Product Line Design Under Capacity and Competition

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ABSTRACT
**HESNA MUGE YAYLA-KULLU: Product Line Design Under Capacity and
Competition**
(Under the direction of Dr. Jayashankar M. Swaminathan)

Firms have long recognized the importance of quality based market segmentation and designed their product lines to make use of this phenomenon. However, product line decisions are traditionally made without regard to capacity limitations. It is usually ignored that a firm has limited resources for offering its products and needs to use these resources efficiently. This dissertation provides managerial insights by simultaneously studying the product line design problem and capacity limitations faced by the firms in a stylized two product setting.

The optimal choice of product mix and pricing of these products when the products have different quality levels is a well-known problem. It has been studied extensively in both the marketing and economics literatures. However, the impact of capacity constraints has never been investigated in these literatures. On the other hand, the effects of product variety on operational decisions and how to mitigate these effects are fundamental questions in the operations literature. However, the effects of segmentation and cannibalization have not been understood well in the operations literature. This dissertation aims to fill these gaps in the literature.

In the first essay, the problem is solved from a monopolist firm's point of view. This solution is compared to a socially efficient solution subject to capacity limitations.

In the second essay, we introduce competition into the model. We characterize the solutions for both duopoly and oligopoly market structures. We investigate how competitor entry changes the optimal product mix, how industry supply and prices of products are affected when the number of competing firms changes, and how these results under capacity limitations are different from the existing literature.

In the third essay, we study firms that have focused product line strategies. We derive the profitability limits of the focused strategy firms under both monopoly and duopoly settings where the competition may be asymmetric.

In the fourth essay, we extend the monopoly model into a multiperiod setting and we study the effects of customer valuation uncertainty. We discuss how the results from the deterministic case compare to the stochastic case and how increasing uncertainty affects the firm's product line decisions.

Dedicated to my daughter,

Lara İnci Kullu,

who brought me lots of joy and happiness

by her arrival during the making of this dissertation. . .

ACKNOWLEDGMENTS

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CHAPTER 1

Introduction

This dissertation studies the management of product variety. In particular, we focus the design of a vertically differentiated product line in the face of capacity constraints. A firm typically has to consider product variety implementation issues at the same time with the product variety design issues as often discussed in the operations literature (Ramdas, 2003). In this dissertation, we contribute to this literature and show that simultaneous consideration of capacity limitations (an implementation issue) and product line choice (a design issue) leads to fundamentally different results than the previous literature.

Quality based segmentation is prevalent in almost all industries. Whether it is the comfort of the seats in the airline industry, or the speed and clarity of connection in the telecommunications industry, or the customer service in a bank, or the index of refraction (sparkle) and intricacy of the cut in the glass industry or the display resolutions in the television industry; there is always an attribute or a combination of attributes that exhibits the "more is better" property in the eyes of the customers. Moreover, each customer is different in his/her willingness to pay for quality. An executive officer who plans to attend a meeting right off the plane surely values the comfort of the seat more than a student traveling to his/her hometown on a tight budget.

Firms have long recognized this quality based segmentation. They design the final product mix taking segmentation opportunities into account. However, traditionally, the

product mix choices are made without regard to the capacity limitations. Indeed, a firm does not have unlimited resources for offering its products and it often faces capacity constraints in different forms: For an airline, the space in an aircraft; for the telecommunications service provider, the total bandwidth; for a bank, number of customer representatives; and for a durable goods manufacturer, amount of available raw material, labor, equipment within the manufacturing facility are limited. These resources are critical for the firm and they have to be allocated among the products efficiently. Moreover, the products of different quality levels generally consume different amounts of the resources. In particular, high quality product usually consumes greater amount from the critical resource: The size of the seat has to be greater to achieve more comfortable seating in an aircraft or a glass artist has to pay more attention and spend more time to cut the crystals into the perfect shape.

Hence, it is essential for the firm to understand the implications of capacity limitations on the optimal product mix and design its product line taking its resources into account.

1.1   Motivation

In this dissertation, we investigate the economic forces that determine multiproduct firms' product line design decisions. There are many conflicting pressures and tradeoffs that firms have to face when making strategic business decisions. Examples of these conflicting pressures are varying demand and customer valuations, changing cost structures, decreasing resources and intensified competition. We analyze the impact of these trade-offs on strategic product mix decisions of the firms.

One of the most important business decisions is the choice of product mix that will be offered in the market. By choosing the right products, firms would like to extract as much profit as possible from the market. They use market segmentation techniques in order to increase the demand. They present multiple products with different quality

levels and different prices; and customers self select from this menu of offerings. This in turn creates the risk that lower priced low quality products may cannibalize the demand for higher priced high quality products. Hence, it is not always beneficial for the firm to segment the market: Firms need to employ well-designed product line strategies in order to increase the overall profitability of the product variety offered in the market.

On the supply side, an increase in the product variety builds pressure on the limited and precious resources of the firms. Often, high quality products consume more resources in return for higher unit profits. On the other hand, lower quality products consume less of these critical resources and leave room for economies of scale. Hence, allocation of the critical resources becomes a strategic business decision for the firm: Firms need to dwell on capacity limitations in order to increase the overall efficiency of the resources.

Moreover, external competition is tougher than ever. New entrants act fast to copy the profitable products and steal market shares of the incumbents. Competitive threats apparently affects the decisions of the firms. Any firm that wants to stay on top of the game needs to respond to the competition and adjust its product variety whenever necessary.

Under such conflicting pressures, firms need to better understand the economic forces behind their decisions and how they relate to each other. As these effects act in opposite ways, it is not clear a priori whether a firm should follow a segmentation or a focus strategy. Increasing the product variety requires a well designed segmentation and resource allocation policy. On the other hand, if the firm chooses to follow a focus strategy, the direction of the focus is unclear: should the firm focus on the high quality market or focus on the low quality market?

We address these questions in Chapters 2 and 3. We focus on the following tradeoffs in our analysis. For symmetric multiproduct firms, (1) there is an intra-firm competition among the products for the demand; (2) there is another intra-firm competition among the products for the resources; and (3) there is an interfirm competition among the firms

for the market shares.

In Chapter 4, we study the firms that follow a pre-determined focused strategy. Some firms may choose to offer only low quality products and some others may choose to offer only high quality products in the market. This may be due to technological capabilities or executive board decisions among other reasons. We investigate the profitability bounds for these focused strategy firms. For example, the airline industry has seen all kinds of product line strategies: airlines focusing on the economy class like JetBlue, airlines focusing on the first class like EOS, and airlines following a traditional strategy and offering all classes like Delta.

Lufthansa has been the first airline to successfully operate a business-class only flight in the market (Lufthansa, 2008). They have redesigned commercial jets into business jets and targeted the executives who have been traveling on the route from New York to Düsseldorf which required at least one connection. The executives were ready to pay the high premium in return to a more comfortable, direct and quicker service. They offer only 48 very spacious and comfortable seats in an Airbus A319 and 44 such seats in a Boeing 737. The routes include Munich-New York, Düsseldorf-New York and Düsseldorf-Chicago as of August 2008 and Munich-Boston, Frankfurt-Dubai, and Frankfurt-Pune as of February 2009.

On the other end, there are economy-class only airlines, like JetBlue Airways and Southwest Airlines. These airlines dedicate their whole capacity to economy class customers. Although a passenger cannot expect complimentary gourmet dinners on board, the cost effectiveness of these airlines allow them to offer lower prices than a traditional airline.

Besides the success of JetBlue, Southwest and Lufthansa Airlines, the market has experienced many failures when it comes to focused strategy firms. In 2005, the introduction of two small-size firms to the market created a stir (Brancatelli, 2005). EOS Airlines and MaxJet Airlines entered the transatlantic flight market during Fall 2005 with

a better quality customer service and competitive prices. MaxJet redesigned Boeing 767 aircrafts, that typically seats around 200 passengers, to seat 102 premium seats (Maxjet, 2008). However, on December 24, 2007, the company announced that "due to a number of material factors such as competitive pressure and operating cost increases", they had filed Chapter 11 of the U.S. Bankruptcy Code. EOS Airlines also filed bankruptcy on April 26, 2008.

As an economy class focused airline, Skybus Airlines was founded to operate on the basis of ultra-low cost structure. They targeted secondary markets rather than heavy traffic airports in order to keep the competition to a minimum. When Skybus had ceased operations on April 5, 2008, its fleet consisted of 13 Airbus A319 each of which were designed to have all economy class seats.

Following these examples of failures and successes of focused strategy firms in practice, we explore the underlying reasons that lead to success or failure of these firms in Chapter 4. We analyze the driving economic factors for focused strategy firms in the existence of capacity constraints and find out the limits of their profitability. We investigate how well a focused strategy firm performs compared to a traditional firm.

The world has been going through a serious recession period. Many customers prefer to be conservative in terms of their expenditures, especially when it comes to luxurious (high quality) goods. These luxurious purchases are substituted by economical (low quality) counterparts. We observe from first hand that customers' willingness to pay for quality has decreased as the recession progressed. This observation is also recorded by the Federal Reserve System's 2009 Beige Book (FederalReserve, 2009): "*Overall economic activity continued to weaken across almost all of the Federal Reserve Districts since the previous reporting period. Most Districts noted reduced or low activity across a wide range of industries... Reports of retail sales during the holiday season were generally negative in most Districts.*"

Moreover, forecasters are quite cautious about predicting what may happen next.

The demand and customer willingness to spend may get better soon especially with the governmental interventions or it may stay low for a longer while. What is clear is the fact that there is uncertainty about the customer spending in all industries.

Mindful of such volatility in the economic climate, firms need to take customer valuation uncertainty into account before making commitments, in particular in terms of capacity building. In Chapter 5, we introduce this uncertainty into the formulation and investigate the problem in a stylized setting where there are multiple periods.

1.2   Overview of the Dissertation

In this dissertation, we study the effects of limited capacity on vertically differentiated product line design decisions of the firms. We analyze the situation in various market structures including monopoly, oligopoly and socially efficient markets.

The firm offers two different product types: high and low quality products. These products also differ in their unit costs and resource consumptions. The firm decides how to allocate its limited capacity among the products and their prices. For example, consider an airline determining how many first and economy class seats to install in an aircraft. First and economy class seats offer different quality of service and they differ in their unit operating costs. Furthermore, each seat type requires different amount of space per unit and the number of total seats is constrained by the size of the aircraft.

In the demand model, we consider a market with heterogenous customers that vary in their willingness to pay for quality. Customer preference is private information, but the firm knows its distribution. Each customer decides among the two products and the no purchase option, and chooses the one that maximizes his/her utility.

In Chapter 2, we solve the resulting equilibrium from a monopolist firm's point of view and study its characteristics. We also look at the problem from a social planner's point of view. Among other results, our key findings are as follows.

When there are increasing costs to quality, i.e., when the unit cost to quality ratio of high quality product is larger than that of the low quality product, we find that the firm may be better off focusing on one product and offering either the low or the high quality product for sufficiently small capacity levels. The focus depends on the maximum margin as well as the resource consumption of each product type. This is in contrast to the existing literature (Mussa and Rosen (1978), Itoh (1983), Moorthy (1984), Desai (2001), Johnson and Myatt (2003)) that argues that the firm should serve both products offering a differentiated product line in this case. When the firm's capacity is sufficiently large, our results coincide with the existing literature.

When there are decreasing costs to quality, the firm's optimal strategy is either to offer both products or only one of the products depending on its capacity. In this case, the existing literature that disregards the capacity constraint shows that the firm should always focus on the high quality product (Johnson and Myatt (2003), Bhargava and Choudhary (2001)). While our results agree with the existing literature when the firm's capacity is sufficiently large, we show that the firm might be better off with a diametrically opposite policy focusing on the low quality product, when its capacity is sufficiently small, and for intermediate capacity levels the firm prefers offering both product types.

We also show that limited capacity can induce a monopoly to offer the higher end customers a better quality product compared to a social planner. Furthermore, a monopolist can cover a greater portion of the market than a social planner. These are in contrast to existing literature which shows that the customers (except those at the high end) get either a lower quality product or nothing at all from a monopoly compared to a social planner's assignment (Mussa and Rosen (1978), Moorthy (1984), Desai (2001)).

In Chapter 3, we study the system in competitive settings. There are $n \geq 2$ symmetric firms that engage in a Cournot competition. We aim to understand the effects of competition on the optimal product lines of firms.

We find that the capacity availability is again a critical component in the competi-

tive firm's product line design decisions. The decisions made by ignoring the capacity constraint would provide insights for the cases when there is abundant resources; as it was done in the existing literature (Gal-Or (1983), DeFraja (1996), Johnson and Myatt (2003), and Johnson and Myatt (2006a)). We show that these decisions may no longer be optimal when the capacity is limited. For instance, when the cost structure favors the high quality product (cost to quality ratio is decreasing), if the firm has unlimited resources, best response to the competition is to focus and offer only the high quality product. In this case, the economic force leading the product line decisions of the firms is to eliminate the intrafirm competition for demand among products. However, if the capacity is limited for the firm and resource consumption is high for the high quality product, efficient use of resources becomes the leading force and low quality product is optimally introduced into the product mix. If the available capacity is even lower, then the firms optimally focus and offer only the low quality product regardless of the fact that the cost to quality ratio favors the high quality product.

We also investigate the influence of external competition on the prices and the product variety supplied in the market in this chapter. The impact depends on the cost structures of the industry as well as the available capacities. As opposed to the acquired wisdom in the literature, we show that the total industry supply may decrease as the number of firms increase in the market for a specific range of total industry capacity levels when the potential profit per unit resource consumed is greater for the high quality product. Moreover, we show that the price of a product may increase as the number of firms increase for a specific range of total industry capacity levels, if the product has a low potential profit per unit resource consumed and when the cost to quality ratio is increasing and resource consumption ratio is small.

Another impact of increasing competition could be increasing the product variety. For instance, if the low quality product has high potential profit per unit resource consumed, for a specific range of total industry capacity levels, increasing competition force the firms

to introduce the low quality product into the product mix, even when the cost structures favor the high quality product (cost to quality ratio is decreasing).

Among other results, we also find that an incumbent firm should revise its product line decisions once a new firm enters the market with extra capacity. The outcome may be deleting or introducing new products to the market depending on the individual firm capacity and cost and resource structure of the products. For example, the firm is forced to delete the low quality product from the product mix in response to entry for a certain range of individual firm capacity levels when the potential profit per unit resource consumed is greater for the low quality product and cost structure favors the high quality product. In this case, both the increase in total industry capacity and existence of interfirm competition affects the optimal decisions of the firms.

In Chapter 4, we study firms that have focused product line strategies. We discuss the profitability limits of the focused strategy firms under both monopoly and duopoly settings. In this essay, competition is between asymmetric firms. While the product line choice is fixed for the focused strategy firm, the competitor firm may respond with any strategy: it has the necessary capability to offer any product combination.

We observe that the high quality focused firm may earn as much as 99.9% of a competitor firm that has the capability to follow the optimal strategy, depending on the cost and resource consumption conditions. We also observe that the profitability of the high quality focused firm could go as low as 6.63% when the capacity is scarce, depending on the cost and resource consumption conditions. If the cost structure in which the high quality focused firm operates is favorable, then it will survive just fine in competitive settings. However, if the cost conditions change in an unfavorable direction, this could be detrimental for the high quality focused firm as observed in our numerical study.

More interestingly, we find that the competitor firm may change its product line in response to the high quality focused firm. We found instances where the competitor firm actually focus on the low quality product for all capacity levels; while the optimal

strategy would be offering both products in a monopoly.

On the other hand, we observe that the firm with a low quality focus may earn as low as 1.18%, depending on the cost and resource consumption conditions. These low profitability levels may help us explain the failures of many low-cost carriers in the airline industry. As the unit cost increases, the marginal profit of the low quality product also decreases dramatically. These changes in the cost structures explain the extremely low profits in the face of competition and the failures of the low quality focused strategy firms.

Until this point in the dissertation, we study the economic forces behind the firms' product line decisions under deterministic assumptions. ***In Chapter 5***, we incorporate a very important dimension to our analysis: the customer valuation uncertainty.

In our model, we recognize the fact that firms make their strategic capacity decisions well before the markets clear for prices. In the first period, the firm decides for the committed capacity for each product based on the available resources and expected customer valuation distribution. In the second period, the specifics of the customer valuation distribution is realized and the firm makes its actual production and sales decisions constrained by the initial capacity commitment.

We first solve the second period problem where the capacity allocations and product line decisions are known. As opposed to the conventional wisdom in the economics literature, we find that when the capacity commitment of the low quality product is below a certain threshold and cost to quality ratio is increasing, the sales decision favors the low quality product. If the market valuations are low, then the firm only sells a limited amount of the low quality product in order to keep its price high. In this case, capacity constraints are not binding. If the valuations get a little better, then the firm optimally sells all the low quality product. Only if the market valuations are high enough, the firm starts selling the high quality product.

When the capacity commitment of the low quality product is above a certain threshold

and cost to quality ratio is increasing, we find that the low quality product is offered for a longer range of valuations. Nevertheless, the firm does not wait to sell all the low quality production but introduces high quality product together with the low quality product.

Given the optimal sales decisions of the second period, we solve the problem to find the optimal capacity allocation in the first period. When capacity constraint is not binding and the cost to quality ratio is increasing, we have shown that the optimal strategy is to differentiate and offer both products in the market as is the case when there is no uncertainty. Nevertheless, there is one important difference: the quantity of the high quality product increases with the level of uncertainty.

When the capacity constraint is binding, achieving the closed form solutions is not analytically tractable. We present some numerical examples and observe how the results of the deterministic models change under uncertainty. When the cost to quality ratio is increasing and marginal profit per unit resource is better for the low quality product; for a medium range of capacity levels, high quality commitment increases as the level of uncertainty increases. However, since the capacity constraint is binding at this level, the required resources are gained from decreasing the commitment of the low quality product. The increase in the high quality commitment comes at the expense of low quality commitment: the commitment levels decrease as uncertainty increases for the low quality product.

CHAPTER 2

Impact of Shared Capacity on Vertically Differentiated Product Lines

In this chapter, we investigate a monopolist firm with limited capacity that serves vertically differentiated products to a heterogeneous customer market. We also look at the problem from a social planner's point of view and compare the monopolist's solution to this socially efficient assignment.

2.1   Related Literature

There is rich literature in marketing and economics on the monopolist's choice of vertically differentiated product lines (e.g., Mussa and Rosen (1978), Itoh (1983), Moorthy (1984), Gabszewicz et al. (1986), Desai (2001), Kim and Chhajed (2002), Johnson and Myatt (2003)). This literature has studied product choice, pricing, market segmentation, and cannibalization among other things. However, it has ignored the effects of capacity limitations on the firm's product line choice. Indeed, a firm does not have unlimited resources for offering its products and it often faces capacity constraints in the form of

time, labor, equipment, space and inventory. With this study, we aim to contribute to the above literature by incorporating such a capacity constraint. We also contribute to the emerging literature in the marketing-manufacturing interface that looks at the effects of operational elements on a firm's product-line. Heese and Swaminathan (2006), Desai et al. (2001) and Kim and Chhajed (2000) study the effects of component commonality on product line design. Netessine and Taylor (2007) characterize the effect of production technology (production to order vs. production to stock). Dobson and Yano (2002) and Chayet et al. (2007) consider a shared resource used for offering a product-line similar to our set-up, however there are fundamental differences. In Dobson and Yano (2002), products have independent demands, there is no cannibalization. In Chayet et al. (2007) high and low quality products have equal resource consumptions (equal production time in their context), while the difference in capacity consumption of product types is one of the key elements of our model.

2.2   Model

We study two product types, high and low quality products. Our main model assumes exogenous quality levels $q_h > q_l$, we later relax this assumption and allow for endogenous quality levels in Section 2.5. Each unit of product i costs c_i and it consumes s_i units of the capacity. We assume $c_h > c_l$ and $s_h > s_l$. For example, consider an airline offering first and economy class seats. First class seats are perceived as better quality by the customers ($q_h > q_l$); they are bigger in size ($s_h > s_l$) and it costs more to operate them ($c_h > c_l$) due to better quality customer service (greater number of flight attendants,

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more expensive food and drinks, private aisle check-in process, airport lounges, etc).

The firm decides how to allocate its limited capacity K to the product types. To serve x_i units of product i, the firm needs to allocate $s_i \cdot x_i$ units of its capacity. We assume that the parameters are such that the trivial case ($x_l = 0$ and $x_h = 0$) is not optimal. The firm sets prices p_i, and this in turn determines the demand of each product D_i, which will be derived in the following. Clearly, the firm sells the minimum of allocated capacity x_i and demand D_i for product i. So, the firm's profit is

$$\Pi^M = (p_h - c_h)\min(D_h, x_h) + (p_l - c_l)\min(D_l, x_l). \tag{2.1}$$

We adopt the classical vertical differentiation demand model (cf. Tirole (1988)). The customers vary in their willingness to pay for quality. Specifically, the customer types θ are uniformly distributed in the unit interval [0,1] with unit total mass. When type θ customer buys product i at price p_i, his utility is equal to

$$U(q_i, p_i, \theta) = \theta q_i - p_i.$$

If the customer does not buy a product, his utility is zero. Thus, each customer has three options, buying the high quality product, buying the low quality product and not buying a product, and he chooses the one that maximizes his utility. This yields $0 \leq \theta_l \leq \theta_h \leq 1$ such that customers in $[0, \theta_l)$ do not buy a product, customers in $[\theta_l, \theta_h)$ buy the low quality product and customers in $[\theta_h, 1]$ buy the high quality product. So, the demand for the high quality and the low quality products are $D_h = 1 - \theta_h$ and $D_l = \theta_h - \theta_l$. It is

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straightforward to show that the marginal customer θ_h who is indifferent between buying the high and the low quality products is given by $\theta_h = (p_h - p_l)/(q_h - q_l)$ and similarly, the marginal customer θ_l who is indifferent between buying the low quality product and not buying a product at all is given by $\theta_l = p_l/q_l$. Thus, we can express the demands for the two product types as follows,

$$D_h(p_l, p_h) = 1 - \frac{p_h - p_l}{q_h - q_l} \qquad D_l(p_l, p_h) = \frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l}. \qquad (2.2)$$

2.3   Monopoly

In this section, we solve the monopoly firm's problem and discuss our findings. The firm chooses the capacity allocations x_i and prices p_i to maximize its profit Π^M given in (2.1) subject to the capacity constraint. Specifically the firm solves,

$$\max_{p_l, p_h, x_l, x_h \geq 0} \quad \Pi^M$$

$$\text{subject to} \quad s_h x_h + s_l x_l \leq K.$$

Without loss of generality, we can restrict the analysis to $x_i = D_i$. If $D_i > x_i$, the firm can increase price p_i and achieve a higher profit, similarly if $D_i < x_i$, the firm can decrease x_i without affecting the profit. Following $x_i = D_i(p_l, p_h)$, the optimal prices can be expressed as a function of capacity allocations x_i, and the above problem can be simplified to

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$$\max_{x_l, x_h \geq 0} \quad (p_h(x_l, x_h) - c_h)x_h + (p_l(x_l, x_h) - c_l)x_l$$

$$\text{subject to} \quad s_h x_h + s_l x_l \leq K.$$

The objective function of this problem is jointly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)).

In the following passages, we characterize a monopoly firm's optimal product line and contrast it to the existing literature. Propositions 1 and 2 describe the optimal policy when there are increasing cost to quality (i.e., $c_h/q_h > c_l/q_l$) and decreasing cost to quality (i.e., $c_h/q_h < c_l/q_l$) respectively. Let us define two threshold capacities,

$$\bar{K}_1^M = \frac{s_l(s_h(q_l - c_l) - s_l(q_h - c_h))}{2q_l(s_h - s_l)} \tag{2.3}$$

$$\bar{K}_2^M = \frac{s_h(s_l(q_h - c_h) - s_h(q_l - c_l))}{2q_h s_l - 2q_l s_h} \tag{2.4}$$

which will be useful for describing the firm's optimal policy. All proofs appear in Appendix A.

Proposition 1 *Suppose $c_l/q_l < c_h/q_h$. For a monopolist, the optimal product line strategy is as follows:*

i) If $q_l - c_l \geq q_h - c_h$, then the optimal strategy for the firm is to focus and offer only the low quality product for all capacity levels.

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ii.a) If $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, *then*

- *if* $K \leq \bar{K}_1^M$, *the optimal product line strategy is to focus and offer only the low quality product.*

- *if* $K > \bar{K}_1^M$, *the optimal product line strategy is to differentiate and offer both products.*

ii.b) If $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, *then*

- *if* $K \leq \bar{K}_2^M$, *the optimal product line strategy is to focus and offer only the high quality product.*

- *if* $K > \bar{K}_2^M$, *the optimal product line strategy to differentiate and offer both products.*

Part (i) in Proposition 1 describes the trivial case where it is always better to sell only the low quality product. Note that the maximum price that can be charged for product i is equal to the willingness to pay of the highest valuation customer ($\theta = 1$), that is equal to the quality of product q_i. Thus, in this case, the maximum profit margin for the low quality product $q_l - c_l$ is larger than that of the high quality product $q_h - c_h$.

Part (ii.a) and (ii.b) describe what happens when the maximum margin for the high quality product $q_h - c_h$ is larger. In this case, the optimal product line of the firm depends on its capacity. The Proposition shows that when the firm has a sufficiently large capacity, it prefers selling a differentiated product line offering both products, which is is in line with the existing literature (Mussa and Rosen (1978), Itoh (1983), Moorthy (1984), Desai (2001), Johnson and Myatt (2003)). Serving a differentiated product line helps the firm

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segment the market, which in turn enables attracting a larger demand with a smaller sacrifice in the profit margin of the high quality product. However, when the firm is already capacity constrained, a larger demand has little value and the firm may not benefit from offering both product types. Indeed, in contrast to existing literature, we show that when the firm has a sufficiently small capacity, it is better off focusing on one product and selling only either the low or the high quality product. The right focus depends on the capacity adjusted maximum margins $\frac{q_i - c_i}{s_i}$, that is, the maximum profit margin per unit capacity.

In part (ii.a) of Proposition 1, the lower quality product has a higher capacity adjusted maximum margin and the firm sells only the low quality product when its capacity is smaller than $\bar{K}_1^M$. Suppose that the airline AX is acting as a monopolist on a flight leg from city A to city B. The potential market size for this city pair is 1,000 customers per day and the customer valuations are such that the highest amount of money any customer would pay is $3,000 for the business class seat and $1,000 for the economy class seat. Also suppose that the interior design of this airline's aircrafts is such that the business class seat occupies 20 sq. ft. and the economy class seat occupies 5 sq. ft. inside the aircraft. It costs $1,500 to operate a business class seat and $300 an economy class seat. If this airline optimizes the product line without taking the capacity availability into account, then the optimal solution is to offer 200 business class seats and 150 economy class seats with a profit of $202,500 and requirement of 4,750 sq. ft. of space. While this requirement could be satisfied with multiple flights in a day, if the airline has only 1,000 sq. ft. plane available for one flight only, then they can only offer 42 business class and

31 economy class seats for a profit of \$76,288. However, if the company would take this capacity limitation into account upfront, they would have found that the real optimal is to focus on the economy class in this single flight. They would have offered 200 economy class seats for a profit of \$100,000 with a dramatic increase of \$23,712 per day.

On the other hand, in part (ii.b), the high quality product has a better capacity adjusted maximum margin and the firm sells only the high quality product when its capacity is smaller than $\bar{K}_2^M$. Suppose that another airline BX is acting as a monopolist on a flight leg from city B to city C. The potential market size for this city pair is also 1,000 customers per day and the customer valuations are such that the highest amount of money any customer would pay is \$4,000 for the business class seat and \$1,000 for the economy class seat. Also suppose that the interior design of this airline's aircrafts is such that the business class seat occupies 15 sq. ft. and the economy class seat occupies 5 sq. ft. inside the aircraft. It costs \$1,500 to operate a business class seat and \$300 an economy class seat. If this airline optimizes the product line without taking the capacity availability into account, then the optimal solution is to offer 300 business class seats and 50 economy class seats with a profit of \$392,500 and requirement of 4,750 sq. ft. of space. While this requirement could be satisfied with multiple flights in a day, if the airline has only 500 sq. ft. plane available for one flight only, then they can only offer 31 business class and 6 economy class seats for a profit of \$77,448. However, if the company would take this capacity limitation into account upfront, they would have found that the real optimal is to focus on the economy class in this single flight. They would have offered 33 economy class seats for a profit of \$78,889 with an increase of \$1,440 per day.

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Proposition 2 *Suppose $c_h/q_h \leq c_l/q_l$. For a monopolist the optimal product line strategy is as follows:*

i) If $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, then

- *if $K \leq \bar{K}_1^M$, the optimal product line strategy is to focus and offer only the low quality product.*

- *if $\bar{K}_1^M < K \leq \bar{K}_2^M$, the optimal product line strategy is to differentiate and offer both products.*

- *if $K > \bar{K}_2^M$, the optimal product line strategy is to focus and offer only the high quality product.*

ii) If $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, then the optimal strategy for the firm is to focus and offer only the high quality product for all capacity levels.

Proposition 2 characterizes the firm's optimal policy when there are decreasing costs to quality i.e., $c_h/q_h \leq c_l/q_l$. For this case, the existing literature (Bhargava and Choudhary (2001), Johnson and Myatt (2003)) has shown that the firm should offer only the high quality product when there are no capacity limitations. Our findings coincide with the existing literature when the firm has a sufficiently large capacity or when the high quality product has a larger capacity adjusted maximum margin. A firm with unlimited capacity does not offer the low quality product due to its cannibalization effect. However, when the low quality product has a larger capacity adjusted maximum margin (i.e., maximum profit margin per unit capacity $\frac{q_i - c_i}{s_i}$), it is more profitable than the high quality product for a capacity constrained firm and the optimal product line choice critically depends on the

firm's capacity. Specifically, the optimal policy can be diametrically opposite: The firm is better off selling only the low quality product when its capacity is sufficiently small (i.e., $K \leq \bar{K}_1^M$). In this case, a firm that disregards the capacity constraint would be offering the wrong product. In addition, for intermediate capacity levels (i.e., $\bar{K}_1^M < K \leq \bar{K}_2^M$), the firm prefers offering both products contrary to the existing literature. It is interesting that its limited capacity induces the firm to expand its product line, offering more product types in this case. The capacity constraint gives incentive to the firm to allocate more resources to the product that has a greater capacity adjusted maximum margin. This leads to adding the low quality product to the portfolio.

Suppose that another airline CX is acting as a monopolist on a flight leg from city C to city D. The potential market size for this city pair is also 1,000 customers per day and the customer valuations are such that the highest amount of money any customer would pay is \$3,000 for the business class seat and \$1,000 for the economy class seat. Also suppose that the interior design of this airline's aircrafts is such that the business class seat occupies 35 sq. ft. and the economy class seat occupies 5 sq. ft. inside the aircraft. It costs \$1,500 to operate a business class seat and \$750 an economy class seat. If this airline optimizes the product line without taking the capacity availability into account, then the optimal solution is to offer 250 business class seats only with a profit of \$187,500 and requirement of 8,750 sq. ft. of space. While this requirement could be satisfied with multiple flights in a day, if the airline has only 100 sq. ft. plane available for one flight only, then they can only offer 3 business class only seats for a profit of \$4,261 or cancel the flight all together. However, if the company would take this capacity limitation into

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account upfront, they would have found that the real optimal is to focus on the economy class in this single flight. They would have offered 20 economy class seats for a profit of $4,600 with almost 8% increase in profits per day.

Our results show that scarcity of the capacity forces a firm to drop the product with the smaller capacity adjusted maximum margin from its product line and dedicate the whole capacity to the more profitable product. This is formally stated in the following Corollary.

Corollary 1 *Suppose $\frac{q_i - c_i}{s_i} < \frac{q_j - c_j}{s_j}$. There exist $\hat{K} > 0$ such that x_i is non-decreasing in K with $x_i = 0$ for $K < \hat{K}$ in the optimal solution.*

2.4 Social Planner

In this section, we solve for the social planner's problem and compare it to the monopoly. Social planner maximizes the sum of customers' and firm's surplus, which is given by,

$$\Pi^S = \int_{1-x_h-x_l}^{1-x_h} (\theta q_l - c_l) d\theta \ + \ \int_{1-x_h}^{1} (\theta q_h - c_h) d\theta \tag{2.5}$$

Thus, the social planner solves

$$\max_{x_h, x_l \geq 0} \quad \Pi^S$$

$$\text{subject to} \quad s_h x_h + s_l x_l \leq K. \tag{2.6}$$

The solution to this problem is fully characterized in Lemma A2 in the Appendix

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A. Here, we discuss our findings and contrast them to the existing literature. We define additional capacity thresholds $\bar{K}_3^M, \bar{K}_4^M, \bar{K}_5^M, \bar{K}_1^S$, and $\bar{K}_2^S$ given in (A-7)-(A-9) and (A-16)-(A-17) in the Appendix A that will be helpful in stating our results.

In the standard vertical differentiation literature, it is well known that each customer gets a lower quality product or nothing at all from a monopolist compared to a social planner except the high end customer who gets the same assignment in both cases (e.g., Mussa and Rosen (1978), Moorthy (1984), Desai (2001)). In contrast, we show that a monopoly firm may offer a higher quality product than that of a social planner to some customer segments due to the capacity constraint. Specifically, when the maximum profit margin of the high quality product is larger, but its capacity adjusted maximum margin is smaller, the high end customers get a higher quality product from the monopolist than the social planner's assignment for a range of capacity levels. This is formally stated in the following proposition.

Proposition 3 *When* $q_h - c_h > q_l - c_l$ *and* $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, *the monopoly offers the high quality product whereas the social planner does not offer it for* $\bar{K}_1^M < K \leq \bar{K}_1^S$.

Under the condition in Proposition 3, the monopolist and the social planner prefer offering only the low quality product for sufficiently small capacity levels (i.e., $K \leq \bar{K}_1^M$) and similarly, they both offer both product types for sufficiently large capacity levels (i.e., $K > \bar{K}_1^S$). However, when $\bar{K}_1^M < K \leq \bar{K}_1^S$, the social planner does not offer the high quality product in order to increase its market coverage whereas the monopolist driven by the higher profit margins offers the high quality product. In this case, while the higher end customer segment is getting better quality under monopoly, the lower end

customer segment is getting worse off: they are getting nothing. The better quality at the high end comes at the expense of the lower customer segment.

