambda$. If Player 2 bids $b_2^2 = v_2$, then Player 1 must bid $v_2 + \Lambda = v_1$. Also, Player 1 must overcome Player 2's advantage. Thus Player 1 will not bid in the interval $(0, \Lambda)$. Thus G_1 and G_2 have supports $x_1^2 \in [\Lambda, v_1]$ and $x_2^2 \in [0, v_2]$.

Based on these supports, $G_1(\Lambda) = 0$, $G_1(v_1) = 1$, $G_2(0) = \alpha$, and $G_2(v_2) = 1$.

Analyze Payoffs: Using mixed strategies defined over the supports above, the payoffs for players are:

$$\pi_1 = v_1 G_2(b_1^2 - \Lambda) - b_1^2 \tag{D.5}$$

$$\pi_2 = v_2 G_1(b_2^2 + \Lambda) - b_2^2 \tag{D.6}$$

Because each player has the same effective bid, the expected profits are:

$$\pi_1 = 0 \tag{D.7}$$

$$\pi_2 = 0 \tag{D.8}$$

Solving equations (D.5), (D.6), (D.7), (D.8) simultaneously yields the distributions:

$$G_1(x) = \frac{x - \Lambda}{v_2} \text{ for } x \in [\Lambda, v_1]$$
$$G_2(x) = \frac{x + \Lambda}{v_1} \text{ for } x \in [0, v_2]$$

This results shows that when Player 2 enters round two with enough of an advantage so that his maximum bid equals that of Player 1, neither player receives positive expected profits.

Case 2: $v_1 > v_2 + \Lambda$

Unlike the first case, the advantage held by Player 2 is not enough to allow his maximum effective bid to equal Player 1's highest effective bid. Therefore, Player 1 has a range of bidding options where he can secure a win. To see this, we will again repeat the exercise above.

Conjecture Support: Player 2 will bid no higher than v_2 regardless of the size of Λ. Player 1 may bid up to $v_2 + \Lambda$. However, Player 1 will not bid below Λ otherwise $\pi_1 < 0$. So, $x_1^2 \in [\Lambda, v_2 + \Lambda]$ and $x_2^2 \in [0, v_2]$. The CDFs of each player evaluated at the upper and lower bounds are $G_1(\Lambda) = 0$, $G_1(v_2 + \Lambda) = 1$, $G_2(0) = \alpha$, and $G_2(v_2) = 1$. Again, because Player 1 has the opportunity to outbid Player 2, there is some positive probability that Player 2 will put a mass at zero.

Analyze Payoffs: The payoffs are the same as those in case 1. However the expected profits differ:

$$\pi_1 = v_1 - v_2 - \Lambda \tag{D.9}$$

$$\pi_2 = 0 \tag{D.10}$$

Solving equations (D.5), (D.6), (D.9), (D.10) simultaneously yields the distributions:

$$G_1(x) = \frac{x - \Lambda}{v_2} \text{ for } x \in [\Lambda, v_2 + \Lambda]$$
$$G_2(x) = \frac{v_1 - v_2 + x}{v_1} \text{ for } x \in [0, v_2]$$

Player 2 puts a mass of $\alpha = \frac{v_1 - v_2}{v_1}$ at zero.

Case 3: $v_1 < v_2 + \Lambda$

Conjecture Support: In this final case of scenario 2, Player 2 enters the second round with enough of an advantage that his maximum effective bid is above Player 1's maximum effective bid, $x_1^2 = v_1$. Therefore, Player 1 will place a positive probability, α, on playing zero. Player 1's support is $x_1^2 \in \{0, [\Lambda, v_1]\}$. And Player 2, aware that Player 1 will not bid above v_1, will not bid above $v_1 - \Lambda$. Hence, Player 2's strategy spans $[0, v_1 - \Lambda]$.

Analyze Payoffs: Using mixed strategies defined over the supports above, the payoffs for players are:

$$\pi_1 = v_1 G_2(b_1^2 + \Lambda) - b_1^2 \tag{D.11}$$

$$\pi_2 = v_2 G_1(b_2^2 - \Lambda) - b_2^2 \tag{D.12}$$

Because it is now Player 2 who has the higher maximum effective bid, he will earn positive expected profits.

$$\pi_1 = 0 \tag{D.13}$$

$$\pi_2 = v_2 - v_1 + \Lambda \tag{D.14}$$

Solving equations (D.11), (D.12), (D.13), (D.14) simultaneously yields the distributions:

$$\begin{aligned} G_1(x) &= \frac{v_2 - v_1 + x}{v_2} \text{ for } x \in \{0, [\Lambda, v_1]\} \\ G_2(x) &= \frac{x + \Lambda}{v_1} \text{ for } x \in [0, v_1 - \Lambda] \end{aligned}$$

And, is this case, Player 1 will place a mass at zero: $\alpha = \frac{v_1 - v_2}{v_1}$

