Conditional Betas: Asymmetric Responses to Good and Bad News

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Abstract

FERNANDO DANIEL CHAGUE: Conditional Betas: Asymmetric Responses to Good and Bad News.
(Under the direction of Eric Ghysels.)

In this dissertation we propose a theoretical model for conditional betas. Within a rational expectation equilibrium model, we provide a precise characterization of the dynamics of betas and the price of beta risk in terms of the model's primitive parameters and state variables. The expressions reveal that during periods of higher uncertainty, the investor requires a higher market premium. Likewise, the conditional betas also respond to levels of uncertainty; depending on the cash-flow properties of the asset, the asset's beta can increase or decrease on higher uncertainty. Because of the connection with uncertainty, conditional betas derive the stochastic properties from investor beliefs. One of such properties is the asymmetric response to positive and negative news.

We also provide empirical evidence of the model's predictions about the dynamics of betas. For this empirical investigation, we propose an econometric specification that provides time-varying estimates of betas and relates them, non-linearly, to investor beliefs. As a by-product, we suggest proxies for investor beliefs and uncertainty that can be extracted from stock returns. The dynamics implied by the estimated parameters confirms the model's prediction that value and growth betas have opposing sensitivity to the levels of uncertainty.

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Chapter 1

Introduction

It has long been acknowledged that the systematic risk of stocks, as measured by the market beta, is time-varying. In empirical applications as early as Fama and MacBeth (1973), betas were already computed from rolling-sample moments. However, the conditional Capital Asset Pricing Model (CAPM), that usually motivates time-varying betas, does not provide any hint on how betas ought to be estimated. In fact, not much is known about what makes betas vary over time and across assets. An evidence of this is that rolling betas are still used in empirical applications. Indeed, based on such rolling betas, Lewellen and Nagel (2006) have condemned the conditional CAPM, by claiming it cannot explain the returns on momentum and book-to-market portfolios. In order to give the conditional CAPM a fair trial, and also to improve the measurement of systematic risk, a better understanding of the dynamics of market betas is urged. The goal of this dissertation is to provide a step in this direction.

In Chapter 2, we derive new theoretical expressions for market betas based on a rational expectation equilibrium model. The central assumption of the model is the uncertainty faced by the investor about the true profitability of the assets, which can take on two forms, depending on the state of the economy. Using the available information, the investor learns about the state of the economy, and optimally allocates wealth across the assets. In the rational expectation equilibrium that results, expected returns derive their stochastic properties from investor beliefs, and can be decomposed into exposures to market risk and hedging risk. The market risk of an asset is derived from its comovement with the market portfolio, and the hedging risk, from its comovement with investor beliefs.

The two peculiarities of this factor decomposition are the following. First, it says what the hedging risk is. The Intertemporal Capital Asset Pricing model (ICAPM) of Merton (1973) does not specify it and, as a result, empirical applications of the ICAPM, and of multifactor models in general, typically justify risk factors from empirical considerations[1]. Second, it provides a functional form for conditional betas and prices of risk. Since the model solves asset prices in closed-form formulas[2], the covariance of returns and prices of risk are also obtainable in closed-form formulas. As a result, conditional betas and prices of risk are linked to the model's primitive parameters and to the stochastic properties of investor beliefs. This, again, contrasts with the lack of characterization of the dynamics of beta risk in the ICAPM and also in the conditional CAPM.

In Chapter 2 we also verify the model's pricing predictions by means of a simulation. We calibrate an economy with five assets, each set to resemble one of the five book-to-market sorted portfolios. For a reasonable choice of the primitive parameters, which include the risk-aversion parameter, the assets cash-flow parameters, and the probabilities driving the states of the economy, the model can reproduce the unconditional excess returns and, to a certain degree, the variance of excess returns of the actual data. Given the difficulties in reconciling, within an equilibrium framework, the equity premium puzzle — that forces a large risk aversion parameter — and the excess volatility puzzle — that results in incompatible volatility of returns and dividends — the fact that the model matches the (unconditional) equity premia of a cross-section of assets with reasonable parameters is remarkable.

The following empirical implications arise from this calibration. First, a conditional CAPM, with the defined beta dynamics, provides an appropriate representation of expected returns. Some empirical studies on the conditional CAPM have assumed away the hedging factors, such as Jagannathan and Wang (1996), but here this approximation is based within a formal model. Second, conditional market betas are time-varying and non-linearly related to investor beliefs.

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[1] For example, the variables from the predictability literature, such as the price-dividend ratio and the term spread, are usually taken as the proxies driving the investment opportunity set. Also, the cross-section anomalies summarized by Fama and French (1993) motivate the size- and value-related factors of risk that now constitute the Fama-French three factor model.

[2] Up to the numerical solution of two ordinary differential equations.

This non-linearity can be approximated by a monotonic relation of betas with uncertainty, where by uncertainty we mean the distance of investor beliefs probabilities from the high certainty cases. Third, the conditional betas of value and growth portfolios have opposing dynamics; value betas are *higher* during high uncertainty periods while growth betas are *lower* during high uncertainty periods.

Our model also suggests a different interpretation[3] for the relevance of return asymmetries to risk and to asset pricing. The particular sign of returns matters because it can signal a potential change in the economic conditions. In particular, the average negative news, weighted by a signal-to-noise ratio, increases uncertainty, whereas the average positive news decreases uncertainty. Since betas and prices of risk depend on the level of uncertainty, asymmetries also arise in expected returns. One of the extra features of this interpretation is that it also identifies which assets are more susceptible to pricing asymmetries — those with cash-flows that are very sensitive to shifts in the economic conditions.

The last two chapters of this dissertation are devoted to the investigation of the model's main empirical implications using data from U.S. stock markets.

In Chapter 3 we explore two aspects of the dynamics of betas: i) how betas relate to different *levels* of investor beliefs, and ii) how betas relate to *changes* in investor beliefs, where changes in beliefs are proxied by shocks in return. The results reveal different asymmetric patterns across portfolios, particularly across those associated with the pricing anomalies. For instance, among book-to-market portfolios, value betas are higher during high uncertainty periods, while growth betas are lower during high uncertainty periods. This empirical finding, albeit marginally significant, corroborates the calibration results in Chapter 2. Among momentum portfolios, a clear pattern emerges that distinguishes the risk dynamics of past-winners and past-losers portfolios. Past-winners betas tend to be lower during periods of high uncertainty, while past-losers betas tend to be higher during periods of high uncertainty.

[3] The usual justification for the relevance of return asymmetries to asset pricing resides on investor asymmetric preferences. We can go back as far as Markowitz (1959), who suggested the mean and the semi-variance as the key parameters to investor risky choices, and to Hogan and Warren (1974), who derive asset pricing implication for such risk preferences. Recent approaches include Ang, Chen, and Xing (2006), who assume the disappointment aversion preferences of Gul, 1991, and Harvey and Siddique (2000), who conjectured investors with skewness aversion.

The asymmetric patterns with respect to changes in beliefs are also different across portfolios. Among size portfolios, small firms are particularly riskier during negative news markets than during positive news markets. Among book-to-market portfolios, value firms also display higher betas during negative news markets, and this asymmetry further increases with the relevance of negative news. Interestingly, the asymmetric patterns on the industry portfolios are less clear, which indicates that the size, book-to-market and momentum anomalies may be, at least partially, related to misspecified beta asymmetries.

In Chapter 4 we dissect betas according to the signs of market and asset returns, and further unveil the asymmetries in betas. The suggested decomposition of betas into signed betas holds, as a special case, the upside and downside betas of Ang, Chen, and Xing (2006). For this exploratory task, we consider all common stocks on the Center for Research in Security Prices (CRSP) dataset, that were listed on the NYSE, Nasdaq and Amex markets. The results point to a potential asymmetry related to the beta computed on positive market and negative asset returns that cannot be explained by the measures of risk commonly considered by the literature, including the coskewness measure of risk of Harvey and Siddique (2000).

Chapter 2

Theoretical Model

2.1 Introduction

The conditional Capital Asset Pricing Model (CAPM) does not impose any structure on how betas should vary. This has largely been tackled from an empirical perspective. Early parametrical approaches include the multivariate GARCH framework (Bollerslev, Engle, and Wooldridge, 1988) and the instrumental variables betas (Harvey (1989), Harvey and Kirby (1996)). Recent parametric models suggest treating conditional betas as latent variables: Adrian and Franzoni (2009) suggest using the Kalman filter while Ang and Chen (2007) apply Markov-chain Monte-Carlo and Gibbs sampling to obtain time varying betas. Non-parametric approaches have been suggested by Andersen, Bollerslev, Diebold, and Wu (2006), who use high-frequency data to estimate betas and Ang, Chen, and Xing (2006), who point out how asymmetries in betas may be important.

As the econometric literature indicates, there is still an ongoing debate as to how conditional betas should be estimated. Ghysels (1998) points out that misspecified conditional betas can result in higher pricing errors than static betas. This is one of the reasons why many empirical works still use the rolling betas of Fama and MacBeth (1973) to avoid taking a stand on an econometric model (Lewellen and Nagel, 2006).

In this chapter we contribute to this debate from an economic theoretical perspective. We investigate the dynamics of conditional betas implied by a rational expectations equilibrium. More specifically, we consider a multiple asset version of the rational expectations equilibrium model of Veronesi (1999) first suggested by Ribeiro and Veronesi (2002). In this model, the investor is uncertain about the true distribution of each asset's cash-flow stream. In particular, the investor does not observe the drift of the continuous process that characterizes cash-flows, which can take on two values according to a Markov-chain process. As a result of this uncertainty, investor decisions, and pricing formulas, are affected by a learning process. Expected returns are decomposed by the asset's exposures to common sources of risk and a similar expression to Merton's (1973) ICAPM is obtained. The extra structure imposed on asset's cash-flows, however, allows for closed-form formulas of conditional market betas and prices of market risk that are not possible with the standard assumptions in the ICAPM.

The main implications to the dynamics of conditional betas are the following. First, at given levels of investor beliefs, conditional betas differ across assets that have distinct cash-flows properties. Assets that are very sensitive to changes in the economic conditions have higher betas during high uncertainty periods. As we show in a calibration exercise, an example of such assets is the value portfolio. The empirical evidence in Petkova and Zhang (2005), who show that value betas tend to be larger during recessions and growth betas tend to be smaller, is supported by our model's predictions.

Second, conditional betas respond asymmetrically changes in beliefs. This result is an extension of the asymmetric response of volatility and covariance to news. These two asymmetries are well known empirical properties of stocks returns, but the empirical evidence of similar asymmetries to news in betas is not as clear (Braun, Nelson, and Sunier, 1995). However, recent empirical evidence by Ang, Chen, and Xing (2006), that points to the relevance of downside betas[1] for the risk premium, relates to our model's predictions about the asymmetric response of betas to news.

This chapter is related to Santos and Veronesi (2004), who derive implications to market betas within a general equilibrium model. In their model, it is assumed that the investor has habit-persistent preferences and that the dividends in the economy are random shares of the

[1] Downside beta is defined in that paper as the beta conditional on negative market returns.

total endowment process of the economy. They find that betas can be decomposed into a cash-flow and a discount risk components and that the dynamics of conditional betas is determined by the component that is relatively most important.

This chapter proceeds as follows: in Section 2.2 we solve the model and discuss the resulting asset pricing formulas. Then, in Section 2.3 we simulate an economy and investigate the model's predictions. First, we calibrate the model with U.S. data and discuss the pricing implications that arise. Then, we simulate time-series of returns and estimate univariate and multivariate GARCH models to assess the dynamics of covariance and market betas. We conclude the chapter in Section 2.4 with a summary of the results and some final remarks.

2.2 The Model

The model is a multiple asset version of the rational expectations equilibrium model of Veronesi (1999), and that was also derived by Ribeiro and Veronesi (2002). The authors show how uncertainty about the state of the world economy can result in the observed excess covariation in international stock markets during downturns. However, they do not address the factor structure of expected returns that arises in that model. In contrast, here we investigate the dynamics of the different components of the risk premia and, in particular, how good and bad news are incorporated into market betas.

The key assumption of the model is the uncertainty the investor faces about the true distribution of the asset's cash-flows. More specifically, the drifts of the continuous stochastic processes that describe cash-flows can on take two values according to an unobserved two state Markov-switching process. It is further assumed that the investor optimally infers the true drifts from cash-flows realizations. This generates a learning process that results in asset prices that bear many of the empirical properties observed in real data.

Apart from the ability to replicate many of the stylized facts about stock returns, the model is appealing for it provides a tractable framework to incorporate a learning dynamic into pricing formulas. For instance, it allows us to assess how news about the economy can change the risk of assets. As we will see below, different cash-flow structures can result in opposite responses of market betas to news of the same sign.

2.2.1 The Economy

The economy has one representative investor that maximizes expected utility subject to a budget constraint. There are $n+1$ financial assets: a risk-free asset that is inelastically supplied with a known rate of return rdt and n risky assets that pay continuous stream of cash-flows given by:

$$dD_{it} = \theta_{it} dt + \sigma_i d\xi_t \quad i = 1, ..., n \tag{2.1}$$

where $d\xi_t$ is a $(n \times 1)$ vector of Brownian motions and σ_i a $(1 \times n)$ vector of diffusion coefficients. The n expressions presented above can be written in matrix notation as $dD_t = \theta_t dt + \Phi d\xi_t$, where θ_t is the $(n \times 1)$ vector of drift terms θ_{it}, and Φ is the $(n \times n)$ matrix that stacks the diffusion terms σ_i. Denote by $\Sigma = \Phi\Phi'$ the cash-flow covariance matrix. The market portfolio cash-flow is defined as the sum of all cash-flows times the available shares of each asset, $D_{mt} \equiv \sum_{i=1}^{n} \omega_i D_{it}$, where $\omega = [\omega_1, ..., \omega_n]'$ are the available shares.

The investor does not observe the random vector $\{\theta_t\}$ but knows it can take two values: $\theta_G = [\theta_{1G}, ..., \theta_{nG}]'$ in the good state and $\theta_B = [\theta_{1B}, ..., \theta_{nB}]'$ in the bad state. This random vector switches between the two states with conditional probabilities that follow a two-state Markov-chain process with parameters μ, the probability of going to a good state from a bad state, and λ, the probability of shifting from the good state to the bad state. Note that the same Markov-switching process governs the shifts of all drifts and thus can be naturally associated with the business cycles shifts. We label asset i cyclical if $\Delta\theta_i \equiv \theta_G - \theta_B > 0$ and countercyclical otherwise.

The investor optimally infers the true drifts of cash-flows from past observations. That is, he conditions his beliefs about the true drifts on the information set $\mathcal{F}_t = \sigma(D_\tau, \tau < t)$. As was shown by Veronesi (1999), the optimal prediction is conveniently described by a stochastic process. The following lemma is an extension of the univariate case for multiple assets.

Lemma 1. *The investor's belief that the economy is in the good state, $\pi_t \equiv Prob(\theta_t = \theta_G | \mathcal{F}_t)$, evolves according to the stochastic process:*

$$d\pi_t \;=\; (\lambda + \mu)(\pi_s - \pi_t) dt + \pi_t (1 - \pi_t) \Delta\theta' \Phi'^{-1} dv_t \tag{2.2}$$

where $\pi_s = \frac{\mu}{\lambda+\mu}$ *is the unconditional probability of* π_t, $\Delta\theta' = [\theta_{1G} - \theta_{1B}, ..., \theta_{nG} - \theta_{nB}]$, *and* $dv_t \equiv \Phi^{-1}(dD_t - E[dD_t|\mathcal{F}_t])$ *is a* $(n \times 1)$ *vector of standard Brownian motions with respect to the filtration* $\mathcal{F}_t$, *with* $E[dD_{it}|\mathcal{F}_t] = \theta_{iG}\pi_t + \theta_{iB}(1-\pi_t)$ *for* $i = 1, ..., n$.

Proof. It follows from theorem 9.3 in Lipster and Shiryaev (2001). □

Note that π_t mean reverts towards its unconditional mean, π_s, at a rate of $\lambda + \mu$. Shocks to dv_t are weighted by a signal to noise ratio, $\Delta\theta'\Phi'^{-1}$, and by the uncertainty level about the state of the economy, $h(\pi_t) \equiv \pi_t(1-\pi_t)$. The closer π_t is to 0.50, the more uncertain the investor is about the true state, and the larger the revisions to the conditional probability are. For ease of notation, let $\alpha_\pi \equiv (\lambda+\mu)(\pi_s - \pi_t)$ and $\sigma_\pi^2 \equiv \pi_t^2(1-\pi_t)^2\Delta\theta'\Sigma^{-1}\Delta\theta$. We will also denote the $(1 \times n)$ vector by $\sigma_\pi \equiv \pi_t(1-\pi_t)\Delta\theta'\Phi'^{-1}$.

As we will see below, the second moments of asset returns will be non-linear functions of uncertainty, $h(\pi_t)$. In order to study the dynamics of these moments, it will be instructive to assess how uncertainty evolves by differentiating $h(\pi_t)$. We define the market at time t as good if $\pi_t \geq 0.5$ and as bad otherwise. The following corollary gives the conditional dynamics of uncertainty.

Corollary 2. *Define uncertainty as* $h(\pi_t) \equiv \pi_t(1-\pi_t)$. *Then the following process describes the evolution of conditional uncertainty over time*

$$dh_t = \begin{cases} \left[\alpha_h - (\mu-\lambda)\sqrt{h^{max} - h_t}\right]dt - \sigma_h dv_t & \text{if the market is good, } \pi_t \geq 0.5 \\ \left[\alpha_h + (\mu-\lambda)\sqrt{h^{max} - h_t}\right]dt + \sigma_h dv_t & \text{if the market is bad, } \pi_t < 0.5 \end{cases} \quad (2.3)$$

where $\alpha_h \equiv 2(\lambda+\mu)(h^{max} - h_t) - h_t^2\Delta\theta'\Sigma^{-1}\Delta\theta$, $\sigma_h \equiv 2h_t\sqrt{h^{max} - h_t}\Delta\theta'\Phi'^{-1}$ *is a* $(1 \times n)$ *row vector and* $h^{max} = \frac{1}{4}$. dv_t *is the same* $(n \times 1)$ *vector of standard Brownian motions with respect to* $\mathcal{F}_t = \sigma(D_\tau, \tau < t)$ *defined in proposition (1).*

Proof. The result follows from the application of Ito's lemma to $h(\pi_t)$. □

Note that the sign on the term σ_h in equation (2.3) shows that positive news in a bad market and negative news in a good market increase uncertainty[2].

---

[2]In what follows, we refer to news as shocks to dv_t times the signal to noise ratio $\Delta\theta'\Phi'^{-1}$. This normalization will help us compare news across assets and simplify our exposition. For instance, a shock to cash-flows from an asset with a very volatile process is not as informative as a shock of the same magnitude to an asset with more

Whenever expansions last longer than recessions, $\lambda < \mu$, the unconditional mean π_s will be greater than 0.50, that is, the market will be good more often than not[3]. As a result, it follows from corollary (2) that increases in uncertainty are more likely to arise after bad news than after good news. We will see below that this asymmetric response of uncertainty to news will also induce asymmetries in sample moments of asset returns, volatility and covariances.

In this economy investor preferences are represented by a constant absolute risk aversion utility function:

$$U(c,t) = -exp\left[-\rho t - \gamma c\right]$$

where γ is the coefficient of absolute risk aversion and ρ the time preference parameter.

Under the incomplete information set, $\mathcal{F}_t$, cash-flows can be written as $dD_t = \alpha_{Dt}dt + \Phi d\mathbf{v}_t$, where $\alpha_{Dt} = [\alpha_{1D,t}, ..., \alpha_{nD,t}]'$ and $\alpha_{iD,t} \equiv \theta_{iG}\pi_t + \theta_{iB}(1-\pi_t)$. The investor's optimization problem is solved by expressing dD_t in terms of the Brownian motion $d\mathbf{v}_t$ and including π_t as a state variable. Pricing formulas are obtained by imposing a market clearing condition on the available shares of the risky assets.