The conventional wisdom in the literature states that the social planner serves a larger portion of the market compared to a monopolist (e.g., Mussa and Rosen (1978)). This is indeed the case when one ignores the capacity constraint. However, we show that, depending on the capacity level, a monopolist may serve a greater portion of the market than a social planner. Specifically, this happens when there are increasing costs to quality and the high quality product has a larger capacity adjusted maximum margin.

Proposition 4 *When $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and $c_l/q_l \leq c_h/q_h$, the monopoly covers a greater portion of the market than the social planner for $\bar{K}_2^M < K < \min\{\bar{K}_2^S, \bar{K}_3^M\}$.*

Under the condition in Proposition 4, both the monopolist and the social planner offer only the high quality product for sufficiently small capacity levels (i.e., $K \leq \bar{K}_2^M$) and similarly they both offer both product types for sufficiently large capacity levels $K > \bar{K}_2^S$ (the monopolist does not use all of its capacity for $K > \bar{K}_3^M$). However, when $\bar{K}_2^M < K < \min\{\bar{K}_2^S, \bar{K}_3^M\}$, only the monopoly serves the low quality product. By offering the low quality product, monopoly serves the high quality product to a smaller market segment and this in turn keeps its price higher. Whereas the social planner is not concerned about prices, it sells only the high quality product. This results in a larger market coverage under monopoly since both the monopoly and the social planner utilize all the capacity and high quality product consumes greater amount of capacity per unit.

Proposition 5 *When the capacity is sufficiently small, all customers get their socially efficient assignment from the monopolist.*

It is well-established in the literature that the monopolist degrades the quality level offered to low valuation customers (e.g. Mussa and Rosen (1978), Moorthy (1984)). However, we show that when the capacity is sufficiently small, the optimal policy for both the monopolist and the social planner is to dedicate all the capacity to the most valuable product type (in terms maximum surplus per unit capacity, i.e., $\frac{q_i - c_i}{s_i}$). Because all the capacity is dedicated to the same product, the segment of customers who get the high quality product, the low quality product and nothing are the same for both the monopolist and the social planner.

2.5   Endogenous Quality Levels

In this section, through numerical examples, we study what happens when quality levels are endogenous. The firm decides whether to offer one or two products in a quality differentiated product line as well as its capacity allocation to the products offered. In addition, the firm determines the quality levels $q_i \in [0, \overline{q}]$, where $\overline{q}$ is the maximum possible quality level. Functions $c(\cdot)$ and $s(\cdot)$ show how cost and required capacity depend on quality. Specifically, each product with quality q_i costs $c(q_i)$ and it requires capacity $s(q_i)$. To maximize its profit, the monopoly solves

$$\max_{x_l, x_h, p_l, p_h \geq 0, \ q_l, q_h \in [0, \overline{q}]} (p_h - c(q_h)) \min(D_h, x_h) + (p_l - c(q_l)) \min(D_l, x_l) \quad (2.7)$$

$$\text{subject to} \quad s(q_h) x_h + s(q_l) x_l \leq K \quad (2.8)$$

where demand for product i, D_i, is given in (2.2). Similarly, the social planner solves

$$\max_{x_l, x_h, p_l, p_h \geq 0, \; q_l, q_h \in [0, \overline{q}]} \quad \int_{1-x_h-x_l}^{1-x_h} (\theta q_l - c(q_l)) d\theta \; + \; \int_{1-x_h}^{1} (\theta q_h - c(q_h)) d\theta$$

$$\text{subject to} \quad s(q_h) x_h + s(q_l) x_l \leq K$$

In our numerical examples, we use polynomial unit costs $c(q) = \alpha q^\beta$ as commonly assumed in the literature (e.g. Moorthy (1984), Desai (2001)). Notice that a strictly convex unit cost function, i.e., $\beta > 1$, results in $c_h/q_h > c_l/q_l$, that is in increasing costs to quality. Similarly, a strictly concave unit cost results in decreasing costs to quality.

In the following passages, we discuss how our results in Sections 2.3 and 2.4 carry over when the quality levels become endogenous decisions. Figure 2.1.a shows the optimal quality levels chosen by a monopolist as a function of its capacity when there are increasing costs to quality. Recall that the existing literature (e.g. Mussa and Rosen (1978), Itoh (1983), Moorthy (1984), Johnson and Myatt (2003)), while disregarding the limited capacity, has shown that the firm should offer a quality differentiated product line in this case. Indeed, Figure 2.1.a is consistent with the existing literature when the firm's capacity is sufficiently large, i.e., when $K > 0.29$. However, when the firm's capacity is small, i.e., when $K < 0.29$, the firm prefers following a focus strategy and offers only one product type as in Proposition 1. The firm is better off dedicating all of its capacity to the high quality product, this enables it to offer the high quality product to more customers. The firm should introduce the low quality product to differentiate only if its capacity is sufficiently high.
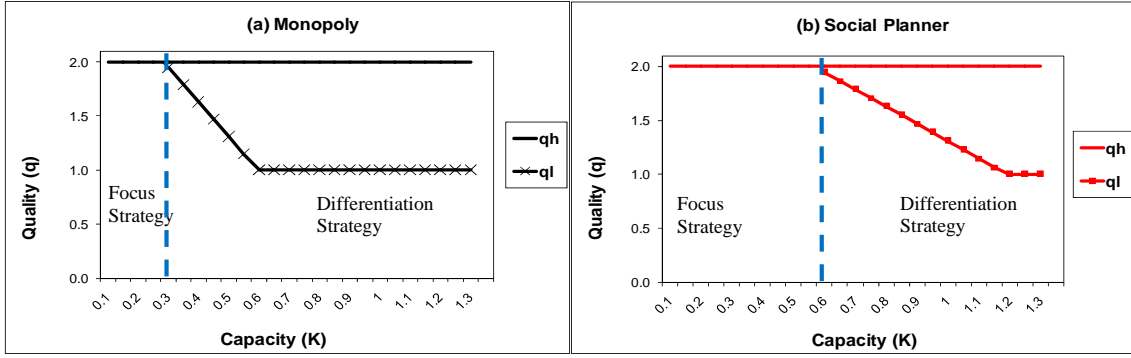
FIGURE 2.1: Optimal quality levels: Monopoly vs. Social Planner ($c(q) = 0.1q^2$, $s(q) = q^{1/2}$ and $\overline{q} = 2$)

Figure 2.3.a plots the optimal quality levels for a monopoly when there are decreasing costs to quality. In this case, the existing literature suggests that the firms should focus on one product, offering only a high quality product to avoid cannibalization (Johnson and Myatt (2003), Bhargava and Choudhary (2001)). Figure 2.3.a coincides with the existing literature when the firm's capacity is large, i.e., when $K > 2$. However, the firm prefers the differentiation strategy offering a high and a low quality product when the capacity is scarce. Offering a low quality product enables the firm to serve a larger market in this case, as the low quality product consumes less capacity. Notice that these findings are consistent with Proposition 2.
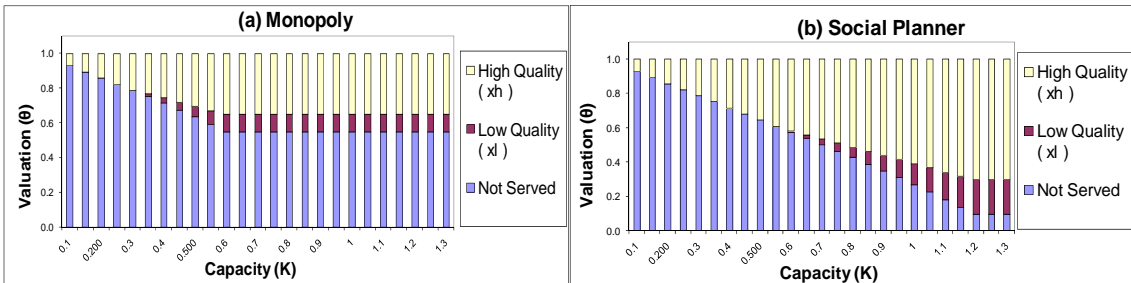
FIGURE 2.2: Market Coverage: Monopoly vs. Social Planner ($c(q) = 0.1q^2$, $s(q) = q^{1/2}$ and $\overline{q} = 2$)

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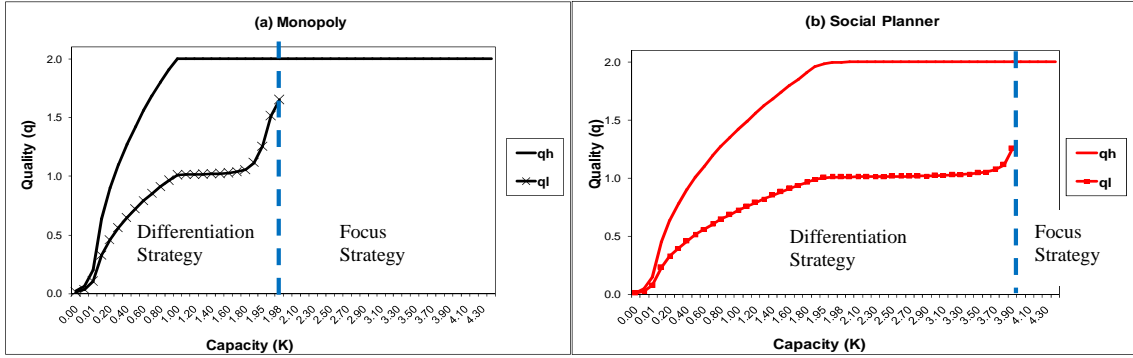
FIGURE 2.3: Optimal quality levels: Monopoly vs. Social Planner ($c(q) = 0.01q^{1/2}$, $s(q) = q^2$ and $\overline{q} = 2$)

Now, we discuss the social welfare results. Figures 2.2 and 2.4 show the optimal market coverage chosen by a monopolist (Figures 2.2.a and 2.4.a) and by a social planner (Figures 2.2.b and 2.4.b) as a function of their capacities. The existing literature shows that compared to a social planner, a monopolist degrades the quality of products offered except to the high-end customers. However, Figures 2.3 and 2.4 show an example where a monopoly offers a higher quality product than a social planner due to the capacity limitation as in Proposition 3. For example, when capacity $K = 1$, the customers that are at the high end ($\theta \geq 0.80$, see Figure 2.4.a and b) get a better quality product from a monopoly ($q_h = 2.0$, see Figure 2.3.a) than from a social planner ($q_h = 1.423$, see
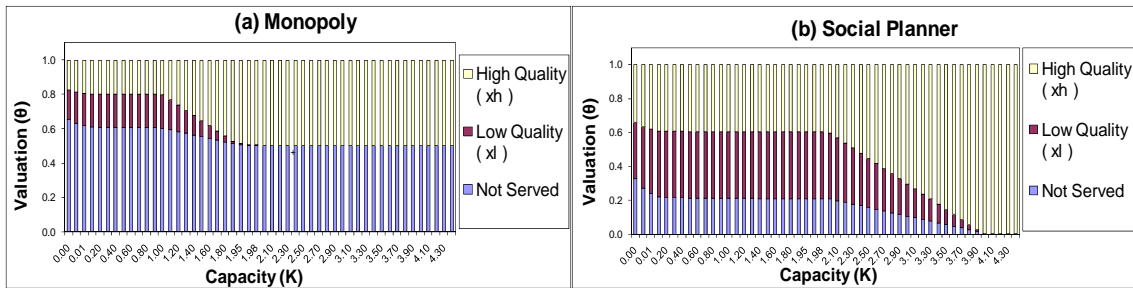
FIGURE 2.4: Market Coverage: Monopoly vs. Social Planner ($c(q) = 0.01q^{1/2}$, $s(q) = q^2$ and $\overline{q} = 2$)

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Figure 2.3.b). This happens because for a given capacity, the social planner can serve more customers at the low end by degrading the quality level at the high end.

The existing literature also shows that the monopoly chooses to serve a smaller market than that of a social planner. Figures 2.2.a and b, similar to Proposition 4, show that this result does not necessarily hold when the capacity constraint is taken into account. In this example, when the capacity is sufficiently small, the monopoly serves a larger market compared to a social planner. For example, in Figure 2.2 at $K = 0.50$, both firms utilize all their capacity and the monopolist serves the customers with valuation $\theta \geq 0.635$ whereas the social planner serves only the customers with valuation $\theta \geq 0.646$. In this case, the social planner offers only high quality product type while the monopolist offers both a high and a low quality product type. This helps the monopolist limit the amount of high quality product offered in the market and keep its prices high. On the other hand, social planner prefers offering only the high quality product type to a larger customer segment and consequently generating greater surplus for the customers. Although this leads to smaller market coverage under social planner, the total surplus of the customers is greater than the monopoly outcome.

In addition, Figures 2.1 and 2.2 show that a monopoly can lead to the socially efficient outcome due to the limited capacity. For example, when capacity $K = 0.2$, both the social planner and the monopolist offer only a single product type at quality $q_h = 2$, they both utilize their whole capacity and serve the same customer segment (i.e. $\theta \geq 0.86$). Notice that these findings are consistent with Proposition 5; although the quality levels are chosen endogenously.

Finally, Figures 2.1 and 2.3 also show that the quality of the products offered depend on the capacity level in a non-trivial way. The quality of the lower quality product can be decreasing in capacity as in Figure 2.1 or it can be increasing in capacity as in Figure 2.3.

CHAPTER 3

Vertically Differentiated Product Line Design Under Competition

In this chapter, we study the vertically differentiated market problem in duopoly and oligopoly markets. We analyze the impact of competitor entry and increasing number of firms on the optimal product line decisions of the firm in the existence of capacity limitations. We also discuss the social welfare effects in competitive settings.

3.1 Related Literature

Competition in vertically differentiated markets have been studied in different contexts. The oligopolistic setting where each firm offers only one distinct product has attracted a lot of attention from the researchers (e.g. Gabszewicz and Thisse (1980), Shaked and Sutton (1982), Gal-Or (1985), Moorthy (1988), Motta (1993), Wauthy (1996), Chambers et al. (2006), Jing (2006)). In addition, there is a line of research where the authors study the effects of vertical differentiation with the effects of horizontal differentiation and brand loyalty (e.g., Gilbert and Matutes (1993), Verboven (1999), Armstrong and Vickers (2001), Desai (2001), Doraszelski and Draganska (2006), Alderighi (2007)). In

our model, we consider multiproduct competition since firms have the options of offering multiple products in their product lines.

The competition in vertically differentiated multiproduct markets have been studied under two separate contexts: price-setting games and quantity-setting games. Fundamental result in the price setting games is that the symmetric firms do not offer symmetric product lines (e.g. Champsaur and Rochet (1989), Rochet and Stole (2002), Schmidt-Mohr and Villas-Boas (2008)). This approach fail to explain the head-to-head competition that is the dominant form of competition in most industries. In practice, firms offer similar products. Firms adjust their product lines in response to competitor entry and try to match the products already offered in the market rather than moving away from them as claimed in the price-setting games. We will follow the quantity game approach which has proved to be effective in explaining the head-to-head competition. We investigate the product line design and capacity allocation problem in a Cournot setting. Moreover, Kreps and Scheinkman (1983) have shown in a general model that the price competition would yield same outcomes as the quantity competition when the capacity is limited for the firms.

The fundamental result in the quantity setting games is that the symmetric firms offer symmetric product lines (Gal-Or (1983), DeFraja (1996), Johnson and Myatt (2003), Johnson and Myatt (2006a)). Although these few papers are related to our work, they do not take capacity limitations into account. The competition between capacity constrained multiproduct firms in vertically differentiated markets have not been studied in the literature. So, we investigate the product line design and capacity allocation problem

in a Cournot setting; extend the existing results and provide new insights for capacity constrained cases.

The competition between capacity constrained multiproduct firms in vertically differentiated markets have not been studied in the literature. Although the following few papers are related to our work, they do not take capacity limitations into account. Gal-Or (1983) presents an oligopolistic framework where she considers simultaneous quality and quantity competition among firms. She studies symmetric equilibria where the average quality supplied to the market decreases with entry of new firms. In contrast, in a duopoly setting, we show that when the capacity is limited for the firm and the high quality product has a better capacity-adjusted profit margin, average quality supplied is greater under competition.

DeFraja (1996) also studies the quantity competition in vertically differentiated markets. He aims to explain the head-to-head competition among firms. The fundamental result of this paper is that any equilibrium is symmetric if firms compete in quantities. We extend this result and show that the symmetric equilibrium is unique even for limited capacity cases when the market structure is a duopoly. DeFraja (1996) also shows that the total quantity supplied for any quality or higher increases with competition. Our findings for high capacity levels coincide with this result. However, when the capacity is limited; capacity adjusted profit of high quality product is greater and there is increasing cost to quality ratio, the total quantity supplied by the monopolist is greater than the total duopoly quantity. Another result discussed in DeFraja (1996) is that the high quality product is always supplied under competition. We find that this result no

longer holds when capacity is limited for the firm: if the capacity adjusted profit of low quality product is greater, then there exists a threshold capacity below which both firms optimally offer only the low quality product.

Johnson and Myatt (2003) and Johnson and Myatt (2006a) provide a comprehensive analysis of the multiproduct quality competition in duopoly and oligopoly settings and aim to explain how the firms adjust their product lines in response to competition. They consider the asymmetric cases as well as different marginal revenue and cost assumptions. When there is decreasing marginal revenues, they show that the incumbent firm does not produce distinct products than its monopoly solution in response to competition. For large capacity levels, our findings are in line with this result. However, when the capacity constraint is binding, the firm may have to adjust its product line in response to competition. The change could be in the form of pruning a product from the line or introducing new products depending on the relationships between size, cost and quality parameters. Johnson and Myatt (2003) and Johnson and Myatt (2006a) also show that if the firms are symmetric, then there exists a pure strategy equilibrium that is symmetric. When there is increasing returns to quality (in our model, decreasing cost to quality ratio), then both firms offer only the highest quality that they can produce. Our findings coincide with this result for high capacity levels. However, there exists a range of low capacity levels where the firms are better off offering only the low quality product when the capacity adjusted profit margin is greater for the low quality product.

3.2 Duopoly

In this chapter, the market structure is a duopoly; there are two firms, Y and Z, competing against each other. The firms simultaneously decide the amount of each product that will be offered in the market. They participate in a quantity (Cournot) competition and then the prices are used to clear the market. This setting is more appropriate for product line design problems with shared capacity where the firm has to decide how to allocate the production capacity among products long before the price games are played between firms. Moreover, the price competition fails to explain the head-to-head competition that exists in many industries. It is shown in the literature that a unique pure strategy equilibrium is not guaranteed under a price competition in a vertically differentiated market with multiproduct duopolists (e.g. Champsaur and Rochet (1989), Rochet and Stole (2002), Schmidt-Mohr and Villas-Boas (2008)). Even for the cases where an equilibrium exist, it is not symmetric. Nevertheless, in practice many firms offer multiple products and target same customer segments as their competitors rather than focusing on segments away from their competitors.

The firms, Y and Z have the same limited capacity (K). In this one-shot game, firms simultaneously decide how to allocate this capacity among the product offerings given the competitor's offerings and customers' self selection constraints. They set quantities, (y_h, y_l) and (z_h, z_l) respectively. Then the prices (p_h,p_l) are set to clear the demand of each product (D_h,D_l). Without loss of generality, we will restrict the analysis to $y_i + z_i = D_i$ as explained in Chapter 2. Following $y_i + z_i = D_i(p_h, p_l)$, the optimal prices of the products can be expressed as a function of capacity allocations y_i and z_i: $p_h(y_h, y_l, z_h, z_l)$

and $p_l(y_h, y_l, z_h, z_l)$ respectively.

3.2.1 Analysis

We will solve the problem for symmetric firms where the available technology and the business strategies of the firms do not put any restrictions on the menu of offerings: Both firms have the ability to choose and offer both products. The firm Y solves the following optimization problem given the best response function (z_h^*, z_l^*) of firm Z, self selection of the customers, and the available capacity K:

$$\max_{y_h, y_l \geq 0} \quad (p_h(y_h, y_l, z_h^*, z_l^*) - c_h)y_h + (p_l(y_h, y_l, z_h^*, z_l^*) - c_l)y_l$$

$$\text{subject to} \quad s_h y_h + s_l y_l \leq K$$

The objective function of this problem is jointly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)).

In the following paragraphs, we characterize the symmetric duopolists' optimal product line in a simultaneous game. We look for a pure strategy (Nash) equilibrium under various unit cost, quality and resource consumption assumptions. Propositions 6 and 7 describe the optimal policy when there are increasing cost to quality (i.e., $c_h/q_h > c_l/q_l$) and decreasing cost to quality (i.e., $c_h/q_h < c_l/q_l$) respectively.

In the literature, Gal-Or (1983), DeFraja (1996), and Johnson and Myatt (2003) have shown that a quantity competition in a vertically differentiated market yields a unique

symmetric equilibrium. Our findings extend this result: Unique symmetric equilibrium exists even for limited capacity levels. All proofs appear in the Appendix B. Additional threshold capacities ($\bar{K}_1^D - \bar{K}_5^D$ and $\bar{K}_1^{TD} - \bar{K}_5^{TD}$) are defined in Appendix B Equations (B-7)-(B-16) which will be useful for describing the firm's optimal policy.

Proposition 6 *Suppose $c_l/q_l < c_h/q_h$. The game has a unique, symmetric pure strategy Nash equilibrium as follows:*

    i) If $q_l - c_l \geq q_h - c_h$, then the optimal strategy for the duopolist firm is to focus and offer only the low quality product for all capacity levels.

ii.a) If $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, then

    - if $K \leq \bar{K}_1^D$, the optimal product line strategy for the duopolist firm is to focus and offer only the low quality product.

    - if $K > \bar{K}_1^D$, the optimal product line strategy for the duopolist firm is to differentiate and offer both products.

ii.b) If $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, then

    - if $K \leq \bar{K}_2^D$, the optimal product line strategy for the duopolist firm is to focus and offer only the high quality product.

    - if $K > \bar{K}_2^D$, the optimal product line strategy for the duopolist firm is to differentiate and offer both products.

Part (i) in Proposition 6 describes the trivial case where it is always better to focus on the economy class. Part (ii.a) and (ii.b) describe what happens when the maximum

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margin for the first class $q_h - c_h$ is greater. In this case, we find that the optimal product line of the duopolist firm depends on its capacity. It is a generalization of the result that is established for the monopoly case.

Johnson and Myatt (2003) characterize the solution for this problem with no capacity constraints and show that the optimal strategy for the firm is to differentiate and produce both products like the monopoly case. The firm offers low quality product together with the high quality product in order to price-discriminate among customers and make higher profits even under competitive threats. Our findings are in line with this result when the capacity is large enough. However, as we have shown for the monopoly and generalize it here for the duopoly that the firm's choice of product offerings could be different when the capacity is limited. Then, the firm should choose the product that potentially brings more profit per unit resource consumed which may not be the high quality product, even under competitive threats.

It is also suggested in the literature that the high quality product is always offered in a duopoly market (DeFraja (1996), Johnson and Myatt (2003)). In contrast, we show that focusing on the low quality product may be the optimal strategy when the firm is capacity constrained and the potential profit per unit resource consumed is greater for the low quality product.

Proposition 7 *Suppose $c_h/q_h \leq c_l/q_l$. The game has a unique, symmetric pure strategy Nash equilibrium as follows:*

i) If $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, then

    - if $K \leq \bar{K}_1^D$, the optimal product line strategy for the duopolist firm is to focus

and offer only the low quality product.

- *if $\bar{K}_1^D < K \le \bar{K}_2^D$, the optimal product line strategy for the duopolist firm is to differentiate and offer both products.*

- *if $K > \bar{K}_2^D$, the optimal product line strategy for the duopolist firm is to focus and offer only the high quality product.*

ii) If $\frac{q_h - c_h}{s_h} \ge \frac{q_l - c_l}{s_l}$, then the optimal strategy for the duopolist firm is to focus and offer only the high quality product for all capacity levels.

Proposition 7 generalizes the result to competitive markets that is discussed for the monopoly case in Proposition 2. Here, we characterize the duopolist firm's optimal policy when there are decreasing costs to quality i.e., $c_h/q_h \le c_l/q_l$. In this case, maximum margin for the first class $q_h - c_h$ is always greater. The result in the literature is to focus and offer only the high quality product under these conditions (DeFraja (1996), Johnson and Myatt (2003)). Our findings are consistent with this result: when the capacity is large enough, the firm should focus and offer only the high quality product even under competition. Nevertheless, we also show that when the capacity is limited, this result no longer holds for some cases. Part (i) describes the case where the potential profit per unit resource consumed is greater for the low quality product. The resources become more important than the self selection and cannibalization hazards; the firm has to focus on the low quality product as opposed to the high quality product and slowly introduce the high quality product to the market. In this case, the firm that disregards the capacity constraint would be offering the wrong product to the market. Part (ii) describes the case where it is always better to focus on the first class.

3.2.2 Effects of Firm Entry

There are many cases in practice where a firm changes its product line when another firm enters the market. These effects of competitor entry on the product line decisions of an incumbent firm are widely discussed in the product line design literature (e.g. DeFraja (1996), Johnson and Myatt (2003)). The incumbent firm either offers new products that are not sold otherwise or deletes some products from its product line. The situation we study in this section is as follows: At the beginning, the incumbent firm is a monopoly. Its solution is as provided in Section 2.3. Then, a symmetric firm enters the market. We assume there are no barriers or fixed costs for entry and the market is stable with these two firms. In response to entry from this new firm, the incumbent firm adjusts its product line. In the following paragraphs, we will compare the incumbent firm's solution under competition to its monopoly solution and try to understand the effects of firm entry on the firm's optimal product line. The market size is kept the same whereas the total capacity in the market doubles with the symmetric entrant acting in the market.

Johnson and Myatt (2003) aim to explain the product line pruning and emergence of new products in response to entry. They provide many examples from a variety of industries including computer hardware, airlines and the market for watches where the firms adjust their product lines in response to entry from other firms. They explain these phenomena through different assumptions for revenue and technology functions. We show that product line pruning and emergence of new products can also be explained by the capacity limitations of the firms.

Proposition 8 *When $(q_h - c_h > q_l - c_l)$, and $(q_h - c_h)/s_h < (q_l - c_l)/s_l$, the incumbent*

firm introduces the high quality product to the product mix in response to entry for $\bar{K}_1^D < K < \bar{K}_1^M$ when there is increasing cost to quality ratio; and for $\bar{K}_1^D < K < \min\{\bar{K}_1^M, \bar{K}_2^D\}$ when there is decreasing cost to quality ratio.

The condition in Proposition 8 covers the instances where the potential profit per unit resource consumed by low quality product is greater. Then, the incumbent firm is better off focusing on the low quality product when it has very limited capacity both as a monopolist and as a duopolist. The thresholds are $\bar{K}_1^M$ and $\bar{K}_1^D$ respectively. The supply is so low below these thresholds such that the focus of the firm is to produce as much as it can and keep the prices as high as possible for the available supply in the market. The prices are too high for the products such that the demand for the high quality product diminishes below these thresholds. However, in the case of duopoly, prices drop a lot faster than in the case of monopoly both due to capacity increase and competitive pressures. With the falling prices, the demand for the high quality product starts to establish at $\bar{K}_1^D$ for the firm under competition and at only $\bar{K}_1^M$ for the firm when it acts as a monopolist. We note that when there is decreasing cost to quality ratio, the firm ceases the low quality production all together at a second capacity threshold ($\bar{K}_2^M$, $\bar{K}_2^D$ respectively where $\bar{K}_2^D < \bar{K}_2^M$). Thus, for the range of capacities presented in the Proposition 8, the firm's best response to competition is to introduce high quality product to the market due to the price decrease as well as the supply increase. In a way, effect of entry of a competitor is relaxing the capacity constraint a little for the firm. The response of the firm may also be introducing low quality products to the market.

Proposition 9 *When $(q_h - c_h)/s_h > (q_l - c_l)/s_l$ and there is increasing cost to quality*

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ratio, the incumbent firm introduces the low quality product to the product mix in response to entry for $\bar{K}_2^D < K < \bar{K}_2^M$.

The condition in Proposition 9 covers the case when there is increasing cost to quality ratio and the potential profit per unit resource consumed by high quality product is greater. Then, the firm sells high quality product for high prices and introduce low quality product when there is enough capacity and demand in the market. Then, the thresholds are $\bar{K}_1^M$ and $\bar{K}_1^D$ respectively for the firm acting as a monopoly and the firm under competition. The supply is so low below these thresholds such that the focus of the firm is to produce as much as it can and keep the prices as high as possible for the available supply in the market. The capacity is so tight that the no capacity remains for the low quality product below these thresholds. However, in the case of duopoly, capacity increases with the entry of the competitor increasing the available supply in the market which eventually drives the prices down a lot faster. With the falling prices, the demand for the low quality product starts to establish at $\bar{K}_2^D$ for the firm under competition and at only $\bar{K}_2^M$ for the firm when it acts as a monopolist. Thus, for the range of capacities presented in the Proposition 9, the firm's best response to competition is to introduce low quality product to the market. Another response of firms may be pruning the products from the product lines.

Proposition 10 *When $(q_h - c_h)/s_h < (q_l - c_l)/s_l$ and there is decreasing cost to quality ratio, the incumbent firm prunes the low quality product from the product mix in response to entry for $\bar{K}_2^D < K < \bar{K}_2^M$.*

When there is decreasing cost to quality ratio, if the firm has large enough capacity, the optimal solution is to focus and offer only the high quality product both under monopoly and duopoly markets. However, if the capacity is below a threshold level, the firm has to offer the low quality product in addition due to capacity limitations. The thresholds are $\bar{K}_2^M$ and $\bar{K}_2^D$ respectively for the firm acting as a monopoly and the firm under competition. For the range of capacities given in Propositions 10 if the firm keeps low quality product even in the existence of competitor, it will only result in the cannibalization of demand for the high quality product due to the excess supply in the market. Thus, the firm ceases the production of low quality product at those capacities in response to competitor entry.

Under the conditions presented in Propositions 8, 9, 10 the literature suggests that the firm does not produce any distinct products than the monopoly solution in response to entry (Johnson and Myatt (2003)). Similarly, in an oligopolistic framework, DeFraja (1996) shows that number of products supplied in the market is non-increasing in the number of firms entering the market. Our findings in a duopoly setting coincide with these results when the capacity is large enough. However, for limited capacity levels, the monopolist may choose to adjust its product line in response to entry. This adjustment could be in the form of pruning some products from the product line or introducing new products. There are two drivers behind these actions: one is the competition and the other is the increased capacity in the market. With the entrant in the market, prices decrease and demand increases for the products as well as the available supply. While in the monopoly market, the firm would be still concerned about the efficient use of

resources due to limited capacity levels; in the duopoly market, the firm has to share the same market with its competitor. More supply is available to the market. With this increased supply, the firm is more concerned with price discrimination policies compared to its strategy as a monopolist.

3.2.3 Effects of Capacity Consolidations

In this section, we will discuss pure effects of competition on the product line decisions of firms through a discussion of firm mergers. Mergers and acquisitions are relevant to many industries. The merger between two firms causes the elimination of the competition from the market. We will compare the solutions of two duopolists to the solution of an integrated firm formed by the merger of these two firms. Hence, the change in the product line could be explained solely by the elimination of competitive pressures. The effect of a merger between two competitive firms can be introducing or pruning some products from the product line.

Proposition 11 *When $(q_h - c_h > q_l - c_l)$, and $(q_h - c_h)/s_h < (q_l - c_l)/s_l$, the integrated firm introduces the high quality product to the product mix whereas the non-cooperative duopolists do not offer it for the total market capacity levels of $\bar{K}_1^M < K < \bar{K}_1^{TD}$ when there is increasing cost to quality ratio; and for the total market capacity levels of $\bar{K}_1^M < K < \min\{\bar{K}_2^M, \bar{K}_1^{TD}\}$ when there is decreasing cost to quality ratio.*

Under the conditions presented in Proposition 11, both the non-cooperative duopolists and the integrated monopolist optimally focus on the low quality product below a specific threshold, $\bar{K}_1^{TD}$ and $\bar{K}_1^M$ respectively. However, the competitive pressure leads the

duopolist firms increase the supply for a given product (low quality product in this case) and drive the prices down. The integrated monopolist has the power to keep the total supply under control and not let the prices go inefficiently low. This way the monopolist can introduce the high quality product and price-discriminate among the customers for lower capacity levels than the non-cooperative duopolists. When there is decreasing cost to quality ratio, the firms actually cease the low quality production all together at a second capacity threshold ($\bar{K}_2^M, \bar{K}_2^{TD}$ respectively where $\bar{K}_2^M < \bar{K}_2^{TD}$). Thus, for the range of capacities presented in the Proposition 11, the elimination of competition leads the firms to introduce the high quality product to the market. In other words, competition requires greater capacity availability to have the price-discrimination policies come into effect. The result of the elimination of competition may also be introducing low quality products to the market.

Proposition 12 *When $(q_h - c_h)/s_h > (q_l - c_l)/s_l$ and there is increasing cost to quality ratio, the integrated firm introduces the low quality product to the product mix whereas the non-cooperative duopolists do not offer it for $\bar{K}_2^M < K < \bar{K}_2^{TD}$.*

The condition in Proposition 12 covers the case when there is increasing cost to quality ratio and the potential profit per unit resource consumed by high quality product is greater. Then, the firms sell high quality product for high prices and introduce low quality product when there is enough capacity and demand in the market. Then, the thresholds are $\bar{K}_2^{TD}$ and $\bar{K}_2^M$ respectively for the non-cooperative duopolists and the integrated firm. The competitive pressure leads the duopolist firms increase the supply for the high quality product and drive the prices down. The integrated monopolist has

the power to keep the total supply under control and not let the prices go inefficiently low. This way the monopolist can introduce the low quality product and keep the price high for the high quality product while increasing the overall demand by introducing the low quality product to the market. Thus, for the range of capacities presented in the Proposition 12, the elimination of competition leads the firms to introduce the low quality product to the market. The result of the elimination of competition may also be pruning low quality products from the product line.

Proposition 13 *When $(q_h - c_h)/s_h < (q_l - c_l)/s_l$ and there is decreasing cost to quality ratio, the integrated firm prunes the low quality product from the product mix whereas the non-cooperative duopolists continue to offer it for $\bar{K}_2^M < K < \bar{K}_2^{TD}$.*

Under the conditions presented in Proposition 13, below the total market capacity threshold $\bar{K}_2^M$, the firms optimally offer both products to the market. Again, the competitive pressure forces the firms to drive prices down and offer inefficiently high supply to the market. On the other hand, the integrated firm can keep the prices and the supply at the optimal level. Thus, for the range of capacities given in Proposition 13, the integrated firm ceases the production of the low quality product, decrease the total supply and increase the prices when the competition is eliminated from the market.