Scenario 3: $\Delta = 0$

In this final scenario, neither player enters round 2 with an advantage. Therefore, the first stage has no impact on how the players behave in the second. This case is developed in Baye, Kovenock, and de Vries (1996). Adapting their multiple player model to this two player model, we have that players will both bid in the range: $x_i^2 \in [0, v_2]$. The payoff functions are as follows:

$$\pi_1 = v_1 G_2(b_1^2) - b_1^2 \quad \text{(D.15)}$$

$$\pi_2 = v_2 G_1(b_2^2) - b_2^2 \quad \text{(D.16)}$$

And, expected profits are given below. Because Player 1 has the higher maximum effective bid, he will earn positive expected profits.

$$\pi_1 = v_1 - v_2 \quad \text{(D.17)}$$

$$\pi_2 = 0 \quad \text{(D.18)}$$

Solving equations (D.15), (D.16), (D.17), (D.18) simultaneously yields the distributions:

$$\begin{aligned} G_1(x) &= \frac{x}{v_2} \text{ for } x \in [0, v_2] \\ G_2(x) &= \frac{v_1 - v_2 + x}{v_1} \text{ for } x \in [0, v_2] \end{aligned}$$

□

Proof of Proposition 4: No mixed strategies. For Player 1 (assuming Player 2 bids[2]), the valuation function is an integral over the range of possible bids from Player 2. Player 1 seeks to maximize this value function, V_1^{N-1} by choosing b_1^{N-1}. To simplify notation, I have dropped the superscripts from the bids. The value function for Player 1 in stage $N-1$ is:

$$\begin{aligned}
V_1^{N-1}(b_1,\Delta^{N-1}) &= -c(b_1)+\int_{\underline{b_1}}^{\overline{b_1}} V_1^N g_2(b_2)db_2 \\
V_1^{N-1}(b_1,\Delta^{N-1}) &= -c(b_1)+ \\
&\int_0^{max(0,b_1+c^{-1}(v_1)-c^{-1}(v_2)+\Delta^{N-1})} (v_1-c(c^{-1}(v_2)-b_1+b_2-\Delta^{N-1}))g_2(b_2)db_2
\end{aligned}$$

$$\begin{aligned}
0 &= \frac{\partial V_1^{N-1}(b_1,\Delta^{N-1})}{\partial b_1} \\
0 &= -c'(b_1)+ \\
&\left(v_1-c(c^{-1}(v_2)-b_1+b_1+c^{-1}(v_1)-c^{-1}(v_2)+\Delta^{N-1}-\Delta^{N-1})\right)*1+ \\
&\left(v_1-c(c^{-1}(v_2)-b_1+0-\Delta^{N-1})\right)*0+ \\
&\int_0^{b_1+c^{-1}(v_1)-c^{-1}(v_2)+\Delta^{N-1}} \left(-c'(c^{-1}(v_2)-b_1+b_2-\Delta^{N-1}))g_2(b_2)\right)db_2 \\
0 &= -c'(b_1)+ \\
&\left(v_1-c(c^{-1}(v_1))\right)*1+ \\
&\left(v_1-c(c^{-1}(v_2)-b_1+0-\Delta^{N-1})\right)*0+ \\
&\int_0^{b_1+c^{-1}(v_1)-c^{-1}(v_2)+\Delta^{N-1}} \left(-c'(c^{-1}(v_2)-b_1+b_2-\Delta^{N-1})g_2(b_2)\right)db_2 \\
0 &= -c'(b_1)+ \\
&(v_1-v_1)*1+ \\
&0+ \\
&\int_0^{b_1+c^{-1}(v_1)-c^{-1}(v_2)+\Delta^{N-1}} \left(-c'(c^{-1}(v_2)-b_1+b_2)g_2(b_2)\right)db_2 \\
c'(b_1) &= -\int_0^{max(0,b_1+c^{-1}(v_1)-c^{-1}(v_2)+\Delta^{N-1})} \left(c'(c^{-1}(v_2)-b_1+b_2-\Delta^{N-1})g_2(b_2)\right)db_2
\end{aligned}$$

---

[2] Δ^{N-1} could be such that Player 2 will not bid. This is the case when $\Delta^{N-1} \geq b_1^{N-1}+c^{-1}(v_1)-c^{-1}(v_2)$

Because c is an increasing function of b_1^{N-1}, there is no b_1^{N-1} that would result in a negative change in $c(b_1^{N-1})$. Therefore, there is no mixed strategy equilibrium for Player 1.

