STRATEGIC OPERATIONAL DECISIONS IN A SUPPLY CHAIN WITH DEMAND AND RECALL RISKS

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A dissertation submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Kenan-Flagler Business School (Operations).

Chapel Hill
2014

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ABSTRACT

GANG WANG: Strategic Operational Decisions in a Supply Chain with Demand and Recall Risks
(Under the direction of Lauren Xiaoyuan Lu and Jayashankar M. Swaminathan)

Among supply chain risks, both demand risk and recall risk have been recognized as critical challenges firms have to face. Making proper operational decisions to mitigate these two types of risks is of great importance to every firm. This dissertation "Strategic Operational Decisions in a Supply Chain with Demand and Recall Risks" focus on capacity related decisions, which tackles the demand risk, and quality related decisions, which tackles the recall risk. Specifically, we conduct our research along three dimensions: (i) optimizing capacity decisions or quality decisions; (ii) the interaction between capacity and quality decisions; (iii) the impact of supply chain factors on these decisions. In Chapter 2, we examine quality choice and capacity timing of start-ups and established firms. In Chapter 3, we focus on procurement contracting under product recall risk to manage product quality and mitigate the financial impact of product recalls. In Chapter 4, we investigate strategies to improve product quality and to make proper recall decisions.

ACKNOWLEDGEMENTS

This is a great opportunity to express my gratitude to a number of individuals listed here as well as countless others who have helped me in achieving the task of completing this dissertation.

My deepest gratitude is to my advisors, Dr. Lauren Xiaoyuan Lu and Dr. Jayashankar M. Swaminathan. I am deeply indebted to them for their supervision, encouragement, patience, and rigorous yet pleasant training during my Ph.D. study. Lauren pays close attention to every detail of my speaking, writing, presenting, positioning and even thinking. I am grateful to Lauren for her great generosity by giving countless hours of her time to answer my questions and give me guidance. Lauren has always been there ready to set up a meeting or talk over phone, anytime. I clearly remember the night when Lauren helped me revise a manuscript to 2am, and this is just one of the many examples of her devotion to my research training. I also appreciate the invaluable instructions from Jay, who is a great mentor and always help me figure out the big picture and the right direction to proceed. Despite his busy schedule due to both academic and administrative affairs, Jay has always been available to support and encourage me. I am blessed to have both of them to be my advisors during my Ph.D. study.

Also, I would like to acknowledge my committee members. In particular, I am grateful to Dr. Ann Marucheck, who provided nothing but constant support, instructions and encouragement through the past five years. I also thank Ann for her two seminar courses, which led me into the giant world of operations management. I am also indebted to Dr. Bin Hu for his kind help and inspirational discussion. Bin also supported me a lot for teaching BUSI 410 in 2013 summer, which I believe is a main reason I could win the Ph.D. best teaching award. I also thank Dr. Tarun L. Kushwaha for agreeing to serve on my dissertation committee and provide valuable suggestions to my research.

I would like to express my appreciation to all the faculty in the Operations area for their feedbacks and instructions. I would also like to thank the environment and resources provided by the Kenan-Flagler Business School, which was the best. My gratitude also goes to my fellow Ph.D. students: Aaron Ratcliffe, Karthik Natarajan, Yen-Ting Lin, Gokce Esenduran, Vidya Mani, Zhe Wang, Adem Orsdemir. their support and friendship enriched my life here. I also want to thank Sharon Parks, Holly Guthrie, and Erin Rimmer for their assistence from the beginning to the end of my Ph.D. study.

Most importantly, none of this would have been possible without the love and patience of my family. My family has been a constant source of love, concern, support and strength all these years. I would like to express my heart-felt gratitude to my family.

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CHAPTER 1 INTRODUCTION

In this dissertation, we study strategic operational decisions in a supply chain with demand and recall risks.

1.1 Overview of Chapter 2

In Chapter 2, we examine quality choice and capacity timing of start-ups and established firms. Many industries have experienced disruptive innovations that create new products and markets displacing old ones. Some innovations are based on a transformative technology that provides novel product features or dimensions appealing to high-end customers, while other innovations target at low-end customer segments initially. Although many disruptive innovations have been led by start-ups, some established firms have also led innovations and brought radical technologies to market. This variation of innovation strategies inspires us to investigate whether start-ups and established firms differ in their quality choice and investment timing of market entry when making innovations.

To this end, we build a stylized duopoly model in which a start-up and an established firm compete in a market with quality differentiation and demand uncertainty. The firms may choose to make a high-quality product or a low-quality product. They may also choose a capacity-leading strategy, i.e., invest before demand realization, or a capacity-lagging strategy, i.e., invest after demand realization. We assume that the start-up maximizes its survival probability whereas the established firm maximizes its expected profit. Our analysis yields the following main insights: (1) When the market size increases, the established firm is more likely to choose high quality. Moreover, the start-up is more likely to choose high quality when the threshold of the start-up's survival probability increases. (2) The presence of a start-up in the market tends to increase quality differentiation. (3) The firms' quality choice and capacity timing are interdependent. Specifically, we find that quality differentiation is more likely to arise when the two firms choose different capacity timing. (4) We also identify an interesting equilibrium in which the established firm with a low-quality product chooses a capacity-leading strategy when the start-up with a high-quality chooses a capacity-lagging strategy. This equilibrium is consistent with anecdotal evidence on established firms being disruptive innovators.

1.2 Overview of Chapter 3

In Chapter 3, we focus on procurement contracting under product recall risk. Product recall is commonly observed in various industries with production outsourcing. Managing product quality and mitigating the financial impact of product recalls pose great challenges to manufacturers due to demand uncertainty and non-contractibility of suppliers' quality effort. To understand the interdependence of supply chain quantity and quality decisions, we develop a procurement contractual framework under both demand and recall risks. We consider a model in which a manufacturer outsources to a supplier the production of a component, which is subject to potential quality failure leading to a product recall. The manufacturer acts as the Stackelberg leader offering a recall cost sharing contract to the supplier. We analyze two settings: a pull system in which the supplier makes the quantity decision and a push system in which the manufacturer makes the quantity decision. We find that the manufacturer achieves a higher production quantity and induces a higher quality effort of the supplier in the push system than in the pull system. Therefore, the manufacturer can improve quality by taking on the demand risk of the supply chain. Moreover, the presence of product recall risk decreases the production quantity in the push system but does not affect the production quantity in the pull system. Interestingly, the manufacturer can improve quality and profit by decreasing her share of the total recall cost without affecting the production quantity of the supply chain in both the push and pull systems.

1.3 Overview of Chapter 4

In Chapter 4, we investigate recall decisions under quality contracting framework. When outsourcing production to suppliers, ensuring product quality is of great challenge to a manufacturer because suppliers' quality effort cannot be directly observed and it is unrealistic to have a clear-cut assignment of fault to each firm involved in the product development and manufacturing processes. Therefore we are interested in studying strategies to improve product quality and to make proper recall decisions. Specifically, we consider a recall cost sharing contract proposed by the manufacturer to align the incentive of the supplier in ensuring the product quality, and also examine two types of recall decisions: (1) voluntary recall, which is issued by the manufacturer voluntarily at a relatvely early stage; (2) mandatory recall, which is forced by the government agency at a relatively late stage. We characterize the firms' decisions in equilibrium. Surprisingly, we find that, as a Stackeberg leader, the manufacturer does not always push all the recall cost to the supplier. His decision depends on whether he will issue a voluntary recall or not. Furthermore, we find two cost sharing percentage thresholds that determines the equilibrium solutions. Specifically, the lower cost sharing percentage is always associated with voluntary recall, while the higher one is always associated to mandatory recall. We conduct comparative statics study to see how the equilibrium solutions evolve with the change of parameters. To faciliate comparision, we study the first-best problem where the supply chain is integrated, and find that the recall cost sharing contract cannot coordinate the supply chain.

CHAPTER 2 HIGH-END OR LOW-END INNOVATION? QUALITY CHOICE AND CAPACITY TIMING OF START-UPS AND ESTABLISHED FIRMS

2.1 Introduction

Many industries have experienced innovations that create new products and markets displacing old ones. Some innovations are based on a transformative technology that provides novel product features or dimensions appealing to high-end customers. One such example is Tesla Motors, which is an American electric vehicle manufacturer. Founded in 2003 by a team of entrepreneurs in Silicone Valley, Telsa was aimed to enter the automotive market with expensive electric vehicles targeted at affluent buyers. It gained widespread attention by introducing the first fully electric vehicle, Tesla Roadster. The success of Telsa has inspired major automakers to enter the electric vehicle market with lower-priced electric cars. Unlike Telsa's strategy to enter a new market at the high end, the disk drive industry has seen innovations that target at price-sensitive customer segments initially. For example, in the early 80's, Seagate, back then still a start-up, introduced the first 5.25-inch disk drive to compete in the established and more expensive 8-inch disk drive market. In both examples, the start-ups led innovations to create new products but differed in their quality positioning in the new market.

In the case of Seagate, the 5-inch disk drive market eventually replaced the 8-inch disk drive market completely, which is a classic example of low-end disruptive innovation (Schmidt and Van Mieghem 2005). Disruptive innovations may not always undertaken by start-ups. In some cases, well-established firms also bring to market radical technologies that create new markets (Yu and Hang 2010). Success stories include HP's inkjet printers, Intel's Centrino chip-sets, as well as Apple's iPods and iPhones.

Whether being a high-end or a low-end innovation, a new product's market potential is often highly unpredictable because historical sales of existing products may not be good predictors for new products due to their novel functionalities and characteristics. Such demand uncertainty poses a great challenge to the capacity planning process of both start-ups and established firms. Two capacity investment timing strategies are commonly adopted by firms: (1) a capacity-leading strategy, i.e., to build capacity in anticipation of demand; (2) a capacity-lagging strategy, i.e., to install capacity only after demand has occurred. The capacity-leading strategy gives a firm the first-mover advantage of commanding a favorable position in the marketplace. However, excess capacity may occur should the prospect of a large market demand fail to materialize, or product shortages may occur should market demand exceed the installed capacity. The strategic need for market leadership and the operational need for matching supply with demand create an apparent tradeoff between the two capacity timing strategies.

Academic research suggests that start-ups and established firms behave differently in making capacity investment decisions (Swinney, Cachon and Netessine 2011). Because start-ups are generally at the early stage of product and market development, they are constrained by limited cash flows, and thus are faced with a vital challenge whether they could earn enough money to sustain until the next round of funding becomes available. Therefore, the ability to reach a sustainable level of profit seems to matter a great deal to start-ups. In contrast, the financial objectives of established firms can be quite different due to factors such as the ability to bear risk, access to credit, and cash availability, etc. In general, established corporations are viewed as profit maximizers.

In light of the different financial objectives of start-ups and established firms, the aforementioned innovation examples raise several interesting questions. First, when do firms choose to target at the high-quality or the low-quality market segment when making innovations? Second, are firms' quality choices and capacity timing strategies interdependent? Third, does a start-up choose a quality level different from a competing established firm? Moreover, does a start-up in the market increase or decrease product differentiation? To answer these questions, we study firms' innovation strategies along two dimensions: quality choice and capacity timing. We build a stylized duopoly model in which a start-up and an established firm compete in a market with quality differentiation and demand uncertainty. The firms may choose to make a high-quality product or a low-quality product. They may also choose to invest early, i.e., before demand realization, or to invest late, i.e., after demand realization. We assume that a start-up maximizes its survival probability, which is defined as the probability that the firm's profit is no smaller than the minimum profit needed for survival. By contrast, an established firm maximizes its expected profit. To simplify analysis, we consider two games: (1) a capacity timing game in which the firms with different quality levels make capacity timing decisions; (2) a quality choice game in which the firms choose their quality levels under different capacity timing scenarios.

Analyzing the capacity timing game allows us to find interesting innovation patterns. For example, we find that the start-up with a low quality product always behaves aggressively and takes a capacity-leading strategy as long as late capacity investment is more expensive than early capacity investment. This pattern is consistent with the classical examples of low-end disruptive innovation by start-ups, such as Seagate's 5.25-inch disk drives and Dell direct in the early 1980's (Govindarajan and Kopalle 2006). However, when the start-up produces a high quality product, the dynamics can be drastically different—a surprising equilibrium may arise in which when the capacity cost increases over time, the start-up with a high quality product invests late whereas the established firm with a low quality product invests early. This observation is consistent with the anecdotal evidence that disruptive innovations may not always be carried out by start-ups, and established firms may also take the lead in innovation with low-end disruption, e.g., Sony's Walkman and HP's inkjet printers (Yu and Hang 2010). Interestingly, we note that this equilibrium would never arise in the model of Swinney et al. (2011), in which the start-up's and the established firm's products are homogenous. The emergence of this equilibrium in our model stems from the fact that quality differentiation affects not only the first-mover advantage of early capacity investment but also the resulting variability of firm profits, thereby impacting start-ups' survival probability.

The equilibrium analysis of the quality choice game also yields fruitful insights about firms' innovation strategies in the presence of demand uncertainty. We find that when the market size increases, the established firm is more likely to choose high quality. Moreover, the start-up is more likely to choose high quality when the threshold of the start-up's survival probability increases. Interestingly, we find that the presence of a start-up in the market tends to increase quality differentiation.

The quality equilibrium patterns also suggest that quality differentiation is more likely to arise when the start-up and the established firm choose different capacity timing, i.e., one invests early while the other invests late. In contrast, as long as the firms' capacity timing decisions are symmetric, investing early or investing late does not seem to make a significant impact on the quality choices.

The rest of the paper is organized as follows. Section 2 conducts a brief literature review on relevant works. We introduce the model in §3. We present the results for the capacity timing game in §4 and the results for the quality choice game in §5. We conclude in §6.

2.2 Literature Review

There are four streams of literatures related to our work. The first stream is the operations literature on timing strategies of capacity investment under uncertainty. For a comprehensive review on this subject, see Van Mieghem (2003). Some researchers investigate whether it is worthwhile to postpone capacity investment until after obtaining accurate demand information (see, e.g., Van Mieghem and Dada 1999, Anand and Girotra 2007, Anupindi and Jiang 2008*a*). They study the effect of postponement strategies in the context of monopoly or duopoly settings, and show that postponement makes the optimal capacity decision more sensitive to uncertainty and mitigates the destructive effect of competition, and the effect of postponement may be diminished by strategic effects.

The second stream our work is related to is the literatures on quality differentiation. The marketing literatures on this topic generally concentrates on the effects of quality differentiation on product line design and market competition. For instance, Moorthy (1984) considers a monopolist firm providing different quality levels to multiple consumer segments with different valuations for quality. Kim and Chhajed (2002) examine a similar problem but incorporate multiple quality-type attributes. Vandenbosch and Weinberg (1995) consider product and price competition in the context of two competing firms with products vertically differentiated in two dimensions. Desai (2001) investigates whether the cannibalization problem affects firms' price and quality decisions in both monopoly and duopoly settings. Our work focuses on examining the effect of quality differentiation on capacity investment decisions for competing firms, which has not been explored by the marketing literatures. Different from these literatures that assume given quality levels, there are also literatures that endogenize quality choices. Chan and Leland (1982) consider the scenario where sellers select price and quality levels, while buyers can acquire price/quality information about sellers at a cost. Wauthy (1996) provide complete description of quality choices in a duopoly model with vertical differentiation where firms first choose the quality of products simultaneously and then compete in prices. This paper also considers endogenized quality choices and examines it as an alternate setting. Our setting is similar as in Wauthy (1996) but differs in that we capture demand uncertainty in our model, and the start up and the established firm compete in capacity quantity instead of price.

Third, our work is related to the strategic management literature on disruptive innovation. Disruptive innovation refers to an improvement in the product or service that radically refines the performance, related costs, or its target market in a new way. A commonly held notion in this literature is that start-ups, instead of established firms, tend to bring innovations to the market. Christensen and Bower (1996), Christensen (1997), and Druehl and Schmidt (2008) construct a framework on disruptive innovation and provide a thorough review of this topic.

Lastly, our work is related to the bankruptcy theory in the economics literature. Radner and Shepp (1996) and Dutta and Radner (1999) both argue that a firm subject to the risk of bankruptcy will fail with probability one if it purely maximizes its expected profit over an infinite horizon. Greenwald and Stiglitz (1990) and Walls and Dyer (1996) adopt a utility function that incorporates both the operating profit and the risk of failure. Chod and Lyandres (2011) point out that private firms (e.g., start-ups) tend to be more sensitive to risk and the chance of failure than public firms (e.g., established firms), and hence it is reasonable for start-ups and established firms to have different objective functions.

There are extensive literatures studying the impact of bankruptcy risk on operational decisions such as inventory decisions, process development, capacity levels, financial subsidies to suppliers, contracting and sourcing strategies. Some representatives of this literature are Archibald, Thomas, Betts and Johnston (2002), Babich, Burnetas and Ritchken (2007), Babich (2008), Swinney et al. (2011), Boyabatli and Toktay (2011), and Tanrisever, Erzurumlu and Joglekar (2008). Our work is most closely related to Swinney et al. (2011). They investigate the impact of bankruptcy risk on capacity investment timing decisions under demand uncertainty. They find that when demand uncertainty is high and capacity investment costs do not decline too severely over time, a start-up tends to invest early while an established firm tends to invest late. The distinction of our work is to incorporate quality differentiation and study the interdependence of a firm's quality choice and capacity timing. Our analysis yields a new capacity timing pattern in which the established firm invest early while the start-up invests late. This pattern is consistent with the anecdotal observation that disruptive innovation may also be carried out by established firms (Yu and Hang 2010). A more important differentiation of our work with respect to Swinney et al. (2011) is the fact that we endogenize quality differentiation and derive insights about a disruptive innovator's quality choice and its dependence on capacity timing.

2.3 Model

We consider a start-up competing against an established firm. Both firms plan to launch a new product. Their products may differ in quality. We denote the quality levels of the start-up and established firm by S_s and S_e, respectively. Assume $S_s, S_e \in \{S_l, S_h\}$, where S_l denotes the low quality level and S_h denotes the high quality level, thus $S_h > S_l$. The firms need to build capacities for making the products. The products' market demands are uncertain, and the firms can follow two different capacity investment strategies: (1) invest *early*, i.e., invest before demand realization; (2) invest *late*, i.e., invest after observing the demand. Depending on the capacity timing strategies, the firms' unit capacity costs may be different and are denoted by C_{ik}, where $i \in \{1, 2\}$ represents investing early and late, respectively, while $k \in \{l, h\}$ represents the firms' quality levels. Let K_l and K_h denote the capacity quantity for the low quality product and the high quality product, respectively. Any capacity investment is irreversible once being made.

We assume that the firms' capacity investment costs are linear in quantity. This is a common assumption in the capacity management literature (see, e.g., Swinney et al. 2011, Van Mieghem 2003). We further assume that the marginal capacity cost of the high quality product is larger than that of the low quality product, i.e., $\frac{C_{ih}}{S_h} \geq \frac{C_{jl}}{S_l}$, where $i, j \in \{1, 2\}$. This assumption ensures the potential existence of quality differentiation in the market. Otherwise, the firm would find it profitable to only produce and sell the high quality product.

2.3.1 Consumer Utility and Market Demand

We derive the inverse demand function from a consumer utility model. Consider a market of consumers with different quality valuations. Let θ denote a consumer's taste parameter, which is heterogenous among the consumers. Assume that consumer tastes are uniformly distributed over an interval $[0, \hat{\theta}]$ with density one, hence the market size is $\hat{\theta}$.[1]

Following the convention of the quality differentiation literature, we specify the consumer utility function:

$$U = \begin{cases} \theta S - P, & \text{if he purchases a good with quality } S \text{ and price } P, \quad (2.3.1) \\ 0, & \text{if he does not purchase.} \quad (2.3.1') \end{cases}$$

Given the two quality levels, S_h and S_l, the above utility function gives rise to the following inverse demand functions:

$$\begin{cases} P_l = S_l\hat{\theta} - S_l Q_l - S_l Q_h, & (2.3.2) \\ P_h = S_h\hat{\theta} - S_l Q_l - S_h Q_h, & (2.3.2') \end{cases}$$

where Q_l and Q_h denote the quantities of the low and high quality products released to the market.

To capture demand uncertainty, we adopt the *additive* demand shock model, which is commonly used in the operations and marketing literatures (see, e.g., Swinney et al. 2011,

[1] To ensure the existence of vertical differentiation in the market, we assume that the customer with the highest taste value $\hat{\theta}$ achieves higher utility by purchasing high quality product at a price of its cost than that from low quality product, i.e., $S_h\hat{\theta} - C_{ih} \geq S_l\hat{\theta} - C_{jl}$, where $i, j \in \{1, 2\}$. In other words, $\hat{\theta} \geq \frac{C_{ih} - C_{jl}}{(S_h - S_l)}$, where $i, j \in \{1, 2\}$.

Desai, Koenigsberg and Purohit 2007, Anupindi and Jiang 2008*b*). To do that, assume $\hat{\theta}$ is a positive random variable defined on the support of $\left[\max_{i,j\in\{1,2\}}\left\{\frac{C_{ih}-C_{jl}}{(S_h-S_l)}\right\},+\infty\right)$, with a continuous distribution function $G(\hat{\theta})$, whose mean $\mu = E(\hat{\theta})$ and variance $\sigma^2 = Var(\hat{\theta})$. We shall point out that this formulation of uncertainty leads to a demand function that closely resembles Swinney et al. (2011)'s linear demand function with additive random shocks. The similarity of the demand formulations makes it reasonable to compare our equilibrium results with those of Swinney et al. (2011). [2]

Naturally, the firms' production decisions are subject to the capacity constraints. To simplify analysis, we assume that the firms adopt a production clearance strategy, i.e., they always produce up to their capacity limits and release all products to the market. This is a common assumption adopted in the capacity-constrained competition models in the operations management literature (see, e.g., Swinney et al. 2011, Goyal and Netessine 2007).

2.3.2 Firm Objective Functions

Start-ups are generally at an early stage of development and tend to face cash-flow constraints. Therefore, earning a sufficient amount of profit to survive is more important than pursuing high expected profit. There are studies (see, e.g., Radner and Shepp 1996, Dutta and Radner 1999) pointing out that a firm prone to bankruptcy will fail with probability one if it simply aims at maximizing expected profit. This intuitively leads to the idea of incorporating the risk of bankruptcy into a start-up's financial objective. In this paper, we borrow a utility structure from the bankruptcy literature (see, e.g.,

[2] An alternative approach to model demand uncertainty is the so-called *multiplicative* demand shock model. In such a model, the taste distribution is fixed, while the market size is a random variable. The economics literature suggests that the additive and the multiplicative approaches capture different market characteristics, but conclusive empirical studies are rare while the theory literature on demand uncertainty continues to use both types of demand shocks (see, e.g., Cowan 2004).

Greenwald and Stiglitz 1990, Walls and Dyer 1996):

$$total\ utility = operating\ profit - cost\ of\ bankruptcy\ \times\ probability\ of\ bankruptcy$$

For analytical simplicity, we examine two extreme forms of this general utility function. If a firm has a very small chance of bankruptcy, the first term in the utility function, i.e., the profit term, dominates the second term, i.e., the bankruptcy cost term. Then the firm may safely ignore the second term and focus on maximizing the expected profit. This leads to profit maximization in the case of an established firm. If the cost of bankruptcy is large compared to the assets of the firm, or the probability of bankruptcy is high, the second term dominates the first term. Under a fixed cost of bankruptcy, the firm should aim at minimizing the probability of bankruptcy. This leads to survival probability maximization in the case of a start-up firm.

Specifically, we assume that a start-up will survive if its total profit is greater than α, an exogenous parameter. The start-up's objective is to maximize its survival probability, i.e., the probability that its total profit is above α. Denote the optimal survival probability for early investment as

$$\psi^* = \max_K \Pr\{\text{profit} \geq \alpha\}. \tag{2.3.3}$$

Similarly, the optimal survival probability for late investment is

$$\psi^* = \Pr\{\max_K \text{profit} \geq \alpha\}, \tag{2.3.4}$$

which is equivalent to maximizing profit. For late investment, the maximum can be moved into the parentheses because the start-up faces no demand uncertainty at the stage of capacity investment, and hence maximizing profit leads to optimal ex-ante survival probability. This formulation of survival probability as a start-up's objective function is identical to the one used in Swinney et al. (2011). Note that if the value of α is very low, the star-up will survive whatever strategy it takes, which is not of our interest. To

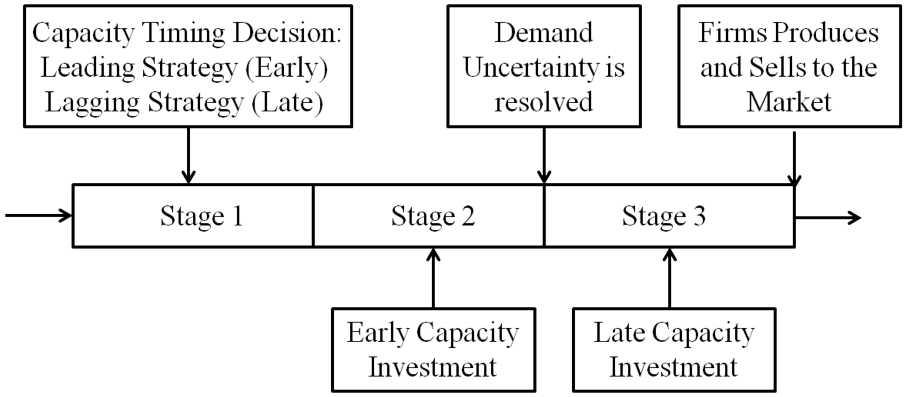


Figure 2.1: Sequence of Events: The Capacity Timing Game

avoid this case, we assume the value of α is reasonably large. Specifically, we assume $\alpha \geq \frac{(S_l C_{ih} - S_h C_{jl})^2}{4S_l(S_h - S_l)^2}$, where $i, j \in \{1, 2\}$.

2.3.3 Sequence of Events

Our goal is to study the interdependence between firms' quality choices and their capacity timing strategies. To do that, we first study a capacity timing game by fixing the quality choices and then consider a quality choice game by fixing the capacity timing decisions.

The Capacity Timing Game

In this setting, the firms' quality choices, i.e., S_s and S_e, are exogenous. The capacity timing game proceeds in three stages, as shown in Figure 2.1. In stage 1, the firms decide on capacity timing in terms of whether to invest *early* or *late*. In stage 2, the firm(s) who have decided to invest early determine their capacity quantities. By the end of this stage, demand uncertainty is resolved. In stage 3, the firm(s) who have decided to invest late determine their capacity quantities.

Two scenarios will be analyzed: (1) the start-up produces a high quality product

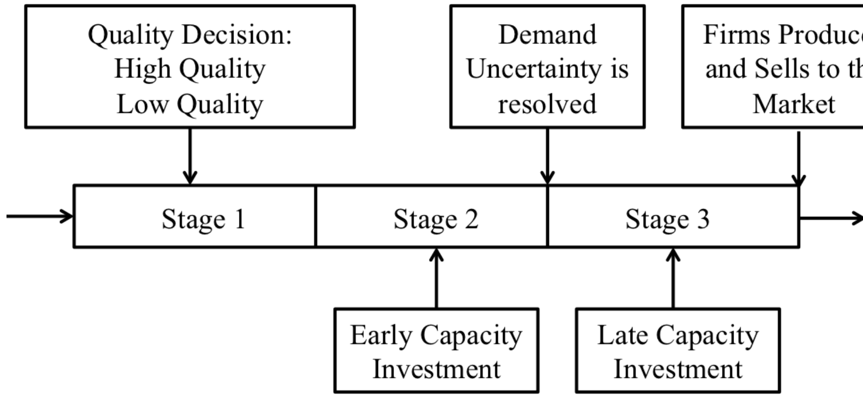


Figure 2.2: Sequence of Events: The Quality Choice Game

while the established firm produces a low quality product; (2) the start-up produces a low quality product while the established firm produces a high quality product. We will also consider the benchmark case where two established firms compete with different quality levels.

The Quality Choice Game

In this setting, the firms' capacity timing decisions are exogenous. The quality choice game proceeds in three stages, as shown in Figure 2.2. In stage 1, both firms choose their quality levels: S_s and S_e, respectively. In stage 2, the firm(s) who invest early determine their capacity quantities. By the end of this stage, demand uncertainty is resolved. In stage 3, the firm(s) who invest late determine their capacity quantities. By the end of stage 3, the firms produce and release the products to the market. We assume the unit capacity cost is given by: $C_{ik} = \beta_i S_k^2$, where $i \in \{1,2\}$, $k \in \{l,h\}$, and β_1 is the coefficient for early capacity investment while β_2 is the coefficient for late capacity investment. To make the problem feasible, we have made an assumption as specified in footnote 3, which can now be transformed into $S_h \leq \frac{\mu}{2\beta_i}$, where $i, j \in \{1,2\}$. We will consider four different timing scenarios, which will be specified later.

2.4 Equilibrium of the Capacity Timing Game

In this section, we analyze the capacity timing game. We also examine the benchmark case wherein two established firms compete against each other. We solve the game by backward induction: given each pair of timing decisions, we solve the capacity quantity subgame first, and subsequently derive the capacity timing equilibrium.

In the following analysis, we use notation T_q^f to denote the capacity investment timing strategy of a firm. $T \in \{E, L\}$ represents the timing of capacity investment being *early* (E) or *late* (L). $f \in \{e, s\}$ denotes the attribute of the firm, where e or s refers to *established firm* or *start-up*, respectively. Lastly $q \in \{l, h\}$ stands for the quality level with l and h denoting *low* and *high* quality, respectively. For instance, (E_l^s, L_h^e) denotes the equilibrium in which the low quality start-up invests early while the high quality established firm invests late. We summarize the notations in Table 2.1.

	High-quality Es.Firm Early	High-quality Es.Firm Late
Low-quality Start-up Early	(E_l^s, E_h^e)	(E_l^s, L_h^e)
Low-quality Start-up Late	(L_l^s, E_h^e)	(L_l^s, L_h^e)
	Low-quality Es.Firm Early	Low-quality Es.Firm Late
High-quality Start-up Early	(E_h^s, E_l^e)	(E_h^s, L_l^e)
High-quality Start-up Late	(L_h^s, E_l^e)	(L_h^s, L_l^e)
	High-quality Es.Firm Early	High-quality Es.Firm Late
Low-quality Es.Firm Early	(E_l^e, E_h^e)	(E_l^e, L_h^e)
Low-quality Es.Firm Late	(L_l^e, E_h^e)	(L_l^e, L_h^e)

Table 2.1: Notations for the Capacity Timing Equilibria

2.4.1 High-quality Established Firm vs. Low-quality Start-up

We first examine the scenario in which the start-up produces a low quality product while the established firm produces a high quality product. We focus on the situations when both firms are able to make profits in this market, which requires an additional restriction on the distribution of $\hat{\theta}$.[3]

	ψ^*	$E(\pi^*)$
(E_l^s, E_h^e)	$1-G\left(2\sqrt{\frac{\alpha}{S_l}}+\frac{S_h\mu-\sqrt{S_l\alpha}-C_{1h}}{2S_h}+\frac{C_{1l}}{S_l}\right)$	$\frac{(S_h\mu-\sqrt{S_l\alpha}-C_{1h})^2}{4S_h}$
(E_l^s, L_h^e)	$1-G\left(2\sqrt{\frac{2(2S_h-S_l)}{S_lS_h}\alpha}-\frac{C_{2h}}{S_h}+\frac{2C_{1l}}{S_l}\right)$	$\frac{S_h}{4}(\mu^2+\sigma^2)-\left(\sqrt{\frac{S_lS_h\alpha}{2(2S_h-S_l)}}+\frac{C_{2h}}{2}\right)\mu$ $+\frac{S_l\alpha}{2(2S_h-S_l)}+\frac{C_{2h}^2}{4S_h}+\sqrt{\frac{S_l\alpha}{2S_h(2S_h-S_l)}}C_{2h}$
(L_l^s, E_h^e)	$1-G\left(\frac{\mu}{2}+\frac{C_{2l}-2C_{1h}}{2(2S_h-S_l)}+\frac{C_{2l}}{S_l}+2\sqrt{\frac{\alpha}{S_l}}\right)$	$\frac{2S_h-S_l}{8}\mu^2+\frac{(C_{2l}-2C_{1h})\mu}{4}+\frac{(C_{2l}-2C_{1h})^2}{8(2S_h-S_l)}$
(L_l^s, L_h^e)	$1-G\left(\frac{2C_{2l}S_h-C_{2h}S_l+(4S_h-S_l)\sqrt{S_l\alpha}}{S_lS_h}\right)$	$S_h\left(1-\frac{2S_h}{4S_h-S_l}\right)^2(\mu^2+\sigma^2)+\left(\frac{C_{2l}-2C_{2h}}{4S_h-S_l}\right)^2 S_h$ $+2S_h\left(1-\frac{2S_h}{4S_h-S_l}\right)\left(\frac{C_{2l}-2C_{2h}}{4S_h-S_l}\right)\mu$

Table 2.2: Optimal Survival Probability and Expected Profit When a Low-quality Start-up Competes Against a High-quality Established Firm

In Table 2, we present the optimal survival probability for the start-up and the optimal profit for the established firm in the capacity quantity subgame given each pair of timing decisions. With these results, we are ready to derive the equilibria of the supergame. Taking one case for instance, we find that when $\psi^*(E_l^s,\, L_h^e) \geq \psi^*(L_l^s,\, L_h^e)$ and $E(\pi^*)(E_l^s,\, L_h^e) \geq E(\pi^*)(E_l^s,\, E_h^e)$ are both satisfied, $(E_l^s,\, L_h^e)$ is an equilibrium. For brevity, we do not list all the equilibria and the corresponding conditions. In the following proposition, we present some important and interesting equilibria.

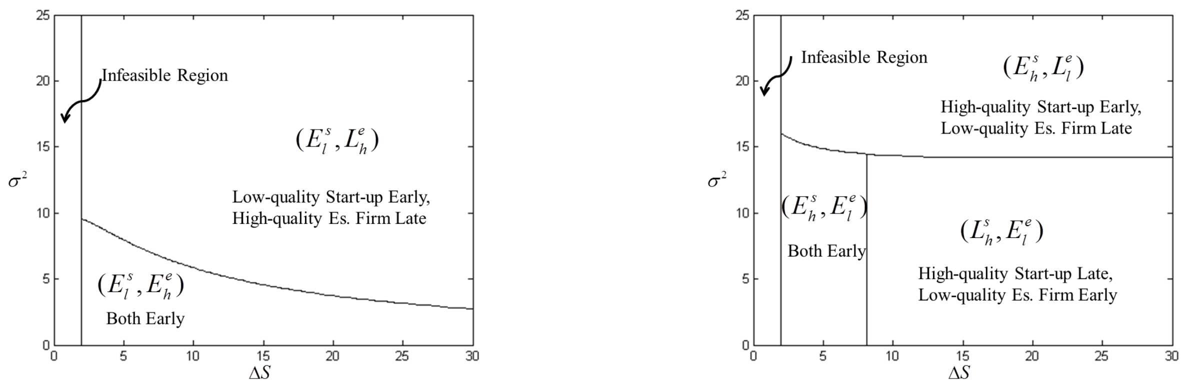
Proposition 2.4.1. *Suppose that a start-up with a low quality product competes against an established firm with a high quality product. If $C_{1l} < C_{2l}$,*

i) There exists a threshold $\bar{\sigma}_{se}$ such that for all $\sigma > \bar{\sigma}_{se}$, a unique equilibrium arises in which the start-up invests early while the established firm invests late.

ii) Neither an equilibrium in which the start-up invests late while the established firm invests early nor an equilibrium in which both firms invest late arises.

[3] To ensure that both firms would make nonnegative profits, we further assume $\hat{\theta} \geq \max\{\sqrt{\frac{2S_l\alpha}{S_h(2S_h-S_l)}}+\frac{C_{2h}}{S_h}, \frac{2C_{2l}S_h-C_{2h}S_l}{S_lS_h}, \frac{\mu}{2}+\frac{C_{2l}-2C_{1h}}{2(2S_h-S_l)}+\frac{C_{2l}}{S_l}, \frac{\sqrt{S_l\alpha}}{S_h}+\frac{C_{1h}}{S_h}, \sqrt{\frac{2\alpha}{2S_h-S_l}}+\frac{C_{2l}}{S_l}, \frac{S_lS_h\mu+C_{2h}S_l-2C_{1l}S_h}{2S_h(2S_h-S_l)}+\frac{C_{2h}}{S_h}, \frac{2C_{1l}S_h-C_{2h}S_l}{S_lS_h}, \sqrt{\frac{\alpha}{S_h}}+\frac{C_{1l}}{S_l}\}$.