2.2.2 Asset Prices and Returns

The following proposition shows that asset prices that solve the investor problem and clear the market are non-linear functions of the investor beliefs and cash-flows.

Proposition 3. *[Ribeiro and Veronesi (2002)] The following asset prices solve the investor problem and clear the market:*

$$P_{it} = p_{0i} + \frac{D_{it}}{r} + p_{\pi i}\pi_t + p_{1i} + S_i(\pi_t) \quad (2.4)$$

stable cash-flows. Also, a positive shock to an countercyclical asset is actually bad news about the state of the economy. Thus, by considering news as $\Delta\theta'\Phi'^{-1}dv_t$, we do not need to be more specific about the cash-flow structure of the assets.

[3] In fact, NBER cycles imply an unconditional mean of around $\pi_s = 0.80$

where S_i is *the solution to a differential equation given in the Appendix and*

$$
\begin{aligned}
p_{0i} &= \frac{\theta_{iB}}{r^2} + \frac{(\theta_{iG} - \theta_{iB})}{r^2 (r + \lambda + \mu)} \mu \\
p_{\pi i} &= \frac{(\theta_{iG} - \theta_{iB})}{r (r + \lambda + \mu)} \\
p_{1i} &= -\frac{\gamma \sigma_{i,m}}{r^2}
\end{aligned}
$$

for $i = 1, ..., n$. *The market portfolio is the aggregate portfolio* $P_{mt} = \sum_{i=1}^{n} \omega_i P_{it}$.

Proof. See Appendix. □

The S_i function in equation (2.4) discounts cyclical assets and inflates countercyclical assets, generating a premium for holding risky assets. This discount (inflation) reaches a minimum (maximum) in the interior of $\pi_t \in (0, 1)$ if the asset is cyclical (countercyclical).

From asset prices, excess returns, variances and covariances can be obtained by direct application of Ito's lemma, as the following proposition shows.

Proposition 4. *Define excess return as* $R_{it}^e \equiv \frac{dP_{it}}{P_{it}} + \frac{D_{it}}{P_{it}} dt - r dt$. *Then the following continuous process describes excess returns in terms of the model's parameters:*

$$
R_{it}^e = \alpha_{iR,t} dt + \sigma_{iR,t} dv_t \tag{2.5}
$$

$$
\begin{aligned}
\alpha_{iR} &= \frac{1}{P_{it}} \left[\frac{\gamma}{r} e_i' \Sigma \omega - r S_i (\pi_t) + S_i' (\pi_t) \alpha_\pi + \frac{1}{2} S_i'' (\pi_t) \sigma_\pi^2 \right] \\
\sigma_{iR} &= \frac{1}{P_{it}} \left[\frac{e_i' \Phi}{r} + \left[S_i' (\pi_t) + p_{\pi i} \right] \pi_t (1 - \pi_t) \Delta\theta' \Phi'^{-1} \right]
\end{aligned}
$$

for $i = 1, ..., n$ *assets, where* e_i *is a* $(n \times 1)$ *vector of zeros and one at the* ith *row. For the market portfolio, set* $i = m$ *and* $e_m \equiv \omega$. *Expected excess returns are then given by* $E_t [R_{it}^e] = \alpha_{iR} dt$ *and covariance between assets* i *and* j, *where* $i, j = 1, ..., n, m$, *by:*

$$
\sigma_{ij,R} = \frac{1}{P_{it} P_{jt}} \left[(A_{ij} + M_{ij}(\pi_t)) \pi_t^2 (1 - \pi_t)^2 + (B_{ij} + N_{ij}(\pi_t)) \pi_t (1 - \pi_t) + C_{ij} \right] dt
$$

where

$$
\begin{aligned}
A_{ij} &= \frac{\Delta\theta_i \Delta\theta_j}{r^2 \left(r+\lambda+\mu\right)^2} \Delta\theta' \Sigma^{-1} \Delta\theta \\
B_{ij} &= 2\frac{\Delta\theta_i \Delta\theta_j}{r^2 \left(r+\lambda+\mu\right)} \\
C_{ij} &= \frac{1}{r^2} cov_t \left(dD_{it}, dD_{jt}\right) \\
M_{ij}(\pi_t) &= \Delta\theta' \Sigma^{-1} \Delta\theta \left[S_i'(\pi_t) S_j'(\pi_t) + \frac{S_i'(\pi_t)\Delta\theta_j + S_j'(\pi_t)\Delta\theta_i}{r\left(\lambda+\mu+r\right)} \right] \\
N_{ij}(\pi_t) &= \frac{\left[S_i'(\pi_t)\Delta\theta_j + S_j'(\pi_t)\Delta\theta_j \right]}{r}
\end{aligned}
$$

The excess return variance of asset i follows by setting both subscripts above to i.

Proof. It follows by applying Ito's lemma to the definition of excess returns. □

If the investor is risk-neutral, the discounting function S is zero and expected returns are proportional to the cash-flow covariance of the asset with the market portfolio, normalized by prices. If we instead assume the investor is risk averse, expected returns will also depend on the conditional probability π_t through the S function. Increases in the discounting of prices, $-rS_i\left(\pi_t\right)$, and in their sensitivity to π_t, $S_i'\left(\pi_t\right)\alpha_\pi$, imply higher expected returns. Also, higher uncertainty will command higher expected returns through the term $\frac{1}{2}S_i''\left(\pi_t\right)\sigma_\pi^2$. In addition to time-varying expected returns, the model also implies that return covariance and volatility are stochastic.

Expected returns can also be expressed in terms of the exposure of the asset to the common sources of risk, or risk factors. In this representation, the risk premium of an asset should equal its quantity of risk, the conditional beta, times the price of such risk. This decomposition is convenient as it splits the difficult task of estimating returns into two separate ones, the estimation of conditional betas and the price of risk. The price of risk is the same for all assets; conditional betas are functions of second moments, potentially easier to estimate (Merton, 1980).

Proposition 5. *Expected returns have the following factor representation:*

$$E_t\left[R_{it}^e\right] = \lambda_{mt}\beta_{im,t} + \lambda_{\pi t}\beta_{i\pi,t} \tag{2.6}$$

where the prices of risk are given by:

$$\begin{aligned}\lambda_{mt} &= r\gamma P_{mt}\sigma^2_{mR,t} \\ \lambda_{\pi t} &= f'(\pi_t) - r\gamma S'_m(\pi_t)\end{aligned}$$

and conditional betas, defined as $\beta_{im,t} \equiv \frac{\sigma_{im,R}}{\sigma^2_{m,R}}$ *and* $\beta_{i\pi,t} \equiv \sigma_{i\pi,R}$*, are given by:*

$$\beta_{im,t} = \frac{P_{mt}}{P_{it}} \times \frac{(A_{im} + M_{im}(\pi_t))\, h(\pi_t)^2 + (B_{im} + N_{im}(\pi_t))\, h(\pi_t) + C_{im}}{(A_{mm} + M_{mm}(\pi_t))\, h(\pi_t)^2 + (B_{mm} + N_{mm}(\pi_t))\, h(\pi_t) + C_{mm}} \quad (2.7)$$

$$\beta_{i\pi,t} = \frac{1}{P_{it}}\left[\left(p_{i\pi} + S'_i(\pi_t)\right) h(\pi_t)^2 + \frac{h(\pi_t)\,\Delta\theta_i}{r}\right] \quad (2.8)$$

where A, B, C, M and N are given in proposition (4). The functions f and S are solutions to differential equations given in the Appendix.

Proof. The expression for expected returns (2.5) follows by rewriting the optimal demand for shares, equation (5.3) in Appendix, in terms of expected returns and substituting for the market clearing condition, $X^*_t = \omega$. After scaling by the market variance, $\sigma^2_{m,R}$, we obtain market betas and prices of market risk. The expressions of betas in terms of the primitive parameters of the model follow after substituting for the covariances and variances given in (4). □

The first component of expression (2.6) is the usual conditional CAPM term, with variable beta and price of risk. The conditional market beta is defined as the ratio of the conditional covariance of asset and market excess returns normalized by the conditional variance of the market excess returns, $\beta_{im,t} = \sigma_{im,R}/\sigma^2_{m,R}$. This measure of risk captures the responsiveness of asset returns to changes in market returns. An asset with a high market beta will be riskier as it amplifies the volatility, or risk, of the investor's portfolio. Indeed, the price of market risk is positive as all elements in λ_{mt} are greater than zero, and assets with high betas reward the investor with higher returns.

As expressions for returns are available (see proposition (4)), we substitute covariances and variances of returns and link market betas to the parameters of the model and the state variables. As equation (2.7) shows, the market beta is a non-linear function of π_t and depends upon the discounting function S that can only be obtained numerically. In Section 2.3 we investigate betas by solving the model for calibrated parameters and computing the S function numerically. Before we proceed with the calibration, the case of a risk-neutral investor is discussed as this obviates the numerical computation of the S function.

The second term of the expected returns expression (2.6) results from the time-varying nature of the investment opportunity set (Merton, 1973). Note that the drift and diffusion terms of stock returns in equation (2.5) are functions of the random variable π_t and are thus stochastic. Assets that can help the investor hedge against future changes in profitability should be more expensive, i.e. have lower expected returns. The exposure of an asset to this source of risk is measured by its factor loading, defined as $\beta_{i\pi,t} \equiv \sigma_{i\pi,R}$, and is also equal to (2.8). We observe that assets that are very sensitive to changes in π_t, and have a large state shift risk, i.e. a large $\Delta\theta_i$, also have large betas.

The price of a unit of such risk is given by $\lambda_{\pi t}$ and it can be positive or negative, depending on the function f and the market discount S_m function. For the parameters selected in the next section, the price of risk is negative at lower values of π_t and positive for higher values.

2.2.3 The Risk-Neutral Case

The risk-neutral investor does not require a premium for uncertainty, the function S is zero and the analytical expressions of returns are simpler to interpret. The risk-neutral expressions still retain some interesting characteristics, *e.g.* time-variation and nonlinearity in π_t, as the investor is still uncertain and has to predict cash-flows.

Consider first the dynamics of the risk-neutral return volatility. Setting S equal to zero in (4), the risk-neutral variance of asset i is given by:

$$\sigma_{iRN}^2 = \left(\frac{1}{P_{it}^{RN}}\right)^2 \left[A_{ii} h\left(\pi_t\right)^2 + B_{ii} h\left(\pi_t\right) + C_{ii}\right]$$

where $P_{it}^{RN} = p_{0i} + D_{it}/r + p_{\pi i}\pi_t$ denotes risk-neutral prices and the constants A_{ii}, B_{ii} and C_{ii} are the same ones defined in proposition (4). Note that these constants are positive for both cyclical and countercyclical assets and so variance is increasing on uncertainty. Furthermore, return volatility of assets with a higher state shift risk is more responsive to changes on uncertainty.

From corollary (2) we argued that: i) good markets are more frequent than bad markets and ii) in good markets, negative news is followed by an increase on uncertainty whereas positive news implies a decrease on uncertainty. Both aspects of the learning process combined with the monotonic relation of volatility and uncertainty results in an asymmetric response of volatility to news. On average, bad news will be followed by a larger increase in volatility than good news. This predicted volatility asymmetry has long been observed in stock returns and was originally attributed to a leverage effect (Black, 1976) – negative stock returns reduce the equity value of the firm and increase the debt-to-equity ratio and the riskiness of the firm, which ultimately increase variance. Here, the mechanism behind is closer to the volatility feedback hypothesis of Campbell and Hentschel (1992) – negative shocks increase the required risk premium which further depreciate price to compensate the increase in the expected return.

As above, risk-neutral covariance of an asset i with the market simplifies to:

$$\sigma_{im,RN} = \frac{1}{P_{it}^{RN} P_{mt}^{RN}} \left[A_{im} h\left(\pi_t\right)^2 + B_{im} h\left(\pi_t\right) + C_{im} \right]$$

If the asset is cyclical, $\Delta\theta_i > 0$, the constants A_{im} and B_{im} are positive, since $\Delta\theta_m > 0$ as the market is by definition cyclical. On the other hand, these constants are negative for countercyclical assets. For simplicity, assume that C_{im}, the covariance of the asset and market cash-flows, is positive. Then, the covariance of asset and market returns increase with uncertainty if the asset is cyclical but decrease if the asset is countercyclical. Since the economy is in the good state most of the time, covariances will also respond asymmetrically to shocks: negative news has a stronger, upwards, effect upon covariances of cyclical assets than positive news. The opposite is true for countercyclical assets.

Finally, we consider how risk-neutral conditional betas respond to news. With S equal to zero, the market betas from equation (2.7) are simplified to:

$$\beta_{im,t}^{RN} = \frac{P_{m,t}^{RN}}{P_{it}^{RN}} \times \frac{A_{im} h\left(\pi_t\right)^2 + B_{im} h\left(\pi_t\right) + C_{im}}{A_{mm} h\left(\pi_t\right)^2 + B_{mm} h\left(\pi_t\right) + C_{mm}} \tag{2.9}$$

As discussed above, for cyclical assets and assuming C_{im} positive, both numerator and denominator are increasing functions of uncertainty. Depending upon which term responds more to uncertainty, the asset's beta will be either increasing or decreasing on uncertainty. An inspection of the constants A_{im}, A_{mm}, B_{im} and B_{mm} indicates that assets with smaller $\Delta\theta_i$ than that of the market's, $\Delta\theta_m$, have a decreasing beta on uncertainty and vice-versa. This pattern is maintained after scaling equation (2.9) by the ratio of prices. For the countercyclical asset, the market covariance is declining in uncertainty and, as a result, conditional betas decline as uncertainty increases. As with the other moments, conditional betas are also expected to respond asymmetrically to news. For assets with large state shift risk, $\Delta\theta_i > \Delta\theta_m$, conditional betas, on average, increase more after negative news than after positive news. However, for countercyclical assets, $\Delta\theta_i < 0$, or assets with low state shift risk, $0 < \Delta\theta_i < \Delta\theta_m$, conditional betas increase, on average, more after positive news than after negative news.

We summarize the findings about risk-neutral moments as follows: first, conditional variance increases on uncertainty, irrespective of the asset's cash flow structure. Second, the conditional covariance of asset returns and market returns increases on uncertainty if the asset is cyclical and decreases if it is countercyclical. Finally, conditional betas of assets with a larger state shift risk than the market's will increase on uncertainty and decrease otherwise. Since these moments are monotonic functions of uncertainty and uncertainty responds asymmetrically to news, risk-neutral variance, covariance and betas also respond asymmetrically to news.

As we will see in the next section, similar patterns are observed when the investor is risk averse. These expressions of returns are also monotonic functions of a "risk adjusted" uncertainty, that attains a maximum point slightly to the right of $\pi_t = 0.5$.

2.3 Simulated Economy

To further investigate the model's predictions, we calibrate an economy with five assets, each following one of the five book-to-market sorted portfolios, with parameters drawn from the U.S. economy. The cash-flow parameters implied by such portfolios varies substantially across quintiles and provide an appropriate framework for this investigation. This variation across quintiles is in line with the perception that low book-to-market firms (growth firms) derive most of their profitabiliy from future cash-flows as opposed to value firms, that derive most of their profitability from current cash-flows and assets and that, as a result, are more susceptible to the current economic conditions. Indeed, general equilibrium models that explain the value premium anomaly[4] often explore the differences in the investment and cash-flow characteristics of those firms (Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003) and Zhang (2005)).

In the Subsection 2.3.1, we perform the calibration and show that the model is able to match reasonably well the unconditional mean and variance of excess returns of the five book-to-market portfolios. Then, in Subsection 2.3.2, we investigate how conditional market betas varies across portfolios with distinct risk characteristics and how news about economic conditions relates to the dynamics of betas.

2.3.1 Calibration

For this calibration, we will set the risk aversion parameter equal to one, $\gamma = 1$, as in Veronesi (2004). The other free parameters of the model will be calibrated from the U.S. economy. The risk-free instantaneous rate is set at $r = 0.045$, a relatively high value but close to the average one month treasury bill rate on the same period (4.9%).

The free parameters of the cash-flow processes are: the drift vectors $\theta_G = [\theta_{1G}, ..., \theta_{5G}]'$ and $\theta_B = [\theta_{1B}, ..., \theta_{5B}]'$, the diffusion matrix Φ, and the scalars of the Markov-switching transition matrix μ and λ, that characterize the random switches of the drifts. For the transition matrix, we select the parameters implied by the NBER cycles data[5] from 1956 to 2010. As shown in Panel A of Table 2.1, the NBER cycles data indicate 83.3% of the months are expansionary, with an average duration of a recession of 11 months and of an expansion of 62 months. These numbers imply[6] the following monthly transition matrix parameters: $\lambda = 0.016$, the probability

[4]The discrepancy of high and low book-to-market portfolios expected returns relative to the static CAPM predictions.

[5]http://www.nber.org/cycles.html

[6]From the NBER average length of an expansion we obtain $\lambda \equiv prob\,(S_{t+1} = Bad|S_{t+1} = Good)$ by setting the average sample duration of an expansions equal to $1/\lambda$. $\mu \equiv prob\,(S_{t+1} = Good|S_{t+1} = Bad)$ is then obtained by setting $\mu/\left(\mu + \lambda\right)$ equal to the proportion of expansionary months in the sample.

of going from the good state to the bad state, and $\mu = 0.080$, the probability of switching to the good state from the bad state.

The drifts of the cash-flows θ_G and θ_B are calibrated[7] using the moments implied by the log-dividend growth of the five book-to-market portfolios data[8], from 1956 to 2010. The log-dividend growth series are constructed from the difference in the monthly returns with and without dividend payouts as in Bansal, Dittmar, and Lundblad (2005). In order to avoid seasonal variations typical to dividend payouts, monthly log-dividend growth are aggregated at the annual frequency. Panel B of Table 2.1 shows the sample means and the standard deviations of log-dividend growth for the five book-to-market portfolios on the full sample as well as the means and standard deviations conditional on recessionary and expansionary years[9]. The log-dividend growth average of the value portfolio varies the most across the two sub-samples, from -0.130 during recessionary years to 0.109 during expansionary years. The difference in the conditional averages is 0.239. On the other hand, the log-dividend growth averages of the growth portfolios change the least, 0.058 and 0.046 in recessions and expansions respectively. The difference in the conditional averages is -0.011.

It should be noted the log-dividend series are very volatile and the distinctive pattern between value and growth portfolios may not be supported on statistical grounds. However, this pattern is roughly monotonic across quintiles, particularly the average log-dividend growth during recessions, indicating this to be an economically meaningful pattern related to the book-to-market ratio. Furthermore, it has been argued that value firms are particularly susceptible to economic downturns, which is in line with our empirical findings. For instance, Fama and French (1993) conjectured that value firms are riskier than growth firms because a higher book-to-market ratio associates most often with distressed firms. Also, Zhang (2005) characterizes value firms as those with costly-to-adjust investments (*e.g.* asset's in place type of investment)

[7]Because of the assumption that cash-flows follow arithmetic Brownian motions and that the data actually show exponential growth of dividends, the parameters used for calibration, based on log-differences, are only valid as approximations, and particularly around cash-flow levels close to one.

[8]The data was obtained from the website of Kenneth French.

[9]A year is considered a recessionary year if five or more months are recessionary months according to the NBER data

and thus those with cash-flow more susceptible to adverse shocks, i.e. recessions, whereas growth firms as those with more flexible investments scales (*e.g.* growth options type of investment) and thus those with cash-flow less sensitive to fluctuations in economic conditions. The empirical findings above combined with the economic theory indicates that a reasonable calibration for the changes in the cash-flow drift, $\Delta\theta_i = \theta_{iG} - \theta_{iB}$, to be larger for the value portfolio and smaller for the growth portfolio.