Player 2 wishes to maximize his value function, V_2. As Player 1 had to include the possible bids of Player 2, Player 2 must also maximize over the possible bids of Player 1.

$$
\begin{aligned}
V_2(b_2,\Delta^{N-1}) &= -c(b_2)+\int_0^{max(0,b_2+c^{-1}(v_2)-c^{-1}(v_1)-\Delta^{N-1})}(v_2-c(c^{-1}(v_1)+b_1-b_2+\Delta^{N-1}))g_1(b_1)db_1 \\
0 &= \frac{\partial V_2(b_2,\Delta^{N-1})}{\partial b_2} \\
0 &= -c'(b_2)+ \\
&\left(v_2-c(c^{-1}(v_1)+b_2+c^{-1}(v_2)-c^{-1}(v_1)-\Delta^{N-1}-b_2+\Delta^{N-1})\right)*1+ \\
&\left(v_2-c(c^{-1}(v_1)+0-b_2+\Delta^{N-1})\right)*0+ \\
&\int_0^{b_2+c^{-1}(v_2)-c^{-1}(v_1)-\Delta^{N-1}}\left(-c'(c^{-1}(v_1)+b_1-b_2+\Delta^{N-1}))g_1(b_1)\right)db_1 \\
0 &= -c'(b_2)+ \\
&\left(v_2-c(c^{-1}(v_2))\right)*1+ \\
&\left(v_2-c(c^{-1}(v_1)+0-b_2+\Delta^{N-1})\right)*0+ \\
&\int_0^{b_2+c^{-1}(v_2)-c^{-1}(v_1)-\Delta^{N-1}}\left(-c'(c^{-1}(v_1)+b_1-b_2+\Delta^{N-1})g_1(b_1)\right)db_1 \\
0 &= -c'(b_2)+ \\
&(v_2-v_2)*1+ \\
&0+ \\
&\int_0^{b_2+c^{-1}(v_2)-c^{-1}(v_1)-\Delta^{N-1}}\left(-c'(c^{-1}(v_1)+b_1-b_2+\Delta^{N-1})g_1(b_1)\right)db_1 \\
c'(b_2) &= -\int_0^{max(0,b_2+c^{-1}(v_2)-c^{-1}(v_1)-\Delta^{N-1})}\left(c'(c^{-1}(v_1)+b_1-b_2+\Delta^{N-1})g_1(b_1)\right)db_1
\end{aligned}
$$

Using the same logic as above, there is no mixed strategy equilibrium for Player 2. □

Proof of Proposition 5: Pure strategy equilibria. Each player must decide whether to bid. This leads to four possible cases: 1. Each player bids; 2. Player 1 bids and Player 2 does not; 3. Player 2 bids and Player 1 does not; and 4. Neither player bids. The following examines each of the four cases in detail to arrive at possible equilibria.

1. Assume $b_1 > 0$, $b_2 > 0$, and $b_1 > b_2 + c^{-1}(v_2) - c^{-1}(v_1) - \Delta^{N-1}$. In this case, Player 1 earns positive profit.

$$
\begin{aligned}
V_1(b_1) &= -c(b_1) + v_1 - c(c^{-1}(v_2) - b_1 + b_2 - \Delta^{N-1}) \\
\underset{\mathbf{b}_1}{\arg\max}\, V_1 & \\
\frac{\partial V_1(b_1)}{\partial b_1} = 0 & \\
0 &= -c'(b_1) + c'(c^{-1}(v_2) - b_1 + b_2 - \Delta^{N-1}) \\
c'(b_1) &= c'(c^{-1}(v_2) - b_1 + b_2 - \Delta^{N-1}) \\
c(b_1) &= c(c^{-1}(v_2) - b_1 + b_2 - \Delta^{N-1}) \\
b_1 &= c^{-1}(v_2) - b_1 + b_2 - \Delta^{N-1} \\
b_1 &= \frac{c^{-1}(v_2) + b_2 - \Delta^{N-1}}{2}
\end{aligned}
$$

Player 2 earns zero profit.

$$
\begin{aligned}
V_2(b_2) &= -c(b_2) + 0 \\
\underset{\mathbf{b}_2}{\arg\max}\, V_2 & \\
\frac{\partial V_2(b_2)}{\partial b_2} = 0 & \\
0 &= -c'(b_2) \\
c'(b_2) &= 0 \\
b_2 &= 0
\end{aligned}
$$

Note that second order conditions are satisfied as $V_i''(b_i) < 0$. However, the assumption that $b_2 > 0$

is violated.