To explain the results of Proposition 2.4.1, we compute a numerical example in Figure 2.3(a). It shows that as the capacity cost for the low quality product increases over time, neither (L_l^s, L_h^e) nor (L_l^s, E_h^e) would ever arise in equilibrium, which implies that the low quality start-up always behaves aggressively and takes a capacity-leading strategy. Furthermore, Figure 2.3(a) indicates that when the demand volatility exceeds some level, (E_l^s, L_h^e) becomes a unique equilibrium, which is consistent with the result in Proposition 2.4.1. In other words, when the demand is highly volatile, the established firm chooses a capacity-lagging strategy. This shows that the established firm cares more about demand uncertainty than the start-up.



(a) Low-quality Start-up and High-quality Es. Firm (b) High-quality Start-up and Low-quality Es. Firm

Figure 2.3: Capacity Investment Timing Equilibria, $C_{1h} < C_{2h}, C_{1l} < C_{2l}, S_l = 5, C_{1h} = 25, C_{1l} = 10, \alpha = 10, \mu = 10$

2.4.2 Low-quality Established Firm vs. High-quality Start-Up

In this scenario, we switch the product quality level of the start-up and the established firm, i.e, the start-up now produces a high quality product while the established firm produces a low quality product. We are curious whether this change would substantially affect the investment dynamics we have observed previously. The results for the capacity subgame are summarized in Table 2.3, and the equilibria are presented in Proposition 2.4.2.

	ψ^*	$E(\pi^*)$
(E_h^s, E_l^e)	$1-G\left(2\sqrt{\frac{\alpha}{S_h}}+\frac{S_l\mu-S_l\sqrt{\alpha/S_h}-C_{1l}}{2S_h}+\frac{C_{1h}}{S_h}\right)$	$\frac{(S_l\mu-S_l\sqrt{\alpha/S_h}-C_{1l})^2}{4S_l}$
(E_h^s, L_l^e)	$1-G\left(2\sqrt{\frac{2\alpha}{2S_h-S_l}}+\frac{2C_{1h}-C_{2l}}{2S_h-S_l}\right)$	$\frac{S_l}{4}(\mu^2+\sigma^2)-\left(S_l\sqrt{\frac{\alpha}{2(2S_h-S_l)}}+\frac{C_{2l}}{2}\right)\mu$ $+\frac{S_l\alpha}{2(2S_h-S_l)}+\frac{C_{2l}^2}{4S_l}+\sqrt{\frac{\alpha}{2(2S_h-S_l)}}C_{2l}$
(L_h^s, E_l^e)	$1-G\left(\frac{S_lS_h\mu+S_lC_{2h}-2S_hC_{1l}}{2S_h(2S_h-S_l)}+\frac{C_{2h}}{S_h}+2\sqrt{\frac{\alpha}{S_h}}\right)$	$\frac{(S_lS_h\mu+S_lC_{2h}-2S_hC_{1l})^2}{8S_lS_h(2S_h-S_l)}$
(L_h^s, L_l^e)	$1-G\left(\frac{-S_h(C_{2l}-2C_{2h})+(4S_h-S_l)\sqrt{S_h\alpha}}{S_h(2S_h-S_l)}\right)$	$\frac{S_lS_h^2}{(4S_h-S_l)^2}(\mu^2+\sigma^2)-\frac{2S_h(2C_{2l}S_h-C_{2h}S_l)}{(4S_h-S_l)^2}\mu$ $+\frac{(2C_{2l}S_h-C_{2h}S_l)^2}{S_l(4S_h-S_l)^2}$

Table 2.3: Optimal Survival Probability and Expected Profit When a Low-quality Established Firm Competes Against a High-quality Start-up

Proposition 2.4.2. *Suppose that an established firm with a low quality product competes against a start-up with a high quality product.*

i) If $C_{1l} < C_{2l}$ and $C_{2h} - C_{1h} < \frac{S_l\mu}{4}$, there exists a threshold $\bar{\sigma}_{es} > 0$ and a threshold $\bar{S}_h > 0$ such that for $\sigma < \bar{\sigma}_{es}$ and $S_h > \bar{S}_h$, is a unique equilibrium arises in which the start-up invests late while the established firm invests early. If $C_{2h} - C_{1h} > \frac{S_l\mu}{4}$, an equilibrium in which the start-up invests late while the established firm invests early never arises.

ii) If $C_{1h} < C_{2h}$, an equilibrium in which both firms invest late never arises. Furthermore, there exists a threshold $\tilde{\sigma}_{es}(\tilde{\sigma}_{es} \geq \bar{\sigma}_{es})$ such that for $\sigma > \tilde{\sigma}_{es}$, a unique equilibrium arises in which the start-up invests early while the established firm invests late.

Proposition 2.4.2 deals with the case when a high quality start-up competes against a low quality established firm. Again, we use a numerical example to illustrate the findings and intuitions, which is displayed in Figure 2.3(b). When the capacity cost for the high quality product increases over time, we can see from the figure that (L_h^s, L_l^e) is never an equilibrium. Among the equilibria displayed in Figure 2.3(b), (L_h^s, E_l^e) is the most interesting one. As both indicated in the Proposition 2.4.2 and Figure 2.3(b), with increasing investment costs over time, (L_h^s, E_l^e) becomes a unique equilibrium when the following conditions hold: the capacity cost for the start-up does not increase too much over time, the quality differential is sufficiently large, and the demand volatility is sufficiently low.

This observation contrasts sharply with the findings in Swinney et al. (2011). In their setting, when a start-up competes against an established firm, and when the capacity cost increases over time, the equilibrium with established firm investing early and start-up investing late never exists. However, when quality differentiation is incorporated into the model, this equilibrium emerges in the scenario described above. We shall emphasize that the condition $C_{2h} - C_{1h} < \frac{S_l\mu}{4}$ in Proposition 2.4.2 is critical in deriving the unique equilibrium (L_h^s, E_l^e). We show this by presenting a numerical counterexample in Figure 3.4. We see that equilibrium (L_h^s, E_l^e) no longer exists when this condition is violated. Equilibrium (E_h^s, L_l^e) is also worth noticing. As stated in Proposition 2.4.2, when demand volatility is sufficiently high, this equilibrium becomes a unique equilibrium. This reflects that the high quality start-up could also be aggressive and adopt a capacity-leading strategy when the demand is highly volatile.

Last but not least, in Figures 2.3(a) and 2.3(b), when demand volatility is high, there exist a unique equilibrium in which the start-up chooses the capacity-leading strategy when facing a highly volatile demand, while the established firm acts as a follower. This result is consistent with the results in Proposition 2.4.1(i) and Proposition 2.4.2(ii). This equilibrium of start-up investing early and established firm investing late replicates the result of Swinney et al. (2011), but in a setting of quality differentiation.

2.4.3 Two Established Firms

We now proceed to the benchmark case of two established firms competing against each other. By comparing it to the start-up vs. established firm scenario, we can isolate

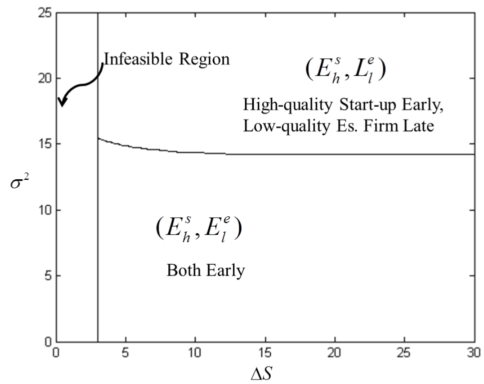


Figure 2.4: A Special Case When a Low-quality Established Firm Competes Against a High-quality Start-Up, $C_{2h} - C_{1h} > \frac{S_l \mu}{4}$

the effect of bankruptcy risk on capacity investment timing strategies. The results for the capacity subgame are summarized in Table 2.4, and the equilibria are presented in Proposition 2.4.3.

	$E(\pi_l^*)$	$E(\pi_h^*)$
(E_l^e, E_h^e)	$S_l\left(\frac{S_h}{4S_h-S_l}\mu - \frac{2C_{1l}S_h - C_{1h}S_l}{S_l(4S_h-S_l)}\right)^2$	$S_h\left(\frac{2S_h-S_l}{4S_h-S_l}\mu + \frac{C_{1l}-2C_{1h}}{4S_h-S_l}\right)^2$
(E_l^e, L_h^e)	$\frac{(S_lS_h\mu+S_lC_{2h}-2S_hC_{1l})^2}{8S_lS_h(2S_h-S_l)}$	$S_h\left(\frac{\mu}{2} - \frac{S_lS_h\mu+S_lC_{2h}-2S_hC_{1l}}{4(2S_h-S_l)S_h} - \frac{C_{2h}}{2S_h}\right)^2 + \frac{S_h}{4}\sigma^2$
(L_l^e, E_h^e)	$S_l\left(\frac{\mu}{4} - \frac{C_{2l}-2C_{1h}}{4(2S_h-S_l)} - \frac{C_{2l}}{2S_l}\right)^2 + \frac{S_l}{4}\sigma^2$	$\left(\frac{2S_h-S_l}{2}\right)\left(\frac{\mu}{2} + \frac{C_{2l}-2C_{1h}}{2(2S_h-S_l)}\right)^2$
(L_l^e, L_h^e)	$S_l\left(\frac{S_h}{4S_h-S_l}\mu - \frac{2C_{2l}S_h - C_{2h}S_l}{S_l(4S_h-S_l)}\right)^2 + \frac{S_lS_h^2}{(4S_h-S_l)^2}\sigma^2$	$S_h\left(\frac{2S_h-S_l}{4S_h-S_l}\mu + \frac{C_{2l}-2C_{2h}}{4S_h-S_l}\right)^2 + S_h\left(\frac{2S_h-S_l}{4S_h-S_l}\right)^2\sigma^2$

Table 2.4: Two Established Firms

Proposition 2.4.3. *When two established firms, one with low quality and the other with high quality, compete against each other, there exists a threshold $\bar{\sigma}_{ee}$ such that for all $\sigma > \bar{\sigma}_{ee}$, a unique equilibrium in which both firms invest late arises.*

Figure 2.5 illustrates the results of Proposition 2.4.3. Notice that (L_l^e, L_h^e) arises as an equilibrium in Figure 2.5, which does not appear in Figures 2.3(a) and 2.3(b). As indicated in Proposition 2.4.3, this unique equilibrium occurs when demand volatility exceeds a threshold value. In Figure 2.5, this equilibrium occurs in the upper right corner. The emergence of this unique equilibrium is caused by the change of competition

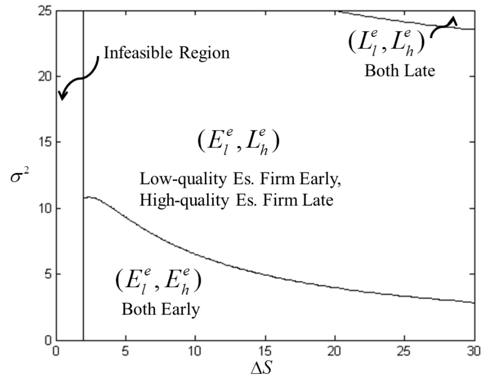


Figure 2.5: Capacity Investment Timing Equilibria, $C_{1h} < C_{2h}, C_{1l} < C_{2l}, S_l = 5, C_{1h} = 25, C_{1l} = 10, \mu = 10$. Two Established Firms

structure from a start-up vs. an established firm to two established firms. From Figures 2.3(a) and 2.3(b), we see that a start-up competing against an established firm always chooses a capacity-leading strategy whenever the demand is highly volatile. But competing established firms choose a capacity-lagging strategy when facing a highly volatile demand.

2.5 Equilibrium of the Quality Choice Game

In this section, we analyze the equilibrium of the quality choice game. Given the two-level quality choices, there are four possible quality equilibria. We use (S_l^s, S_h^e) to represent the equilibrium in which the start-up chooses the low quality level while the established firm chooses the high quality level. The other three equilibria are similarly specified in Table 2.5.

	Es.Firm chooses Low-quality	Es.Firm chooses High-quality
Start-up chooses Low-quality	(S_l^s, S_l^e)	(S_l^s, S_h^e)
Start-up chooses High-quality	(S_h^s, S_l^e)	(S_h^s, S_h^e)

Table 2.5: Notations for the Quality Equilibria

We solve the problem backwardly by solving the subgame first. That is, given the quality choices of both firms, we solve the capacity quantity subgame and derive equilibria. Note that when both firms choose different quality levels, the subgame is identical to what we have solved in the previous section. But we need to consider a new scenario here, that is, when both firms choose the same quality level, i.e., $S_s = S_e$. In this case, the subgame is reduced to a Cournot game with no product differentiation(see Appendix A.1).

There are four different capacity timing scenarios: (1) (E^s, E^e), representing that both the start-up and the established firm invest early; (2) (L^s, L^e), representing that both the start-up and the established firm invest late; (3) (E^s, L^e), representing that the start-up invests early while the established firm invests late; (4) (L^s, E^e), representing that the start-up invest late while the established firm invest early.

In this section we focus on the analysis for (E^s, E^e), i.e., both the start-up and the established firm invest early. This case is relatively more interesting than the other cases because it keeps the firms symmetric in term of capacity timing and at the same time preserves the effect of demand uncertainty, which helps us better understand the outcome of the quality choice game. The subgame equilibrium analysis of the (E^s, E^e) scenario is given in Table 2.6. For brevity, we provide the subgame analysis of the other three timing scenarios in the Appendix.

Quality Choices	ψ^*	$E(\pi^*)$
(S_l^s, S_l^e)	$1 - G\left(2\sqrt{\frac{\alpha}{S_l}} + \frac{S_l\mu - \sqrt{S_l\alpha} - \beta_2 S_l^2}{2S_l} + \beta_1 S_l\right)$	$\frac{(S_l\mu - \sqrt{S_l\alpha} - \beta_2 S_l^2)^2}{4S_l}$
(S_l^s, S_h^e)	$1 - G\left(2\sqrt{\frac{\alpha}{S_l}} + \frac{S_h\mu - \sqrt{S_l\alpha} - \beta_2 S_h^2}{2S_h} + \beta_1 S_l\right)$	$\frac{(S_h\mu - \sqrt{S_l\alpha} - \beta_2 S_h^2)^2}{4S_h}$
(S_h^s, S_l^e)	$1 - G\left(2\sqrt{\frac{\alpha}{S_h}} + \frac{S_l\mu - S_l\sqrt{\alpha/S_h} - \beta_2 S_l^2}{2S_h} + \beta_1 S_h\right)$	$\frac{(S_l\mu - S_l\sqrt{\alpha/S_h} - \beta_2 S_l^2)^2}{4S_l}$
(S_h^s, S_h^e)	$1 - G\left(2\sqrt{\frac{\alpha}{S_h}} + \frac{S_h\mu - \sqrt{S_h\alpha} - \beta_2 S_h^2}{2S_h} + \beta_1 S_h\right)$	$\frac{(S_h\mu - S_h\sqrt{\alpha/S_h} - \beta_2 S_h^2)^2}{4S_h}$

Table 2.6: The Start-Up's Optimal Survival Probability and The Established Firm's Expected Profit When Both Firms Make Early Capacity Investment

2.5.1 Asymmetric and Symmetric Quality Equilibria

Next we characterize the equilibria of the quality choice game. Note that (S_l^s, S_h^e) is an equilibrium if and only if $\psi^*(S_l^s,\, S_h^e) \geq \psi^*(S_h^s,\, S_h^e)$ and $E(\pi^*)(S_l^s,\, S_h^e) \geq E(\pi^*)(S_l^s,$

S_l^e) are both satisfied. Similar arguments can be applied to the other three equilibria: (S_l^s, S_l^e), (S_h^s, S_l^e) and (S_h^s, S_h^e). An immediate observation is that if there exist multiple equilibria, at most two equilibria are possible: either the symmetric multiple equilibria: (S_l^s, S_l^e) and (S_h^s, S_h^e), or the asymmetric multiple equilibria: (S_l^s, S_h^e) and (S_h^s, S_l^e). The next proposition characterizes conditions under which an asymmetric equilibrium arises.

Proposition 2.5.1. *Suppose both firms invest early. When the following conditions are satisfied, an asymmetric equilibrium arises. Specifically,*

1. *An equilibrium arises in which the start-up chooses the low quality level while the established firm chooses the high quality level if* $S_l > (\frac{5\sqrt{\alpha}}{4\beta_1})^{2/3}$ *and* $S_h < \frac{\mu}{3\beta_1}$. *Furthermore, this equilibrium is unique if* $S_l > \underline{S}_l$ *or* $S_h \in [\underline{S}_h, \bar{S}_h]$ *holds, where* $\underline{S}_l$ *is the solution to* $\mu(\underline{S}_l)^{-1} + \sqrt{\alpha}(\underline{S}_l)^{-\frac{3}{2}} = 3\beta_1$ *and* $\underline{S}_h, \bar{S}_h$ *are the solutions to* $S_h + \frac{\sqrt{\alpha}}{\sqrt{S_h}} \frac{1}{3\beta_1} = \frac{\mu}{3\beta_1}$.
2. *A unique equilibrium arises in which the start-up chooses the high quality level, while the established firm chooses the low quality level if* $S_l > \frac{\mu}{3\beta_1}$ *and* $S_h < (\frac{3\sqrt{\alpha}}{4\beta_1})^{2/3}$.

Proposition 2.5.1 provides sufficient conditions for the existence of asymmetric equilibria. Comparing the conditions of part 1 and part 2 of Proposition 2.5.1 yields two interesting observations. First, when μ, the market size, increases, the conditions of part 1 are more likely to be satisfied while the conditions of part 2 are less likely to be satisfied. This implies that (S_l^s, S_h^e), the equilibrium in which the start-up chooses low quality and the established firm chooses high quality, is more likely to arise when the market size increases. Second, when α, the threshold of the start-up's survival probability, increases, the the conditions of part 1 are less likely to be satisfied while the conditions of part 2 are more likely to be satisfied. This implies that (S_h^s, S_l^e), the equilibrium in which the start-up chooses high quality and the established firm chooses low quality, is more likely to arise when the threshold increases.

Proposition 2.5.2. *Suppose both firms invest early. When the following conditions are satisfied, a symmetric equilibrium arises. Specifically,*

1. *An equilibrium arises in which both firms choose high quality if* $S_h < (\frac{5\sqrt{\alpha}}{4\beta_1})^{2/3}$ *and* $S_h \in [\underline{S}_h, \bar{S}_h]$*, where* $\underline{S}_h$ *and* $\bar{S}_h$ *are the solutions to* $S_h + \frac{\sqrt{\alpha}}{\sqrt{S_h}} \frac{1}{3\beta_1} = \frac{\mu}{3\beta_1}$*. Furthermore, this equilibrium is unique if* $S_h < (\frac{3\sqrt{\alpha}}{4\beta_1})^{2/3}$ *or* $S_h < \frac{\mu}{3\beta_1}$ *holds.*
2. *A unique equilibrium arises in which both firms choose low quality if* $S_l > \frac{\mu}{2\beta_1}$ *and* $S_l > \underline{S}_l$*, where* $\underline{S}_l$ *is the solution to* $\mu(\underline{S}_l)^{-1} + \sqrt{\alpha}(\underline{S}_l)^{-\frac{3}{2}} = 3\beta_1$*.*

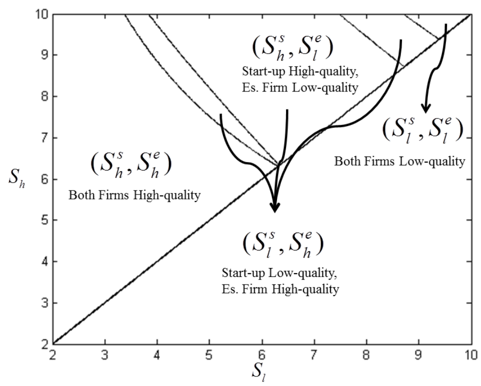
Proposition 2.5.2 provides sufficient conditions for the existence of symmetric equilibria. By comparing the conditions of part 1 and part 2 of Proposition 2.5.2, we observe that when μ or α increases, the conditions of part 1 are more likely to be satisfied while the conditions of part 2 are less likely to be satisfied. This implies that (S_h^s, S_h^e), the equilibrium in which both firm choose high quality, is more likely to arise when the market size or the threshold of the start-up's survival probability increases.

Taken together, Propositions 2.5.1 and 2.5.2 suggest that *when the market size increases, the established firm is more likely to choose high quality. Moreover, the start-up is more likely to choose high quality when the threshold of the start-up's survival probability increases.*

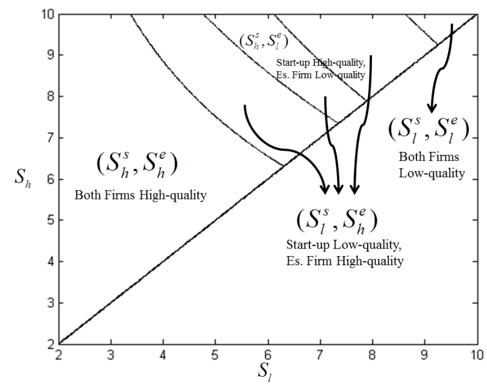
2.5.2 Comparative Statics of Quality Levels

So far we have assumed the two quality levels are given. Next, we conduct comparative statics of quality levels. We use Figure 2.6 to help illustrate the results. The equilibria are marked in the upper triangular region since S_h is assumed to be larger than S_l. As indicated in the figure, when S_h is relatively small (note that relatively small S_h also implies relatively small S_l), or when S_l is extremely small and S_h is relatively high, a unique symmetric equilibrium occurs in which both firms choose high quality. As

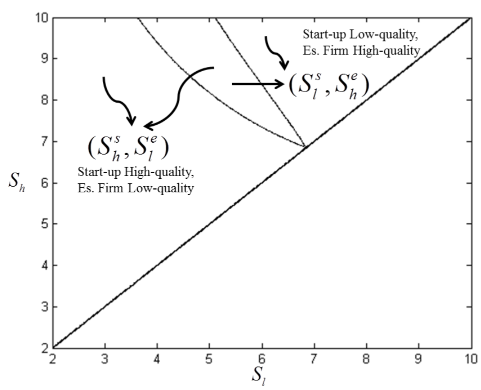
S_l and S_h increase, the equilibrium changes to the asymmetric case, in which quality differentiation helps both firms improve profits. Interestingly, in part of this region, we also observe two equilibria co-exist, both of which are asymmetric. Finally, in the right upper corner of the triangular area, the unique symmetric equilibrium arises with both firms choosing the low quality.



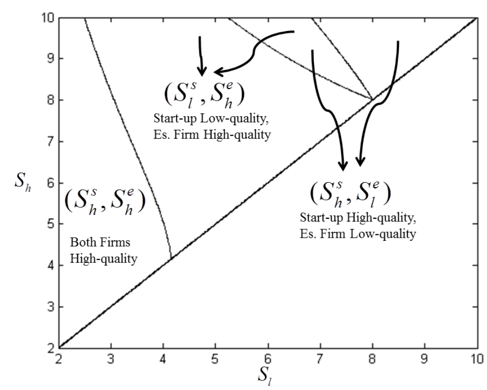
(a) Early Start-up and Early Es. Firm



(b) Late Start-up and Late Es. Firm



(c) Early Start-up and Late Es. Firm



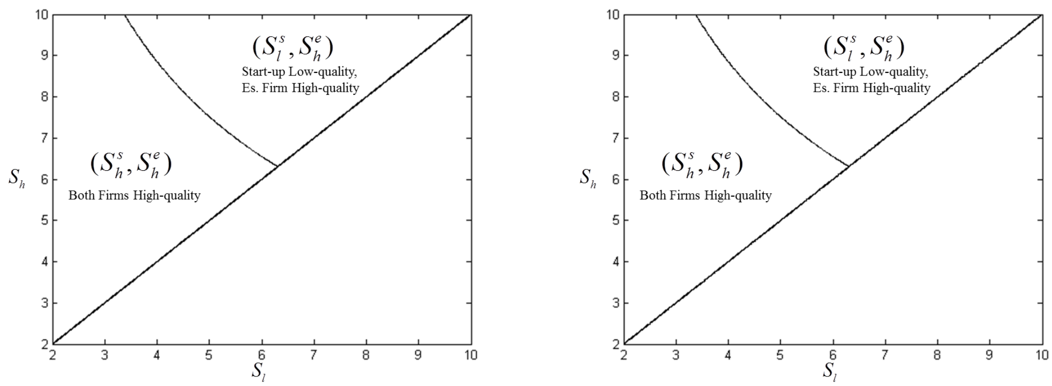
(d) Late Start-up and Early Es. Firm

Figure 2.6: Quality Equilibria under the Four Capacity Timing Scenarios: Relatively Small Demand, $\mu = 6$, $\sigma^2 = 1$, $\alpha = 10$, $\beta_1 = 0.25$

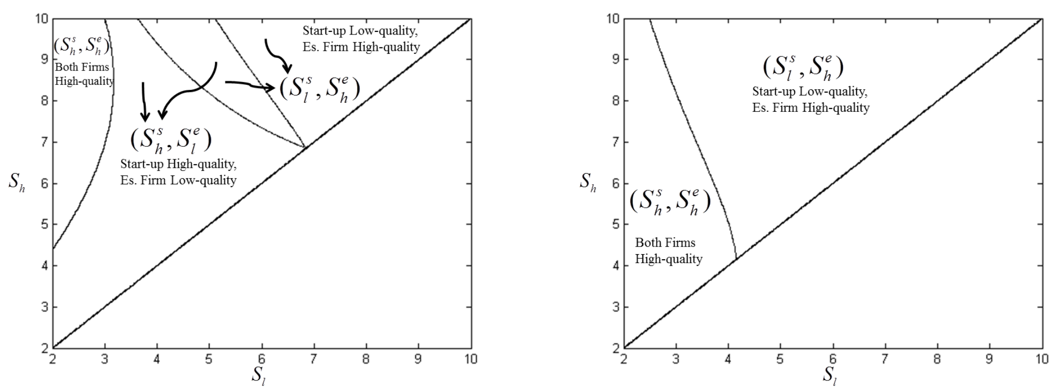
2.5.3 The Interdependence between Quality Choice and Capacity Timing

Next we study the interdependence between quality choice and capacity timing. To this end, we carry out similar numerical analysis for the other three capacity timing scenarios. The results are presented in Figure 2.6 (b), (c) and (d).

Comparing Figure 2.6 (a) and (c) leads to an interesting observation: the $(S_h^s,\ S_h^e)$ region in (a) mostly changes to $(S_h^s,\ S_l^e)$ in (c). This implies when the start-up invests early, the established firm's low quality choice seems to be correlated with its late investment timing. In this situation, quality differentiation arises in the market. Interestingly when the potential quality differentiation (i.e., when the difference between S_h and S_l is large), the start-up tends to choose high quality while the established firm tends to choose low quality. Moreover, we observe that symmetric quality equilibria do not exist in (c), which suggests that *asymmetric quality equilibria are correlated with asymmetric capacity timing.*



(a) Early Start-up and Early Es. Firm (b) Late Start-up and Late Es. Firm



(c) Early Start-up and Late Es. Firm (d) Early Start-up and Late Es. Firm

Figure 2.7: Quality Equilibria under the Four Capacity Timing Scenarios: Relatively High Demand, $\mu = 10$, $\sigma^2 = 1$, $\alpha = 10$, $\beta_1 = 0.25$

Comparing Figure 2.6 (a) and (b), we observe that the equilibrium patterns are similar, except that the region with multiple equilibria shrinks. This implies that as long as the firms' capacity timing decisions are symmetric, investing early or investing late do not seem to make a significant impact on the quality choices.

The asymmetric quality equilibria in Figure 2.6 (c) are particularly interesting because the (E^s, L^e) capacity timing pattern resembles disruptive innovations in various industries. Empirical evidence suggests that start-ups may take either high-end or low-end encroachment strategies. Our results suggest that two conditions may make start-ups more likely to choose high-end encroachment strategies: (1) when the potential quality differentiation (i.e., when the difference between S_h and S_l) is large; (2) when the potential quality differentiation is small and the absolute quality level is small.

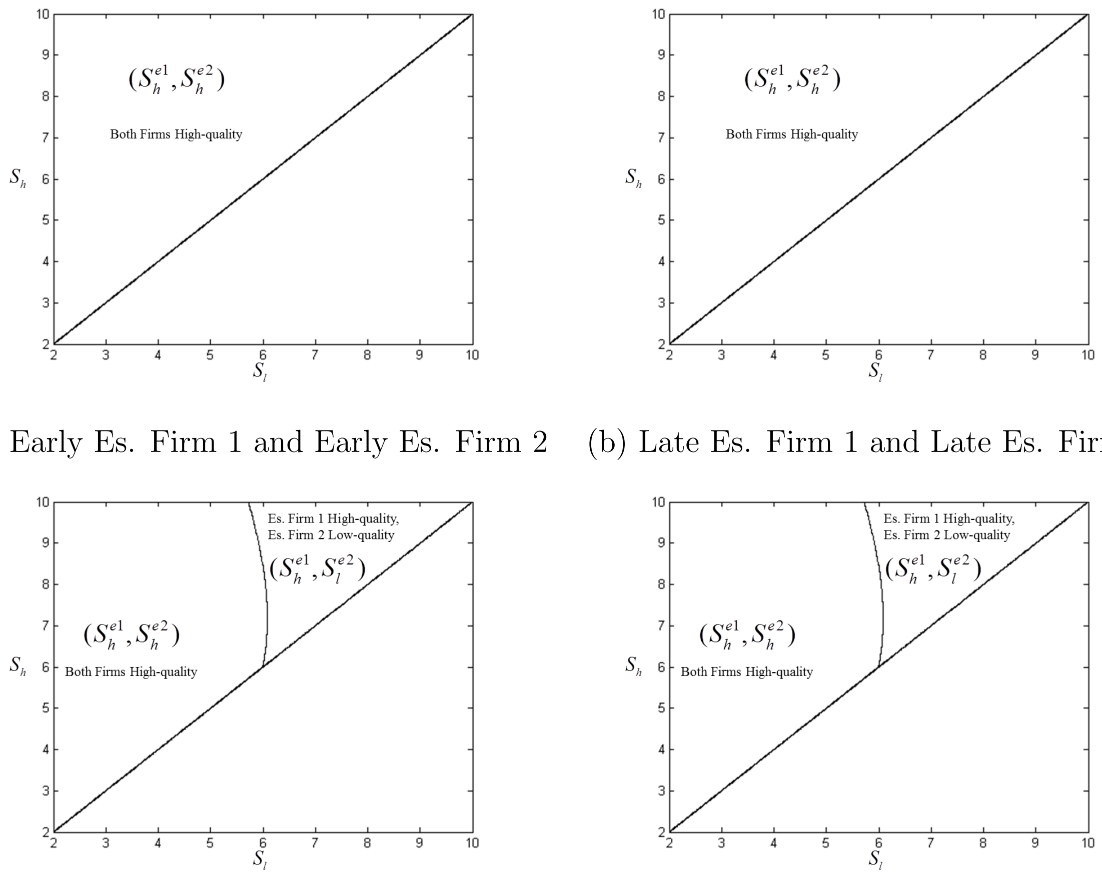
Next we examine how quality choices of firms depend on the market size. The numerical results are displayed in Figure 2.7. Comparing it with Figure 2.6 suggests that (S_h^s, S_h^e) and (S_l^s, S_h^e) are more likely to arise when the market size increases. This observation is consistent with the results of Proposition 2.5.1 and Proposition 2.5.2. (see the discussion after Proposition 2.5.1 and Proposition 2.5.2)[4]

2.5.4 Quality Choices of Two Established Firms

One of research questions asks whether the presence of a start-up in the market increase or decrease product differentiation. To answer this question, we analyze the benchmark of two established firms. The results are displayed in Figure 2.8. Comparing the quality choice equilibria of this figure with Figure 2.7 yields the following observation:

[4]For Figure 2.7 (c), there are more than two equilibria displayed in the graph. If we continue to increase μ, (S_h^s, S_h^e) and (S_l^s, S_h^e) will be the only two equilibria left.

the presence of start-up increase product differentiation. For instance, Figure 2.7(a) has the equilibrium in which the start-up chooses high quality and the established firm chooses low quality (S_l^s, S_h^e). This asymmetric equilibrium does not exists in Figure 2.8(a), which only has the symmetric equilibrium in which both firms choose high quality.



(a) Early Es. Firm 1 and Early Es. Firm 2 (b) Late Es. Firm 1 and Late Es. Firm 2

(c) Early Es. Firm 1 and Late Es. Firm 2 (d) Late Es. Firm 1 and Early Es. Firm 2

Figure 2.8: Two Established Firms, Quality Equilibria under the Four Capacity Timing Scenarios: Relatively High Demand, $\mu = 10$, $\sigma^2 = 1$, $\alpha = 10$, $\beta_1 = 0.25$

2.6 Concluding Remarks

Inspired by various disruptive innovations observed in practice, we study the innovation strategies of start-ups and established firms along two dimensions: quality choice and capacity timing. We build a stylized duopoly model in which a start-up and an established firm compete in a market with quality differentiation and demand uncertainty.

Our analysis generate valuable insights that can be applied to understand the drivers of high-end vs. low-end disruptive innovations taken by either start-ups or established firms in the market. For instance, we find that when the market size increases, the established firm is more likely to choose high quality. Moreover, the start-up is more likely to choose high quality when the threshold of the start-up's survival probability increases. Second, we find that the presence of a start-up in the market tends to increase quality differentiation. Third, our results suggest that the firms' quality choice and capacity timing are interdependent. Specifically, we find that quality differentiation is more likely to arise when the two firms choose different capacity timing. Fourth, we also identify an interesting equilibrium in which the established firm with a low-equality product chooses a capacity-leading strategy when the start-up with a high-quality chooses a capacity-lagging strategy. This equilibrium is consistent with anecdotal evidence on established firms being disruptive innovators.

Limitations exist in our model. First, our model does not capture the setting when both the quality choices and the capacity timing decisions are endogenous. Endogenizing both decisions in our model with demand uncertainty inevitably leads to an intractable model. To overcome this, we focus on the quality choice game and the capacity timing game separately, but consider different timing and quality scenarios in the two games, respectively. This simplification leads to interesting managerial insights about the interdependence between quality choice and capacity timing. One issue may arise when endogenizing both the quality choices and the timing decisions together in a model—it is not clear whether the quality choices should be determined before or after the timing decisions, or both should be determined concurrently. Relevant anecdotal evidence needs to be collected to validate such an assumption on the sequence of events.

Second, we adopt a production clearance strategy. A more realistic assumption would be that the production quantity is bounded by the capacity level, and hence is not necessarily equal to it. Thus, holdback may happen. Although we find it difficult to obtain analytical results under such an assumption, further work in this area is worthwhile.

Third, the objective functions we adopt for start-ups and established firms only describe the extreme cases, i.e., start-ups only care about bankruptcy risk while established firms only care about profit. Nevertheless, in reality, all firms would care about not only profit but also bankruptcy risk. And they may differ in the relative weight of these two factors.[5]

[5] Swinney et al. (2011) show that the main results still hold when using a combined form of objective function.

CHAPTER 3 PROCUREMENT CONTRACTING UNDER PRODUCT RECALL RISK

3.1 Introduction

In today's competitive global markets, product quality and consumer satisfaction are raised to an unparalleled status. However, product recalls are commonly observed in various industries. When a major product quality failure is detected, either due to design flaws or production defects, the manufacturer may recall all the affected batches of products that may potentially contain the defect. Leading manufacturers, renowned for high quality, may still suffer from massive product recalls. As a legend in the automobile industry, Toyota has long been recognized as a leader in manufacturing and quality management. However, it ran into serious challenges in recent years: a series of recalls were issued by Toyota between 2009 and 2010. The first recall, happened in September 2009, including eight models and 3.8 million vehicles, was due to gas pedals sticking on floor mats that may cause dangerous sudden acceleration. Later that year, two additional recalls followed and 4.2 million more vehicles were called back. At the beginning of 2010, Toyota issued two additional recalls which amount to 3.39 million cars in the U.S., and then decided to expand the recall to cover over 1.8 million cars across Europe and China. Meanwhile, Toyota temporarily suspended US sales and production of all the eight models involved in the recall. As a result, Toyota lost five percent market share in the US, which may further rise as an aftershock of the crisis. To make the matter worse, a group of law firms sued Toyota to compensate losses due to deaths and injuries related to the quality issues. Product recalls are not unique to the automobile industry, they are also observed in the food industry (e.g. (Thomsen and McKenzie 2001)) and the medical device industry (e.g. (Thirumalai and Sinha 2011)). Even in the aerospace industry, for example, Boeing announced in early 2013 that they were recalling all of their 50 manufactured 787 Dreamliner Aircraft due to a faulty Lithium Ion Battery outsourced from a supplier. As a consequence, over 1900 flights were rescheduled. Product recalls are not necessarily restricted to manufacturers, they can also happen to retailers. An example is Lululemon Athletica, an athletic apparel retailer, which recalled its yoga pants in the spring of 2013 [1]. An extensive review on product recalls can be found in (Marucheck, Greis, Mena and Cai 2011).