The numbers chosen for the drifts θ_G and θ_B, and diffusion matrix Φ follow the patterns observed by the data but also are such that the model's implied unconditional excess return are similar to real data sample averages. Because the model cannot exclude *a priori* negative prices, this calibration strategy ensures that prices, and therefore returns, are within a reasonable range.

Table 2.2 shows the parameters chosen for assets 1 to 5, that respectively mimic the lowest to highest book-to-market quintile portfolios. Asset 1 (A1), that resembles the growth portfolio, has the lowest state-shift risk among the assets, $\Delta\theta_1 = -0.01$, the lowest unconditional drift, $\bar{\theta}_1 = 0.04$, and the largest volatility $\sigma_1 = 0.16$. On the other side is asset 5 (A5), that resembles the value portfolio. It has the highest state-shift risk, $\Delta\theta_5 = 0.23$, but also the highest unconditional drift $\bar{\theta}_5 = 0.062$ and the lowest volatility, $\sigma_5 = 0.09$. Note that this diffusion term is smaller than the one implied by the data. This was needed to match sample and theoretical returns, a result of our calibration strategy discussed above[10]. The correlation parameters, ρ_{ij}, were set equal to 0.25, 0.15, 0.10 and 0.05 for $|i - j|$ equal to 1, 2, 3 and 4, respectively, and sets a higher correlation to portfolios with similar book-to-market values. Table 2.2 also shows the expected excess returns and deviations at $\pi_t = \pi_s$ implied by the model, *i.e.* the unconditional moments, as well as the sample counterparts of the five book-to-market portfolios[11]. A comparison of the values on Panel B and Panel C shows that the model reproduces the cross-section dispersion on expected returns of the book-to-market portfolios

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[10]If we imposed a higher variance for A5 cash-flow and kept the state-risk spread, this would have resulted in a very risky asset with an incompatible high expected returns. We preferred to keep the state-risk spread but reduce the diffusion risk.

[11]Cash-flow levels are set at 1. At this point, the drift parameters better approximate the log-dividend changes that were used to calibrate them.

for reasonable parameters.

2.3.2 Conditional Betas

Given the parameters that calibrated the model, functions f and S can be computed numerically and the pricing formulas in equation (4) follows. Next, we discuss the properties of these formulas. First, we discuss how conditional betas differ across asset for given levels of beliefs. Then, we discuss how conditional betas differ across assets given changes in investor beliefs, that is, following the arrival of news.

2.3.2.1 Cross-Section Asymmetries

Figure 2.1 shows the model's main expressions for all possible values of π_t and fixed cash-flows at $D_t = 1$. On the top-left plot, we see that A1 has the highest price on almost all the domain of π_t. This asset is the least profitable, as it has the lowest unconditional drift among all cash-flows, but also the least susceptible to changes in the economic conditions and so less risky. At the other extreme is asset A5, which is the most profitable one, but also the most risky and discounted one, with the lowest price on almost all the domain of π_t. In the top-right plot we see that expected returns for asset A5 is the most sensitive to π_t, changing from 3%, when π_t is close to one, to almost 15%, when uncertainty is higher. All the other cyclical assets also have increasing expected returns on uncertainty, but the change is less substantial. The expected return on the countercyclical asset A1 slightly declines on uncertainty. The second row in Figure 2.1 shows covariances of the assets with the market as well as the variances of asset returns. The shapes are similar to the ones implied by the risk-neutral case, but peaking slightly to the right of the maximum uncertainty point, around $\pi_t = 0.6$.

The last four plots in Figure 2.1 show all the elements in the factor decomposition of excess returns (5), the market betas, $\beta_{im,t}$, hedging betas, $\beta_{i\pi,t}$, and their corresponding prices, $\lambda_{m,t}$ and $\lambda_{\pi,t}$, as functions of all possible values of π_t and given cash-flows. First, we observe that the premium for exposure to market risk is more important than the premium for exposure to hedging risk. The most sensitive asset to the hedging factor, A5, has the highest absolute hedging premium at $\pi_t = 0.75$ when $\lambda_{\pi t}\beta_{5\pi,t} = -1\%$. At the same point, the risk premium for market risk is much larger, around $\lambda_{mt}\beta_{5m,t} = 15\%$. This confirms Merton's (1980) observation that the market portfolio is likely the most important factor determining expected returns and justifies the assumption made by Jagannathan and Wang (1996) of hedging motives not being sufficiently important.

Second, as we previously noted analytically for the risk-neutral case, market beta of assets with a high and positive $\Delta\theta_i$, such as A5, increases as uncertainty about the state of the economy increases. Since the investor is now risk-averse, the market beta of asset A5 peaks slightly above[12] the point of maximum uncertainty, taking its maximum value of $\beta_{5m,t} = 1.80$ at around $\pi_t = 0.60$. On the other hand, the beta of asset A1, declines as π_t moves away from 0 and 1, reaching a minimum of $\beta_{1m,t} = 0.40$ also around $\pi_t = 0.60$. We note also that there is enough variation in betas to make A1 riskier than A5. In periods of low uncertainty, *e.g.* when $\pi_t > 0.95$, the beta of A1 is higher than that of A5.

Third, the price of market risk, or the market premium, is positive and also increasing on uncertainty. It reaches a maximum of about $\lambda_{mt} = 8\%$ at $\pi_t = 0.60$ and a minimum of $\lambda_{mt} = 3\%$ at $\pi_t = 1$. At $\pi_s = 0.83$, the unconditional or long run mean of the random variable π_t, the price of market risk is 6.5% and close to its historical sample mean[13].

Finally, time-variation of market betas is relevant to some assets but less important to others. Figure 2.1 indicates that for A1 and A5, both the conditional market beta and price of market risk are equally important for the asset's risk premium. Consider a shift to investor beliefs from $\pi_1 = 0.90$ to $\pi_2 = 0.50$. The price of market risk, λ_{mt}, increases from 4.93% to 7.81%, a change of 58%. Likewise, asset A5 beta also change significantly, from 1.33 to 1.87, an increase of 41%. The change in the asset A1 beta is also important but to the opposite direction, from 0.72 to 0.42, a decrease of 42%. The variation in the betas is less important than the variation in the price of market risk for the other assets, as we clearly observe from the plots.

The above expressions are for all possible values of $\pi_t \in [0, 1]$, but not all are equally likely. For the chosen parametrization, in particularly λ and μ that matches the U.S. business

[12] This rightward shift resulting from increases in risk aversion was also observed, in the single asset case, by Veronesi (1999).

[13] The price of market risk is often estimated by the sample mean of the market portfolio excess return.

cycles, most of the mass of the π_t distribution is above 0.50, since the economy is most often in expansionary periods. Thus, on average, negative news about the economic conditions increases uncertainty while positive news decreases it. Consequently, the response of market betas[14] to news is asymmetric due its (approximately) monotonic relation to uncertainty.

Another aspect of the π_t distribution under this parametrization is that periods of higher uncertainty most often occur during the bad state. The persistence of learning process coupled with its shorter duration results in a higher proportion of uncertainty periods during the bad state. Therefore, volatility of asset returns, the covariance of cyclical asset with the market returns and the price of market risk, will tend to be higher during the bad state, as these variables typically increase on uncertainty. The model thus provides theoretical justification to the empirical findings that such variables tend to increase during recessions. Another implication of the model is that the value premium should be higher during the bad state, as the difference in market betas of value and growth portfolios is also higher during high-uncertainty periods. This countercyclicality of the value premium predicted by the model has also been observed empirically (Petkova and Zhang, 2005).

2.3.2.2 Time-Series Asymmetries

To assess the relevant portion of the pricing formulas, we generate time-series of the variables in our model using the same parameters discussed in the calibration. Once cash-flows are generated according to (2.1), beliefs are computed as indicated by the optimal filtration equation (1). These state variables, π_t and D_t, are then used to determine the model's pricing formulas. The length of the generated series is equivalent to a sample of six years of daily data. We avoid a sample larger than 6 years because of our assumption that cash-flows follow an arithmetic Brownian motion. As the cash-flow level moves away from its starting value, $D_0 = 1$, the drift of the stochastic process becomes a worse approximation of percentage changes, the scale used for the calibration. On the other hand, we do not select a smaller sample because of the duration of recessions and expansions implied by the transition matrix parameters. The six year time frame allows an expansionary period lasting five years and a recession lasting one

[14] And also the variance of returns and the covariance of returns with the market.

year, which implies average durations and proportions of good and bad months that are similar to the ones imposed by the calibration of λ and μ.

In order to assess how the proportion of good and bad states can have an impact, we consider three different combinations of good and bad states. First, we consider the case of no bad state and six years of good state. We will refer this case as the bull market case. In the second scenario the economic conditions are average, with five years of the good state and one year of the bad state. In the third scenario, two out of the six years the economy is in the bad state. We refer this as the bear market case. For all three scenarios, the bull, bear and regular markets, 500 histories are generated, each with 1584 observation (six years of daily observations).

The top two panels in Table 2.3 show the averages and standard deviations across the 500 histories of the expected and realized excess returns. We observe that for the bull and bear markets, expected excess returns do not coincide with realized excess returns. In the bull market case, asset A5, that resembles the value portfolio, outperforms and has higher realized returns than would be expected; the annual average realized return of asset A5 is around 9% and the average annual expected returns is 4%. On the other hand, in the bear market, A5 underperforms, with an annual average realized return of 4.8% against an average expected return of 7.6%. The opposite holds for asset A1, the asset that resembles the growth portfolio. It underperforms in the bull market but overperforms in bear market. When the economic conditions are the ones implied by the calibration, the second scenario, expected returns are similar to the realized ones. The values do not coincide because of sample variation and of the approximation imposed by the assumption that cash-flows follow arithmetic Brownian motions[15].

Since expected excess returns also have a factor characterization in our model, such discrepancies or anomalies observed above for expected returns are also observed in the factor regression. The pricing errors captured by the intercepts of static CAPM regressions, the realized alphas in the lower-left panel of Table 2.3, indicates the existence of a value premium when

[15]Veronesi (2004), that investigates the properties of the univariate version of our model, also faces similar approximation errors.

sample are generated by bull markets. The average intercept across all histories is negative for asset A1 and positive for asset A5. The opposite is observed for bear markets, where a growth premium arises. We have deliberately ignored the hedging factor in the regressions of the static CAPM, as its contribution to risk premium is much smaller than the exposure to market risk. As Table 2.3 shows, the average price of the hedging risk is around $\lambda_\pi = -1\%$ and the average quantity of hedging risk is 0.6 for the asset A5, resulting in an average hedging premium of about -0.6%, less than a tenth of the market risk premium, 6.5%.

These results point to an interpretation of the forces behind the value premium related to biased sampling. A similar argument has been employed by Veronesi (2004) to explain the equity premium puzzle. The author, using the univariate version of our model, attributes the apparent puzzle that market average returns are too high relatively to its observed (realized) riskiness, to a rational premium required by the investor to account for a peso-problem type of event – a very unfavorable event, which the investor is aware of, that has never happened, at least in the particular sample considered.

The importance of the sample to the value premium was also observed by Ang and Chen (2007). They argue that most studies that find a value premium on the U.S. stock market generally consider only the post-1963 period, mainly due to the ready availability of data, and that the omission of previous years is key to finding a value premium. In fact, they show that the alphas in the static CAPM regressions turn out to be insignificant when the sample is extended to include the months from 1926 to 1962. Since our objective is to explore the dynamics of market betas implied by our theory and not to propose a solution to the value premium, we restrict our analysis to the unbiased histories generated under the second scenario.

For the task of unveiling the dynamics of market betas, we first fit an univariate asymmetric GARCH(1,1) model to the simulated returns to discuss the dynamics of volatility. The specification of conditional volatility, also referred to as GJR-GARCH model (Glosten, Jagannathan, and Runkle, 1993), is the following:

$$\begin{aligned} r_{it} &= \alpha_i + u_{it} \\ \sigma_{it} &= \kappa + \delta\sigma_{it-1} + \gamma u_{it-1} + \gamma_{-} 1_{[u_{it-1}<0]} u_{it-1} \end{aligned} \tag{2.10}$$

where $u_{it} = \sigma_{it}\epsilon_{it}$, $\epsilon_{it} \overset{iid}{\sim} N(0,1)$, and $1_{[.]}$ the indicator function. This specification is appropriate here as it allows past shocks to influence future volatility, in line with the model assumption that investor beliefs are based on past information, and also because the sign of shocks, consistent with the learning feature of the model, can be informative about shifts in the economic conditions.

We use mean-adjusted past excess returns as proxies for cash-flows news, which is what actually drives beliefs. This can be justified by the following. From equations (1) and (4) we observe that both beliefs and excess returns are driven by same the standard Brownian motion, dv_t. Furthermore, both diffusion terms in those equation, σ_π and σ_{iR}, indicate that shocks to returns will be positively related to shocks to beliefs when the term $S_i'(\pi_t) + p_{\pi i}$ is positive, which is the case for all cyclical assets on most values of π_t (the term $S_i'(\pi_t)$ can be negative and offset $p_{\pi i}$ for lower values of π_t, which only seldom occur).

Table 2.4 shows the averages, standard deviations and quantiles of the estimated parameters of (2.10) across the 500 histories for each asset. First, we observe that conditional volatilities are very persistent, the δ's are high and close to one, a well known stylized fact about stock returns. Second, negative shocks to returns are more important to future volatility than positive shocks, as for all assets the coefficient γ_- is positive. This asymmetric response of volatility to past shocks has long been observed empirically and referred to as the leverage effect (Black, 1976). Finally, assets with cash-flows that are more exposed to shifts have stronger asymmetries. The average coefficient γ_- across the 500 histories for asset A5 is 0.089 while for asset A1 it is only 0.016. This was expected, as A5's expected returns is the most responsive one to changes in uncertainty. Furthermore, shocks to A5 are also the most informative ones, as it has the highest signal to noise ratio among all cash-flows.

We now turn to the question of how the covariances respond to past shocks. In order to do so, we fit an asymmetric multivariate GARCH model to the simulated data. More precisely, we follow the BEKK specification of Engle and Kroner (1995) but also introduce asymmetric terms as in Hafner and Herwartz (1998). For computational convenience, we focus on bivariate models of asset excess returns and market excess returns.

Denote as before demeaned excess returns by u_{it} for $i = 1, ..., 5$ and $i = m$, the market portfolio, the vector as $u_t = [u_{it}, u_{mt}]'$ and let $\mathcal{G}_t$ be the information set at time t. The conditional joint distribution is assumed to be $u_t|\mathcal{G}_{t-1} \sim \left(0, \Sigma_{t|t-1}\right)$ with conditional covariance given by

$$\begin{aligned}\Sigma_{t|t-1} &= C'C + A'\Sigma_{t-1|t-2}A + B'u_{t-1}u_{t-1}'B \\ &\quad +1_{[u_{it-1}<0]}D_1'u_{t-1}u_{t-1}'D_1 + 1_{[u_{mt-1}<0]}D_2'u_{t-1}u_{t-1}'D_2\end{aligned} \tag{2.11}$$

where A, B, D_1 and D_2 are 2×2 matrices and C an upper triangular 2×2 matrix. Matrices D_1 and D_2 are new to the original BEKK formulation and add the needed flexibility to capture asymmetric responses of the covariance matrix to shocks. Assuming that the joint distribution is normal, parameters are estimated by maximizing the log-likelihood function.

The estimated parameters[16] of equation (2.11) for a simulated[17] history are shown in Table 2.5. We also show the log-likelihood ratio (LR) statistics that compares the full model (2.11) with an specification with just one asymmetric matrix, D_2, and another with no asymmetric matrices. The LR-test p-values indicates that the difference in the likelihoods of the symmetric BEKK and the asymmetric BEKK, with matrices D_1 and D_2, are statistically significant. The LR-test also shows that asymmetries at the asset level are not statistically relevant, except for asset A5. This was expected, as A5 is the most informative asset about the state of the nature and so its returns shocks relate more closely to changes in aggregate returns.

We can also see that asymmetries are relevant by noting that the parameters on matrices D_1 and D_2 are significant and relatively large. However, in order to make sense of these numbers, we compute impulse response functions (IRFs). First, we need to rewrite the matrices of parameters in vector form using the *vec* operator that stacks columns:

$$vec\left(\Sigma_t\right) = \overline{C} + \overline{A}vec\left(\Sigma_{t-1}\right) + \overline{B}vec\left(u_t u_t'\right) + \overline{D_1}1_{[u_{it-1}<0]}vec\left(u_t u_t'\right) + \overline{D_2}1_{[u_{mt-1}<0]}vec\left(u_t u_t'\right)$$

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[16]The asymptotic distribution of the estimates is generally unknown and the results can only provide a description of the dataset (Herwartz and Lutkepohl, 2000).

[17]The precise results can vary, depending on the particular history chosen. We have selected the history where the results of the unconditional market betas that follows from the estimation are similar to the ones predicted by the theory.

where $\overline{C} = (C \otimes C)' \, vec\,(I_2)$, $\overline{A} = (A \otimes A)'$,$\overline{B} = (B \otimes B)'$, $\overline{D_1} = (D_1 \otimes D_1)'$, $\overline{D_2} = (D_2 \otimes D_2)'$ and I_2 is a (2×2) identity matrix. Here, $vec\,(\Sigma_t)$ will then be a (4×1) vector, with the first element being the asset return conditional variance, the second and third elements the conditional covariance of the asset return with the market return and the last term the market return conditional variance. Hafner and Herwartz (1998) define the IRF as $V_t\,(\xi_0) = E\,[vec\,(\Sigma_t)\,|\xi_0, \Sigma_0]$, which can be computed by starting the above auto-regression at the long run value of the covariance matrix, Σ, and perturbing it with standardized shocks, ξ_0. At $t = 1$ we have

$$V_1\,(\xi_0) \quad = \quad \overline{C} + \left(\overline{B} + 1_{[\xi_{0,i}<0]}\overline{D_1} + 1_{[\xi_{0,m}<0]}\overline{D_2}\right) vec\left(\Sigma^{1/2}\xi_0\xi_0'\Sigma^{1/2}\right) + \overline{A}vec\,(\Sigma)$$

and for $t \geq 2$

$$V_t\,(\xi_0) \quad = \quad \overline{C} + \left(\overline{A} + \overline{B} + \frac{\overline{D_1}}{2} + \frac{\overline{D_2}}{2}\right) V_{t-1}\,(\xi_0)$$

Impulse response functions for betas easily follow from the ratio of the covariance and market variance IRFs:

$$\beta_{it}\,(\xi_0) \quad = \quad \frac{V_{im,t}\,(\xi_0)}{V_{m,t}\,(\xi_0)}$$

where $V_{im,t}\,(\xi_0)$ and $V_{m,t}\,(\xi_0)$ are the second and fourth elements of the vector $V_t\,(\xi_0)$.

Figure 2.2 shows the IRFs of assets A1 and A5 22 days after the initial shock. The first column shows the variances IRFs. The upper plot shows the responses to shocks in the market portfolio, leaving asset return unperturbed, and the lower plot responses to shocks in the assets, leaving market return unperturbed. The asymmetric response to shocks is clear, particularly to A5, as was noted in the univariate estimation above. We observe that negative shocks to both market and the assets returns result in a larger change in the volatility than positive shocks.

The second column in Figure 2.2 shows the IRFs of the covariance of assets A1 and A5 with the market portfolio, where we have written the y-axis in terms of percentage changes to the initial position. The upper plot shows how shocks to the market portfolio changes future covariances. Negative shocks, particularly larger than two standard deviations, increase the covariance of A5 with the market portfolio substantially. On the other hand, when the shocks are positive, the covariance declines slightly. This shape is in line with our previous discussion on the relation of covariances with uncertainty and how uncertainty changes with news. The covariance of A1 with the market portfolio is relatively stable, and slightly increases with negative shocks and slightly decreases after positive shocks in the market portfolio. The lower plot on the second column, shows a similar response of the covariance of A5 to shocks on its own returns, but with changes of smaller magnitude.