When there is increasing cost to quality ratio and the capacity adjusted profit margin is greater for the high quality product, the total quantity sold by the integrated firm is greater than the total quantity sold by the non-cooperative duopolists for a range of capacity levels. Moreover, average quality may increase with competition as opposed to the results presented in the literature.

Proposition 14 *When* $(q_h - c_h)/s_h > (q_l - c_l)/s_l$ *and there is increasing cost to quality ratio,* $\sum_i x_i > \sum_i y_i + z_i$ *for* $\bar{K}_2^M < K < \min\{\bar{K}_3^M, \bar{K}_2^{TD}\}$. *Moreover, the average quality of the products supplied increases as a result of the competition.*

We showed that the actual amount of products supplied in the market is greater for the monopolist. This is in contrast with the literature where the monopoly quantity is shown to be smaller than the total duopoly quantity (DeFraja (1996)). In addition, we show that the average quality of the products supplied under the integrated firm is less than the average quality of the products supplied in the duopoly market. This is in contrast to Gal-Or (1983) who shows that the average quality supplied to the market decreases with entry of new firms. When the capacity and resource consumptions are taken into account, under the conditions presented in Proposition 14, the competition forces the duopolist firms decrease prices lower than most profitable levels and no capacity is left to price-discriminate the customer base. However, the integrated monopolist has the power to keep the prices high and use the remaining capacity for low quality production. Since both the duopolists and the integrated monopolist are capacity constrained at the given range of capacities and low quality product consumes less resources, the actual amount of products supplied in the market is greater for the monopolist.

3.3 Oligopoly

In this section, the market structure is an oligopoly; there are $n \geq 2$ firms. The firms simultaneously decide the amount of each product that will be offered in the market. They participate in a quantity (Cournot) competition and then the prices are used to

clear the market. The firms have the same limited capacity (K). Since the market size is normalized to the unit mass, the capacity parameter in this formulation could also be regarded as the capacity-to-market size ratio. In the thesis, limited capacity should be perceived as small capacity relative to the market size.

In this one-shot game, firms simultaneously decide how to allocate their capacity among the product offerings given the competitors' offerings and customers' self selection constraints. The available technology and the business strategies of the firms do not put any restrictions on the menu of offerings: All firms have the ability to choose and offer both products. The firm W sets quantities, $(w_h$ and $w_l)$ given the best response functions $(t(-w)_h^*, t(-w)_l^*)$ of all firms except W, self selection of the customers, and the available capacity K. Then the prices (p_h, p_l) are set to clear the demand of each product (D_h, D_l). Since this is a single period game with no inventory or backlog considerations and since the capacity allocation is a costly action, we will restrict the analysis to $D_i = t(-w)_i^* + w_i^* = t_i^*$ without loss of generality. Then, the optimal prices of the products can be expressed as a function of capacity allocations: $p_h(w_h^*, w_l^*, t(-w)_h^*, t(-w)_l^*)$ and $p_l(w_h^*, w_l^*, t(-w)_h^*, t(-w)_l^*)$ respectively. Then the firm W solves the following optimization problem given the best response functions and self selection of the customers subject to the available capacity K:

$$\max_{w_h, w_l \geq 0} \quad (p_h(w_h, w_l, t(-w)_h^*, t(-w)_l^*) - c_h)w_h + (p_l(w_h, w_l, t(-w)_h^*, t(-w)_l^*) - c_l)w_l$$

subject to $\quad s_h w_h + s_l w_l \leq K$

We look for a symmetric pure strategy Nash equilibrium under various unit cost, quality and resource consumption assumptions. Since the objective function of this problem is jointly concave on a convex set defined by linear constraints, the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)). In the following paragraphs, we characterize the symmetric oligopolists' optimal product line in a simultaneous game. Propositions 15 and 16 describe the optimal policy when there are increasing cost to quality (i.e., $c_h/q_h > c_l/q_l$) and decreasing cost to quality (i.e., $c_h/q_h < c_l/q_l$) respectively. Additional threshold capacities that depend on the number of firms (n) ($\bar{K}_1^{(n)} - \bar{K}_5^{(n)}$ and $\bar{K}_1^{T(n)} - \bar{K}_5^{T(n)}$) are defined in Appendix B Equations (B-17)-(B-26) which will be useful for describing the firm's optimal policy.

Proposition 15 *Suppose $c_l/q_l < c_h/q_h$. The game has a symmetric pure strategy Nash equilibrium as follows:*

  i) If $q_l - c_l \geq q_h - c_h$, then the optimal strategy for the oligopolist firm is to focus and offer only the low quality product for all capacity levels.

ii.a) If $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, then

  - *if $K \leq \bar{K}_1^{(n)}$, the optimal product line strategy for the oligopolist firm is to focus and offer only the low quality product.*

  - *if $K > \bar{K}_1^{(n)}$, the optimal product line strategy for the oligopolist firm is to differentiate and offer both products.*

ii.b) If $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, then

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- if $K \leq \bar{K}_2^{(n)}$, the optimal product line strategy for the oligopolist firm is to focus and offer only the high quality product.

- if $K > \bar{K}_2^{(n)}$, the optimal product line strategy for the oligopolist firm is to differentiate and offer both products.

Part (i) in Proposition 15 describes the trivial case where it is always better to focus on the low quality product. Part (ii.a) and (ii.b) describe what happens when the maximum margin for the high quality product $q_h - c_h$ is greater. In this case, we find that the optimal product line of the oligopolist firm depends on its capacity.

Johnson and Myatt (2003) and Johnson and Myatt (2006a) characterize the solution for this problem with no capacity constraints and show that the optimal strategy for the firm is to differentiate and produce both products. The firm offers low quality product together with the high quality product in order to price-discriminate among customers and make higher profits. Our findings are in line with this result when the capacity is large enough. Firms choose to price discriminate to keep the prices of the high quality product high but at the same time benefit from the low costs and high demand for the low quality product.

However, the proposition suggest that the firm's choice of product offerings could be different when the capacity is limited. For low capacity levels, the firm should choose the product that potentially brings more profit per unit resource consumed which may not be the high quality product. Below a threshold capacity level, the economic force leading the firm's decision is no longer the benefits of price discrimination nor the competitive threats: it is the intrafirm competition of the products for the limited resources. The

resources of the firm is so scarce that the demand even for all those firms is high enough such that all that can be produced will surely be sold. At that point the firm should decide which product has more potential to bring more profit with the limited resources at hand and make that decision even if it means ceasing the production of the high quality product.

Proposition 16 *Suppose $c_h/q_h \leq c_l/q_l$. The game has a symmetric pure strategy Nash equilibrium as follows:*

i) If $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, then

- *if $K \leq \bar{K}_1^{(n)}$, the optimal product line strategy for the oligopolist firm is to focus and offer only the low quality product.*

- *if $\bar{K}_1^{(n)} < K \leq \bar{K}_2^{(n)}$, the optimal product line strategy for the oligopolist firm is to differentiate and offer both products.*

- *if $K > \bar{K}_2^{(n)}$, the optimal product line strategy for the oligopolist firm is to focus and offer only the high quality product.*

ii) If $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, then the optimal strategy for the oligopolist firm is to focus and offer only the high quality product for all capacity levels.

Proposition 16 characterizes the oligopolist firm's optimal policy when there are decreasing costs to quality i.e., $c_h/q_h \leq c_l/q_l$. In this case, maximum margin for the high quality product $(q_h - c_h)$ is always greater. The result in the literature is to focus and offer only the high quality product under these conditions (DeFraja (1996), Johnson and Myatt (2003), Johnson and Myatt (2006a)). Our findings are consistent with this result:

when the capacity is large enough, the firm should focus and offer only the high quality product. In this case, the cannibalization hazards outweighs the benefits of price discrimination and the firms choose to eliminate the low quality product from their product lines. High quality product wins the intrafirm competition for demand due to higher expected profits.

Nevertheless, we also show that when the capacity is limited, this result no longer holds for some cases. Part (i) describes the case where the potential profit per unit resource consumed is greater for the low quality product. Economic forces that lead the firms' decisions change below $\bar{K}_2^{(n)}$. At that capacity level, the firm's solution is already capacity constrained which means that the demand is high enough for the industry such that all production will surely be sold. Then, the intrafirm competition starts to prevail. We show that below $\bar{K}_2^{(n)}$, the intrafirm competition for the resources is the economic force behind the firms' actions. The firm has to offer low quality product which consumes less resources together with the high quality product. The firm has to benefit from the economies of scale that could be realized with the production of the low quality product. Moreover, we show that there exists another threshold $\bar{K}_1^{(n)}$ belwo which the firm has no longer have the luxury to offer high profit high quality product. When the capacity is so limited, the firm is better off offering only the low quality product. Low quality product wins the competition for resources in spite of the lower unit profits due to higher potential profits per unit resource consumed. In this case, remember that the firm that disregards the capacity constraint would be offering only the high quality product which is the wrong product to the market. Part (ii) describes the case where it is always better

to focus on the high quality product.

3.3.1   Effects of Competition From a Firm's Point of View

We will look for the generalization of the results that we have obtained for the duopoly case. The situation we study in this section is as follows: At the beginning, the firm is competing with (n-1) firms; hence, there are n firms in the market with capacity K. Then, another symmetric firm enters the market; now there are (n+1) firms in the market with capacity K. The market size is kept the same whereas the total capacity in the market increases with the symmetric entry to the market. In response to entry from this new firm, we try to understand whether the firms adjust their product lines. In the following paragraphs, we will compare the firm's solution under competition with (n-1) firms to its solution under competition with n firms.

Proposition 17 *When $(q_h - c_h > q_l - c_l)$, and $(q_h - c_h)/s_h < (q_l - c_l)/s_l$, the firm introduces the high quality product to the product mix in response to increasing competition for $\bar{K}_1^{(n+1)} < K < \bar{K}_1^{(n)}$ when there is increasing cost to quality ratio; and for $\bar{K}_1^{(n+1)} < K < \min\{\bar{K}_1^{(n)}, \bar{K}_2^{(n+1)}\}$ when there is decreasing cost to quality ratio.*

The condition in Proposition 17 covers the instances where the potential profit per unit resource consumed by low quality product is greater. Then, the firm is better off focusing on the low quality product when it has very limited capacity in both cases. The thresholds are $\bar{K}_1^{(n)}$ and $\bar{K}_1^{(n+1)}$ respectively. The supply is so low below these thresholds such that the focus of the firm is to produce as much as it can. However, in the case with n+1 firms, prices drop a lot faster than in the case with n firms both due to increasing

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capacity and competitive pressures. We note that when there is decreasing cost to quality ratio, the firm ceases the low quality production all together at a second capacity threshold ($\bar{K}_2^{(n)}, \bar{K}_2^{(n+1)}$ respectively where $\bar{K}_2^{(n+1)} < \bar{K}_2^{(n)}$). Thus, for the range of capacities presented in the Proposition 17, the firm's best response to increasing competition is to introduce high quality product to the market due to the price decrease as well as the supply increase. In a way, effect of more competition is relaxing the capacity constraint a little for the firm. The response of the firm may also be introducing low quality products to the market.

Proposition 18 *When $(q_h - c_h)/s_h > (q_l - c_l)/s_l$ and there is increasing cost to quality ratio, the firm introduces the low quality product to the product mix in response to increasing competition for $\bar{K}_2^{(n+1)} < K < \bar{K}_2^{(n)}$.*

The condition in Proposition 18 covers the case when there is increasing cost to quality ratio and the potential profit per unit resource consumed by high quality product is greater. Then, the firm sells high quality product for high prices and introduce low quality product when there is enough capacity and demand in the market. Then, the thresholds are $\bar{K}_1^{(n)}$ and $\bar{K}_1^{(n+1)}$ respectively for the firm in two markets. The capacity is so tight that the no capacity remains for the low quality product below these thresholds. However, in the case of n+1 firms, capacity increases with the entry of new firm increasing the available supply in the market which eventually drives the prices down a lot faster. With the falling prices, the demand for the low quality product starts to establish at $\bar{K}_2^{(n+1)}$. Thus, for the range of capacities presented in the Proposition 18, the firm's best response to increasing competition is to introduce low quality product to the market.

Another response of firms may be pruning the products from the product lines.

Proposition 19 *When $(q_h - c_h)/s_h < (q_l - c_l)/s_l$ and there is decreasing cost to quality ratio, the firm prunes the low quality product from the product mix in response to increasing competition for $\bar{K}_2^{(n+1)} < K < \bar{K}_2^{(n)}$.*

When there is decreasing cost to quality ratio, if the firm has large enough capacity, the optimal solution is to focus and offer only the high quality product. However, if the capacity is below a threshold level, the firm has to offer the low quality product in addition due to capacity limitations. The thresholds are $\bar{K}_2^{(n)}$ and $\bar{K}_2^{(n+1)}$ respectively.

Under the conditions presented in Propositions 17, 18, 19, DeFraja (1996) shows that number of products supplied in the market is non-increasing in the number of firms entering the market. Our findings coincide with these results when the capacity is large enough. However, for limited capacity levels, the firm may choose to adjust its product line in response to increasing competition. This adjustment could be in the form of pruning some products from the product line or introducing new products. There are two drivers behind these actions: one is the competition and the other is the increased capacity in the market. With the new firm in the market, prices decrease and demand increases for the products as well as the available supply. We also note that the thresholds have greater impact when n is small. The effect of increasing competition diminishes as n increases.

In general, the supply of a single firm decreases as new firms enter the market. We find that under some conditions, individual supply of the high quality product may actually increase as the number of firms increases in the market.

Proposition 20 *When $(q_h - c_h)/s_h < (q_l - c_l)/s_l$, $\partial_n w_h^* \geq 0$ for $0 < K < \bar{K}_3^{(n)}$ when $c_h/q_h > c_l/q_l$; and for $0 < K < \bar{K}_4^{(n)}$ when $c_h/q_h \leq c_l/q_l$.*

In particular, when the potential profit per unit resource consumed by high quality product is smaller than the low quality product, the threshold is $\bar{K}_3^{(n)}$ when there are increasing costs to quality; and it is $\bar{K}_4^{(n)}$ when there are decreasing costs to quality. Below either threshold, the dominating economic force is the efficient allocation of resources. We would expect that the increasing competition should decrease the production for both products. The competition is tougher on the more efficient product, i.e. low quality product. The individual production of the low quality product decreases because the whole industry supply increases and prices decrease as well. With the decrease in the production of one product, the firm has excess resources that it can dedicate for production of the less preferred product, i.e. the high quality product. As a result, individual supply of the high quality product increases as the number of firms increases in the market. We also find that individual supply of the low quality product may also increase as the number of firms increase in the market.

Proposition 21 *When $(q_h - c_h)/s_h > (q_l - c_l)/s_l$ and $c_h/q_h > c_l/q_l$, $\partial_n w_l^* \geq 0$ for $0 < K < \bar{K}_3^{(n)}$.*

On the other hand, when the potential profit per unit resource consumed by low quality product is smaller than the high quality product, the threshold is $\bar{K}_3^{(n)}$ when there are increasing costs to quality; and the low quality production is no longer optimal when there are decreasing costs to quality. With the similar argument, the competition is tougher on the more efficient product, i.e. high quality product. The individual

production of the high quality product decreases because the whole industry supply increases and prices decrease as well. With the decrease in the production of one product, the firm has excess resources that it can dedicate for production of the less preferred product, i.e. the low quality product. As a result, individual supply of the low quality product increases as the number of firms increases in the market.

3.3.2 Effects of Competition on Industry Supply

We discuss the effects of market concentration on the product line decisions of firms. In this case, the total market capacity is kept constant at K in order to have a fair comparison on the total supply quantities between different concentrations of the market. In this setting, if there are n firms, then each firm has capacity K/n. When the number of firms acting in the market increases, market concentration decreases and each firm has smaller capacity. The following propositions discuss the effect of changing market concentration on the product variety. The effect of increasing the market concentration can be introducing or pruning some products from the market.

Proposition 22 *When $(q_h - c_h > q_l - c_l)$, and $(q_h - c_h)/s_h < (q_l - c_l)/s_l$, the n+1 firm market do not offer high quality product whereas the n-firm market offer it for the total capacity levels of $\bar{K}_1^{T(n)} < K < \bar{K}_1^{T(n+1)}$ when there is increasing cost to quality ratio; and for the total capacity levels of $\bar{K}_1^{T(n)} < K < \min\{\bar{K}_2^{T(n)}, \bar{K}_1^{T(n+1)}\}$ when there is decreasing cost to quality ratio.*

Under the conditions presented in Proposition 22, both markets optimally focus on the low quality product below a specific threshold $\bar{K}_1^{T(n)}$. Below this threshold, firms do

not have the luxury to think about the benefits of segmentation but they need to use the limited resources most efficiently and choose to focus on the product that potentially brings more profit per unit resource it consumes, i.e. low quality product. However, when the n-firm market reaches $\bar{K}_1^{T(n)}$ total capacity level, the firms find it more profitable to start segmenting the market and offer high quality product together with the low quality product. For the (n+1)-firm market, the competitive forces dominate the decision process leading to an increase in the current supply of the low quality product further. This increase in the total supply leaves no room for the high quality production in terms of resources. The firms do not start the production of the high quality product if total capacity is less than $\bar{K}_1^{T(n+1)}$. When there is decreasing cost to quality ratio, since the industry supply of low quality product is zero beyond $\bar{K}_2^{T(n)}$ for the n-firm market and the order of $\bar{K}_2^{T(n)}$ and $\bar{K}_1^{T(n+1)}\}$ is ambiguous, the upper limit of the capacity range is presented as the minimum of the two. Thus, due to increasing interfirm competitive pressures (decreasing market concentration), the product variety offered in the market is different for the ranges presented in Proposition 22. In this case, competitive forces delay the high quality production. The result of the increasing competition may also be delaying the low quality production as presented in the following proposition.

Proposition 23 *When $(q_h - c_h)/s_h > (q_l - c_l)/s_l$ and there is increasing cost to quality ratio, the n+1 firm market do not offer the low quality product whereas the n-firm market offers it for $\bar{K}_2^{T(n)} < K < \bar{K}_2^{T(n+1)}$.*

The condition in Proposition 23 covers the case when there is increasing cost to quality ratio and the unit profit as well as the potential profit per unit resource consumed by

high quality product is greater. Then, the firms sell high quality product for efficient use of resources and introduce low quality product when there is enough capacity compared to the demand in the market. The specific thresholds for n-firm and (n+1)-firm markets are $\bar{K}_2^{T(n)}$ and $\bar{K}_2^{T(n+1)}$ respectively. The increasing competitive pressure leads the individual firms decrease the production for the high quality product while the total supply increase in the industry which drives the prices down. The less competitive market has comparatively more power to keep the total supply under control. This way they can keep the price for the high quality product higher while increasing the overall demand by introducing the low quality product to the market for the remaining capacity. Thus, for the range of capacities presented in the Proposition 23, competitive forces delay the low quality production. In contrast, the result of decreasing market concentration may also be increasing the product variety as presented in the following proposition.

Proposition 24 *When $(q_h - c_h)/s_h < (q_l - c_l)/s_l$ and there is decreasing cost to quality ratio, the n+1 firm market offer the low quality product whereas the n-firm market prunes it for $\bar{K}_2^{T(n)} < K < \bar{K}_2^{T(n+1)}$.*

Under the conditions presented in Proposition 24, below the total market capacity threshold $\bar{K}_2^{T(n)}$, the firms in both markets optimally offer both products to the market. In that case, the intrafirm competition for the resources is the economic force behind the firms' actions as presented in Proposition 16. The firm has to offer low quality product which consumes less resources together with the high quality product regardless of the fact that cost to quality ratio favors the high quality product in the market. Beyond $\bar{K}_2^{T(n)}$, n-firm market discontinues the low quality production and profitably focus on the

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high quality product. Under the set of conditions presented in Proposition 24, when the demand becomes small relative to the available capacity, the optimal decision for the firm is to focus on the high quality production. Beyond the capacity threshold $\bar{K}_2^{T(n+1)}$, the leading economic force is the fact that the low quality may cannibalize the demand for the high quality product. However, greater competition in the (n+1) firm market results in lower prices which leads the firms to increase the production for the the product that potentially brings more profit per unit resource it consumes, i.e. low quality product. This behavior in turn delays the firms' decision to cease low quality production. This in turn leads to greater product variety for the range of capacities presented in the Proposition 24 due to decreasing market concentration.

In the standard textbook explanation of the effects of market concentration, it says that the total market supply of a product increases as the number of firms acting in the market increases for homogeneous goods industries. It also mentions that the market clearing prices of the products decrease as the market concentration decreases. The literature investigating vertically differentiated industries has found similar results. DeFraja (1996) shows that the total quantity supplied for each quality and higher quality products increase and the price of each quality product strictly decreases with entry. Johnson and Myatt (2006a) also find an expansion of total supply of each quality and higher quality products and reduction in each of the prices of the products in a symmetric oligopoly. They also mention that under some conditions the low quality supplies may fall; but this only leaves room for more increase in the supply of higher qualities. Thus, in their setting, in a two product market, the supply of the high quality product would never

decrease as the number of firms increase. However, neither of these papers look at the capacity constrained case. In the following propositions, we show that industry supply of a particular quality type may decrease and particular price of a product type may increase as the number of firms increase in the market as opposed to the established wisdom in the literature.

Proposition 25 *Suppose $(q_i - c_i)/s_i < (q_j - c_j)/s_j$. There exists $\hat{K} > 0$ such that t_i^* is non-increasing in n for $K < \hat{K}$ at the equilibrium.*

In particular, when the potential profit per unit resource consumed by high quality product is smaller than the low quality product, the threshold is $\bar{K}_3^{T(n)}$ when there are increasing costs to quality; and it is $\bar{K}_4^{T(n)}$ when there are decreasing costs to quality. Below either threshold, the dominating economic force is the efficient allocation of resources. At this range of capacity, there is enough demand to sell all the production that the firm is capable of. We would expect that the increasing competition should increase the industry supply for both products. However, since the capacity constrained is binding, there is no excess resource to allocate in order to accommodate that. At that point, the supply of the more efficient product, i.e. low quality product is preferred to be increased with the trade off of decreasing the less efficient product, i.e. high quality product. On the other hand, when the potential profit per unit resource consumed by low quality product is smaller than the high quality product, the threshold is $\bar{K}_3^{T(n)}$ when there are increasing costs to quality; and the low quality production is no longer optimal when there are decreasing costs to quality. With the similar argument, the supply of the more efficient product, i.e. high quality product is preferred to be increased with

the trade off of decreasing the less efficient product, i.e. low quality product wherever possible.

Suppose that on a flight leg from city E to city F, there are 2 airlines competing for the demand of 1,000 customers per day with a combined capacity of 4,000 sq ft. The customer valuations are such that the highest amount of money any customer would pay is $3,000 for the business class seat and $1,000 for the economy class seat. Also suppose that the interior design of this airline's aircrafts is such that the business class seat occupies 20 sq. ft. and the economy class seat occupies 5 sq. ft. inside the aircraft. It costs $1,500 to operate a business class seat and $300 an economy class seat. Assuming both airlines are rational and implement the optimal strategy, the customers get 139 business class seats and 242 economy class seats offered in the market in total. Now, suppose that on a flight leg from city G to city H, there are 3 airlines competing with a combined capacity of 4,000 sq ft. Assume that all other conditions are equal to city pair E-F in the market. In this lower concentration market, the customers get 130 business class seats and 282 economy class seats. While the only difference is the number of firms in the market, the results have changed. The impact is not trivial: We observe that while the number of economy class seats increased, the number of business class seats have decreased as opposed to the conventional wisdom in the literature.

The acquired wisdom maintains that the market clearing price of a particular product decreases as the number of firms acting in that market increase. We find that under some conditions, price of a product may actually increase as the number of firms increase in the market as presented in the following proposition.

Proposition 26 *When $c_h/q_h > c_l/q_l$ and $q_h/s_h > q_l/s_l$,*

i) If $(q_h - c_h)/s_h < (q_l - c_l)/s_l$, then p_h^* is non-decreasing in n for $0 < K < \bar{K}_3^{T(n)}$.

ii) If $(q_h - c_h)/s_h > (q_l - c_l)/s_l$, then p_l^* is non-decreasing in n for $0 < K < \bar{K}_3^{T(n)}$.

The main condition in Proposition 26 covers the case when the cost ratio of the products (c_h/c_l) is strictly greater than the quality ratio $(c_h/c_l > q_h/q_l)$ and the resource consumption ratio is the smallest of the three $(c_h/c_l > q_h/q_l > s_h/s_l)$. It suggests that the resource differential between the two products is small relative to the quality and cost differentials. Under these conditions, same economic forces explained in the Proposition 25 is active: Capacity constraint is binding; efficient allocation of resources is the dominant factor; and the interfirm competition forces the industry to produce more of the product that has a potential to bring more profit per unit resource consumed. However, in this case, making room by decreasing some of the production of the less efficient product alone is not enough: more resources has to be created. The solution is found by decreasing the production further for the less efficient product which in turn creates an upward turn in the price for this product which help decrease the demand in the market. Thus, for the range of capacities presented in Proposition 26, both the industry supply decrease and price increase is realized with decreasing market concentration as opposed to the conventional wisdom established in the literature.

Suppose that on a flight leg from city P to city R, there are 10 airlines competing for the demand of 1,000 customers per day with a combined capacity of 8,000 sq ft. The customer valuations are such that the highest amount of money any customer would pay is $4,000 for the business class seat and $1,000 for the economy class seat. Also suppose

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that the interior design of this airline's aircrafts is such that the business class seat occupies 15 sq. ft. and the economy class seat occupies 5 sq. ft. inside the aircraft. It costs $1,500 to operate a business class seat and $300 an economy class seat. Assuming both airlines are rational and implement the optimal strategy, the customers get 509 business class seats and 73 economy class seats offered in the market in total. The prices are $1,890.91 and $418.18 respectively for business class and economy class seats.

Now, suppose that on a flight leg from city R to city S, there are 15 airlines competing with a combined capacity of 8,000 sq ft. Assume that all other conditions are equal to city pair P-R in the market. In this lower concentration market, the customers get 511 business class seats and 68 economy class seats. The prices are $1,889.29 and $421.43 respectively for business class and economy class seats. While the only difference is the number of firms in the market, the results have changed. The impact is not trivial: We observe that while the number of business class seats increased, the number of economy class seats have decreased with increasing competition. Moreover, while we would expect that all prices decrease, in this case price of the economy class seat has increased with increasing competition. Too much decrease in the production of the low quality product (due to capacity limitations and increase in the production of high quality product) eventually resulted in an increase in the prices as shown in the Proposition 26.

As a final remark, we note that the effects of market concentration diminishes as the number of firms increase in the industry. The greatest impact mentioned in the above propositions are realized when the market is highly concentrated. In other words, the rate of change of the capacity thresholds decrease as the number of firms increase in

the market. Bresnahan and Reiss (1991) empirically show that postentry competition increases at a rate that decreases with the number of incumbents. Although they study homogeneous products, we found similar results as n increases in a vertically differentiated model.

3.4 Welfare Effects in Competitive Settings

We also study the social welfare implications in the competitive markets. This analysis requires comparing the solutions of the n-firms ($n \geq 2$) to the social planner's solution. We have shown that when there is only one firm acting in the market, the distortion no longer exists when the capacity is very limited for all cases (Proposition 5). Here, we generalize this result for competitive markets as well.

In the standard vertical differentiation literature, it is well known that each customer gets a lower quality product or nothing at all from a monopolist compared to a social planner except the high end customer who gets the same assignment in both cases (e.g., Mussa and Rosen (1978), Moorthy (1984), Desai (2001)). In contrast, we show that a profit maximizing firm may offer a higher quality product than that of a social planner to some customer segments due to the capacity constraint even under competitive pressures. Specifically, when the maximum profit margin of the high quality product is larger, but its capacity adjusted maximum margin is smaller, the high end customers get a higher quality product from the firm than the social planner's assignment for a range of capacity levels. This is formally stated in the following proposition.

Proposition 27 *When $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, the oligopoly market offers*

the high quality product whereas the social planner does not offer it for $\bar{K}_1^{T(n)} < K \leq \bar{K}_1^S$.

Under the condition in Proposition 27, the oligopolist firm and the social planner prefer offering only the low quality product for sufficiently small capacity levels (i.e., $K \leq \bar{K}_1^{T(n)}$) and similarly, they both offer both product types for sufficiently large capacity levels (i.e., $K > \bar{K}_1^S$). However, when $\bar{K}_1^{T(n)} < K \leq \bar{K}_1^S$, the social planner does not offer the high quality product in order to increase its market coverage whereas the oligopolist driven by the higher profit margins offers the high quality product. In this case, while the higher end customer segment is getting better quality under oligopoly, the lower end customer segment is getting worse off: they are getting nothing. The better quality at the high end comes at the expense of the lower customer segment, as in the case of the monopoly.

The conventional wisdom in the literature states that the social planner serves a larger portion of the market compared to a monopolist (e.g., Mussa and Rosen (1978)). This is indeed the case when one ignores the capacity constraint. However, we show that, depending on the capacity level, an oligopolist may serve a greater portion of the market than a social planner. Specifically, this happens when there are increasing costs to quality and the high quality product has a larger capacity adjusted maximum margin.

Proposition 28 *When $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and $c_l/q_l \leq c_h/q_h$, the oligopoly market covers a greater portion of the market than the social planner for $\bar{K}_2^{T(n)} < K < \min\{\bar{K}_2^S, \bar{K}_3^{T(n)}\}$.*

Under the condition in Proposition 28, both the oligopolist and the social planner offer only the high quality product for sufficiently small capacity levels (i.e., $K \leq \bar{K}_2^{T(n)}$) and similarly they both offer both product types for sufficiently large capacity levels $K > \bar{K}_2^S$

(the oligopoly market does not use all of their capacities for $K > \bar{K}_3^{T(n)}$). However, when $\bar{K}_2^{T(n)} < K < \min\{\bar{K}_2^S, \bar{K}_3^{T(n)}\}$, only the oligopoly serves the low quality product. By offering the low quality product, oligopoly firms serve the high quality product to a smaller market segment and this in turn keeps the price higher. Whereas the social planner is not concerned about prices, it sells only the high quality product. This results in a larger market coverage under oligopoly since both the markets utilize all the capacity and high quality product consumes greater amount of capacity per unit.

Proposition 29 *When the capacity is sufficiently small, all customers get their socially efficient assignment from the oligopoly market.*

It is well-established in the literature that the profit maximizing firms degrade the quality level offered to low valuation customers. However, we show that when the capacity is sufficiently small, the optimal policy for both the profit maximizing firms and the social planner is to dedicate all the capacity to the most valuable product type (in terms maximum surplus per unit capacity, i.e., $\frac{q_i - c_i}{s_i}$). Because all the capacity is dedicated to the same product, the segment of customers who get the high quality product, the low quality product and nothing are the same for both the profit maximizing firms and the social planner.

Proposition 30 $Limit_{n \to \infty} \bar{K}_i^{T(n)} = \bar{K}_i^S$

We also find that as the number of competing firms goes to infinity, the solution converges to the solution of the social planner's. Thus, the firms lose any power on the prices and the quantities if there is infinitely large competition in the market.

CHAPTER 4

Product Line Design Issues For Focused Strategy Firms

In this chapter, we study the firms with focused strategies in more detail. The firm either offers only low or only high quality product in the market. We investigate their profitability levels. We provide analytical bounds for the monopoly case and numerical bounds for the asymmetric duopoly cases.

4.1   Related literature

This study is closely related to the literature that investigates competition in vertically differentiated industries. The oligopolistic setting where each firm offers only one distinct product has attracted a lot of attention from the researchers (e.g. Gabszewicz and Thisse (1980), Shaked and Sutton (1982), Gal-Or (1985), Moorthy (1988), Motta (1993), Wauthy (1996), Mazzeo (2002), Chambers et al. (2006), Jing (2006)). In our model, we extend these models and consider an asymmetric multiproduct competition where one firm is a single-product firm while the other firm has the option of offering multiple products. The multiproduct competition have been studied under two sepa-

rate frameworks: price-setting (Bertrand) games and quantity-setting (Cournot) games. Fundamental result in the price setting games is that the firms move away from their competitors and offer non-overlapping product lines at the equilibrium (e.g. Champsaur and Rochet (1989), Desai (2001), Rochet and Stole (2002), Schmidt-Mohr and Villas-Boas (2008)). However, in practice, firms offer similar products and the competition over prices fail to explain this head-to-head competition. In many industries, firms try to match the products already offered in the market rather than moving away from them as claimed in the price-setting games. We will follow the quantity game approach which has proved to be effective in explaining the head-to-head competition. In addition, Haskel and Martin (1994) shows empirically that if the firms are capacity constrained, the appropriate way to model competition is the Cournot model. So, the quantity competition setting is a more appropriate tool to model our focus on capacity allocation decisions of firms under competitive pressures.

The fundamental result in the quantity setting games is that the symmetric firms offer symmetric product lines (Gal-Or (1983), DeFraja (1996), Johnson and Myatt (2003)). However, competition models with asymmetric firms are rare. There are notable exceptions. Champsaur and Rochet (1989) study firms with different technology capabilities in a duopolistic setting. Due to the nature of the price competition, they find that there is always a gap between the product lines at the equilibrium. Johnson and Myatt (2003) study how firms adjust their product lines in response to entry in a quantity game. In particular, they compare a monopolist's solution to its decision in a duopolistic setting. When the entrant has lower technological capabilities than the incumbent and enters

the market with low-end products, the incumbent firm may also introduce low-end products to the market in response to entry. On the other hand, if the entrant has better technological capabilities, the incumbent may choose to exit the low-end markets.

Among the few papers investigating asymmetric technological capabilities, none has looked at the impact of capacity limitations. Moreover, we discuss the profitability levels of focus strategy firms when they have to face the more diverse competitors at the market place which was never done before in the literature.

4.2 Monopoly

In this section we will investigate the profitability levels of focused strategy firms when they act as a monopoly in a market. After we solve the problem for these focused strategy monopolists, we compare their results with the case of a multiproduct monopolist. The model and its assumptions are presented below which is similar to the previous chapters.