Typically, the financial impact of product recalls is enormous in magnitude. Once a recall is issued, the number of products involved could be numerous. There are direct costs from repairs, recall logistics, and litigation fees, and indirect costs due to lost sales, damaged reputation, and manufacturing downtime. (see also (Hendricks and Singhal 2003), (Rupp 2004) and (Jarrell and Peltzman 1985)). Product recalls are generally caused by product quality failures. In many cases, the defects lie in the components or the products outsourced from an external supplier. In the presence of outsourcing, ensuring product quality become particularly challenging because quality failures could result from the use of low-quality raw materials, design flaws, or manufacturing defects, which are managed by different parties of a supply chain. As a result, information asymmetry and moral hazard make it impossible for manufacturers to rely on supplier's self-motivation to achieve quality. There are different ways to induce suppliers to invest in quality. One way is through quality inspection, but it can be costly and time consuming. An alternative and complementary way is providing financial incentives to suppliers through structured contracts on quality. The challenge of quality contracting is that in general the suppliers quality effort cannot be directly observed and the quality output is uncertain. What complicates quality contracting even further is the fact that it is often times unrealistic

[1] http://online.wsj.com/article/SB10001424127887323415304578369812787114262.html

to have a clear-cut assignment of fault to each firm involved in the product development and manufacturing processes. It is not uncommon for a manufacturer and a supplier to point fingers at each other when product failures occur. Lululemons yoga pants were recalled in 2013 because they became too thin when being stretched. The apparel retailer blamed the defective batches of yoga pants on its Taiwanese supplier, Eclat Textile, who manufactured the products. The supplier, however, refused to accept this blame and argued that it followed Lululemons design specifications in choosing the fabrics, and the products went through a certification process approved by the retailer. Such quality disputes may also arise when product architecture is complex and ambiguity in fault determination. A classical story was the one between Ford and its tire supplier Firestone in a massive recall of Ford Explorer in 2000.[2] Ford claimed that the tires were faulty and could, under certain conditions, cause vehicles to roll over. Firestone claimed that the problem was with the vehicle. Some suspected that the quality failure lied in the interfacing area of the vehicle and the tires.

Due to the challenge of directly contracting on quality effort and output, some manufactures have used recall cost sharing contracts to incentivize suppliers quality effort. For instance, in August 2013, General Motors announced that it would start sharing recall costs with its suppliers even if their products initially passed GM's quality inspection but later were found to be defective.[3] In this paper, we will focus on recall cost sharing contracts and investigate their effectiveness in improving quality.

Naturally, the financial impact of product recalls also depends on the total sales, creating an inherent dependence between quality and quantity decisions. As a result,

[2] http://www.people.vcu.edu/~dbromley/firestoneLink.htm, http://en.wikipedia.org/wiki/Firestone_and_Ford_tire_controversy

[3] http://www.autonews.com/article/20130805/OEM10/308059934/gm-presses-suppliers-for-future-recall-costs

demand uncertainty further complicates manufacturers effort in managing product recall risks. With the demand risk, a careful quantity decision needs to be made, and who decides the quantity makes a difference. In a push system, the manufacturer commits to an order quantity before observing demand; while in a pull system, the manufacturer places an order after demand uncertainty is resolved while the supplier commits to a production quantity before demand realization.

Motivated by GM's practice, we center our research questions on how to mitigate recall risk under demand uncertainty. What is the interaction between quality and quantity decisions in the supply chain? To address this problem, we consider a model in which a manufacturer outsources to a supplier the production of a component, which is subject to potential quality failure leading to a product recall. The manufacturer acts as the Stackelberg leader offering a recall cost sharing contract to the supplier. We consider two regimes: (1) a push system in which the manufacturer sets both the wholesale price and the quantity before observing demand, leaving only the quality effort decision to the supplier; (2) a pull system, in which the manufacturer only determines the wholesale price while the supplier makes both the quantity decision and the quality effort decision before demand realization. The essential difference between the two regimes lies in who bears the demand risk. We find that the manufacturer achieves a higher production quantity and induces a higher quality effort of the supplier in the push system than in the pull system. Therefore, the manufacturer can improve quality by taking on the demand risk of the supply chain. Moreover, the presence of product recall risk decreases the production quantity in the push system but does not affect the production quantity in the pull system. Interestingly, the manufacturer can improve quality and profit by decreasing her share of the total recall cost without affecting the production quantity of the supply chain in both the push and pull systems.

The rest of the paper is organized as follows. Section 2 provides a review of related literatures. Section 3 presents the model while Sections 4 and 5 present the analysis for the model with and without the product recall risk, respectively. Section 6 concludes our work.

3.2 Literature Review

By incorporating both the recall risk and the demand risk in a procurement contracting setting, our work contributes to two streams of literatures in supply chain contracting: quality contracting and newsvendor contracting. There exists a considerable amount of related literature on newsvendor contracting literature in supply chains. This stream of literature mainly focuses on demand risk. (Lariviere and Porteus 2001) consider a simple wholesale price contract in which a manufacturer sells to a retailer facing a newsvendor problem. (Cachon and Lariviere 2001) study a manufacturer-retailer outsourcing setting in which the supplier need to construct capacity in advance of receiving order from the manufacturer. They consider both forced compliance and voluntary compliance regimes and study contracts that allow the supply chain to share demand forecasts credibly. (Cachon 2004) investigate both push and pull systems based on wholesale price based contracts and focus on how the allocation of inventory risk influences a supply chains performance and its division of profit. There are a number of papers which study more complicated supply chain coordinating contracts (buy-back contracts, revenue sharing contracts et al.). The focus is on how to design a contract between downstream and upstream players to eliminate double marginalization and maximize supply chain efciency. (Cachon 2003) provides an excellent review on the management of incentive conflicts with contracts in various newsvendor settings. There is a growing literature on quality contracting in supply chains. This stream of literature focuses on modeling the quality improvement incentives of supply chain members. Quality could be improved either by inspection or by investment from multiple players. (Reyniers and Tapiero 1995) consider a model in which a supplier makes a unobservable quality-related choice while Producer independently decides on his inspection policy for both non-cooperative and cooperative settings. (Lim 2001) also consider a product quality inspection problem. They investigate the contract design of a producer when he purchases parts from a supplier, and there is incomplete information regarding the quality of the parts. Similarly, (Baiman, Fischer and Rajan 2000) and (Baiman, Fischer and Rajan 2001) consider a setting where a supplier, who incurs quality improvement costs, sells an intermediate product to a buyer, who incurs appraisal costs to identify defects, and examine the relationship between product architecture, supply-chain performance metrics, and supply-chain efficiency. There are papers modeling quality improvement investment by both the supplier and the buyer. (Balachandran and Radhakrishnan 2005) examine a supply chain in which the nal product consists of components made by a buyer and a supplier in a double moral hazard situation. (Zhu, Zhang and Tsung 2007) focus on supply risk and consider a buyer who designs a product and outsources the production to a supplier and both players have options to invest in quality improvement. They also investigate the interaction between quality-improvement decisions interact and operational quantity decisions such as the buyer's order quantity and the supplier's production lot size using an EOQ model. In addition, all papers listed out above examine the fixed sharing rate contracts covering the external quality costs. As a more general case, (Chao, Iravani and Savaskan 2009) focus on recall instances, and discuss two external quality cost sharing contracts, in which product recall costs can be shared between a manufacturer and a supplier to induce effort from both sides to improve final product quality. They characterize the quality improvement effort decisions which are subject to moral hazard and even when there is information asymmetry regarding to the existing process capability. To summarize our contribution to the existing literatures, we are the first paper to establish the linkage between the product recall risk and the demand risk. Secondly we derive novel managerial insights into quality decisions in both push and pull systems.

r	$=$	market selling price
c	$=$	unit quantity cost
u	$=$	unit recall cost
s	$=$	quality effort marginal cost
λ	$=$	exponential rate
$\exp\{-\lambda e\}$	$=$	recall probability
g	$=$	density distribution
G	$=$	demand distribution
$\bar{G}$	$=$	complementary demand distribution
d	$=$	demand mean
$\underline{l}$	$=$	demand lower bound
θ	$=$	recall cost sharing percentage
w	$=$	wholesale price
e	$=$	quality effort by the supplier
q	$=$	order quantity
$\tilde{q}$	$=$	order quantity for the setting without product recall risk
$S(q)$	$=$	$E_D[\min(q, D)] = \int_0^q \bar{G}(x)dx =$ expected sales
Π	$=$	manufacturer's profit
π	$=$	supplier's profit
Σ	$=$	supply chain profit

Table 3.1: Notations

3.3 Model

We consider a single period, single sourcing setting where a downstream manufacturer outsources to an upstream supplier the production of a customized component. After receiving the components from the supplier, the manufacturer finishes assembling and releases the finished products to the market at price r. The market demand is assumed to be random with distribution function $G(x)$ and density function $g(x)$. Assume $G(x)$ to be continuous and differentiable. Let $\bar{G}(x) = 1 - G(x)$ denote complementary demand distribution.

The components produced by the supplier are subject to potential quality failure. Once a quality failure occurs, the manufacturer need to issue a product recall. This product recall risk is characterized by a recall probability in an exponential form of $\exp\{-\lambda e\}$, where e denotes the supplier's quality effort, and λ is a given constant. Note that the recall probability is decreasing and convex in e, which also resembles the realistic case. By deciding how much to invest in the quality effort e, the supplier can affect the possibility of product recall. Besides, we assume that e is not contractible, which implies the manufacturer, as a Stackelberg leader, faces a moral hazard problem and need to incentivize the supplier to invest more in quality effort.

A linear cost structure is adopted with unit quantity cost c, unit recall cost u, and unit quality effort cost s. In terms of the contract that the manufacturer provides to the supplier, we consider a simple recall cost sharing contract (wholesale price based contract), which is commonly used in the literatures. There are two parameters in this contract, the unit wholesale price w and the recall cost sharing percentage θ. In other words, the manufacturer offers the supplier with a wholesale price and the percentage of the total recall cost that he will share in case a recall happens.

In this paper we study and compare two schemes, namely the push system and the pull system ((Cachon 2004)). The sequence of events are as follows (displayed in Figure 3.1). We begin with the push system. At the beginning of the time horizon, as a Stackelberg leader, the manufacturer offers a recall cost sharing contract to the supplier, including the unit wholesale price w and the recall cost sharing percentage θ. We treat the cost sharing percentage θ as an exogenous decision first and then investigate the impact of θ. The reason lies in the observation that the cost sharing percentage is often a long-term strategic decision, while both the quantity and quality decisions are short-term decisions and need to be revised periodically.

In the pull system, the manufacturer determines both the wholesale price w and the order quantity q, leaving only the quality effort decision to the supplier. After the supplier chooses the quality effort level, the demand D is realized. Then $\min\{D, q\}$ units of products are sold to the market and the manufacturer is subject to recall risk. Once a recall occurs, both parties (manufacturer and supplier) share the total recall

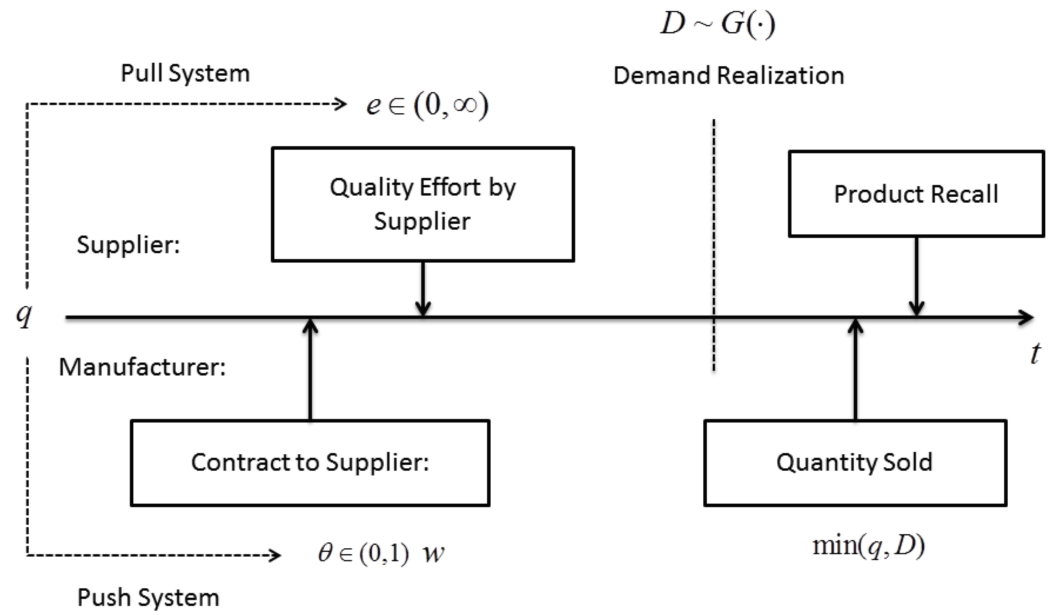


Figure 3.1: Sequence of Events

cost according to the sharing scheme specified in the contract. In the pull system, the demand risk is transferred from the downstream manufacturer in the former setting to the upstream supplier, who now decides the quantity decision q before demand realization, while the manufacturer does not order until the demand uncertainty is resolved.

3.4 Analysis

In this section, we first analyze the first-best benchmark case and then examine the push and pull systems separately. After that we study the impact of recall risk and cost sharing percentage. We close the section with a discussion on the range of cost sharing percentage.

Denote the Π and π as the profits for the downstream manufacturer and the upstream supplier, respectively. Let Σ be the total profit for the supply chain, i.e. the sum of both manufacturer's and supplier's profits. In the following analysis, we use superscript FB to denote first-best variables, use superscript S to denote push system variables, and use superscript L to denote pull system variables.

3.4.1 First-Best Benchmark

To facilitate our understanding of the two schemes, we establish the first-best benchmark first, where the integrated supply chain maximizes the sum of both the manufacturer's and supplier's profits. The decision variables are the order quantity q and the quality effort e, and the first-best optimization problem is given by

$$\Sigma^{FB} = \max_{e,q} rS(q) - \exp\{-\lambda e\}uS(q) - cq - se,$$

where

$$S(q) = E_D[\min(q, D)] = \int_0^q \bar{G}(x)dx.$$

In the following lemma, we provide the first-best optimal solution.

Lemma 3.4.1. *If the demand distribution satisfies the IFR property and $\underline{l} \geq s/u\lambda$ holds, the optimal solution $\{e^{FB}, q^{FB}\}$ for the first-best benchmark is given by*

$$e^{FB} = \frac{1}{\lambda} \ln\left(\frac{S(q^{FB})u\lambda}{s}\right),$$

and

$$r\bar{G}(q^{FB}) - \frac{s}{\lambda}\frac{\bar{G}(q^{FB})}{S(q^{FB})} - c = 0.$$

The following corollary presents how the solution set $\{e^{FB}, q^{FB}\}$ change when parameters alter.

Corollary 3.4.1. *In the integrated supply chain, there are,*

(i) q^{FB} is increasing in r and λ, decreasing in c and s, and independent of u.

(ii) e^{FB} is increasing in u and r, and decreasing in c and s.

Corollary 3.4.1 provide the monotone properties of order quantity and quality effort when the parameters change. The dependence on r and c accord with the results in classic newsvendor problem. When unit quantity cost c increases, the overage cost for the newsvendor is essentially higher, hence q^{FB} decreases accordingly. With less order quantity, the incentive for putting in quality effort is lowered, so e^{FB} is reduced too. Similarly, a higher selling price r means a higher underage cost, and therefore q^{FB} and e^{FB} increases with r. The recall related parameters are λ, s and u. As the quality effort marginal cost s increases, it costs more for the supplier to put in same amount of effort than before. Therefore e^{FB} decreases in s, resulting in higher recall probability. Similarly, q^{FB} decreases in s too. As λ increases, with same amount of effort as before, the recall probability is lower, so the manufacturer has the incentive to release more product to the market. With higher unit recall cost u, the potential penalty from recall is higher, which incentivizes the supplier to put in more quality effort to reduce the possibility of recall. Interestingly, we find that the unit recall cost u does not affect q^{FB}. Intuitively, higher unit recall cost should hinder from ordering more. However, the probability of recall is reduced in the meantime, which contributes to reduce the expected recall cost and encourage a higher product quantity. The effects of these two conflicting factors cancel out.

3.4.2 Push System

With first-best benchmark established, we now move to examine the decentralized settings. We start with the push system first, i.e., the manufacturer decides the order quantity, which can be formulated as a stylized principle-agent problem, with individual rationality (IR) and incentitive compatibility (IC) constraints:

$$\begin{aligned} \Pi^S = \quad & \max_{w,q} \quad \Pi(w, e^*, q) \\ & \text{s.t.} \quad \pi(w, e^*, q) \geq 0 \quad \text{(IR)} \\ & \qquad\quad e^* \in \operatorname*{argmax}_e \pi(w, e, q) \quad \text{(IC)}, \end{aligned}$$

where

$$\begin{aligned}
\Pi(w,e,q) &= rS(q) - \exp\{-\lambda e\}\theta u S(q) - wq, \\
\pi(w,e,q) &= wq - \exp\{-\lambda e\}(1-\theta)uS(q) - cq - se.
\end{aligned}$$

We solve this problem backwardly by first solving the supplier's problem given wholesale price w and order quantity q provided by the manufacturer, and then substituting the supplier's reactive function into the manufacturer's problem to determine the optimal solutions. We characterize the optimal solutions in the following proposition.

Lemma 3.4.2. *If the demand distribution satisfies the IFR property and $\theta < \bar{\theta} = 1 - s/(u\lambda \underline{l})$ holds, then the unique optimal solution is given by the following equations:*

$$r\bar{G}(q^S) - \frac{s}{\lambda}\frac{\bar{G}(q^S)}{S(q^S)} - c = 0,$$
$$e^S = \frac{1}{\lambda}\ln\left(\frac{S(q^S)(1-\theta)u\lambda}{s}\right),$$

and

$$w^S = c + \frac{s}{\lambda q^S} + \frac{s}{\lambda q^S}\ln\left(\frac{S(q^S)(1-\theta)u\lambda}{s}\right).$$

Note that the first equation in Lemma 3.4.2 is the same as that in the first-best benchmark, and the second equation in Lemma 3.4.2 is different by only a coefficient of $(1-\theta)$ from that in the first-best benchmark. By comparing with the first-best solution in previous section, we further obtain the following result:

Lemma 3.4.3. *In the push system, we have*

(i) $q^{FB} = q^S$, and $e^{FB} > e^S$, thus the recall cost sharing contract cannot attain the first-best solution.

(ii) $\pi^S = 0$, and $\Sigma^S = \Pi^S$, i.e., the supplier's profit is pushed to zero, and the manufacturer takes all the supply chain's profit.

Lemma 3.4.3 shows that, under the decentralized push system when the manufacturer bears the demand risk, he or she choose to order the same quantity as in the first-best case, but the supplier chooses a lower quality effort than in the first-best case. Note that the equations for determining q^S and e^S have the same structures as those in the first-best benchmark, therefore all the qualitative properties in Corollary 3.4.1 carry over here. Apart from the solution, we are also interested in the profit allocation in the push system. According to Lemma 3.4.3, the supplier ends up with zero profit under the optimal solution. Therefore, under this setting, the maunfacuter's objective is aligned with the supply chain profit, and thus she, as a stackberlg leader, has the incentive to push both quantity and quality decisions as close as possible to the first-best solution. With both quantity q and wholesale price w as decision variables, the manufacturer has more flexibility to incentivize the supplier compared with the pull system we are about to examine later. To summarize, q^S is chosen to equal q^{FB}, however, the suppier's decision e^S is smaller than e^{FB} due to the effect of double marginization. As a consequence, recall cost sharing contract cannot attain the first-best, or in other words, this contract is not efficient. Next we aim to further study this gap. Define $\delta^{S(L)} = \Sigma^{FB} - \Sigma^{S(L)}$, which represents the supply chain profit loss (compared with integrated system) for the push (pull) system, and we have the following result:

Proposition 3.4.1. *In the push system, the supply chain profit loss is given by*

$$\delta^S = \frac{s}{\lambda}\left(\ln(1-\theta) + \frac{\theta}{1-\theta}\right).$$

Remark 3.4.1. Note that $\delta^S \geq 0$, $\frac{d\delta^S}{d\theta} \geq 0$, and both inequalities are binding if and only if $\theta = 0$. Therefore, the supply chain profit loss is increasing in θ. In other words, the larger percentage of recall costs the manufacturer is paying, the larger the supply chain profit loss is. Ideally, the supply chain become most efficient if the manufacturer can to the largest extent push recall risk upward the supply chain to the supplier.

3.4.3 Pull System

Next we move to the pull system. Now the decision of q is up to the upstream supplier, who takes the demand risk in the current setting. The problem becomes convoluted since the agent now faces a two-dimensional decision problem, which is rarely seen in most supply chain contracting literatures. Traditionally the agent is often faced with an one-dimensional problem, which is easy to be solved with an explicit solution so that it can be substituted back into the principle's problem.

We formulate decentralized optimization problem as follows:

$$\begin{aligned} \Pi^L = \quad & \max_{w} \quad \Pi(w, e^*, q^*) \\ & \text{s.t.} \quad \pi(w, e^*, q^*) \geq 0 \quad \text{(IR)} \\ & \qquad\ (e^*, q^*) \in \operatorname*{argmax}_{e,q} \pi(w, e, q) \quad \text{(IC)}, \end{aligned}$$

where

$$\Pi(w, e, q) = rS(q) - \exp\{-\lambda e\}\theta u S(q) - wS(q), \tag{3.4.1}$$

$$\pi(w, e, q) = wS(q) - \exp\{-\lambda e\}(1-\theta)uS(q) - cq - se. \tag{3.4.2}$$

Note that the agent's objective in our case is challenging too, because it is not concave, and not even unidomal except that in some cases its first derivative is unimodal. Nevertheless, we are able to prove both the existence and the uniqueness for the optimal solutions in this setting.

We follow the similar procedure as in the push system, and characterize the optimal solution for the pull system first.

Lemma 3.4.4. *If the demand distribution satisfies the IFR property and* $\theta < \bar{\theta} = 1 -$

$s/(u\lambda\underline{l})$ holds, then the unique optimal solution is given by

$$r\bar{G}(q^L) - \frac{cg(q^L)S(q^L)}{\bar{G}^2(q^L)} - c = 0, \tag{3.4.3}$$

$$e^L = \frac{1}{\lambda}\ln\left(\frac{S(q^L)(1-\theta)u\lambda}{s}\right), \tag{3.4.4}$$

and

$$w^L = \frac{c}{\bar{G}(q^L)} + \frac{s}{\lambda S(q^L)}. \tag{3.4.5}$$

As before, we examine how the optimal solutions change as the parameters vary. The results are presented in the following corollary.

Corollary 3.4.2. *In the pull system, we have*

(i) q^L is decreasing in c, increasing in r, and is independent of u, s, and λ.

(ii) e^L is increasing in u and r, and decreasing in c and s.

Compared with both the first-best benchmark and the push system, the changes are the dependence of q^L and e^L on s and λ. As quality effort marginal cost s increase, the supplier's quality effort is likely to decrease, and hence lead to potentially higher recall cost. Therefore the supplier has the incentive to reduce order quantity. However, as the manufacuter's profit is increasing in q^L, he has the incentive to provide higher wholesale price to compensate the supplier and obstruct him from lowering the order quantity. As a result, q^L appears to be independent of s. The argument for λ is similar. As λ decreases, same amount of quality effort results in larger recall probability, so the supplier is more conservative towards ordering quantity and want to choose lower q^L. But again, the manufacturer has the incentive to stop him from lowering it, so q^L also appears to be independent of λ.

We further compare the solutions and profits in the pull system with those in the first-best benchmark, and the results are summarized in the following lemma.

Lemma 3.4.5. *The pull system yields lower order quantity and quality effort than the integrated supply chain, and the supplier could yield a nonzero profit under this setting:*

(i) $q^{FB} > q^L$, *and* $e^{FB} > e^L$, *thus the recall cost sharing contract cannot attain the first-best solution.*

(ii) $\pi^L > 0$, *and* $\Sigma^L > \Pi^L$.

Therefore we see that when the supplier bears the demand risk, both the quantity and quality effort are lower than the first-best solution. Here in the pull system, we obtain the similar conclusion that the contract cannot coordinate the supply chain. Both product quantity and quality effort cannot attain first-best. By taking the demand risk, the supplier in the pull system can leverage on its quantity decision and avoid being pushed to zero profit. Essentially the supplier is better off in the pull system compared with the push system.

Similar as in the push system, noting the contract cannot achieve first-best, we examine the supply chain profit loss again and find that the supply chain profit is even worse off in the pull system. Hence the supply chain profit in the pull system is lower than that in the push system. Recall that the supplier's profit is zero in the push system, and that Lemma 3.4.5 (ii) shows that the supplier ends with a positive profit, therefore we must have, the manufacturer's profit in the pull system is lower than that in the push system. We present these results formally in the following Proposition.

Proposition 3.4.2. *Compared with the push system, the pull system have larger supply chain profit loss, lower supply chain's profit, higher supplier's profit, and lower manufacturer's profit:*

$$\delta^L > \delta^S, \Sigma^S > \Sigma^L, \pi^S < \pi^L, \text{ and } \Pi^S > \Pi^L.$$

Next we compare the optimal solutions between the push system and the pull system, and we obtain the following proposition:

Proposition 3.4.3. *The pull system yields both smaller order quantity and lower quality effort: $q^S > q^L$ and $e^S > e^L$.*

By Proposition 3.4.3, it can be seen that the manufacturer achieves a higher output (quantity) level and induces a higher quality effort from the supplier in the push system, i.e., when the manufacturer makes the quantity decision. This result reveals that manufacturers could take on demand risk in exchange for higher quality effort by suppliers. Note that the manufacturer also achieves a higher profit and an output level closer to first-best solution under the pull system.

Managerial Insights #1: The manufacturer achieves a higher production quantity and induces a higher quality effort of the supplier in the push system than in the pull system.

Risk aversion is commonly observed in reality, i.e., the supply chain players tend to avoid taking risks. This also partially explains why pull system is more popular among manufacturers, because it protects him from demand risk. However, our result shows that, when product recall is incorporated, the manufacturer may interestingly choose to take on demand risk in order to alleviate the recall risk.

3.4.4 The Impact of Recall Risk on Quantity Decisions

To understand what the potential recall affect the quantity decision and the quality decision in the supply chain, we further introduce the scenario without product recall risk. Similar as previous analysis, we consider the push system and the pull system, as well as the first-best benchmark. By comparing the results with those in previous sections, we can isolate the effect of the feature of product recall in the model. Without potential recall risk, the centralized first-best problem and decentralized principle-agent problem are both generic and we can establish the optimal solutions and related properties in a similar procedure as we did before. The problems for the push system and the pull system are given by

$$\begin{aligned} \tilde{\Pi}^S = \max_{w,q} & \quad rS(q) - wq \\ s.t. & \quad (w-c)q \geq 0, \end{aligned}$$

and

$$\begin{aligned} \tilde{\Pi}^L = \max_{c \leq w \leq r} & \quad (r-w)S(q^*) \\ s.t. & \quad q^* = \operatorname*{argmax}_q wS(q) - cq, \end{aligned}$$

respectively.

Lemma 3.4.6. *If the demand distribution satisfies the IFR property,*

(i) for the push system, the unique optimal solution $\{\tilde{w}^S, \tilde{q}^S\}$ for the no-recall benchmark is given by

$$\begin{aligned} \tilde{q}^S &= \bar{G}^{-1}\left(\frac{c}{r}\right), \\ \tilde{w}^S &= c. \end{aligned}$$

(ii) For the pull system, the unique optimal solution $\{\tilde{w}^L, \tilde{q}^L\}$ for the no-recall benchmark is given by

$$\begin{aligned} r\bar{G}(\tilde{q}^L) - \frac{cg(\tilde{q}^L)S(\tilde{q}^L)}{\bar{G}^2(\tilde{q}^L)} - c = 0, \\ \tilde{w}^L = \frac{c}{\bar{G}(\tilde{q}^L)}. \end{aligned}$$

We can also solve the integrated first-best benchmark when there is no product recall risk. The problem is given by

$$\tilde{\Pi}^{FB} = \max_{w,q} \quad rS(q) - cq,$$

the solution to which is denoted as $\tilde{q}^{FB}$. It is easy to derive $\tilde{q}^{FB} = \bar{G}^{-1}\left(\frac{c}{r}\right)$.

Now we are ready to compare the solutions of the systems under consideration. The results are summarized in Proposition 3.4.4.

Proposition 3.4.4. $\tilde{q}^{FB} = \tilde{q}^S > q^{FB} = q^S > q^L = \tilde{q}^L$.

From Proposition 3.4.4, the first observation we find is that the first-best output level without recall is higher than that with recall. The intuition behind it is that, with recall incorporated into the model, the underage cost for the newsvendor is essentially lower due to the potential recall cost, and hence by the classic newvendor factile solution, the optimal output level becomes lower. Besides, it turns out that, the potential recall risk affects the push and pull systems differently. In the push system, q^S is less than $\tilde{q}^S$, which represents the output level in the push system without product recall risk. This suggests that when the manufacturer bears the demand risk, the potential recall risk decreases the optimal output level. This is rather intuitive since incoparating the recall risk decreases the underage cost, which makes the newsvendor, who is the manufacturer in this setting, become more conservative. In contrast, in the pull system, we show q^L is larger or equal to $\tilde{q}^L$, which suggests when it is the supplier who bears the demand risk, the potential recall risk either increase or does not affect the output level. Although by incorporating recall, the underage cost for the newsvendor is also reduced, the newsvendor in the pull system is the supplier, who is the stackelberg follower and cannot fully control the output level by himself. The manufacturer want to keep the optimal output level at the same level as in the case without recall, since at that output level, the manufacturer's profit is maximized. We summarize these interpretations as the following managerial insights.

Managerial Insights #2: The presence of product recall risk decreases the production quantity in the push system but does not affect the production quantity in the pull system.

3.4.5 The Impact of Cost Sharing Percentage

In this subsection, we focus on studying the cost sharing percentage θ, which is the key element in the contract we considered. For now we follow the previous analysis by restricting θ to be no greater than an upperbound $\bar{\theta}$ and studying the impact of θ on the profitability, quality and quantity decisions for the manufacturer, the supplier, and the supply chain as a whole, respectively. Later we will discuss why we impose the restriction and further demonstrate our results by analyzing the push system with θ ranging from 0 to 1.

Assume that θ continue to satisfy $0 \leq \theta \leq \bar{\theta}$, then we obtain the following qualitative properties.

Proposition 3.4.5. *Given that θ is within the region of $[0, \bar{\theta}]$,*

(i) q^S and q^L are indepedent of θ;

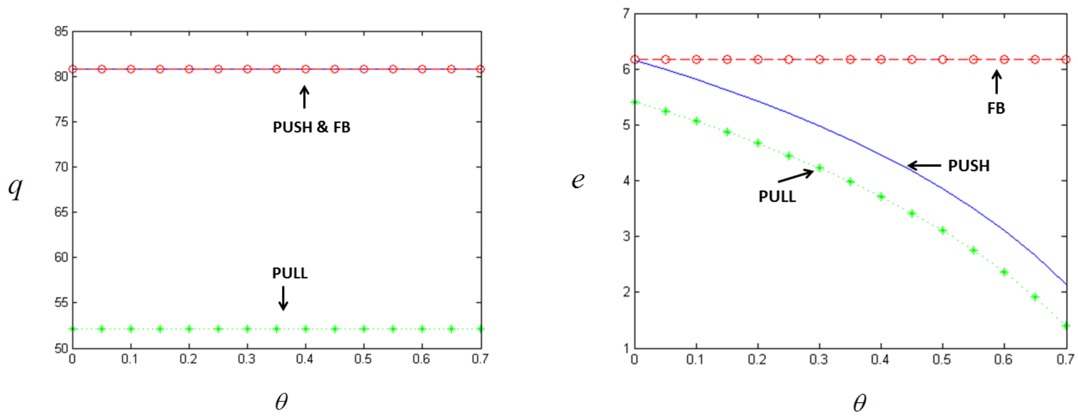
(ii) e^S and e^L are decresing in θ;

(iii) Π^S, Π^L, Σ^S and Σ^L are decresing in θ, and π^S is zero, while π^L is incresing in θ.

Next we use a numerical example to illustrate Proposition 3.4.5 (see Figures 3.2 and 3.3).

Figure 3.2(a) illustrates the impact of θ on quantity decisions for both settings. To facilitate comparison, we also plot first-best benchmark. As the figure shows, the quantity decision is constant in both settings, which accords with Proposition 3.4.5.

Figure 3.2(b) illustrates the impact of θ on quality decisions for both settings. The quality effort levels in both settings decrease when theta is increasing, i.e., the supplier shares less. This is rather intuitive, because the supplier has less incentitive to invest in



(a) q vs θ (b) e vs θ

Figure 3.2: The impact of θ on q and e

quality effort when he shares less risk. Note that the quality effort in the pull system is consistently lower than that in the push system.

Figure 3.3 presents the impact of θ on profits for both settings and the first-best benchmark is also included. As θ increases, i.e. when the manufacturer shares more of the total recall cost, both the manufacturer and the whole supply chain are worse off in terms of profits regardless of push or pull system, while in contrast the supplier's profit is always pushed to zero in the push system, but is positive as well as increasing in the pull system. These observations are also consistent with Proposition 3.4.5. We summarize them into the managerial insights below.

Managerial Insights #3: The manufacturer can improve her product quality and profit by decreasing her share of the total recall cost without affecting the production quantity of the supply chain in both the push and pull systems.

3.4.6 The Range of Cost Sharing Percentage

We restrict our previous analysis to the scenario when θ is in the range of $[0, \bar{\theta}]$ instead of considering the general $[0, 1]$ range. Because when θ is greater than $\bar{\theta}$, the objective function could have two local maximizers: one corresponds to the boundary solution

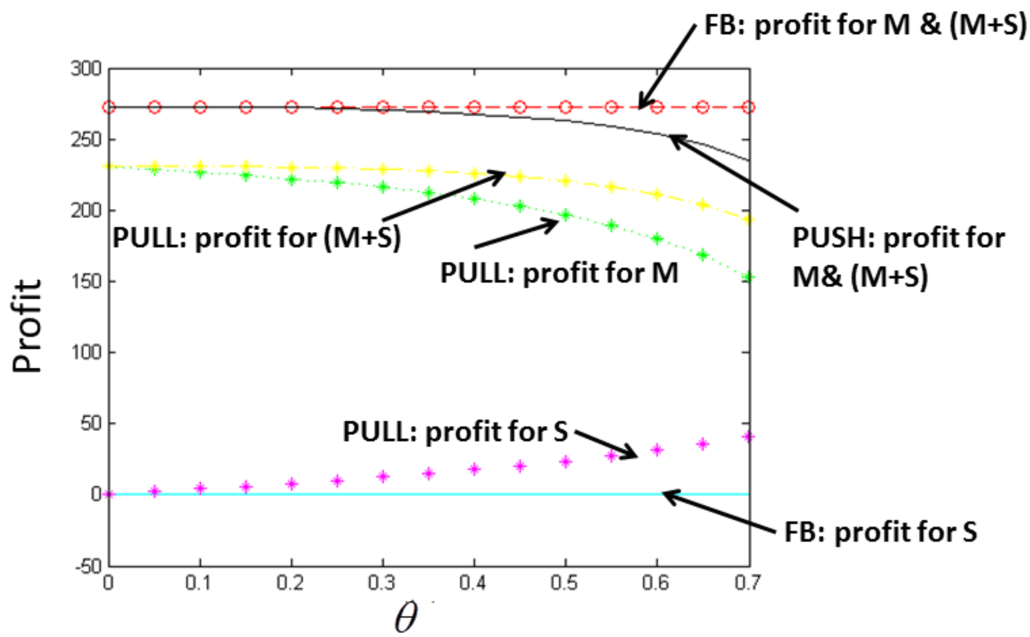


Figure 3.3: The impact of θ on profits

when the supplier chooses zero effort , and the other one corresponds to the interior solution when the supplier chooses a positive effort level. To determine which one is the global maximizer, we need to compare the objective function values at those two points. Figure 3.4 displays an example for the pull system where the optimal solution could either be an interior solution or a boundary solution depending on the parameter values. This comparison could cause tractability issues in solving the problem. Besides, we are more interested in the nontrivial case of interior solution, therefore we chose to restrict θ to the range to ensure the optimal solution to be always an interior soluiton.