Finally, the last column in Figure 2.2 shows the betas IRF for both assets A1 and A5. In the top plot we see how betas respond to shocks in the market. As discussed analytically, we have that negative shocks to market returns result in an increase in the market beta of the asset A5 but a decrease in the market betas of A1. A5 beta increases by about 30% on large negative news, while A1 beta decreases by about 30% on large negative news. On the other hand, positive news have a much smaller impact on betas.

2.4 Conclusion

The implications of the model for the variance, covariance and conditional market betas of asset returns confirm many empirical facts and also suggests new results. First, as pointed by Veronesi (1999) for the market portfolio in the univariate model, excess returns display the predicted volatility asymmetry so pervasive in real data. This has been originally attributed by Black (1976) to a leverage effect, but in our model the justification is closer to the volatility feedback hypothesis of Campbell and Hentschel (1992). A novel implication for the dynamics of volatility is that assets that are very sensitive to the economic conditions should also display stronger asymmetric responses to news.

Second, the covariance of asset returns with the market portfolio also responds asymmetrically to the arrival of news, a result also verified empirically, for instance, in the contagion literature of international markets (Ribeiro and Veronesi, 2002). Our model shows that negative news increases the covariance of cyclical assets with the market portfolio by a larger magnitude than positive news. Again, this asymmetry will be larger the more sensitive the asset is to changes in economic conditions. In the case of a countercyclical asset, the model predicts an opposite asymmetric response of covariances to news.

Third, the conditional market betas respond asymmetrically to news. Market betas of assets that are very sensitive to changes in the economic conditions increase during high uncertainty cases as opposed to less sensitive assets. A concrete example of assets with such opposing risk dynamics, as shown in the calibration exercise, are the value and growth portfolios.

The empirical evidence regarding the asymmetric response of market betas to news is unclear (Braun, Nelson, and Sunier, 1995). Nonetheless, the difficulty of assessing the opposite response of betas to news from realized returns in a multivariate GARCH framework was also present in our investigation under the controlled environment of simulated returns. Despite the analytical equations indicating that asymmetries are relevant, the parameters estimated from an asymmetric GARCH model did show such asymmetries but for some histories only. Since the forces behind the beta asymmetry are the same ones behind the asymmetry of variance and covariances, which are two well known empirical facts about stock returns, the lack of empirical evidence of beta asymmetry could be a result of econometric misspecification as opposed to economic irrelevance.

2.5 Graphs and Tables

Table 2.1: NBER Business Cycles and Book-to-Market Portfolios Log-Dividend Growth

Panel A reports the average number of months and proportion of recessions and expansions according to the NBER business cycles data. Panel A also shows the Markov-switching transition matrix parameters that is implied by NBER data, where: $\lambda \equiv prob\,(S_{t+1} = Bad|S_t = Good)$ is obtained by setting the average sample duration of expansion equal to $1/\lambda$; and $\mu \equiv prob\,(S_{t+1} = Good|S_t = Bad)$ is obtained by setting $\mu/\left(\mu + \lambda\right)$ equal to the proportion of expansionary months in the sample. Panel B reports the unconditional and conditional sample moments of the annual log-dividend growth on the 5 book-to-market portfolios. The lowest quintile portfolio is the growth portfolio and the highest quintile, the value portfolio. Log-dividend growth series are constructed from monthly returns with and without dividend payouts, as in pg. 1648 of Bansal, Dittmar, and Lundblad (2005). Log-dividend growth is aggregate at the annual frequency to avoid seasonal variations. Mean and standard deviations are computed for the full sample and conditional to recessionary and expansionary years, where a year is recessionary if five or more months in it were recessionary according to NBER.

Panel A: NBER Cycles (1956 - 2010)

	Sample (Months)			**Implied Transition Matrix**	
	Recessions	**Expansions**	**t | t+1**	**Good State**	**Bad State**
Average	11.22	62.22	**Good State**	0.984	0.016
Proportion	0.17	0.83	**Bad State**	0.080	0.920

Panel B: Log-Dividend Growth Sample Moments (1956 - 2010)

		Mean			**Standard Deviation**	
	Full Sample	**Recession**	**Expansion**	**Full Sample**	**Recession**	**Expansion**
Growth	0.0487	0.0579	0.0464	0.0888	0.0742	0.0927
Qnt 2	0.0479	0.0324	0.0506	0.1040	0.1089	0.1039
Qnt 3	0.0558	-0.0026	0.0704	0.1021	0.1053	0.0970
Qnt 4	0.0519	-0.0192	0.0697	0.1196	0.1985	0.0848
Value	0.0610	-0.1305	0.1089	0.1769	0.2438	0.1173

Table 2.2: Calibration Parameters, Model and Sample Moments

Panel A reports the parameters that calibrated the cash-flow processes, defined by the equation (2.1). $\theta_{G,i}$ ($\theta_{B,i}$) is the cash-flow drift of asset i in the good (bad) state. $\sigma_{D,i}$ is the standard deviation of asset i's cash-flow. The correlation parameters, ρ_{ij}, are set equal to 0.25, 0.15, 0.10 and 0.05 for $|i-j|$ equal to 1, 2, 3 and 4, respectively. Both standard deviation vector and correlation matrix are used to determine the diffusion term Φ, which is the same across the two states. Panel B reports the main numbers that result from the calibration and from the model's pricing equations. $\Delta\theta_i \equiv \theta_{G,i} - \theta_{B,i}$ is a measure of the asset's sensitivity to shifts. $\bar{\theta}_i = \pi_s \theta_{G,i} + (1-\pi_s)\,\theta_{B,i}$ is the unconditional or long-run drift, where $\pi_s \equiv \mu/(\mu+\lambda) = 0.83$. $E\left[r_i^{ex}\right]$ is the unconditional excess return or, equivalently, the conditional excess return when $\pi_t = \pi_s$. $\sigma_{r,i}$ is the unconditional standard deviation of asset's i excess return or, equivalently, the conditional standard deviation when $\pi_t = \pi_s$. Panel C reports the sample moments of the empirical counterparts in Panel B. The sample moments from the five book-to-market portfolios monthly were computed from monthly data from 1956 to 2010, and then annualized.

Panel A: Calibrated Parameters

	A1	**A2**	**A3**	**A4**	**A5**
$\theta_{G,i}$	0.040	0.059	0.066	0.074	0.100
$\theta_{B,i}$	0.050	0.000	-0.020	-0.040	-0.130
$\sigma_{D,i}$	0.160	0.135	0.130	0.130	0.090

Panel B: Model's Implied Parameters

	A1	**A2**	**A3**	**A4**	**A5**
$\Delta\theta_i$	-0.010	0.059	0.086	0.114	0.230
$\bar{\theta}_i$	0.042	0.049	0.052	0.055	0.062
$E\left[r_i^{ex}\right]$	3.49%	4.85%	5.51%	5.95%	8.22%
$\sigma_{r,i}$	0.151	0.135	0.141	0.152	0.213

Panel C: Book-to-Market Sample Parameters

	Growth	**Qnt 2**	**Qnt 3**	**Qnt 4**	**Value**
$\bar{\theta}_i$	0.049	0.048	0.056	0.052	0.061
$\sigma_{D,i}$	0.089	0.104	0.102	0.120	0.177
$E\left[r_i^{ex}\right]$	3.50%	4.81%	5.96%	6.53%	8.03%
$\sigma_{r,i}$	0.190	0.166	0.165	0.181	0.198

Table 2.3: Time Series Simulations

500 histories are simulated, each with 6 years of daily data for three different combination of recessions and expansions. In case A there is no recession. In case B, there is one recession in the first year and then 5 years of expansions. In case C there are two years of recessions. The Table shows the averages and standard deviations of the 500 histories sample means. Expected excess returns, expected market betas and expected hedging betas, and the prices of market risk, λ_m, and of hedging risk, λ_π, are ex-ante variables. Realized excess returns is the ex-post average return generated by the model. Realized betas and alphas are the average coefficients obtained from running regressions with realized returns for each history. A1 through A5 are the calibrated assets that resembles the growth through value portfolios and M is the market portfolio.

	case A		**case B**		**case C**		**case A**		**case B**		**case C**	
	mean	**std.**	**mean**	**std.**	**mean**	**std.**	**mean**	**std.**	**mean**	**std.**	**mean**	**std.**
	Expected Excess Returns						**Realized Excess Returns**					
A1	0.035	0.002	0.034	0.001	0.034	0.002	0.035	0.052	0.038	0.052	0.040	0.054
A2	0.037	0.002	0.041	0.003	0.044	0.003	0.050	0.038	0.045	0.042	0.040	0.045
A3	0.039	0.003	0.045	0.004	0.049	0.004	0.056	0.038	0.048	0.046	0.038	0.045
A4	0.039	0.004	0.047	0.005	0.053	0.006	0.062	0.038	0.053	0.041	0.040	0.042
A5	0.040	0.007	0.062	0.013	0.076	0.013	0.091	0.024	0.075	0.025	0.048	0.030
M	0.033	0.003	0.054	0.005	0.046	0.005	0.056	0.021	0.048	0.023	0.036	0.025
	Expected Market Beta						**Realized Market Betas**					
A1	0.972	0.293	0.826	0.298	0.734	0.285	0.867	0.210	0.689	0.166	0.611	0.179
A2	1.005	0.002	0.958	0.003	0.917	0.003	0.997	0.126	0.929	0.126	0.906	0.133
A3	1.052	0.087	1.039	0.105	1.027	0.100	1.038	0.118	1.017	0.154	1.010	0.143
A4	1.042	0.086	1.068	0.099	1.084	0.105	1.067	0.135	1.097	0.147	1.112	0.157
A5	1.070	0.341	1.331	0.462	1.505	0.475	1.162	0.206	1.509	0.221	1.649	0.263
λ_m	0.039	0.012	0.047	0.017	0.051	0.017						
	Expected Hedging Beta						**Realized Alphas**					
A1	-0.031	0.023	-0.026	0.022	-0.021	0.020	-0.012	0.050	0.006	0.049	0.018	0.054
A2	0.168	0.123	0.148	0.121	0.122	0.107	-0.005	0.034	0.001	0.037	0.007	0.037
A3	0.247	0.181	0.221	0.179	0.185	0.162	-0.002	0.033	0.000	0.041	0.002	0.038
A4	0.315	0.230	0.285	0.227	0.241	0.206	0.002	0.035	0.001	0.035	0.000	0.037
A5	0.618	0.435	0.616	0.463	0.542	0.431	0.024	0.033	0.002	0.037	-0.012	0.042
λ_π	-0.011	0.011	-0.012	0.013	-0.015	0.014						

Table 2.4: Asymmetric Volatility

This Table shows the descriptive statistics of the coefficients estimated across the 500 histories of simulated returns of the GJR-GARCH(1,1) model of asymmetric conditional variance,

$$\sigma_{it+1} \quad = \quad \kappa + \delta\sigma_{it} + \gamma u_{it} + \gamma_{-} 1_{[u_{it}<0]} u_{it}$$

where $u_{it} = r_{it} - E[r_{it}]$, r_{it} is the excess returns in one of the 5 assets calibrated to resemble the book-to-market portfolios, A1 after the growth portfolio and A5 after the value porftolio. $1_{[]}$ is the indicator function.

		mean	**std.**	**min.**	**5%**	**25%**	**50%**	**75%**	**95%**	**max**
persistence (δ)	**A1**	0.939	0.163	0.000	0.608	0.979	0.985	0.988	0.990	0.993
	A2	0.945	0.162	0.000	0.678	0.979	0.984	0.987	0.990	0.993
	A3	0.958	0.126	0.000	0.953	0.976	0.982	0.985	0.989	0.991
	A4	0.979	0.009	0.824	0.967	0.976	0.980	0.984	0.987	0.991
	A5	0.952	0.008	0.921	0.937	0.947	0.953	0.958	0.964	0.973
	M	0.966	0.007	0.922	0.953	0.962	0.967	0.971	0.977	0.983
news (γ)	**A1**	0.006	0.012	0.000	0.000	0.000	0.001	0.007	0.026	0.083
	A2	0.005	0.009	0.000	0.000	0.000	0.001	0.007	0.019	0.063
	A3	0.005	0.007	0.000	0.000	0.000	0.002	0.008	0.018	0.057
	A4	0.004	0.006	0.000	0.000	0.000	0.001	0.007	0.016	0.030
	A5	0.002	0.005	0.000	0.000	0.000	0.000	0.001	0.012	0.038
	M	0.008	0.008	0.000	0.000	0.000	0.007	0.013	0.022	0.034
leverage (γ_{-})	**A1**	0.016	0.017	-0.078	-0.006	0.012	0.017	0.022	0.035	0.102
	A2	0.017	0.014	-0.063	0.002	0.012	0.018	0.022	0.031	0.131
	A3	0.022	0.011	-0.043	0.006	0.016	0.021	0.026	0.039	0.102
	A4	0.026	0.009	-0.013	0.011	0.020	0.026	0.031	0.039	0.082
	A5	0.089	0.017	0.036	0.060	0.077	0.088	0.100	0.117	0.135
	M	0.036	0.012	-0.011	0.017	0.029	0.036	0.045	0.055	0.071

Table 2.5: Asymmetric Covariance

This Table shows the coefficients of a bivariate asymmetric BEKK model estimated for each simulated asset and the market portfolio excess returns:

$$\Sigma_{t|t-1} = C'C + A'\Sigma_{t-1|t-2}A + B'u_{t-1}u'_{t-1}B + 1_{[u_{it-1}<0]}D'_1 u_{t-1}u'_{t-1}D_1 + 1_{[u_{mt-1}<0]}D'_2 u_{t-1}u'_{t-1}D_2$$

for $i = 1, ..., 5$, where $u_{it} = r_{it} - E[r_{it}]$, r_{it} is the excess return in one of the 5 assets calibrated to resemble the book-to-market portfolios, A1 after the growth portfolio and A5 after the value porftolio, and r_{mt} is the market portfolio. Conditional on $y_t = \sigma(u_\tau, \tau < t)$, it is assumed that $u_t|y_t \sim (0, \Sigma_{t|t-1})$ is jointly normal, and parameters are obtained by the maximum likelihood method. The sample comprises of simulated data, with 1584 observations, which is equivalent to 6 years of daily data. Standard deviations are in brackets. Log-likelihood ratio (LR) test is performed with respect to the model with both asymmetries included, D1 and D2. The constants estimated are multiplied by 100.

	C		A		B		D_1		D_2		**Max LogLik**		**LR test**	
A1 and M	0.234	0.014	0.930	0.011	0.006	-0.045	-0.040	0.023	-0.232	0.005	**Full Model**	11539.2	**LR stat**	**p-value**
	[0.086]	[0.029]	[0.042]	[0.006]	[0.023]	[0.020]	[0.075]	[0.028]	[0.071]	[0.008]	$D_1 = 0$	11538.2	2.0	0.73
		0.000	0.023	0.975	-0.196	0.071	-0.137	0.036	0.015	-0.238	$D_1, D_2 = 0$	11514.3	49.8	0.00
		[0.004]	[0.012]	[0.006]	[0.071]	[0.042]	[0.070]	[0.069]	[0.021]	[0.033]				
A2 and M	0.126	0.040	0.991	0.003	-0.013	-0.003	-0.129	-0.005	-0.052	-0.016	**Full Model**	11610.3	**LR stat**	**p-value**
	[0.146]	[0.059]	[0.031]	[0.014]	[0.009]	[0.023]	[0.114]	[0.014]	[0.098]	[0.030]	$D_1 = 0$	11608.2	4.1	0.39
		-0.036	-0.021	0.977	-0.005	0.024	0.078	0.019	0.309	0.246	$D_1, D_2 = 0$	11587.1	46.3	0.00
		[0.016]	[0.016]	[0.010]	[0.015]	[0.012]	[0.165]	[0.017]	[0.150]	[0.053]				
A3 and M	0.046	0.048	0.995	0.000	-0.009	0.000	-0.029	0.000	-0.008	0.000	**Full Model**	11977.5	**LR stat**	**p-value**
	[0.029]	[0.022]	[0.012]	[0.008]	[0.016]	[0.006]	[0.041]	[0.014]	[0.024]	[0.009]	$D_1 = 0$	11975.6	3.9	0.42
		0.026	-0.009	0.981	0.072	0.011	-0.141	-0.077	0.161	0.219	$D_1, D_2 = 0$	11952.7	49.6	0.00
		[0.022]	[0.022]	[0.011]	[0.139]	[0.059]	[0.069]	[0.071]	[0.052]	[0.035]				
A4 and M	0.070	0.035	0.974	-0.001	0.125	0.053	0.010	0.088	0.150	0.017	**Full Model**	12092.8	**LR stat**	**p-value**
	[0.028]	[0.036]	[0.022]	[0.009]	[0.068]	[0.082]	[0.026]	[0.055]	[0.123]	[0.080]	$D_1 = 0$	12091.2	3.2	0.52
		0.044	0.005	0.979	-0.012	-0.062	0.222	0.044	-0.025	0.166	$D_1, D_2 = 0$	12063.4	58.8	0.00
		[0.024]	[0.037]	[0.014]	[0.013]	[0.094]	[0.106]	[0.137]	[0.073]	[0.071]				
A5 and M	0.017	-0.085	0.958	0.003	-0.114	0.035	-0.282	-0.087	0.022	0.056	**Full Model**	11500.9	**LR stat**	**p-value**
	[0.106]	[0.056]	[0.037]	[0.027]	[0.182]	[0.072]	[0.150]	[0.068]	[0.034]	[0.058]	$D_1 = 0$	11492.2	17.4	0.00
		0.000	0.033	0.960	0.149	-0.076	0.118	0.123	0.431	0.154	$D_1, D_2 = 0$	11458.5	84.8	0.00
		[0.115]	[0.112]	[0.058]	[0.153]	[0.094]	[0.311]	[0.133]	[0.095]	[0.104]				