Now assume $b_1 > 0$, $b_2 > 0$, and $b_2 > b_1 + c^{-1}(v_1) - c^{-1}(v_2) + \Delta^{N-1}$. Here, Player 1 earns zero profit.

$$\begin{aligned}
V_1(b_1) &= -c(b_1) + 0 \\
\arg\max_{\mathbf{b}_1} V_1 & \\
\frac{\partial V_1(b_1)}{\partial b_1} = 0 & \\
0 &= -c'(b_1) \\
c'(b_1) &= 0 \\
b_1 &= 0
\end{aligned}$$

Player 2 earns positive profit.

$$\begin{aligned}
V_2(b_2) &= -c(b_2) + v_2 - c(c^{-1}(v_1) + b_1 - b_2 + \Delta^{N-1}) \\
\arg\max_{\mathbf{b}_2} V_2 & \\
\frac{\partial V_2(b_2)}{\partial b_2} = 0 & \\
0 &= -c'(b_2) + c'(c^{-1}(v_1) + b_1 - b_2 + \Delta^{N-1}) \\
c'(b_2) &= c'(c^{-1}(v_1) + b_1 - b_2 + \Delta^{N-1}) \\
c(b_2) &= c(c^{-1}(v_1) + b_1 - b_2 + \Delta^{N-1}) \\
b_2 &= c^{-1}(v_1) + b_1 - b_2 + \Delta^{N-1} \\
b_2 &= \frac{c^{-1}(v_1) + b_1 + \Delta^{N-1}}{2}
\end{aligned}$$

Now, the assumption that $b_1 > 0$ is violated. Hence, it cannot be the case that both player bid a positive amount.

2. Now, assume Player 1 bids $b_1 > 0$ and Player 2 bids zero. Also assume that $b_1 > c^{-1}(v_2) - c^{-1}(v_1) - \Delta^{N-1}$, so that Player 1 crosses the threshold to earn positive expected profit in the final

round. Player 1 chooses b_1 to maximize profit:

$$
\begin{aligned}
V_1(b_1) &= -c(b_1) + v_1 - c(c^{-1}(v_2) - b_1 - \Delta^{N-1}) \\
\arg\max_{\mathbf{b}_1} V_1 & \\
\frac{\partial V_1(b_1)}{\partial b_1} = 0 & \\
0 &= -c'(b_1) + c'(c^{-1}(v_2) - b_1 - \Delta^{N-1}) \\
c'(b_1) &= c'(c^{-1}(v_2) - b_1 - \Delta^{N-1}) \\
c(b_1) &= c(c^{-1}(v_2) - b_1 - \Delta^{N-1}) \\
b_1 &= c^{-1}(v_2) - b_1 - \Delta^{N-1} \\
b_1^* &= \frac{c^{-1}(v_2) - \Delta^{N-1}}{2}
\end{aligned}
$$

Now I verify that b_1^* satisfies the assumption above:

$$
\begin{aligned}
b_1^* &> c^{-1}(v_2) - c^{-1}(v_1) - \Delta^{N-1} \\
\frac{c^{-1}(v_2) - \Delta^{N-1}}{2} &> c^{-1}(v_2) - c^{-1}(v_1) - \Delta^{N-1} \\
c^{-1}(v_1) &> \frac{c^{-1}(v_2) - \Delta^{N-1}}{2}
\end{aligned} \tag{D.19}
$$

Next I verify that the overall profit is positive:

$$
\begin{aligned}
V_1(\frac{c^{-1}(v_2) - \Delta^{N-1}}{2}) &= -c(\frac{c^{-1}(v_2) - \Delta^{N-1}}{2}) + v_1 - c(c^{-1}(v_2) - \frac{c^{-1}(v_2) - \Delta^{N-1}}{2}) \\
&= -c(\frac{c^{-1}(v_2) - \Delta^{N-1}}{2}) + v_1 - c(c^{-1}(v_2) - \frac{c^{-1}(v_2) - \Delta^{N-1}}{2}) \\
&= -c(\frac{c^{-1}(v_2) - \Delta^{N-1}}{2}) + v_1 - c(\frac{c^{-1}(v_2) - \Delta^{N-1}}{2}) \\
&= v_1 - 2c(\frac{c^{-1}(v_2) - \Delta^{N-1}}{2})
\end{aligned}
$$

Therefore, overall profit for Player 1 is positive as long as:

$$\begin{aligned} v_1 &> 2c(\frac{c^{-1}(v_2) - \Delta^{N-1}}{2}) \\ c^{-1}(\frac{v_1}{2}) &> \frac{c^{-1}(v_2) - \Delta^{N-1}}{2} \end{aligned} \tag{D.20}$$

Note if the overall profitability requirement, (D.20), is met, then the threshold requirement, (D.19), is also satisfied.[3]