To demonstrate what we have argued, we conduct analysis for the push system allowing θ to take values in full range of $[0, 1]$. Similar procedure also applies for the pull system, except that more complex argument is required for that case. For clarity, we demonstrate the analysis for the push system only.

To avoid repetition, we define two sets of equations that determine boundary solutions and interior solutions, respectively:

$$\begin{aligned}
q^S &= \bar{G}^{-1}\left(\frac{c}{r-u}\right),\\
e^S &= 0,\\
w^S &= c + \frac{(1-\theta)uS(q^S)}{q^S}.
\end{aligned}$$

and

$$\begin{aligned}
&r\bar{G}(q^S) - \frac{s}{\lambda}\frac{\bar{G}(q^S)}{S(q^S)} - c = 0,\\
&e^S = \frac{1}{\lambda}\ln\left(\frac{S(q^S)(1-\theta)u\lambda}{s}\right),\\
&w^S = c + \frac{s}{\lambda q^S} + \frac{s}{\lambda q^S}\ln\left(\frac{S(q^S)(1-\theta)u\lambda}{s}\right).
\end{aligned}$$

In the following we refer to these equations sets as ES1 and ES2, respectively.

The solutions for the push system allowing full range of cost sharing percentage are summarized in Proposition 3.4.6.

Proposition 3.4.6. *Suppose the demand distribution satisfies the IFR property. For the push system, there always exists a unique optimal solution. Specifically,*

(i) If

$$rS(q_2^S) - \tfrac{s}{(1-\underline{\theta}_1)\lambda} - cq_2^S - \tfrac{s}{\lambda}\ln\left(\tfrac{S(q_2^S)(1-\underline{\theta}_1)u\lambda}{s}\right) \le (r-u)S(q_1^S) - cq_1^S,$$

(i-i) further if $\theta \in [0, \underline{\theta}_1]$, the solution is given by the solution to ES2,

(i-ii) further if $\theta \in [\underline{\theta}_1, 1]$, the solution is given by the solution to ES1.

(ii) If

$$(r-u)S(q_1^S) - cq_1^S \le rS(q_2^S) - \frac{s}{(1-\underline{\theta}_2)\lambda} - cq_2^S - \frac{s}{\lambda}\ln\left(\frac{S(q_2^S)(1-\underline{\theta}_2)u\lambda}{s}\right),$$

(ii-i) further if $\theta \in [0, \underline{\theta}_2]$, the solution is given by the solution to ES2.

(ii-ii) further if $\theta \in [\underline{\theta}_2, 1]$, the solution is given by the solution to ES1.

(iii) If

$$rS(q_2^S) - \tfrac{s}{(1-\underline{\theta}_1)\lambda} - cq_2^S - \tfrac{s}{\lambda}\ln\left(\tfrac{S(q_2^S)(1-\underline{\theta}_1)u\lambda}{s}\right) \geq (r-u)S(q_1^S) - cq_1^S$$
$$\geq rS(q_2^S) - \tfrac{s}{(1-\underline{\theta}_2)\lambda} - cq_2^S - \tfrac{s}{\lambda}\ln\left(\tfrac{S(q_2^S)(1-\underline{\theta}_2)u\lambda}{s}\right),$$

then there exists a $\underline{\theta}_3 \in [\underline{\theta}_1, \underline{\theta}_2]$, which satisfies

$$rS(q_2^S) - \frac{s}{(1-\theta)\lambda} - cq_2^S - \frac{s}{\lambda}\ln\left(\frac{S(q_2^S)(1-\underline{\theta}_3)u\lambda}{s}\right) = (r-u)S(q_1^S) - cq_1^S.$$

(iii-i) further if $\theta \in [0, \underline{\theta}_3]$, the solution is given by the solution to ES2,

(iii-ii) further if $\theta \in [\underline{\theta}_3, 1]$, the solution is given by the solution to ES1,

where we define $q_1^S = \bar{G}^{-1}\left(\frac{c}{r-u}\right)$, q_2^S is the solution to $r\bar{G}(q_2^S) - \frac{s}{\lambda}\frac{\bar{G}(q_2^S)}{S(q_2^S)} - c = 0$, $\underline{\theta}_1 = 1 - \frac{s}{u\lambda S(\bar{G}^{-1}(c/\{r-u\}))}$, and $\underline{\theta}_2 = 1 - \frac{s}{u\lambda d}$ (Note that $\bar{\theta} < \underline{\theta}_1 < \underline{\theta}_3 < \underline{\theta}_2$).

Proposition 3.4.6 shows that θ plays a crucial role in determing the optimal solution structure. When $\theta < \underline{\theta}_1$, the opitmal solution is always an interior solution with a positive effort level. In contrast, when $\theta < \underline{\theta}_2$, the opitmal solution is always a boundary solution with zero effort. Intuitively the manufacturer always prefers a lower cost sharing percentage in order to push more potential recall risk to the upstream supplier, and incentivize him to put in higher quality effort. The manufacter should be aware that if this θ is high enough, the supplier has no incentive to put in any quality effort.

In the following we move our focus to an extreme scenario when the cost sharing percentage is relatively high. We find that under such context, it always results in boundary solutions. The results are presented in the following proposition.

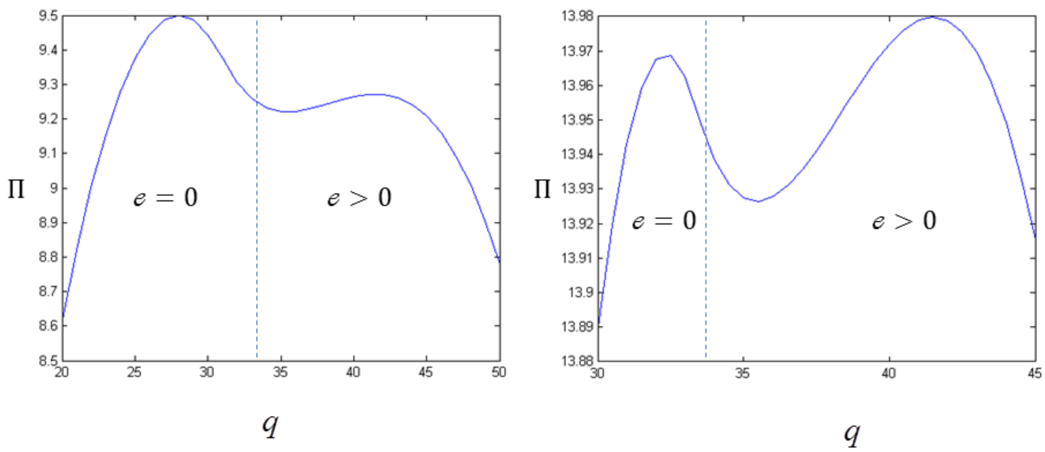


Figure 3.4: Profit Function

Corollary 3.4.3. *Suppose $\theta \geq \underline{\theta}_2 = 1 - \frac{s}{u\lambda d}$, then we have,*

(i) for the push system, there exists a unique optimal solution $\{w^S, e^S, q^S\}$, which is given by

$$q^S = \bar{G}^{-1}\left(\frac{c}{r-u}\right),$$
$$e^S = 0,$$
$$w^S = c + \frac{(1-\theta)uS(q^S)}{q^S}.$$

(ii) for the pull system, there exists a unique optimal solution $\{w^L, e^L, q^L\}$, which is given by

$$(r-u)\bar{G}(q^L) - \frac{cg(q^L)S(q^L)}{\bar{G}^2(q^L)} - c = 0,$$
$$e^L = 0,$$
$$w^L = \frac{c}{\bar{G}(q^L)} + (1-\theta)u.$$

Corollary 3.4.3 demonstrates that when the cost sharing percentage is relatively high, the supplier chooses not to put in any effort. This arises because when the cost sharing percentage is high, the supplier shares less of the product recall cost, and hence does not have enough incentive to invest in the quality effort.

3.4.7 Extentions

In this section, we focus on the push system supply chain efficiency and consider two main exntentions. In the first part we aim to design more complicated contracts under which both the manufacturer and the supplier make decisions in an efficient way compared with the centralized supply chain setting, and the second part we investigate the impact of futher market share and competition between suppliers on the supply chain efficiency. We first consider two more variations of the recall cost sharing contract: (i) two-part recall cost sharing contract, which is characterized by w, θ and q same as in the recall cost sharing contract plus a lump-sum recall cost T decided by the manufacturer that the supplier need to pay him once a recall happens, and (ii) recall-contingent revenue sharing contract, which is characterized by w, θ and q same as in the recall cost sharing contract plus a percentage α decided by the manufacturer that this part of his revenue will be awarded to the supplier if no recall happens. Here we relaxed the restriction of the exponential form of the recall probability form and assume the recall probability is $\bar{F}(e) = 1 - F(e)$, where $F(e)$ ranges on $[0, 1]$ with derivative $f(e)$. Assume $F(e)$ to be increasing concave, i.e., $f(e)$ is decreasing and nonnegative. In addition, we use superscript TP representing the two-part recall cost sharing contract and RS representing the recall-contingent revenue sharing contract.

Two-part Recall Cost Sharing Contract

Under the two-part recall cost sharing contract, the stylized principle-agent problem could be modeled as below:

$$\Pi^{TP} = \max_{q,w,T}(r - \bar{F}(e^*)\theta u)S(q) - wq + T\bar{F}(e^*)$$

$$\text{s.t.} \quad (w-c)q - \bar{F}(e^*)(1-\theta)uS(q) - se^* - T\bar{F}(e^*) \geq 0 \quad \text{(IR)}$$
$$e^* \in \underset{e}{\operatorname{argmax}}(w-c)q - \bar{F}(e)(1-\theta)uS(q) - se - T\bar{F}(e) \quad \text{(IC)}$$

Note that there is a transferred payment $T\bar{F}(e^*)$ which is the penalty collected by the manufacturer from the supplier once product recall happens. By solving the suppler's problem, we could obtain the following result:

Lemma 3.4.7. *The optimal solution satisfies the following equation:*

$$f(e^{TP}) = \frac{s}{T + (1-\theta)S(q^{TP})u}. \tag{3.4.6}$$

By comparing with the first-best benchmark solution Lemma 3.4.1 analyzed previously, we have the following finding:

Proposition 3.4.7. *Two-part Recall Cost Sharing Contract can achieve the first-best solution by letting $T = \theta uS(q^{FB})$, $e^{TP} = e^{FB}$ and $q^{TP} = q^{FB}$.*

Recall-contingent Revenue Sharing Contract

Under the recall-contingent revenue sharing contract, the stylized principle-agent problem could be modeled as below:

$$\Pi^{RS} = \max_{q,w,\alpha}(r - \bar{F}(e^*)\theta u)S(q) - wq - \alpha rS(q)F(e^*)$$
$$\text{s.t.} \quad (w-c)q - \bar{F}(e^*)(1-\theta)uS(q) - se^* + \alpha rS(q)F(e^*) \geq 0 \quad \text{(IR)}$$
$$e^* \in \underset{e}{\operatorname{argmax}}(w-c)q - \bar{F}(e)(1-\theta)uS(q) - se + \alpha rS(q)F(e) \quad \text{(IC)}$$

Note that the manufacturer will share the revenue of $\alpha rS(q)F(e^*)$ to the supplier if there is product recall happen. By solving the suppler's problem, we could obtain the following result:

Lemma 3.4.8. *The optimal solution satisfies the following equation:*

$$f(e^{RS}) = \frac{s}{(1-\theta)S(q^{RS})u + \alpha r S(q^{RS})}. \tag{3.4.7}$$

Again compare Lemma 3.4.8 and Lemma 3.4.1, we could obtain the following proposition.

Proposition 3.4.8. *Recall-contingent Revenue Sharing Contract can achieve the first-best solution by letting $\alpha = \theta u / r$, $e^{RS} = e^{FB}$ and $q^{RS} = q^{FB}$.*

In summary, we find that both the two-part recall cost sharing contract and the recall-contingent revenue sharing contract could help improve the supply chain efficiency.

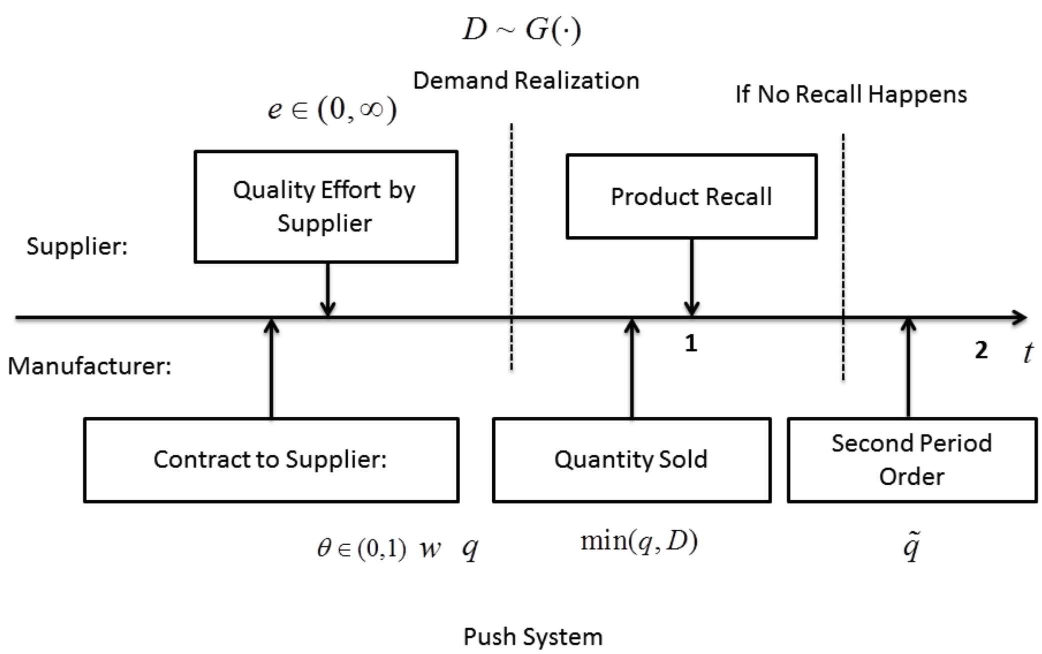


Figure 3.5: Sequence of Event for Two-Period Model

Single Sourcing: Two Periods

Next we investigate the impact of futher market share and competition between suppliers on the supply chain efficiency. We start with the consideration of the future market share. In reality, it is common that once a recall happens, the supplier will lose the future market share from the manufacturer, which may prevent the supplier from making low quality effort. To examine this effect on operational decisions, we consider a two period extension of the basic model. If a recall happens by the end of the first period, there is no market for the manufacturer in the second period, and no production is needed for the second period. To isolate the effect of future market share from other factors, we suppress the possibility of recall in the second period, i.e., no recall cost incurs in the second period. If no recall happens by the end of the first period, the manufacturer and the supplier involve in the second period contracting and production. And the supplier no longer needs to make effort decision for the second period since there is no potential product recalls in the second period. This also captures the reality in that supplier invests quality effort in the first contracting term, and should be able to carry over the skills or experiences to the following contracting term. For simplicity, the demand $\tilde{q}$ in the second period is assumed to be fixed. Besides, we use superscript ST representing this single souring two-period setting.

To facilitate the understanding of the supply chain efficiency, we need to establish first-best benchmark for the two-period model. The first-best problem is defined as below:

$$\Sigma^{FB} = \max_{e,q}(r - \bar{F}(e)u)S(q) - cq - se + F(e)(r - c)\tilde{q}$$

Immediately we could come up with the following result:

Lemma 3.4.9. *The optimal solution satisfies the following equation:*

$$f(e^{FB}) = \frac{s}{S(q^{FB})u + (r-c)\tilde{q}}.$$

Now we could model the decentralized supply chain problem. Note we focus on the simplest recall cost sharing contract.

$$\begin{aligned}
\Pi^{ST} = \quad & \max_{q,w}(r - \bar{F}(e^*)\theta u)S(q) - wq + F(e^*)(r-w)\tilde{q} \\
\text{s.t.} \quad & (w-c)q - \bar{F}(e^*)(1-\theta)uS(q) - se^* + F(e^*)(w-c)\tilde{q} \geq 0 \quad \text{(IR)} \\
& e^* \in \operatorname*{argmax}_{e}(w-c)q - \bar{F}(e)(1-\theta)uS(q) - se + F(e)(w-c)\tilde{q} \quad \text{(IC)}
\end{aligned}$$

Then we derive the following lemma:

Lemma 3.4.10. *The optimal solution satisfies the following equation:*

$$f(e^{ST}) = \frac{s}{(1-\theta)S(q^{ST})u + (w^{ST}-c)\tilde{q}}.$$

To compare Lemma 3.4.10 and Lemma 3.4.9, we could establish the following proposition.

Proposition 3.4.9. *In the single sourcing two-period setting, recall cost sharing contract cannot achieve the first-best solution.*

In the single-period problem, we find that the recall cost sharing contract cannot coordinate the supply chain. Even we consider the effect of the futher market share on the supplier's quality effort and extend the single-period model to the two-period model, the conclusion is unchanged. The recall cost sharing contract, although great to implement, is not a supply chain coordinated contract.

Dual Sourcing: Two Periods

Next we consider a dual sourcing strategy for the manufacturer and investigate the competition effect on the supply chain efficiency. And it is realistic in the sense that a manufacturer keeps on cooperation with reliable suppliers and stop ordering from less reliable ones. Therefore we further extend the two period model from the single sourcing setting into dual sourcing setting, i.e., adding a competitive supplier to the original single sourcing model. Now the manufacturer have two choices to source from. Suppose the two suppliers are identical, who share equally the order from the manufacturer. If no recall happens by the end of the first period, the two suppliers keep on splitting the order from the manufacturer equally as in period 1. If a recall happens due to the failure of components from supplier 1 (2), while the components from supplier 2 (1) work well, the manufacturer stops sourcing from supplier 1 (2) in the following period, and all of the manufacturer's order goes to supplier 2 (1). However, if the recall is due to quality failures from both suppliers' components in the first period, then the market is lost for the manufacturer and he stops production in the second period. We use superscript DT representing this dual souring two-period setting. Again a new first-best benchmark needs to be established:

$$\begin{aligned}\Sigma^{FB} = \quad & \max_{e_1,e_2,q} rS(q) - \bar{F}(e_1)u\frac{1}{2}S(q) - \bar{F}(e_2)u\frac{1}{2}S(q) - cq \\ & -se_1 - se_2 + (r-c)\tilde{q}(1-\bar{F}(e_1)\bar{F}(e_2))\end{aligned}$$

Similarly we need to derive the relationship between the first-best solution variables.

Lemma 3.4.11. *The optimal solution satisfies the following equations:*

$$S(q^{FB}) = \frac{s-(r-c)\tilde{q}f(e^{FB})\bar{F}(e^{FB})}{f(e^{FB})u/2}.$$

Next we consider the decentralized model, note that the two suppliers are indentical, thus they face the symmetric optimization problem.

$$
\begin{aligned}
\Pi^{DT} = \quad & \max_{q,w}(r - \bar{F}(e_1^*)\theta u)\frac{1}{2}S(q) - \frac{1}{2}wq \\
& +(r - \bar{F}(e_2^*)\theta u)\frac{1}{2}S(q) - \frac{1}{2}wq \\
& +(r-w)\tilde{q}(1 - \bar{F}(e_1^*)\bar{F}(e_2*)) \\
\text{s.t.} \quad & (w-c)\frac{q}{2} - \bar{F}(e_1^*)(1-\theta)uS(q)\frac{1}{2} - se_1^* + F(e_1^*)\bar{F}(e_2^*)(w-c)\tilde{q} \\
& +F(e_1^*)F(e_2^*)(w-c)\frac{\tilde{q}}{2} \geq 0 \quad \text{(IR1)} \\
& e_1^* \in \operatorname*{argmax}_{e_1}(w-c)\frac{q}{2} - \bar{F}(e_1)(1-\theta)uS(q)\frac{1}{2} - se_1 \\
& +F(e_1)\bar{F}(e_2^*)(w-c)\tilde{q} + F(e_1)F(e_2^*)(w-c)\frac{\tilde{q}}{2} \quad \text{(IC1)} \\
\text{and} \quad & (w-c)\frac{q}{2} - \bar{F}(e_2^*)(1-\theta)uS(q)\frac{1}{2} - se_2 + F(e_2^*)\bar{F}(e_1^*)(w-c)\tilde{q} \\
& +F(e_2^*)F(e_1^*)(w-c)\frac{\tilde{q}}{2} \geq 0 \quad \text{(IR2)} \\
& e_2^* \in \operatorname*{argmax}_{e_2}(w-c)\frac{q}{2} - \bar{F}(e_2)(1-\theta)uS(q)\frac{1}{2} - se_2 \\
& +F(e_2)\bar{F}(e_1^*)(w-c)\tilde{q} + F(e_2)F(e_1^*)(w-c)\frac{\tilde{q}}{2} \quad \text{(IC2)}
\end{aligned}
$$

Note that the two suppliers are symmetric, and we could obtain the following lemma. We write $e_1^{DT} = e_2^{DT} = e^{DT}$.

Lemma 3.4.12. *The optimal solution satisfies the following equations:*

$$
S(q^{DT}) = \frac{s - (w^{DT} - c)\tilde{q}f(e^{DT})(1 + \bar{F}(e^{DT}))/2}{f(e^{DT})(1-\theta)u/2}.
$$

By compare Lemma 3.4.11 and Lemma 3.4.12, we have the following interesting finding:

Proposition 3.4.10. *In the dual sourcing two-period setting, recall cost sharing contract can achieve the first-best solution.*

This result is interesting because we previously show that the effect of futher market share is not enough to incentivize the supplier to make quality effort in a supply chain efficient way. However, if we add the competition into the model, then the combined effect from both the futher market share and the competition will help the recall cost sharing contract coordinate the supply chain. The intuition behind is that the manufacturer could use dual scouring strategy to reduce the product recall risk.

3.5 Concluding Remarks

With increasing product recall issued in various industries, and with the aftermath of product recall being recogonized, how to avoid the potential recall and how to assign the fault after a recall takes place are questions that worth investigating. Noting the challanges of demand uncertainty and non-contractability of suppliers' quality effort, we develop a procurement contractual framework in this paper to examine how a manufacturer contracts its supplier to mitigate recall risk under demand uncertainty, and how quality and quantity decisions interact with each other in the supply chain. We focus on examining a cost sharing contract, which is easy to implement and has industrial applications. Two settings are analyzed here: a pull system where the supplier makes the quantity decision and a push system where the manufacturer makes the quantity decision. We also establish the centralized benchmark and no-recall benchmark, and prove both the existence and the uniqueness for the equilibrium solutions of these two regimes.

Compared with the pull system, we find that the manufacturer achieves a higher production quantity and induces a higher quality effort of the supplier in a push system, where the manufacturer takes on demand risk and induces higher quality effort of the supplier. Pull and push systems have been recogonized for a long time both by the academia and industry. There are a couple of trade-offs in comparing these two systems including demand uncertainty, difficulty of forecast, flexibility in adjusting inventory, transportation lead times, etc. In recent years, it is more popular to implement a pull system. Some successful industrial examples of introducing pull system in recent decades include Dell and Toyota, which achieve lower inventory levels and hence reduce costs through using pull system. Our results indicate that quality is another dimention to investigate when comparing these two systems. When there exists the possiblity of recall, push system could be favored due to the incentive it provides to the supplier in inducing higher quality effort.

Moreover, the presence of product recall risk decreases the production quantity in the push system but does not affect the production quantity in the pull system. The decrease in the push system accords with the intuitive explanation that the presence of potential recall decrease the underage cost for the newsvendor, who hereafter reduces the quantity. On the other hand, the counterintuitve increase in the pull system demonstrates that the quantity decision need to be made with more caution by taking whether the system is push or pull into consideration.

In addition, We also examine the impact of cost sharing percentage on the profitability, and quality and quantity decisions for the manufacturer, the supplier, and the supply chain. We find that the manufacturer can improve her product quality and profit as well as the supply chain's profit by decreasing her share of the botal recall cost without affecting the production quantity in both the pull and push systems. This result offers a plausible explanation for why GM insisted on sharing recall costs with its supplier even if their products passed GM's quality inspection. Note that the supplier is worse off, but since the whole supply chain is better off, there must exists a more complex mechanism with cost sharing feature such that both the manufacturer's and supplier's profit are improved, which is beyond the scope of this paper and hence left as a direction for future research.

CHAPTER 4 VOLUNTARY RECALL? INCENTIVES FOR QUALITY EFFORT

4.1 Introduction

Product recall is commonly observed worldwide across various industries. Famous examples include Toyota's vehicle recalls during 2009-2012 in the automobile industry, Boeing's 787 Dreamliner aircraft recall in early 2013 in the aerospace industry, and AngioScore's balloon catheter recall in 2010 in the medical device industry. Moreover, product recalls are not necessarily restricted to manufacturers, they can also happen among retailers. For example, Lululemon Athletica, an athletic apparel retailer, recalled its yoga pants in the spring of 2013.

The consequences of product recalls are known as serious because they are usually associated with safety issues. In this case, the recalls are highly focused by customers and social media. Once a recall is issued, many sorts of cost occur immediately due to repairs, recall logistics and downtime of the manufacturing. On top of this, the recall is very likely associated with potential law suits and litigation fees. High expenses, damaged reputation and lost sales will come up as the result. If the safety issues are involved, the close attention from social media is often inevitable, and hence it could lead to the widespread social impact that has long term damage to brand image. (see also (Hendricks and Singhal 2003), (Rupp 2004) and (Jarrell and Peltzman 1985)). Due to the negative consequences above, firms are keen on developing strategies to prevent and alleviate the severe crisis that recall might cause. To achieve the goal, there are two critical time points: one is during the quality control before sales, and the other is by the time of responsive product recall decisions when there are complaints and problem reports after sales.

For the former time point, the complexity typically stems from the outsourcing structure of supply chain. The manufacturers outsource the production of components to suppliers. In this case, the effort suppliers put in to ensure the products' quality is unobservable and uncontractible. As a result, the risk of information asymmetry and moral hazard arises. In addition, once a recall happens, determining which party causes the problem and allocating the responsibility for the recall are even more difficult questions. Due to such issues, disputes between supply chain players are common. For example, the debate between Lululemon and its yoga pants supplier Eclat Texitile in 2013, as well as between Ford and its tire supplier Firestone in 2000. Previous literatures propose inspection to tackle the issue, which does help improve the quality level. However, once a recall takes place, the inspection will no longer help the allocation of duty. Our work aims to address this problem through contracting. Although quality effort is not contractible, we consider a cost sharing contract that specifies the percentage of recall cost the supplier needs to undertake once a recall is issued. In this way, the supplier has more incentive to ensure the product quality because according the contract part of the recall cost will be paid through the responsible suppliers.

For the latter time point, the problem is also nontrivial. When there are accumulating certain amount of complaints and problem reports, it is challenge of manufacturer's ethic and capability to achieve the settlement. At this time, the problem is often not publicly known. Therefore, it is a critical decision for the manufacturer whether or not issue a recall and hence disclose the private information about products to the customers. According to the complex issues related to recall listed above, the recall strategies to deal with product quality problems are various.

An up-to-date example is General Motors, who is recently penalized by the U.S. Transportation Department for failing to report defects of ignition switches in 2.6 million

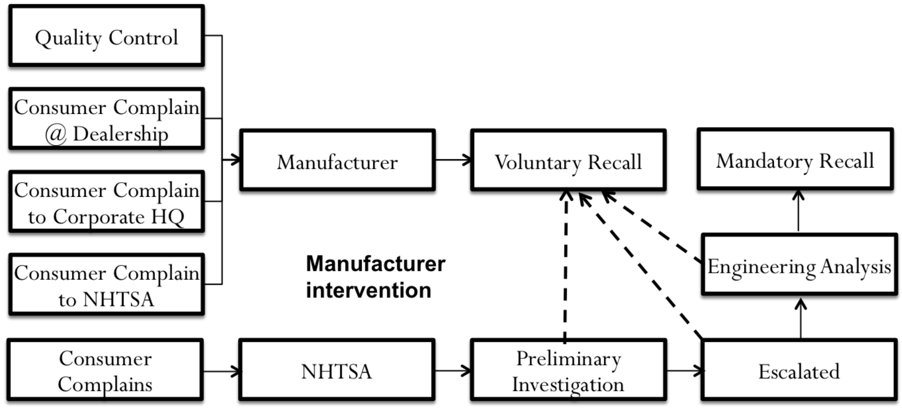


Figure 4.1: Recall Process

cars since 2001 before being forced to start a recall early this year. This is the largest single penalty issued by the government under the Tread Act. So far GM has ordered recalls affecting nearly 13 million vehicles. Back to 2005, despite hundreds of complaints and evidences, GM officials decided not to recall the problematic vehicles and redesign the switches. Because they evaluated the savings on warranty costs and found that they could not offset the cost associated with recall. Different from General Motors who try to defer the recall until the government forces it to do, what Toyota did is to issue the recall voluntarily. Although during the series of recalls Toyota initiated in the past years, it suffered great loss in terms of recall cost, Toyota has been regaining the trust of customers and rebuilt its brand image by handling the recalls well and taking measures in making the product quality information more transparent to public.

Noting the different product recall decisions observed in reality, we are interested to see what is a typical recall process, which can be illustrated using the automobile industry as an instance.

The Safety Act of 1966 requires a manufacturer to notify the National Highway and Traffic Safety Administration (NHTSA) if he has the knowledge that the vehicle or equipment fails to satisfy the federal safety standards. A typical process that involves recall is as follows. At first, the consumers' complaints accumulate. The complaints could go to either the manufacturer or NHSTA. The manufacturer can choose to recall voluntarily based on his private information of the product's quality. If the NHTSA receive certain amount of complaints, the safety agency would initiate a preliminary investigation. If there is no apparent violation of safety standard, the investigation is ended. Otherwise, the investigation escalates into engineering analysis, which takes approximately one year to complete. During this process, the manufacturer can choose either to recall voluntarily, or wait for the result of the engineering analysis. If he chooses to wait and the result indicates that there are indeed safety standards violated, the manufacturer is forced to issue a recall, otherwise, he does not issue a recall at all. The manufacturer is not required to issue a press release announcing the quality problem if he chooses to recall voluntarily. Being forced to recall is often associated with bad social impact and extra penalty, such as the example of General Motor we have addressed before. However, choosing to wait could avoid any recall in the hope that the engineering analysis indicates that no recall is needed, which prevents the manufacturer from the loss of recall cost.

Motivated by these industrial practices, we are interested in studying two types of product recall decisions: voluntary recall and mandatory recall. Voluntary recall is issued by the manufacturer voluntarily at an early stage when the problem is noticed but not revealed to the public yet, while mandatory recall is forced by the government agency at later stage when the problem becomes serious and widespread. Specifically, we are interested in answering these four research questions:

- When the manufacturer has the opportunity to issue an early recall voluntarily, will he take it or not?
- How does the supplier choose her quality effort when she knows the manufacturer may issue an early voluntary recall?

- How will the cost sharing percentage offered by the manufacturer be affected under such configuration?
- What is the interdependence between designing the recall cost sharing contract and making product recall decisions?

To address these questions, we consider a recall cost sharing contract proposed by the manufacturer, as a Stackelberg leader facing a moral hazard problem, to align the incentive of the supplier in ensuring the product quality, and also examine two types of recall decisions by the manufacturer: (1) voluntary recall, which is issued by the manufacturer voluntarily at a relatively early stage; (2) mandatory recall, which is forced by the government agency at a relatively late stage. The mandatory recall is characterized by a recall probability which depends on the supplier's quality effort. By solving the model, we characterize the firms' decisions in equilibrium. Surprisingly, we find that, as a Stackeberg leader, the manufacturer does not always push all the recall cost to the supplier. His decision depends on whether he will issue a voluntary recall or not. Furthermore, we find two cost sharing percentage thresholds that determines the equilibrium solutions. Specifically, the lower cost sharing percentage is always associated with voluntary recall, while the higher one is always associated to mandatory recall. However, we do find that when it is more costly for the supplier to make quality effort, the manufacturer will choose a cost sharing percentage level such that it induces a voluntary recall. We conduct comparative statics study to see how the equilibrium solutions evolve with the change of parameters. To facilitate comparison, we study the first-best problem where the supply chain is integrated, and find that the recall cost sharing contract cannot coordinate the supply chain. We also consider an extension model with an stochastic recall probability and conduct numerical studies on equilibrium solutions.

The rest of the paper is organized as follows. Section 2 provides a review of related literatures. Section 3 presents the model while Sections 4 present the analysis for the model. Section 5 concludes our work.

4.2 Literature Review

By considering both the quality incentives in reducing the product recall risk and the product recall decisions, our work naturally contributes two streams of literatures: supply chain contracting literature and the product recall literature. There is a growing literature on quality contracting in supply chains. This stream of literature focuses on modeling the quality improvement incentives of supply chain members. Quality could be improved either by inspection or by investment from multiple players. (Reyniers and Tapiero 1995) consider a model in which a supplier makes a unobservable quality-related choice while a producer independently decides on his inspection policy for both non-cooperative and cooperative settings. (Lim 2001) also consider a product quality inspection problem. They investigate the contract design of a producer when he purchases parts from a supplier, and there is incomplete information regarding the quality of the parts. Similarly, (Baiman et al. 2000) and (Baiman et al. 2001) consider a setting where a supplier, who incurs quality improvement costs, sells an intermediate product to a buyer, who incurs appraisal costs to identify defects, and examine the relationship between product architecture, supply-chain performance metrics, and supply-chain efficiency. There are papers modeling quality improvement investment by both the supplier and the buyer. (Balachandran and Radhakrishnan 2005) examine a supply chain in which the final product consists of components made by a buyer and a supplier in a double moral hazard situation. (Zhu et al. 2007) focus on supply risk and consider a buyer who designs a product and outsources the production to a supplier and both players have options to invest in quality improvement. They also investigate the interaction between quality-improvement decisions interact and operational quantity decisions such as the buyer's order quantity and the supplier's production lot size using an EOQ model. In addition, all papers listed out above examine the fixed sharing rate contracts covering the external quality costs. As a more general case, (Chao et al. 2009) focus on recall instances, and discuss two external quality cost sharing contracts, in which product recall costs can be shared between a manufacturer and a supplier to induce effort from both sides to improve final product quality. They characterize the quality improvement effort decisions which are subject to moral hazard and even when there is information asymmetry regarding to the existing process capability. There are some empirical papers studying on the topic of product recall risk, although very limited analytical research. This stream of literature help understand how product recall is conducted by various players and what is the impact of product recall on various operations performance. (Rupp and Taylor 2002) provide an excellent overview of product recall process and introduce different types of product recall strategies. (Hendricks and Singhal 2003), (Rupp 2004) and (Jarrell and Peltzman 1985) focus on product recall costs and point out that once a recall is issued, there are immediate costs due to repairs, recall logistics and lost sales as well as the potential litigation fees and damaged reputation. (Kalaignanam, Kushwaha and Eilert 2013) suggest that recall magnitude lead to decreases in future number of injuries and recalls, and also summarize the findings of the impact of product recall on firm's performance, future recall rates, consumer responses, market accidents and product reliability. (Marucheck et al. 2011) explore how the field of operations management can provide fresh views and insights in addressing the problems of product safety in the context of global supply chain.