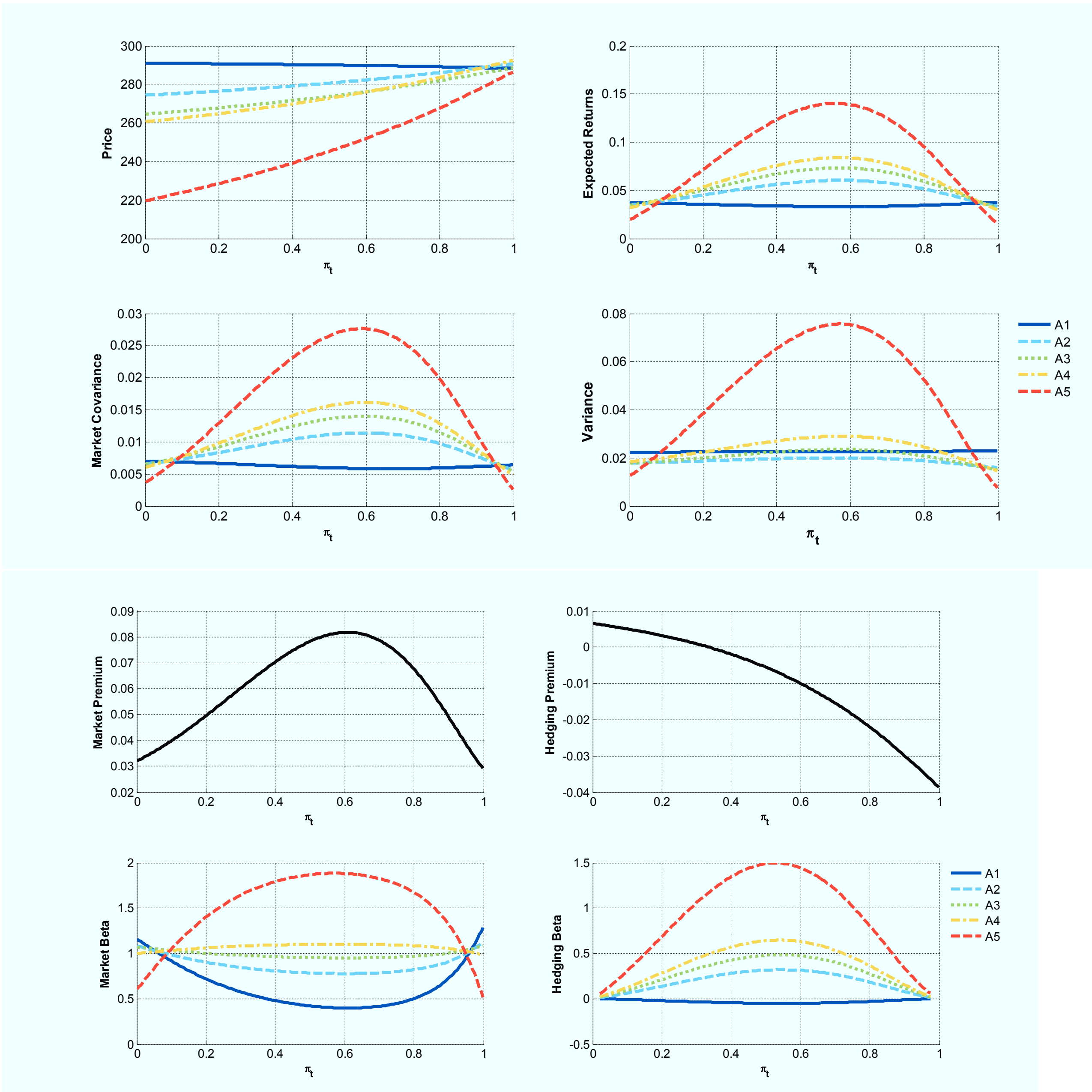

Figure 2.1: Theoretical Expressions Conditional on π_t

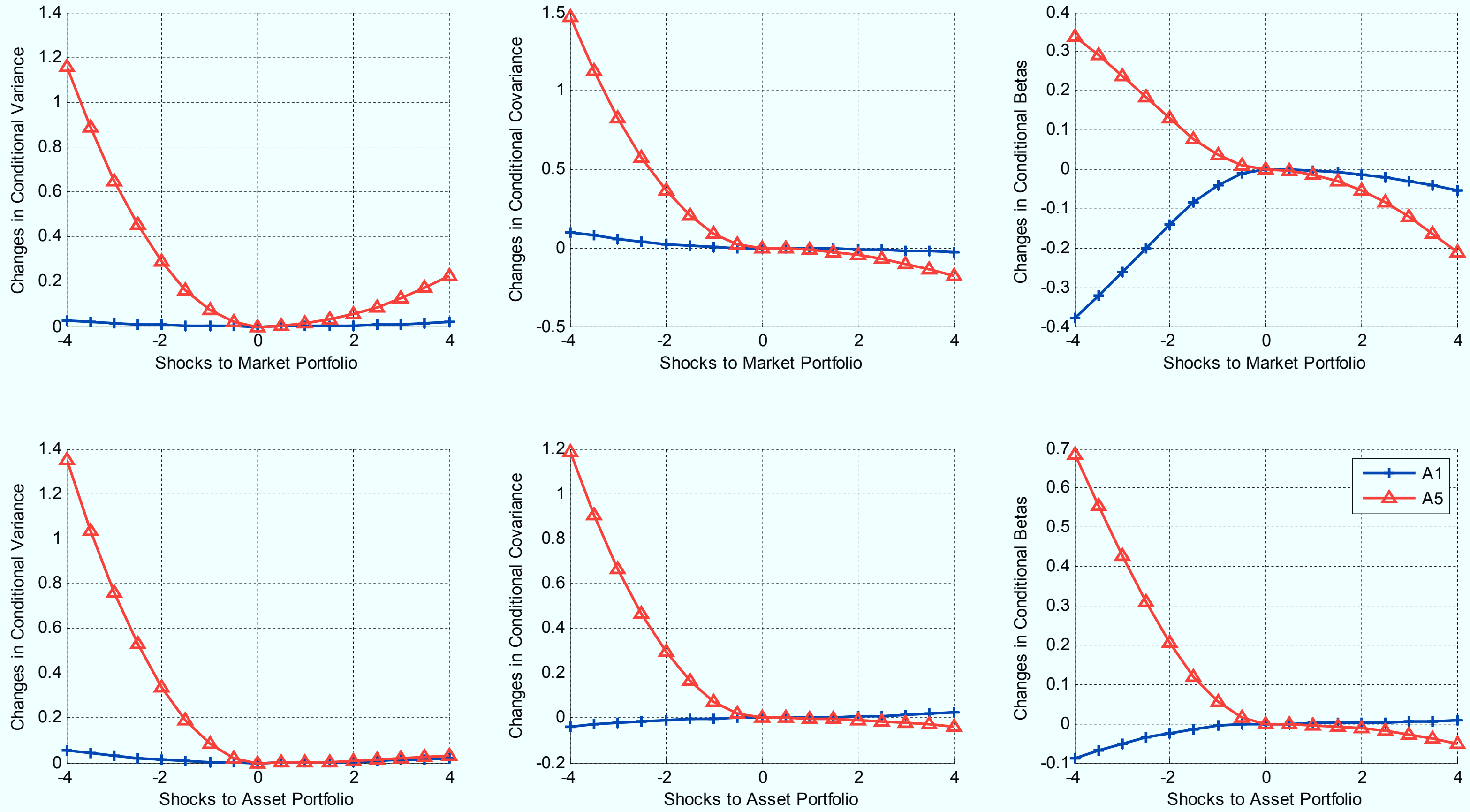

Figure 2.2: Impulse Response Functions

Chapter 3

Empirical Analysis

3.1 Introduction

The price of a risky asset — a claim to an uncertain stream of cash-flows — is the present value of its payments discounted by an appropriate rate. A large portion of the asset pricing literature is concerned with the properties of such discounting rate, the so-called stochastic discount factor (SDF). For instance, the equity premium puzzle generated a voluminous debate in the literature as to how the marginal utility of consumption relates to the SDF. In this chapter, we rather focus on the implications to asset pricing of varying investor uncertainty about such payments. The investigation here is an empirical one, drawing from the theoretical results laid out in Chapter 2.

The standard assumption in asset pricing models is that the investor does not observe future cash-flows, but knows the distribution from which they are generated. For example, the Intertemporal Capital Asset Pricing Model (ICAPM) of Merton (1973), assumes that cash-flows are drawn from a continuous Gaussian process. In contrast, the theoretical framework investigated here, and that was originally suggested by Ribeiro and Veronesi (2002), adds an extra layer of uncertainty. Cash-flows are assumed to originate from a continuous Gaussian process with a drift that can take on two different values, each according to the state of the economy. The states, unobserved by the investor, follow a Markov-switching process that takes on two values, one associated with business cycles expansions, where the average profitability is high, and the other with recessions, where the average profitability is low. Since the investor has an incomplete characterization of the cash-flow distributions, he optimally infers them from past observations, and allocates his wealth among assets according to such beliefs. The resulting rational expectation equilibrium that follows the imposition of the market clearing conditions provides the pricing implications that are empirically verified here.

This extra layer of cash-flow uncertainty creates a dynamic learning process, and imposes a factor structure on expected returns that is richer than the ICAPM: it defines the relevant state variables and the functional forms of market betas and of price of risk. This extra structure provides many testable restrictions on the joint distribution of stock returns. In this chapter, we investigate the dynamics of conditional betas across portfolios, formed with U.S. stocks, that are sorted according to the firm's characteristics, such as size, book-to-market and industry portfolios, and according to past performance of the stocks.

Two aspects of the dynamics of market betas are investigated: first, how betas relate to different *levels* of investor beliefs; second, how betas relate to signs of news, or equivalently, how betas relate to *changes* in investor beliefs.

To relate betas to investor beliefs, we propose an econometric model that projects betas on proxies of investor beliefs and uncertainty. The proxy for beliefs follows the suggestion by Ozoguz (2009), who derives it from a two state Markov-switching model fitted on market returns. For investor uncertainty, we suggest two proxies, one based on the distance of probabilities of investor beliefs, and the other based on the risk-neutral variance of option prices.

To assess how betas relate to changes in investor beliefs, we follow Ang and Chen (2002) and estimate upside and downside betas from sample moments conditional on opposing sides of the joint distribution of returns. We use the statistic proposed by Hong, Tu, and Zhou (2007) to verify if there are statistically significant asymmetries across upside and downside betas.

We summarize our findings about the cross-sectional differences in the relation of beta risk and levels of beliefs as follows. First, we observe an opposing pattern across portfolios sorted by book-to-market ratio. Value betas tend to increase during periods of high uncertainty, while growth betas tend to decrease on uncertainty. Since the price of market risk is positively related to uncertainty, this opposing response of betas goes in the direction of explaining the value premium puzzle. These results also imply a counter-cyclical value premium, also found by

Petkova and Zhang (2005), as periods of higher uncertainty are typically recessionary periods. This cross-sectional asymmetric response of value and growth betas confirms the theoretical predictions in Chapter 2.

Second, we find a clear pattern across momentum betas. The betas of portfolios formed by past-losers stocks are higher in periods of high uncertainty and lower in periods of lower uncertainty. In contrast, the betas of portfolios formed by past-winners stocks are lower in periods of high uncertainty and higher in periods of low uncertainty. Past-winners are riskier on lower uncertainty because in such periods the economy is typically doing well, which increases correlations with the market. A similar argument follows to past-losers betas during high uncertainty periods.

Similar qualitative results hold for the different functional forms of market betas suggested, as well as for the different proxies of investor beliefs used. Interestingly, the two proxies for investor uncertainty used, one based on the distance of investor beliefs from the high-probabilities and the other obtained from option prices, resulted in similar conclusions. The later proxy is motivated by the monotonic relation between risk-neutral variance and uncertainty found in Chapter 2 and can be obtained directly from the VIX volatility index, calculated by the Chicago Board Options Exchange (CBOE). Thus, our results also point to the relevance of the VIX index in explaining market returns, as observed recently by Bollerslev, Tauchen, and Zhou (2009). In that paper, the authors show that the difference between implied and realized variation on the market portfolio, what they call the variance risk premium, explains a portion of future market returns, particularly at the quarterly frequency. Here, the evidence points that the risk-neutral variance helps explaining systematic risk. The particular horizon of predictability is also conformable with our assumption about investor's learning process, as it is reasonable to expect the uncertainty resulting from a shift in the economic conditions to dissipate only after some months of data, but before not too many months, as economic recessions last an year on average.

Our findings about the asymmetries of betas to news are the following. First, as was predicted by the model, value betas are more sensitive to negative news than to positive news. The downside beta — the beta conditional on returns being below a negative threshold —

is higher than its corresponding upside beta — the beta conditional on returns being above a positive threshold. Furthermore, the asymmetry becomes more important with the size of news, as the difference between upside and downside betas increases with the threshold values. In contrast, the growth portfolio does not show important asymmetries; upside and downside betas are equal, and both decrease with the size of news.

Second, size and momentum sortings also result in asymmetric betas across deciles, but each with a different associated pattern. The lowest decile, with smaller firms, has a downside beta that is substantially higher than its upside beta. In contrast, the highest decile, with larger firms, has an upside beta slightly higher than its downside beta. The highest momentum decile, formed with past-winner stocks, has a slightly higher downside beta than its upside beta. Also, past-winners beta decreases as thresholds increase, regardless of sign, indicating that past-winners systematic risk decreases with the size of news. Asymmetries were largely absent in the industry sortings, an indication that the size, book-to-market and momentum sortings, usually taken as evidence of pricing anomalies, may in fact be associated with different aspects of a misspecified dynamics of beta risk.

Our investigation is closest in purpose to Ozoguz (2009). The author also analyzes the implications of investor uncertainty to the cross-section of expected returns using U.S. data. Also, the main theoretical implications in that paper are drawn from Veronesi (1999), who derives the univariate version of the model investigated here. However, the paper does not analyze the implications of uncertainty to conditional betas.

This Chapter proceeds as follows. Section 3.2 presents the theoretical expressions of Chapter 2 that are investigated here and discusses testable functional forms of market betas and prices of risk. Section 3.3 describes the estimation of beliefs, and reports our empirical results on the relation of conditional betas and beliefs. Section 3.4 reports and discusses how conditional betas respond to news. Section 3.5 presents our conclusions.

3.2 Asset Pricing Formulas

Under the assumptions of Chapter 2, asset i expected excess return, $r_{i,t+1}$, can be decomposed in two terms, each corresponding to risk premia for exposure to market and the hedging risks:

$$E_t\left[r_{i,t+1}\right] = \lambda_{m,t}\beta_{im,t} + \lambda_{\pi,t}\beta_{i\pi,t} \tag{3.1}$$

where the prices of risk are $\lambda_{m,t}$ and $\lambda_{\pi,t}$, and the quantities of risk, or betas, are defined as $\beta_{im,t} \equiv cov_t\left(r_{i,t+1}, r_{m,t+1}\right)/var_t\left(r_{m,t+1}\right)$ and $\beta_{i\pi,t} \equiv cov_t\left(r_{i,t+1}, \pi_{t+1}\right)/var_t\left(\pi_{t+1}\right)$. $\pi_{t+1} \equiv Pr_t\left(state_{t+1} = Good\right)$ is the probability that the economy is in the good state at time $t+1$. The subscript t denote conditional moments with respect to the information available to the investor at time t. This particular interpretation of the hedging factor is new and follows from the particular set of assumptions in the model. In the ICAPM of Merton (1973), the hedging factor is left unspecified and is typically justified by the empirical observation that investment opportunities are time-varying. In practice, empirical research has either assumed away the hedging factor or proposed variables with some predicting power over returns as proxies for the state variables driving investment opportunities, with no necessary theoretical justification. In our empirical research, we follow the model's implications, shown by means of a calibration in Section 2.3, that a conditional CAPM with an appropriately defined conditional market beta is sufficient to explain expected returns, without the need of including hedging factors.

Since the model is solved and closed-form formulas[1] for returns are obtained, the dynamics of conditional betas and prices of risk are precisely characterized. This also contrasts with the ICAPM, and with the conditional CAPM, that do not impose any structure on the dynamics of beta. Empirical research traditionally imposes that betas are constant. Time-varying attempts to estimate the ICAPM derive functional forms from empirical regularities, such as multivariate GARCH models that explore the clustering feature of returns second moments (Bali and Engle (2010)), or by projecting betas on a set of instrumental variables (Brandt and Wang (2010)), just to mention some of the recently proposed approaches.

[1] Up to the numerical solution of two ordinary differential equations.

Figure 2.1 depicts the model's market betas and prices of market risk, conditional on the relevant state variable, π_t, of five assets calibrated to resemble the five book-to-market sorted portfolios. We observe that the conditional market beta of asset A5, that resembles the value portfolio, increases as π_t moves away from 0 and 1, towards points where uncertainty, measured by $\pi_t(1-\pi_t)$, is higher. Eventually, the market beta of A5 reaches a maximum at around $\pi_t = 0.60$. We observe a similar response of betas to changes in uncertainty in all the other assets. For instance, the beta of asset A1, that resembles the growth portfolio, decreases on uncertainty and reaches a minimum at $\pi_t = 0.60$.

A functional form that would capture this non-linear relation of conditional market betas and investor beliefs, π_t, but that is flexible enough so not to impose it *a priori,* is the following:

$$\beta_{mi,t} = a_{1,i} + a_{2,i}\pi_t + a_{3,i}\pi_t^2 \tag{3.2}$$

where $a_{k,i}$, $k = 1, 2, 3$, are parameters to be estimated. This functional form, which can be seen as a reduced form of the true one given in Proposition (2.7), allows several different shapes of the relation between market betas and beliefs. For instance, if both coefficients $a_{2,i}$ and $a_{3,i}$ are zero, we have the constant beta implied by the static CAPM. If $a_{3,i}$ is zero, we have an affine relation of market betas and investor beliefs; it would either be increasing or decreasing on beliefs, depending on the sign of $a_{2,i}$. Only when all the tree parameters $a_{k,i}$, $i = 1, 2, 3$, are non-zero, we have a non-linear relation between betas and π_t. For instance, for some positive values of $a_{2,i}$ and negative values of $a_{3,i}$, market beta initially increases on π_t and then decreases, as for larger values of π_t the term π_t^2 becomes more important. Thus, in terms of the model's predictions, a combination of $a_{2,i} > 0$ and $a_{3,i} < 0$ is expected for value betas, and a combination with opposite signs, $a_{2,i} < 0$ and $a_{3,i} > 0$, is expected growth betas.

Another possible functional form imposes symmetry of betas with respect to uncertainty. This, more parsimonious form, is obtained by setting $a_{2,i} = -a_{3,i}$ in equation (3.2) so the affine relation of betas and uncertainty results:

$$\beta_{mi,t} = a_{4,i} + a_{5,i}\pi_t(1-\pi_t) \tag{3.3}$$

Apart from having fewer parameters, this functional form requires the specification of a proxy for uncertainty, $\pi_t(1-\pi_t)$, which may be easier to find, as shown in the next section.

3.3 Cross-Section Asymmetries

In this section we estimate betas from monthly returns of portfolios formed with stocks traded in the NYSE, Nasdaq and Amex markets. Before we proceed with the estimation of the suggested functional forms, we need to find appropriate proxies for investor beliefs and uncertainty.

3.3.1 Investor Beliefs

We follow Ozoguz (2009) and infer investor beliefs from market returns. This is done by fitting a two-state Markov-switching model to the conditional mean and variance of the market returns. The resulting filtered probability, $\hat{\pi}_{t+1} = Pr_t\left(state_{t+1} = good\right)$, is then used as a proxy for investor beliefs about the economy being in the good state. This state is identified as the state with longer duration.

The econometric specification of the two-state Markov-switching model follows Perez-Quiros and Timmermann (2000):

$$\begin{aligned} r_{m,t} &= c_{0,s_t} + c_{1,s_t} Def_{t-1} + c_{2,s_t} Term_{t-1} + c_{3,s_t} I_{t-1} + c_4 Yield_{t-1} + \epsilon_t \\ \epsilon_t &\sim N\left(0, h_{S_t}\right), ln\left(h_{S_t}\right) = c_{5,s_t} + c_{6,s_t} I_{t-1} \end{aligned}$$

where the explanatory variables of the conditional mean of market excess returns, $r_{m,t}$, are taken from the predictability literature: the dividend yield on the market portfolio ($Yield_t$), the spread on the yields of the U.S. 10 year and 1 year treasury bonds ($Term_t$), the spread on the corporate bonds rated BAA and AAA by Moody's (Def_t), and the interest rate on the 3 months treasury bill (I_t). The error term is assumed to be conditionally normal with a time-switching variance, which is an affine function of the short-run interest rates. The Markov-switching transition matrix parameters are specified as follows:

$$\begin{aligned} p_t &= Pr\left(s_t = good | s_{t-1} = good, z_{t-1}\right) = \phi\left(d_0 + d_1 \Delta CLI_{t-1}\right) \\ q_t &= Pr\left(s_t = bad | s_{t-1} = bad, z_{t-1}\right) = \phi\left(d_0 + d_2 \Delta CLI_{t-1}\right) \end{aligned}$$

where p_t is the probability that at time t the economy is in state $s_t = good$, conditional on available information, z_{t-1}, and on the previous states being good. $1 - p_t$ is then probability of switching to the bad state, $s_t = bad$, conditional on the same information. Likewise, q_t is the conditional probability that the economy remains in the bad state and $1 - q_t$ that it shifts to the good state. The transition probabilities have a t subscript because they are allowed to change over time according to the year-over-year changes of the variable ΔCLI_t, a leading indicator of business cycles turning points. ϕ is the cumulative normal distribution and ensures that the probabilities p_t and q_t are numbers between 0 and 1.