We study a single product in this case. The firm offers either a high or a low quality product. Given a focused strategy on product type i, the firm decides its optimal production quantity subject to the limited capacity K. To serve x units of product i, the firm needs to have at least $s \cdot x$ units of capacity. Clearly, the firm sells the minimum of its capacity K/s and demand D.

We adopt the classical vertical differentiation demand model (cf. Tirole (1988)). The customers vary in their willingness to pay for quality. Specifically, the customer types θ are uniformly distributed in the unit interval [0,1] with unit total mass. When type θ customer buys the product at price p, his utility is equal to $U(q, p, \theta) = \theta q - p$. If the

customer does not buy a product, his utility is zero. Thus, customers in $[0, \theta)$ do not buy the product and customers in $[\theta, 1]$ buy it where $\theta = p/q$. Then, we can express the demand as follows: $D(p) = 1 - \frac{p}{q}$.

Given this demand, the firm chooses the quantity x and price p to maximize its profit subject to the capacity constraint. Specifically the firm that has a strategy to focus on product type i solves,

$$\max_{x \geq 0} \quad \pi = (p(x) - c)x$$

$$\text{subject to} \quad sx \leq K.$$

The objective function of this problem is strictly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)).

4.2.1 High Quality Focused Firm

In practice, a firm may choose to focus on high quality for a variety of reasons including the promising high profitability levels, better customer service with a focus on luxury, etc. At the time of capacity building and technology choice, the market may favor this high quality focus. We analyzed in detail the cases where the focus on high quality might be the optimal choice. For instance, when there is decreasing cost to quality ratio and abundant capacity, then it is optimal for the firm to focus on the high quality product type. In the following proposition, we provide the optimal solution for the high quality focused firm.

Proposition 31 *For a high quality focused monopolist, the optimal production schedule is as follows:* $x = \frac{K}{s}$, *if* $K \leq \bar{K}_4^M$; *and* $x = \frac{q-c}{2q}$, *if* $K > \bar{K}_4^M$.

The solution presented in Proposition 31 was proved to be the optimal solution for a multiproduct monopolist under various set of conditions which was presented in Chapter 2. However, after the execution of this strategy, the economic conditions may change and this high quality advantage may become a disadvantage for the firm. For instance, if the cost to quality ratio changes in favor of the low quality product type, it may be better for the firm to offer both products to the market. However, since the firm has dedicated all its resources to the high quality production, they may not be able to offer the low quality product easily and obliged to stick with the focus strategy. In such a sub-optimal case, we study the worst case profitability levels of the firms. Let π_h^* be the optimal profit that can be achieved by the high quality focused firm and π^* be the optimal profit that is achieved by a multiproduct firm that has the capability to follow an optimal strategy.

Proposition 32 *Suppose* $q_l - c_l > q_h - c_h$ *and* $c_h/c_l > q_h/q_l > s_h/s_l$. *Then,* $\frac{\pi_h^*}{\pi^*} \geq$ $\frac{q_l \ (q_h - c_h)^2}{q_h \ (ql - cl)^2}$.

A high quality focused firm would do worst in a parametric setting where the optimal strategy is to focus on the low quality. As analyzed in the Chapter 2, this setting is the one when the traditional profit margin of the low quality product is better than that of high quality product $(q_l - c_l > q_h - c_h)$. The worst profit ratio calculated in this setting would provide a worst-case bound for all other parametric settings.

4.2.2 Low Quality Focused Firm

On the other hand, a firm may choose to focus on low quality for a bunch of other reasons such as technology limitations, positioning away from competition, etc. At the time of capacity building and technology choice, the market may favor this low quality focus. We analyzed in detail the cases where the focus on low quality might be the optimal choice. For instance, when there is increasing cost to quality ratio and limited capacity, it is optimal for the firm to focus on the low quality product type. In the following proposition, we provide the optimal solution for the low quality focused firm.

Proposition 33 *For a low quality focused monopolist, the optimal production schedule is as follows:* $x = \frac{K}{s}$, *if* $K \leq \bar{K}_5^M$; *and* $x = \frac{q-c}{2q}$, *if* $K > \bar{K}_5^M$.

The solution presented in Proposition 33 was proved to be the optimal solution for a multiproduct monopolist under various set of conditions which was presented in Chapter 2. However, after the execution of this strategy, the economic conditions may change and this strategy may become a disadvantage for the firm. For instance, if the cost to quality ratio changes in favor of the high quality product type, it may be better for the firm to focus on the high quality product. However, since the firm has dedicated all its resources to the low quality production, it may not be able to offer the high quality product easily and obliged to stick with the focus strategy. In such a sub-optimal case, we study the worst case profitability levels of the firms. Let π_l^* be the optimal profit that can be achieved by the high quality focused firm and π^* be the optimal profit that is achieved by a multiproduct firm that has the capability to follow an optimal strategy.

Proposition 34 *Suppose* $(q_l - c_l)/s_l < (q_h - c_h)/s_h$ *and* $q_h/q_l > c_h/c_l > s_h/s_l$. *Then,*

$$\frac{\pi_l^*}{\pi^*} \geq \frac{q_h \ (q_l - c_l)^2}{q_l \ (q_h - c_h)^2}.$$

A low quality focused firm would do worst in a parametric setting where the optimal strategy is to focus on the high quality. As analyzed in the Chapter 2, this setting is the one when the capacity adjusted profit margin of the high quality product is better than that of low quality product $((q_l - c_l)/s_l < (q_h - c_h)/s_h)$. Then, the worst profit ratio calculated in such a setting would provide a worst-case bound for all parametric settings.

4.3   Duopoly

In this section, we will study the focused strategy firm's performance under competition. We assume that the competitor firm has the ability to produce any product type. This means that the competitor can respond to the focused strategy firm in any way. This situation best reflects the problems of focused strategy firms in practice. For example, MaxJet had to compete with British Airlines on the non-stop route from London to Las Vegas. This market is a duopoly where a business class focused airline had to face an airline with multiple class capability under rapidly changing cost structure of the airline industry. We aim to gain insights for the profitability of the focus strategy firm in such a situation. The model and its assumptions are presented below which is similar to the previous chapters.

As in the case of monopoly, we study two product types, high and low quality products with quality levels $q_h > q_l$. Each unit of product i costs c_i and it consumes s_i units of the capacity. We assume $c_h > c_l$ and $s_h > s_l$. We adopt the classical vertical differentiation

demand model (cf. Tirole (1988)). The customers vary in their willingness to pay for quality. Specifically, the customer types θ are uniformly distributed in the unit interval [0,1] with unit total mass. When type θ customer buys product i at price p_i, his utility is equal to $U(q_i, p_i, \theta) = \theta q_i - p_i$. If the customer does not buy a product, his utility is zero. Thus, each customer has three options, buying the high quality product, buying the low quality product and not buying a product, and he chooses the one that maximizes his utility. This yields $0 \le \theta_l \le \theta_h \le 1$ such that customers in $[0, \theta_l)$ do not buy a product, customers in $[\theta_l, \theta_h)$ buy the low quality product and customers in $[\theta_h, 1]$ buy the high quality product. So, the demand for the high quality and the low quality products are $D_h = 1 - \theta_h$ and $D_l = \theta_h - \theta_l$. It is straightforward to show that the marginal customer θ_h who is indifferent between buying the high and the low quality products is given by $\theta_h = (p_h - p_l)/(q_h - q_l)$ and similarly, the marginal customer θ_l who is indifferent between buying the low quality product and not buying a product at all is given by $\theta_l = p_l/q_l$. Thus, we can express the demands for the two product types as follows,

$$D_h(p_l, p_h) = 1 - \frac{p_h - p_l}{q_h - q_l} \qquad\qquad D_l(p_l, p_h) = \frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l}.$$

The firms simultaneously decide the amount of each product that will be offered in the market. They participate in a quantity (Cournot) competition and then the prices are used to clear the market. The firms have the same limited capacity (K). In this one-shot game, firms simultaneously decide how to allocate their capacity among the product offerings given the competitors' offerings and customers' self selection constraints. We investigate the competition profits of both a high quality focused firm and a low quality

focused firm in each of the below subsections.

4.3.1 High Quality Focused Firm

The high quality focused firm Z chooses only the quantity of the high quality product. The competitor firm Y has the ability to choose among products and allocate its capacity as a best response to firm Z's decisions. Then, Firm Z solves the following optimization problem given the best response function (y_h^*, y_l^*) of firm Y, self selection of the customers, and the available capacity K:

$$\max_{z_h \geq 0} \quad (p_h(z_h, y_h^*, y_l^*) - c_h)z_h$$

$$\text{subject to} \quad s_h z_h \leq K$$

The objective function of the problem is strictly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)). The best response functions and feasibility conditions are provided in the Appendix C Lemma C1. On the other hand, firm Y solves the following optimization problem given the best response function (z_h^*) of firm Z, self selection of the customers, and the symmetric capacity K:

$$\max_{y_h, y_l \geq 0} \quad (p_h(y_h, y_l, z_h^*) - c_h)y_h + (p_l(y_h, y_l, z_h^*) - c_l)y_l$$

$$\text{subject to} \quad s_h y_h + s_l y_l \leq K$$

The objective function of the problem is strictly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)). The best response functions and feasibility conditions are provided in the Appendix C Lemma C2. After obtaining the best response functions of both firms, the equilibriums are found by solving them simultaneously.

Since the analytical analysis of these equilibriums get intractable, we conduct comprehensive numerical experiments to find out the bounds on the profitability of these focused strategy firms. First we study the parametric setting $(q_l - c_l > q_h - c_h)$ where the optimal strategy in the symmetric case is focusing on the low quality product.

The bounds achieved from 135 examples are presented in Figure 4.1. We observe that the profitability of the high quality firm could go as low as 0.24% under such circumstances. These low profitability levels may help us explain the recent failures of MaxJet, EOS and Silverjet. As the cost numbers increase, if the marginal profits of the business class seats has decreased below the marginal profits of the economy class seats, then the firms may have experienced extremely low profits in the face of competition from the traditional firms and failed as a result of these changes in the cost structures.

We also study the parametric setting $(q_l - c_l < q_h - c_h)$ where the optimal strategy in the symmetric case is offering both products for high capacity levels whereas focusing on the low quality product for low capacity levels. Although this setting does not provide the worst case boundaries for the profitability of the firms, it is the most common setting studied in the literature.

The bounds achieved from 230 examples are presented in Figure 4.2. The firm with a
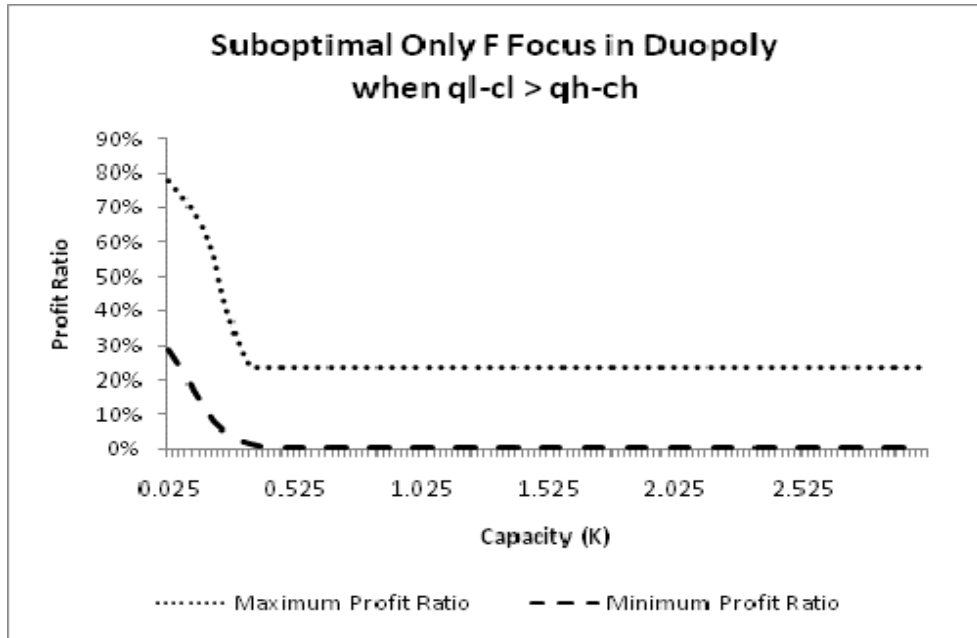
FIGURE 4.1: Profitability Bounds for High Quality Focused Firm in Duopoly when $q_l - c_l > q_h - c_h$
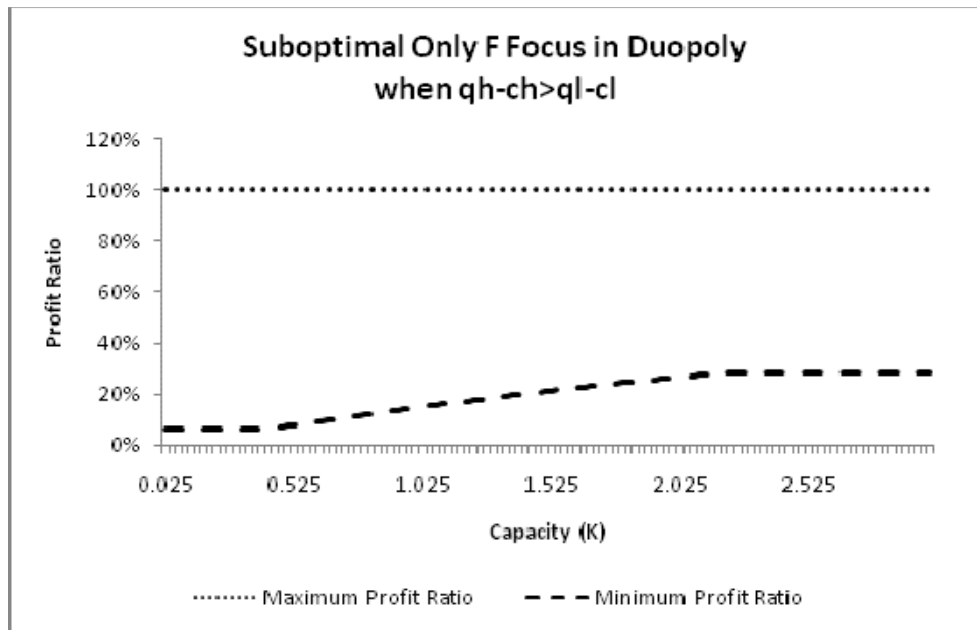
FIGURE 4.2: Profitability Bounds for High Quality Focused Firm in Duopoly when $q_l - c_l < q_h - c_h$

high quality focus may earn as much as 99.9% of a firm that has the capability to follow the optimal strategy. We also observe that the profitability of the high quality firm could go as low as 6.63% when the capacity is scarce.

More interestingly, we find that the competitor firm may adjust its product line in response to the high quality focused firm. While the optimal strategy for the competitor would be offering both products for high capacity levels in the symmetric game; we found instances where the competitor firm may adjust its strategy in response to high quality focused firm and actually focus on the low quality product for all capacity levels in this asymmetric game.

4.3.2   Low Quality Focused Firm

The low quality focused firm Z chooses only the quantity of the low quality product. The competitor firm Y has the ability to choose among products and allocate its capacity as a best response to firm Z's decisions. Then, Firm Z solves the following optimization problem given the best response function (y_h^*, y_l^*) of firm Y, self selection of the customers, and the available capacity K:

$$\max_{z_l \geq 0} \quad (p_l(z_l, y_h^*, y_l^*) - c_l)z_l$$

$$\text{subject to} \quad s_l z_l \leq K$$

The objective function of the problem is strictly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first

order conditions (Bazaraa et al. (2006)). The best response functions and feasibility conditions are provided in the Appendix D. On the other hand, firm Y solves the following optimization problem given the best response function (z_h^*) of firm Z, self selection of the customers, and the symmetric capacity K:

$$\max_{y_h, y_l \geq 0} \quad (p_h(y_h, y_l, z_l^*) - c_h)y_h + (p_l(y_h, y_l, z_l^*) - c_l)y_l$$

$$\text{subject to} \quad s_h y_h + s_l y_l \leq K$$

The objective function of the problem is strictly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)). The best response functions and feasibility conditions are provided in the Appendix. After obtaining the best response functions of both firms, the equilibriums are found by solving them simultaneously. Since the analytical analysis of these equilibriums get intractable, we conduct comprehensive numerical experiments to find out the bounds on the profitability of these focused strategy firms.

In this case, we study the parametric setting $((q_h - c_h)/s_h > (q_l - c_l)/s_l)$ where the optimal strategy in the symmetric case is focusing on the high quality product. It is also the most common setting studied in the literature. The bounds achieved from 255 examples are presented in Figure 4.3. We observe that the profitability of the low quality firm could go as low as 1.18% under such circumstances.

These low profitability levels may help us explain the failures of many low-cost carriers in the airline industry. As the cost numbers increase, the marginal profits of the
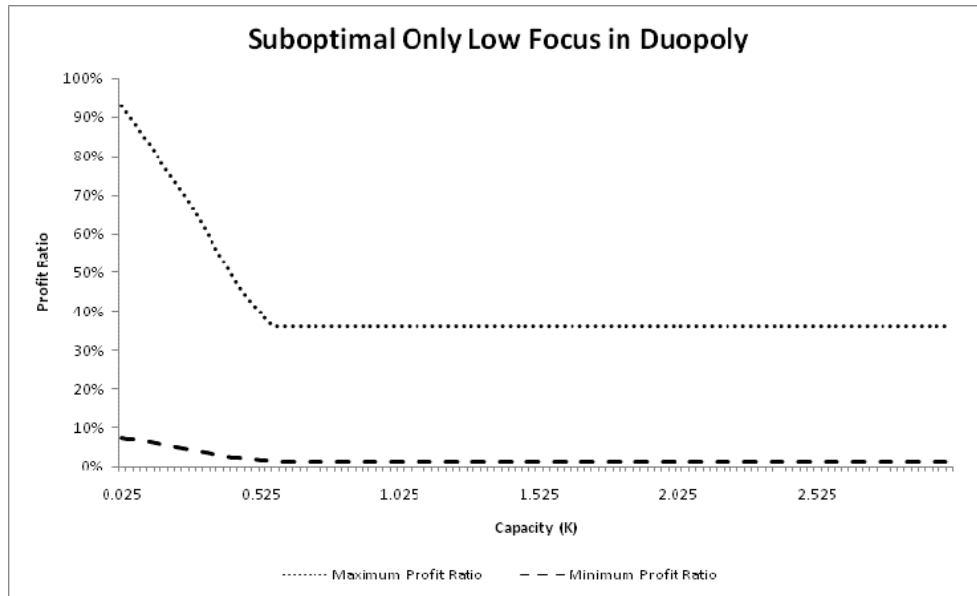
FIGURE 4.3: Profitability Bounds for Low Quality Focused Firm in Duopoly when $(q_h - c_h)/s_h > (q_l - c_l)/s_l$

economy class seats has also decreased extensively. Then, the firms may have experienced

extremely low profits in the face of competition from the traditional firms and failed as

a result of these changes in the cost structures.

CHAPTER 5

Vertically Differentiated Product Line Design Under Uncertainty

In a recent business report, IBM Consulting Services discuss how careful consideration of customer valuation during the planning process can increase profitability (Meckley and Toscano, 2005). They note that *"improved decision-making for spending scarce resources can have significant impacts on growth, risk and profitability"*. In this chapter, we extend the model along these lines. We analyze the system in a multiperiod setting and study the effects of customer valuation uncertainty on the product line decisions of the capacity constrained firms.

In our model, we recognize the fact that firms make their strategic capacity decisions well before the markets clear for prices. The amount of capacity that is initially allocated to a product is a constraint on the number of those products produced and sold later in the market. In this initialization period, firm decides for the committed capacity for each product based on the available resources and expectations on the customers' valuations. During the later period(s), customers' valuation distribution is realized and the firm

makes its actual production and sales decisions constrained by the initial production capacity commitment.

Then, we further extend the model to investigate the impact of use of revenue management techniques. In this multiperiod formulation, the initialization period is again the product line decision and capacity allocation stage. Later periods have different demand distributions which presents opportunities for the implementation of revenue management techniques.

5.1   Related Literature

Operations literature study the revenue management issues rather extensively. Talluri and vanRyzin (2004b) provide a broad review of revenue management theory, applications and history. The aim of the traditional revenue management literature is to devise pricing mechanisms for firms where the product mix is already given. Moreover, the literature ignores the cannibalization effects of offering multiple products to the customer base. We address the segmentation and complex pricing decisions jointly in this model. McGill and van Ryzin (1999) provide a review on revenue management. One of the key aspects they emphasize with regard to the advance of research is the integration of revenue management decisions with other planning decisions such as the product design and pricing. Bitran and Caldentey (2003) further review the literature on pricing models for revenue management. The authors state that analysis of optimal pricing policies for the multi-product case is a challenging and practically important venue of research.

Consumer behavior is an aspect which has recently started to be studied by the oper-

ations literature. In a recent survey, Shen and Su (2007) reviews the emerging literature on customer behavior modeling in revenue management context. Talluri and vanRyzin (2004a) study the revenue management under a general discrete choice model of consumer behavior. They formulate the problem as a dynamic program and study the nested allocation policies. However, the model assumes exogenous prices which is a key aspect of our formulation and primary driver of the cannibalization phenomenon. Ng (2006) also proposes strategies for the firms to follow vertically differentiated segmentation strategies together with the traditional revenue management strategies. Following this promising line of research, we study the revenue management problem jointly with segmentation decisions.

On the other hand, economics and marketing literatures lack the thorough discussion of the effects of uncertainty in customer valuations on the firm's product choice. There is a line of research where the authors study the effects of uncertainty in quality levels of product. Bester (1998) studies how consumers' uncertainty about the quality of products effects the firm's incentives for horizontal differentiation. Cavaliere (2005) also works out the uncertainty faced by customers with regard to the quality of the products where the firms use prices as a signaling mechanism of their qualities. Casado-Izaga (2000) and Meagher and Zauner (2005) also study the consumer valuation uncertainty in the context of horizontal differentiation. They show that the existence of uncertainty raises the degree of product differentiation in this context.

Consumer valuations uncertainty was approached from different angles in the vertical differentiation context. Saak (2008) studies the setting where the consumers themselves

lack precise knowledge of their valuations. Johnson and Myatt (2006b) studies how the shifts in consumer valuations change the product line choices of the firms. They study an extensive amount of functions and discuss their effects on the firm's profits. However, there is no uncertainty related to the valuation functions. The shift is known to the firm and the authors discuss how the optimal strategy changes in existence of such a shift.

Since there is no study in the literature studying the effects of customer valuation uncertainty in the vertical differentiation context, we aim to fill this gap with this study.

5.2   Single Recourse Analysis

We reformulate the problem to incorporate uncertainty. In the first period, capacity allocations will be determined. In the second period, committed to these capacity allocations, prices and sales are determined.

5.2.1   Model

There are two periods: In period 1, the firm makes the decision on how to allocate its capacity among the products (x_i). For example, during the construction stage, the firm has to decide how to allocate the budget (K) among different types of production facilities: the facility (x_h) that manufactures the high quality product would cost more than the facility (x_l) that manufactures the low quality product $(s_h > s_l)$. At this period the valuations of the customers are not known by the firm. In a deterministic case, the firm would know the distribution of the valuations of the customers. However, in this model, the specifics of the distribution is also unknown to the firm. The firm knows that

the market size is one and distributed uniformly between $[\bar{\theta} - 1, \bar{\theta}]$ where $\bar{\theta}$ is distributed uniformly between $[1 - \epsilon, 1 + \epsilon]$.

At the beginning of period 2, the specifics of the customer valuation distribution is revealed $(\bar{\theta} = \hat{\theta})$. Given this information, the firm decides the production quantities and prices (y_i and p_i) subject to the capacity commitments (x_i) made in the first period. The mathematical formulation of the problem is as follows:

$$\max_{x_i \geq 0} \quad E_{\bar{\theta}}[Q(x, \theta)]$$

$$\text{subject to} \quad \sum_i s_i x_i \leq K$$

$$\text{where} \quad Q(x, \hat{\theta}) = \max_{y_i \geq 0} \sum_i y_i(p_i(\hat{\theta}, y) - c_i)$$

$$\text{subject to} \quad y_i \leq x_i \; \forall i$$

The objective function of this problem is jointly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions Bazaraa et al. (2006).

5.2.2 Single Product

We first study the case where the firm has to make decisions for a single product with quality q, unit cost c, and resource consumption rate s. Given the decisions of the first period, and after the customer valuation distribution's specifics are revealed in the second period, the firm has to decide the quantity that will be sold in the market. In the following paragraphs, we present the results of this analysis.

We will first solve the second period problem where the firm knows the specifics of the distribution for the customer valuations. After these specifics are revealed, the firm has to decide the production (sales) quantity constrained by the capacity commitment made during the first period.

Proposition 35 *Suppose the customer valuation is distributed uniformly between $[\hat{\theta} - 1, \hat{\theta}]$. Then, the optimal sales of a monopolist is as follows:*

If $max\{1 - \epsilon, c/q\} \leq \hat{\theta} < min\{(2qx + c)/q, 1 + \epsilon\}$, then $y = (\hat{\theta}q - c)/2q$. Else if $\hat{\theta} \geq max\{1 - \epsilon, (2qx + c)/q\}$, then $y = x$.

If the market valuations are relatively low compared to the capacity commitment, then the firm only sells a limited amount of the product in order to keep its price high. In that case, capacity constraint is not binding $(y < x)$. On the other hand, if the valuations are high enough, the firm is better off selling all he could and adjust (increase) the prices to get a demand that is equal to its maximum production $(y = x)$.

Given these second period solutions, one could take the expectation over the valuation realizations and find the optimal solution in the first period.

Proposition 36   *i. When $K < \frac{s(q - c + q\epsilon)}{2q}$, the optimal strategy for the monopolist firm is to dedicate all the capacity to production: $x = K/s$.*

ii. When $K \geq \frac{s(q - c + q\epsilon)}{2q}$, the optimal strategy for the monopolist firm is to produce a certain amount and leave the remaining capacity excess: $x = q - c + q\epsilon/2q$.

We see that the optimal capacity allocation $(x = q - c + q\epsilon/2q)$ under uncertainty increases by $\epsilon/2$ amount when compared to the deterministic case $(x = q - c/2q)$. The

risk neutral firm would like to go after the higher valuation that has a positive probability of occurrence in order to get the higher profits in return. Another point is that the first period decision comes at no expense to the firm. Then, an increase in the capacity commitment helps the firm attract customers when the valuations are high.

5.2.3   Two Differentiated Products

We will now look at the problem when there are two products to share a single resource and a heterogenous customer base.

i. Second Period

Given the decisions of the first period, i.e. the capacity allocations (x_h, x_l), and after the customer valuation distribution's specifics are revealed in the second period, the firm has to decide the quantities that will be sold in the market. In the following paragraphs, we present the results of this analysis.

Proposition 37 *Suppose $c_l/q_l > c_h/q_h$; and the customer valuation is distributed uniformly between $[\hat{\theta} - 1, \hat{\theta}]$. Then, the optimal sales of a monopolist is as follows:*

If $max\{1 - \epsilon, c_h/q_h\} \le \hat{\theta} < min\{2x_h + c_h/q_h, 1 + \epsilon\}$, then $y_h = (\hat{\theta}q_h - c_h)/2q_h$. Else if $\hat{\theta} \ge max\{1 - \epsilon, 2x_h + c_h/q_h\}$, then $y_h = x_h$.

If $max\{1 - \epsilon, c_h/q_h\} \le \hat{\theta} < min\{2x_h + c_l/q_l, 1 + \epsilon\}$, then $y_l = 0$. Else if $max\{1 - \epsilon, 2x_h + c_l/q_l\} \le \hat{\theta} < min\{2(x_l + x_h) + c_l/q_l, 1 + \epsilon\}$, then $y_l = (\hat{\theta}q_l - c_l - 2q_l x_h)/2q_l$. Else if $\hat{\theta} \ge max\{1 - \epsilon, 2(x_l + x_h) + c_l/q_l\}$, then $y_l = x_l$.

The market clearing prices are $p_h = q_h (\hat{\theta} - y_h) - q_l y_l$ and $p_l = q_l (\hat{\theta} - y_h - y_l)$ respectively.

This proposition covers the instances where the cost to quality ratio favors the high quality product. Indeed, the second stage favors the high quality product as well. If the market valuations are too low compared to the capacity allocations, then the firm only sells a limited amount of the high quality product in order to keep its price high. In this case, capacity constraints are not binding $(y_i < x_i)$. If the valuations get a little better, then the firm optimally sells all the high quality product and does not cannibalize its demand with low quality product for a while. Only if the market valuations are high enough, the firm starts selling the low quality product. If the valuations are high enough, the firm is better off selling all he could and adjusting (increase) the prices to get a demand that is equal to its capacity $(y_i = x_i)$. The sales would have a different priority when the cost to quality ratio is reversed.

Proposition 38 *Suppose $c_l/q_l < c_h/q_h$ and $x_l \geq \frac{c_h q_l - c_l q_h}{2 q_l (q_h - q_l)}$; and the customer valuation is distributed uniformly between $[\hat{\theta} - 1, \hat{\theta}]$. Then, the optimal sales of a monopolist is as follows:*

If $max\{1 - \epsilon, c_l/q_l\} \leq \hat{\theta} < min\{(c_h - c_l)/(q_h - q_l), 1 + \epsilon\}$, then $y_h = 0$. Else if $max\{1 - \epsilon, (c_h - c_l)/(q_h - q_l)\} \leq \hat{\theta} < min\{2x_h + (c_h - c_l)/(q_h - q_l), 1 + \epsilon\}$, then $y_h = \frac{(\hat{\theta} q_h - c_h) - (\hat{\theta} q_l - c_l)}{2(q_h - q_l)}$. Else if $\hat{\theta} \geq max\{1 - \epsilon, 2x_h + (c_h - c_l)/(q_h - q_l)\}$, then $y_h = x_h$.

If $max\{1 - \epsilon, c_l/q_l\} \leq \hat{\theta} < min\{(c_h - c_l)/(q_h - q_l), 1 + \epsilon\}$, then $y_l = (\hat{\theta} q_l - c_l)/2q_l$. Else if $max\{1 - \epsilon, (c_h - c_l)/(q_h - q_l)\} \leq \hat{\theta} < min\{2x_h + (c_h - c_l)/(q_h - q_l), 1 + \epsilon\}$, then $y_l = \frac{c_h q_l - c_l q_h}{2 q_l (q_h - q_l)}$. Else if $max\{1 - \epsilon, 2x_h + (c_h - c_l)/(q_h - q_l)\} \leq \hat{\theta} < min\{2(x_l + x_h) + c_l/q_l, 1 + \epsilon\}$, then $y_l = (\hat{\theta} q_l - c_l - 2q_l x_h)/2q_l$. Else if $\hat{\theta} \geq max\{1 - \epsilon, 2(x_l + x_h) + c_l/q_l\}$, then $y_l = x_l$.

The market clearing prices are $p_h = q_h (\hat{\theta} - y_h) - q_l y_l$ and $p_l = q_l (\hat{\theta} - y_h - y_l)$

respectively.

This proposition covers the instances where the cost to quality ratio favors the low quality product. In addition, the solution is valid when the capacity allocation of the low quality product is below a certain threshold. We observe that the second stage also favors the low quality product. If the market valuations are too low compared to the capacity allocations, then the firm only sells a limited amount of the low quality product in order to keep its price high. In this case, capacity constraints are not binding $(y_i < x_i)$. If the valuations get a little better, then the firm optimally sells all the low quality product. Only if the market valuations are high enough, the firm starts selling the high quality product. If the valuations are high enough, the firm is better off selling all he could and adjusting (increase) the prices to get a demand that is equal to its capacity $(y_i = x_i)$. The sales may be different when the capacity allocation for the low quality product is greater.

Proposition 39 *Suppose $c_l/q_l < c_h/q_h$ and $x_l \leq \frac{c_h q_l - c_l q_h}{2q_l(q_h - q_l)}$; and the customer valuation is distributed uniformly between $[\hat{\theta} - 1, \hat{\theta}]$. Then, the optimal sales of a monopolist is as follows:*

If $max\{1 - \epsilon, c_l/q_l\} \leq \hat{\theta} < min\{(2q_l x_l + c_h)/q_h, 1 + \epsilon\}$, then $y_h = 0$. Else if $max\{1 - \epsilon, (2q_l x_l + c_h)/q_h\} \leq \hat{\theta} < min\{2(x_h + x_l q_l/q_h) + c_h)/q_h, 1 + \epsilon\}$, then $y_h = \frac{(\hat{\theta} q_h - c_h) - 2q_l x_l)}{2q_h}$. Else if $\hat{\theta} \geq max\{1 - \epsilon, 2(x_h + x_l q_l/q_h) + c_h)/q_h\}$, then $y_h = x_h$.

If $max\{1 - \epsilon, c_l/q_l\} \leq \hat{\theta} < min\{2x_l + c_l/q_l, 1 + \epsilon\}$, then $y_l = (\hat{\theta} q_l - c_l)/2q_l$. Else if $\hat{\theta} \geq max\{1 - \epsilon, 2x_l + c_l/q_l\}$, then $y_l = x_l$.

The market clearing prices are $p_h = q_h (\hat{\theta} - y_h) - q_l \, y_l$ and $p_l = q_l (\hat{\theta} - y_h - y_l)$

respectively.

This solution is valid when the capacity allocation of the low quality product is above a certain threshold. We observe that the low quality product is still favored over the high quality product and it is offered for a longer range of valuations. Nevertheless, the firm does not wait to sell all the low quality production but introduces high quality before the full capacity production. Only if the valuations are high enough, the firm is better off selling all he could and adjusting (increase) the prices to get a demand that is equal to its capacity $(y_i = x_i)$.

ii. First Period

In the first period, the firm does not know the specifics of the customer valuations distribution. The firm only knows that the market size is one and distributed uniformly between $[\bar{\theta} - 1, \bar{\theta}]$ where $\bar{\theta}$ is distributed uniformly between $[1 - \epsilon, 1 + \epsilon]$. The firm needs to make capacity commitment decisions under this uncertainty. Once the distribution is revealed in the second period, based on the product line decisions made in the first period, the firm can decide its production quantity and prices. In this section, we will investigate the firm's first period decisions where the product mix choices and capacity commitments are finalized.