4.3 Model

We consider a single period, single sourcing setting where a downstream manufacturer outsources the production of a customized component to an upstream supplier. After receiving the components from the supplier, the manufacturer finishes assembling and releases the finished products to a market with fixed size. The components are subject to potential quality failure, and the customers who incur such problem may issue complaints

$C_v(e)$	$=$	unit voluntary recall cost
$C_m(e)$	$=$	unit mandatory recall cost
α_v	$=$	unit voluntary recall cost parameter
α_m	$=$	unit mandatory recall cost parameter
s	$=$	quality effort marginal cost
λ	$=$	mandatory recall probability parameter
$\exp\{-\lambda e\}$	$=$	recall probability
f	$=$	density distribution of λ
F	$=$	cumulative distribution of λ
d_1	$=$	demand before recall decision point
d_2	$=$	demand after recall decision point
$\bar{C}$	$=$	supplier's reservation cost
θ	$=$	recall cost sharing percentage
e	$=$	quality effort by the supplier

Table 4.1: Notations

to the manufacturer. In the middle of the period when there are certain amount of complaints and quality failure reports, the manufacturer may conduct a private investigation with the supplier and obtain information about the component's quality. Based on the information he collects, the manufacturer has the option to issue a voluntary recall and fix the problem. Otherwise, he may withhold the information and keep selling the product, however, by the end of the period, the product may face mandatory recall enforced by the government agency due to either too many complaints noticed by the public or the spot test by the government agency. If the manufacturer chooses not to issue a voluntary recall, the mandatory recall takes place with a recall probability characterized by an exponential form of $\exp\{-\lambda e\}$, where e denotes the supplier's quality effort, and λ is a given constant. Note that the recall probability is decreasing and convex in e, which also resembles the realistic case. By deciding how much to invest in the quality effort e, the supplier can affect the possibility of product recall. Besides, we assume that e is not contractible, which implies the manufacturer, as a Stackelberg leader, faces a moral hazard problem and need to incentivize the supplier to invest more in quality effort. Both types of recall- voluntary and mandatory- are costly. However, the unit voluntary recall cost is lower than that of mandatory recall because the problem often become more serious as time goes by, and the government could penalize the manufacturer additionally for withholding the information and do not recall voluntarily. Denote $C_v(e)$ and $C_m(e)$ to be the unit voluntary recall cost and unit mandatory recall cost, which are both functions of supplier's effort e. Assume $C_v(e) < C_m(e)$, $C_v'(e) < 0$, $C_m'(e) < 0$, $C_v''(e) > 0$, $C_m''(e) > 0$, which reveals that the unit recall costs are decreasing convex in the supplier's effort. In addition we assume $C_v(e)/C_m(e) = \alpha_v/\alpha_m$. The supplier incurs marginal quality cost s for the quality effort she invests in. In terms of the contract that the manufacturer provides to the supplier, we consider a simple recall cost sharing contract, which is commonly used in the literatures. There is only one parameter in the contract, the recall cost sharing percentage θ, where $0 \leq \theta \leq 1$. In other words, the manufacturer offers the supplier with a percentage of the total recall cost that he will share in case a recall happens.

The sequence of events is as follows: The manufacturer, as a stackelberg Leader, offers a cost sharing percentage θ, and then the supplier decides his quality effort e. In the middle of the period when there are certain amount of complaints and quality failure reports, the manufacturer makes the decision of whether or not to issue a voluntary recall based on the evaluation of the voluntary recall cost and expected mandatory recall cost. Once a voluntary recall is issued, assume the problem is completely fixed by the recall process and no mandatory recall will take place later. Otherwise, by the end of the period, the mandatory recall happens with the probability $\exp\{-\lambda e\}$.

4.4 Analysis

In this section, we first analyze the decentralized setting, where both the manufacturer and the supplier make decisions separately with the objectives of minimizing their own costs. The problem is a stylized principle-agent problem under such setting. To facilitate comparison with this setting, we next consider the centralized supply chain setting, in which all decisions are made by a central planner with the objective of minimizing supply

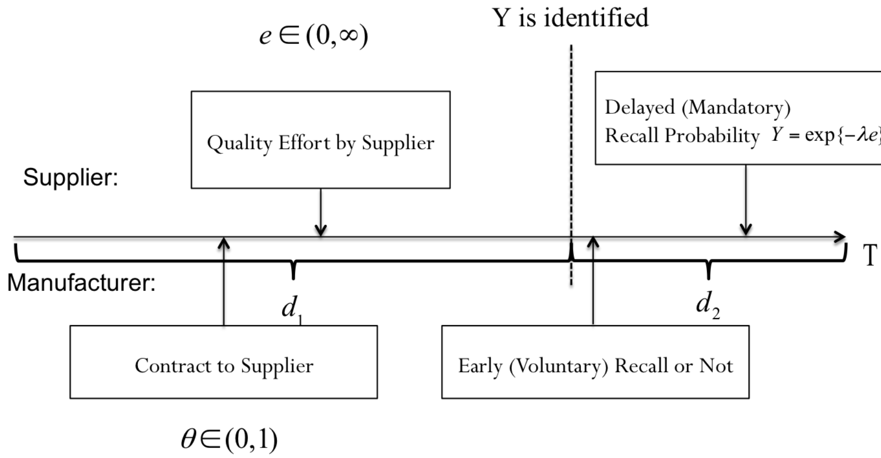


Figure 4.2: Sequence of Events

chain's cost. Finally, we will extend the previous model to allow λ to be random.

4.4.1 Decentralized Supply Chain

We solve the problem by backward induction. After the manufacturer identifies the recall probability, he is faced with the decision of whether or not to issue a voluntary recall. To make the decision he needs to compare the cost of voluntary recall and the expected cost of mandatory recall. To issue a voluntary recall results in a cost of $C_v(e)d_1$, while if the manufacturer chooses to wait, he will face a expected cost of $C_m(e)(d_1 + d_2)\exp\{-\lambda e\}$ due to the potential mandatory recall later. Specifically, the problem can be expressed as the following principle-agent problem:

$$\begin{aligned}
\min_{\theta} \quad & \theta H(e^*(\theta)) \\
\text{s.t.} \quad & se^*(\theta) + (1-\theta)H(e^*(\theta)) \le \bar{C} \quad \text{(IR)} \\
& e^*(\theta) = \arg\min_{e} \; se + (1-\theta)H(e) \quad \text{(IC)},
\end{aligned}$$

where $H(e)$ is the expected recall cost:

$$H(e) = \begin{cases} C_v(e)d_1, & \text{if the manufacturer issues a voluntary recall;} \\ C_m(e)(d_1+d_2)\exp\{-\lambda e\}, & \text{if the manufacturer does not issue a voluntary recall.} \end{cases}$$

and $\bar{C}$ is the reservation cost for the supplier.

Given θ and e, the manufacturer will compare the issue a voluntary recall if

$$\theta C_v(e)d_1 < [\theta C_m(e)(d_1 + d_2)] \exp\{-\lambda e\}.$$

The above assumptions immediately yields that the manufacturer will place a voluntary recall if

$$e \in \left[0, \ln\left(\frac{d_1 + d_2}{d_1}\frac{\alpha_m}{\alpha_v}\right)\Big/\lambda\right], \tag{4.4.1}$$

and not place a voluntary recall if

$$e \in \left[\ln\left(\frac{d_1 + d_2}{d_1}\frac{\alpha_m}{\alpha_v}\right)\Big/\lambda, +\infty\right]. \tag{4.4.2}$$

Denote

$$\tilde{e} = \ln\left(\frac{d_1 + d_2}{d_1}\frac{\alpha_m}{\alpha_v}\right)\Big/\lambda,$$

then we have the following lemma.

Lemma 4.4.1. *The manufacturer will place a voluntary recall if he identifies $e < \tilde{e}$, otherwise, he chooses to face mandatory recall risk.*

Note that the manufacturer's decision of whether to issue a voluntary recall or not only depends on the supplier's effort, and is independent of the cost sharing percentage θ.

Next we move to the supplier's decision of quality effort. Given a cost sharing percentage θ specified by the manufacturer, the supplier determines her optimal quality effort to invest in by minimizing her cost, which could come from either the voluntary recall cost or the mandatory recall cost, depending on the manufacturer's decision based on the her effort level. Then the optimization problem for the supplier becomes:

$$\min_e se + (1 - \theta)H(e),$$

where

$$
se + (1-\theta)H(e) = \begin{cases} se + (1-\theta)d_1 C_v(e), & \text{if } \; e \in [0, \tilde{e}], \\ se + (1-\theta)(d_1+d_2)C_m(e)\exp\{-\lambda e\}, & \text{if } \; e \in [\tilde{e}, +\infty). \end{cases}
$$

Denote $e_1^*(\theta)$ and $e_2^*(\theta)$ satisfying the following equations:

$$
s + (1-\theta)d_1 {C_v}'(e_1^*(\theta)) = 0,
$$

$$
s + (1-\theta)(d_1+d_2)\exp\{-\lambda e_2^*(\theta)\}(C_m'(e_2^*(\theta)) - \lambda C_m(e_2^*(\theta))) = 0.
$$

Then we can show that the optimal quality effort for the supplier exists and is unique, which is presented in the following lemma.

Lemma 4.4.2. *Given θ, the optimal solution $e^*(\theta)$ for the supplier exists and is unique. Specifically,*

(i) if $s + d_1 C_v'(\tilde{e}) < 0$, there exists a threshold $\bar{\theta}$ such that

$$
e^*(\theta) = \begin{cases} e_1^*(\theta), & \text{if } \; \theta \in [\bar{\theta}, 1], \\ e_2^*(\theta), & \text{if } \; \theta \in [0, \bar{\theta}], \end{cases}
$$

where

$$
s(e_1^*(\bar{\theta}) - e_2^*(\bar{\theta})) + (1-\bar{\theta})[d_1 C_v(e_1^*(\bar{\theta})) - (d_1+d_2)C_m(e_2^*(\bar{\theta}))\exp\{-\lambda e_2^*(\bar{\theta})\}] = 0;
$$

(ii) if $s + (d_1+d_2)\exp\{-\lambda\tilde{e}\}(C_m'(\tilde{e}) - \lambda C_m(\tilde{e})) > 0$, for any $\theta \in [0, 1]$,

$$
e^*(\theta) = e_1^*(\theta);
$$

(iii) if $s + d_1 C_v'(\tilde{e}) > 0$ and $s + (d_1+d_2)\exp\{-\lambda\tilde{e}\}(C_m'(\tilde{e}) - \lambda C_m(\tilde{e})) < 0$, there exists a threshold $\min\{\tilde{\theta}, \bar{\theta}\}$ such that

$$
e^*(\theta) = \begin{cases} e_1^*(\theta), & \text{if } \; \theta \in [\min\{\tilde{\theta}, \bar{\theta}\}, 1], \\ e_2^*(\theta), & \text{if } \; \theta \in [0, \min\{\tilde{\theta}, \bar{\theta}\}], \end{cases}
$$

where

$$s(e_1^*(\bar{\theta}) - e_2^*(\bar{\theta})) + (1-\bar{\theta})[d_1 C_v(e_1^*(\bar{\theta})) - (d_1+d_2) C_m(e_2^*(\bar{\theta})) \exp\{-\lambda e_2^*(\bar{\theta})\}] = 0,$$

and

$$s + (1-\tilde{\theta})(d_1+d_2)\exp\{-\lambda \tilde{e}\}(C_m'(\tilde{e}) - \lambda C_m(\tilde{e})) = 0.$$

Lemma 4.4.2 indicates that the optimal solution $e^*(\theta)$'s structure depends on the parameters. Specifically, the parameter space is divided into three regions, each of which corresponds to one type of optimal $e^*(\theta)$. The inequalities that describe the three regions can be viewed as conditions of s, i.e., when s is small, moderate, and large. When s is small or moderate, corresponding to (i) and (iii), there are two forms of $e^*(\theta)$'s expression depending on the θ's value. When θ is small, $e^*(\theta) = e_2^*(\theta)$, which induces the manufacturer to choose to wait; when θ is large, $e^*(\theta) = e_1^*(\theta)$, which induces the manufacturer to recall voluntarily. When s is large, $e^*(\theta)$ always equals to $e_1^*(\theta)$, i.e., the manufacturer will always choose to recall voluntarily.

Next we want to discuss the relation between θ and $e^*(\theta)$, and we have the following corollary.

Lemma 4.4.3. $e^*(\theta)$ *is decreasing in* θ*.*

Lemma 4.4.3 indicates that the supplier has less incentive to invest in quality effort when she needs to share less of recall costs.

Now we focus on solving the manufacturer's problem, which is

$$\min_{\theta} \ \theta H(e^*(\theta)).$$

Since $e^*(\theta)$ is decreasing in θ, thus there is one-on-one relationship between these two decision variables. We could use e^* represents θ, and it is equivalent for the manufacturer to decide on e^* instead of θ. The feasible region of e is such that the corresponding θ lies in the interval $[0, 1]$. The manufacturer's problem could be written as below:

$$\min_{e^*} \ \theta(e^*)H(e^*).$$

First, we have the following two lemmas on the monotonicity of the cost of manufacturer and supplier with respect to θ. For the manufacturer, we have

Lemma 4.4.4. *Manufacturer's cost is increasing in θ, respectively in the voluntary recall and the mandatory recall settings. Namely, both $\theta H(e_1^*(\theta))$ and $\theta H(e_2^*(\theta))$ are increasing in θ.*

For the manufacturer, we have

Lemma 4.4.5. *Supplier's cost is decreasing in θ, respectively in the voluntary recall and the mandatory recall settings. Namely, both $se_1^*(\theta) + (1-\theta)d_1 C_v(e_1^*(\theta))$ and $se_2^*(\theta) + (1-\theta)(d_1 + d_2)C_m(e_2^*(\theta)) \exp\{-\lambda e_2^*(\theta)\}$ are decreasing in θ.*

Now we are ready to characterize the optimal solution structure for the decentralized model, and the conclusion is presented by the following proposition.

Proposition 4.4.1. *The optimal solution θ^* for the manufacturer exists and is unique. Specifically,*

(i) if $s + (d_1 + d_2)\exp\{-\lambda\tilde{e}\}(C'_m(\tilde{e}) - \lambda C_m(\tilde{e})) < 0$, when

$$\begin{aligned}(d_1 + d_2)C_m(e_2^*(\underline{\theta}_2)) \exp\{-\lambda e_2^*(\underline{\theta}_2)\} &- \frac{sC_m(e_2^*(\underline{\theta}_2))}{\lambda C_m(e_2^*(\underline{\theta}_2)) - C'_m(e_2^*(\underline{\theta}_2))} \\ &< d_1 C_v(e_1^*(\underline{\theta}_1)) + \frac{sC_v(e_1^*(\underline{\theta}_1))}{C'_v(e_1^*(\underline{\theta}_1))}\end{aligned}$$

is satisfied, then $\theta^* = \underline{\theta}_2$; otherwise, $\theta^* = \underline{\theta}_1$.

where $\underline{\theta}_1$ and $\underline{\theta}_2$ are solutions to the following equations respectively:

$$se_1^*(\underline{\theta}_1) + (1 - \underline{\theta}_1)d_1 C_v(e_1^*(\underline{\theta}_1)) = \bar{C};$$

$$se_2^*(\underline{\theta}_2) + (1 - \underline{\theta}_2)(d_1 + d_2)C_m(e_2^*(\underline{\theta}_2)) \exp\{-\lambda e_2^*(\underline{\theta}_2)\} = \bar{C}.$$

(ii) if $s + (d_1 + d_2)\exp\{-\lambda\tilde{e}\}(C'_m(\tilde{e}) - \lambda C_m(\tilde{e})) > 0$, $\theta^* = \underline{\theta}_1$.

Immediately followed the proof of Proposition 4.4.1, we further have the following corollary.

Corollary 4.4.1. *The voluntary recall is always associated with* $\underline{\theta}_1$*, while the mandatory recall is always associated with* $\underline{\theta}_2$*.*

According to Proposition 4.4.1 and Corollary 4.4.1, there are two cost sharing percentage thresholds that determines the equilibrium solutions. Specifically, the lower cost sharing percentage is always associated with voluntary recall, while the higher one is always associated to mandatory recall. However, we do find that when it is more costly for the supplier to make quality effort, the manufacturer will choose a cost sharing percentage level such that it induces a voluntary recall.

By adopting the specific form of unit voluntary recall cost and unit mandatory recall cost, i.e., $C_v(e)$ and $C_m(e)$, we can further obtain a close-form expressions for $\underline{\theta}_1$ and $\underline{\theta}_2$.

Proposition 4.4.2. *Assume* $C_v(e) = \alpha_v/e$, $C_m(e) = \alpha_m/e$, *we have the following expressions for the two optimal cost sharing percentages.*

$$\underline{\theta}_1 = 1 - \frac{\bar{C}^2}{4sd_1\alpha_v},$$

and

$$\underline{\theta}_2 = 1 - \frac{\bar{C}^2\lambda^2 - (2s - \sqrt{4s^2 + \bar{C}^2\lambda^2})^2}{4s\lambda^2(d_1 + d_2)\alpha_m \exp\{-\frac{\bar{C}\lambda - 2s + \sqrt{4s^2 + \bar{C}^2\lambda^2}}{2s}\}}.$$

Furthermore, we have

$$e_1^* = \frac{\bar{C}}{2s},$$

and

$$e_2^* = \frac{\bar{C}\lambda - 2s + \sqrt{4s^2 + \bar{C}^2\lambda^2}}{2s\lambda}.$$

By the closed-form expressions of the cost sharing percentage θ by the manufacturer and quality effort e by the supplier, we are interesting how these optimal solutions change with respect to operational parameters. The results are summarized by the following proposition.

Proposition 4.4.3. *In the decentralized supply chain model,*

(i) e_1^* *is decreasing in* s*, and irrelevant to* λ, d_1, d_2 *and* α_m*;*

(ii) e_2^* *is increasing in* λ*, decreasing in* s*, and irrelevant to* d_1, d_2 *and* α_m*;*

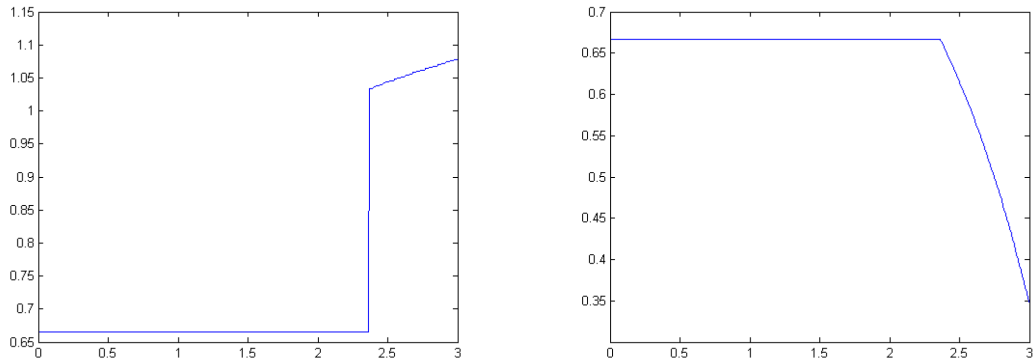
(iii) $\underline{\theta}_1$ *is increasing in* s, d_1*, and irrelevant to* λ, d_2 *and* α_m*;*

(iv) $\underline{\theta}_2$ *is increasing in* s, d_1, d_2, α_m*, decreasing in* λ*.*

We leave the discussion of intuitions to the numerical part later in order to gain better understanding with the aid of numerical graphs.

Comparative Statics

Next we examine how the equilibrium solutions change as the parameters change. Specifically, we want to study how θ^* and e^* evolve with the parameters' values. There are five parameters we are interested in: mandatory recall probability parameter λ, demand parameters d_1 and d_2, and cost parameters s and α_m. We start with λ first and the result is presented in Figure 4.3, where (a) and (b) show the change of e^* and θ^* with λ, respectively. The horizon axis represents the value of λ, and the vertical axis refers to e^* in (a) or θ^* in (b). With larger λ, the resulting mandatory recall probability with same amount of effort becomes lower. As is shown by Figure 4.3, both curve are flat when λ is relatively small, implying both e^* and θ^* are independent of the value of λ in such case. The reason is that, the mandatory recall probability is relatively high when λ is small, and hence leads to a high expected mandatory recall cost. Therefore in such case, the manufacturer and supplier choose the e^* and θ^* such that they induce a



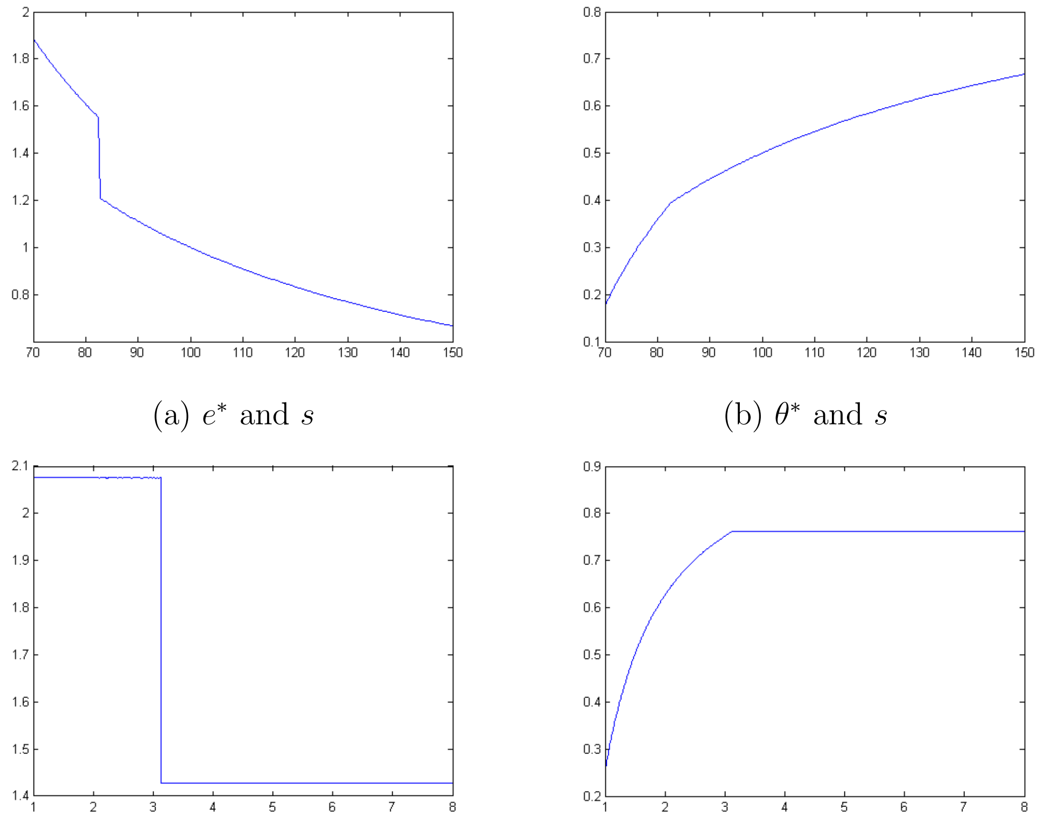
(a) e^* and λ (b) θ^* and λ

Figure 4.3: Equilibria Analysis with λ

voluntary recall, and hence are independent of the specific values of λ. However, when λ is relatively large, e^* is increasing in λ, while θ^* is decreasing in λ. The intuition behind this is similar as before. With larger λ, the mandatory recall probability is small, so the manufacturer and the supplier choose e^* and θ^* in equilibrium such that mandatory recall is induced. With smaller expected mandatory recall cost, the manufacturer can push more percentage of recall cost to the supplier while not violating the individual rationality condition. As a result, the supplier has the incentive to input higher quality effort since his share of recall cost is higher.

Figure 4.4 (a) and (b) display how the equilibrium solutions evolve with the change of quality marginal cost s. With higher quality marginal cost s, the supplier tends to put in less quality effort, therefore the manufacturer has the incentive to increase θ to counteract the supplier's tendency to decrease quality effort. Note that there is jump in the supplier's effort level, which corresponds to the change from voluntary recall to mandatory recall.

Figure 4.4 (c) and (d) present how the equilibrium solutions change as the unit mandatory recall cost parameter α_m increase. When α_m is small, the manufacturer and the supplier will induce the mandatory recall instead of voluntary recall. In this region, as the



(a) e^* and s

(b) θ^* and s

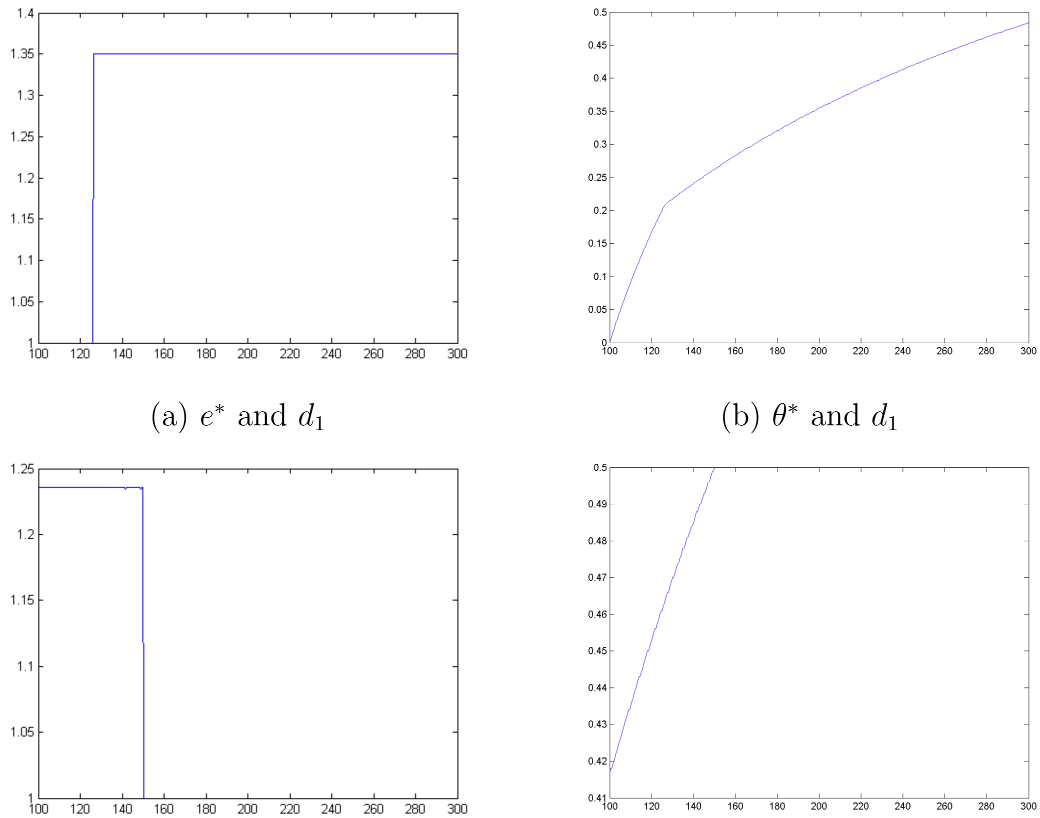
(c) e^* and α_m

(d) θ^* and α_m

Figure 4.4: Equilibria Analysis with Cost Parameters

unit mandatory recall cost α_m increases, the the manufacturer need to increase his cost share percentage θ so that the supplier's individual rationality condition is not violated. On the other hand, the supplier's effort exibits to be constant due to two conflicting factors: the decreasing cost share percentage induces a lower effort, while the increasing unit mandatory recall cost promotes a higher effort. Therefore overall, it behaves as a constant effort level. While as α_m becomes large enough, the voluntary recall would always be preferred to mandatory recall for the manufacturer and the supplier. Therefore, they will choose e^* and θ^* so as to induce voluntary recall. Hence the equilibrium solutions are independent of the specific value of α_m in this region.

Figure 4.5 displays the comparative statics of equilibrium solutions as the demand parameters d_1 and d_2 change. When d_1 is small or d_2 is large, i.e., the voluntary recall



(a) e^* and d_1

(b) θ^* and d_1

(c) e^* and d_2

(d) θ^* and d_2

Figure 4.5: Equilibria Analysis with Demand Parameters

cost is low or the mandatory recall cost is high, the voluntary recall is preferred. In such scenario, the supplier has no incentive to increase her quality effort level since it does not affect the voluntary recall cost but increase her cost. Otherwise, when d_1 is large or d_2 is small, the supplier will increase her effort level since now mandatory recall is preferred, and hence her quality effort affects the recall cost she is going to share. In this region, as d_1 and d_2 increase, the expected mandatory recall cost increase so the manufacturer needs to increase his share of recall cost to guarantee the IR condition to continue to hold. However within this region, the effort level is flat and the reason again comes from two conflicting forces: the manufacturer increases his share of recall cost, i.e., the cost percentage of supplier becomes smaller, while the expected mandatory recall cost increases as d_1 and d_2 increase.

Note that in all the figures we have discussed, the curves have a turning point which divides the curve into two parts, which corresponds to voluntary recall and mandatory recall, respectively. This is consistent with our findings in Propositions 4.4.1, 4.4.2 and 4.4.3.

4.4.2 Integrated Supply Chain

Next we move on to the integrated supply chain when a central planner takes care of all decisions and he aims to minimize the supply chain's total cost. Note that under this setting, cost sharing percentage θ is no longer needed because the manufacturer and the supplier do not consider their own cost separately.

Denote the integrated supply chain problem as below:

$$\min_{e} \quad se + \min\{C_v(e)d_1, C_m(e)(d_1 + d_2)\} \exp\{-\lambda e\}.$$

We first present the optimal solution of supplier's quality effort in this integrated setting.

Lemma 4.4.6. *The optimal solution e^{FB} for the central planner exists and is unique. Specifically,*

(i) if $s + d_1 C_v'(\tilde{e}) < 0$, then $e^{FB} = e_2^{FB}$,

where

$$s + (d_1 + d_2) \exp\{-\lambda e_2^{FB}\}(C_m'(e_2^{FB}) - \lambda C_m(e_2^{FB})) = 0;$$

(ii) if $s + (d_1 + d_2) \exp\{-\lambda e\}(C_m'(e) - \lambda C_m(e)) > 0$, then $e^{FB} = e_1^{FB}$,

where

$$s + d_1 C_v'(e_1^{FB}) = 0;$$

(iii) if $s + d_1 C_v'(\tilde{e}) > 0$ *and* $s + (d_1 + d_2)\exp\{-\lambda e\}(C_m'(e) - \lambda C_m(e)) < 0$*, when*

$$se_1^{FB} + d_1 C_v(e_1^{FB}) < se_2^{FB} + (d_1 + d_2) C_m(e_2^{FB} \exp\{-\lambda e_2^{FB}\}$$

is satisfied, then $e^{FB} = e_1^{FB}$*; otherwise,* $e^{FB} = e_2^{FB}$*.*

Similarly as in Lemma 4.4.2, the parameter space is also divided into three regions here, and the regions are exactly the same as before. In addition, e_1^{FB} corresponds to the effort level when the central planner would also choose to voluntary recall, and e_2^{FB} corresponds to the one when he would wait and bear the potential mandatory recall. Interestingly, we find that under the integrated setting, when s is relatively small, voluntary recall is never preferred, unlike the decentralized case where the decision depends on the cost sharing percentage.

Next we establish the comparison between the decentralized supply chain setting and the integrated supply chain setting and yield the following proposition,

Proposition 4.4.4. $e_1^* < e_1^{FB}$ *and* $e_2^* < e_2^{FB}$*, therefore in the decentralized supply chain setting, the supplier's optimal quality effort is lower than that in the integrated supply chain setting, therefore the recall cost sharing contract cannot coordinate the supply chain.*

This proposition shows that the quality effort made by the supplier in the decentralized supply chain setting is always smaller than that made by the central planner, which implies that the recall cost sharing contract cannot coordinate the supply chain.

4.4.3 Decentralized Supply Chain with Random λ

So far we have assumed λ to be deterministic for the sake of tractability. However, we do notice the limitation that the supplier can fully anticipate the mandatory recall probability based on the effort level she chooses. Therefore, in this section, we introduce randomness to the recall probability by allowing λ to be random, and the randomness is

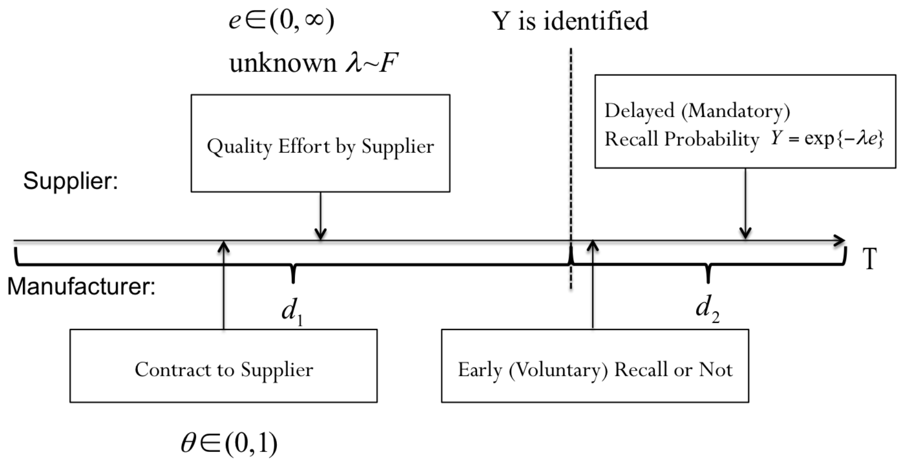


Figure 4.6: Sequence of Events with Random λ

resolved by the time of the decision of whether to issue a voluntary recall or not. Now the sequence of events is changed as follows:

The manufacturer, as a Stackelberg leader, offers a cost sharing percentage θ, and then the supplier decides her quality effort e. At this point, λ for the mandatory recall probability is random. By the time of the decision of voluntary recall or not, λ is realized and the mandatory recall probability is observed. The manufacturer makes the decision of whether or not to issue a voluntary recall based on the evaluation of the voluntary recall cost and expected mandatory recall cost. Once a voluntary recall is issued, assume the problem is completely fixed by the recall process and no mandatory recall will take place later. Otherwise, by the end of the period, the mandatory recall happens with the probability $\exp\{-\lambda e\}$.

We follow a similar backward induction procedure as before to solve this problem. Given θ, e and realized λ, the manufacturer will issue a voluntary recall if

$$\theta C_v(e) d_1 < [\theta C_m(e)(d_1 + d_2)] \exp\{-\lambda e\},$$

which is equivalent to

$$\exp\{-\lambda e\} > \frac{\alpha_v d_1}{\alpha_m (d_1 + d_2)},$$

or,

$$\lambda < \frac{1}{e} ln \left(\frac{\alpha_m (d_1 + d_2)}{\alpha_v d_1} \right).$$

Define

$$\alpha(e) = \frac{1}{e} ln \left(\frac{\alpha_m (d_1 + d_2)}{\alpha_v d_1} \right),$$

note that $\alpha(e)$ is always positive, and the manufacturer chooses voluntary recall if $\lambda \leq \alpha(e)$, and chooses to wait otherwise.

Now we are ready to analyze the supplier's problem. For the supplier, given θ,

$$\min_e se + (1 - \theta) H(e),$$

$$\text{where} \qquad H(e) = C_v(e) d_1 F_\lambda(\alpha(e)) + \int_{\alpha(e)}^{\infty} C_m(e)(d_1 + d_2) \exp\{-\lambda e\} f(\lambda) d\lambda.$$

Let $e^*(\theta)$ denote the optimal solution to the supplier's cost minimization problem above.