Condition (D.20) assumes that Player 2 bid zero. Player 2 will only bid a positive amount if it can cross the threshold for profitability given b_1^*. So, assume that $b_2 > c^{-1}(v_1) - c^{-1}(v_2) + b_1^* + \Delta^{N-1}$. Given this, Player 2 will select b_2 maximize profit:

$$\begin{aligned} V_2(b_2) &= -c(b_2) + v_2 - c(c^{-1}(v_1) + b_1^* - b_2 + \Delta^{N-1}) \\ V_2(b_2) &= -c(b_2) + v_2 - c(c^{-1}(v_1) + \frac{c^{-1}(v_2) - \Delta^{N-1}}{2} - b_2 + \Delta^{N-1}) \\ V_2(b_2) &= -c(b_2) + v_2 - c(c^{-1}(v_1) + \frac{c^{-1}(v_2) + \Delta^{N-1}}{2} - b_2) \\ \arg\max_{\mathbf{b}_2} V_2 & \\ \frac{\partial V_2(b_2)}{\partial b_2} = 0 & \\ 0 &= -c'(b_2) + c'(c^{-1}(v_1) + \frac{c^{-1}(v_2) + \Delta^{N-1}}{2} - b_2) \\ c'(b_2) &= c'(c^{-1}(v_1) + \frac{c^{-1}(v_2) + \Delta^{N-1}}{2} - b_2) \\ c(b_2) &= c(c^{-1}(v_1) + \frac{c^{-1}(v_2) + \Delta^{N-1}}{2} - b_2) \\ b_2 &= c^{-1}(v_1) + \frac{c^{-1}(v_2) + \Delta^{N-1}}{2} - b_2 \\ b_2^* &= \frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2) + \Delta^{N-1}}{4} \end{aligned}$$

Now, to satisfy the assumption above, I derive the requirements for $b_2^* > c^{-1}(v_1) - c^{-1}(v_2) + b_1^* +$

[3]Also note that $c^{-1}(\frac{v_1}{2}) > \frac{c^{-1}(v_2)}{2}$. Therefore, Δ^{N-1} must be greater than or equal to $c^{-1}(v_2) - 2c^{-1}(\frac{v_1}{2})$ which is a negative value because $c(v_1) > c(v_2)$ and $2c^{-1}(\frac{v_1}{2}) > c(v_1)$.

Δ^{N-1}.

$$\begin{aligned}
b_2^* &> c^{-1}(v_1) - c^{-1}(v_2) + b_1^* + \Delta^{N-1} \\
\frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2) + \Delta^{N-1}}{4} &> c^{-1}(v_1) - c^{-1}(v_2) + \frac{c^{-1}(v_2) - \Delta^{N-1}}{2} + \Delta^{N-1} \\
\frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2)}{4} + \frac{\Delta^{N-1}}{4} &> c^{-1}(v_1) - \frac{c^{-1}(v_2)}{2} + \frac{\Delta^{N-1}}{2} \\
\frac{3}{4}c^{-1}(v_2) - \frac{\Delta^{N-1}}{4} &> \frac{c^{-1}(v_1)}{2} \\
\frac{3}{2}c^{-1}(v_2) &> c^{-1}(v_1) + \frac{\Delta^{N-1}}{2} \\
\Delta^{N-1} &< 3c^{-1}(v_2) - 2c^{-1}(v_1) \qquad \text{(D.21)}
\end{aligned}$$

Finally, if (D.21) holds, does Player 2 earn overall positive profits? Profit for Player 2 is:

$$\begin{aligned}
V_2(b_1^*, b_2^*) &= -c(b_2^*) + v_2 - c(c^{-1}(v_1) + b_1^* - b_2^* + \Delta^{N-1}) \\
&= -c(\frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2) + \Delta^{N-1}}{4}) + v_2 \\
&\quad -c(c^{-1}(v_1) + \frac{c^{-1}(v_2) - \Delta^{N-1}}{2} - \frac{c^{-1}(v_1)}{2} - \frac{c^{-1}(v_2) + \Delta^{N-1}}{4} + \Delta^{N-1}) \\
&= -c(\frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2) + \Delta^{N-1}}{4}) + v_2 - c(\frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2) + \Delta^{N-1}}{4}) \\
&= v_2 - 2c(\frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2) + \Delta^{N-1}}{4})
\end{aligned}$$

$V_2(b_1^*, b_2^*)$ is positive if:

$$\begin{aligned}
v_2 - 2c(\frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2) + \Delta^{N-1}}{4}) &\geq 0 \\
v_2 &\geq 2c(\frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2) + \Delta^{N-1}}{4}) \\
\frac{v_2}{2} &\geq c(\frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2) + \Delta^{N-1}}{4}) \\
c^{-1}\left(\frac{v_2}{2}\right) &\geq \frac{c^{-1}(v_1)}{2} + \frac{c^{-1}(v_2) + \Delta^{N-1}}{4} \\
c^{-1}\left(\frac{v_2}{2}\right) - \frac{c^{-1}(v_1)}{2} - \frac{c^{-1}(v_2)}{4} &\geq \frac{\Delta^{N-1}}{4} \\
\Delta^{N-1} &\leq 4c^{-1}\left(\frac{v_2}{2}\right) - 2c^{-1}(v_1) - c^{-1}(v_2) \qquad \text{(D.22)}
\end{aligned}$$