Proposition 40 *When $c_l/q_l < c_h/q_h$, $(q_h - c_h) \geq (q_l - c_l)$, the monopolist firm optimally offers two products at positive quantities:*

$$x_h = \frac{(q_h - c_h) - (q_l - c_l) + (q_h - q_l)\epsilon}{2(q_h - q_l)} \qquad x_l = \frac{c_h q_l - c_l q_h}{2q_l(q_h - q_l)}$$

when $K \geq \frac{c_h q_l(-s_h+s_l)+c_l(q_l s_h-q_h s_l)+(q_h-q_l)q_l s_h(1+\epsilon)}{2(q_h-q_l)q_l}$.

Proposition 40 studies the cases where the cost to quality ratio is increasing and capacity constraint is not binding. In this case, we have shown that the optimal strategy is to differentiate and offer both products in the market when there is no uncertainty. In this proposition, we show that the strategy remains the same; nevertheless, with one difference: the quantity of the high quality product increases with the level of uncertainty. Note that there is a positive probability that the customer valuation distribution may shift upwards with higher valuations at the high end of the customer segment. The risk neutral firm would like to go after these high end customers who may have higher valuations. In case the valuations shift downwards, the firm does not lose anything since the first period commitment comes at no expense when the capacity is not binding.

When the capacity constraint is binding, achieving the closed form solutions is not analytically tractable. We will present some numerical examples and observe how the results of the deterministic models change under uncertainty.

Figure 5.1 shows an example where the cost to quality ratio is increasing as is the case in Proposition 40. However, we know from the deterministic case that when the capacity constraint is binding, potential profit per unit resource consumed becomes an important economic driver behind the firm's decisions. In this example, marginal profit per unit resource is better for the low quality product ($q_l - c_l/s_l > q_h - c_h/s_h$). In Figure Figure 5.1, the graph on the left is the optimal capacity commitment for the high quality product for a given capacity level (K) and the graph on the right is the one for the low quality product. Please note that the far end of each plot shows the capacity commitments at
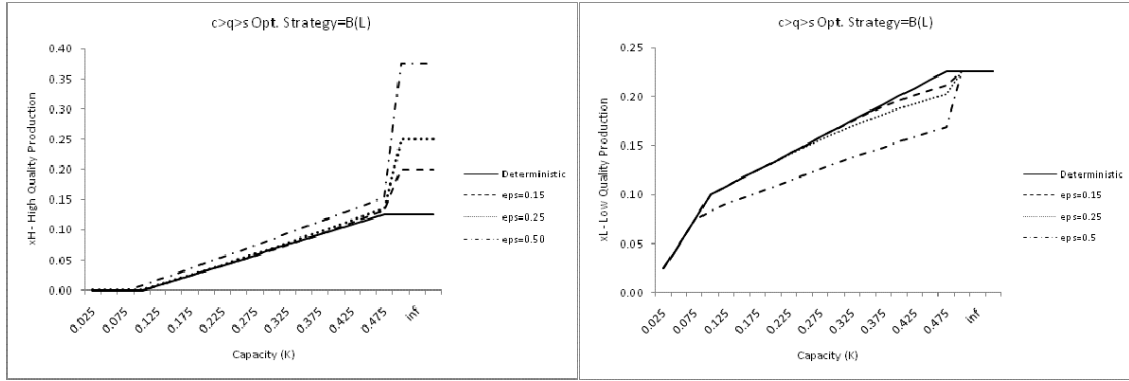
FIGURE 5.1: Product Line Under Uncertainty when $c_l/q_l < c_h/q_h$ and $q_l - c_l/s_l > q_h - c_h/s_h$

infinite capacity which creates a break in the line although there is none. Each graph presents multiple lines each of which are plotted using different uncertainty levels. As ϵ (eps) increases, the level of uncertainty increases as well.

In this case, the deterministic results suggest that the firm should focus on the low quality product for scarce capacity whereas both products are offered above a threshold capacity. We observe the same behavior when there is uncertainty about the market, too: The firm focuses on the low quality product for scarce capacity levels. We do not observe any high quality commitment below a certain threshold. However, as the capacity increases, the high quality product is offered in the product line together with the low quality product. In this medium range of capacity availability, the level of uncertainty has a similar effect as the Proposition 40. High quality commitment increases as the level of uncertainty increases. However, since the capacity constraint is binding at this level, the required resources are gained from decreasing the commitment of the low quality product. Hence we observe that increasing uncertainty has an effect to increase the high quality production and decrease the low quality production for a range of capacity levels.

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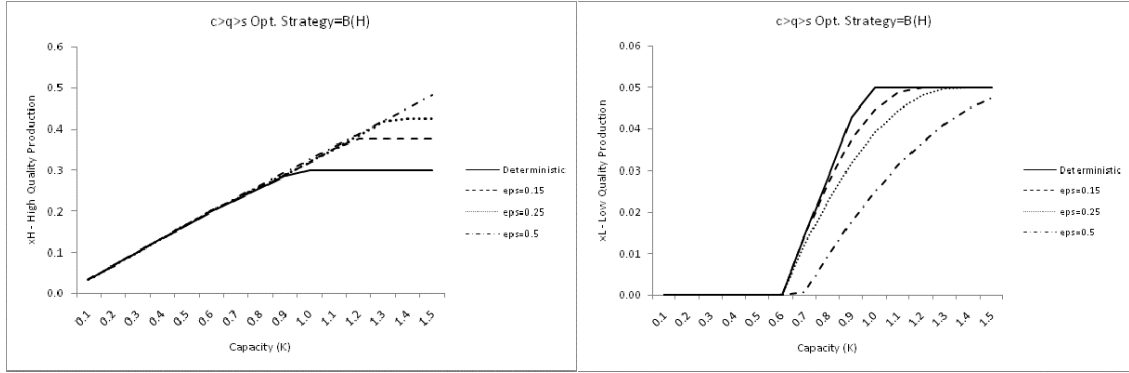
FIGURE 5.2: Product Line Under Uncertainty when $c_l/q_l < c_h/q_h$ and $q_l - c_l/s_l < q_h - c_h/s_h$

We also note that for large capacity levels (as seen in the far end of the graphs), the low quality production remains the same for all uncertainty levels as shown earlier analytically. Again, the high quality commitment increases with uncertainty when capacity is not binding.

Figure 5.2 shows another example where the cost to quality ratio is increasing. Nevertheless, in this example, marginal profit per unit resource is better for the high quality product ($q_l - c_l/s_l < q_h - c_h/s_h$). Although we expect that the large capacity levels should behave as presented in Proposition 40, it is not clear a priori how the uncertainty will impact the results when the capacity constraint is binding.

In this case, the deterministic results suggest that the firm should focus on the high quality product for scarce capacity whereas both products are offered above a threshold capacity. We observe the same behavior when there is uncertainty about the market, too: The firm focuses on the high quality product for scarce capacity levels. The product with less potential for profit per unit resource consumed is dropped from the product line.

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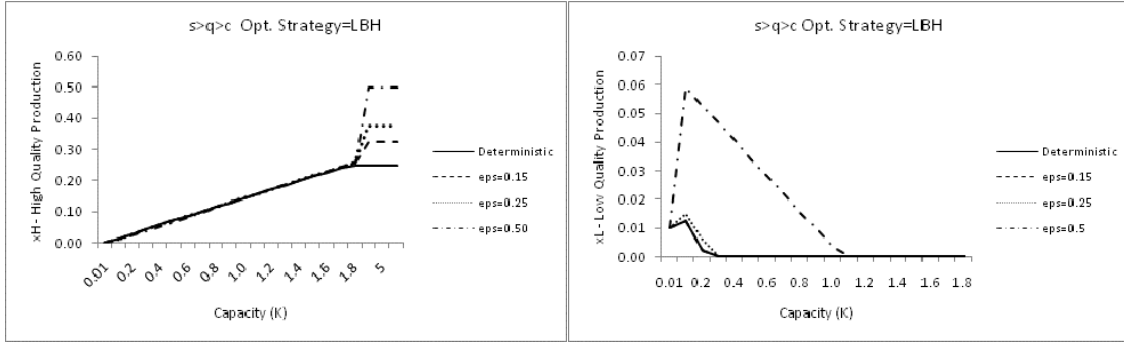
FIGURE 5.3: Product Line Under Uncertainty when $c_l/q_l > c_h/q_h$ and $q_l - c_l/s_l > q_h - c_h/s_h$

We observe that increasing uncertainty has an effect to increase the high quality commitment for medium capacity levels like the previous example. This increase in the high quality commitment comes at the expense of low quality commitment: the commitment levels decrease as uncertainty increases for the low quality product.

Figure 5.3 shows an example where the cost to quality ratio is decreasing. This case favors the high quality production when capacity is large enough. Yet, marginal profit per unit resource is better for the low quality product $(q_l - c_l/s_l > q_h - c_h/s_h)$ which has an adverse effect when capacity is less.

In this case, the deterministic results suggest that the firm should focus on the low quality product for scarce capacity whereas both products are offered at medium capacity levels and only high quality focus is optimal for large capacities. We observe the same behavior when there is uncertainty about the customer valuations, too: The firm focuses on the low quality product for scarce capacity levels, introduces high quality to the product mix as capacity increases and eventually ceases the production of low quality product all together above a certain threshold. As shown earlier analytically, the high

quality commitment increases with uncertainty when capacity is not binding.

However, we observe that the low quality commitment increases with uncertainty for a range of capacity levels. This increase comes at the expense of a decrease in the high quality commitment at that range. Interestingly, in this case, the firm chooses to increase the commitment for the product that has a greater potential to bring profit per unit resource consumed at the expense of the high quality product. This example shows that the effect of increasing uncertainty is not trivial for medium capacity levels.

5.3   Multiperiod Analysis

We further revise the stochastic market model and study a multiperiod setting where the firm will have the option of changing prices in response to changing demand in each period.

5.3.1   Model

We investigate the revenue management implications of the problem under uncertainty. We extend the problem to three periods: In period 1, the firm makes the decision on how to allocate its capacity among the products (x_i). The firm also decides the reservation limits $(b_i < x_i)$ in the first period. Reservation limit is the upper bound on the amount of sales in the second period $(y_i^1 < b_i)$. At this period the valuations of the customers are not known by the firm. In a deterministic case, the firm would know the distribution of the valuations of the customers. However, in this model, the specifics of the distribution is also unknown to the firm. The firm knows that the market size is one: half of this

market will arrive in the second period and remaining half will arrive in the third period, possibly with different valuation distributions. The customers are distributed uniformly between $[\bar{\theta} - 1, \bar{\theta}]$ where $\bar{\theta}$ is stochastic and can take values between $[1 - \epsilon, 1 + \epsilon]$.

We follow a scenario based approach in this case: there are 6 (2 high, 2 medium and 2 low) scenarios that could occur between $[1-\epsilon, 1+\epsilon]$. In the second period, low and medium scenarios could occur equally likely and in the third period, high and medium scenarios could occur equally likely. At the beginning of each following period, the specifics of the customer valuation distribution is revealed ($\bar{\theta} = \hat{\theta}$). Given this information, the firm decides the production quantities and prices (y_i and p_i) subject to the capacity commitments (x_i) and reservation limits (b_i) made in the first period. The mathematical formulation of the problem is as follows:

$$\max_{x_i, b_i \geq 0} \quad E_{\bar{\theta}^1}[Q1(x, b, \bar{\theta})]$$

$$\text{subject to} \quad \sum_i s_i x_i \leq K$$

$$b_i \leq x_i \quad \forall i$$

$$\text{where} \quad Q1(x, b, \bar{\theta}^1) = \max_{y_i^1 \geq 0} \sum_i y_i^1(p_i^1(\bar{\theta}^1, y) - c_i) + E_{\bar{\theta}^2}[Q2(x, b, \bar{\theta})]$$

$$\text{subject to} \quad y_i^1 \leq b_i \quad \forall i$$

$$\text{where} \quad Q2(x, y, \bar{\theta}^2) = \max_{y_i^2 \geq 0} \sum_i y_i^2(p_i^2(\bar{\theta}^2, y) - c_i)$$

$$\text{subject to} \quad y_i^2 \leq x_i - y_i^1 \quad \forall i$$
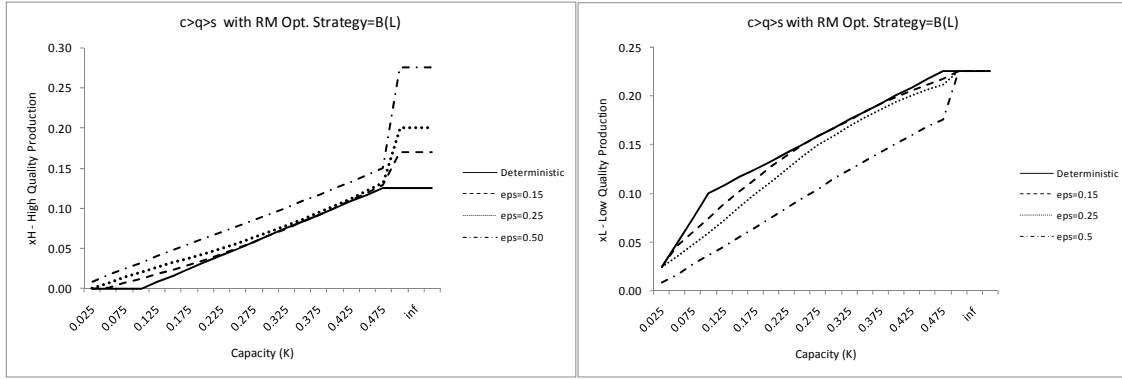
FIGURE 5.4: Product Line with Revenue Management when $c_l/q_l < c_h/q_h$ and $q_l - c_l/s_l > q_h - c_h/s_h$

5.3.2 Analysis

Figure 5.4 shows an example where the cost to quality ratio is increasing. Marginal profit per unit resource is also better for the low quality product ($q_l - c_l/s_l > q_h - c_h/s_h$).

In this case, both the deterministic results and 2-period uncertainty model suggest that the firm should focus on the low quality product for scarce capacity whereas both products are offered above a threshold capacity. We observe the same behavior when there are multiple periods and more complex pricing options (i.e., revenue management), too: The firm focuses on the low quality product for scarce capacity levels. We do not observe any high quality commitment below a certain threshold. However, as the capacity increases, the high quality product is offered in the product line together with the low quality product. In this medium range of capacity availability, the level of uncertainty has a similar effect as the Proposition 40 and 2-period uncertainty model. High quality commitment increases as the level of uncertainty increases. However, since the capacity constraint is binding at this level, the required resources are gained from decreasing the commitment to the low quality product. Hence we observe that increasing uncertainty has

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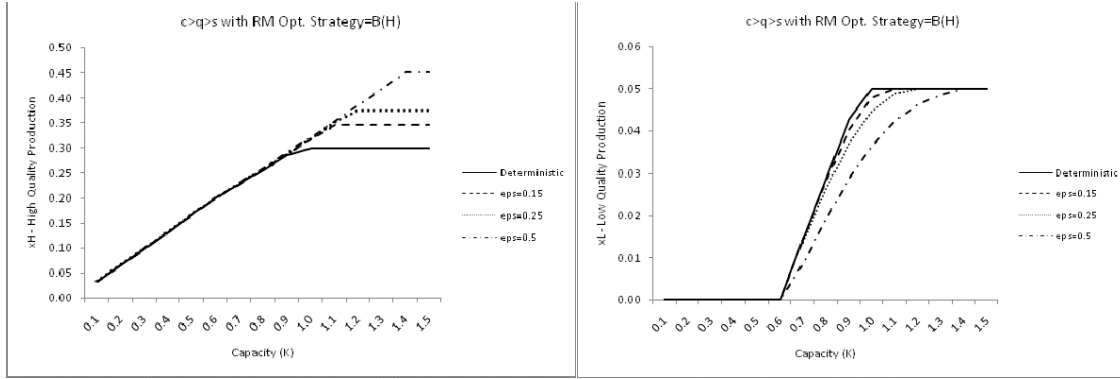
FIGURE 5.5: Product Line with Revenue Management when $c_l/q_l < c_h/q_h$ and $q_l - c_l/s_l < q_h - c_h/s_h$

an effect to increase the high quality production and decrease the low quality production for a range of capacity levels. We also note that for large capacity levels (as seen in the far end of the graphs), the low quality production remains the same for all uncertainty levels as shown earlier analytically. Again, the high quality commitment increases with uncertainty when capacity is not binding.

Figure 5.5 shows another example where the cost to quality ratio is increasing. In this example, marginal profit per unit resource is better for the high quality product $(q_l - c_l/s_l < q_h - c_h/s_h)$.

In this case, the deterministic results and 2-period uncertainty model suggest that the firm should focus on the high quality product for scarce capacity whereas both products are offered above a threshold capacity. We observe the same behavior when there are multiple periods and more complex pricing options (i.e., revenue management), too: The firm focuses on the high quality product for scarce capacity levels. The product with less potential for profit per unit resource consumed is dropped from the product line. We also observe that increasing uncertainty has an effect to increase the high quality commitment

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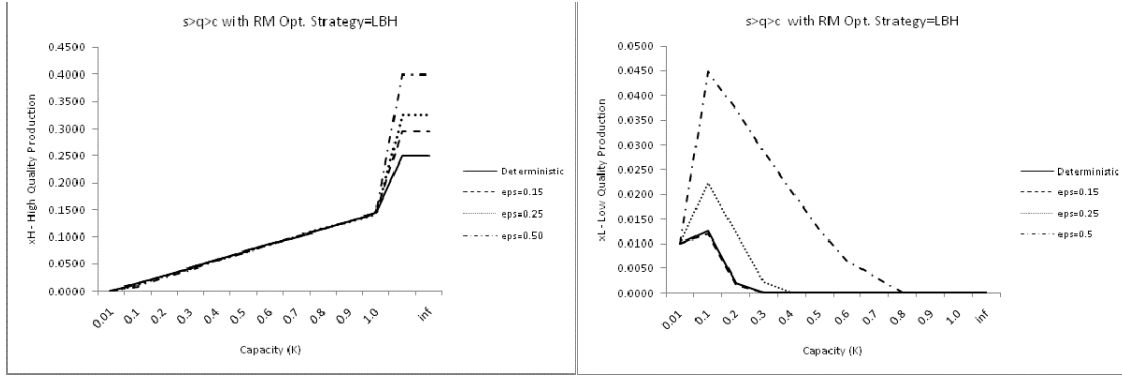
FIGURE 5.6: Product Line with Revenue Management when $c_l/q_l > c_h/q_h$ and $q_l - c_l/s_l > q_h - c_h/s_h$

for medium capacity levels like the previous example. This increase in the high quality commitment comes at the expense of low quality commitment: the commitment levels decrease as uncertainty increases for the low quality product.

Figure 5.6 shows an example where the cost to quality ratio is decreasing. This case favors the high quality production when capacity is large enough. However, marginal profit per unit resource is better for the low quality product $(q_l - c_l/s_l > q_h - c_h/s_h)$. In this case, the deterministic results and single-recourse model suggest that the firm should focus on the low quality product for scarce capacity whereas both products are offered at medium capacity levels and only high quality focus is optimal for large capacities. We observe the same behavior when there are multiple periods and more complex pricing options (i.e., revenue management), too: The firm focuses on the low quality product for scarce capacity levels, introduces high quality to the product mix as capacity increases and eventually ceases the production of low quality product all together above a certain threshold. We also observe that the high quality commitment increases with uncertainty when capacity is not binding as is the case before.

However, we observe that the low quality commitment increases with uncertainty for a range of capacity levels. This increase comes at the expense of a decrease in the high quality commitment at that range. Interestingly, in this case, the firm chooses to increase the commitment for the product that has a greater potential to bring profit per unit resource consumed at the expense of the high quality product. This example shows that the effect of increasing uncertainty is not trivial for medium capacity levels.

CHAPTER 6

Conclusions

In this dissertation, we study the optimal product line decisions of firms that are constrained by capacity. We characterize the conditions on the capacity and the costs that lead to significant deviations in the results from the existing literature. We solve the problem in various market structures: monopoly, duopoly, oligopoly, and socially efficient markets. We also introduced uncertainty into the formulation.

Among other results, our key findings in Chapter 2 are as follows.

When there are increasing costs to quality, i.e., when the unit cost to quality ratio of high quality product is larger than that of the low quality product, we find that the firm may be better off focusing on one product and offering either the low or the high quality product for sufficiently small capacity levels. The focus depends on the maximum margin as well as the capacity consumption per unit of each product type. This is in contrast to the existing literature that argues that the firm should serve both products offering a differentiated product line in this case. When the firm's capacity is sufficiently large, our results coincide with the existing literature.

When there are decreasing costs to quality, the firm's optimal strategy is either to

offer both products or only one of the products depending on its capacity. In this case, the existing literature that disregards the capacity constraint shows that the firm should always focus on the high quality product. While our results agree with the existing literature when the firm's capacity is sufficiently large, we show that the firm might be better off with a diametrically opposite policy focusing on the low quality product, when its capacity is sufficiently small, and for intermediate capacity levels the firm prefers offering both product types.

We also show that limited capacity can induce a monopoly to offer the higher end customers a better quality product compared to a social planner. Furthermore, a monopolist can cover a greater portion of the market than a social planner. These are in contrast to existing literature which shows that the customers (except those at the high end) get either a lower quality product or nothing at all from a monopoly compared to a social planner's assignment.

In Chapter 3, we extend the study for competitive markets. Among other results, our key findings are as follows.

There exists a unique symmetric pure strategy Nash equilibrium for symmetric duopolist firms even for limited capacity levels.

In competitive markets, the high quality product does not have to be offered in all equilibria as claimed in the literature. If the capacity adjusted profit margin of the low quality product is greater, then there exists a threshold capacity below which all firms optimally offer only the low quality product.

In response to increasing competition in the market, the incumbent firm may intro-

duce or prune some products from the product line. When there is increasing cost to quality ratio and unit profit is greater for the high quality product, the firm introduces new products to the market for a range of capacities in response to an increasing competition. When there is decreasing cost to quality ratio and the capacity adjusted profit margin is greater for the low quality product, the firm prunes the low quality products from the product mix in response to competitive pressures.

As opposed to the acquired wisdom in the literature, we show that the total industry supply may decrease as the number of firms increase in the market for a specific range of capacity levels when the potential profit per unit resource consumed is greater for the high quality product. Moreover, we show that the price of a product may increase as the number of firms increase for a specific range of capacity levels if the product has a low potential profit per unit resource consumed and when the cost to quality ratio is increasing and resource consumption ratio is small. On the other hand, the impact of increasing competition could be increasing the product variety. For instance, when the cost structures favor the high quality product, for a specific range of capacity levels increasing competition force the firms to introduce the low quality product into the product mix.

In Chapter 4, we study the profitability issues of the focused strategy firms.

We observe that the firm with a high quality focus may earn almost as good as a multiproduct firm. We also observe that the profitability of the high quality firm could go below 10% when the capacity is scarce. If the cost structure of the time that the firm operates is supportive, then the firm will survive just fine in competition with the

traditional airlines. However, if the cost conditions change, this could be detrimental for the firm.

We also find that the multiproduct competitor firm may adjust its product line in response to the high quality focused firm. While the optimal strategy for the competitor would be offering both products for high capacity levels in the symmetric game; we found instances where the competitor firm may adjust its strategy in response to high quality focused firm and actually focus on the low quality product for all capacity levels in this asymmetric game.

We also observe that the results for the low quality focused firms are particularly worse which leads to a conclusion that these firms should be extra careful to make sure that they operate at the optimal range of cost, quality and capacity ranges.

In Chapter 5, we introduced uncertainty into the model. We observe similar behavior as the deterministic cases when there is uncertainty about the market.

When cost to quality ratio favors the high quality product but the marginal per unit resource is better for the low quality product, the firm focuses on the low quality product for scarce capacity levels, introduces high quality to the product mix as capacity increases and eventually ceases the production of low quality product all together above a certain threshold. On the other hand, when cost to quality ratio favors the low quality product, the firm focuses on the low quality product for scarce capacity levels and optimally offers both products to the market when there is ample capacity.

In the recourse stage, when the capacity commitment of the low quality product is below a certain threshold and cost to quality ratio is increasing, we observe that the firm

only sells a limited amount of the low quality product and no high quality products if the market valuations are too low compared to the capacity commitments. If the valuations get a little better, then the firm optimally sells all the low quality product. Only if the market valuations are high enough, the firm starts selling the high quality product.

In the first stage, when the cost to quality ratio is increasing and capacity constraint is not binding, we have shown that the optimal strategy is to differentiate and offer both products in the market as is the case when there is no uncertainty. Nevertheless, there is one difference: the quantity of the high quality product increases with the level of uncertainty.

When the capacity constraint is binding, we present some numerical examples and observe how the results of the deterministic models change under uncertainty. When the cost to quality ratio is increasing and marginal profit per unit resource is better for the low quality product; for a medium range of capacity levels, high quality commitment increases as the level of uncertainty increases. However, since the capacity constraint is binding at this level, the required resources are gained from decreasing the commitment of the low quality product. The increase in the high quality commitment comes at the expense of low quality commitment: the commitment levels decrease as uncertainty increases for the low quality product.

As a conclusion, we find that the scarcity of capacity plays a critical role in determining the optimal product line. It can lead to expanding the product line with an additional product type, and it can also lead to a reduction in the product line serving fewer product types.

Appendix A: Appendix for Chapter

2

Proof of Proposition 1. We will first solve the general problem in Lemma A1 and then present the solution that corresponds to the parameters given in the proposition. Following the fact that $x_i = D_i$, the price-quantity equations (2.2) for the firm can be solved for prices as follows:

$$p_h = q_h \cdot (1 - x_h) - q_l \cdot x_l \qquad p_l = q_l \cdot (1 - x_l - x_h)$$

Then, the formulation takes the following final form where the Lagrangian variables that will help with the solution are provided in the parentheses:

$$\max \Pi^M = x_h \cdot (q_h \cdot (1 - x_h) - q_l \cdot x_l - c_h)$$
$$+ x_l \cdot (q_l \cdot (1 - x_l - x_h) - c_l)$$

$$\text{subject to}$$

$$x_h \cdot s_h + x_l \cdot s_l \leq K \qquad (\lambda)$$

$$x_h \geq 0 \qquad (\mu_h)$$

$$x_l \geq 0 \qquad (\mu_l)$$

Note that $Hessian(\Pi^M) = \begin{bmatrix} -2q_h & -2q_l \\ -2q_l & -2q_l \end{bmatrix}$. Given that $q_h > q_l$, the $Hessian(\Pi^M)$ is negative definite. Since the objective function of this problem is jointly concave on a convex set defined by linear constraints, the optimal solution can be obtained by solving the first order conditions together with the feasibility conditions citepbazaraa06. First order conditions are as follows for this problem:

$$-c_h + q_h - 2q_h x_h - 2q_l x_l - s_h \lambda + \mu_h = 0 \tag{A-1}$$

$$-c_l + q_l - 2q_l x_h - 2q_l x_l - s_l \lambda + \mu_l = 0 \tag{A-2}$$

$$(K - s_h x_h - s_l x_l)\lambda = 0 \tag{A-3}$$

$$x_l \mu_l = 0 \tag{A-4}$$

$$x_h \mu_h = 0 \tag{A-5}$$

where the feasibility conditions are as given below:

$$x_h \geq 0 \qquad x_l \geq 0 \qquad \lambda \geq 0 \qquad \mu_h \geq 0 \qquad \mu_l \geq 0 \qquad K \geq x_h \cdot s_h + x_l \cdot s_l \tag{A-6}$$

Lemma A1 *All solutions of the first order conditions are as follows:*

- *Solution 1:* $x_h = \frac{-c_h + c_l + q_h - q_l}{2(q_h - q_l)}$; $\qquad x_l = \frac{-c_l q_h + c_h q_l}{2(q_h - q_l)q_l}$;

  $\lambda = 0$; $\qquad \mu_h = 0$; $\qquad \mu_l = 0$.

- *Solution 2:* $x_h = -\frac{-2Kq_l s_h + 2Kq_l s_l - c_l s_h s_l + q_l s_h s_l + c_h s_l^2 - q_h s_l^2}{2(q_l s_h^2 - 2q_l s_h s_l + q_h s_l^2)}$;

  $x_l = -\frac{2Kq_l s_h + c_l s_h^2 - q_l s_h^2 - 2Kq_h s_l - c_h s_h s_l + q_h s_h s_l}{2(q_l s_h^2 - 2q_l s_h s_l + q_h s_l^2)}$;

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$$\lambda = -\frac{-2Kq_hq_l+2Kq_l^2-c_hq_ls_h+c_lq_ls_h+q_hq_ls_h-q_l^2s_h-c_lq_hs_l+c_hq_ls_l}{-q_ls_h^2+2q_ls_hs_l-q_hs_l^2} \; ; \qquad \mu_h = 0 \; ; \qquad \mu_l = 0.$$

- *Solution 3:* $x_h = 0 \; ; \qquad x_l = \frac{q_l-c_l}{2q_l} \; ;$

  $\lambda = 0 \; ; \qquad \mu_h = c_h - c_l - q_h + q_l \; ; \qquad \mu_l = 0.$

- *Solution 4:* $x_h = 0 \; ; \qquad x_l = \frac{K}{s_l} \; ;$

  $\lambda = -\frac{2Kq_l+c_ls_l-q_ls_l}{s_l^2} \; ; \qquad \mu_h = c_h - q_h + \frac{2Kq_l}{s_l} - \frac{s_h(2Kq_l+c_ls_l-q_ls_l)}{s_l^2} \; ; \qquad \mu_l = 0.$

- *Solution 5:* $x_h = \frac{-c_h+q_h}{2q_h} \; ; \qquad x_l = 0 \; ;$

  $\lambda = 0 \; ; \qquad \mu_h = 0 \; ; \qquad \mu_l = \frac{c_lq_h-c_hq_l}{q_h}.$

- *Solution 6:* $x_h = \frac{K}{s_h} \; ; \qquad x_l = 0 \; ;$

  $\lambda = -\frac{2Kq_h+c_hs_h-q_hs_h}{s_h^2} \; ; \qquad \mu_h = 0 \; ; \qquad \mu_l = c_l - q_l + \frac{2Kq_l}{s_h} - \frac{(2Kq_h+c_hs_h-q_hs_h)s_l}{s_h^2}.$

Proof. Since there are 3 constraints, there are 8 (2^3) possible solutions to the problem. Among these, there are two cases that give the trivial solution ($x_h = 0$ and $x_l = 0$). Thus, there are 6 solutions as listed. ∎

Following threshold capacities are defined in addition to thresholds (2.3)-(2.4) to facilitate the presentation of the solution:

$$\bar{K}_3^M = \frac{q_l(q_l - q_h)s_h + c_hq_l(-s_l + s_h) + c_l(q_hs_l - q_ls_h)}{2q_l(q_l - q_h)} \tag{A-7}$$

$$\bar{K}_4^M = \frac{(q_h - c_h)s_h}{2q_h} \tag{A-8}$$

$$\bar{K}_5^M = \frac{(q_l - c_l)s_l}{2q_l} \tag{A-9}$$

When $(c_h/c_l > q_h/q_l)$, for a monopolist the optimal product line configuration is as follows:

i) For parameters $q_l - c_l \geq q_h - c_h$, the solution is characterized as follows:

For $K < \bar{K}_5^M$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 4* of Lemma A1: $x_h = 0$ and $x_l = \frac{K}{s_l}$. For $K \geq \bar{K}_5^M$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 3* of Lemma A1: $x_h = 0$ and $x_l = \frac{q_l - c_l}{2q_l}$. Hence, the result follows.

ii.a) For parameters $q_l - c_l < q_h - c_h$, and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_1$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 4* of Lemma A1: $x_h = 0$ and $x_l = \frac{K}{s_l}$. For $\bar{K}_1 \leq K < \bar{K}_3^M$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 2* of Lemma A1: $x_h = \frac{2Kq_l(-s_l+s_h)+s_l(-c_hs_l+q_hs_l+c_ls_h-q_ls_h)}{2(q_hs_l^2+q_ls_h(-2s_l+s_h))}$ and $x_l = \frac{2K(q_hs_l-q_ls_h)+s_h(c_hs_l-q_hs_l-c_ls_h+q_ls_h)}{2(q_hs_l^2+q_ls_h(-2s_l+s_h))}$. For $K \geq \bar{K}_3^M$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 1* of Lemma A1: $x_h = \frac{(q_h-c_h)-(q_l-c_l)}{2(q_h-q_l)}$ and $x_l = \frac{q_lc_h-q_hc_l}{2q_l(q_h-q_l)}$. Hence, the result follows.

ii.b) For parameters $q_l - c_l < q_h - c_h$, and $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_2$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 6* of Lemma A1: $x_h = \frac{K}{s_h}$ and $x_l = 0$. For $\bar{K}_2 \leq K < \bar{K}_3^M$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 2* of Lemma A1: $x_h = \frac{2Kq_l(-s_l+s_h)+s_l(-c_hs_l+q_hs_l+c_ls_h-q_ls_h)}{2(q_hs_l^2+q_ls_h(-2s_l+s_h))}$ and $x_l = \frac{2K(q_hs_l-q_ls_h)+s_h(c_hs_l-q_hs_l-c_ls_h+q_ls_h)}{2(q_hs_l^2+q_ls_h(-2s_l+s_h))}$. For $K \geq \bar{K}_3^M$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 1* of Lemma A1: $x_h = \frac{(q_h-c_h)-(q_l-c_l)}{2(q_h-q_l)}$ and $x_l = \frac{q_lc_h-q_hc_l}{2q_l(q_h-q_l)}$. Hence, the result follows.

■

Proof of Proposition 2. Since the objective function of this problem is jointly concave on a convex set defined by linear constraints, the optimal solution can be obtained by solving the first order conditions (A-1)-(A-5) together with the feasibility conditions (A-6) (Bazaraa et al. (2006)), where all feasible solutions are provided in Lemma A1 given in the proof of Proposition 1.

In the following, we characterize the solutions that correspond to the parameters in Proposition 2. Note that we use threshold capacities defined in equations (2.3)-(2.4) and (A-7)-(A-9) to facilitate the presentation of the solutions.