The parameters estimated by maximum likelihood on the monthly sample from 1956 to 2010 are shown in Table 3.1. Our proxy of investor beliefs, the filtered probability shown in Figure 3.1 along with shaded areas denoting NBER recessions, captures reasonably well changes in the states of the economy. A measure for uncertainty is directly obtained from this proxy of beliefs by computing $h(\hat{\pi}_t) = \hat{\pi}_t (1 - \hat{\pi}_t)$.

We also consider another proxy for investor uncertainty based on the implied volatility of option contracts on the market portfolio, the VIX index computed by the Chicago Board Options Exchange (CBOE)[2]. This index is calculated from the S&P 500 index options and measures the risk-neutral variance implied by the contracts, with a fixed 30-day maturity. This uncertainty proxy based on option prices can be justified theoretically, since the market's risk neutral expected volatility is positively and monotonically related to uncertainty, as seen in Section 2.2.3. The advantage of this proxy is that it is model-free, and can be obtained directly from prices of traded contracts. In this regard, since such prices ultimately reflect true investors beliefs, this proxy is likely more appropriate than $h(\hat{\pi}_t)$. The disadvantage is the limited sample size, monthly data is only available since 1990.

The choice of this proxy is also motivated by the recent empirical evidence pointing to the predictive power of the VIX index over stock returns. Bollerslev, Tauchen, and Zhou (2009) have shown that the difference between implied and realized volatility on the market portfolio, what they call the variance risk premium, is able to explain future market returns, particularly

[2]http://www.cboe.com/micro/VIX/historical.aspx

at the quarterly frequency, that cannot be accounted for by the traditional predicting variables, such as the price-earnings ratio and default spread. However, the theoretical justification for the predicting properties of the risk-neutral variance is different than the one suggested here. The authors introduce time-variation in the volatility consumption process, *i.e.* the volatility of consumption volatility, which is equivalent, in their endowment economy, to the volatility of cash-flow volatility. In our context, risk-neutral variance, like other variables of interest, is a function of investor uncertainty. And because the risk-neutral variance is also an exact monotonic function of uncertainty, it works as an appropriate proxy for the unobserved investor uncertainty.

Table 3.2 shows the descriptive statistics of the proxies for belief and uncertainty. The two proxies for uncertainty are correlated only to a certain extent.

3.3.2 Estimation of Conditional Betas

First, we estimate the parameters in the factor regression $R_{i,t} = \alpha_i + \beta_{mi,t} R_{m,t} + \epsilon_{i,t}$, $i = 1, ..., N$, by imposing the less restrictive functional form (3.2) on $\beta_{mi,t}$. To account for heteroskedasticity and autocorrelation in errors and across assets, we jointly estimate the parameters by the Generalized Method of Moments (GMM) by specifying the following set of moments conditions:

$$E\left\{\left[R_{i,t} - \alpha_i - \left(a_{1,i} + a_{2,i}\hat{\pi}_t + a_{3,i}\hat{\pi}_t^2\right) R_{m,t}\right] \otimes \ Z_t \right\} = \mathbf{0} \tag{3.4}$$

where the instruments are $Z_t = \begin{bmatrix} 1 & R_{m,t} & \hat{\pi}_t R_{m,t} & \hat{\pi}_t^2 R_{m,t} \end{bmatrix}'$. In this specification, the system is exactly identified, with same number of parameters and restrictions, $N \times 4$.

We first use the same dataset that calibrated the model in Section 2.3.1. This dataset consists of monthly excess returns from 1956 to 2010 of five book-to-market sorted portfolios and the value-weighted market portfolio from the Center for Research in Security Prices (CRSP). We expect parameters to corroborate the patterns predicted by the model, shown in Figure 2.1.

In the top panel of Table 3.3, the estimated parameters of the specification (3.4) have the predicted signs, but are statistically insignificant. The estimated functional forms depicted in Figure 3.2 indicate that conditional beta of the low book-to-market portfolio decreases on uncertainty whereas that of high book-to-market portfolio increases on uncertainty. Figure 3.2 also shows the implied dynamics for conditional betas on the sample that excludes the years 1997 to 2001, a period where the growth portfolio constituted a larger portion of the stock market. In this case, the market beta of the value portfolio also increases on uncertainty and, additionally, is higher than that of the growth portfolio for some levels of π_t, as predicted. We also observe that portfolios with similar book-to-market ratios share similar patterns; the beta of the fourth quintile (4 Qnt) also increases on uncertainty, and the beta of the second quintile (2 Qnt) decreases on uncertainty. However, the variation in the conditional betas, particularly on the two extreme portfolios, the value and growth ones, does not have the amplitude suggested by the theory.

We now extend the analysis of beta dynamics to deciles portfolios and sortings by different characteristics. We estimate the market betas of 10 portfolios sorted on book-to-market ratio, 10 momentum portfolios sorted according to past performance, 10 size portfolios sorted by firm's market capitalization, and 10 industry portfolios sorted by firm's businesses. The sample period is the same as before, monthly returns from 1956 to 2010, and the source for this dataset is also the website of Kenneth French.

The market betas of the 40 portfolios, jointly estimated by GMM, are shown in Table 3.4. First, we note that the patterns discussed above for the five book-to-market sorted portfolios extend to the decile sort. The market betas of portfolios with lower ratios seem to respond negatively to increases in uncertainty as opposed to portfolios with higher ratios. The estimated coefficients for $a_{2,i}$ are generally negative for the lowest deciles and positive for the highest deciles portfolios. The opposite holds for the coefficients $a_{3,i}$, positive for the lowest deciles and negative for the highest deciles. This opposing response of market betas to uncertainty across book-to-market portfolios is best noted on Figure 3.5. Because of the flexible functional form imposed to betas, some of the estimated patterns are very different from the ones implied by the theory, but typically market betas do seem to either increase or decrease on uncertainty. It should be pointed, however, that the parameters t-statistics, which account for heteroskedasticity, autocorrelation and correlation across errors, are generally not statistically different than zero.

The second panel of Table 3.4 shows the estimated parameters of size sorted portfolios. Interestingly, the patterns implied by the parameters and plotted on Figure 3.6 are similar across portfolios; the betas generally increase with the belief that the economy is in the good state, and are thus positively related to $\hat{\pi}_t$. Among the betas, that of smaller firms is the most variable one; it reaches a minimum of 0.90 at $\hat{\pi}_t = 0$ and a maximum of 1.20 at $\hat{\pi}_t = 1$. The beta of largest firms vary the least; it reaches a minimum of 0.87 a $\hat{\pi}_t = 0$ and a maximum of 0.94 at $\hat{\pi}_t = 0.60$. Again, these patterns cannot be sustained on statistical grounds.

The third panel of Table 3.4 reveals some interesting dynamics related to the momentum sorting. The shapes of betas implied by such parameters and shown in Figure 3.7 follow a clear pattern across deciles. Except for the sixth decile, all betas are clearly symmetric around $\pi_t = 0.50$ and thus monotonically related to uncertainty. The response of betas to uncertainty is also clearly positive for the lowest deciles and clearly negative for the highest deciles. Furthermore, the portfolios on the two extreme deciles are also the ones with the most variation in conditional betas. As opposed to the book-to-market and size sorted portfolios, such dynamics of betas can be sustained statistically, as the coefficients are significantly different than zero.

Finally, the last panel of Table 3.4 shows the parameters estimated for 10 industry betas. The implied shapes shown in Figure 3.8 are very diverse, as would be expected for a sorting based on industries categories. High-technology is the riskier industry, has the highest beta of 1.40 at $\hat{\pi}_t = 0.60$, and most sensitive to changes in uncertainty. Utilities is the least risky portfolio, with a maximum beta of 0.64, and an increasing beta on uncertainty. Other portfolios have market betas decreasing on uncertainty, such as Non-Durables, Energy and Health. Retail and Manufacturing have betas increasing betas on $\hat{\pi}_t$ while Durables and Telecom decreasing on $\hat{\pi}_t$. The slope coefficients, however, are not statistically different than zero.

We have seen that many of the predicted patterns for the book-to-market betas are observed on the data, but that cannot be supported statistically. We now impose the more restrictive and simpler functional form of market betas in an attempt to obtain more precise estimators. We will also consider two proxies for uncertainty that are constructed in very different ways,

$\hat{\pi}_t(1-\hat{\pi}_t)$ and VIX_t, to assess to which extent the results depend on the proxy chosen. As before, we estimate the parameters by GMM to obtain robust covariances by imposing the following set of moment conditions:

$$E\left\{ [R_{i,t} - \alpha_i - (a_{4,i} + a_{5,i}UC_t) R_{m,t}] \otimes \ Z_t \right\} \ = \ \mathbf{0} \tag{3.5}$$

where the instruments are $Z_t = \begin{bmatrix} 1 & R_{m,t} & UC_t R_{m,t} \end{bmatrix}'$. As before, the system is exactly identified, with $N \times 3$ parameters and moment restrictions.

Initially, we estimate the symmetric betas on the same dataset that calibrated the model in Section 2.3.1, as we can contrast the results with the theory predictions. The second, third and fourth panels of Table 3.3 confirm the predicted patterns about betas and uncertainty. For two different proxies of uncertainty, $\hat{\pi}_t(1-\hat{\pi}_t)$ and VIX_t, and two different sample sizes, from March 1956 to December 2010 and January 1990 to December 2010, the estimated coefficients $a_{5,i}$ are generally positive for value portfolios and negative for growth portfolios. Furthermore, if for the proxy, $\hat{\pi}_t(1-\hat{\pi}_t)$, the coefficients are not significant, for the model-free proxy, VIX_t, the coefficients are significantly different than zero. Based on the covariance matrix of the parameters from the regression with VIX_t, most of the joint confidence region of the slope parameters for growth and value betas, shown in Figure 3.4, lays on the second quadrant, and indicates that these portfolios have opposite sensitivity to uncertainty.

For a robust assessment of our previous results, we re-estimate the market betas of the 40 portfolios, this time using as a proxy for uncertainty VIX_t. The results are shown in Table 3.5. The slope coefficient, $a_{5,i}$, of the 10 book-to-market sorted portfolios is lowest and negative for growth portfolios and positive and highest for value portfolios, with some of the estimates statistically significant. The average slope of the 3 lowest deciles is -0.26, of the 4 mid-deciles is 0.59 and of the 3 highest deciles is 1.09. This pattern, predicted by our model, corroborates the findings above with different proxy for uncertainty and functional form of betas.

The second panel of Table 3.5 shows how market betas of size portfolios depend on uncertainty. The slope coefficient, $a_{5,i}$, is higher and significant for small firms, but closer to zero and insignificant for large firms. The average slope of the 3 lowest deciles is 0.53, of the 4 mid-deciles 0.31, and of the 3 highest deciles 0.16. This pattern is in-line with the intuition that small firms are riskier than larger firms, particularly during periods of high uncertainty.

The third panel of Table 3.5 confirms the findings of Table 3.4 that betas of past-losers are more sensitive to changes in uncertainty than betas of past-winners. The average response to uncertainty of the 3 lowest deciles is 1.47, of the 4 mid-deciles is 0.52 and of the 3 highest deciles is -0.65. Note that when there is no uncertainty, only the coefficient $a_{4,i}$ matters for risk, and, in this case, past-winners would be riskier than past-losers, as they are likely to be more correlated with the market portfolio. The average $a_{4,i}$ coefficient of the 3 lowest deciles is 0.91, of the 4 mid-deciles is 0.70 and of the 3 highest deciles is 1.14.

Finally, the last panel of Table 3.5 shows the estimated coefficients for the 10 industry portfolios. We see that for the Durables, Manufacturing and Other portfolios, the coefficient $a_{5,i}$ is now positive and statistically significant.

In this subsection we have seen how the dynamics of risk differs across portfolios. Our empirical findings suggest that market betas of value and growth portfolio do respond differently to changes in uncertainty, as predicted by our theory. Both the more flexible functional form estimated from (3.4) and the affine function of uncertainty estimated from (3.5) result in value betas increasing on uncertainty and growth betas decreasing on uncertainty. The qualitative results remain when another proxy for uncertainty is used, VIX_t, but with stronger statistical significance. The amplitude of the variation in conditional betas, however, is smaller than the one predicted by the model. Furthermore, a value premium still remains as can be observed by the significance of the intercepts, α_i. Thus, despite correctly capturing the model's main implications regarding the dynamics of market betas, and thus going on the direction of solving the value premium, the proposed estimators may not completely capture the cross-section variation in expected returns. This agrees with Petkova and Zhang (2005) that, despite conditional betas going in the right direction in explaining the value premium puzzle, they cannot completely explain it. A point argued by Lewellen and Nagel (2006).

Another empirical result, perhaps not surprising, is the clear opposite risk dynamics of momentum portfolios. We have seen that portfolios formed with past-loser stocks are riskier than past-winner stocks during periods of higher uncertainty. However, during periods of lower uncertainty, the opposite is true, past-winners are riskier than past-losers and have higher betas. Apparently contradictory, this result is in fact intuitive: during expansions past-winner stocks are likely to be highly correlated with the market portfolios, and thus riskier, but during recessions, past-loser stocks are likely to be highly correlated with the market portfolio, and thus riskier. Thus, time-varying betas are essential to explain returns on momentum portfolios.

The fact that some of the intercepts in the time-series regressions are significantly different than zero can still be reconciled with the theory, as the model predicts that the price of risk is also time-varying. In the next section, we estimate the price of risk and assess if its time-variation is related to investors beliefs as noted by theory.

3.3.3 Estimation of Price of Market Risk

In this section we investigate another implication of the model. Namely, we assess whether the price of market risk is also a function of investor beliefs about the state of the economy. As discussed above, the model predicts that, as investor uncertainty increases, the price of one unit of market risk, the so-called market premium, should also increase and reach a maximum price around the point of maximum uncertainty.

As opposed to the estimation of betas above, the task of estimating the price of market risk is more complicated, particularly because it requires the joint estimation of betas and prices of risk. The traditional approach of Fama and MacBeth (1973) involves a two-step procedure, where in the first step time-series regressions are used to estimate betas, and in the second, cross-section regressions with the estimated betas as regressands are used to obtain an estimate of the price of risk. Despite the computational simplicity, this procedure requires, for correct statistical inference, the adjustment of t-statistics to account for the existence of error-in-variables problem.

To impose fewer restrictions on the distribution of returns and avoid the problems associated with the two-step approach, we jointly estimate the parameters by GMM. The GMM framework is also flexible enough to allow us investigate time-varying functional forms for the price of market risk. However, as pointed out by Shanken and Zhou (2007), the moment restrictions typically imposed on the cross-section of stock returns, such as those in chapter 12 of Cochrane

(2005), are difficult to solve numerically, and the convergence, when possible, depends on the initial values chosen. These computational complications arise from the need of joint estimation of the constants on the time-series factor regressions and the constants on the cross-section pricing restriction. To avoid such difficulties, we follow Shanken and Zhou (2007) and estimate the parameters sequentially, imposing the following set of moments suggested by Harvey and Kirby (1996):

$$E\begin{bmatrix} r_{i,t}-\mu_i \\ r_{m,t}-\mu_m \\ (r_{m,t}-\mu_m)^2-\sigma_m^2 \\ r_{i,t}-\lambda_0-\lambda_{m,t}\frac{(r_{i,t}-\mu_i)(r_{m,t}-\mu_m)}{\sigma_m^2} \end{bmatrix} = 0 \qquad (3.6)$$

where $i = 1, ..., N$. The first $N + 2$ set of moment restrictions exactly identify the $N + 2$ parameters μ_i, $i = 1, .., N$, μ_m and σ_m^2 and are estimated separately, in the initial step. In the second step, the remaining N moment restrictions are used to estimate λ_0 and the parameters of $\lambda_{m,t}$. Here, we allow the price of market risk to be time-varying with the suggested functional forms, $\lambda_{m,t} = b_1 + b_2\hat{\pi}_t + b_3\hat{\pi}_t^2$ and $\lambda_{m,t} = b_4 + b_5 UC_t$. The proxy of beliefs, $\hat{\pi}_t$, is the same as before, and also the two proxies for uncertainty, $UC_t = \hat{\pi}_t(1-\hat{\pi}_t)$ and $UC_t = VIX_t$. The GMM estimators, $\hat{\lambda}_0, \hat{b}_j$, $j = 1, ..., 5$, are obtained analytically. For more details on this sequential procedure, the reader is referred to Shanken and Zhou (2007).

Table 3.6 shows the GMM estimated parameters $\hat{\lambda}_0, \hat{b}_j$, $j = 1, 2, 3$, implied by the same 40 portfolios used in previously. To simplify our analysis, however, the betas of such portfolios are assumed static here. Model (1) is the classic CAPM cross-sectional regression. The intercept is positive and significantly different than zero and the price of market risk is negative and insignificant. This economically inconsistent result usually arises when portfolios sorted on book-to-market and size are used in the regressions. The specifications on Model (2) and Model (3) allow the price of market risk to be time-varying, by projecting $\lambda_{m,t}$ on beliefs, $\hat{\pi}_t$, and squared beliefs, $\hat{\pi}_t^2$. The significance of the coefficients indicates that time-variation is a statistically relevant characteristic of market premium. The point estimates, however, indicate a price dynamics that is at odds with our theory. The parameters imply that the price of market risk is actually lower during periods of high uncertainty. Model (5) controls for other two commonly used risk factors, the HML and SMB factors of the Fama and French 3 factor model. In this case, all the coefficients are insignificant, which suggests an over-specification of the dynamics of price of risk.

Table 3.7 shows the GMM estimated parameters $\hat{\lambda}_0, \hat{b}_j,\ j = 4, 5$, with a more restrictive functional form on the price of market risk. Two different sample sizes and proxies for uncertainty are considered. The coefficients in Model (6) confirm the results on Table 3.6 that the price of market risk is time-varying and also decreasing on levels of uncertainty. The coefficient $\hat{b}_5$ is negative, -0.1799, and statistically significant with a t-statistic well below -3 when $\hat{\pi}_t\left(1 - \hat{\pi}_t\right)$ is the uncertainty proxy, and on the monthly sample from 1956 to 2010. The inclusion of the High-Minus-Low (HML) and Small-Minus-Big (SMB) risk factors of the three-factor Fama-French model does not change its negative sign, but reduces its statistical significance. In Model (8) a different proxy, VIX_t, as well as a different sample period, from January 1990 to December 2010, are used. The point estimates cannot be compared because of the different scales of the proxies, but the qualitative results are the same, as the negative sign remains. Thus, the evidence also points to a decreasing equity premium on uncertainty in this restricted sample periods and for the model-free proxy of uncertainty, although this time not statistically different than zero.

The dynamics of market risk price revealed by the data does not match our model predictions. This inconsistent result is typically found in similar empirical investigations. For instance, a large body of literature has long debated on what is the appropriate econometric approach to assess the risk versus return trade-off on the market portfolio. Depending on the chosen approach, a negative relation between return and market variance can be found (Whitelaw (1994) and Brandt and Kang (2004)). The task of assessing the price of market risk from the cross-section of returns, which is essentially the same as finding a positive risk-return trade-off on the market portfolio, has also presented with contradictory results, as evidenced by the murderings and resurrectings of the CAPM (Fama and French (1996) and Lettau and Ludvigson (2001)).

3.4 Time-Series Asymmetries

In the previous section we empirically investigated how the systematic risk of different assets relates to *levels* of investor beliefs and uncertainty about the state of the economy. In particular, we have confirmed the prediction of our model that assets with opposing cash-flow sensitivities, such as those of low and high book-to-market portfolios, have different levels of risk for given beliefs. In this section, we explore the impact of *changes* in beliefs on betas.