Based on the above analysis, we now can move to the manufacturer's decision of θ and write out the whole principle-agent problem as follows:

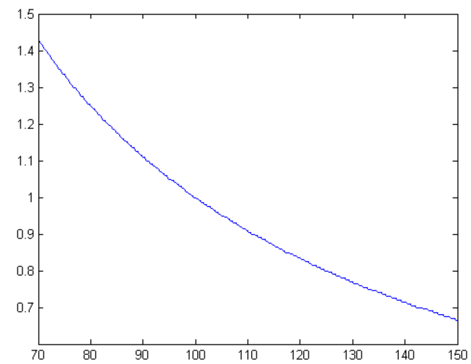
$$\begin{aligned} \min_\theta \quad & \theta H(e^*(\theta)) \\ \text{s.t.} \quad & se^*(\theta) + (1 - \theta) H(e^*(\theta)) \leq \bar{C} \quad \text{(IR)} \\ & e^*(\theta) = \arg\min_e \ se + (1 - \theta) H(e) \quad \text{(IC)}, \end{aligned}$$

where $H(e)$ is the expected recall cost:

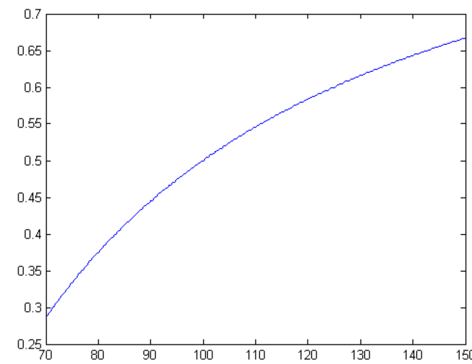
$$H(e) = C_v(e) d_1 F_\lambda(\alpha(e)) + \int_{\alpha(e)}^{\infty} C_m(e)(d_1 + d_2) \exp\{-\lambda e\} f(\lambda) d\lambda,$$

and $\bar{C}$ is the reservation cost for the supplier.

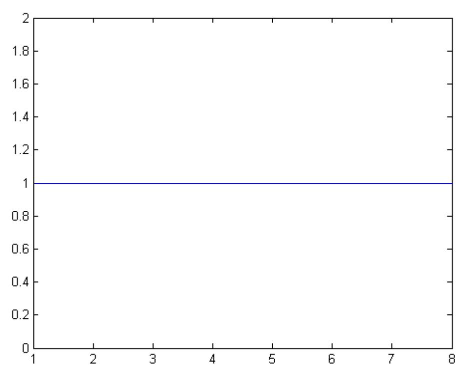
Next we conduct comparative statics analysis to help analyze this problem. Here we assume λ is according to a uniform distribution, and the results are presented in Figures 4.7 and 4.8. Since λ is random now, a given set of θ and e could result in the choice of voluntary recall or mandatory recall depending on the realization of λ by the time when



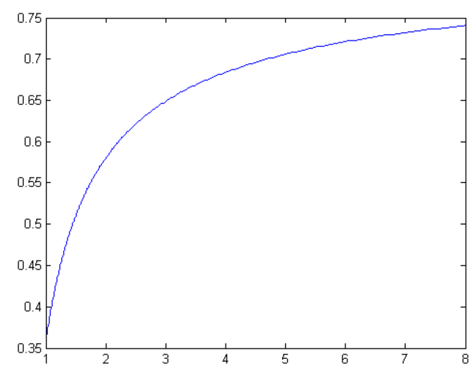
(a) e^* and s



(b) θ^* and s



(c) e^* and α_m



(d) θ^* and α_m

Figure 4.7: Equilibria Analysis with Cost Parameters under Random λ

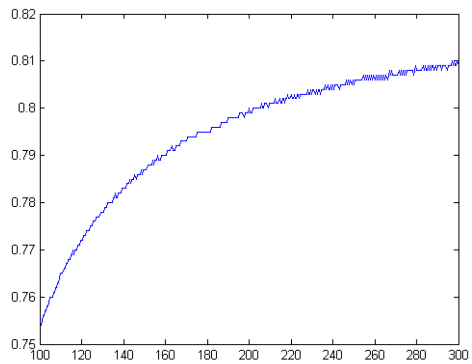
the manufacturer needs to make the decision. Therefore, we no longer have the turning point in the curves as we did in the fixed λ case, where the regions divided by the turning point corresponded to preference of voluntary recall and mandatory recall, respectively.

Figure 4.7 (a) and (b) presents the trend of equilibrium solutions as the change of quality marginal cost s. As s increases, the supplier has the incentive to decrease his effort level, and the manufacturer will increase his cost share percentage. Figure 4.7 (c) and (d) display the trend when the unit mandatory recall cost α_m changes. As α_m increases, the expected mandatory recall cost increases. Since before the realization of λ, mandatory recall could always be preferred with some probability, the manufacturer needs to increase his cost share percentage to ensure the IR condition to hold. While for the supplier, the effect of manufacturer's decreasing θ and the effect of increased mandatory recall cost cancel out, and therefore result in constant effort level.

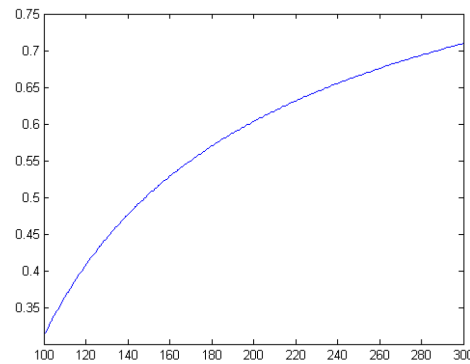
Figure 4.8 shows the dependence of equilibrium solutions as the demand parameters d_1 and d_2 change. The general trend accords with the case with fixed λ except for there is no turning point in the curve, and the curves are strictly monotone when λ is uncertain. As d_1 increases, both voluntary recall cost and expected mandatory recall cost increase. The manufacturer again increases θ to keep the supplier in the game, and the supplier increases his effort level to counteract the increase of mandatory recall cost. The argument for d_2 is similar by noting that a higher d_2 leads to higher expected mandatory recall cost.

4.5 Conclusion

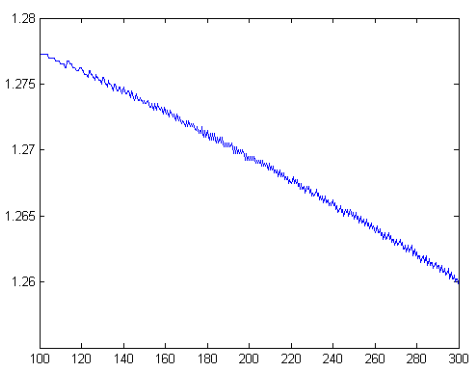
In this paper, we consider a recall cost sharing contract proposed by the manufacturer, as a Stackelberg leader facing a moral hazard problem, to align the incentive of the supplier in ensuring the product quality, and also examine two types of recall decisions by the manufacturer, i.e., voluntary recall and mandatory recall. We consider two settings,



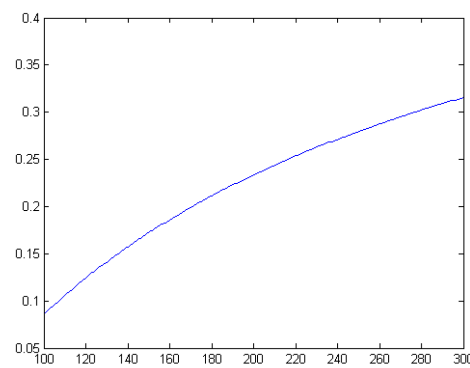
(a) e^* and d_1



(b) θ^* and d_1



(c) e^* and d_2



(d) θ^* and d_2

Figure 4.8: Equilibria Analysis with Demand Parameters under Random λ

both with realistic applications: we first consider the setting when the mandatory recall probability solely depends on the supplier's quality effort, and then consider the setting when there is extra randomness involved in the mandatory recall probability. To facilitate comparison, we also consider the first best problem where the supply chain is integrated. By solving the model, we characterize the firms' decisions in equilibrium. Surprisingly, we find that, as a Stackelberg leader, the manufacturer does not always push all the recall cost to the supplier. His decision depends on whether he will issue a voluntary recall or not. Furthermore, we find two cost sharing percentage thresholds that determines the equilibrium solutions. Specifically, the lower cost sharing percentage is always associated with voluntary recall, while the higher one is always associated to mandatory recall. We conduct comparative statics study to see how the equilibrium solutions evolve with the change of parameters. In addition, we find that the recall cost sharing contract cannot coordinate the supply chain.

There are certain limitations and possible extensions for future research with the models considered in this paper. In our work we focus more on the decisions within the supply chain and the interdependence between designing the product recall contract and making product recall decisions. Actually, there are many other angles to study the process of recall. For example, the games between the supply chain players and the government agency are potentially interesting research questions for future study. Besides, more analytical results for the decentralized supply chain with random λ would be contributing.

Appendix A SUPPLEMENTARY RESULTS AND PROOFS FOR CHAPTER 2

Start-Up and Established Firm - No Vertical Differentiation

We first examine the new capacity subgame when both firms choose the same quality level, either S_l or S_h, and then compete with each other in capacity quantity. It is similar as the classic Cournot game, however, one key difference is that here two firms are not symmetric (start up vs established firm) regarding to the objectives, besides, different timing structures of the capacity investment are also considered. The demand function is:

$$P = -SQ_s - SQ_e + S\hat{\theta},$$

where Q_s and Q_e denote the product quantities released to the market by the start-up and the established firm. The production clearance strategy is carried over here, i.e., $Q_s = K_s, Q_e = K_e$, where K_s, K_e are the capacity quantities of start-up and established firm. Following standard procedures, we obtain the following results[1]:

	ψ^*	$E(\pi^*)$
$(E^s,\, E^e)$	$1 - G\left(2\sqrt{\frac{\alpha}{S}} + \frac{S\mu - \sqrt{S\alpha} - \beta_2 S^2}{2S} + \beta_1 S\right)$	$\frac{(S\mu - S\sqrt{\alpha/S} - \beta_2 S^2)^2}{4S}$
$(E^s,\, L^e)$	$1 - G\left(2\sqrt{\frac{2\alpha}{S}} + (2\beta_1 - \beta_4)S\right)$	$\frac{S}{4}(\mu^2 + \sigma_2) - \left(\sqrt{S\alpha} + \frac{\beta_4 S^2}{2}\right)\mu + \frac{\alpha}{2} + \frac{\beta_4^2}{4}S + \sqrt{\frac{\alpha}{2S}}\beta_4 S^2$
$(L^s,\, E^e)$	$1 - G\left(\frac{\mu}{2} + \frac{3\beta_3 - 2\beta_2}{2}S + 2\sqrt{\frac{\alpha}{S}}\right)$	$\frac{1}{8}S\mu^2 + \frac{\beta_3 - 2\beta_2}{4}\mu S^2 + \frac{(\beta_3 - 2\beta_2)^2}{8}S^3$
$(L^s,\, L^e)$	$1 - G\left(2\beta_3 S - \beta_4 S + 3\sqrt{\frac{\alpha}{S}}\right)$	$\frac{S}{9}(\mu^2 + \sigma_2) + \left(\frac{\beta_3 - 2\beta_4}{3}\right)^2 S^3 + \frac{2}{9}(\beta_3 - 2\beta_4)S^2\mu$

Table A.1: No Vertical Differentiation

Table A.1 presents the survival probability of the start-up and the expected profit for the established firm in the capacity quantity subgame, respectively, for four different timing structures. Note that in the table we use S which could be either S_l or S_h.

	ψ^*	$E(\pi^*)$
(S_l^s, S_l^e)	$1-G\left(2\beta_3 S_l-\beta_4 S_l+3\sqrt{\frac{\alpha}{S_l}}\right)$	$\frac{S_l}{9}(\mu^2+\sigma_2)+\left(\frac{\beta_3-2\beta_4}{3}\right)^2 S_l^3+\frac{2}{9}(\beta_3-2\beta_4)S_l^2\mu$
(S_l^s, S_h^e)	$1-G\left(\frac{2\beta_3 S_l^2 S_h-\beta_4 S_h^2 S_l+(4S_h-S_l)\sqrt{S_l\alpha}}{S_h S_l}\right)$	$S_h\left(1-\frac{2S_h}{4S_h-S_l}\right)^2(\mu^2+\sigma^2)$ $+2S_h\left(1-\frac{2S_h}{4S_h-S_l}\right)\left(\frac{\beta_3 S_l^2-2\beta_4 S_h^2}{4S_h-S_l}\right)\mu+\left(\frac{\beta_3 S_l^2-2\beta_4 S_h^2}{4S_h-S_l}\right)^2 S_h$
(S_h^s, S_l^e)	$1-G\left(\frac{2\beta_3 S_h^3-\beta_4 S_l^2 S_h+(4S_h-S_l)\sqrt{S_h\alpha}}{S_h(2S_h-S_l)}\right)$	$\frac{S_l S_h^2}{(4S_h-S_l)^2}(\mu^2+\sigma^2)-\frac{2S_h(2\beta_4 S_l^2 S_h-\beta_3 S_h^2 S_l)}{(4S_h-S_l)^2}\mu+\frac{(2\beta_4 S_l^2 S_h-\beta_3 S_h^2 S_l)^2}{S_l(4S_h-S_l)^2}$
(S_h^s, S_h^e)	$1-G\left(2\beta_3 S_h-\beta_4 S_h+3\sqrt{\frac{\alpha}{S_h}}\right)$	$\frac{S_h}{9}(\mu^2+\sigma_2)+\left(\frac{\beta_3-2\beta_4}{3}\right)^2 S_h^3+\frac{2}{9}(\beta_3-2\beta_4)S_h^2\mu$

Table A.2: The Start-Up's Optimal Survival Probability and The Established Firm's Expected Profit When Both Firms Make Late Capacity Investment

	ψ^*	$E(\pi^*)$
(S_l^s, S_l^e)	$1-G\left(2\sqrt{\frac{2\alpha}{S_l}}+(2\beta_1-\beta_4)S_l\right)$	$\frac{S_l}{4}(\mu^2+\sigma_2)-\left(\sqrt{S_l\alpha}+\frac{\beta_4 S_l^2}{2}\right)\mu+\frac{\alpha}{2}+\frac{\beta_4^2}{4}S_l+\sqrt{\frac{\alpha}{2S_l}}\beta_4 S_l^2$
(S_l^s, S_h^e)	$1-G\left(2\sqrt{\frac{2(2S_h-S_l)\alpha}{(S_h S_l)}}-\beta_4 S_h+2\beta_1 S_l\right)$	$\frac{S_h}{4}(\mu^2+\sigma_2)-\left(\sqrt{\frac{S_l S_h\alpha}{2(2S_h-S_l)}}+\frac{\beta_4 S_h^2}{2}\right)\mu+\frac{S_l\alpha}{2(2S_h-S_l)}$ $+\frac{\beta_4^2}{4}S_h+\sqrt{\frac{S_l\alpha}{2S_h(2S_h-S_l)}}\beta_4 S_h^2$
(S_h^s, S_l^e)	$1-G\left(2\sqrt{\frac{2\alpha}{(2S_h-S_l)}}+\frac{2\beta_1 S_h^2-\beta_4 S_l^2}{2S_h-S_l}\right)$	$\frac{S_l}{4}(\mu^2+\sigma_2)-\left(S_l\sqrt{\frac{\alpha}{2(2S_h-S_l)}}+\frac{\beta_4 S_l^2}{2}\right)\mu+\frac{S_l\alpha}{2(2S_h-S_l)}$ $+\frac{\beta_4^2}{4}S_l+\sqrt{\frac{\alpha}{2(2S_h-S_l)}}\beta_4 S_l^2$
(S_h^s, S_h^e)	$1-G\left(2\sqrt{\frac{2\alpha}{S_h}}+(2\beta_1-\beta_4)S_h\right)$	$\frac{S_h}{4}(\mu^2+\sigma_2)-\left(\sqrt{S_h\alpha}+\frac{\beta_4 S_h^2}{2}\right)\mu+\frac{\alpha}{2}+\frac{\beta_4^2}{4}S_h+\sqrt{\frac{\alpha}{2S_h}}\beta_4 S_h^2$

Table A.3: The Start-Up's Optimal Survival Probability and The Established Firm's Expected Profit When The Start-Up Invests Early and The Established Firm Invests Late

Capacity Quantity Subgame Equilibria

Proof of Table 2.2

Proof. In case of (E_l^s, L_h^e), i.e., the low quality start-up invests early while the high quality established firm invests late. $\pi_{es.}=(P_h-C_{2h})K_h=(-S_lK_l-S_hK_h+S_h\hat{\theta}-C_{2h})K_h=-S_hK_h^2+(S_h\hat{\theta}-S_lK_l-C_{2h})K_h$. Derive the partial derivative with respect to K_h, $K_h^*=\frac{S_h\hat{\theta}-S_lK_l-C_{2h}}{2S_h}$, and $\pi_{es.}(K_l)=\frac{(S_h\hat{\theta}-S_lK_l-C_{2h})^2}{4S_h}$. Note that $\pi_{st.}(K_l)=(P_l-C_{1l})K_l=\frac{S_lK_l}{2}\hat{\theta}-S_lK_l^2+\frac{S_l^2}{2S_h}K_l^2+\frac{S_lC_{2h}}{2S_h}K_l-C_{1l}K_l$, and $\Pr\{\pi_{st.}(K_l)\geq\alpha\}=1-G\left(2K_l-\frac{S_lK_l}{S_h}-\frac{C_{2h}}{S_h}+\frac{2C_{1l}}{S_l}+\frac{2\alpha}{S_lK_l}\right)$. Maximizing $\Pr\{\pi_{st.}(K_l)\geq\alpha\}$ over K_l generates $K_l^*=\sqrt{\frac{2S_h\alpha}{S_l(2S_h-S_l)}}$, $K_h^*=\frac{\hat{\theta}}{2}-\sqrt{\frac{S_l\alpha}{2S_h(2S_h-S_l)}}$. Then we get $\psi^*=1-G\left(2\sqrt{\frac{2(2S_h-S_l)}{S_lS_h}\alpha}-\frac{C_{2h}}{S_h}+\frac{2C_{1l}}{S_l}\right)$,

[1]The demand model is essentially the same as the one in (Swinney et al. 2011) except for the scale parameter S

	ψ^*	$E(\pi^*)$
(S_l^s, S_l^e)	$1-G\left(\frac{\mu}{2}+\frac{3\beta_3-2\beta_2}{2}S_l+2\sqrt{\frac{\alpha}{S_l}}\right)$	$\frac{1}{8}S_l\mu^2+\frac{\beta_3-2\beta_2}{4}\mu S_l^2+\frac{(\beta_3-2\beta_2)^2}{8}S_l^3$
(S_l^s, S_h^e)	$1-G\left(\frac{\mu}{2}+\frac{\beta_3 S_l^2-2\beta_2 S_h^2}{2(2S_h-S_l)}+\beta_3 S_l+2\sqrt{\frac{\alpha}{S_l}}\right)$	$\frac{2S_h-S_l}{8}\mu^2+\frac{(\beta_3 S_l^2-2\beta_2 S_h^2)\mu}{4}+\frac{(\beta_3 S_l^2-2\beta_2 S_h^2)^2}{8(2S_h-S_l)}$
(S_h^s, S_l^e)	$1-G\left(\frac{S_l S_h\mu+\beta_3 S_l S_h^2-2\beta_2 S_l^2 S_h}{2S_h(2S_h-S_l)}+\beta_3 S_h+2\sqrt{\frac{\alpha}{S_h}}\right)$	$\frac{S_h S_l(\mu+\beta_3 S_h-2\beta_2 S_l)^2}{8(2S_h-S_l)}$
(S_h^s, S_h^e)	$1-G\left(\frac{\mu}{2}+\frac{3\beta_3-2\beta_2}{2}S_h+2\sqrt{\frac{\alpha}{S_h}}\right)$	$\frac{1}{8}S_h\mu^2+\frac{\beta_3-2\beta_2}{4}\mu S_h^2+\frac{(\beta_3-2\beta_2)^2}{8}S_h^3$

Table A.4: The Start-Up's Optimal Survival Probability and The Established Firm's Expected Profit When The Start-Up Invests Late and The Established Firm Invests Early

$\pi^*_{es.}=\frac{S_h}{4}\hat{\theta}^2-\left(\sqrt{\frac{S_l S_h\alpha}{2(2S_h-S_l)}}+\frac{C_{2h}}{2}\right)\hat{\theta}+\frac{S_l\alpha}{2(2S_h-S_l)}+\frac{C_{2h}^2}{4S_h}+\sqrt{\frac{S_l\alpha}{2S_h(2S_h-S_l)}}C_{2h}$, thus $E(\pi^*)=\frac{S_h}{4}(\mu^2+\sigma^2)-\left(\sqrt{\frac{S_l S_h\alpha}{2(2S_h-S_l)}}+\frac{C_{2h}}{2}\right)\mu+\frac{S_l\alpha}{2(2S_h-S_l)}+\frac{C_{2h}^2}{4S_h}+\sqrt{\frac{S_l\alpha}{2S_h(2S_h-S_l)}}C_{2h}$.

In case of (L_l^s, L_h^e), i.e., both the start-up invests and the established firm invest late. $\pi_{es.}=(P_h-C_{2h})K_h=(-S_lK_l-S_hK_h+S_h\hat{\theta}-C_{2h})K_h=-S_hK_h^2+(S_h\hat{\theta}-S_lK_l-C_{2h})K_h$, $K_h^*=\frac{S_h\hat{\theta}-S_lK_l-C_{2h}}{2S_h}$. $\pi_{st.}=(P_l-C_{2l})K_l=-S_lK_l^2+(S_l\hat{\theta}-S_lK_h-C_{2l})K_l$, $K_l^*=\frac{S_l\hat{\theta}-S_lK_h-C_{2l}}{2S_l}$. Combine these two first order conditions, and we get $K_h^*=\left(1-\frac{2S_h}{4S_h-S_l}\right)\hat{\theta}+\frac{C_{2l}-2C_{2h}}{4S_h-S_l}$, $K_l^*=\left(\frac{S_h}{4S_h-S_l}\right)\hat{\theta}-\frac{2C_{2l}S_h-C_{2h}S_l}{S_l(4S_h-S_l)}$. $\pi^*_{es.}=S_h\left(1-\frac{2S_h}{4S_h-S_l}\right)^2\hat{\theta}^2+2S_h\left(1-\frac{2S_h}{4S_h-S_l}\right)\left(\frac{C_{2l}-2C_{2h}}{4S_h-S_l}\right)\hat{\theta}+\left(\frac{C_{2l}-2C_{2h}}{4S_h-S_l}\right)^2S_h$, thus $E(\pi^*)=S_h\left(1-\frac{2S_h}{4S_h-S_l}\right)^2(\mu^2+\sigma^2)+2S_h\left(1-\frac{2S_h}{4S_h-S_l}\right)\left(\frac{C_{2l}-2C_{2h}}{4S_h-S_l}\right)\mu+\left(\frac{C_{2l}-2C_{2h}}{4S_h-S_l}\right)^2S_h$. $\psi^*=1-G\left(\frac{2C_{2l}S_h-C_{2h}S_l+(4S_h-S_l)\sqrt{S_l\alpha}}{S_lS_h}\right)$.

In case of (L_l^s, E_h^e), i.e., the start-up invests late while the established firm invests early. $\pi_{st.}=(P_l-C_{2l})K_l=-S_lK_l^2+(S_l\hat{\theta}-S_lK_h-C_{2l})K_l$, $K_l^*=\frac{S_l\hat{\theta}-S_lK_h-C_{2l}}{2S_l}$. $\pi_{es.}=(P_h-C_{1h})K_h=-S_hK_h^2+(S_h\hat{\theta}-S_lK_l-C_{1h})K_h$. Substituting K_l^* into $\pi_{es.}$ generates $\pi_{es.}=\left(\frac{S_l}{2}-S_h\right)K_h^2+\left(S_h\hat{\theta}-\frac{S_l}{2}\hat{\theta}+\frac{C_{2l}}{2}-C_{1h}\right)K_h$, thus $E(\pi_{es.})=\left(\frac{S_l}{2}-S_h\right)K_h^2+\left(S_h\mu-\frac{S_l}{2}\mu+\frac{C_{2l}}{2}-C_{1h}\right)K_h$. Since $\frac{S_l}{2}-S_h<0$, we get $K_h^*=\frac{\mu}{2}+\frac{C_{2l}-2C_{1h}}{2(2S_h-S_l)}$, $K_l^*=\frac{\hat{\theta}}{2}-\frac{\mu}{4}-\frac{C_{2l}-2C_{1h}}{4(2S_h-S_l)}-\frac{C_{2l}}{2S_l}$.$E(\pi^*)=\frac{2S_h-S_l}{8}\mu^2+\frac{(C_{2l}-2C_{1h})\mu}{4}+\frac{(C_{2l}-2C_{1h})^2}{8(2S_h-S_l)}$, and $\psi^*=1-G\left(\frac{\mu}{2}+\frac{C_{2l}-2C_{1h}}{2(2S_h-S_l)}+\frac{C_{2l}}{S_l}+2\sqrt{\frac{\alpha}{S_l}}\right)$.

In case of (E_l^s, E_h^e), i.e., both the start-up and the established firm invest early.

$\pi_{st.}(K_l, K_h)=(P_l-C_{1l})K_l=-S_lK_l^2+(S_l\hat{\theta}-S_lK_h-C_{1l})K_l$, thus $\psi=\Pr\{\pi_s^*\geq\alpha\}=$

$1-G\left(K_l+\frac{\alpha}{S_lK_l}+\frac{S_lK_h+C_{1l}}{S_l}\right)$. Denote $K=K_l+\frac{\alpha}{S_lK_l}+\frac{S_lK_h+C_{1l}}{S_l}$, then $\frac{\partial K}{\partial K_l}=0$ generates $K_l^*=\sqrt{\frac{\alpha}{S_l}}$, and $K^*=2\sqrt{\frac{\alpha}{S_l}}+\frac{S_lK_h+C_{1l}}{S_l}$. $\pi_{es.}=(P_h-C_{1h})K_h=-S_hK_h^2+(S_h\hat{\theta}-S_lK_l-C_{1h})K_h$, thus $E(\pi_{es.})=(P_h-C_{1h})K_h=-S_hK_h^2+(S_h\mu-S_lK_l-C_{1h})K_h$, and $\frac{\partial E(\pi_{es.})}{\partial K_h}=0$ generates $K_h^*=\frac{S_h\mu-S_lK_l-C_{1h}}{2S_h}$. Substituting K_l^* into K_h^* leads to $K_h^*=\frac{S_h\mu-\sqrt{\alpha S_l}-C_{1h}}{2S_h}$, $K^*=2\sqrt{\frac{\alpha}{S_l}}+\frac{S_h\mu-\sqrt{\alpha S_l}-C_{1h}}{2S_h}+\frac{C_{1l}}{S_l}$, $E(\pi^*)=\frac{(S_h\mu-S_lK_l-C_{1h})^2}{4S_h}$, and $\psi^*=1-G\left(2\sqrt{\frac{\alpha}{S_l}}+\frac{S_h\mu-\sqrt{\alpha S_l}-C_{1h}}{2S_h}+\frac{C_{1l}}{S_l}\right)$ $\square$

Proof of Proposition 2.4.1

Proof. (1) We first show that (E_l^s,L_h^e) is an equilibrium under specific conditions. $\psi^*(E_l^s,L_h^e)>\psi^*(L_l^s,L_h^e)$ could be written as $2\sqrt{\frac{2(2S_h-S_l)}{S_lS_h}\alpha}-\frac{C_{2h}}{S_h}+\frac{2C_{1l}}{S_l}<\frac{2C_{2l}S_h-C_{2h}S_l+(4S_h-S_l)\sqrt{S_l\alpha}}{S_lS_h}$, which could be further simplified as $\left(2\sqrt{\frac{2(2S_h-S_l)}{S_h}}-\frac{(4S_h-S_l)}{S_h}\right)\sqrt{\frac{\alpha}{S_l}}+\frac{2(C_{1l}-C_{2l})}{S_l}<0$. We can check the $2\sqrt{\frac{2(2S_h-S_l)}{S_h}}<\frac{(4S_h-S_l)}{S_h}$, thus if $C_{1l}<C_{2l}$, the inequality holds. Next we derive the conditions under which $E[\pi^*(E_l^s,L_h^e)]>E[\pi^*(E_l^s,E_h^e)]$ is satisfied, i.e. $\frac{S_h}{4}(\mu^2+\sigma^2)-\left(\sqrt{\frac{S_lS_h\alpha}{2(2S_h-S_l)}}+\frac{C_{2h}}{2}\right)\mu+\frac{S_l\alpha}{2(2S_h-S_l)}+\frac{C_{2h}^2}{4S_h}+\sqrt{\frac{S_l\alpha}{2S_h(2S_h-S_l)}}C_{2h}>\frac{(S_h\mu-\sqrt{S_l\alpha}-C_{1h})^2}{4S_h}$. The above inequality is equivalent to $\frac{S_h}{4}\sigma^2-\left(\sqrt{\frac{S_lS_h\alpha}{2(2S_h-S_l)}}+\frac{C_{2h}}{2}\right)\mu+\frac{S_l\alpha}{2(2S_h-S_l)}+\frac{C_{2h}^2}{4S_h}+\sqrt{\frac{S_l\alpha}{2S_h(2S_h-S_l)}}C_{2h}>-(\frac{\sqrt{S_l\alpha}+C_{1h}}{2})\mu+\frac{S_l\alpha}{4S_h}+\frac{C_{1h}^2}{4S_h}+\frac{\sqrt{S_l\alpha}}{2S_h}C_{1h}$. Note that left hand side is monotone increasing in σ (when $\sigma>0$), thus there exists a threshold σ_1 such that for all $\sigma>\sigma_1$, the inequality holds. Following the proof above it is clear that neither (L_l^s,L_h^e) nor (E_l^s,E_h^e) is a possible equilibrium. Next we need to show (L_l^s,E_h^e) is not possible equilibrium as well. Note that $E[\pi^*(L_l^s,L_h^e)]>E[\pi^*(L_l^s,E_h^e)]$ is equivalent to $S_h\left(1-\frac{2S_h}{4S_h-S_l}\right)^2(\mu^2+\sigma^2)+2S_h\left(1-\frac{2S_h}{4S_h-S_l}\right)\left(\frac{C_{2l}-2C_{2h}}{4S_h-S_l}\right)\mu+\left(\frac{C_{2l}-2C_{2h}}{4S_h-S_l}\right)^2S_h>\frac{2S_h-S_l}{8}\mu^2+\frac{(C_{2l}-2C_{1h})\mu}{4}+\frac{(C_{2l}-2C_{1h})^2}{8(2S_h-S_l)}$. Similarly left hand side is monotone increasing in σ (when $\sigma>0$), thus there exists another threshold σ_2 such that for all $\sigma>\sigma_2$, the inequality holds. Take $\bar{\sigma}_{se}=\max\{\sigma_1,\sigma_2\}$, as we concluded, (E_l^s,L_h^e) is an unique equilibrium under specific conditions.

(2) For (L_l^s,L_h^e) part, see proof above. For (L_l^s,E_h^e), We will show $\psi^*(L_l^s,E_h^e)<$

$\psi^*(E_l^s, E_h^e)$ under specific conditions. It reduces to show that $\frac{\mu}{2}+\frac{C_{2l}-2C_{1h}}{2(2S_h-S_l)}+\frac{C_{2l}}{S_l}+2\sqrt{\frac{\alpha}{S_l}} > 2\sqrt{\frac{\alpha}{S_l}}+\frac{S_h\mu-\sqrt{S_l\alpha}-C_{1h}}{2S_h}+\frac{C_{1l}}{S_l}$, i.e. $\frac{C_{2l}-2C_{1h}}{2(2S_h-S_l)}+\frac{C_{1h}}{2S_h}+\frac{\sqrt{S_l\alpha}}{2S_h}+\frac{C_{2l}-C_{1l}}{S_l} > 0$. If $C_{1l} < C_{2l}$, write $\frac{C_{2l}-2C_{1h}}{2(2S_h-S_l)}+\frac{C_{1h}}{2S_h}+\frac{\sqrt{S_l\alpha}}{2S_h}$ as $\frac{C_{2l}S_h-C_{1h}S_l+(2S_h-S_l)\sqrt{S_l\alpha}}{2(2S_h-S_l)S_h}$. Since we have the assumption $\alpha > \frac{(S_lC_{1h}-S_hC_{1l})^2}{4S_l(S_h-S_l)^2}$, thus $C_{2l}S_h-C_{1h}S_l+(2S_h-S_l)\sqrt{S_l\alpha} > C_{2l}S_h-C_{1h}S_l+\frac{(2S_h-S_l)(S_lC_{1h}-S_hC_{1l})}{2(S_h-S_l)} > C_{2l}S_h-C_{1h}S_l+S_lC_{1h}-S_hC_{1l} > 0$, thus the inequality holds. □

Proof of Proposition 2.4.2

Proof. (1) We first show that (L_h^s, E_l^e) is an equilibrium under specific conditions. Consider $E[\pi^*(L_h^s, E_l^e)] > E[\pi^*(L_h^s, L_l^e)]$, i.e. $\frac{(S_lS_h\mu+S_lC_{2h}-2S_hC_{1l})^2}{8S_lS_h(2S_h-S_l)} > \frac{S_lS_h^2}{(4S_h-S_l)^2}(\mu^2+\sigma^2) - \frac{2S_h(2C_{2l}S_h-C_{2h}S_l)}{(4S_h-S_l)^2}\mu+\frac{(2C_{2l}S_h-C_{2h}S_l)^2}{S_l(4S_h-S_l)^2}$. Note that the right hand side is monotone increasing in σ (when $\sigma > 0$). As long as the inequality above with $\sigma = 0$ holds, there would exist some threshold σ_1^* such that for all $\sigma < \sigma_1^*$, the original inequality holds. Note that $\frac{(S_lS_h\mu+S_lC_{2h}-2S_hC_{1l})^2}{8S_lS_h(2S_h-S_l)} > \frac{(S_lS_h\mu+S_lC_{2h}-2S_hC_{1l})^2}{S_l(4S_h-S_l)^2}$, thus it is sufficient to check whether $\frac{(S_lS_h\mu+S_lC_{2h}-2S_hC_{1l})^2}{S_l(4S_h-S_l)^2} > \frac{S_lS_h^2}{(4S_h-S_l)^2}\mu^2 - \frac{2S_h(2C_{2l}S_h-C_{2h}S_l)}{(4S_h-S_l)^2}\mu+\frac{(2C_{2l}S_h-C_{2h}S_l)^2}{S_l(4S_h-S_l)^2}$ is satisfied, which could be further simplified as $(C_{1l}-C_{2l})\left(\frac{S_h}{S_l}(C_{1l}+C_{2l})-S_h\mu-C_{2h}\right) > 0$. Note that we have the assumption $2C_{2l}\frac{S_h}{S_l}-S_h\mu-C_{2h} < 0$. If $C_{1l} < C_{2l}$, the inequality would be satisfied. Consider $\psi^*(L_h^s, E_l^e) > \psi^*(E_h^s, E_l^e)$, i.e. $\frac{S_lS_h\mu+S_lC_{2h}-2S_hC_{1l}}{2S_h(2S_h-S_l)}+\frac{C_{2h}}{S_h}+2\sqrt{\frac{\alpha}{S_h}} < 2\sqrt{\frac{\alpha}{S_h}}+\frac{S_l\mu-S_l\sqrt{\alpha/S_h}-C_{1l}}{2S_h}+\frac{C_{1h}}{S_h}$, which could be written as $C_{2h}-C_{1h} < \frac{S_l\mu}{2}-\frac{S_l}{2}\sqrt{\frac{\alpha}{S_h}} - \frac{C_{1l}}{2}-\frac{S_lS_h\mu}{2(2S_h-S_l)}-\frac{S_lC_{2h}}{2(2S_h-S_l)}+\frac{C_{1l}S_h}{2S_h-S_l}$. Note that the right hand side in monotone increasing in S_h, and it converges to $\frac{S_l\mu}{4}$ as $S_h \to +\infty$. As long as $C_{2h}-C_{1h} < \frac{S_l\mu}{4}$, there would exist some threshold $\bar{S}_h$ such that for all $S_h > \bar{S}_h$, the original inequality holds. Next we establish conditions which ensure that (L_h^s, E_l^e) as a unique equilibrium. It is clear that (E_h^s, E_l^e) or (L_h^s, L_l^e) cannot be possible equilibrium under conditions specified above. Check whether $E[\pi^*(E_h^s, L_l^e)] < E[\pi^*(E_h^s, E_l^e)]$, i.e., $\frac{S_l}{4}(\mu^2+\sigma^2)-\left(S_l\sqrt{\frac{\alpha}{2(2S_h-S_l)}}+\frac{C_{2l}}{2}\right)\mu+\frac{S_l\alpha}{2(2S_h-S_l)}+\frac{C_{2l}^2}{4S_l}+\sqrt{\frac{\alpha}{2(2S_h-S_l)}}C_{2l} < \frac{(S_l\mu-S_l\sqrt{\alpha/S_h}-C_{1l})^2}{4S_l}$, could be satisfied under specific conditions. Note that the left hand side in monotone increasing in σ(when $\sigma > 0$). As long as the inequality above with $\sigma = 0$ holds, there would exist some threshold σ_2^*

such that for all $\sigma < \sigma_2^*$, the original inequality holds. Note that we have the assumption $\mu > \sqrt{\frac{2\alpha}{(2S_h - S_l)}} + \frac{C_{2l}}{S_l}$, we only need to check $(C_{2l} - C_{1l})\left(\sqrt{\frac{\alpha}{2(2S_h - S_l)}} - \frac{1}{2}\sqrt{\frac{\alpha}{S_h}}\right) + \frac{C_{2l}^2 + C_{1l}^2 - 2C_{1l}C_{2l}}{4S_l} + S_l\alpha\left(\frac{1}{2(2S_h - S_l)} + \frac{1}{4S_h} - \sqrt{\frac{1}{2S_h(2S_h - S_l)}}\right) \geq 0$, and it is clear to be satisfied if $C_{2l} > C_{1l}$. Take $\bar{\sigma}_{es} = \min\{\sigma_1^*, \sigma_2^*\}$, then we can guarantee (L_h^s, E_l^e) as a unique equilibrium under specific conditions stated.