When $(c_h/c_l \leq q_h/q_l)$, for a monopolist the optimal product line configuration is as follows:

i) For parameters $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_1$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 4* of Lemma A1: $x_h = 0$ and $x_l = \frac{K}{s_l}$. For $\bar{K}_1 \leq K < \bar{K}_2$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 2* of Lemma A1: $x_h = \frac{2Kq_l(-s_l + s_h) + s_l(-c_h s_l + q_h s_l + c_l s_h - q_l s_h)}{2(q_h s_l^2 + q_l s_h(-2s_l + s_h))}$ and $x_l = \frac{2K(q_h s_l - q_l s_h) + s_h(c_h s_l - q_h s_l - c_l s_h + q_l s_h)}{2(q_h s_l^2 + q_l s_h(-2s_l + s_h))}$. For $\bar{K}_2 \leq K < \bar{K}_4^M$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 6* of Lemma A1: $x_h = \frac{K}{s_h}$ and $x_l = 0$. For $K \geq \bar{K}_4^M$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 5* of Lemma A1: $x_h = \frac{q_h - c_h}{2q_h}$ and $x_l = 0$. Hence, the result follows.

ii) For parameters $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_4^M$, the only feasible solution to the equations (A-1)-(A-6) is the *Solution 6*

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of Lemma A1: $x_h = \frac{K}{s_h}$ and $x_l = 0$. For $K \geq \bar{K}_4^M$, the only feasible solution to

the equations (A-1)-(A-6) is the *Solution* 6 of Lemma A1: $x_h = \frac{q_h - c_h}{2q_h}$ and $x_l = 0$.

Hence, the result follows.

■

Proof of Corollary 1. The proof directly follows from the proofs of Propositions 1 and

2. It is also straightforward to show that the derivative with respect to K is non-negative

in each case. ■

Proof of Proposition 3. We will first characterize the solution of the social planner's

problem in Lemma A2, then the result in the Proposition will follow from this Lemma.

The social planners' problem in 2.6 leads to the following after solving for the integral

in the objective function (2.5), where the Lagrangian variables that will help with the

solution are provided in the parentheses:

$$\max \Pi^S = x_h \cdot (q_h \cdot (1 - \frac{x_h}{2} - c_h))$$
$$+ x_l \cdot (q_l \cdot (1 - x_h - \frac{x_l}{2}) - c_l)$$

subject to

$$s_h \cdot x_h + s_l \cdot x_l \leq K \qquad (\lambda)$$
$$x_h \geq 0 \qquad (\mu_h)$$
$$x_l \geq 0 \qquad (\mu_l)$$

$Hessian(\Pi^S) = \begin{bmatrix} -q_h & -q_l \\ -q_l & -q_l \end{bmatrix}$. Given that $q_h > q_l$, the $Hessian(\Pi^S)$ is negative definite.

Since the objective function of this problem is jointly concave on a convex set defined

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by linear constraints, the optimal solution can be obtained by solving the first order conditions together with the feasibility conditions citepbazaraa06. First order conditions are as follows for this problem:

$$-c_h + q_h - q_h x_h - q_l x_l - s_h \lambda + \mu_h = 0 \qquad \text{(A-10)}$$

$$-c_l - q_l(-1 + x_h + x_l) - s_l \lambda + \mu_l = 0 \qquad \text{(A-11)}$$

$$(K - s_h x_h - s_l x_l)\lambda = 0 \qquad \text{(A-12)}$$

$$x_l \mu_l = 0 \qquad \text{(A-13)}$$

$$x_h \mu_h = 0 \qquad \text{(A-14)}$$

where the feasibility conditions are given as below:

$$x_h \geq 0 \qquad x_l \geq 0 \qquad \lambda \geq 0 \qquad \mu_h \geq 0 \qquad \mu_l \geq 0 \qquad K \geq x_h \cdot s_h + x_l \cdot s_l \quad \text{(A-15)}$$

We define the following threshold capacities to facilitate the presentation of the solution:

$$\bar{K}_1^S = 2\bar{K}_1^M \qquad \text{(A-16)}$$

$$\bar{K}_2^S = 2\bar{K}_2^M \qquad \text{(A-17)}$$

$$\bar{K}_3^S = 2\bar{K}_3^M \qquad \text{(A-18)}$$

$$\bar{K}_4^S = 2\bar{K}_4^M \qquad \text{(A-19)}$$

$$\bar{K}_5^S = 2\bar{K}_5^M \qquad \text{(A-20)}$$

Lemma A2   *a) For the parameters $q_l - c_l \geq q_h - c_h$, the solution is as follows:*

For $K < \bar{K}_5^S$, $x_h = 0$ and $x_l = \frac{K}{s_l}$; for $K \geq \bar{K}_5^S$, $x_h = 0$ and $x_l = \frac{q_l - c_l}{q_l}$.

b) For the parameters $q_l - c_l < q_h - c_h$, and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$:

i) If $(c_h/c_l > q_h/q_l)$, the solution is as follows:

For $K < \bar{K}_1^S$, $x_h = 0$ and $x_l = \frac{K}{s_l}$;

for $\bar{K}_1^S \leq K < \bar{K}_3^S$, $x_h = \frac{Kq_l(-s_l + s_h) + s_l(-c_h s_l + q_h s_l + c_l s_h - q_l s_h)}{q_h s_l^2 + q_l s_h(-2s_l + s_h)}$ and

$x_l = \frac{K(q_h s_l - q_l s_h) + s_h(c_h s_l - q_h s_l - c_l s_h + q_l s_h)}{q_h s_l^2 + q_l s_h(-2s_l + s_h)}$;

for $K \geq \bar{K}_3^S$, $x_h = \frac{(q_h - c_h) - (q_l - c_l)}{q_h - q_l}$ and $x_l = \frac{q_l c_h - q_h c_l}{q_l(q_h - q_l)}$.

ii) If $(c_h/c_l \leq q_h/q_l)$, the solution is as follows:

For $K < \bar{K}_1^S$, $x_h = 0$ and $x_l^{SP} = \frac{K}{s_l}$;

for $\bar{K}_1^S \leq K < \bar{K}_2^S$, $x_h = \frac{Kq_l(-s_l + s_h) + s_l(-c_h s_l + q_h s_l + c_l s_h - q_l s_h)}{q_h s_l^2 + q_l s_h(-2s_l + s_h)}$ and

$x_l = \frac{K(q_h s_l - q_l s_h) + s_h(c_h s_l - q_h s_l - c_l s_h + q_l s_h)}{q_h s_l^2 + q_l s_h(-2s_l + s_h)}$;

for $\bar{K}_2^S \leq K < \bar{K}_4^S$, $x_h = \frac{K}{s_h}$ and $x_l = 0$;

for $K \geq \bar{K}_4^S$, $x_h = \frac{q_h - c_h}{q_h}$ and $x_l = 0$.

c) For the parameters $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$:

i) If $(c_h/c_l > q_h/q_l)$, the solution is as follows:

For $K < \bar{K}_2^S$, $x_h = \frac{K}{s_h}$ and $x_l = 0$;

for $\bar{K}_2^S \leq K < \bar{K}_3^S$, $x_h = \frac{Kq_l(-s_l + s_h) + s_l(-c_h s_l + q_h s_l + c_l s_h - q_l s_h)}{q_h s_l^2 + q_l s_h(-2s_l + s_h)}$ and

$x_l = \frac{K(q_h s_l - q_l s_h) + s_h(c_h s_l - q_h s_l - c_l s_h + q_l s_h)}{q_h s_l^2 + q_l s_h(-2s_l + s_h)}$;

for $K \geq \bar{K}_3^S$, $x_h = \frac{(q_h - c_h) - (q_l - c_l)}{q_h - q_l}$ and $x_l = \frac{q_l c_h - q_h c_l}{q_l(q_h - q_l)}$.

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ii) If $(c_h/c_l \leq q_h/q_l)$, the solution is as follows:

For $K < \bar{K}_4^S$, $x_h = \frac{K}{s_h}$ and $x_l = 0$; for $K \geq \bar{K}_4^S$, $x_h = \frac{q_h - c_h}{q_h}$ and $x_l = 0$.

Proof. Proof follows the same method as in Lemma A1 and Propositions 1 and 2. ∎

Following Propositions 1 and 2 and Lemma A2, when $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, both the monopoly firm and the social planner offers only the low quality product below a threshold capacity, but they offer both product types (high and low quality) above that threshold. The threshold for the monopoly firm and the social planner are $\bar{K}_1$ and $\bar{K}_1^S$ respectively where $\bar{K}_1^S = 2\bar{K}_1$. Thus, for all $\bar{K}_1 < K < \bar{K}_1^S$, the monopolist serve the high quality product while the social planner does not serve it, hence the result follows.

∎

Proof of Proposition 4. Following Propositions 1 and 2 and Lemma A2, when $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and $c_l/q_l \leq c_h/q_h$, the strategies for both the monopoly firm and the social planner is to offer only high quality product below a threshold capacity, but to offer both product types above that threshold. The threshold for the monopoly firm is $\bar{K}_2$ and it is $\bar{K}_2^S$ for the social planner where $\bar{K}_2^S = 2\bar{K}_2$. For $\bar{K}_2 < K < min\{\bar{K}_2^S, \bar{K}_3^M\}$, the optimal strategy for the monopolist firm is to offer both high and low quality product types ($x_h^M > 0$ and $x_l^M > 0$), while the social planner serves only the high quality product ($x_h^{SP} > 0$ and $x_l^{SP} = 0$). Notice that both the monopolist and the social planner use their whole capacity in this case. Thus, $s_h x_h^M + s_l x_l^M = K = s_h x_h^{SP} \Rightarrow x_h^M + \frac{x_l^M}{s_h/s_l} = x_h^{SP} \Rightarrow x_h^M + x_l^M > x_h^{SP}$. ∎

Proof of Proposition 5. The proof follows from Propositions 1 and 2 and Lemma A2:

i) When $q_l - c_l \geq q_h - c_h$, both monopoly and social planner assignments are same ($x_h = 0$ and $x_l = K/s_l$) for all $K < \bar{K}_5^M$.

ii) When $q_l - c_l < q_h - c_h$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, both monopoly and social planner assignments are same ($x_h = 0$ and $x_l = K/s_l$) for all $K < \bar{K}_1$.

iii) When $\frac{q_h - c_h}{s_h} = \frac{q_l - c_l}{s_l}$ and $c_h/c_l > q_h/q_l$, both monopoly and social planner assignments are same ($x_h^M = \frac{2Kq_l(-s_l+s_h)+s_l(-c_h s_l + q_h s_l + c_l s_h - q_l s_h)}{2(q_h s_l^2 + q_l s_h(-2s_l+s_h))} =$

$\frac{Kq_l(-s_l+s_h)+s_l(-c_h s_l + q_h s_l + c_l s_h - q_l s_h)}{q_h s_l^2 + q_l s_h(-2s_l+s_h)} = x_h^{SP}$ and $x_l^M = \frac{2K(q_h s_l - q_l s_h)+s_h(c_h s_l - q_h s_l - c_l s_h + q_l s_h)}{2(q_h s_l^2 + q_l s_h(-2s_l+s_h))}$

$= \frac{K(q_h s_l - q_l s_h)+s_h(c_h s_l - q_h s_l - c_l s_h + q_l s_h)}{q_h s_l^2 + q_l s_h(-2s_l+s_h)} = x_l^{SP}$) for all $K < \bar{K}_3^M$.

iv) When $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and $c_h/c_l > q_h/q_l$, both monopoly and social planner assignments are same ($x_h = K/s_h$ and $x_l = 0$) for all $K < \bar{K}_2$.

v) When $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and $c_h/c_l < q_h/q_l$, both monopoly and social planner assignments are same ($x_h = K/s_h$ and $x_l = 0$) for all $K < \bar{K}_4^M$.

∎

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Appendix B: Appendix for Chapter

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Lemma B1 *Asymmetric pure strategy Nash equilibriums are not optimal for the game described in Section 3.2.1.*

Proof.

The objective function 3.1 is strictly concave with linear constraints. Hence, following KKT conditions are necessary and sufficient to find the optimal quantities:

$$\partial_{y_f} V_1 = 0$$

$$\partial_{y_e} V_1 = 0$$

$$\lambda \left(K - s_e \, y_e - s_f \, y_f \right) = 0$$

$$\mu_f \, y_f = 0$$

$$\mu_e \, y_e = 0$$

The best response functions for Firm 1 and corresponding Lagrangian multipliers are as follows:

A: Strategy is to offer both classes at the unconstrained quantity.

$$y_f = -\frac{c_e - c_f + (q_e - q_f)(-1 + z_f)}{2(q_e - q_f)} \; ; \; y_e = \frac{-c_f \, q_e + c_e \, q_f + q_e \, (-q_e + q_f) \, z_e}{2 \, q_e \, (q_e - q_f)}$$

$$\mu_f = 0 \; ; \; \mu_e = 0 \; ; \; \lambda = 0$$

B: Strategy is to offer both classes at the capacity constrained quantity.

$$y_f = -\frac{2\,K\,q_e\,(s_e - s_f) + s_e\,(c_f\,s_e - c_e\,s_f + q_e\,s_f + q_e\,s_e\,z_e - q_e\,s_f\,z_e + q_f\,s_e\,(-1 + z_f) - q_e\,s_f\,z_f)}{2\,(q_f\,s_e^2 + q_e\,s_f\,(-2\,s_e + s_f))}$$

$$y_e = \frac{2\,K\,(q_f\,s_e - q_e\,s_f) + s_f\,(c_f\,s_e - c_e\,s_f + q_e\,s_f + q_e\,s_e\,z_e - q_e\,s_f\,z_e + q_f\,s_e\,(-1 + z_f) - q_e\,s_f\,z_f)}{2\,(q_f\,s_e^2 + q_e\,s_f\,(-2\,s_e + s_f))}$$

$$\mu_f = 0 \; ; \; \mu_e = 0$$

$$\lambda = \frac{2\,K\,q_e\,(q_e - q_f) - c_e\,q_f\,s_e + c_f\,q_e\,(s_e - s_f) + c_e\,q_e\,s_f - q_e^2\,s_f + q_e\,q_f\,s_f + q_e^2\,s_e\,z_e - q_e\,q_f\,s_e\,z_e + q_e^2\,s_f\,z_f - q_e\,q_f\,s_f\,z_f}{q_f\,s_e^2 + q_e\,s_f\,(-2\,s_e + s_f)}$$

C: Strategy is to offer only high quality product at the unconstrained quantity.

$$y_f = -\frac{c_f + q_e\,z_e + q_f\,(-1 + z_f)}{2\,q_f} \; ; \; y_e = 0$$

$$\mu_f = 0 \; ; \; \lambda = 0$$

$$\mu_e = \frac{-c_f\,q_e + c_e\,q_f + q_e\,(-q_e + q_f)\,z_e}{q_f}$$

D: Strategy is to offer nothing.

$$y_f = 0 \; ; \; y_e = 0 \; ; \; \lambda = 0$$

$$\mu_f = c_f + q_e\,z_e + q_f\,(-1 + z_f) \; ; \; \mu_e = c_e + q_e\,(-1 + z_e + z_f)$$

E: Strategy is to offer only high quality product at the capacity constrained quantity.

$$y_f = \frac{K}{s_f} \; ; \; y_e = 0$$

$$\lambda = -\frac{2\,K\,q_f + s_f\,(c_f + q_e\,z_e + q_f\,(-1 + z_f))}{s_f^2} \; ; \; \mu_f = 0$$

$$\mu_e = c_e - q_e + \frac{2\,K\,q_e}{s_f} + q_e\,z_e - \frac{s_e\,(2\,K\,q_f + s_f\,(c_f + q_e\,z_e + q_f\,(-1 + z_f)))}{s_f^2} + q_e\,z_f$$

F: Strategy is to offer only low quality product at the unconstrained quantity.

$$y_f = 0 \; ; \; y_e = -\frac{c_e + q_e\,(-1 + z_e + z_f)}{2\,q_e}$$

$$\mu_f = -c_e + c_f - (q_e - q_f)(-1 + z_f) \; ; \; \mu_e = 0 \; ; \; \lambda = 0$$

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G: Strategy is to offer only low quality product at the capacity constrained quantity.

$$y_f = 0 \; ; \; y_e = \frac{K}{s_e}$$

$$\mu_f = c_f - q_f + \frac{2 \, K \, q_e}{s_e} + q_e \, z_e + q_f \, z_f - \frac{s_f \, (2 \, K \, q_e + s_e \, (c_e + q_e \, (-1 + z_e + z_f)))}{s_e^2}$$

$$\mu_e = 0 \; ; \; \lambda = -\frac{2 \, K \, q_e + s_e \, (c_e + q_e \, (-1 + z_e + z_f))}{s_e^2}$$

We analyze the asymmetric equilibriums one by one as follows: We assign different strategies for each firm and solve the best response functions together to get the closed form solutions for the equilibrium quantities. Then, we check the feasibility and optimality conditions (the Lagrangian multipliers, the capacity constraint and the nonnegativity constraints) where applicable. If there are contradictions among these conditions, then the equilibrium is infeasible. In some cases where we cannot prove contradiction, we showed that there is incentive for either or both firms to move away from the equilibrium. Hence, the suggested equilibrium is not stable even if it were feasible. In the following part of the proof, W.L.O.G. we have assigned $q_e = 1$ and $q_f = q$ representing the quality ratio and $s_e = 1$ and $s_f = s$ representing the capacity usage ratio.

- **Firm 1: Strategy A, Firm 2: Strategy B**

  $\lambda \geq 0$ requires $c_f + (c_e + q) \, s \geq 3 \, K \, (-1 + q) + c_e \, q + s + c_f \, s$

  On the contrary; $y_e + s \, y_f < K$ requires $c_f + (c_e + q) \, s < 3 \, K \, (-1 + q) + c_e \, q + s + c_f \, s$

- **Firm 1: Strategy A, Firm 2: Strategy C**

  $y_e > 0$ requires $c_e \, q < c_f$

  On the contrary; $\mu_e \geq 0$ requires $c_e \, q \geq c_f$.

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- **Firm 1: Strategy A, Firm 2: Strategy D**

  $\mu_f \geq 0$ and $\mu_e \geq 0$ cannot be satisfied.

- **Firm 1: Strategy A, Firm 2: Strategy E**

  Equilibrium quantities and corresponding profit functions are as follows:

  $$y_e = -\frac{c_f \, q_e - c_e \, q_f}{2 \, q_e^2 - 2 \, q_e \, q_f}; \; y_f = \frac{K \, (-q_e + q_f) + (-c_e + c_f + q_e - q_f) \, s_f}{2 \, (q_e - q_f) \, s_f}; \; z_f = \frac{K}{s_f}; \; z_e = 0$$

  $$V_1 = \frac{K^2 \, (-1+q) \, q + 2 \, K \, (c_f - q) \, (-1+q) \, s + (c_f^2 - 2 \, c_f \, (-1 + c_e + q) + q \, (-1 + c_e^2 + q)) \, s^2}{4 \, (-1+q) \, s^2};$$

  $$V_2 = -\frac{K(K \, q + (c_f - q) \, s)}{2 \, s^2}$$

  We claim that Firm 2 can do better if they decrease the amount of high quality products by $\Delta > 0$ amount while increasing low quality products by Δ amount ($z_e' = \Delta$ and $z_f' = \frac{K}{s} - \Delta$). In the new scenario, both the prices and the profits of the firms change. We need to know whether there are any incentives (increase in the profit) under the new quantities for Firm 2.

  $$V_2' = \frac{K^2 \, q + s^2 \, \Delta \, (-1 + c_e - c_f + q + 2(-1+q)\Delta) + K \, s \, (c_f - q + 3 \, \Delta - 3 \, q \, \Delta)}{2 \, s^2}$$

  $$V_2' > V_2 \; \Leftrightarrow \; K > \frac{s \, (c_e - c_f + (-1+q) \, (1 + 2 \, \Delta))}{3 \, (-1+q)}$$

  $s \, y_f + y_e < K$ requires $K > \frac{c_f - c_e \, q - c_f \, s + (-1 + c_e + q) \, s}{3 \, (-1+q)}$

  $$K > \frac{c_f - c_e \, q - c_f \, s + (-1 + c_e + q) \, s}{3 \, (-1+q)} \geq \frac{s \, (c_e - c_f + (-1+q) \, (1 + 2 \, \Delta))}{3 \, (-1+q)} \; \Leftrightarrow \; c_f > c_e \, q + 2 \, (-1 + q) \, s \, \Delta$$

  Due to $y_e > 0$ condition; we know that $c_f > c_e \, q$. Then there exists small enough $\Delta > 0$ such that $c_f > c_e \, q + 2 \, (-1 + q) \, s \, \Delta$. This proves that $(V_2' > V_2)$ and Firm 2 has incentive to move away from this asymmetric equilibrium and earn more profit. Hence, this asymmetric equilibrium is not stable, hence it is not an optimal equilibrium.

- **Firm 1: Strategy A, Firm 2: Strategy F**

$y_f > 0$ requires $q - c_f > 1 - c_e$

On the contrary; $\mu_f \geq 0$ requires $q - c_f \leq 1 - c_e$.

- **Firm 1: Strategy A, Firm 2: Strategy G**

$\mu_f \geq 0$ requires $\frac{c_f - q + s - c_e\, s}{3\,(-1+s)} \geq K$

$y_e + s\, y_f < K$ requires $K > \frac{c_f - c_f\, s + (-1+q)\, s + c_e\,(-q+s)}{3\,(-1+q)}$

$\Rightarrow \quad \frac{c_f - q + s - c_e\, s}{3\,(-1+s)} > \frac{c_f - c_f\, s + (-1+q)\, s + c_e\,(-q+s)}{3\,(-1+q)} \quad \Rightarrow \quad 1 - c_e > q - c_f$

However, $y_f > 0$ requirement $1 - c_e < q - c_f$ is a contradiction.

- **Firm 1: Strategy C, Firm 2: Strategy B**

$\lambda \geq 0$ requires $K \leq \frac{q\,(-3\,c_e\,(q-s)+2\,(-1+q)\,s)-c_f\,(s+q\,(-3+2\,s))}{6\,(-1+q)\,q}$

$z_e > 0$ requires $K > -\frac{s\,(c_f\,(q+s)+q\,(-q+s-2\,c_e\,s))}{3\,q\,(q-s)}$

$\Rightarrow \quad \frac{q\,(-3\,c_e\,(q-s)+2\,(-1+q)\,s)-c_f\,(s+q\,(-3+2\,s))}{6\,(-1+q)\,q} > -\frac{s\,(c_f\,(q+s)+q\,(-q+s-2\,c_e\,s))}{3\,q\,(q-s)} \quad \Rightarrow \quad \frac{cf-ce\,q}{q-s} > 0$

$\mu_e \geq 0$ requires $K \geq \frac{(-1+q)\,(q-s)\,s-c_e\,(3\,q^2+2\,q\,(-3+s)\,s+s^2)+c_f\,(-q\,(-3+s)+s\,(-5+3\,s))}{3\,(-1+q)\,(q-s)}$

$\Rightarrow \quad \frac{q\,(-3\,c_e\,(q-s)+2\,(-1+q)\,s)-c_f\,(s+q\,(-3+2\,s))}{6\,(-1+q)\,q} \geq$

$$\frac{(-1+q)\,(q-s)\,s-c_e\,(3\,q^2+2\,q\,(-3+s)\,s+s^2)+c_f\,(-q\,(-3+s)+s\,(-5+3\,s))}{3\,(-1+q)\,(q-s)}$$

$\Rightarrow \quad \frac{cf-ce\,q}{q-s} \leq 0$ is a contradiction.

- **Firm 1: Strategy D, Firm 2: Strategy B**

$\lambda \geq 0$ requires $K \leq \frac{c_f - c_f\, s + (-1+q)\, s + c_e\,(-q+s)}{2\,(-1+q)}$

On the contrary; $\mu_e \geq 0$ requires $K \geq \frac{c_f - c_f\, s - (-1+c_e)(-3+s)\, s + q\,(1-2\,c_e+s)}{2\,(-1+q)}$

However, $K \geq \frac{c_f - c_f \, s - (-1 + c_e)(-3 + s) \, s + q \, (1 - 2 \, c_e + s)}{2 \, (-1 + q)} > \frac{c_f - c_f \, s + (-1 + q) \, s + c_e \, (-q + s)}{2 \, (-1 + q)} \geq K$ is a

contradiction.

- **Firm 1: Strategy E, Firm 2: Strategy B**

$z_e > 0$ requires $3 \, K \, (q - s) > s \, (q - c_f - s(1 - c_e))$

On the contrary; $\mu_e \geq 0$ requires $3 \, K \, (q - s) \leq s \, (q - c_f - s(1 - c_e))$.

- **Firm 1: Strategy F, Firm 2: Strategy B**

$\lambda \geq 0$ requires $K \leq -\frac{-1 + q + c_e \, (1 + 2 \, q - 3 \, s) + 3 \, c_f \, (-1 + s) + 3 \, s - 3 \, q \, s}{6 \, (-1 + q)}$

$z_f > 0$ requires $K > \frac{1 + 2 \, c_f - 2 \, q + s - c_e \, (1 + s)}{3 \, (-1 + s)}$

$\Rightarrow \quad -\frac{-1 + q + c_e \, (1 + 2 \, q - 3 \, s) + 3 \, c_f \, (-1 + s) + 3 \, s - 3 \, q \, s}{6 \, (-1 + q)} > \frac{1 + 2 \, c_f - 2 \, q + s - c_e \, (1 + s)}{3 \, (-1 + s)}$

$\Rightarrow \quad q - c_f > 1 - c_e$

$\mu_f \geq 0$ requires $K \geq \frac{2 \, q^2 + (-1 + ce) \, s \, (-5 + 3 \, s) - q \, (2 + c_e \, (-3 + s) + 5 \, s - 3 \, s^2) - c_f \, (1 + 2 \, q - 6 \, s + 3 \, s^2)}{3 \, (-1 + q) \, (-1 + s)}$

$\Rightarrow \quad -\frac{-1 + q + c_e \, (1 + 2 \, q - 3 \, s) + 3 \, c_f \, (-1 + s) + 3 \, s - 3 \, q \, s}{6 \, (-1 + q)} \geq$

$$\frac{2 \, q^2 + (-1 + ce) \, s \, (-5 + 3 \, s) - q \, (2 + c_e \, (-3 + s) + 5 \, s - 3 \, s^2) - c_f \, (1 + 2 \, q - 6 \, s + 3 \, s^2)}{3 \, (-1 + q) \, (-1 + s)}$$

$\Rightarrow \quad q - c_f \leq 1 - c_e$ is a contradiction.

- **Firm 1: Strategy G, Firm 2: Strategy B**

$z_f > 0$ requires $K > \frac{s \, (1 - c_e) - (q - c_f)}{3 \, (s - 1)}$

On the contrary; $\mu_f \geq 0$ requires $K \leq \frac{s \, (1 - c_e) - (q - c_f)}{3 \, (s - 1)}$.

- **Firm 1: Strategy D, Firm 2: Strategy C**

$\mu_f \geq 0$ cannot be satisfied.

- **Firm 1: Strategy E, Firm 2: Strategy C**

$\lambda \geq 0$ requires $K \leq \frac{s\,(q-c_f)}{3\,q}$

On the contrary; $z_e + s\,z_f < K$ requires $K > \frac{s\,(q-c_f)}{3\,q}$.

- **Firm 1: Strategy F, Firm 2: Strategy C**

$\mu_e \geq 0$ requires $c_e \geq \frac{1+3\,c_f-q}{1+2\,q}$

$\mu_f \geq 0$ requires $c_e \leq \frac{c_f+2\,c_f\,q-2\,(-1+q)\,q}{3\,q}$

However, $c_e \geq \frac{1+3\,c_f-q}{1+2\,q} > \frac{c_f+2\,c_f\,q-2\,(-1+q)\,q}{3\,q} \geq c_e$ is a contradiction.

- **Firm 1: Strategy G, Firm 2: Strategy C**

We have the following feasibility conditions for this problem:

$z_f > 0$ requires $q - c_f > K$

$s_f\,z_f < K$ requires $K > \frac{(-c_f+q)\,s}{2\,q+s}$

$\mu_e \geq 0$ requires $K \geq \frac{c_f-c_e\,q}{-1+q}$

$\mu_f \geq 0$ requires $K \leq \frac{c_f\,(q+s)+q\,(-q+s-2\,c_e\,s)}{-s+q\,(-3+4\,s)}$

$\lambda \geq 0$ requires $K \leq \frac{c_f+q-2\,c_e\,q}{-1+4\,q}$

All these feasibility conditions create the following conditions on the parameter sets:

$$q - c_f > \frac{c_f - c_e\,q}{-1+q} \tag{B-1}$$

$$q - c_f > \frac{(-c_f+q)\,s}{2\,q+s} \tag{B-2}$$

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$$\frac{c_f \, (q + s) + q \, (-q + s - 2 \, c_e \, s)}{-s + q \, (-3 + 4 \, s)} \geq \frac{c_f - c_e \, q}{-1 + q} \tag{B-3}$$

$$\frac{c_f \, (q + s) + q \, (-q + s - 2 \, c_e \, s)}{-s + q \, (-3 + 4 \, s)} > \frac{(-c_f + q) \, s}{2 \, q + s} \tag{B-4}$$

$$\frac{c_f + q - 2 \, c_e \, q}{-1 + 4 \, q} \geq \frac{c_f - c_e \, q}{-1 + q} \tag{B-5}$$

$$\frac{c_f + q - 2 \, c_e \, q}{-1 + 4 \, q} > \frac{(-c_f + q) \, s}{2 \, q + s} \tag{B-6}$$

For each parameter set, there is a different condition where we fail to find a feasible K level. When $(q > c)$, conditions (B-4) and (B-6) fail to hold. When $(c > q$ and $1 - c_e > q - c_f)$, conditions (B-1) and (B-5) fail to hold. When $(c > q > s$ and $q - c_f > 1 - c_e)$, condition (B-3) fails to hold.

When $(c > q$ and $s > q$ and $q - c_f > 1 - c_e)$, condition (B-5) holds if

$$(q - c_f) - (1 - c_e) \geq 2 \, c_e \, (c - q).$$

However, condition (B-6) holds if

$$2 \, c_e \, (c - q) > s \, (q - c_f - 1 + c_e) + (s - 1) \, (q - c_f) > (q - c_f) - (1 - c_e)$$

which is a contradiction to condition (B-5).

- **Firm 1: Strategy E, Firm 2: Strategy D**

$\mu_f \geq 0$ requires $K \geq \frac{s \, (q - c_f)}{q}$

On the contrary; $\lambda \geq 0$ requires $K \leq \frac{s \, (q - c_f)}{2 \, q}$.

However, $K \geq \frac{s \, (q - c_f)}{q} > \frac{s \, (q - c_f)}{2 \, q} \geq K$ is a contradiction.

- **Firm 1: Strategy F, Firm 2: Strategy D**

$\mu_e \geq 0$ cannot be satisfied.

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- **Firm 1: Strategy G, Firm 2: Strategy D**

$\mu_e \geq 0$ requires $K \geq 1 - c_e$

On the contrary; $\lambda \geq 0$ requires $K \leq (1 - c_e)/2$.

However, $K \geq 1 - c_e > (1 - c_e)/2 \geq K$ is a contradiction.

- **Firm 1: Strategy E, Firm 2: Strategy F**

Equilibrium quantities and corresponding profit functions are as follows:

$$y_e = 0; \ y_f = \frac{K}{s_f}; \ z_f = 0; \ z_e = \frac{1}{2}\left(1 - \frac{c_e}{q_e} - \frac{K}{s_f}\right)$$

$$V_1 = \frac{K\left(K\left(q_e - 2\,q_f\right) + \left(c_e - 2\,c_f - q_e + 2\,q_f\right)s_f\right)}{2\,s_f^2}; \ V_2 = \frac{\left(K\,q_e + \left(c_e - q_e\right)s_f\right)^2}{4\,q_e\,s_f^2}$$

We claim that Firm 1 can do better if they decrease the amount of high quality products by $\Delta > 0$ amount while increasing low quality products by Δ amount ($y_e' = \Delta$ and $y_f' = \frac{K}{s_f} - \Delta$). In the new scenario, both the prices and the profits of the firms change. We need to know whether there are any incentives (increase in the profit) under the new quantities for Firm 1.

$$V_1' = \frac{K^2\left(q_e - 2\,q_f\right) + K\,s_f\left(c_e - 2\,c_f - q_e + 2\,q_f - 4\,q_e\,\Delta + 4\,q_f\,\Delta\right) + 2\,s_f^2\,\Delta\left(-c_e + c_f + \left(q_e - q_f\right)\left(1 + \Delta\right)\right)}{2\,s_f^2}$$

$$V_1' > V_1 \ \Leftrightarrow \ K > \frac{s\left(q - c_f - 1 + c_e + \left(q - 1\right)\Delta\right)}{2\left(-1 + q\right)}$$

Due to $\mu_f \geq 0$ condition; $K \geq \frac{s\left(q - c_f - 1 + c_e\right)}{\left(-1 + q\right)} > \frac{s\left(q - c_f - 1 + c_e\right)}{2\left(-1 + q\right)}$. Then there exists small enough $\Delta > 0$ such that $K > \frac{s\left(q - c_f - 1 + c_e + \left(q - 1\right)\Delta\right)}{2\left(-1 + q\right)}$. This proves that Firm 1 has incentive to move away from this asymmetric equilibrium and earn more profit. Hence, this asymmetric equilibrium is not an optimal, stable equilibrium.

- **Firm 1: Strategy G, Firm 2: Strategy E**

$\mu_f \geq 0$ requires $K (q - s (2 s - 1)) \geq s (q - c_f - s (1 - c_e))$

On the contrary; $\mu_e \geq 0$ requires $s (q - c_f - s (1 - c_e)) \geq K (2 q - s (1 + s))$.

$\Rightarrow K (q - 2 s^2 + s) \geq K (2 q - s^2 - s) \Rightarrow (q - 2 s^2 + s) \geq (2 q - s^2 - s) \Rightarrow 0 \geq q + s^2 - 2 s > 0$ is a contradiction.

- **Firm 1: Strategy G, Firm 2: Strategy F**

$z_e > 0$ requires $K < 1 - c_e$

On the contrary; $z_e + s z_f < K$ requires $K > 1 - c_e$.

■

Proof of Proposition 6. In Lemma B1, we proved that the solution is not asymmetric. Then, we can solve the problem with symmetric best response functions and characterize the solution for different cost, quality and size parameters. We will solve the problem in Lemma B2 for all cases and then find out which ones correspond to each case listed in the Proposition 6.