The empirical approach will also be different in this section. Instead of defining a proxy for investor beliefs and uncertainty, we will use the fact that asset returns, and in special returns on the market portfolio, can be informative about investor beliefs. As was discussed in Section 2.2, asset excess returns that result from that rational expectation equilibrium model (see Proposition (4)) share a common Brownian motion with investor beliefs (dv_t in the expression (1)). Furthermore, the excess returns diffusion term multiplying the common term is typically positive, meaning that positive returns should be on average positively related to positive updates on beliefs.

The model's predictions regarding the response of conditional betas to news were discussed in Section 2.3.1. We fitted a multivariate asymmetric BEKK model to the simulated series and, with the estimated parameters, impulse response functions (IRF) for conditional betas were computed. The dynamics seen on the IRFs revealed that assets that have higher levels of systematic risk during uncertain periods, such as the value portfolio, should have conditional betas increasing on negative news. On the other hand, assets that have lower levels of systematic risk during uncertain periods, such as the growth portfolio, should have conditional betas decreasing on negative news.

In order to assess to which extent the betas of book-to-market portfolios have the predicted asymmetries, and how betas of portfolios with different sortings respond to news, we obtain estimates of market betas directly from sample moments that are conditional on returns being above or below certain thresholds. If asymmetries are indeed relevant, such conditioned betas should differ. And, if our model is correct, the direction of such asymmetries should also have the predicted directions. The value beta conditional on negative returns should be higher than the value beta conditional on positive returns. Likewise, the growth beta should be lower conditional on negative returns but higher conditional on positive returns. This approach to assessing asymmetries has the advantage of being straightforward to interpret and computationally easier than the asymmetric BEKK to estimate.

A similar exercise is performed by Ang and Chen (2002), who investigate the asymmetries of correlations of several portfolio sortings with the market portfolio. The authors show that correlations are typically larger in downside markets than in upside markets. Furthermore, they find that small stocks, value stocks and past-loser stocks are more susceptible to such asymmetric correlations with the market.

Following Ang and Chen (2002), we define upside beta, $\beta_{+}^{+}(c)$, and downside beta, $\beta_{-}^{-}(c)$, in the following way:

$$
\begin{aligned}
\beta_{+}^{+}(c) &= \frac{cov\left(\tilde{r}_{i,t}, \tilde{r}_{m,t} | \tilde{r}_{i,t} > c, \tilde{r}_{m,t} > c\right)}{var\left(\tilde{r}_{m,t} | \tilde{r}_{i,t} > c, \tilde{r}_{m,t} > c\right)} \\
\beta_{-}^{-}(c) &= \frac{cov\left(\tilde{r}_{i,t}, \tilde{r}_{m,t} | \tilde{r}_{i,t} < -c, \tilde{r}_{m,t} < -c\right)}{var\left(\tilde{r}_{m,t} | \tilde{r}_{i,t} < -c, \tilde{r}_{m,t} < -c\right)}
\end{aligned}
$$

where c is the threshold. The return on portfolio i, $\tilde{r}_{i,t}$, and the return on the market portfolio, $\tilde{r}_{m,t}$, are normalized to have zero mean and unit variance. The thresholds will take values between 0 and 1.5, and not larger to avoid too restricted samples. Two other conditionings are also considered, one restricts the sample to positive market returns, $\beta_{+}(c)$, and the other to positive portfolio returns, $\beta^{+}(c)$:

$$
\begin{aligned}
\beta^{+}(c) &= \frac{cov\left(\tilde{r}_{i,t}, \tilde{r}_{m,t} | \tilde{r}_{i,t} > c\right)}{var\left(\tilde{r}_{m,t} | \tilde{r}_{i,t} > c\right)} \\
\beta_{+}(c) &= \frac{cov\left(\tilde{r}_{i,t}, \tilde{r}_{m,t} | \tilde{r}_{m,t} > c\right)}{var\left(\tilde{r}_{m,t} | \tilde{r}_{m,t} > c\right)}
\end{aligned}
$$

and similarly for $\beta^{-}(c)$ and $\beta_{-}(c)$.

In order to improve the estimation of betas on the restricted samples, we increase the number of observations by considering weekly returns as opposed to the monthly frequency of previous section. The first row of plots in Figure 3.9 shows the conditional betas of the size portfolios for values of c between -1.5 and 1.5. The lines show a discontinuity at $c = 0$ and indicate a clear asymmetric pattern across positive and negative thresholds, c. The betas of the smallest firms decile, ME1, are higher when news is negative than when news is positive, for all of the three conditionings. The betas of the largest firms decile, M10, also displays asymmetries, but in the opposite direction. The betas of large firms are slightly higher on positive news than on negative news. This is consistent across all threshold levels and conditioning specifications. The dynamics on the deciles between the lowest and highest, M3, M5 and M8, confirms the increasing asymmetric pattern towards the lowest deciles of downside betas being higher than upside betas.

The second row of plots in Figure 3.9 shows different asymmetric patterns across book-to-market sorted portfolios. The downside beta of the value portfolio, BE10, is typically higher than the upside betas across all thresholds and conditioning specifications. The plot on the first column, with the conditioning that both returns are above and below the thresholds, $\beta_{+}^{+}(c)$ and $\beta_{-}^{-}(c)$, shows that the asymmetry becomes more pronounced as we move towards the (odd-quadrants) tails of their joint distribution. This is in-line with the model's prediction that value portfolios are particularly riskier during bad news markets. In contrast, the betas of the growth portfolio, BE1, do not display asymmetries across positive and negative thresholds. For a given threshold, the upside and downside growth betas are similar across the three conditionings. As before, the dynamics on the deciles between the lowest and highest book-to-market, BE3, BE5 and BE8, confirms the opposing patterns of value and growth betas.

Finally, the third row in Figure 3.9 shows that some asymmetric patterns are also present on the momentum sorted portfolios. The downside betas of past-winners are higher than their corresponding upside betas at all values of c. Also, as was the case with growth portfolios, past-winners are less risky in extreme news markets. On the other hand, the betas of past-losers do not display strong asymmetries across values of c, but increase in riskiness on extreme news markets. The difference in risk across past-winners and past-losers is most important during good markets. This is more clearly seen with the betas of portfolio M3, which should share some of the characteristics of M1.

In order to see if these results are statistically relevant, we need a formal test to verify if such asymmetries persist after accounting for sample variation. For this task, we use the test suggested by Hong, Tu, and Zhou (2007). The advantages of the test are that it is model-free and relatively simple to compute. The null hypothesis of symmetric betas across thresholds, c, is tested against no asymmetries for some c:

$$\begin{aligned} H_0 &: \beta_+^+(c) = \beta_-^-(c)\,,\ for\, all\, c \geq 0 \\ H_a &: \beta_+^+(c) \neq \beta_-^-(c)\,,\ for\, some\, c \geq 0 \end{aligned}$$

To compute a statistic to test such hypothesis, let n thresholds $c_1, ..., c_n$ and define the $(n \times 1)$ vector $\beta_+^+ - \beta_-^- = \left[\beta_+^+(c_1) - \beta_-^-(c_1), ..., \beta_+^+(c_n) - \beta_-^-(c_n)\right]'$. The test statistic is the following:

$$J_\beta = T\left(\beta_+^+ - \beta_-^-\right)' \hat{\Psi}^{-1}\left(\beta_+^+ - \beta_-^-\right)$$

where $\hat{\Psi} = \sum_{l=1-T}^{T-1} k(l/p)\,\hat{\gamma}_l$ is a weighted sum of $\hat{\gamma}_l$, an $N \times N$ matrix with $(i,j)-th$ element given by $\hat{\gamma}_l(c_i, c_j) = T^{-1}\sum_{t=|l|+1}^{T-1} \hat{\xi}_t(c_i)\,\hat{\xi}_{t-|l|}(c_j)$ and

$$\begin{aligned} \hat{\xi}_t(c_i) &= \frac{T-T_+^+}{T_+^+}\left[\frac{\left(\tilde{r}_{j,t} - \mu_{+j}^+(c_i)\right)\left(\tilde{r}_{m,t} - \mu_{+m}^+(c_i)\right)}{\sigma_{+m}^{+2}(c_i)} - \beta_+^+(c_i)\right] 1\left(\tilde{r}_{j,t} > c_i, \tilde{r}_{m,t} > c_i\right) \\ &\quad - \frac{T-T_-^-}{T_-^-}\left[\frac{\left(\tilde{r}_{j,t} - \mu_{-j}^-(c_i)\right)\left(\tilde{r}_{m,t} - \mu_{-m}^-(c_i)\right)}{\sigma_{-m}^{-2}(c_i)} - \beta_-^-(c_i)\right] 1\left(\tilde{r}_{j,t} < -c_i, \tilde{r}_{m,t} < -c_i\right) \end{aligned}$$

where T_+^+ is the number of observations when both returns are above c_i, $\mu_+^+(c_i)$ and $\sigma_+^{+2}(c_i)$ are the mean and variance conditional on both returns above c_i. Define likewise the variables associated with the $-c_i$ threshold. The statistic is asymptotically chi-square distributed with n degrees of freedom, $J_\beta \sim \chi_n^2$. The same test statistic can be applied to the other difference of conditional betas, $\beta^+ - \beta^-$ and $\beta_+ - \beta_-$.

Table 3.8 shows the results of the asymmetry tests on the betas conditional on both returns being above and below c's. The tests are conducted for three sets of thresholds, $c = [0]$, $c = [0, 0.5, 1.0, 1.5]$ and $c = [0, 0.1, ..., 1.4, 1.5]$ on the same 40 portfolios studied on the previous section. The dataset used for the tests begins on July 1963 and the weekly returns are standardized. The p-values of the J_β statistics for each portfolio and for each of three sets of thresholds are shown.

The p-values on the first panel of Table 3.8 show that asymmetries are generally not statistically significant on the book-to-market portfolios. However, the averages of differences in betas, $\overline{\beta_+^+ - \beta_-^-}$, show the monotonic pattern predicted by our theory. The downside beta of the value portfolio is higher than its upside beta. In contrast, the downside beta of the growth portfolio is slightly smaller than its upside beta.

As was observed graphically, the asymmetries on the small firms are the most important ones. The p-values are below 10% for the lowest 4 deciles, and the averages of the differences in betas, $\overline{\beta_+^+ - \beta_-^-}$, are negative and large. The asymmetries are not statistically relevant for larger firms portfolios. However, the negative asymmetries become less important as we move towards highest deciles and eventually turn positive.

The p-values also indicate that beta asymmetry is significant for the past-winners portfolio. The negative sign on the average of differences shows that downside beta is higher than upside betas on this portfolio. This negative asymmetry is consistent with the negative asymmetries on the other high-deciles portfolios, although for these they are not statistically significant. This result contrasts with our findings about the relation of momentum betas and the level of uncertainty in the economy. As seen in the previous section, the evidence pointed that the market beta of past-winners is lower during periods of high uncertainty and, as discussed, should decrease on negative news.

Finally, some significant asymmetries arise on the industry portfolios for the case with multiple thresholds. In this case, the beta of Non-Durables and High-Tech portfolios show negative asymmetries while the beta of Health and Utilities portfolios show positive asymmetries.

The Tables 3.9 and 3.10 report the same statistics but for two different conditionings. The qualitative results regarding the sign of the asymmetries are generally the same, but with some of the p-values now falling inside a rejection range. The asymmetries that arise from conditioning on market returns only result in greater statistical significance on the lower deciles of the size portfolios. For all the other sortings, conditioning betas to asset returns thresholds result in an overall increase in the rejection of the null hypothesis of no asymmetry. In particular, Table 3.10 now shows that the asymmetries of values betas are statistically, with downside betas being significantly higher than upside betas.

3.5 Conclusion

In this chapter we investigated the time-varying features of market betas of portfolios sorted by size, book-to-market and momentum and also of industry portfolios. Two aspects of the variation were considered: first, how betas relate to different *levels* of investor beliefs; second, how betas relate to signs of news, or equivalently, how betas relate to *changes* in investor beliefs.

In the case of the book-to-market betas, we contrasted the empirical findings with those predicted by the theory in Chapter 2 and concluded that the patterns found confirm the predictions. Despite the statistical evidence being marginal — in some of the specifications of betas parameters were not statistically different from zero — the estimates pointed to a consistent pattern emerging across all deciles of the sort. Value betas tend to be higher during periods of high uncertainty and lower otherwise. In contrast, growth betas tend to be higher during periods of low uncertainty and lower during periods of high uncertainty. Asymmetries were also found with respect to the signs of news. For the value portfolio, a downside beta, defined as a beta conditional on negative news markets, is higher than the opposing upside beta, defined as a beta conditional on positive news markets. Furthermore, this difference in upside and downside betas increases as the conditioning is made on more significant news markets.

We observed a clear and significant relation between levels of investor beliefs and betas of momentum portfolios. Past-winners betas tend to be higher during periods of low uncertainty but lower during periods of higher uncertainty. In contrast, past-losers betas tend to be higher during periods of high uncertainty and lower during periods of low uncertainty. Interestingly, a contradictory pattern emerged on the relation of betas and changes in investor beliefs. The downside betas of past-winners are higher than their corresponding upside betas. This opposes the prediction that assets that are less risky during high-uncertainty periods, as is the case of past-winners, negative news should be followed by lower betas as opposed to higher betas.

The betas of size portfolios, particularly the smallest firms, show the most asymmetry with respect to changes in investor beliefs. The downside beta of the lowest decile is substantially and statistically larger than their corresponding upside betas. This result is in line with the perception that small firms are more susceptible to changes in the market conditions.

Finally, the asymmetries of betas to news that arise in the industry sortings are less pronounced and generally occur conditional on extreme news. This difference in asymmetric patterns may be an indication that the size, book-to-market and momentum sortings, usually taken as evidence of pricing anomalies, are effectively capturing different aspects of a non-trivial dynamic of beta risk.

3.6 Graphs and Tables

Table 3.1: Markov-Switching Model

This Table shows the estimated parameters of the Markov-switching model for the excess return on the market portfolio. The conditional mean and variance are specified as:

$$\begin{aligned} r_{m,t} &= c_{0,s_t} + c_{1,s_t} Def_{t-1} + c_{2,s_t} Term_{t-1} + c_{3,s_t} I_{t-1} + c_4 Yield_{t-1} + \epsilon_t \\ \epsilon_t &\sim N\left(0, h_{s_t}\right), ln\left(h_{s_t}\right) = c_{5,s_t} + c_{6,s_t} I_{t-1} \end{aligned}$$

$$\begin{aligned} p_t &= Pr\left(s_t = good | s_{t-1} = good, z_{t-1}\right) = \phi\left(d_0 + d_{1,1} \Delta CLI_{t-1}\right) \\ q_t &= Pr\left(s_t = bad | s_{t-1} = bad, z_{t-1}\right) = \phi\left(d_0 + d_{1,2} \Delta CLI_{t-1}\right) \end{aligned}$$

where $r_{m,t}$ is the CRSP value-weighted market portfolio, Def_t the default premium, $Term_t$ the term spread, $Yield_t$ the dividend yield on the market portfolio, I_t the short-run interest rate. The transition probabilities q_t and p_t are allowed to vary over time and are functions of ΔCLI_t, the year-over-year change in the composite leading indicator. The subscript s_t indicates the parameter switches with state $s_t \in \{good, bad\}$. The data are at the monthly frequency, from January of 1956 to December of 2010. The coefficient c_4 is the same on the two states; the estimated coefficient is on the left column and the standard deviation on the right column.

		Good State		**Bad State**	
		Parameter	**Std. Dev.**	**parameter**	**Std. Dev.**
Mean	c_0	-0.007	0.006	-0.029	0.042
	c_1	1.70	0.53	-3.68	1.80
	c_2	0.58	0.36	3.10	1.80
	c_3	-2.76	1.03	3.25	7.18
	c_4	0.32		0.22	
Variance	c_5	-7.28	0.17	-5.85	0.28
	c_6	127.23	34.84	118.96	86.82
Transition probabilities	d_0	-1.89	0.27	-1.12	0.38
	d_1	-11.80	13.16	-10.31	11.53
Log-likelihood value		1172.54			

Table 3.2: Descriptive Statistics of Beliefs and Uncertainty Proxies

This Table shows the descriptive statistics of the proxies for investor beliefs and uncertainty. The beliefs proxy, $\hat{\pi}_t$, are the probabilities that the economy is in the good state implied by a two-state Markov-switching model fitted to the excess return on the market portfolio. The uncertainty proxy $\hat{\pi}_t(1-\hat{\pi}_t)$ is directly computed from investor beliefs, $\hat{\pi}_t$. The other proxy for uncertainty is VIX, the Chicago Board Options Exchange (CBOE) volatility index, which is a measure of the implied volatility of S&P 500 index options.

	Starting Sample	**Obs**	**Mean**	**Std**	**Min**	**Max**
$\hat{\pi}_t$	Jan-1957	648	0.84	0.20	0.08	0.99
$\hat{\pi}_t(1-\hat{\pi}_t)$	Jan-1957	648	0.09	0.06	0.01	0.25
VIX	Jan-1990	252	20.39	7.87	10.42	59.89

Correlations

	$\hat{\pi}_t$	$\hat{\pi}_t(1-\hat{\pi}_t)$	VIX
$\hat{\pi}_t$	1.00	-0.77	-0.62
$\hat{\pi}_t(1-\hat{\pi}_t)$		1.00	0.52
VIX			1.00

Table 3.3: 5 Book-to-Market Portfolios Conditional Betas

This Table shows the GMM estimated parameters from the monthly excess returns on the five book-to-market portfolios from 1956 to 2010 that results from the moment restrictions:

$$E\left\{\left[r_{i,t} - \alpha_i - \left(a_{1,i} + a_{2,i}\hat{\pi}_t + a_{3,i}\hat{\pi}_t^2\right) r_{m,t}\right] \otimes \;\; Z_t \;\; \right\} \quad = \quad \mathbf{0}$$

for $i = 1, ..., 5$. Two different specifications for market betas are considered: $\beta_{i,t} = a_{1,i} + a_{2,i}\hat{\pi}_t + a_{3,i}\hat{\pi}_t^2$, denoted model (M1), and $\beta_{i,t} = a_{4,i} + a_{5,i}UC_t$, denoted model (M2). $\hat{\pi}_t$, is a proxy for investor beliefs implied by the probabilities of a two-state Markov-switching model fitted to the excess return on the market portfolio. In model (M2), UC_t can be one of two investor uncertainty proxies: $UC_t = \hat{\pi}_t (1 - \hat{\pi}_t)$, directly obtained from $\hat{\pi}_t$, or $UC_t = VIX_t$, the Chicago Board Options Exchange (CBOE) volatility index, a measure of the implied volatility of S&P 500 index options. The instruments used to estimate the parameters in (M1) are $Z_t = \begin{bmatrix} 1 & r_{m,t} & \hat{\pi}_t r_{m,t} & \hat{\pi}_t^2 r_{m,t} \end{bmatrix}'$ and the instruments in (M2) are $Z_t = \begin{bmatrix} 1 & r_{m,t} & UC_t r_{m,t} \end{bmatrix}'$. Both models are exactly identified. The long-run covariance matrix of the moments is estimated with the Newey-West kernel and the optimal bandwidth.