(2) We first show that if $C_{1h} < C_{2h}$, $\psi^*(L_h^s, L_l^e) < \psi^*(E_h^s, L_l^e)$, thus (L_h^s, L_l^e) is not an equilibrium. $\psi^*(L_h^s, L_l^e) < \psi^*(E_h^s, L_l^e)$, i.e. $\frac{-S_h(C_{2l} - 2C_{2h}) + (4S_h - S_l)\sqrt{S_h\alpha}}{S_h(2S_h - S_l)} > 2\sqrt{\frac{2\alpha}{2S_h - S_l}} + \frac{2C_{1h} - C_{2l}}{2S_h - S_l}$, could be simplified to $0 > \frac{2(C_{1h} - C_{2h})}{2S_h - S_l} + \frac{\sqrt{S_h\alpha}(2\sqrt{S_h}\sqrt{4S_h - 2S_l} - 4S_h + S_l)}{(2S_h - S_l)S_h}$. Since $2\sqrt{S_h}\sqrt{4S_h - 2S_l} < 4S_h - S_l$, if $C_{1h} < C_{2h}$, the right hand side is negative, thus the inequality holds. For the (E_h^s, L_l^e) part, it is clear that $\psi^*(E_h^s, L_l^e) > \psi^*(L_h^s, L_l^e)$ according to proof above. Next under specific conditions we need to show $E[\pi^*(E_h^s, L_l^e)] > E[\pi^*(E_h^s, E_l^e)]$, i.e. $\frac{S_l}{4}(\mu^2 + \sigma^2) - \left(S_l\sqrt{\frac{\alpha}{2(2S_h - S_l)}} + \frac{C_{2l}}{2}\right)\mu + \frac{S_l\alpha}{2(2S_h - S_l)} + \frac{C_{2l}^2}{4S_l} + \sqrt{\frac{\alpha}{2(2S_h - S_l)}}C_{2l} > \frac{(S_l\mu - S_l\sqrt{\alpha/S_h} - C_{1l})^2}{4S_l}$. Note the left hand side is monotone increasing in σ (when $\sigma > 0$), there exists some threshold σ' such that for all $\sigma > \sigma'$, the inequality holds. Take $\tilde{\sigma}_{es} = \max\{\sigma', \sigma_1^*\}$ (σ_1^* from (1)), then (E_h^s, L_l^e) would be guaranteed as a unique equilibrium when $\sigma > \tilde{\sigma}_{es}$. □

Proof of Proposition 2.4.3

Proof. We first show that (L_l^e, L_h^e) is an equilibrium, i.e., $E[\pi_l^*(L_l^e, L_h^e)] > E[\pi_l^*(E_l^e, L_h^e)]$ and $E[\pi_h^*(L_l^e, L_h^e)] > E[\pi_h^*(L_l^e, E_h^e)]$. For the first inequality, the equivalent expression is $S_l\left(\frac{S_h}{4S_h - S_l}\mu - \frac{2C_{2l}S_h - C_{2h}S_l}{S_l(4S_h - S_l)}\right)^2 + \frac{S_l S_h^2}{(4S_h - S_l)^2}\sigma^2 > \frac{(S_l S_h\mu + S_l C_{2h} - 2S_h C_{1l})^2}{8S_l S_h(2S_h - S_l)}$. Since the left hand side is monotone increasing in σ (when $\sigma > 0$), there exists some threshold σ_1^*, such that for all $\sigma > \sigma_1^*$, the inequality holds. Then we consider the second inequality, which could be written as $S_h\left(\frac{2S_h - S_l}{4S_h - S_l}\mu + \frac{C_{2l} - 2C_{2h}}{4S_h - S_l}\right)^2 + S_h\left(\frac{2S_h - S_l}{4S_h - S_l}\right)^2\sigma^2 > \left(\frac{2S_h - S_l}{2}\right)\left(\frac{\mu}{2} + \frac{C_{2l} - 2C_{1h}}{2(2S_h - S_l)}\right)^2$, similarly, there exists some threshold σ_2^* such that for all $\sigma > \sigma_2^*$, the inequality holds. It's clearly neither (E_l^e, L_h^e) nor (L_l^e, E_h^e) is an equilibrium. We next show that (E_l^e, E_h^e) is not a possible equilibrium by proving $E[\pi_1^*(E_l^e, E_h^e)] < E[\pi_1^*(L_l^e, E_h^e)]$, i.e., $S_l\left(\frac{S_h}{4S_h - S_l}\mu - \frac{2C_{1l}S_h - C_{1h}S_l}{S_l(4S_h - S_l)}\right)^2 < S_l\left(\frac{\mu}{4} - \frac{C_{2l} - 2C_{1h}}{4(2S_h - S_l)} - \frac{C_{2l}}{2S_l}\right)^2 + \frac{S_l}{4}\sigma^2$. Similarly, there exists some threshold σ_3^* such that for all

$\sigma > \sigma_3^*$, the inequality holds. Take $\bar{\sigma}_{ee} = \max\{\sigma_1^*, \sigma_2^*, \sigma_3^*\}$, and our proof is complete. $\square$

Proof of Proposition 2.5.1

Proof of 2.5.1.1. When the following two inequalities hold

$$2\sqrt{\frac{\alpha}{S_l}} + \frac{S_h\mu - \sqrt{S_l\alpha} - \beta_1 S_h^2}{2S_h} + \beta_1 S_l < 2\sqrt{\frac{\alpha}{S_h}} + \frac{S_h\mu - \sqrt{S_h\alpha} - \beta_1 S_h^2}{2S_h} + \beta_1 S_h \quad \text{(A.0.1)}$$

and

$$\frac{(S_l\mu - \sqrt{S_l\alpha} - \beta_1 S_l^2)^2}{4S_l} < \frac{(S_h\mu - \sqrt{S_l\alpha} - \beta_1 S_h^2)^2}{4S_h}, \quad \text{(A.0.2)}$$

we could have an equilibrium (S_l^s, S_h^e), in which the start up chooses low quality while the established firm chooses high quality.

For Inequality A.0.1, moving the right hand side to the left and then divided by $\sqrt{S_h} - \sqrt{S_l}$ which is great than 0. It is equivalent to $\frac{2\sqrt{\alpha}}{\sqrt{S_h S_L}} + \frac{\sqrt{\alpha}}{S_h} - \beta_1(\sqrt{S_h} + \sqrt{S_l}) < 0$. Further calculations yields $\sqrt{S_h S_L}(\sqrt{S_h} + \sqrt{S_l}) > \frac{5\sqrt{\alpha}}{4\beta_1}$. And $S_l > (\frac{5\sqrt{\alpha}}{4\beta_1})^{2/3}$ is a sufficient condition for this inequality to hold.

For Inequality A.0.2, it is equivalent to prove $\sqrt{S_l}\mu - \sqrt{\alpha} - \beta_1 S_l^{3/2} < \sqrt{S_h}\mu - \sqrt{\alpha\frac{S_l}{S_h}} - \beta_1 S_h^{3/2}$. And it will be sufficient to show $\sqrt{S_l}\mu - \beta_1 S_l^{3/2} < \sqrt{S_h}\mu - \sqrt{\alpha} - \beta_1 S_h^{3/2}$. Consider function $f(x) = \mu\sqrt{x} - \beta_1 x^{3/2}$, which is increasing in $(0, \frac{\mu}{3\beta_1}]$ and decreasing in $[\frac{\mu}{3\beta_1}, \infty)$. Thus a sufficient condition for Inequality A.0.2 to hold is $S_h < \frac{\mu}{3\beta_1}$. We proved the first statement of Proposition 6.1.

Next, to show (S_l^s, S_h^e) is the unique equilibrium, we need to find sufficient conditions to the following two inequalities, violation of either of which will result in non-existence of (S_h^s, S_l^e) equilibrium, which lead to the uniqueness of (S_l^s, S_h^e).

$$2\sqrt{\frac{\alpha}{S_h}} + \frac{S_l\mu - S_l\sqrt{\alpha/S_h} - \beta_1 S_l^2}{2S_h} + \beta_1 S_h > 2\sqrt{\frac{\alpha}{S_l}} + \frac{S_l\mu - \sqrt{S_l\alpha} - \beta_1 S_l^2}{2S_l} + \beta_1 S_l \quad \text{(A.0.3)}$$

$$\frac{(S_l\mu - S_l\sqrt{\alpha/S_h} - \beta_1 S_l^2)^2}{4S_l} < \frac{(S_h\mu - S_h\sqrt{\alpha/S_h} - \beta_1 S_h^2)^2}{4S_h}. \quad \text{(A.0.4)}$$

For Inequality A.0.3, it is sufficient to show that $2\sqrt{\frac{\alpha}{S_h}} + \frac{S_l\mu - S_l\sqrt{\alpha/S_l} - \beta_1 S_l^2}{2S_h} + \beta_1 S_h > 2\sqrt{\frac{\alpha}{S_l}} + \frac{S_l\mu - \sqrt{S_l\alpha} - \beta_1 S_l^2}{2S_l} + \beta_1 S_l$. It is equivalent to $\frac{S_l\mu - \sqrt{\alpha S_l} - \beta_1 S_l^2}{2S_h S_l} + \frac{2\sqrt{\alpha}}{\sqrt{S_h S_L}(\sqrt{S_h}+\sqrt{S_l})} < \beta_1$. One sufficient condition is to show $\frac{S_l\mu - \sqrt{\alpha S_l} - \beta_1 S_l^2 + 2\sqrt{\alpha S_l}}{2S_l^2} < \beta_1$, which is equivalent to $\mu(S_l)^{-1} + \sqrt{\alpha}(S_l)^{-\frac{3}{2}} < 3\beta_1$. Noticing the function of the left hand side of S_l is decreasing, we have an sufficient condition for Inequality A.0.3 is $S_l > \underline{S}_l$, where $\underline{S}_l$ is the solution to $\mu(\underline{S}_l)^{-1} + \sqrt{\alpha}(\underline{S}_l)^{-\frac{3}{2}} = 3\beta_1$.

For Inequality A.0.4, it is equivalent to $\sqrt{S_h}\mu - \sqrt{\alpha} - \beta_1 S_h^{3/2} > \sqrt{S_l}\mu - \sqrt{\alpha\frac{S_l}{S_h}} - \beta_1 S_l^{3/2}$. Further calculations yields that the proceeding inequality is equivalent to $\beta_1(S_h + S_l + \sqrt{S_h S_l}) + \sqrt{\alpha}/\sqrt{S_h} < \mu$. A sufficient condition will be $3\beta_1 S_h + \sqrt{\alpha}/\sqrt{S_h} < \mu$. We study the function $f(x) = 3\beta_1 x + \sqrt{\alpha}/\sqrt{x}$, which is decreasing in $(0, (\frac{\sqrt{\alpha}}{6\beta_1})^{3/2}]$ and increasing in $[(\frac{\sqrt{\alpha}}{6\beta_1})^{3/2}, \infty)$, with its minimum achieved at $(\frac{\sqrt{\alpha}}{6\beta_1})^{3/2}$. Thus a sufficient condition for Inequality A.0.4 to hold is $S_h \in [\underline{S}_h, \bar{S}_h]$, where $\underline{S}_h, \bar{S}_h$ are the solutions to $S_h + \frac{\sqrt{\alpha}}{\sqrt{S_h}}\frac{1}{3\beta_1} = \frac{\mu}{3\beta_1}$. □

Proof of 2.5.1.2. When the following two inequalities hold

$$2\sqrt{\frac{\alpha}{S_h}} + \frac{S_l\mu - S_l\sqrt{\alpha/S_h} - \beta_1 S_l^2}{2S_h} + \beta_1 S_h < 2\sqrt{\frac{\alpha}{S_l}} + \frac{S_l\mu - \sqrt{S_l\alpha} - \beta_1 S_l^2}{2S_l} + \beta_1 S_l \quad \text{(A.0.5)}$$

and

$$\frac{(S_l\mu - S_l\sqrt{\alpha/S_h} - \beta_1 S_l^2)^2}{4S_l} > \frac{(S_h\mu - S_h\sqrt{\alpha/S_h} - \beta_1 S_h^2)^2}{4S_h}, \quad \text{(A.0.6)}$$

we could have an equilibrium (S_h^s, S_l^e), in which the start up chooses high quality while the established firm chooses low quality.

For Inequality A.0.5, it is sufficient to show that $2\sqrt{\frac{\alpha}{S_h}} + \frac{S_l\mu - S_l\sqrt{\alpha/S_h} - \beta_1 S_l^2}{2S_l} + \beta_1 S_h < 2\sqrt{\frac{\alpha}{S_l}} + \frac{S_l\mu - \sqrt{S_l\alpha} - \beta_1 S_l^2}{2S_l} + \beta_1 S_l$, which is equivalent to $\frac{3}{2}\sqrt{\frac{\alpha}{S_h}} + \beta_1 S_h < \frac{3}{2}\sqrt{\frac{\alpha}{S_l}} + \beta_1 S_l$. Consider function $f(x) = \frac{3}{2}\sqrt{\frac{\alpha}{x}} + \beta_1 x$ which is decreasing in $(0, (\frac{3\sqrt{\alpha}}{4\beta_1})^{2/3}]$. Thus $S_h < (\frac{3\sqrt{\alpha}}{4\beta_1})^{2/3}$ is a sufficient condition for Inequality A.0.5 to hold.

For Inequality A.0.6, it is equivalent to $\sqrt{S_l}\mu - \sqrt{\alpha\frac{S_l}{S_h}} - \beta_1 S_l^{3/2} > \sqrt{S_h}\mu - \sqrt{\alpha} - \beta_1 S_h^{3/2}$.

Thus it is enough to show that $\sqrt{S_l}\mu - \beta_1 S_l^{3/2} > \sqrt{S_h}\mu - \sqrt{\alpha} - \beta_1 S_h^{3/2}$. Consider function $f(x) = \mu\sqrt{x} - \beta_1 x^{3/2}$, which is increasing in $(0, \frac{\mu}{3\beta_1}]$ and decreasing in $[\frac{\mu}{3\beta_1}, \infty)$. Thus a sufficient condition for Inequality A.0.6 to hold is $S_l > \frac{\mu}{3\beta_1}$.

Next, to show $(S_h^s,\ S_l^e)$ is the unique equilibrium, we need to find sufficient conditions to the following two inequalities, violation of either of which will result in non-existence of $(S_l^s,\ S_h^e)$ equilibrium, which leads to the uniqueness of $(S_h^s,\ S_l^e)$.

$$2\sqrt{\frac{\alpha}{S_l}} + \frac{S_h\mu - \sqrt{S_l\alpha} - \beta_1 S_h^2}{2S_h} + \beta_1 S_l > 2\sqrt{\frac{\alpha}{S_h}} + \frac{S_h\mu - \sqrt{S_h\alpha} - \beta_1 S_h^2}{2S_h} + \beta_1 S_h \quad \text{(A.0.7)}$$

$$\frac{(S_l\mu - \sqrt{S_l\alpha} - \beta_1 S_l^2)^2}{4S_l} > \frac{(S_h\mu - \sqrt{S_l\alpha} - \beta_1 S_h^2)^2}{4S_h}. \quad \text{(A.0.8)}$$

For Inequality A.0.7, it is equivalent to $2\frac{\sqrt{\alpha}}{\sqrt{S_h S_l}} + \frac{\sqrt{\alpha}}{2S_h} > \beta_1(\sqrt{S_h} + \sqrt{S_l})$. One sufficient condition will be $2\frac{\sqrt{\alpha}}{S_h} + \frac{\sqrt{\alpha}}{2S_h} > 2\beta_1\sqrt{S_h}$, which is equivalent to $S_h < (\frac{5\sqrt{\alpha}}{4\beta_1})^{2/3}$.

For Inequality A.0.8, it is sufficient to show $\frac{(S_l\mu - \sqrt{S_l\alpha} - \beta_2 S_l^2)^2}{4S_h} > \frac{(S_h\mu - \sqrt{S_l\alpha} - \beta_2 S_h^2)^2}{4S_h}$, which is equivalent to $S_l\mu - \beta_1 S_l^2 > S_h\mu - \beta_1 S_h^2$. One sufficient condition will be $S_l > \frac{\mu}{2\beta_1}$.

The sufficient condition for Inequality A.0.7 is automatically satisfied given the sufficient condition for existence of $(S_h^s,\ S_l^e)$, thus it is also unique under the sufficient condition.

□

Proof of Proposition 2.5.2

Proof of 2.5.2.1. When Inequalities A.0.7 and A.0.4 hold, we could have an equilibrium $(S_h^s,\ S_h^e)$, in which both the start up and established firms choose high quality. Refer to the proof of Proposition 2.5.1, the sufficient condition will be $S_h < (\frac{5\sqrt{\alpha}}{4\beta_1})^{2/3}$ and $S_h \in [\underline{S}_h, \bar{S}_h]$, where $\underline{S}_h, \bar{S}_h$ are the solutions to $S_h + \frac{\sqrt{\alpha}}{\sqrt{S_h}}\frac{1}{3\beta_1} = \frac{\mu}{3\beta_1}$.

Furthermore, if either A.0.5 or A.0.2 holds, then $(S_l^s,\ S_l^e)$ will not be an equilibrium, which leads to the uniqueness of $(S_h^s,\ S_h^e)$. And according to the proof of Proposition 2.5.1, the condition corresponds to either $S_h < (\frac{3\sqrt{\alpha}}{4\beta_1})^{2/3}$ or $S_h < \frac{\mu}{3\beta_1}$. □

Proof of 2.5.2.2. When Inequalities A.0.3 and A.0.8 hold, we could have an equilibrium $(S_l^s,\ S_l^e)$, in which both the start up and established firms choose low quality. Refer to the proof of Proposition 2.5.1, the sufficient condition will be $S_l > \frac{\mu}{2\beta_1}$ and $S_l > \underline{S}_l$, where $\underline{S}_l$ is the solution to $\mu(\underline{S}_l)^{-1} + \sqrt{\alpha}(\underline{S}_l)^{-\frac{3}{2}} = 3\beta_1$.

Furthermore, if either A.0.1 or A.0.6 holds, then $(S_h^s,\ S_h^e)$ will not be an equilibrium, which leads to the uniqueness of $(S_l^s,\ S_l^e)$. And according to the proof of Proposition 2.5.1, the condition corresponds to either $S_l > (\frac{5\sqrt{\alpha}}{4\beta_1})^{2/3}$ or $S_l > \frac{\mu}{3\beta_1}$, and this is automatically satisfied given the sufficient condition for existance of $(S_l^s,\ S_l^e)$, thus it is also unique under the sufficient condition. □

Appendix B PROOFS FOR CHAPTER 3

Proof of Lemma 3.4.1

Proof of Lemma 3.4.1. The first-best optimization problem is given by

$$\Sigma^{FB} = \max_{e,q} rS(q) - \exp\{-\lambda e\}uS(q) - cq - se.$$

Denote $\Sigma(e,q) = rS(q) - \exp\{-\lambda e\}uS(q) - cq - se$. Now consider the partial derivatives with respect to e, and we have

$$\frac{\partial \Sigma(e,q)}{\partial e} = u\lambda \exp\{-\lambda e\}S(q) - s,$$
$$\frac{\partial^2 \Sigma(e,q)}{\partial e^2} = -u\lambda^2 \exp\{-\lambda e\}S(q) < 0,$$

Note $\Sigma(e,q)$ is concave in e for every q, hence, we can first set q and then e to achieve the optimization sequentially. Since for the demand there exists a lower bound $\underline{l}$ satisfying $S(\underline{l}) \geq s/u\lambda$. For any order quantity q chosen, it is obvious that $q \geq \underline{l}$, which yields $S(q) \geq s/u\lambda$, then set

$$\frac{\partial \Sigma(e,q)}{\partial e} = u\lambda \exp\{-\lambda e\}S(q) - s = 0,$$

we have

$$e^*(q) = \frac{1}{\lambda} \ln\left(\frac{S(q)u\lambda}{s}\right). \tag{B.0.1}$$

Then,

$$\Sigma(q) = rS(q) - cq - \frac{s}{\lambda} - \frac{s}{\lambda} \ln\left(\frac{S(q)u\lambda}{s}\right). \tag{B.0.2}$$

The first derivative is as follow:

$$\frac{d\Sigma(q)}{dq} = r\bar{G}(q) - \frac{s}{\lambda}\frac{\bar{G}(q)}{S(q)} - c. \tag{B.0.3}$$

Note the similarity between the function (B.0.3) and (B.0.9), thus if we have IFR assumption, i.e. $g(q)^2 + g^{'}(q)\bar{G}(q) \geq 0$, there is a unique maximizer for q, which could be obtained from

$$r\bar{G}(q^{FB}) - \frac{s}{\lambda}\frac{\bar{G}(q^{FB})}{S(q^{FB})} - c = 0. \tag{B.0.4}$$

Now we could conclude that the first-best optimal solution could be given by

$$e^{FB} = \frac{1}{\lambda}\ln\left(\frac{S(q^{FB})u\lambda}{s}\right),$$
$$r\bar{G}(q^{FB}) - \frac{s}{\lambda}\frac{\bar{G}(q^{FB})}{S(q^{FB})} - c = 0.$$

□

Proof of Corollary 3.4.1

Proof of Corollary 3.4.1. First consider q^{FB}, it is obvious that q^{FB} is independent of u given the relation of (B.0.4). Let $f(r, q^{FB}) = r\bar{G}(q^{FB}) - \frac{s}{\lambda}\frac{\bar{G}(q^{FB})}{S(q^{FB})} - c$, then by the implicit function theorem, we have

$$\frac{\partial q^{FB}}{\partial r} = -\left(\frac{\partial f(r,q)}{\partial q}\right)^{-1}\frac{\partial f(r,q)}{\partial r}\bigg|_{f(r,q^{FB})=0}.$$

From the arguments in the proof of Lemma 3.4.2, we know that $\frac{\partial f(r,q)}{\partial r}\Big|_{f(r,q^{FB})=0} < 0$. Along with the fact that $\frac{\partial f(r,q)}{\partial r}\Big|_{f(r,q^{FB})=0} = \bar{G}(q^{FB}) > 0$, we have $\frac{\partial q^{FB}}{\partial r} > 0$ which means that q^{FB} is increasing in r. Similar arguments will yield q^{FB} is increasing in λ, and decreasing in c and s.

Next consider e^{FB}. Function (B.0.1) is increasing in q, u and decreasing in s. Combining this with the monotonicity of q^{FB}, we have that e^{FB} is increasing in u, r, and decreasing in c and s.

□

Proof of Lemma 3.4.2

Proof of Lemma 3.4.2. We first consider the supplier's problem. Since w and q are determined by the manufacturer, the supplier's problem could be written by

$$\begin{aligned} \max\nolimits_e \quad & \pi(e) \\ \text{s.t.} \quad & \pi(e) \geq 0. \end{aligned}$$

Consider the first derivative

$$\frac{d\pi}{de} = (1-\theta)u\lambda \exp\{-\lambda e\}S(q) - s.$$

Since for the demand there exists a lower bound $\underline{l}$ satisfying $S(\underline{l}) \geq s/(1-\theta)u\lambda$. For any order quantity q chosen by the manufacturer, it is obvious that $q \geq \underline{l}$, which yields $S(q) \geq s/(1-\theta)u\lambda$, then set

$$\frac{d\pi}{de} = (1-\theta)u\lambda \exp\{-\lambda e\}S(q) - s = 0,$$

we have

$$e^*(q) = \frac{1}{\lambda}\ln\left(\frac{S(q)(1-\theta)u\lambda}{s}\right). \tag{B.0.5}$$

Now the manufacturer's problem would be considered as below:

$$\Pi(w,q) = rS(q) - \frac{\theta s}{(1-\theta)\lambda} - wq. \tag{B.0.6}$$

Since the manufacturer's profit is monotone decreasing in w for every q, thus w is chosen by the manufacturer as lower as possible. Also note that we need to guarantee the supplier's profit should be non-negative, i.e.,

$$\pi(e^*) = (w-c)q - \frac{s}{\lambda} - \frac{s}{\lambda}\ln\left(\frac{S(q)(1-\theta)u\lambda}{s}\right) \geq 0, \tag{B.0.7}$$

and the supplier's profit is monotone increasing in w for every q, therefore w is chosen as below:

$$c + \frac{s}{\lambda q} + \frac{s}{\lambda q}\ln\left(\frac{S(q)(1-\theta)u\lambda}{s}\right).$$

In terms of profit, supplier's objective function is binding. Substituting it into equation (B.0.6), we could get

$$\Pi(q) = rS(q) - \frac{\theta s}{(1-\theta)\lambda} - cq - \frac{s}{\lambda} - \frac{s}{\lambda}\ln\left(\frac{S(q)(1-\theta)u\lambda}{s}\right). \tag{B.0.8}$$

The derivatives are as follows:

$$\frac{d\Pi(q)}{dq} = r\bar{G}(q) - \frac{s}{\lambda}\frac{\bar{G}(q)}{S(q)} - c, \tag{B.0.9}$$

$$\frac{d^2\Pi(q)}{dq^2} = -rg(q) + \frac{s}{\lambda}\frac{\bar{G}^2(q) + g(q)S(q)}{S^2(q)}, \tag{B.0.10}$$

$$\frac{d^3\Pi(q)}{dq^3} = -rg^{'}(\hat{q}) + \frac{s}{\lambda}\frac{-2\bar{G}^3(q) - 3g(q)\bar{G}(q)S(q) + g^{'}(q)S^2(q)}{S^3(q)}. \tag{B.0.11}$$

Next we evaluate the third derivative at the points where $\frac{d^2\Pi(q)}{dq^2} = 0$, we have

$$\left.\frac{d^3\Pi(q)}{dq^3}\right|_{\frac{d^2\Pi(q)}{dq^2}=0} = -rg(q)\frac{\bar{G}(q)S(q)(g^2(q) + g^{'}(q)\bar{G}(q)) + 2g^2(q)\bar{G}(q)S(q) + 2g(q)\bar{G}^3(q)}{(g(q)S(q) + \bar{G}^2(q))S(q)}.$$

If we have weak IFR assumption, i.e. $g^2(q) + g^{'}(q)\bar{G}(q) \geq 0$, then $\left.\frac{d^3\Pi(q)}{dq^3}\right|_{\frac{d^2\Pi(q)}{dq^2}=0} < 0$, combined with $\left.\frac{d\Pi(q)}{dq}\right|_{q=\infty} = -c < 0$, which together imply $\frac{d\Pi(q)}{dq}$ is either monotone or unimodal (first increasing and then decreasing). If it is the monotone case, at most there is a unique solution to $d\Pi(q)/dq = 0$, which is a maximizer; and if it is the unimodal case, there are at most two solutions to $d\Pi(q)/dq = 0$, and the the larger one always corresponds to the unique maximizer. We could obtain the unique maximizer q^S from

$$r\bar{G}(q^S) - \frac{s}{\lambda}\frac{\bar{G}(q^S)}{S(q^S)} - c = 0. \tag{B.0.12}$$

Now we could conclude that the optimal solution set for this scenario could be given by

$$r\bar{G}(q^S) - \frac{s}{\lambda}\frac{\bar{G}(q^S)}{S(q^S)} - c = 0, \tag{B.0.13}$$

$$e^S = \frac{1}{\lambda}\ln\left(\frac{S(q^S)(1-\theta)u\lambda}{s}\right), \tag{B.0.14}$$

$$w^S = c + \frac{s}{\lambda q^S} + \frac{s}{\lambda q^S}\ln\left(\frac{S(q^S)(1-\theta)u\lambda}{s}\right). \tag{B.0.15}$$

□

Proof of Lemma 3.4.3

Proof of Lemma 3.4.3. (i)From Lemma 3.4.1 and 3.4.2, it is clearly $q^{FB} = q^S$, since they satisfy the same equation. Given $q^{FB} = q^S$ and $1 - \theta < 1$, we have $e^{FB} > e^S$.

(ii)From the proof of Lemma 3.4.2, supplier's profit objective function is binding, that is $\pi^S = 0$. Thus $\Sigma^S = \Pi^S$. □

Proof of Proposition 3.4.1. By definition, $\delta^S = \Sigma^{FB} - \Sigma^S$, where $\Sigma^{FB} = rS(q^{FB}) - \frac{s}{\lambda} - cq^{FB} + \frac{s}{\lambda}\ln\frac{s}{\lambda u S(q^{FB})}$ and $\Sigma^S = rS(q^S) - \frac{s\theta}{\lambda(1-\theta)} - cq^S - \frac{s}{\lambda} + \frac{s}{\lambda}\ln\frac{s}{\lambda u(1-\theta)S(q^S)}$. From Lemma

3.4.3, we have $q^{FB} = q^S$. Thus we have

$$\begin{aligned}\delta^S =& \frac{s\theta}{\lambda(1-\theta)} + \frac{s}{\lambda}(\ln\frac{s}{\lambda u S(q^{FB})} - \ln\frac{s}{\lambda u(1-\theta)S(q^S)})\\ =& \frac{s\theta}{\lambda(1-\theta)} + \frac{s}{\lambda}\ln(1-\theta)\\ =& \frac{s}{\lambda}\left(\ln(1-\theta) + \frac{\theta}{1-\theta}\right).\end{aligned}$$

□

Proof of Lemma 3.4.4

Proof of Lemma 3.4.4. We first consider the supplier's problem. Since w is determined by the manufacturer, the supplier faces a two dimension optimization problem, which is given by

$$\begin{aligned}\max_{e,q} \quad & \pi(e,q)\\ \text{subject to} \quad & \pi(e,q) \geq 0.\end{aligned}$$

Clearly the objective function is concave in e for every q (may not jointly concave in both e and q), hence, we can first set q and then e to achieve the optimization sequentially. Consider the function

$$\frac{\partial\pi}{\partial e} = \exp\{-\lambda e\}(1-\theta)u\lambda S(q) - s.$$

Since $\theta < \bar{\theta} = 1 - s/(u\lambda\underline{l})$, the demand has a lower bound $\underline{l}$ satisfying $\underline{l} \geq s/(1-\theta)u\lambda$. As long as the supplier chooses a order quantity q, it is obvious that $q \geq \underline{l}$, which yields $S(q) \geq s/(1-\theta)u\lambda$, then set

$$\frac{\partial\pi}{\partial e} = \exp\{-\lambda e\}(1-\theta)u\lambda S(q) - s = 0,$$

we have

$$e^*(q) = \frac{1}{\lambda}\ln\left(\frac{S(q)(1-\theta)u\lambda}{s}\right). \tag{B.0.16}$$

Taking (B.0.16) back into equation (3.4.2), we get

$$\pi(q) = (w - \frac{s}{\lambda S(q)})S(q) - \frac{s}{\lambda}\ln\left(\frac{S(q)(1-\theta)u\lambda}{s}\right) - cq. \tag{B.0.17}$$

The derivatives are as follows:

$$\frac{d\pi(q)}{dq} = w\bar{G}(q) - \frac{s}{\lambda}\frac{\bar{G}(q)}{S(q)} - c, \tag{B.0.18}$$

$$\frac{d^2\pi(q)}{dq^2} = -wg(q) + \frac{s}{\lambda}\frac{\bar{G}^2(q) + g(q)S(q)}{S^2(q)}, \tag{B.0.19}$$

$$\frac{d^3\pi(q)}{dq^3} = -wg^{'}(\hat{q}) + \frac{s}{\lambda}\frac{-2\bar{G}^3(q) - 3g(q)\bar{G}(q)S(q) + g^{'}(q)S^2(q)}{S^3(q)}. \tag{B.0.20}$$

Next we evaluate the third derivative at the points where $\frac{d^2\pi(q)}{dq^2} = 0$, we have

$$\left.\frac{d^3\pi(q)}{dq^3}\right|_{\frac{d^2\pi(q)}{dq^2}=0} = -wg(q)\frac{\bar{G}(q)S(q)(g^2(q) + g^{'}(q)\bar{G}(q)) + 2g^2(q)\bar{G}(q)S(q) + 2g(q)\bar{G}^3(q)}{(g(q)S(q) + \bar{G}^2(q))S(q)}.$$

If we have weak IFR assumption, i.e. $g^2(q) + g^{'}(q)\bar{G}(q) \geq 0$, then $\left.\frac{d^3\pi(q)}{dq^3}\right|_{\frac{d^2\pi(q)}{dq^2}=0} < 0$, combined with $\left.\frac{d\pi(q)}{dq}\right|_{q=\infty} = -c < 0$, which together imply $\frac{d\pi(q)}{dq}$ is either monotone or unimodal (first increasing and then decreasing). If it is the monotone case, at most there is a unique solution to $d\pi(q)/dq = 0$, which is a maximizer; and if it is the unimodal case, there are at most two solutions to $d\pi(q)/dq = 0$, and the the larger one always corresponds to the unique maximizer. We could obtain the unique maximizer q^* from

$$w\bar{G}(q^*) - \frac{s}{\lambda}\frac{\bar{G}(q^*)}{S(q^*)} - c = 0 \tag{B.0.21}$$

Next we need to consider the manufacturer's problem. Note that w and $(e^*(q^*), q^*)$ are one-to-one corresponded. As we did in the previous proof, we could substitute w by q^* in the manufacturer's objective function. Therefore, from equation (B.0.21) we have

$$w(q^*) = \frac{c}{\bar{G}(q^*)} + \frac{s}{\lambda S(q^*)}.$$

Note that $w(q^*)|_{q^*=0} = \infty$, $w(q^*)|_{q^*=\infty} = \infty$ and $\frac{d^2w(q^*)}{dq^{*2}} > 0$ under IFR assumption, which implies the function $w(q^*)$ is convex in q^*, and there exists a unique minimizer q^*_{min}. Furthermore, q^*_{min} could be obtained by setting $\frac{dw(q^*)}{dq^*} = 0$, i.e.,

$$\frac{cg(q^*_{min})}{\bar{G}^2(q^*_{min})} - \frac{s\bar{G}(q^*_{min})}{\lambda S^2(q^*_{min})} = 0. \tag{B.0.22}$$

Since we have $c \leq w \leq r$, note that $w(q^*_{min}) > c$, thus

(a) if $c \le w < w(q^*_{min})$, $q^* = 0$;

(b) if $w(q^*_{min}) \le w \le r$, the corresponding range for q^* would be $[q^*_{min}, q^*_{max}]$,and q^*_{max} could be obtained by the larger solution to

$$r = \frac{c}{\bar{G}(q^*_{max})} + \frac{s}{\lambda S(q^*_{max})}.$$

In addition, we can get

$$\left.\frac{dw(q^*)}{d(q^*)}\right|_{q^* \in [q^*_{min}, q^*_{max}]} = \frac{cg(q^*)}{\bar{G}^2(q^*)} - \frac{s\bar{G}(q^*)}{\lambda S^2(q^*)} > 0. \tag{B.0.23}$$

Next substituting $w(q^*)$ by the expression above into equation (3.4.1), we have

$$\Pi(w) = \Pi(q^*) = rS(q^*) - c\frac{S(q^*)}{\bar{G}(q^*)} - \frac{s}{(1-\theta)\lambda}. \tag{B.0.24}$$

Next take the first and second derivatives of $\Pi(q^*)$ with respect to q^*:

$$\frac{d\Pi(q^*)}{dq^*} = r\bar{G}(q^*) - \frac{cg(q^*)S(q^*)}{\bar{G}^2(q^*)} - c, \tag{B.0.25}$$

$$\frac{d^2\Pi(q^*)}{dq^{*2}} = -rg(q^*) - \frac{c(g(q^*)\bar{G}(q^*) + g'(q^*)S(q^*))\bar{G}^2(q^*) + 2g^2(q^*)\bar{G}(q^*)S(q^*)}{\bar{G}^4(q^*)} \tag{B.0.26}$$

By similar induction as in the previous proof, we could find $\Pi(q^*)$ is unimodal (first increasing and then decreasing), and there exists a unique interior global maximizer q^L given by

$$r\bar{G}(q^L) - \frac{cg(q^L)S(q^L)}{\bar{G}^2(q^L)} - c = 0. \tag{B.0.27}$$

By evaluating $\left.\frac{d\Pi(q^*)}{dq^*}\right|_{q^*=q^*_{min}} > 0$ and $\left.\frac{d\Pi(q^*)}{dq^*}\right|_{q^*=q^*_{max}} < 0$, we could find

$$q^*_{min} < q^L < q^*_{max}.$$

$\square$

Proof of Corollary 3.4.2

Proof of Corollary. 3.4.2 Consider q^L, which satisfies equation (B.0.27). It is obvious that q^L is independent of u, s and λ. Let $f(r,q) = r\bar{G}(q) - \frac{cg(q)S(q)}{\bar{G}^2(q)} - c$, then by the implicit function theorem, we have

$$\frac{\partial q^L}{\partial r} = -\left(\frac{\partial f(r,q)}{\partial q}\right)^{-1} \frac{\partial f(r,q)}{\partial r}\bigg|_{f(r,q^L)=0}.$$

From the arguments in the proof of Lemma 3.4.4, we know that $\frac{\partial f(r,q)}{\partial r}\Big|_{f(r,q^L)=0} < 0$. Along with the fact that $\frac{\partial f(r,q)}{\partial r}\Big|_{f(r,q^L)=0} = \bar{G}(q^L) > 0$, we have $\frac{\partial q^L}{\partial r} > 0$ which means that q^L is increasing in r. Similar arguments will yield q^L is decreasing in c.