Lemma B2 *The equilibrium quantities and feasibility conditions are as follows:*

AA: *Strategy is to offer both classes at the unconstrained quantity.*

$$y_f = z_f = \frac{-c_e + c_f + q_e - q_f}{3 (q_e - q_f)} \; ; \; y_e = z_e = -\frac{c_f q_e - c_e q_f}{3 q_e^2 - 3 q_e q_f}$$

$$c_e + q > 1 + c_f \; ; \; c_e q < c_f \; ; \; s_f y_f + s_e y_e < K$$

BB: *Strategy is to offer both classes at the capacity constrained quantity.*

$$y_f = z_f = \frac{3 K q_e (-s_e + s_f) + s_e (-c_f s_e + q_f s_e + c_e s_f - q_e s_f)}{3 (q_f s_e^2 + q_e s_f (-2 s_e + s_f))}$$

$$y_e = z_e = \frac{3 K (q_f s_e - q_e s_f) + s_f (c_f s_e - q_f s_e - c_e s_f + q_e s_f)}{3 (q_f s_e^2 + q_e s_f (-2 s_e + s_f))}$$

$$q + 3\,K\,(-1 + s) + c_e\,s > c_f + s \ ; \ 3\,K\,q + s\,(c_f + s) > s\,(3\,K + q + c_e\,s) \ ;$$

$$c_f + (c_e + q)\,s \geq 3\,K\,(-1 + q) + c_e\,q + s + c_f\,s$$

CC: *Strategy is to offer only high quality product at the unconstrained quantity.*

$$y_f = z_f = \frac{q_f - c_f}{3\,q_f} \ ; \ y_e = z_e = 0$$

$$c_e\,q \geq c_f \ ; \ s_f\,y_f < K$$

DD: *Strategy is to offer nothing.*

This is not a feasible equilibrium since both $\mu_f < 0$ and $\mu_e < 0$.

EE: *Strategy is to offer only high quality product at the capacity constrained quantity.*

$$y_f = z_f = \frac{K}{s_f} \ ; \ y_e = z_e = 0$$

$$s\,(-c_f + q + (-1 + c_e)\,s) \geq 3\,K\,(q - s) \ ; \ 3\,K\,q + c_f\,s \leq q\,s$$

FF: *Strategy is to offer only low quality product at the unconstrained quantity.*

$$y_f = z_f = 0 \ ; \ y_e = z_e = \frac{q_e - c_e}{3\,q_e}$$

$$1 + c_f \geq c_e + q \ ; \ s_e\,y_e < K$$

GG: *Strategy is to offer only low quality product at the capacity constrained quantity.*

$$y_f = z_f = 0 \ ; \ y_e = z_e = \frac{K}{s_e}$$

$$c_f + s \geq q + 3\,K\,(-1 + s) + c_e\,s \ ; \ c_e + 3\,K <= 1$$

Proof. Proof follows directly from the proof of Lemma B1. ∎

Following additional threshold capacities are defined to facilitate the presentation of the solution:

$$\bar{K}_1^D = 2/3\bar{K}_1^M \tag{B-7}$$

$$\bar{K}_2^D = 2/3\bar{K}_2^M \tag{B-8}$$

$$\bar{K}_3^D = 2/3\bar{K}_3^M \tag{B-9}$$

$$\bar{K}_4^D = 2/3\bar{K}_4^M \tag{B-10}$$

$$\bar{K}_5^D = 2/3\bar{K}_5^M \tag{B-11}$$

When $(c_h/c_l > q_h/q_l)$, for a duopolist, the optimal product line configuration is as follows:

i) For parameters $q_l - c_l \geq q_h - c_h$, the solution is characterized as follows:

For $K < \bar{K}_5^D$, the only feasible solution is the Solution GG of Lemma B2.

For $K \geq \bar{K}_5^D$, the only feasible solution is the Solution FF of Lemma B2. Hence, the result follows.

ii.a) For parameters $q_l - c_l < q_h - c_h$, and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_1^D$, the only feasible solution is the Solution GG of Lemma B2.

For $\bar{K}_1^D \leq K < \bar{K}_3^D$, the only feasible solution is the Solution BB of Lemma B2.

For $K \geq \bar{K}_3^D$, the only feasible solution is the Solution AA of Lemma B2. Hence, the result follows.

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ii.b) For parameters $q_l - c_l < q_h - c_h$, and $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_2^D$, the only feasible solution is the Solution EE of Lemma B2.

For $\bar{K}_2^D \leq K < \bar{K}_3^D$, the only feasible solution is the Solution BB of Lemma B2.

For $K \geq \bar{K}_3^D$, the only feasible solution is the Solution AA of Lemma B2. Hence, the result follows.

∎

Proof of Proposition 7. All feasible solutions of this problem are provided in Lemma B2 given in the proof of Proposition 6. In the following, we characterize the solutions that correspond to the parameters in Proposition 7. Note that we use threshold capacities defined in equations (B-7)-(B-8) and (B-9)-(B-11) to facilitate the presentation of the solutions.

When $(c_h/c_l \leq q_h/q_l)$, for a duopolist the optimal product line configuration is as follows:

i) For parameters $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_1^D$, the only feasible solution is the Solution GG of Lemma B2.

For $\bar{K}_1^D \leq K < \bar{K}_2^D$, the only feasible solution is the Solution BB of Lemma B2.

For $\bar{K}_2^D \leq K < \bar{K}_4^D$, the only feasible solution is the Solution EE of Lemma B2.

For $K \geq \bar{K}_4^D$, the only feasible solution is the Solution CC of Lemma B2. Hence, the result follows.

ii) For parameters $\frac{q_h - c_h}{s_h} \geq \frac{q_l - c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_4^D$, the only feasible solution is the Solution EE of Lemma B2.

For $K \geq \bar{K}_4^D$, the only feasible solution is the Solution CC of Lemma B2. Hence, the result follows.

∎

Proof of Proposition 8.

Following Propositions 1, 2, 6 and 7, when $q_h - c_h > q_l - c_l$ and $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$, both in the monopoly case and the duopoly cases, the firm offers only the low quality product below a threshold capacity; $\bar{K}_1$ and $\bar{K}_1^D$ respectively where $\bar{K}_1^D = 2/3 \, \bar{K}_1$. When there is increasing cost to quality ratio, the solution is to offer both product types (high and low quality) above those thresholds. Thus, there is increasing cost to quality ratio, for all $\bar{K}_1^D < K < \bar{K}_1$, the solution in the duopoly market is to offer both products while the solution in the monopoly market remains to be to offer only low quality product. When there is decreasing cost to quality ratio, the solution in the duopoly case is to offer only high quality product when $K > \bar{K}_2^D$. Since $\bar{K}_1^D < \bar{K}_2^D$, when there is decreasing cost to quality ratio, it is clear that in the range $\bar{K}_1^D < K < \min\{\bar{K}_1, \bar{K}_2^D\}$, the solution in the duopoly market for the firm is to offer both products while the solution in the monopoly market remains to be to offer only low quality product. Hence, the result follows. ∎

Proof of Proposition 9.

Following Propositions 1, 2, 6 and 7, when $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and there is increasing cost to quality ratio, both the monopoly case and the duopoly case solutions are to offer only the high quality product below a threshold capacity; $\bar{K}_2$ and $\bar{K}_2^D$ respectively where $\bar{K}_2^D = 2/3 \, \bar{K}_2$. The firm offers both product types (high and low quality) above those

thresholds. Thus, for all $\bar{K}_2^D < K < \bar{K}_2$, in the duopoly case both products are offered while in the monopoly case solution remains to be to offer only high quality product. Hence, the result follows. ■

Proof of Proposition 10.

Following Propositions 1, 2, 6 and 7, when $(q_h - c_h)/s_h < (q_l - c_l)/s_l$ and there is decreasing cost to quality ratio, both the monopoly case and the duopoly case solutions are to offer only the high quality product above a threshold capacity; $\bar{K}_2$ and $\bar{K}_2^D$ respectively where $\bar{K}_2^D = 2/3 \; \bar{K}_2$. The firm offers both product types (high and low quality) below those thresholds. Thus, for all $\bar{K}_2^D < K < \bar{K}_2$, in the duopoly case offering only high quality product is the optimal solution while in the monopoly case solution remains to be offering both products. Hence, the result follows. ■

Proof of Proposition 11. Following threshold capacities are defined in addition to thresholds presented earlier to facilitate the presentation of the proof:

$$\bar{K}_1^{TD} = 2\bar{K}_1^D \tag{B-12}$$

$$\bar{K}_2^{TD} = 2\bar{K}_2^D \tag{B-13}$$

$$\bar{K}_3^{TD} = 2\bar{K}_3^D \tag{B-14}$$

$$\bar{K}_4^{TD} = 2\bar{K}_4^D \tag{B-15}$$

$$\bar{K}_5^{TD} = 2\bar{K}_5^D \tag{B-16}$$

Following Propositions 1, 2, 6 and 7, when $(q_h - c_h > q_l - c_l)$, and $(q_h - c_h)/s_h < (q_l - c_l)/s_l$, both in the monopoly case and the duopoly cases, the firm offers only the

low quality product below a threshold capacity; $\bar{K}_1$ and $\bar{K}_1^{TD}$ respectively where $\bar{K}_1^{TD} = 4/3 \ \bar{K}_1$. When there is increasing cost to quality ratio, the solution is to offer both product types (high and low quality) above those thresholds. Thus, there is increasing cost to quality ratio, for all $\bar{K}_1 < K < \bar{K}_1^{TD}$, the solution in the merger is to offer both products while the solution in the duopoly market is to offer only low quality product. When there is decreasing cost to quality ratio, the solution in the merger case is to offer only high quality product when $K > \bar{K}_2$. Since $\bar{K}_1 < \bar{K}_2$, when there is decreasing cost to quality ratio, it is clear that in the range $\bar{K}_1 < K < \min\{\bar{K}_2, \bar{K}_1^{TD}\}$, the solution in the monopoly market for the firm is to offer both products while the solution in the duopoly market remains to be to offer only low quality product. Hence, the result follows. ∎

Proof of Proposition 12. Following Propositions 1, 2, 6 and 7, when $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and there is increasing cost to quality ratio, both the monopoly case and the duopoly case solutions are to offer only the high quality product below a threshold capacity; $\bar{K}_2$ and $\bar{K}_2^{TD}$ respectively where $\bar{K}_2^{TD} = 4/3 \ \bar{K}_2$. The optimal offerings are both product types (high and low quality) above those thresholds. Thus, for all $\bar{K}_2 < K < \bar{K}_2^{TD}$, in the merger case both products are offered while in the duopoly case solution remains to be to offer only high quality product. Hence, the result follows. ∎

Proof of Proposition 13. Following Propositions 1, 2, 6 and 7, when $(q_h - c_h)/s_h < (q_l - c_l)/s_l$ and there is decreasing cost to quality ratio, both the monopoly case and the duopoly case solutions are to offer only the high quality product above a threshold capacity; $\bar{K}_2$ and $\bar{K}_2^{TD}$ respectively where $\bar{K}_2^{TD} = 4/3 \ \bar{K}_2$. The optimal offerings are both product types (high and low quality) below those thresholds. Thus, for all $\bar{K}_2 < K <$

$\bar{K}_2^{TD}$, in the merger case offering only high quality product is the optimal solution while in the duopoly case solution remains to be offering both products. Hence, the result follows. ∎

Proof of Proposition 14.

Following Propositions 1, 2, 6 and 7, when $(q_h - c_h)/s_h > (q_l - c_l)/s_l$ and there is increasing cost to quality ratio, the strategies for both the monopoly firm and the duoplists is to offer only high quality product below a threshold capacity, but to offer both product types above that threshold. The threshold for the monopoly firm is $\bar{K}_2$ and it is $\bar{K}_2^{TD}$ for the merger where $\bar{K}_2^{TD} = 4/3\bar{K}_2$. For $\bar{K}_2 < K < min\{\bar{K}_2^{TD}, \bar{K}_3\}$, the optimal strategy for the monopolist firm is to offer both high and low quality product types ($x_h > 0$ and $x_l > 0$), while the duopolist serve only the high quality product ($y_h > 0, z_h > 0$ and $y_l = 0, z_l = 0$). Notice that both the monopolist and the duopolists use their whole capacity in this case. Thus, $s_h x_h + s_l x_l = K = s_h(y_h + z_h) \Rightarrow x_h + \frac{x_l}{s_h/s_l} = y_h + z_h \Rightarrow x_h + x_l > y_h + z_h$.

The average quality offered in the market when there are two competing duopolists is exactly q_h. On the other hand, when there is only one integrated monopolist, the average quality offered in the market is $(x_h q_h + x_l q_l)/(x_h + x_l) < q_h$.

∎

Proof of Proposition 15. We solve the problem with symmetric best response functions and characterize the solution for different cost, quality and size parameters. Following additional threshold capacities are defined to facilitate the presentation of the

solution:

$$\bar{K}_1^{(n)} = 2/(n+1)\bar{K}_1^M \tag{B-17}$$

$$\bar{K}_2^{(n)} = 2/(n+1)\bar{K}_2^M \tag{B-18}$$

$$\bar{K}_3^{(n)} = 2/(n+1)\bar{K}_3^M \tag{B-19}$$

$$\bar{K}_4^{(n)} = 2/(n+1)\bar{K}_4^M \tag{B-20}$$

$$\bar{K}_5^{(n)} = 2/(n+1)\bar{K}_5^M \tag{B-21}$$

$$\bar{K}_1^{T(n)} = 2n/(n+1)\bar{K}_1^M \tag{B-22}$$

$$\bar{K}_2^{T(n)} = 2n/(n+1)\bar{K}_2^M \tag{B-23}$$

$$\bar{K}_3^{T(n)} = 2n/(n+1)\bar{K}_3^M \tag{B-24}$$

$$\bar{K}_4^{T(n)} = 2n/(n+1)\bar{K}_4^M \tag{B-25}$$

$$\bar{K}_5^{T(n)} = 2n/(n+1)\bar{K}_5^M \tag{B-26}$$

$$\tag{B-27}$$

When $(c_h/c_l > q_h/q_l)$, for an oligopolist, the optimal product line configuration is as follows:

i) For parameters $q_l - c_l \geq q_h - c_h$, the solution is characterized as follows:

For $K < \bar{K}_5^{(n)}$, the only feasible solution is $w_h = 0$ and $w_l = K/se$.

For $K \geq \bar{K}_5^{(n)}$, the only feasible solution is $w_h = 0$ and $w_l = \frac{qe-ce}{(n+1)qe}$. Hence, the result follows.

ii.a) For parameters $q_l - c_l < q_h - c_h$, and $\frac{q_h-c_h}{s_h} < \frac{q_l-c_l}{s_l}$, the solution is characterized as

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follows:

For $K < \bar{K}_1^{(n)}$, the only feasible solution is $w_h = 0$ and $w_l = K/se$.

For $\bar{K}_1^{(n)} \leq K < \bar{K}_3^{(n)}$, the only feasible solution is

$$w_h = \frac{-K(1+n)qe(se-sf)+se(-cfse+qfse+cesf-qesf)}{(1+n)(qfse^2+qesf(-2se+sf))}$$

and $w_l = \frac{K(1+n)(qfse-qesf)+sf(cfse-qfse-cesf+qesf)}{(1+n)(qfse^2+qesf(-2se+sf))}$.

For $K \geq \bar{K}_3^{(n)}$, the only feasible solution is $w_h = \frac{-ce+cf+qe-qf}{(1+n)(qe-qf)}$ and $w_l = \frac{-cfqe+ceqf}{(1+n)qe(qe-qf)}$.

Hence, the result follows.

ii.b) For parameters $q_l - c_l < q_h - c_h$, and $\frac{q_h-c_h}{s_h} \geq \frac{q_l-c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_2^{(n)}$, the only feasible solution is $w_h = K/sf$ and $w_l = 0$.

For $\bar{K}_2^{(n)} \leq K < \bar{K}_3^{(n)}$, the only feasible solution is

$$w_h = \frac{-K(1+n)qe(se-sf)+se(-cfse+qfse+cesf-qesf)}{(1+n)(qfse^2+qesf(-2se+sf))}$$

and $w_l = \frac{K(1+n)(qfse-qesf)+sf(cfse-qfse-cesf+qesf)}{(1+n)(qfse^2+qesf(-2se+sf))}$.

For $K \geq \bar{K}_3^{(n)}$, the only feasible solution is $w_h = \frac{-ce+cf+qe-qf}{(1+n)(qe-qf)}$ and $w_l = \frac{-cfqe+ceqf}{(1+n)qe(qe-qf)}$.

Hence, the result follows.

∎

Proof of Proposition 16. We characterize the solutions that correspond to the parameters in Proposition 7. When $(c_h/c_l \leq q_h/q_l)$, for an oligopolist the optimal product line configuration is as follows:

i) For parameters $\frac{q_h-c_h}{s_h} < \frac{q_l-c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_1^{(n)}$, the only feasible solution is $w_h = 0$ and $w_l = K/se$.

For $\bar{K}_1^{(n)} \leq K < \bar{K}_2^{(n)}$, $w_h = \frac{-K(1+n)qe(se-sf)+se(-cfse+qfse+cesf-qesf)}{(1+n)(qfse^2+qesf(-2se+sf))}$ and $w_l = $

$\frac{K(1+n)(qfse-qesf)+sf(cfse-qfse-cesf+qesf)}{(1+n)(qfse^2+qesf(-2se+sf))}$.

For $\bar{K}_2^{(n)} \leq K < \bar{K}_4^{(n)}$, the only feasible solution is $w_h = K/sf$ and $w_l = 0$.

For $K \geq \bar{K}_4^{(n)}$, the only feasible solution is $w_h = \frac{qf-cf}{(n+1)qf}$ and $w_l = 0$. Hence, the

result follows.

ii) For parameters $\frac{q_h-c_h}{s_h} \geq \frac{q_l-c_l}{s_l}$, the solution is characterized as follows:

For $K < \bar{K}_4^{(n)}$, the only feasible solution is $w_h = K/sf$ and $w_l = 0$.

For $K \geq \bar{K}_4^{(n)}$, the only feasible solution is $w_h = \frac{qf-cf}{(n+1)qf}$ and $w_l = 0$. Hence, the

result follows.

■

Proof of Proposition 17.

Following Propositions 15 and 16, when $q_h - c_h > q_l - c_l$ and $\frac{q_h-c_h}{s_h} < \frac{q_l-c_l}{s_l}$, the

firm offers only the low quality product below a threshold capacity; $\bar{K}(n)_1$ and $\bar{K}_1^{(n+1)}$

respectively where $\bar{K}_1^{(n+1)} = (n+1)/(n+2) \, \bar{K}_1^{(n)}$. When there is increasing cost to quality

ratio, the solution is to offer both product types (high and low quality) above those

thresholds. Thus, there is increasing cost to quality ratio, for all $\bar{K}_1^{(n+1)} < K < \bar{K}_1^{(n)}$, the

solution in the n+1 firm market is to offer both products while the solution in the n-firm

market remains to be to offer only low quality product. When there is decreasing cost to

quality ratio, the solution in the duopoly case is to offer only high quality product when

$K > \bar{K}_2^{(n+1)}$. Since $\bar{K}_1^{(n+1)} < \bar{K}_2^{(n+1)}$, when there is decreasing cost to quality ratio, it is

clear that in the range $\bar{K}_1^{(n+1)} < K < \min\{\bar{K}_1^{(n)}, \bar{K}_2^{(n+1)}\}$, the solution in the n+1 firm market is to offer both products while the solution in the n-firm market remains to be to offer only low quality product. Hence, the result follows. ∎

Proof of Proposition 18.

Following Propositions 15 and 16, when $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and there is increasing cost to quality ratio, both the n-firm case and the n+1 firm case solutions are to offer only the high quality product below a threshold capacity; $\bar{K}_2^{(n)}$ and $\bar{K}_2^{(n+1)}$ respectively where $\bar{K}_2^{(n+1)} = (n+1)/(n+2) \, \bar{K}_2^{(n)}$. The firm offers both product types (high and low quality) above those thresholds. Thus, for all $\bar{K}_2^{(n+1)} < K < \bar{K}_2^{(n)}$, in the n+1 firm case both products are offered while in the n-firm case solution remains to be to offer only high quality product. Hence, the result follows. ∎

Proof of Proposition 19.

Following Propositions 15 and 16, when $(q_h - c_h)/s_h < (q_l - c_l)/s_l$ and there is decreasing cost to quality ratio, both the n firm and the n+1 firm case solutions are to offer only the high quality product above a threshold capacity; $\bar{K}_2^{(n)}$ and $\bar{K}_2^{(n+1)}$ respectively where $\bar{K}_2^{(n+1)} = (n+1)/(n+2) \, \bar{K}_2^{(n)}$. The firm offers both product types (high and low quality) below those thresholds. Thus, for all $\bar{K}_2^{(n+1)} < K < \bar{K}_2^{(n)}$, in the n+1 firm case offering only high quality product is the optimal solution while in the n-firm case solution remains to be offering both products. Hence, the result follows. ∎

Proof of Proposition 20. When $(q_h - c_h)/s_h < (q_l - c_l)/s_l$,

If $c_h/q_h > c_l/q_l$, then following Proposition 15 when each firm has K capacity: For $0 < K < \bar{K}_1^{(n)}$, $w_h^* = 0 \Rightarrow \partial_n w_h^* = 0$.

For $\bar{K}_1^{(n)} \le K < \bar{K}_3^{(n)}$, $w_h^* = \frac{-K(1+n)q_l(s_l-s_h)+s_l(-c_h s_l+q_h s_l+c_l s_h-q_l s_h)}{(1+n)(q_h s_l^2+q_l s_h(-2s_l+s_h))} \Rightarrow \partial_n w_h^* > 0$. For $K \ge \bar{K}_3^{(n)}$, $\partial_n w_h^* < 0$.

If $c_h/q_h \le c_l/q_l$, then following Proposition 16 when each firm has K capacity: For total industry capacity of $0 < K < \bar{K}_1^{(n)}$, $w_h^* = 0 \Rightarrow \partial_n w_h^* = 0$. For $\bar{K}_1^{(n)} \le K < \bar{K}_2^{(n)}$,

$w_h^* = \frac{-K(1+n)q_l(s_l-s_h)+s_l(-c_h s_l+q_h s_l+c_l s_h-q_l s_h)}{(1+n)(q_h s_l^2+q_l s_h(-2s_l+s_h))} \Rightarrow \partial_n w_h^* > 0$. For $\bar{K}_2^{(n)} \le K < \bar{K}_4^{(n)}$, $w_h^* = K/s_h \Rightarrow \partial_n w_h^* = 0$. For $K \ge \bar{K}_4^{(n)}$, $\partial_n t_h^* < 0$.

Hence, the result follows.

∎

Proof of Proposition 21.

When $(q_h - c_h)/s_h > (q_l - c_l)/s_l$ and $c_h/q_h > c_l/q_l$, then following Proposition 15 when each firm has K capacity: For $0 < K < \bar{K}_2^{(n)}$, $w_l^* = 0 \Rightarrow \partial_n w_l^* = 0$. For $\bar{K}_2^{(n)} \le K < \bar{K}_3^{(n)}$,

$w_l^* = \frac{K(1+n)(q_h s_l-q_l s_h)+s_h(c_h s_l-q_h s_l-c_l s_h+q_l s_h)}{(1+n)(q_h s_l^2+q_l s_h(-2s_l+s_h))} \Rightarrow \partial_n w_l^* > 0$. For $K \ge \bar{K}_3^{(n)}$, $\partial_n w_l^* < 0$. Hence,

the result follows.

∎

Proof of Proposition 22. Following Propositions 15 and 16, when $(q_h - c_h > q_l - c_l)$, and $(q_h - c_h)/s_h < (q_l - c_l)/s_l$, the firm offers only the low quality product below a threshold capacity; $\bar{K}_1^{T(n)}$ and $\bar{K}_1^{T(n+1)}$ respectively. When there is increasing cost to quality ratio, the solution is to offer both product types (high and low quality) above those thresholds. Thus, there is increasing cost to quality ratio, for all $\bar{K}_1^{T(n)} < K < \bar{K}_1^{T(n+1)}$, the solution in the n-firm case is to offer both products while the solution in the n+1 firm market is to offer only low quality product. When there is decreasing cost to quality ratio, the solution in the n-firm case is to offer only high quality product when $K > \bar{K}_2^{T(n)}$.

Since $\bar{K}_1^{T(n)} < \bar{K}_2^{T(n)}$, when there is decreasing cost to quality ratio, it is clear that in the range $\bar{K}_1^{T(n)} < K < \min\{\bar{K}_2^{T(n)}, \bar{K}_1^{T(n+1)}\}$, the solution in the n-firm market for the firm is to offer both products while the solution in the n+1 firm market remains to be to offer only low quality product. Hence, the result follows. ∎

Proof of Proposition 23. Following Propositions 15 and 16, when $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$ and there is increasing cost to quality ratio, both the n-firm case and the n+1 firm case solutions are to offer only the high quality product below a threshold capacity; $\bar{K}_2^{T(n)}$ and $\bar{K}_2^{T(n+1)}$ respectively. The optimal offerings are both product types (high and low quality) above those thresholds. Thus, for all $\bar{K}_2^{T(n)} < K < \bar{K}_2^{T(n+1)}$, in the n-firm case both products are offered while in the n+1 case solution remains to be to offer only high quality product. Hence, the result follows. ∎

Proof of Proposition 24. Following Propositions 15 and 16, when $(q_h - c_h)/s_h < (q_l - c_l)/s_l$ and there is decreasing cost to quality ratio, both the n-firm case and the n+1 firm case solutions are to offer only the high quality product above a threshold capacity; $\bar{K}_2^{T(n)}$ and $\bar{K}_2^{T(n+1)}$ respectively. The optimal offerings are both product types (high and low quality) below those thresholds. Thus, for all $\bar{K}_2^{T(n)} < K < \bar{K}_2^{T(n+1)}$, in the n firm case offering only high quality product is the optimal solution while in the n+1 firm case solution remains to be offering both products. Hence, the result follows. ∎

Proof of Proposition 25.

i) When $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$:

- If $c_h/q_h > c_l/q_l$, then following Proposition 15 when each firm has K/n capacity and $t_h^* = n w_h^*$: For total industry capacity of $0 < K < \bar{K}_1^{T(n)}$, $\partial_n t_h^* = 0$. For

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total industry capacity of $\bar{K}_1^{T(n)} \leq K < \bar{K}_3^{T(n)}$, $\partial_n t_h^* < 0$. For total industry

capacity of $K \geq \bar{K}_3^{T(n)}$, $\partial_n t_h^* > 0$. Hence, the result follows.

- If $c_h/q_h \leq c_l/q_l$, then following Proposition 16 when each firm has K/n capac-

  ity and $t_h^* = nw_h^*$: For total industry capacity of $0 < K < \bar{K}_1^{T(n)}$, $\partial_n t_h^* = 0$.

  For total industry capacity of $\bar{K}_1^{T(n)} \leq K < \bar{K}_2^{T(n)}$, $\partial_n t_h^* < 0$. For total indus-

  try capacity of $\bar{K}_2^{T(n)} \leq K < \bar{K}_4^{T(n)}$, $\partial_n t_h^* = 0$. For total industry capacity of

  $K \geq \bar{K}_4^{T(n)}$, $\partial_n t_h^* > 0$. Hence, the result follows.

ii) When $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$:

- If $c_h/q_h > c_l/q_l$, then following Proposition 15 when each firm has K/n capac-

  ity and $t_l^* = nw_l^*$: For total industry capacity of $0 < K < \bar{K}_2^{T(n)}$, $\partial_n t_l^* = 0$. For

  total industry capacity of $\bar{K}_2^{T(n)} \leq K < \bar{K}_3^{T(n)}$, $\partial_n t_l^* < 0$. For total industry

  capacity of $K \geq \bar{K}_3^{T(n)}$, $\partial_n t_l^* > 0$. Hence, the result follows.

- If $c_h/q_h \leq c_l/q_l$, then following Proposition 16 for all capacity levels $w_l^* = 0$,

  hence $\partial_n t_l^* = 0$. Hence, the result follows.

■

Proof of Proposition 26. When $c_h/q_h > c_l/q_l$ and $q_h/s_h > q_l/s_l$;

i) If $\frac{q_h - c_h}{s_h} < \frac{q_l - c_l}{s_l}$; then following Proposition 15 when each firm has K/n capacity

and $p_h^* = q_h(1 - nw_h^*) - q_l(nw_l^*)$: For total industry capacity of $0 < K < \bar{K}_1^{T(n)}$,

$\partial_n p_h^* = 0$. For total industry capacity of $\bar{K}_1^{T(n)} < K < \bar{K}_3^{T(n)}$, $\partial_n p_h^* > 0$. For total

industry capacity of $K \geq \bar{K}_3^{T(n)}$, $\partial_n p_h^* < 0$. Hence, the result follows.

ii) If $\frac{q_h - c_h}{s_h} > \frac{q_l - c_l}{s_l}$; then following Proposition 15 when each firm has K/n capacity and

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$p_l^* = q_l(1 - nw_h^* - nw_l^*)$: For total industry capacity of $0 < K < \bar{K}_2^{T(n)}$, $\partial_n p_l^* = 0$.

For total industry capacity of $\bar{K}_2^{T(n)} < K < \bar{K}_3^{T(n)}$, $\partial_n p_l^* > 0$. For total industry

capacity of $K \geq \bar{K}_3^{T(n)}$, $\partial_n p_l^* < 0$. Hence, the result follows.

■

Appendix C: Appendix for Chapter

4

Proof of Proposition 31. The problem can be re-written as follows:

$$\max_{x_h \geq 0} \quad \pi_h = (p_h(x_h) - c_h)x_h$$

$$\text{subject to} \quad s_h x_h \leq K.$$

The objective function of this problem is strictly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)). First order conditions are as follows for this problem:

$$-c_h + q_h - 2q_h x_h - s_h \lambda + \mu = 0$$

$$(K - s_h x_h)\lambda = 0$$

$$x_h \mu = 0$$

where the feasibility conditions are as given below:

$$x_h \geq 0 \qquad \lambda \geq 0 \qquad \mu \geq 0 \qquad K \geq s_h x_h$$

The solution of the first order conditions yield 3 alternatives:

Solution 1: $x_h = \frac{q_h - c_h}{2q_h}; \lambda = 0; \mu = 0.$

Solution 2: $x_h = \frac{K}{s_h}; \lambda = \frac{-2Kq_h + (-c_h + q_h)s_h}{s_h^2}; \mu = 0.$

Solution 3: $x_h = 0; \lambda = 0; \mu = c_h - q_h.$

Combined with the feasibility conditions, the result in the proposition follows.

■

Proof of Proposition 32. The parametric set we investigate is $q_l - c_l > q_h - c_h$ and $c_h/c_l > q_h/q_l > s_h/s_l$. In this parametric set, the intuitive and trivial optimal solution is focusing on the low quality. Then, the lower bound that is achieved under these conditions would be a lower bound for all other cost conditions. Let's start by calculating the profit ratios for all capacity levels. There are 3 regions that needs to be studied in this case: $0 < K \leq \bar{K}_4^M$, $\bar{K}_4^M < K \leq \bar{K}_5^M$, and $K > \bar{K}_5^M$.

For $0 < K \leq \bar{K}_4^M$, $\pi_h^* = -\frac{K(Kq_h + (c_h - q_h)s_h)}{s_h^2}$ and $\pi^* = -\frac{K(Kq_l + (c_l - q_l)s_l)}{s_l^2}$. In this region, although both profits are increasing in K, the ratio $\frac{\pi_h^*}{\pi^*}$ is decreasing.

At $\bar{K}_4^M$, π_h^* assumes its highest value at $\pi_h^* = \frac{(q_h - c_h)^2}{4q_h}$ and remains the same thereafter.

For $\bar{K}_4^M < K \leq \bar{K}_5^M$, π^* keeps increasing. This leads to further decrease of the ratio $\frac{\pi_h^*}{\pi^*}$.

At $\bar{K}_5^M$, π^* assumes its highest value at $\pi^* = \frac{(q_l - c_l)^2}{4q_l}$ and remains the same thereafter.

Hence, the smallest profit ratio is achieved in this region at $\frac{\pi_h^*}{\pi^*} = \frac{q_l\,(q_h - c_h)^2}{q_h\,(ql - cl)^2}$. Hence, the result follows.

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■

Proof of Proposition 33.   The problem can be re-written as follows:

$$\max_{x_l \geq 0} \quad \pi_l = (p_l(x_l) - c_l)x_l$$

$$\text{subject to} \quad s_l x_l \leq K.$$

The objective function of this problem is strictly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)). First order conditions are as follows for this problem:

$$-c_l + q_l - 2q_l x_l - s_l \lambda + \mu = 0$$

$$(K - s_l x_l)\lambda = 0$$

$$x_l \mu = 0$$

where the feasibility conditions are as given below:

$$x_l \geq 0 \qquad \lambda \geq 0 \qquad \mu \geq 0 \qquad K \geq s_l x_l$$

The solution of the first order conditions yield 3 alternatives:

Solution 1: $x_l = \frac{q_l - c_l}{2q_l}; \lambda = 0; \mu = 0.$

Solution 2: $x_l = \frac{K}{s_l}; \lambda = \frac{-2Kq_l + (-c_l + q_l)s_l}{s_l^2}; \mu = 0.$

Solution 3: $x_l = 0; \lambda = 0; \mu = c_l - q_l.$

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Combined with the feasibility conditions, the result in the proposition follows. ■

Proof of Proposition 34. The parametric set we investigate is $q_h/q_l > c_h/c_l > s_h/s_l$. In this parametric set, the intuitive and trivial optimal solution is focusing on the high quality. Then, the lower bound that is achieved under these conditions would be a lower bound for all other cost conditions. Let's start by calculating the profit ratios for all capacity levels. There are 3 regions that needs to be studied in this case: $0 < K \leq \bar{K}_5^M$, $\bar{K}_5^M < K \leq \bar{K}_4^M$, and $K > \bar{K}_4^M$.

For $0 < K \leq \bar{K}_5^M$, $\pi_l^* = -\frac{K(Kq_l+(c_l-q_l)s_l)}{s_l^2}$ and $\pi^* = -\frac{K(Kq_h+(c_h-q_h)s_h)}{s_h^2}$. In this region, although both profits are increasing in K, the ratio $\frac{\pi_l^*}{\pi^*}$ is decreasing.

At $\bar{K}_5^M$, π_l^* assumes its highest value at $\pi_h^* = \frac{(q_l-c_l)^2}{4q_l}$ and remains the same thereafter.