The condition for overall profitability, (D.22), is more stringent than the threshold condition, (D.21). To see this note that:

$$\begin{aligned}
4c^{-1}\left(\frac{v_2}{2}\right) - 2c^{-1}(v_1) - c^{-1}(v_2) &< 3c^{-1}(v_2) - 2c^{-1}(v_1) \\
4c^{-1}\left(\frac{v_2}{2}\right) &< 4c^{-1}(v_2) \\
c^{-1}\left(\frac{v_2}{2}\right) &< c^{-1}(v_2)
\end{aligned}$$

Therefore, for Player 2 to bid a positive amount, $\Delta^{N-1} \leq 4c^{-1}\left(\frac{v_2}{2}\right) - 2c^{-1}(v_1) - c^{-1}(v_2)$. And, if not, then Player 2 will bid zero.

To summarize, if $\Delta^{N-1} > c^{-1}(v_2) - 2c^{-1}(\frac{v_1}{2})$ and $\Delta^{N-1} > 4c^{-1}\left(\frac{v_2}{2}\right) - 2c^{-1}(v_1) - c^{-1}(v_2)$, then an equilibrium exists where:

$$\begin{aligned}
b_1^* &= \frac{c^{-1}(v_2) - \Delta^{N-1}}{2} \\
b_2^* &= 0
\end{aligned}$$

3. Consider the possibility that Player 1 bids zero and Player 2 bids $b_2 > 0$. Player 2 will only bid a positive amount if $b_2 > c^{-1}(v_1) - c^{-1}(v_2) + \Delta^{N-1}$ so that expected profit in the final round is positive. Assume this is the case. Player 2 selects b_2 to maximize profit:

$$\begin{aligned}
V_2(b_2) &= -c(b_2) + v_2 - c(c^{-1}(v_1) - b^2 + \Delta^{N-1}) \\
\underset{\mathbf{b}_2}{\arg\max}\, V_2 & \\
\frac{\partial V_2(b_2)}{\partial b_2} = 0 & \\
0 &= -c'(b_2) + c'(c^{-1}(v_1) - b_2 + \Delta^{N-1}) \\
c'(b_2) &= c'(c^{-1}(v_1) - b_2 + \Delta^{N-1}) \\
c(b_2) &= c(c^{-1}(v_1) - b_2 + \Delta^{N-1}) \\
b_2 &= c^{-1}(v_1) - b_2 + \Delta^{N-1} \\
b_2^* &= \frac{c^{-1}(v_1) + \Delta^{N-1}}{2}
\end{aligned}$$

Now I verify that b_2^* satisfies the assumption above:

$$\begin{aligned}
b_2 &> c^{-1}(v_1) - c^{-1}(v_2) + \Delta^{N-1} \\
\frac{c^{-1}(v_1) + \Delta^{N-1}}{2} &> c^{-1}(v_1) - c^{-1}(v_2) + \Delta^{N-1} \\
c^{-1}(v_2) &> \frac{c^{-1}(v_1) + \Delta^{N-1}}{2} \qquad \text{(D.23)}
\end{aligned}$$

However, a more restrictive condition is necessary for Player 2 to earn overall positive profit:

$$\begin{aligned}
V_2(b_2) &\geq 0 \\
-c\left(\frac{c^{-1}(v_1) + \Delta^{N-1}}{2}\right) + v_2 - c\left(c^{-1}(v_1) - \frac{c^{-1}(v_1) + \Delta^{N-1}}{2}\right) &\geq 0 \\
v_2 &\geq 2c\left(\frac{c^{-1}(v_1) + \Delta^{N-1}}{2}\right) \\
c^{-1}\left(\frac{v_2}{2}\right) &\geq \frac{c^{-1}(v_1) + \Delta^{N-1}}{2} \\
\Delta^{N-1} &\leq 2c^{-1}\left(\frac{v_2}{2}\right) - c^{-1}(v_1) \qquad \text{(D.24)}
\end{aligned}$$

This can happen depending on the magnitude of Δ^{N-1}, the convexity of c, and the separation between v_1 and v_2. So Player 2 may bid a positive amount.

Can Player 1 place a bid that profitably deviates from bidding zero? For any positive bid to yield positive profit, $b_1 > c^{-1}(v_2) - c^{-1}(v_1) + b_2 - \Delta^{N-1}$. Based on Player 2's bid of $b_2^* = \frac{c^{-1}(v_1)+\Delta^{N-1}}{2}$, Player 1 bids b_1 to maximizes V_1.