Next consider e^L. Function (B.0.1) is increasing in q, u and decreasing in s. Combining this with the monotonicity of q^L, we have that e^L is increasing in u, r, and decreasing in c and s. □

Proof of Lemma 3.4.5

Proof of Lemma 3.4.5. (i) q^L will satisfy equation (B.0.21), with a w less than r. From the proof of Lemma 3.4.4, by the monotonicity of w and q, we have $q^{FB} > q^L$. Given $q^{FB} > q^L$ and $1 - \theta < 1$, we have $e^{FB} > e^L$.

(ii)From the proof of Lemma 3.4.4, we have $\pi^L > 0$, i.e. $\Sigma^L > \Pi^L$. □

Proof of Proposition 3.4.2

Proof of Proposition 3.4.2. By definition, $\delta^L = \Sigma^{FB} - \Sigma^L$, where $\Sigma^{FB} = rS(q^{FB}) - \frac{s}{\lambda} - cq^{FB} + \frac{s}{\lambda}\ln\frac{s}{\lambda u S(q^{FB})}$ and $\Sigma^L = rS(q^L) - \frac{s}{\lambda(1-\theta)} - cq^L + \frac{s}{\lambda}\ln\frac{s}{\lambda u(1-\theta)S(q^L)}$. Define function $f(q) = rS(q) - \frac{s}{\lambda(1-\theta)} - cq + \frac{s}{\lambda}\ln\frac{s}{\lambda u(1-\theta)S(q)}$, then $\Sigma^L = f(q^L)$. Taking derivative of f with respect to q yields $f'(q) = r\bar{G}(q) - c - \frac{s\bar{G}(q)}{\lambda S(q)}$. From the proof of Lemma 3.4.4, for $q \in [q^L, q^{FB}]$, $f'(q) > 0$. Thus we have $\Sigma^L = f(q^L) < f(q^{FB})$. Notice that

$\Sigma^{FB} - f(q^{FB}) = \delta^S$, thus we have $\delta^L > \delta^S$. By definition of δ, we have $\Sigma^S > \Sigma^L$. Since $\pi^S = 0$, we further have $\pi^S < \pi^L$ and $\Pi^S > \Pi^L$. $\square$

Proof of Proposition 3.4.3

Proof of Proposition 3.4.3. From Lemma 3.4.3 and 3.4.5, $q^{FB} = q^S > q^L$. Thus $e^S > e^L$ $\square$

Proof of Lemma 3.4.6

Proof of Lemma 3.4.6. (i) $\tilde{w}^S$ and $\tilde{q}^S$ are obtained by solving the following problem.

$$\begin{aligned} \max_{w,q} \quad & rS(q) - wq \\ s.t. \quad & (w-c)q \geq 0 \end{aligned}$$

The solution is $\tilde{w}^S = c$, making the supplier's profit equal to zero. Then maximize $rS(q) - cq$ yields $\tilde{q}^S = \bar{G}^{-1}(\frac{c}{r})$.

(ii)In the pull system, $\tilde{w}^L$ and $\tilde{q}^L$ are obtained by solving the following problem.

$$\begin{aligned} \max_{c \leq w \leq r} \quad & (r-w)S(q*) \\ s.t. \quad & q* = \operatorname*{argmax}_q wS(q) - cq \end{aligned}$$

We can easily get the solution by a similar arguments along the proof of Lemma 4. $\square$

Proof of Proposition 3.4.4

Proof of Proposition 3.4.4. It is clearly that $\tilde{q}^{FB} = \tilde{q}^S$. Since $\bar{G}$ is a decreasing function, and $c + \frac{s\bar{G}(q^{FB})}{\lambda S(q^{FB})} > c$, we have $\tilde{q}^{FB} = \tilde{q}^S > q^{FB} = q^S$. From the proof of Lemma 4, we know that $\tilde{q}^L = q^L$. $\square$

Proof of Proposition 3.4.5

Proof of Proposition 3.4.5. From the function (B.0.12) and function (B.0.27), it is clear that both q^S and q^S are indepedent of θ. Futhermore, from the function (B.0.5) and the function (B.0.16), both q^S and q^S are decreasing in θ. For the push sytem, we have shown δ^S is increasing in θ, which implies that Σ^S is decreasing in θ. Besides, $\Pi^S = \Sigma^S$, so does Π^S. For the pull system, from the proof of Lemma 3.4.4, we could get

$$\pi(q^L) = c\left(\frac{S(q^L)}{\bar{G}(q^L)} - q^L\right) - \frac{s}{\lambda}\ln\left(\frac{S(q^L)(1-\theta)u\lambda}{s}\right).$$

Since q^L is indepedent of θ, π^L is increasing in θ. Besides, Π^L is decreasing in θ by By the function (B.0.24). Sum up Π^L and π^L, we could get

$$\Sigma^L = rS(q^L) - cq^L - \frac{s}{\lambda}\left(\frac{1}{1-\theta} + \ln\left(\frac{S(q^L)(1-\theta)u\lambda}{s}\right)\right).$$

Take the first deriative of θ, we find that $\frac{d\Sigma^L}{d\theta} < 0$, which implies Σ^L is decreasing in θ. $\square$

Proof of Corollary 3.4.3

Proof of Corollary 3.4.3. Following the proofs of Lemma 3.4.2 and Lemma 3.4.4, if $\theta \geq \underline{\theta}_2 = 1 - \frac{s}{u\lambda d}$, since $S(q) \leq d$, then we have $\frac{d\pi}{de} < 0$ for all e, which means $e^*(q) = 0$.

(i) For the push sytem, next we need to consider the manufacturer's problem. Consider

$$\Pi(w,q) = (r-\theta u)S(q) - wq. \tag{B.0.28}$$

Since the manufacturer's profit function is monotone decreasing in w for every q (may not jointly concave in both w and q), hence, we can first set q and then w to achieve the optimization sequentially and w is chosen by the manufacturer as lower as possible. . In addition, we need to guarantee the supplier's profit should be non-negative, i.e.,

$$\pi(e^* = 0) = (w-c)q - (1-\theta u)S(q) \geq 0, \tag{B.0.29}$$

and the supplier's profit is monotone increasing in w for every q, therefore w is chosen as below:

$$c + \frac{(1-\theta u)S(q)}{q}.$$

In terms of profit, the supplier's objective function is binding. Substituting it into equation (B.0.28), we could get

$$\Pi(q) = (r - u)S(q) - cq. \tag{B.0.30}$$

The derivatives are as follows:

$$\frac{d\Pi(q)}{dq} = (r - u)\bar{G}(q) - c, \tag{B.0.31}$$

$$\frac{d^2\Pi(q)}{dq^2} = -(r - u)g(q). \tag{B.0.32}$$

Since $\frac{d^2\Pi(q)}{dq^2} < 0$, there is a unique optimal solution:

$$q^S = \bar{G}^{-1}\left(\frac{c}{r - u}\right) \tag{B.0.33}$$

(ii) For the pull sytem, the supplier's profit function could be simplified as below:

$$\pi(q) = (w - (1 - \theta)u)S(q) - cq. \tag{B.0.34}$$

The derivatives are as follows:

$$\frac{d\pi(q)}{dq} = (w - (1 - \theta)u)\bar{G}(q) - c, \tag{B.0.35}$$

$$\frac{d^2\pi(q)}{dq^2} = -(w - (1 - \theta)u)g(q), \tag{B.0.36}$$

Since we have the assumption $c \leq w \leq r$,

(a) if $c \leq w < (1 - \theta)u + c$, considering $S(q) \leq q$, then $q^* = 0$;

(b) if $w(1 - \theta)u + c \leq w \leq r$, considering $\frac{d^2\pi(q)}{dq^2} \leq 0$, there is a unique optimal solution:

$$q^* = \bar{G}^{-1}\left(\frac{c}{w - (1 - \theta)u}\right). \tag{B.0.37}$$

Note that the corresponding range for q^* is $[0, \bar{G}^{-1}(\frac{c}{r-(1-\theta)u})]$. Next we need to consider the principal's optimization problem. For part (a), $\Pi(w) = 0$. For part (b), note that w and $(e^*(q^*), q^*)$ are one-to-one corresponded. In order to find the explicit value for w, it is equivalent to substitute w by $(e^*(q^*), q^*)$ in the manufacturer's profit function. Therefore, from equation (B.0.37) we have

$$w = \frac{c}{\bar{G}(q^*)} + (1-\theta)u.$$

Substituting w by the expression above into function (3.4.1), we have

$$\Pi(w) = \Pi(q^*) = (r-u)S(q^*) - c\frac{S(q^*)}{\bar{G}(q^*)}.$$

Then take the first and second derivatives of $\Pi(q^*)$ with respect to q^*:

$$\frac{d\Pi(q^*)}{dq^*} = (r-u)\bar{G}(q^*) - \frac{cg(q^*)S(q^*)}{\bar{G}^2(q^*)} - c, \tag{B.0.38}$$

$$\frac{d^2\Pi(q^*)}{dq^{*2}} = (u-r)g(q^*) - \frac{c(g(q^*)\bar{G}(q^*) + g'(q^*)S(q^*))\bar{G}^2(q^*) + 2g^2(q^*)\bar{G}(q^*)S(q^*)}{\bar{G}^4(q^*)}. \tag{B.0.39}$$

Next we evaluate the second derivative at the point where $\frac{d\Pi(q^*)}{dq^*} = 0$, we have

$$\left.\frac{d^2\Pi(q^*)}{dq^{*2}}\right|_{\frac{d\Pi(q^*)}{dq^*}=0} = -c\left(\frac{2g(q^*)\bar{G}^2(q^*) + 2g^2(q^*)S(q^*) + (g^2(q^*) + g'(q^*)\bar{G}(q^*))S(q^*)}{\bar{G}^3(q^*)}\right).$$

If we have weak IFR assumption, i.e. $g^2(q) + g^{'}(q)\bar{G}(q) \geq 0$, then $\left.\frac{d^2\Pi(q^*)}{dq^{*2}}\right|_{\frac{d\Pi(q^*)}{dq^*}=0} < 0$, combined with $\Pi(q^*)|_{q^*=0} = 0$, $\Pi(q^*)|_{q^*=\infty} = -\infty$ and $\left.\frac{d\Pi(q^*)}{dq^*}\right|_{q^*=0} = r-c-u > 0$, which together imply $\Pi(q^*)$ is unimodal (first increasing and then decreasing), and in the support of $[0,\infty)$ there exists a unique interior global maximizer q^L given by

$$(r-u)\bar{G}(q^L) - \frac{cg(q^L)S(q^L)}{\bar{G}^2(q^L)} - c = 0.$$

Now we need to compare q^L and the upper bound of the feasible region, i.e. $\bar{G}^{-1}(\frac{c}{r-(1-\theta)u})$. Note that $\left.\frac{d\Pi(q^*)}{dq^*}\right|_{q^*=\bar{G}^{-1}(\frac{c}{r-(1-\theta)u})} = -\theta u\bar{G}(q^*) - \frac{cg(q^*)S(q^*)}{\bar{G}^2(q^*)} < 0$, which implies $q^L < \bar{G}^{-1}(\frac{c}{r-(1-\theta)u})$. Furthermore, note that $\pi(q^L) > \pi(q^*)|_{q^*=0} = 0$, which has ruled out the consideration of part (a), thus now we could conclude that the optimal solution set for this scenario could be given by

$$(r-u)\bar{G}(q^L) - \frac{cg(q^L)S(q^L)}{\bar{G}^2(q^L)} - c = 0, \tag{B.0.40}$$

$$e^L = 0, \tag{B.0.41}$$

$$w^L = \frac{c}{\bar{G}(q^L)} + (1-\theta)u. \tag{B.0.42}$$

□

Proof of Lemma 3.4.7

Proof of Lemma 3.4.7. Using the IC Condition, taking derivative with respect to e and set it equal to zero, we have the result. □

Proof of Proposition 3.4.7

Proof of Proposition 3.4.7. Comparing Lemma 3.4.7 with Lemma 3.4.1, we have the proposition. □

Proof of Lemma 3.4.8

Proof of Lemma 3.4.8. Using the IC Condition, taking derivative with respect to e and set it equal to zero, we have the result. □

Proof of Proposition 3.4.8

Proof of Proposition 3.4.8. Comparing Lemma 3.4.8 with Lemma 3.4.1, we have the proposition. □

Proof of Lemma 3.4.9

Proof of Lemma 3.4.9. Taking derivative of Σ^{FB} with respect to e and set it equal to zero, we have the result. □

Proof of Lemma 3.4.10

Proof of Lemma 3.4.10. Using the IC Condition, taking derivative with respect to e and set it equal to zero, we have the result. $\square$

Proof of Proposition 3.4.9

Proof of Proposition 3.4.9. Comparing Lemma 3.4.10 with Lemma 3.4.9, we have the proposition. $\square$

Proof of Lemma 3.4.11

Proof of Lemma 3.4.11. Taking derivative of Σ^{FB} with respect to e_1 and set it equal to zero. We also set $e_1^{FB} = e_2^{FB} = e^{FB}$, and the result follows. $\square$

Proof of Lemma 3.4.12

Proof of Lemma 3.4.12. Taking derivative of $(IC1)$ with respect to e_1 and set it equal to zero. We also set $e_1^{DT} = e_2^{DT} = e^{DT}$, and the result follows. $\square$

Proof of Proposition 3.4.10

Proof of Proposition 3.4.10. Comparing Lemma 3.4.12 with Lemma 3.4.11, we have the proposition. $\square$

Appendix C PROOFS FOR CHAPTER 4

Proof of Lemma 4.4.1

Proof of Lemma 4.4.1. See the decentralized supply chain analysis part. □

Proof of Lemma 4.4.2

Proof of Lemma 4.4.2. (i) $s + d_1 C_v'(\tilde{e}) < 0$.

Denote $h_1(e) = se + (1-\theta)d_1 C_v(e)$ and $h_2(e) = se + (1-\theta)(d_1+d_2)C_m(e)\exp\{-\lambda e\}$. When $e \in (0, \tilde{e}]$,

$$\begin{aligned} {h_1}'(e) &= s + (1-\theta)d_1 {C_v}'(e), && \text{(C.0.1)} \\ {h_1}''(e) &= (1-\theta)d_1 {C_v}''(e). && \text{(C.0.2)} \end{aligned}$$

It shows that $h_1(e)$ is a strictly convex function. Now define e_1^* which satisfies

$$s + (1-\theta)d_1 {C_v}'(e_1^*) = 0.$$

Note that $h_1(e) \to +\infty$, when $e \to 0$. Therefore, when $e \in [0, \tilde{e}]$, the optimal solution e^* for supplier is given by $\min\{e_1^*, \tilde{e}\}$.

Similarly, when $e \in [\tilde{e}, +\infty)$,

$$\begin{aligned} {h_2}'(e) &= s + (1-\theta)(d_1+d_2)\exp\{-\lambda e\}(C_m'(e) - \lambda C_m(e)), && \text{(C.0.3)} \\ {h_2}''(e) &= (1-\theta)(d_1+d_2)\exp\{-\lambda e\}[({C_m}'(e) - \lambda {C_m}''(e)) + (-\lambda)({C_m}'(e) - \lambda C_m(e))]. && \text{(C.0.4)} \end{aligned}$$

It shows that $h_2(e)$ is a strictly convex function. Now define e_2^* which satisfies

$$s + (1-\theta)(d_1+d_2)\exp\{-\lambda e_2^*\}(C_m'(e_2^*) - \lambda C_m(e_2^*)) = 0.$$

Note that $h_2(e) \to +\infty$, when $e \to +\infty$. Therefore, when $e \in [\tilde{e}, +\infty]$, the optimal solution e^* for supplier is given by $\max\{e_2^*, \tilde{e}\}$.

First we note that

$$h_1{}'(\tilde{e}) - h_2{}'(\tilde{e}) = (1-\theta)d_1\lambda C_v(\tilde{e}) \geq 0.$$

When $\theta = 1$, we have

$$h_1{}'(\tilde{e}) = h_2{}'(\tilde{e}) = s > 0.$$

Hence the unique optimal solution is obtained at e_1^*.

When $\theta = 0$, under the assumption that $s + d_1 C_v{}'(\tilde{e}) < 0$, we have

$$h_2'(\tilde{e}) < h_1'(\tilde{e}) < 0.$$

Hence the unique optimal solution is obtained at e_2^*.

Because of continuality, there exist two numbers which are denote by θ_1 and θ_2, such that

$$\text{when } \theta \in [\theta_2, 1], \text{the unique optimal solution is reached at } e_1^*,$$
$$\text{when } \theta \in [0, \theta_1], \text{the unique optimal solution is reached at } e_2^*.$$

Next we need to compare the $h_1(e_1^*)$ and $h_2(e_2^*)$ in the interval $[\theta_1, \theta_2]$. Define

$$\begin{aligned} \bar{h}(\theta) &= h_1(e_1^*(\theta)) - h_2(e_2^*(\theta)), \\ &= s(e_1^* - e_2^*) + (1-\theta)[d_1 C_v(e_1^*) - (d_1+d_2)C_m(e_2^*)\exp\{-\lambda e_2^*\}]. \end{aligned} \quad \text{(C.0.5)}$$

Therefore,

$$\begin{aligned} \frac{d\bar{h}}{d\theta} &= s\left(\frac{de_1^*}{d\theta} - \frac{de_2^*}{d\theta}\right) - (d_1 C_v(e_1^*) - (d_1+d_2)C_m(e_2^*)\exp\{-\lambda e_2^*\}) \\ &\quad +(1-\theta)\left\{d_1 C_v{}'(e_1^*)\frac{de_1^*}{d\theta} - (d_1+d_2)\left[C_m{}'(e_2^*) + C_m(e_2^*)(-\lambda)\right]\exp\{-\lambda e_2^*\}\frac{de_2^*}{d\theta}\right\}. \end{aligned}$$

By the definition of e_1^* and e_2^*, the above equation can be simplified as following:

$$\begin{aligned}
\frac{d\bar{h}}{d\theta} &= \frac{d\bar{h}}{d\theta} = s\left(\frac{de_1^*}{d\theta} - \frac{de_2^*}{d\theta}\right) - \left[d_1 C_v(e_1^*) - \frac{sC_m(e_2^*)}{(1-\theta)(\lambda C_m(e_2^*) - C_m{}'(e_2^*))}\right] \\
&\quad +(1-\theta)\left(\frac{s}{\theta-1}\frac{de_1^*}{d\theta} - \frac{s(C_m{}'(e_2^*) - \lambda C_m(e_2^*))}{(1-\theta)(\lambda C_m(e_2^*) - C_m{}'(e_2^*))}\frac{de_2^*}{d\theta}\right), \\
&= -\left[d_1 C_v(e_1^*) - \frac{sC_m(e_2^*)}{(1-\theta)(\lambda C_m(e_2^*) - C_m{}'(e_2^*))}\right], \\
&= -\left[d_1 C_v(e_1^*) - (d_1+d_2)C_m(e_2^*)\exp\{-\lambda e_2^*\}\right]. \qquad \text{(C.0.6)}
\end{aligned}$$

Consider the equation C.0.5, if $d_1 C_v(e_1^*) - (d_1+d_2)C_m(e_2^*)\exp\{-\lambda e_2^*\} < 0$, we have $\bar{h} < 0$. Hence $\bar{\theta} = \theta_1$.

On the other hand, if $d_1 C_v(e_1^*) - (d_1+d_2)C_m(e_2^*)\exp\{-\lambda e_2^*\} > 0$, we have $\frac{d\bar{h}}{d\theta} < 0$. Let $\bar{h}(\theta) = 0$. Hence we obtain $\bar{\theta}$ by solving this equation.

The result follows.

(ii)$s + (d_1+d_2)\exp\{-\lambda\tilde{e}\}(C_m'(\tilde{e}) - \lambda C_m(\tilde{e})) > 0$.

In this setting, $h_1'(\tilde{e}) > h_2'(\tilde{e}) > 0$. Thus for all $\theta \in [0,1]$, e_1^* is the unique minimizer.

(iii) $s + d_1 C_v'(\tilde{e}) > 0$ and $s + (d_1+d_2)\exp\{-\lambda e\}(C_m'(e) - \lambda C_m(e)) < 0$.

In this setting, $h_1'(\tilde{e}) > 0$, $h_2'(\tilde{e})$ could be either positive or negative. Denote $\tilde{\theta}$ which satisfies $s + (1-\tilde{\theta})(d_1+d_2)\exp\{-\lambda e\}(C_m'(e) - \lambda C_m(e)) = 0$. And we have for $\theta \in [0,\tilde{\theta})$, $h_2(\tilde{e}) < 0$, for $\theta \in [\tilde{\theta},1]$, $h_2(\tilde{e}) > 0$.

In the case $\tilde{\theta} < \bar{t}heta$, we have if $\theta \in [\tilde{\theta},1]$, e_1^* is the minimizer and if $\theta \in [0,\tilde{\theta}]$, e_2^* is the minimizer.

In the case $\tilde{\theta} > \bar{t}heta$, we have if $\theta \in [\tilde{\theta},1]$, e_1^* is the minimizer. If $\theta \in [0,\bar{\theta}]$, e_2^* is the minimizer. And if $\theta \in [\bar{\theta},\tilde{\theta}]$, e_1^* is the minimizer.

Summarizing the above results yield part (iii) of Lemma 4.4.2. $\square$

Proof of Lemma 4.4.3

Proof of Lemma 4.4.3. When $e < \tilde{e}$,

$$\theta = \frac{s}{d_1 C'_v(e_1^*)} + 1.$$

Thus θ is decreasing in e_1^*, which also means e_1^* is decreasing in θ.

When $e > \tilde{e}$,

$$\theta = 1 - \frac{s}{(d_1 + d_2)\exp\{-\lambda e_2^*\}(\lambda C_m(e_2^*) - C'_m(e_2^*))}.$$

Thus θ is decreasing in e_2^*, which also means e_2^* is decreasing in θ.

Combining the above two arguments and the fact that $e_2^* > e_1^*$, we have $e^*(\theta)$ is decreasing in θ. □

Proof of Lemma 4.4.4

Proof of Lemma 4.4.4. For $\theta H(e_1^*(\theta))$, we have

$$\begin{aligned}
\theta H(e_1^*(\theta)) =& \theta d_1 C_v(e_1^*) \\
=& \left(1 + \frac{s}{d_1 C'_v(e_1^*)}\right) d_1 C_v(e_1^*) \\
=& d_1 C_v(e_1^*) + \frac{s C_v(e_1^*)}{C'_v(e_1^*)} \\
\equiv& W_1
\end{aligned}$$

Taking derivatives of W_1 with respect to e_1^*, we have

$$\frac{dW_1}{de_1^*} = d_1 C'_v(e_1^*) + \frac{s[C'^2_v(e_1^*) - C_v(e_1^*)C''_v(e_1^*)]}{C'^2_v(e_1^*)} < 0.$$

With the conclusion of Lemma 4.4.3, we have $\frac{dW_1}{d\theta} > 0$.

For $\theta H(e_2^*(\theta))$, we have

$$\begin{aligned}
\theta H(e_2^*(\theta)) =& \theta(d_1+d_2)C_m(e_2^*)\exp\{-\lambda e_2^*\} \\
=& \left(1 - \frac{s}{(d_1+d_2)\exp\{-\lambda e_2^*\}(\lambda C_m(e_2^*) - C_m'(e_2^*))}\right)(d_1+d_2)C_m(e_2^*)\exp\{-\lambda e_2^*\} \\
=& (d_1+d_2)C_m(e_2^*)\exp\{-\lambda e_2^*\} - \frac{s}{\lambda - \frac{C_m'(e_2^*)}{C_m(e_2^*)}} \\
\equiv& W_2
\end{aligned}$$

Taking derivitives of W_2 with respect to e_2^*, we have

$$\begin{aligned}
\frac{dW_2}{de_2^*} =& (d_1+d_2)(C_m'(e_2^*)\exp\{-\lambda e_2^*\} + C_m(e_2^*)(-\lambda)\exp\{-\lambda e_2^*\}) \\
&+ \frac{s}{\left(\lambda - \frac{C_m'(e_2^*)}{C_m(e_2^*)}\right)^2}(-1)\frac{C_m''(e_2^*)C_m(e_2^*) - C_m'^2(e_2^*)}{C_m^2(e_2^*)}
\end{aligned}$$

Given $C_m(e) = \frac{\alpha_m}{e}$, $C_m'(e) = -\frac{\alpha_m}{e^2}$, $C_m''(e) = \frac{2\alpha_m}{e^3}$ and $C_m''(e)C_m(e) - C_m'^2(e) > 0$, we have $\frac{dW_2}{de_2^*} < 0$. Thus $\frac{dW_2}{d\theta} > 0$. □

Proof of Lemma 4.4.5

Proof of Lemma 4.4.5. First consider

$$\begin{aligned}
\frac{dh_1(e_1^*(\theta))}{d\theta} =& \frac{d}{d\theta}[se_1^* + (1-\theta)d_1C_v(e_1^*)] \\
=& s\frac{de_1^*(\theta)}{d\theta} + (-1)d_1(C_v(e_1^*) + (1-\theta)d_1C_v'(e_1^*)\frac{de_1^*(\theta)}{d\theta} \\
=& s\frac{de_1^*(\theta)}{d\theta} + (-1)d_1(C_v(e_1^*) - s\frac{de_1^*(\theta)}{d\theta} \\
=& -d_1(C_v(e_1^*) < 0.
\end{aligned}$$

Thus $h_1(e_1^*(\theta))$ is decreasing in θ.

Next,

$$\begin{aligned}
\frac{dh_2(e_2^*(\theta))}{d\theta} =& \frac{d}{d\theta}(se_2^* + (1-\theta)(d_1+d_2)C_m(e_2^*)\exp\{-\lambda e_2^*\}) \\
=& (-1)(d_1+d_2)(C_m(e_2^*)\exp\{-\lambda e_2^*\} < 0.
\end{aligned}$$

Thus $h_2(e_2^*(\theta))$ is also decreasing in θ. □

Proof of Proposition 4.4.1

Proof of Proposition 4.4.1. (i) When $s + d_1 C_v'(\tilde{e}) < 0$, consider θ from $[0, \bar{\theta}]$ and $[\bar{\theta}, 1]$ separately. According to Lemma 4.4.4, the minimum of $\theta H(e^*(\theta))$ will be achieved at $\theta = 0$ or $\theta = \bar{\theta}$, depending on parameters. But we also need to take care of the IR constraint.

From Lemma 4.4.5, $h_2(e_2^*(\theta))$ is decreasing in θ, and there exists a $\underline{\theta}_2$ in $[0, \bar{\theta}]$ which satisfies $h_2(e_2^*(\underline{\theta}_2)) = \bar{C}$. And for all $\theta \in [\underline{\theta}_2, \bar{\theta}]$, $h_2(e_2^*(\theta)) \leq \bar{C}$. Similarly, $h_1(e_1^*(\theta))$ is decreasing in θ, and there exists a $\underline{\theta}_1$ in $[\bar{\theta}, 1]$ which satisfies $h_1(e_1^*(\underline{\theta}_1)) = \bar{C}$. And for all $\theta \in [\underline{\theta}_1, 1]$, $h_1(e_1^*(\theta)) \leq \bar{C}$.

Then we have the solution to the principle agent problem is $\theta = \underline{\theta}_2$, or $\underline{\theta}_1$, depending on parameters.

When $s + d_1 C_v'(\tilde{e}) > 0$ and $s + (d_1 + d_2)\exp\{-\lambda\tilde{e}\}(C_m'(\tilde{e}) - \lambda C_m(\tilde{e})) < 0$. In this setting, for $\theta \in [0, \min\{\tilde{\theta}, \bar{\theta}\}]$, e_2^* is the minimizer. And for $\theta \in [\min\{\tilde{\theta}, \bar{\theta}\}, 1]$, e_1^* is the minimizer. Thus for the principle agent problem, the optimal solution will be $\underline{\theta}_2$, or $\underline{\theta}_1$, depending on parameters.

Combining the above two scenarios, we have when $s + (d_1 + d_2)\exp\{-\lambda\tilde{e}\}(C_m'(\tilde{e}) - \lambda C_m(\tilde{e})) < 0$, the optimal solution will be $\underline{\theta}_2$, or $\underline{\theta}_1$.

(ii)$s + (d_1 + d_2)\exp\{-\lambda\tilde{e}\}(C_m'(\tilde{e}) - \lambda C_m(\tilde{e})) > 0$.

In this setting, for all $\theta \in [0, 1]$, e_1^* is the unique minimizer. Thus for the principle agent problem, the optimal solution will be $\underline{\theta}_1$. And in this setting, the manufacture will always voluntarily recall. $\square$

Proof of Proposition 4.4.2

Proof of Proposition 4.4.2. First, consider $e_1^*(\underline{\theta}_1)$. Given

$$se_1^*(\underline{\theta}_1) + (1 - \underline{\theta}_1)d_1C_v(e_1^*(\underline{\theta}_1)) = \bar{C}$$

and

$$s + (1 - \underline{\theta}_1)d_1{C_v}'(e_1^*(\underline{\theta}_1)) = 0.$$

We could solve that

$$e_1^*(\underline{\theta}_1) = \frac{\bar{C}}{2s}.$$

Next, consider $e_2^*(\underline{\theta}_2)$. Given

$$se_2^*(\underline{\theta}_2) + (1 - \underline{\theta}_2)(d_1 + d_2)C_m(e_2^*(\underline{\theta}_2))\exp\{-\lambda e_2^*(\underline{\theta}_2)\} = \bar{C}$$

and

$$s + (1 - \underline{\theta}_2)(d_1 + d_2)\exp\{-\lambda e_2^*(\underline{\theta}_2)\}(C_m'(e_2^*(\underline{\theta}_2)) - \lambda C_m(e_2^*(\underline{\theta}_2))) = 0.$$

We have

$$se_2^*(\underline{\theta}_2) + \frac{se_2^*(\underline{\theta}_2)}{1 + \lambda e_2^*(\underline{\theta}_2)} = \bar{C}.$$

Further calculations yield

$$e_2^*(\underline{\theta}_2) = \frac{\bar{C}\lambda - 2s + \sqrt{4s^2 + \bar{C}^2\lambda^2}}{2s\lambda}.$$

From

$$se_1^*(\underline{\theta}_1) + (1 - \underline{\theta}_1)d_1C_v(e_1^*(\underline{\theta}_1)) = \bar{C}$$

and plug in that $e_1^*(\underline{\theta}_1) = \frac{\bar{C}}{2s}$. We have

$$\underline{\theta}_1 = 1 - \frac{\bar{C}^2}{4sd_1\alpha_v}.$$

Next, from

$$se_2^*(\underline{\theta}_2) + (1 - \underline{\theta}_2)(d_1 + d_2)C_m(e_2^*(\underline{\theta}_2))\exp\{-\lambda e_2^*(\underline{\theta}_2)\} = \bar{C}$$

and

$$e_2^*(\underline{\theta}_2) = \frac{\bar{C}\lambda - 2s + \sqrt{4s^2 + \bar{C}^2\lambda^2}}{2s\lambda}.$$

Through calculations, we have

$$\underline{\theta}_2 = 1 - \frac{\bar{C}^2\lambda^2 - (2s - \sqrt{4s^2 + \bar{C}^2\lambda^2})^2}{4s\lambda^2(d_1 + d_2)\alpha_m \exp\{-\frac{\bar{C}\lambda - 2s + \sqrt{4s^2 + \bar{C}^2\lambda^2}}{2s}\}}.$$

□

Proof of Proposition4.4.3

Proof of Proposition4.4.3. This proposition follows immediately after Proposition 4.4.2.

□

Proof of Lemma4.4.6

Proof of Lemma 4.4.6. (i) $s + d_1 C_v'(\tilde{e}) < 0$.

For the first best solutions, recall

$$\tilde{e} = \ln\left(\frac{d_1 + d2}{d_1}\frac{C_m}{C_v}\right) \Big/ \lambda,$$

When $e < \tilde{e}$,

$$\min_e l_1(e) = \min_e d_1 C_v(e) + se.$$

We have

$$l_1'(e) = s + d_1 C_v'(e),$$

$$l_1''(e) = d_1 C_v''(e) > 0.$$

Thus there exists a unique minimizer, denoted as e_1^{FB}, which satisfies $C_v'(e_1^{FB}) = -\frac{s}{d_1}$.

When $e > \tilde{e}$,

$$\min_e l_2(e) = \min_e se + (d_1 + d_2)C_m(e)\exp\{-\lambda e\}).$$

We have

$$l_2'(e) = s + (d_1 + d_2)\left(C_m'(e)\exp\{-\lambda e\} - \lambda C_m(e)\exp\{-\lambda e\}\right)$$
$$= s + (d_1 + d_2)\exp\{-\lambda e\}(C_m'(e) - \lambda C_m(e))$$

$$l_2''(e) = (d_1 + d_2)\left[\exp\{-\lambda e\}(C_m''(e) - \lambda C_m'(e)) + \exp\{-\lambda e\}(-\lambda)(C_m'(e) - \lambda C_m(e))\right] > 0$$

Thus there exists a unique minimizer, denoted as e_2^{FB}, which satisfies $l_2'(e_2^{FB}) = 0$. Under the assumption $s + d_1 C_v'(\tilde{e}) < 0$, we have $l_1'(\tilde{e}) < 0$ and $l_2'(\tilde{e}) < 0$. Thus there exists a unique solution to the first best problem, which is e_2^{FB}, which means the manufacture do not voluntarily recall.

(ii) $s + (d_1 + d_2)\exp\{-\lambda\tilde{e}\}(C_m'(\tilde{e}) - \lambda C_m(\tilde{e})) > 0$.

In this setting, $l_1'(\tilde{e}) > l_2'(\tilde{e}) > 0$. Thus for all $\theta \in [0, 1]$, e_1^{FB} is the unique minimizer. Note that $e_1^{FB} > e_1^*$ and $e_2^{FB} > e_2^*$

(iii)$s + d_1 C_v'(\tilde{e}) > 0$ and $s + (d_1 + d_2)\exp\{-\lambda\tilde{e}\}(C_m'(\tilde{e}) - \lambda C_m(\tilde{e})) < 0$.

In this setting, $l_1'(\tilde{e}) > 0$, $l_2'(\tilde{e}) < 0$, the optimal solution depends on which of $l_1(e_1^{FB})$ and $l_2(e_2^{FB})$ is smaller. □

Proof of Proposition4.4.4

Proof of Proposition4.4.4. This proposition could be obtained after Lemma 4.4.2 and Lemma 4.4.6. □

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