For $\bar{K}_5^M < K \leq \bar{K}_4^M$, π^* keeps increasing. This leads to further decrease of the ratio $\frac{\pi_l^*}{\pi^*}$.

At $\bar{K}_4^M$, π^* assumes its highest value at $\pi^* = \frac{(q_h-c_h)^2}{4q_h}$ and remains the same thereafter. Hence, the smallest profit ratio is achieved in this region at $\frac{\pi_l^*}{\pi^*} = \frac{q_h}{q_l}\frac{(q_l-c_l)^2}{(q_h-c_h)^2}$. Hence, the result follows.

■

Lemma C1 *The best response functions of the high quality focused firm Z are as follows:*

Solution 1: $z_h = -\frac{c_h+q_l y_l+q_h(-1+y_h)}{2q_h}$ *and* $\lambda_z = 0$ *and* $\mu_z = 0$.

Solution 2: $z_h = \frac{K}{s_h}$ *and* $\lambda_z = -\frac{2Kq_h+s_h(c_h+q_l y_l+q_h(-1+y_h))}{s_h^2}$ *and* $\mu_z = 0$.

Solution 3: $z_h = 0$ *and* $\lambda_z = 0$ *and* $\mu_z = c_h + q_l y_l + q_h(-1+y_h)$.

Moreover, the feasibility conditions are as given below:

$$z_h \geq 0 \qquad \lambda_z \geq 0 \qquad \mu_z \geq 0 \qquad K \geq s_h z_h$$

Proof. The Lagrangian of the problem can be written as follows:

$$\pi_z = -z_h(c_h + q_l y_l + q_h(-1 + y_h + z_h)) + \lambda_z(K - s_h z_h) + \mu_z z_h$$

The objective function of the problem is strictly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)). First order conditions are as follows for this problem:

$$-c_h + q_h - q_l y_l - q_h y_h - 2q_h z_h - s_h \lambda_z + \mu_z = 0$$

$$\lambda_z(K - s_h z_h) = 0$$

$$\mu_z z_h = 0$$

The solution of the first order conditions yield 3 alternatives as presented in the lemma. ∎

Lemma C2 *The best response functions of the multiproduct firm Y to the high quality firm Z are as follows:*

Solution 1: $y_l = -\frac{c_h q_l - c_l q_h}{2q_l^2 - 2q_l q_h}$ *and* $y_h = -\frac{c_l - c_h + (q_l - q_h)(-1 + z_h)}{2(q_l - q_h)}$ *and* $\lambda_y = 0$ *and* $\mu_h = 0$ *and* $\mu_l = 0$.

Solution 2: $y_l = \frac{2K(q_h s_l - q_l s_h) + s_h(c_h s_l - s_h(c_l + q_l(-1 + z_h)) + q_h s_l(-1 + z_h))}{2(q_h s_l^2 + q_l s_h(-2s_l + s_h))}$ *and*

$y_h = -\frac{2K q_l(s_l - s_h) + s_l(c_h s_l - s_h(c_l + q_l(-1 + z_h)) + q_h s_l(-1 + z_h))}{2(q_h s_l^2 + q_l s_h(-2s_l + s_h))}$ *and*

$\lambda_y = \frac{(2K q_l(q_l - q_h) - c_l q_h s_l + c_h q_l s_l + c_l q_l s_h - q_l^2 s_h + q_l q_h s_h + q_l^2 s_h z_h - q_l q_h s_h z_h)}{(q_h s_l^2 + q_l s_h(-2s_l + s_h))}$ *and* $\mu_h = 0$ *and* $\mu_l = 0$.

Solution 3: $y_l = 0$ *and* $y_h = -\frac{c_h + q_h(-1 + z_h)}{2q_h}$ *and* $\lambda_y = 0$ *and* $\mu_h = 0$ *and* $\mu_l = c_l - \frac{c_h q_l}{q_h}$.

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Solution 4: $y_l = 0$ *and* $y_h = 0$ *and* $\lambda_y = 0$ *and* $\mu_h = c_h + q_h(-1 + z_h)$ *and* $\mu_l =$

$c_l + q_l(-1 + z_h)$.

Solution 5: $y_l = 0$ *and* $y_h = K/s_h$ *and* $\lambda_y = -\frac{2Kq_h + s_h(c_h + q_h(-1+z_h))}{s_h^2}$ *and* $\mu_h = 0$ *and*

$\mu_l = c_l - q_l + \frac{2Kq_l}{s_h} - \frac{s_l(2Kq_h + s_h(c_h + q_h(-1+z_h)))}{s_h^2} + q_l z_h$.

Solution 6: $y_l = -\frac{c_l + q_l(-1+z_h)}{2q_l}$ *and* $y_h = 0$ *and* $\lambda_y = 0$ *and* $\mu_h = -c_l + c_h - (q_l -$

$q_h)(-1 + z_h)$ *and* $\mu_l = 0$.

Solution 7: $y_l = K/s_l$ *and* $y_h = 0$ *and* $\lambda_y = -\frac{2Kq_l + s_l(c_l + q_l(-1+z_h))}{s_l^2}$ *and*

$\mu_h = \frac{2Kq_l(s_l - s_h) + s_l(c_h s_l - s_h(c_l + q_l(-1+z_h)) + q_h s_l(-1+z_h))}{s_l^2}$ *and* $\mu_l = 0$.

Moreover, the feasibility conditions are as given below:

$$y_h \geq 0 \qquad y_l \geq 0 \qquad \lambda_y \geq 0 \qquad \mu_h \geq 0 \qquad \mu_l \geq 0 \qquad K \geq s_h y_h + s_l y_l$$

Proof. The Lagrangian of the problem can be written as follows:

$$\pi_y = -y_h(c_h + q_l y_l + q_h(-1 + y_h + z_h)) + y_l(-c_l - q_l(-1 + y_l + y_h + z_h)) +$$

$$(K - s_l y_l - s_h y_h)\lambda_y + y_l \mu_l + y_h \mu_h$$

The objective function of the problem is jointly concave on a convex set defined by

linear constraints, therefore the optimal solution can be obtained by solving the first

order conditions (Bazaraa et al. (2006)). First order conditions are as follows for this

problem:

$$-c_h - 2q_l y_l - q_h(-1 + 2y_h + z_h) - s_h \lambda_y + \mu_h \;=\; 0$$

$$-c_l - q_l(-1 + 2y_l + 2y_h + z_h) - s_l \lambda_y + \mu_l \;=\; 0$$

$$\lambda_y(K - s_l y_l - s_h y_h) \;=\; 0$$

$$\mu_h y_h \;=\; 0$$

$$\mu_l y_l \;=\; 0$$

The solution of the first order conditions yield 7 alternatives as presented in the lemma.

∎

Lemma C3 *The best response functions of the low quality focused firm Z are as follows:*

Solution 1: $z_l = -\frac{c_l + q_l(-1 + y_l + y_h)}{2q_l}$ *and* $\lambda_z = 0$ *and* $\mu_z = 0$.

Solution 2: $z_l = \frac{K}{s_l}$ *and* $\lambda_z = -\frac{2Kq_l + s_l(c_l + q_l(-1 + y_l + y_h))}{s_l^2}$ *and* $\mu_z = 0$.

Solution 3: $z_l = 0$ *and* $\lambda_z = 0$ *and* $\mu_z = c_l + q_l(-1 + y_l + y_h)$.

Moreover, the feasibility conditions are as given below:

$$z_l \geq 0 \qquad \lambda_z \geq 0 \qquad \mu_z \geq 0 \qquad K \geq s_l z_l$$

Proof. The Lagrangian of the problem can be written as follows:

$$\pi_z = -c_l z_l - q_l z_l(-1 + y_l + y_h + z_l) + \lambda_z(K - s_l z_l) + \mu_z z_l$$

The objective function of the problem is strictly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)). First order conditions are as follows for this

problem:

$$-c_l - q_l(-1 + y_l + y_h + 2z_l) - s_l\lambda_z + \mu_z = 0$$

$$\lambda_z(K - s_l z_l) = 0$$

$$\mu_z z_l = 0$$

The solution of the first order conditions yield 3 alternatives as presented in the lemma. ∎

Lemma C4 *The best response functions of the multiproduct firm Y to the low quality firm Z are as follows:*

Solution 1: $y_l = \frac{-c_h q_l + c_l q_h + q_l(-q_l+q_h)z_l}{2q_l(q_l-q_h)}$ and $y_h = \frac{-c_l+c_h+q_l-q_h}{2(q_l-q_h)}$ and $\lambda_y = 0$ and $\mu_h = 0$ and $\mu_l = 0$.

Solution 2: $y_l = \frac{(2K(q_h s_l - q_l s_h) + s_h(c_h s_l - q_h s_l - c_l s_h + q_l s_h + q_l s_l z_l - q_l s_h z_l))}{(2(q_h s_l^2 + q_l s_h(-2s_l + s_h)))}$ and

$y_h = \frac{(2Kq_l(-s_l+s_h) + s_l(-c_h s_l + q_h s_l + c_l s_h - q_l s_h - q_l s_l z_l + q_l s_h z_l))}{(2(q_h s_l^2 + q_l s_h(-2s_l+s_h)))}$ and

$\lambda_y = \frac{(2Kq_l(q_l-q_h) - c_l q_h s_l + c_h q_l(s_l-s_h) + c_l q_l s_h - q_l^2 s_h + q_l q_h s_h + q_l^2 s_l z_l - q_l q_h s_l z_l)}{(q_h s_l^2 + q_l s_h(-2s_l+s_h))}$ and $\mu_h = 0$ and $\mu_l = 0$.

Solution 3: $y_l = 0$ and $y_h = 0$ and $\lambda_y = 0$ and $\mu_h = c_h - q_h + q_l z_l$ and $\mu_l = c_l + q_l(-1 + z_l)$.

Solution 4: $y_l = 0$ and $y_h = -\frac{c_h - q_h + q_l z_l}{2q_h}$ and $\lambda_y = 0$ and $\mu_h = 0$ and $\mu_l = \frac{-c_h q_l + c_l q_h + q_l(-q_l+q_h)z_l}{q_h}$.

Solution 5: $y_l = 0$ and $y_h = K/s_h$ and $\lambda_y = -\frac{2Kq_h + s_h(c_h - q_h + q_l z_l)}{s_h^2}$ and $\mu_h = 0$ and $\mu_l = c_l - q_l + \frac{2Kq_l}{s_h} + q_l z_l - \frac{s_l(2Kq_h + s_h(c_h - q_h + q_l z_l))}{s_h^2}$.

Solution 6: $y_l = -\frac{c_l + q_l(-1+z_l)}{2q_l}$ and $y_h = 0$ and $\lambda_y = 0$ and $\mu_h = -c_l + c_h + q_l - q_h$ and

$\mu_l = 0$.

Solution 7: $y_l = K/s_l$ and $y_h = 0$ and $\lambda_y = -\frac{2Kq_l + s_l(c_l + q_l(-1 + z_l))}{s_l^2}$ and $\mu_h = c_h - q_h +$

$\frac{2Kq_l}{s_l} - \frac{s_h(2Kq_l + s_l(c_l + q_l(-1 + z_l)))}{s_l^2} + q_l z_l$ *and $\mu_l = 0$.*

Moreover, the feasibility conditions are as given below:

$$y_h \geq 0 \qquad y_l \geq 0 \qquad \lambda_y \geq 0 \qquad \mu_h \geq 0 \qquad \mu_l \geq 0 \qquad K \geq s_h y_h + s_l y_l$$

Proof. The Lagrangian of the problem can be written as follows:

$$\pi_y = -y_h(c_h + q_h(-1 + y_h) + q_l(y_l + z_l)) + y_l(-c_l - q_l(-1 + y_l + y_h + z_l)) +$$

$$(K - s_l y_l - s_h y_h)\lambda_y + y_l \mu_l + y_h \mu_h$$

The objective function of the problem is jointly concave on a convex set defined by linear constraints, therefore the optimal solution can be obtained by solving the first order conditions (Bazaraa et al. (2006)). First order conditions are as follows for this problem:

$$-c_h + q_h - 2q_l y_l - 2q_h y_h - q_l z_l - s_h \lambda_y + \mu_h = 0$$

$$-c_l - q_l(-1 + 2y_l + 2y_h + z_l) - s_l \lambda_y + \mu_l = 0$$

$$\lambda_y(K - s_l y_l - s_h y_h) = 0$$

$$\mu_h y_h = 0$$

$$\mu_l y_l = 0$$

The solution of the first order conditions yield 7 alternatives as presented in the lemma.

∎

Appendix D: Appendix for Chapter

5

Proof of Proposition 35. We can re-write the second period problem that the firm

has to solve as follows:

$$Q(x, \hat{\theta}) = \quad \max_{y \geq 0, p \geq 0} y(p - c)$$

$$\text{subject to} \quad y \leq x$$

$$y \leq D(p)$$

$$\text{where} \quad D(p) = \hat{\theta} - \frac{p}{q}$$

This problem can be solved through Lagrangian methods since it is a concave function

on linear constraints. Then, the Lagrangian is as follows:

$$Lagrangian = (p - c)y + \lambda(x - y) + \lambda_m((\hat{\theta} - \frac{p}{q}) - y) + \mu_p p + \mu_y y$$

.

The first order conditions are as follows:

$$y - \frac{\lambda_m}{q} + \mu_p = 0 \; ; \; -c + p - \lambda - \lambda_m + \mu_y = 0 \; ; \; \lambda(x - y) = 0 \; ; \; \lambda_m((\hat{\theta} - \frac{p}{q}) - y) = 0 \; ;$$

$\mu_p p = 0$; $\mu_y y = 0$.

The feasibility conditions are as follows:

$y \geq 0$; $p \geq 0$; $\lambda \geq 0$; $\lambda_m \geq 0$; $\mu_p \geq 0$; $\mu_y \geq 0$; $y \leq x$; $y \leq \hat{\theta} - \frac{p}{q}$.

The solution of the first order conditions together with the feasibility conditions yield

the following alternatives:

Solution 1: $y = x$; $p = q(\hat{\theta} - x)$; $\lambda = -c + q(\hat{\theta} - 2x)$; $\lambda_m = qx$; $\mu_p = 0$; $\mu_y = 0$

Solution 2: $y = \frac{\hat{\theta}q - c}{2q}$; $p = 1/2(\hat{\theta}q + c)$; $\lambda = 0$; $\lambda_m = 1/2(\hat{\theta}q - c)$; $\mu_p = 0$; $\mu_y = 0$

Further check of the feasibility conditions yield the result in the Proposition. ∎

Proof of Proposition 36. Given the second period solutions by Proposition 35, we

take the expectation over the valuation realizations and find the optimal solution for the

first period.

$E_{\bar{\theta}}[Q(x, \theta)] = \int_{(c/q \text{ or } 1-\epsilon)}^{\frac{2qx+c}{q}} (p_2 - c) y_2(\frac{1}{2\epsilon}) d\theta + \int_{\frac{2qx+c}{q}}^{1+\epsilon} (p_1 - c) y_1(\frac{1}{2\epsilon}) d\theta$.

where $y_1 = x$; $p_1 = q(\theta - x)$; $y_2 = \frac{\theta q - c}{2q}$; $p_2 = 1/2(\theta q + c)$.

We can find the unconstrained solution by taking the first order derivatives of the

above expectation.

$\partial_x E_{\bar{\theta}}[Q(x, \theta)] = \frac{(c + q(-1 + 2x - \epsilon))^2}{4q\epsilon} = 0. \Rightarrow x^* = \frac{-c + q + q\epsilon}{2q}$.

A firm that implements a capacity-unconstrained solution needs to have at least

$K \geq sx^* = s(\frac{-c+q+q\epsilon}{2q})$ level of availability. Since the profits are increasing in x, the firm

dedicates all its capacity to production ($x^* = K/s$) below that threshold $K < s(\frac{-c+q+q\epsilon}{2q})$.

Hence, the result follows.

∎

Lemma D1 *Following are the alternative solutions for a 2-product monopolist subject*

to the capacity commitments (x_h, x_l) made in the first period:

Solution 1: $y_h = \frac{K - s_l x_l}{s_h}$; $y_l = 1/2(\hat{\theta} - c_l/q_l - 2K/s_h + (2s_l x_l)/(s_h))$;

$p_h = 1/2 s_h(-2Kq_h + 2Kq_l + c_l s_h + 2\hat{\theta}q_h s_h - \hat{\theta}q_l s_h + 2q_h s_l x_l - 2q_l s_l x_l)$; $p_l = 1/2(c_l + \hat{\theta}q_l)$

$\lambda_h = 1/s_h(-2Kq_h + 2Kq_l - c_h s_h + c_l s_h + \hat{\theta}q_h s_h - \hat{\theta}q_l s_h + 2q_h s_l x_l - 2q_l s_l x_l)$; $\lambda_l = 0$;

$\lambda_{mh} = 1/2(-c_l + \hat{\theta}q_l) + \frac{(q_h - q_l)(K - s_l x_l)}{s_h}$; $\lambda_{ml} = 1/2(-c_l + \hat{\theta}q_l)$;

$\mu_{ph} = 0$; $\mu_{pl} = 0$; $\mu_{yh} = 0$; $\mu_{yl} = 0$.

Solution 2: $y_h = -\frac{c_h - \hat{\theta}q_h + 2q_l x_l}{2q_h}$; $y_l = x_l$;

$p_h = 1/2(c_h + \hat{\theta}q_h)$; $p_l = \frac{q_l(c_h + \hat{\theta}q_h + 2(-q_h + q_l)x_l)}{2q_h}$;

$\lambda_h = 0$; $\lambda_l = \frac{-c_l q_h + q_l(c_h + 2(-q_h + q_l)x_l)}{q_h}$;

$\lambda_{mh} = 1/2(-c_h + \hat{\theta}q_h)$; $\lambda_{ml} = \frac{q_l(-c_h + \hat{\theta}q_h + 2(q_h - q_l)x_l)}{2q_h}$;

$\mu_{ph} = 0$; $\mu_{pl} = 0$; $\mu_{yh} = 0$; $\mu_{yl} = 0$.

Solution 3: $y_h = \frac{K - s_l x_l}{s_h}$; $y_l = x_l$;

$p_h = \frac{-Kq_h + \hat{\theta}q_h s_h - q_l s_h x_l + q_h s_l x_l}{s_h}$; $p_l = \frac{q_l(-K + \hat{\theta}s_h + (-s_h + s_l)x_l)}{s_h}$;

$\lambda_h = \frac{-2Kq_h - c_h s_h + \hat{\theta}q_h s_h - 2q_l s_h x_l + 2q_h s_l x_l}{s_h}$; $\lambda_l = \frac{-2Kq_l - c_l s_h + q_l(\hat{\theta}s_h - 2s_h x_l + 2s_l x_l)}{s_h}$;

$\lambda_{mh} = \frac{Kq_h + q_l s_h x_l - q_h s_l x_l}{s_h}$; $\lambda_{ml} = \frac{q_l(K + (s_h - s_l)x_l)}{s_h}$;

$\mu_{ph} = 0$; $\mu_{pl} = 0$; $\mu_{yh} = 0$; $\mu_{yl} = 0$.

Solution 4: $y_h = 0$; $y_l = (\hat{\theta}q_l - c_l)/(2q_l)$;

$p_h = 1/2(c_l + 2\hat{\theta}q_h - \hat{\theta}q_l)$; $p_l = 1/2(c_l + \hat{\theta}q_l)$;

$\lambda_h = 0$; $\lambda_l = 0$;

$\lambda_{mh} = 1/2(-c_l + \hat{\theta}q_l)$; $\lambda_{ml} = 1/2(-c_l + \hat{\theta}q_l)$;

$\mu_{ph} = 0$; $\mu_{pl} = 0$; $\mu_{yh} = c_h - c_l - \hat{\theta}q_h + \hat{\theta}q_l$; $\mu_{yl} = 0$.

Solution 5: $y_h = 0$; $y_l = x_l$;

$$p_h = \hat{\theta}q_h - q_l x_l \;;\; p_l = q_l(\hat{\theta} - x_l) \;;$$

$$\lambda_h = 0 \;;\; \lambda_l = -c_l + q_l(\hat{\theta} - 2x_l) \;;$$

$$\lambda_{mh} = q_l x_l \;;\; \lambda_{ml} = q_l x_l \;;$$

$$\mu_{ph} = 0 \;;\; \mu_{pl} = 0 \;;\; \mu_{yh} = c_h - \hat{\theta}q_h + 2q_l x_l \;;\; \mu_{yl} = 0.$$

Solution 6: $y_h = \frac{-c_h + c_l + \hat{\theta}(q_h - q_l)}{2(q_h - q_l)} \;;\; y_l = -\frac{c_l q_h - c_h q_l}{2q_h q_l - 2q_l^2} \;;$

$$p_h = 1/2(c_h + \hat{\theta}q_h) \;;\; p_l = 1/2(c_l + \hat{\theta}q_l) \;;$$

$$\lambda_h = 0 \;;\; \lambda_l = 0 \;;$$

$$\lambda_{mh} = 1/2(-c_h + \hat{\theta}q_h) \;;\; \lambda_{ml} = 1/2(-c_l + \hat{\theta}q_l) \;;$$

$$\mu_{ph} = 0 \;;\; \mu_{pl} = 0 \;;\; \mu_{yh} = 0 \;;\; \mu_{yl} = 0.$$

Solution 7: $y_h = (\hat{\theta}q_h - c_h)/2q_h \;;\; y_l = 0 \;;$

$$p_h = 1/2(c_h + \hat{\theta}q_h) \;;\; p_l = \frac{(c_h + \hat{\theta}q_h)q_l}{2q_h} \;;$$

$$\lambda_h = 0 \;;\; \lambda_l = 0 \;;$$

$$\lambda_{mh} = 1/2(-c_h + \hat{\theta}q_h) \;;\; \lambda_{ml} = \frac{-c_h q_l + \hat{\theta}q_h q_l}{2q_h} \;;$$

$$\mu_{ph} = 0 \;;\; \mu_{pl} = 0 \;;\; \mu_{yh} = 0 \;;\; \mu_{yl} = c_l - \frac{c_h q_l}{q_h}.$$

Solution 8: $y_h = (K - s_l x_l)/s_h \;;\; y_l = 0 \;;$

$$p_h = \frac{q_h(-K + \hat{\theta}s_h + s_l x_l)}{s_h} \;;\; p_l = \frac{q_l(-K + \hat{\theta}s_h + s_l x_l)}{s_h} \;;$$

$$\lambda_h = \frac{-2Kq_h - c_h s_h + \hat{\theta}q_h s_h + 2q_h s_l x_l}{s_h} \;;\; \lambda_l = 0 \;;$$

$$\lambda_{mh} = \frac{q_h(K - s_l x_l)}{s_h} \;;\; \lambda_{ml} = \frac{q_l(K - s_l x_l)}{s_h} \;;$$

$$\mu_{ph} = 0 \;;\; \mu_{pl} = 0 \;;\; \mu_{yh} = 0 \;;\; \mu_{yl} = \frac{2Kq_l + c_l s_h - \hat{\theta}q_l s_h - 2q_l s_l x_l}{s_h}.$$

Moreover, the feasibility conditions are as given below:

$$y_h \geq 0 \;;\; y_l \geq 0 \;;\; p_h \geq 0 \;;\; p_l \geq 0 \;;\; \lambda_h \geq 0 \;;\; \lambda_l \geq 0 \;;\; \lambda_{mh} \geq 0 \;;\; \lambda_{ml} \geq 0 \;;\; \mu_{ph} \geq 0 \;;$$

$$\mu_{pl} \geq 0 \;;\; \mu_{yh} \geq 0 \;;\; \mu_{yl} \geq 0 \;;\; y_h \leq x_h \;;\; y_l \leq x_l \;;\; y_h \leq \hat{\theta} - \frac{p_h - p_l}{q_h - q_l} \;;\; y_l \leq \frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l}.$$

Proof. We can re-write the second period problem that the firm has to solve as follows:

$$Q(x, \hat{\theta}) = \max_{y_i \geq 0} \sum_i y_i(p_i(\hat{\theta}, y) - c_i)$$

$$\text{subject to} \qquad y_i \leq x_i \quad \forall i$$

$$y_h \leq \hat{\theta} - \frac{p_h - p_l}{q_h - q_l}$$

$$y_l \leq \frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l}$$

This problem can be solved through Lagrangian methods since it is a concave function on linear constraints. Then, the Lagrangian is as follows:

$$Lagrangian = (p_h - c_h)y_h + (p_l - c_l)y_l + \lambda_h(x_h - y_h) + \lambda_l(x_l - y_l) + \lambda_{mh}((\hat{\theta} - \frac{p_h - p_l}{q_h - q_l}) - y_h)$$

$$+ \lambda_{ml}((\frac{p_h - p_l}{q_h - q_l}) - \frac{p_l}{q_l}) - y_l) + \mu_{ph}p_h + \mu_{pl}p_l + \mu_{yh}y_h + \mu_{yl}y_l$$

The first order conditions are as follows:

$$y_h - \frac{\lambda_{mh}}{q_h - q_l} + \frac{\lambda_{ml}}{q_h - q_l} + \mu_{ph} = 0 \ ;$$

$$y_l + \frac{\lambda_{mh}}{q_h - q_l} - \lambda_{ml}(\frac{1}{q_h - q_l} + \frac{1}{q_l}) + \mu_{pl} = 0 \ ;$$

$$-c_l + p_l - \lambda_l - \lambda_{ml} + \mu_{yl} = 0 \ ;$$

$$-c_h + p_h - \lambda_h - \lambda_{mh} + \mu_{yh} = 0 \ ;$$

$$\lambda_h(x_h - y_h) = 0 \ ; \ \lambda_l(x_l - y_l) = 0 \ ; \ \lambda_{mh}((\hat{\theta} - \frac{p_h - p_l}{q_h - q_l}) - y_h) = 0 \ ;$$

$$\lambda_{ml}((\frac{p_h - p_l}{q_h - q_l}) - \frac{p_l}{q_l}) - y_l) = 0 \ ; \ \mu_{ph}p_h = 0 \ ; \ \mu_{pl}p_l = 0 \ ; \ \mu_{yh}y_h = 0 \ ; \ \mu_{yl}y_l = 0.$$

The feasibility conditions are as follows:

$y_h \geq 0$; $y_l \geq 0$; $p_h \geq 0$; $p_l \geq 0$; $\lambda_h \geq 0$; $\lambda_l \geq 0$; $\lambda_{mh} \geq 0$; $\lambda_{ml} \geq 0$; $\mu_{ph} \geq 0$;

$\mu_{pl} \geq 0$; $\mu_{yh} \geq 0$; $\mu_{yl} \geq 0$; $y_h \leq x_h$; $y_l \leq x_l$; $y_h \leq \hat{\theta} - \frac{p_h - p_l}{q_h - q_l}$; $y_l \leq \frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l}$.

The solution of the first order conditions and careful investigation of the feasibility conditions yield the result in the Lemma.

∎

Proof of Proposition 37. Suppose $c_l/q_l > c_h/q_h$; and the customer valuation is distributed uniformly between $[\hat{\theta} - 1, \hat{\theta}]$. Then, the optimal sales of a monopolist is as follows:

If $\max\{1 - \epsilon, c_h/q_h\} \leq \hat{\theta} < min\{2x_h + c_h/q_h, 1 + \epsilon\}$, then following Lemma D1, the only feasible solution to the problem is Solution 7.

If $max\{2x_h + c_h/q_h, 1 - \epsilon\} \leq \hat{\theta} < min\{1 + \epsilon, 2x_h + c_l/q_l\}$, then following Lemma D1, the only feasible solution to the problem is Solution 8.

If $max\{2x_h + c_l/q_l, 1 - \epsilon\} \leq \hat{\theta} < min\{1 + \epsilon, 2(x_l + x_h) + c_l/q_l\}$, then following Lemma D1, the only feasible solution to the problem is Solution 1.

If $max\{2(x_l + x_h) + c_l/q_l, 1 - \epsilon\} \leq \hat{\theta}$, then following Lemma D1, the only feasible solution to the problem is Solution 3.

Hence, the result follows. ∎

Proof of Proposition 38. Suppose $c_l/q_l < c_h/q_h$ and $x_l \geq \frac{c_h q_l - c_l q_h}{2q_l(q_h - q_l)}$; and the customer valuation is distributed uniformly between $[\hat{\theta} - 1, \hat{\theta}]$. Then, the optimal sales of a monopolist is as follows:

If $\max\{1 - \epsilon, c_l/q_l\} \leq \hat{\theta} < min\{(c_h - c_l)/(q_h - q_l), 1 + \epsilon\}$, then following Lemma D1, the only feasible solution to the problem is Solution 4.

If $max\{(c_h - c_l)/(q_h - q_l), 1 - \epsilon\} \leq \hat{\theta} < min\{1 + \epsilon, 2x_h + (c_h - c_l)/(q_h - q_l)\}$, then following Lemma D1, the only feasible solution to the problem is Solution 6.

If $max\{2x_h + (c_h - c_l)/(q_h - q_l), 1 - \epsilon\} \leq \hat{\theta} < min\{1 + \epsilon, 2(x_l + x_h) + c_l/q_l\}$, then following Lemma D1, the only feasible solution to the problem is Solution 1.

If $max\{2(x_l + x_h) + c_l/q_l, 1 - \epsilon\} \leq \hat{\theta}$, then following Lemma D1, the only feasible solution to the problem is Solution 3.

Hence, the result follows.

■

Proof of Proposition 39. Suppose $c_l/q_l < c_h/q_h$ and $x_l \leq \frac{c_h q_l - c_l q_h}{2q_l(q_h - q_l)}$; and the customer valuation is distributed uniformly between $[\hat{\theta} - 1, \hat{\theta}]$. Then, the optimal sales of a monopolist is as follows:

If $max\{1 - \epsilon, c_l/q_l\} \leq \hat{\theta} < min\{2x_l + c_l/q_l, 1 + \epsilon\}$, then following Lemma D1, the only feasible solution to the problem is Solution 4.

If $max\{2x_l + c_l/q_l, 1 - \epsilon\} \leq \hat{\theta} < min\{1 + \epsilon, (2q_l x_l + c_h)/q_h\}$, then following Lemma D1, the only feasible solution to the problem is Solution 5.

If $max\{(2q_l x_l + c_h)/q_h, 1 - \epsilon\} \leq \hat{\theta} < min\{1 + \epsilon, 2(x_h + x_l(q_l/q_h)) + c_h/q_h\}$, then following Lemma D1, the only feasible solution to the problem is Solution 2.

If $max\{2(x_h + x_l(q_l/q_h)) + c_h/q_h, 1 - \epsilon\} \leq \hat{\theta}$, then following Lemma D1, the only feasible solution to the problem is Solution 3.

Hence, the result follows. ■

Proof of Proposition 40.

Given the second period solutions by Proposition 38, we take the expectation over

the valuation realizations and find the optimal solution for the first period.

$$E_{\bar\theta}[Q(x,\theta)] = \int_{(c_l/q_l \text{ or } 1-\epsilon)}^{\frac{c_h-c_l}{q_h-q_l}}((p_{h1}-c_h)y_{h1}+(p_{l1}-c_l)y_{l1})(\tfrac{1}{2\epsilon})d\theta + \int_{\frac{c_h-c_l}{q_h-q_l}}^{2x_h+\frac{c_h-c_l}{q_h-q_l}}((p_{h2}-c_h)y_{h2}+$$

$$(p_{l2}-c_l)y_{l2})(\tfrac{1}{2\epsilon})d\theta + \int_{2x_h+\frac{c_h-c_l}{q_h-q_l}}^{2(x_l+x_h)+c_l/q_l}((p_{h3}-c_h)y_{h3}+(p_{l3}-c_l)y_{l3})(\tfrac{1}{2\epsilon})d\theta + \int_{2(x_l+x_h)+c_l/q_l}^{1+\epsilon}((p_{h4}-$$

$$c_h)y_{h4}+(p_{l4}-c_l)y_{l4})(\tfrac{1}{2\epsilon})d\theta \ .$$

where $y_{l1} = \frac{\theta q_l-c_l}{2q_l}$; $y_{h1}=0$; $p_{h1}=1/2(2\theta q_h-(\theta q_l-c_l))$; $p_{l1}=\frac{\theta q_l+c_l}{2}$.

$y_{h2}=\frac{(\theta q_h-c_h)-(\theta q_l-c_l)}{2(q_h-q_l)}$; $y_{l2}=\frac{c_h q_l-c_l q_h}{2q_l(q_h-q_l)}$; $p_{h2}=1/2(\theta q_h+c_h)$; $p_{l2}=1/2(\theta q_l+c_l)$.

$y_{h3}=x_h$; $y_{l3}=\frac{\theta q_l-c_l-2q_l x_h}{2q_l}$; $p_{h3}=\theta q_h-x_h(q_h-q_l)-1/2(\theta q_l-c_l)$; $p_{l3}=1/2(\theta q_l+c_l)$.

$y_{h4}=x_h$; $y_{l4}=x_l$; $p_{h4}=q_h(\theta-x_h)-q_l x_l$; $p_{l4}=q_l(\theta-x_h-x_l)$.

We can find the unconstrained solution by taking the first order derivatives of the above expectation.

$$\partial_{x_h}E_{\bar\theta}[Q(x,\theta)] = \frac{1}{4(q_h-q_l)q_l\epsilon}\ (c_l^2 q_h-2c_l q_l(c_h+2(-q_h+q_l)x_l)+q_l(c_h^2+2c_h(q_h-q_l)(-1+$$

$$2x_h-\epsilon)+q_h^2(1-2x_h+\epsilon)^2+4q_l^2 x_l(1-2x_h-x_l+\epsilon)-q_h q_l(4x_h^2-4x_l^2+4x_l(1+\epsilon)+(1+$$

$$\epsilon)^2-4x_h(1+2x_l+\epsilon)))) = 0.$$

$$\partial_{x_l}E_{\bar\theta}[Q(x,\theta)] = \frac{(c_l+q_l(-1+2x_h+2x_l-\epsilon))^2}{4q_l\epsilon} = 0.$$

Simultaneous solution of these conditions yield the optimal solution for the unconstrained case: $x_h = \frac{c_h-c_l-q_h+q_l-q_h\epsilon+q_l\epsilon}{2(-q_h+q_l)}$ and $x_l = \frac{-c_l q_h+c_h q_l}{2(q_h-q_l)q_l}$. Since the firm needs $K \geq s_h x_h + s_l x_l$ capacity in order to be able to offer these quantities, the result follows.

∎

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