$$
\begin{aligned}
V_1(b_1, b_2^*) &= -c(b_1) + v_1 - c(c^{-1}(v_2) - b_1 + b_2^* - \Delta^{N-1}) \\
&= -c(b_1) + v_1 - c\left(c^{-1}(v_2) - b_1 + \frac{c^{-1}(v_1) + \Delta^{N-1}}{2} - \Delta^{N-1}\right) \\
&= -c(b_1) + v_1 - c\left(c^{-1}(v_2) - b_1 + \frac{c^{-1}(v_1) - \Delta^{N-1}}{2}\right)
\end{aligned}
$$

$$\underset{\mathbf{b}_1}{\arg\max}\, V_1$$

$$\frac{\partial V_1(b_1, b_2^*)}{\partial b_1} = 0$$

$$
\begin{aligned}
0 &= -c'(b_1) + c'\left(c^{-1}(v_2) - b_1 + \frac{c^{-1}(v_1) - \Delta^{N-1}}{2}\right) \\
c'(b_1) &= c'\left(c^{-1}(v_2) - b_1 + \frac{c^{-1}(v_1) - \Delta^{N-1}}{2}\right) \\
b_1 &= c^{-1}(v_2) - b_1 + \frac{c^{-1}(v_1) - \Delta^{N-1}}{2} \\
b_1^* &= \frac{c^{-1}(v_2)}{2} + \frac{c^{-1}(v_1) - \Delta^{N-1}}{4}
\end{aligned}
$$

Note that the following threshold must be crossed for Player 1 to receive a payoff:

$$
\begin{aligned}
b_1^* &> c^{-1}(v_2) - c^{-1}(v_1) + b_2^* - \Delta^{N-1} \\
b_1^* &> c^{-1}(v_2) - c^{-1}(v_1) + \frac{c^{-1}(v_1) + \Delta^{N-1}}{2} - \Delta^{N-1} \\
b_1^* &> c^{-1}(v_2) - \frac{c^{-1}(v_1) + \Delta^{N-1}}{2}
\end{aligned}
$$

b_1^* crosses this threshold if:

$$\begin{aligned}
\frac{c^{-1}(v_2)}{2}+\frac{c^{-1}(v_1)-\Delta^{N-1}}{4} &> c^{-1}(v_2)-\frac{c^{-1}(v_1)+\Delta^{N-1}}{2} \\
c^{-1}(v_2)+\frac{c^{-1}(v_1)-\Delta^{N-1}}{2} &> 2c^{-1}(v_2)-c^{-1}(v_1)-\Delta^{N-1} \\
\frac{3}{2}c^{-1}(v_1)+\frac{\Delta^{N-1}}{2} &> c^{-1}(v_2) \\
\Delta^{N-1} &> 2c^{-1}(v_2)-3c^{-1}(v_1) \qquad \text{(D.25)}
\end{aligned}$$

Because $v_1 > v_2$ and c is an increasing function, this threshold is always a negative number. Finally, we must ensure that b_1^* yields an overall profit for Player 1. Player 1's overall profit at b_1^* and b_2^* is:

$$\begin{aligned}
V_1(b_1^*, b_2^*) &= -c(b_1^*)+v_1-c(c^{-1}(v_2)-b_1^*+b_2^*-\Delta^{N-1}) \\
&= -c\left(\frac{c^{-1}(v_2)}{2}+\frac{c^{-1}(v_1)-\Delta^{N-1}}{4}\right)+v_1 \\
&\quad -c\left(c^{-1}(v_2)-\frac{c^{-1}(v_2)}{2}-\frac{c^{-1}(v_1)-\Delta^{N-1}}{4}+\frac{c^{-1}(v_1)+\Delta^{N-1}}{2}-\Delta^{N-1}\right) \\
&= -c\left(\frac{c^{-1}(v_2)}{2}+\frac{c^{-1}(v_1)-\Delta^{N-1}}{4}\right)+v_1-c\left(\frac{c^{-1}(v_2)}{2}+\frac{c^{-1}(v_1)-\Delta^{N-1}}{4}\right) \\
&= -2c\left(\frac{c^{-1}(v_2)}{2}+\frac{c^{-1}(v_1)-\Delta^{N-1}}{4}\right)+v_1
\end{aligned}$$

This is non-negative if:

$$
\begin{aligned}
-2c\left(\frac{c^{-1}(v_2)}{2}+\frac{c^{-1}(v_1)-\Delta^{N-1}}{4}\right)+v_1 &\geq 0 \\
v_1 &\geq 2c\left(\frac{c^{-1}(v_2)}{2}+\frac{c^{-1}(v_1)-\Delta^{N-1}}{4}\right) \\
\frac{v_1}{2} &\geq c\left(\frac{c^{-1}(v_2)}{2}+\frac{c^{-1}(v_1)-\Delta^{N-1}}{4}\right) \\
c^{-1}(\frac{v_1}{2}) &\geq \frac{c^{-1}(v_2)}{2}+\frac{c^{-1}(v_1)-\Delta^{N-1}}{4} \\
\frac{\Delta^{N-1}}{4} &\geq \frac{c^{-1}(v_2)}{2}+\frac{c^{-1}(v_1)}{4}-c^{-1}(\frac{v_1}{2}) \\
\Delta^{N-1} &\geq 2c^{-1}(v_2)+c^{-1}(v_1)-4c^{-1}(\frac{v_1}{2}) \qquad \text{(D.26)}
\end{aligned}
$$

Note that the overall profitablity condition, (D.26), is more strict than the threshold condition, (D.25). Therefore, if $\Delta^{N-1} \leq 2c^{-1}\left(\frac{v_2}{2}\right) - c^{-1}(v_1)$ and $\Delta^{N-1} < 2c^{-1}(v_2) + c^{-1}(v_1) - 4c^{-1}(\frac{v_1}{2})$:

$$
\begin{aligned}
b_1^* &= 0 \\
b_2^* &= \frac{c^{-1}(v_1)+\Delta^{N-1}}{2}
\end{aligned}
$$

4. Finally, note that it cannot be the case that both players bid zero. Due to convexity in the cost function, the player who has the advantage prefers to spread the cost over rounds. Therefore, at least one player will bid a positive amount. □

Proof of Proposition 6: Valuation functions in period N-k. The result is established by induction. I begin with the set of value functions where Player 1 earns positive profit.

1. For the $k = 0$ case:

$$V_1^N = v_1 - c(c^{-1}(v_2) - \Delta^N)$$

Note that this is the value function detailed in Table 2.2.

2. Assuming the result is true for the $k = n$ case, I show that it is also true when $k = n + 1$.

Assume:

$$V_1^{N-n} = v_1 - (n+1)c\left(\frac{c^{-1}(v_2) - \Delta^{N-n}}{n+1}\right)$$

In the n+1 case, Player 1 selects $b_1^{N-(n+1)}$ to maximize the value function:

$$\begin{aligned} V_1^{N-(n+1)} &= -c(b_1^{N-(n+1)}) + V_1^{N-n} \\ &= -c(b_1^{N-(n+1)}) + v_1 - (n+1)c\left(\frac{c^{-1}(v_2) - \Delta^{N-n}}{n+1}\right) \end{aligned}$$

The result of this maximization is that:

$$\begin{aligned}
\frac{\partial V_1(b_1^{N-(n+1)})}{\partial b_1^{N-(n+1)}} &= 0 \\
-c'(b_1^{N-(n+1)}) + c'\left(\frac{c^{-1}(v_2) - \Delta^{N-n}}{n+1}\right) &= 0 \\
c'(b_1^{N-(n+1)}) &= c'\left(\frac{c^{-1}(v_2) - \Delta^{N-n}}{n+1}\right) \\
b_1^{N-(n+1)} &= \frac{c^{-1}(v_2) - \Delta^{N-n}}{n+1} \\
b_1^{N-(n+1)} &= \frac{c^{-1}(v_2) - b_1^{N-(n+1)} + b_2^{N-(n+1)} - \Delta^{N-(n+1)}}{n+1} \\
(n+1)b_1^{N-(n+1)} + b_1^{N-(n+1)} &= c^{-1}(v_2) + b_2^{N-(n+1)} - \Delta^{N-(n+1)} \\
(n+2)b_1^{N-(n+1)} &= c^{-1}(v_2) + b_2^{N-(n+1)} - \Delta^{N-(n+1)} \\
b_1^{N-(n+1)} &= \frac{c^{-1}(v_2) + b_2^{N-(n+1)} - \Delta^{N-(n+1)}}{n+2}
\end{aligned}$$

Under conditions where $b_2^{N-(n+1)} = 0$, we have the following:

$$\begin{aligned}
V_1^{N-(n+1)} &= -c\left(\frac{c^{-1}(v_2) - \Delta^{N-(n+1)}}{n+2}\right) + v_1 - (n+1)c\left(\frac{c^{-1}(v_2) - \frac{c^{-1}(v_2) - \Delta^{N-(n+1)}}{n+2} - \Delta^{N-(n+1)}}{n+1}\right) \\
&= -c\left(\frac{c^{-1}(v_2) - \Delta^{N-(n+1)}}{n+2}\right) + v_1 - (n+1)c(\frac{c^{-1}(v_2) - \Delta^{N-(n+1)}}{n+2}) \\
&= v_1 - (n+2)c\left(\frac{c^{-1}(v_2) - \Delta^{N-(n+1)}}{n+2}\right) \\
&= v_1 - ((n+1)+1)c\left(\frac{c^{-1}(v_2) - \Delta^{N-(n+1)}}{(n+1)+1}\right)
\end{aligned}$$

Recall that $k = n+1$. Substituting this into the equation above yields the result that:

$$V_1^{N-k} = v_1 - (k+1)c\left(\frac{c^{-1}(v_2) - \Delta^{N-k}}{k+1}\right)$$

The equilibrium are symmetrical, so the above holds for the second set of value functions as well.

□

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