			Growth	**Qnt 2**	**Qnt 3**	**Qnt 4**	**Value**
M1	α_i	**param**	0.0031	0.0045	0.0057	0.0063	0.0072
Mar-56 to Dec-10		**t-stat**	4.47	6.54	7.24	5.87	5.95
	$a_{1,i}$	**param**	1.11	1.00	1.03	0.90	0.94
		t-stat	12.92	10.76	14.43	5.74	5.55
	$a_{2,i}$	**param**	-0.26	-0.51	-0.63	0.15	0.41
		t-stat	-0.99	-1.29	-1.60	0.22	0.55
	$a_{3,i}$	**param**	0.20	0.55	0.54	-0.22	-0.45
		t-stat	0.97	1.74	1.49	-0.39	-0.70
M2	α_i	**param**	0.0031	0.0045	0.0057	0.0063	0.0072
$UC_t = \hat{\pi}_t (1 - \hat{\pi}_t)$		**t-stat**	4.44	6.64	7.15	5.85	5.94
Mar-56 to Dec-10	$a_{4,i}$	**param**	1.05	1.05	0.93	0.82	0.90
		t-stat	41.60	39.69	23.72	18.69	15.45
	$a_{5,i}$	**param**	-0.05	-0.69	-0.29	0.39	0.52
		t-stat	-0.21	-3.58	-0.86	0.90	1.00
M2	α_i	**param**	0.0028	0.0045	0.0042	0.0046	0.0053
$UC_t = \hat{\pi}_t (1 - \hat{\pi}_t)$		**t-stat**	2.39	3.74	2.62	2.22	2.52
Jan-90 to Dec-10	$a_{4,i}$	**param**	1.03	0.98	0.90	0.81	0.82
		t-stat	22.98	17.99	11.18	9.11	8.23
	$a_{5,i}$	**param**	-0.14	-0.57	-0.17	0.30	0.88
		t-stat	-0.39	-1.67	-0.27	0.39	1.03
M2	α_i	**param**	0.0021	0.0045	0.0049	0.0058	0.0060
$UC_t = VIX_t$		**t-stat**	1.81	3.18	2.68	2.59	2.51
Jan-90 to Dec-10	$b_{4,i}$	**param**	1.13	0.93	0.77	0.62	0.77
		t-stat	24.97	11.32	7.35	4.67	5.36
	$b_{5,i}$	**param**	-0.42	-0.09	0.38	0.77	0.59
		t-stat	-3.29	-0.42	1.46	2.47	1.61

Table 3.4: Book-to-Market, Size, Momentum and Industry Portfolios Betas (Beliefs)

This Table shows the GMM estimated parameters from the monthly excess returns on 10 book-to-market, 10 size, 10 momentum and 10 industry portfolios from 1956 to 2010 that results from the moment restrictions:

$$E\left\{\left[r_{i,t} - \alpha_i - \left(a_{1,i} + a_{2,i}\hat{\pi}_t + a_{3,i}\hat{\pi}_t^2\right) r_{m,t}\right] \otimes \quad Z_t \quad \right\} \quad = \quad \mathbf{0}$$

for $i = 1, ..., 40$ where $\beta_{i,t} = a_{1,i} + a_{2,i}\hat{\pi}_t + a_{3,i}\hat{\pi}_t^2$. $\hat{\pi}_t$, is a proxy for investor beliefs implied by the probabilities of a two-state Markov-switching model fitted to the excess return on the market portfolio. The instruments used to estimate the parameters in (M1) are $Z_t = \left[\begin{array}{cccc} 1 & r_{m,t} & \hat{\pi}_t r_{m,t} & \hat{\pi}_t^2 r_{m,t} \end{array}\right]'$ and so the model is exactly identified. The long-run covariance matrix of the moments is estimated with the Newey-West kernel and the optimal bandwidth.

	params				**t-stat**					**params**				**t-stat**			
Portfolios	α_i	$a_{1,i}$	$a_{2,i}$	$a_{3,i}$	α_i	$a_{1,i}$	$a_{2,i}$	$a_{3,i}$	**Portfolios**	α_i	$a_{1,i}$	$a_{2,i}$	$a_{3,i}$	α_i	$a_{1,i}$	$a_{2,i}$	$a_{3,i}$
Low BEME	-0.018	1.000	0.158	-0.085	-1.711	8.66	0.40	-0.26	**Low Mom.**	-0.124	0.820	4.141	-3.976	-6.051	2.97	3.56	-3.96
2	-0.002	1.021	-0.324	0.345	-0.279	13.54	-0.85	1.04	**2**	-0.047	0.778	2.851	-2.772	-3.142	3.79	2.98	-3.33
3	0.004	0.798	0.154	0.092	0.551	5.39	0.31	0.24	**3**	-0.016	0.840	1.522	-1.560	-1.319	4.75	1.97	-2.42
4	0.005	1.037	-0.550	0.553	0.543	8.49	-1.00	1.17	**4**	-0.005	0.897	0.581	-0.617	-0.447	8.81	1.20	-1.49
5	0.016	0.902	-0.346	0.393	1.585	8.81	-0.70	0.89	**5**	-0.005	0.772	0.528	-0.418	-0.557	7.07	0.99	-0.91
6	0.017	0.961	-0.305	0.281	1.905	6.50	-0.56	0.64	**6**	0.002	0.837	-0.011	0.156	0.243	7.60	-0.02	0.38
7	0.020	0.594	1.155	-0.948	1.673	3.42	1.45	-1.43	**7**	0.007	1.022	-1.251	1.281	0.816	9.89	-2.64	3.14
8	0.031	0.931	-0.024	-0.039	2.366	3.36	-0.02	-0.05	**8**	0.027	1.027	-1.155	1.191	3.447	8.96	-2.38	2.94
9	0.036	0.790	0.701	-0.624	2.943	4.35	0.91	-0.97	**9**	0.029	1.028	-1.235	1.385	2.941	7.80	-2.37	3.11
High BEME	0.039	0.966	0.722	-0.715	2.222	3.15	0.67	-0.83	**High Mom.**	0.061	1.077	-1.231	1.592	3.924	5.37	-1.45	2.25
Low ME	0.016	0.853	0.400	-0.074	0.790	3.89	0.48	-0.11	**Non-Dur.**	0.033	0.803	-0.321	0.375	2.469	6.32	-0.42	0.54
2	0.008	0.947	0.463	-0.177	0.532	5.06	0.56	-0.25	**Durables**	-0.014	1.267	-0.500	0.318	-0.796	3.32	-0.40	0.34
3	0.016	0.995	0.332	-0.100	1.206	5.29	0.44	-0.16	**Manuf.**	-0.001	0.973	0.047	0.044	-0.109	8.45	0.09	0.10
4	0.013	1.006	0.177	0.021	1.022	5.80	0.24	0.04	**Energy**	0.026	1.061	-1.386	1.215	1.439	6.27	-1.76	1.79
5	0.015	0.888	0.786	-0.532	1.482	5.51	1.24	-1.02	**High-Tec.**	-0.009	1.004	1.452	-1.323	-0.520	4.63	1.52	-1.61
6	0.014	0.805	0.798	-0.495	1.508	6.91	1.73	-1.30	**Telecom**	0.011	0.997	-0.498	0.190	0.693	4.11	-0.55	0.26
7	0.015	0.983	0.206	-0.075	1.917	8.26	0.47	-0.21	**Retail**	0.010	0.894	0.183	-0.074	0.764	5.30	0.23	-0.11
8	0.011	0.894	0.652	-0.494	1.471	10.25	1.80	-1.60	**Health**	0.032	1.012	-1.028	0.953	2.018	4.28	-1.12	1.27
9	0.010	0.963	0.065	-0.027	1.884	9.62	0.19	-0.10	**Utilities**	0.021	0.565	0.329	-0.421	1.412	3.10	0.38	-0.57
High ME	-0.003	0.869	0.242	-0.209	-0.545	10.72	0.96	-1.09	**Others**	-0.007	0.867	0.708	-0.500	-0.621	7.77	1.25	-1.00

Table 3.5: Book-to-Market, Size, Momentum and Industry Portfolios Betas (Uncertainty)

This Table shows the GMM estimated parameters from the monthly excess returns on 10 book-to-market, 10 size, 10 momentum and 10 industry portfolios from 1956 to 2010 that results from the moment restrictions:

$$E\left\{\left[r_{i,t}-\alpha_i-\left(a_{4,i}+a_{5,i}UC_t\right)r_{m,t}\right]\otimes \quad Z_t \quad \right\} \quad = \quad \mathbf{0}$$

for $i=1,...,40$ where $\beta_{i,t}=a_{4,i}+a_{5,i}UC_t$. UC_t can be one of two investor uncertainty proxies: i) $UC_t=\hat{\pi}_t\left(1-\hat{\pi}_t\right)$, directly obtained from $\hat{\pi}_t$, the proxy for investor beliefs implied by the probabilities of a two-state Markov-switching model fitted to the excess return on the market portfolio; and ii) $UC_t=VIX_t$, the Chicago Board Options Exchange (CBOE) volatility index, a measure of the implied volatility of S&P 500 index options. The instruments are $Z_t=\left[\begin{array}{ccc}1 & r_{m,t} & UC_t r_{m,t}\end{array}\right]'$ and so the model is exactly identified. The long-run covariance matrix of the moments is estimated with the Newey-West kernel and the optimal bandwidth.

	params			t-stat				params		t-stat		
Port.	α_i	$a_{4,i}$	$a_{5,i}$	α_i	$a_{4,i}$	$a_{5,i}$	**Port.**	α_i	$a_{4,i}$	$a_{5,i}$	α_i	$a_{4,i}$	$a_{5,i}$
Low BEME	-0.014	1.17	-0.39	-0.82	16.04	-1.81	**Low Mom.**	-0.102	1.13	1.90	-1.99	3.15	1.43
2	0.002	0.96	-0.03	0.12	13.36	-0.22	**2**	-0.012	0.82	1.60	-0.36	4.03	2.46
3	0.005	1.02	-0.35	0.37	8.52	-0.76	**3**	-0.002	0.77	0.92	-0.09	6.68	2.81
4	0.040	0.68	0.89	1.87	6.70	3.79	**4**	0.023	0.66	0.90	0.84	4.40	1.93
5	0.027	0.73	0.49	1.16	5.17	1.08	**5**	0.010	0.69	0.61	0.52	5.30	1.26
6	0.024	0.68	0.79	1.25	7.34	3.53	**6**	0.014	0.66	0.63	0.95	6.62	2.09
7	0.024	0.77	0.19	0.85	5.66	0.70	**7**	0.015	0.80	-0.06	0.95	8.26	-0.28
8	0.046	0.44	1.42	1.79	3.12	3.03	**8**	0.039	0.86	-0.14	2.97	10.05	-0.81
9	0.034	0.75	0.52	1.35	5.75	1.84	**9**	0.002	1.02	-0.49	0.13	8.76	-1.23
High BEME	0.052	0.66	1.32	1.44	3.70	1.97	**High Mom.**	0.016	1.55	-1.32	0.50	11.10	-3.55
Low BE	0.043	0.80	0.83	1.19	5.98	2.61	**Non-Dur.**	0.036	0.56	0.20	1.35	3.96	0.68
2	0.012	1.09	0.35	0.40	8.64	0.89	**Durables**	0.010	0.72	1.63	0.26	4.31	3.43
3	0.017	1.05	0.40	0.67	10.62	1.75	**Manuf.**	0.034	0.83	0.50	1.81	10.49	2.33
4	0.005	1.05	0.30	0.23	11.99	1.30	**Energy**	0.053	0.54	0.41	1.93	4.34	1.31
5	0.016	1.07	0.31	0.86	13.23	1.32	**High-Tec.**	-0.033	1.73	-0.82	-0.95	8.69	-1.29
6	0.017	1.02	0.20	1.02	16.44	0.96	**Telecom**	-0.041	1.03	-0.21	-1.43	8.55	-0.60
7	0.029	0.93	0.44	1.89	17.33	3.07	**Retail**	0.010	0.88	-0.04	0.43	7.22	-0.16
8	0.015	1.03	0.20	1.07	23.28	1.69	**Health**	0.019	0.76	-0.33	0.67	5.22	-0.95
9	0.024	0.90	0.33	2.25	15.65	1.70	**Utilities**	0.031	0.37	0.10	1.01	2.54	0.24
High ME	-0.007	0.95	-0.04	-0.82	13.97	-0.15	**Others**	0.009	0.79	0.90	0.37	5.85	2.07

Table 3.6: Conditional Price of Risk – Beliefs

This Table shows the sequential GMM estimates of the risk premia parameters from the monthly excess returns on 10 book-to-market, 10 size, 10 momentum and 10 industry portfolios from 1956 to 2010, with the pricing restriction imposed in the second step being:

$$E\left[\; r_{i,t} - \lambda_0 - \textstyle\sum_j \lambda_{j,t} \frac{(r_{i,t}-\mu_i)(r_{j,t}-\mu_j)}{\sigma_j^2} \;\right] \quad = \quad 0$$

where $i = 1, ..., 40$ and $j = MKT, HML, SMB$, respectively the market portfolio, high-minus-low and small-minus-big risk factors of the Fama and French 3-factor model. Risk premia, $\lambda_{j,t}$, are allowed to vary with the functional form $\lambda_{m,t} = b_1 + b_2\hat{\pi}_t + b_3\hat{\pi}_t^2$. $\hat{\pi}_t$, is a proxy for investor beliefs implied by the probabilities of a two-state Markov-switching model fitted to the excess return on the market portfolio.

			MKT			**HML**			**SMB**			
Model		λ_0	$const.$	$\hat{\pi}_t$	$\hat{\pi}_t^2$	$const.$	$\hat{\pi}_t$	$\hat{\pi}_t^2$	$const.$	$\hat{\pi}_t$	$\hat{\pi}_t^2$	$J-stat$
(1)	param	0.0063	-0.0015									117.65
	t-stat	4.62	-0.72									0.00
(2)	param	0.0064	-0.0211	0.0262								112.047
	t-stat	4.23	-2.09	1.97								0.00
(3)	param	0.0057	0.0420	-0.2598	0.2376							97.36
	t-stat	3.50	1.57	-2.51	2.84							0.00
(4)	param	0.0063	-0.0008			0.0030			0.0027			107.289
	t-stat	4.11	-0.33			2.35			2.39			0.00
(5)	param	0.0059	0.0384	-0.1675	0.1362	0.0311	-0.0832	0.0553	-0.0040	-0.0462	0.0619	86.523
	t-stat	3.33	1.00	-1.17	1.19	1.17	-0.86	0.73	-0.09	-0.32	0.56	0.00

Table 3.7: Conditional Price of Risk – Uncertainty

This Table shows the sequential GMM estimates of the risk premia parameters from the monthly excess returns on 10 book-to-market, 10 size, 10 momentum and 10 industry portfolios from 1956 to 2010, with the pricing condition imposed in the second step being:

$$E\left[\; r_{i,t} - \lambda_0 - \sum_j \lambda_{j,t} \frac{(r_{i,t}-\mu_i)(r_{j,t}-\mu_j)}{\sigma_j^2} \;\right] \;=\; 0$$

where $i = 1, ..., 40$ and $j = MKT, HML, SMB$, respectively the market portfolio, high-minus-low and small-minus-big risk factors of the Fama and French 3-factor model. Risk premia, $\lambda_{j,t}$, are allowed to vary with the functional form $\lambda_{m,t} = b_4 + b_5 UC_t$. UC_t can be one of two investor uncertainty proxies: i) $UC_t = \hat{\pi}_t\,(1 - \hat{\pi}_t)$, directly obtained from $\hat{\pi}_t$, the proxy for investor beliefs implied by the probabilities of a two-state Markov-switching model fitted to the excess return on the market portfolio; and ii) $UC_t = VIX_t$, the Chicago Board Options Exchange (CBOE) volatility index, a measure of the implied volatility of S&P 500 index options.

				MKT		**HML**		**SMB**		
proxy	**model**		λ_0	*const.*	UC_t	*const.*	UC_t	*const.*	UC_t	$J-stat$
$UC_t = \hat{\pi}_t\,(1-\hat{\pi}_t)$	**(6)**	param	0.0060	0.0185	-0.1799					103.56
		t-stat	3.99	2.79	-3.42					0.00
	(7)	param	0.0058	0.0082	-0.0821	0.0022	0.0109	0.0141	-0.1044	85.6976
		t-stat	3.34	0.79	-0.99	0.40	0.25	2.18	-1.75	0.00
$UC_t = VXI$	**(8)**	param	0.0068	0.0027	-0.0028					47.6514
		t-stat	3.62	0.54	-1.06					0.11
	(9)	param	0.0073	0.0020	-0.0041	-0.0001	0.0004	0.0008	0.0040	48.8237
		t-stat	3.70	0.33	-1.03	-0.02	0.13	0.30	0.67	0.04

Table 3.8: Asymmetric Betas – $\beta^+_+(c)$ and $\beta^-_-(c)$

This Table reports the p-values of the statistic $J_\beta = T\left(\beta^+_+ - \beta^-_-\right)' \hat{\Psi}^{-1}\left(\beta^+_+ - \beta^-_-\right)$, where $\beta^+_+(c) = \frac{cov(\tilde{r}_{i,t}, \tilde{r}_{m,t} | \tilde{r}_{i,t} > c, \tilde{r}_{m,t} > c)}{var(\tilde{r}_{m,t} | \tilde{r}_{i,t} > c, \tilde{r}_{m,t} > c)}$ and $\beta^-_-(c) = \frac{cov(\tilde{r}_{i,t}, \tilde{r}_{m,t} | \tilde{r}_{i,t} < -c, \tilde{r}_{m,t} < -c)}{var(\tilde{r}_{m,t} | \tilde{r}_{i,t} < -c, \tilde{r}_{m,t} < -c)}$, with $\tilde{r}$ denoting standardized returns and c thresholds. The null hypothesis is $H_0 : \beta^+_+(c) = \beta^-_-(c)\,,\ for\,all\,c \geq 0$ versus $H_a : \beta^+_+(c) \neq \beta^-_-(c)\,,\ for\,some\,c \geq 0$. $\overline{\beta^+_+ - \beta^-_-}$ refers to the average of the vector $\beta^+_+ - \beta^-_- = \left[\beta^+_+(c_1) - \beta^-_-(c_1), ..., \beta^+_+(c_n) - \beta^-_-(c_n)\right]'$.

	$c = [0]$		$c = [0, 0.5, 1, 1.5]$		$c = [0, 0.1, ..., 1.5]$	
Portfolios	**p-value**	$\overline{\beta^+_+ - \beta^-_-}$	**p-value**	$\overline{\beta^+_+ - \beta^-_-}$	**p-value**	$\overline{\beta^+_+ - \beta^-_-}$
Low BEME	0.001	-0.380	0.000	-0.502	0.000	-0.504
2	0.043	-0.207	0.061	-0.320	0.121	-0.309
3	0.067	-0.178	0.052	-0.291	0.078	-0.278
4	0.093	-0.164	0.040	-0.273	0.483	-0.266
5	0.226	-0.117	0.279	-0.198	0.821	-0.186
6	0.395	-0.076	0.326	-0.095	0.005	-0.092
7	0.325	-0.091	0.261	-0.127	0.298	-0.119
8	0.617	-0.044	0.895	-0.079	0.286	-0.081
9	0.880	0.014	0.987	-0.019	0.937	-0.010
High BEME	0.334	0.078	0.219	0.102	0.212	0.095
Low ME	0.738	0.030	0.983	0.013	0.754	0.018
2	0.632	0.041	0.905	0.058	0.372	0.060
3	0.707	0.032	0.695	0.030	0.468	0.029
4	0.993	-0.001	0.962	0.011	0.533	0.016
5	0.893	-0.013	0.461	-0.008	0.486	-0.006
6	0.773	-0.029	0.980	-0.095	0.668	-0.093
7	0.743	0.032	0.009	0.037	0.027	0.032
8	0.638	-0.059	0.826	-0.142	0.611	-0.134
9	0.325	-0.096	0.464	-0.153	0.003	-0.145
High ME	0.324	-0.119	0.409	-0.192	0.137	-0.194
Low Mom.	0.669	0.052	0.786	-0.020	0.200	0.012
2	0.298	0.110	0.800	0.120	0.138	0.133
3	0.160	0.157	0.223	0.154	0.115	0.173
4	0.306	0.105	0.898	0.216	0.937	0.220
5	0.297	0.096	0.261	0.189	0.023	0.192
6	0.471	0.065	0.938	0.083	0.270	0.088
7	0.736	-0.030	0.917	-0.044	0.924	-0.046
8	0.890	-0.012	0.924	0.002	0.821	-0.008
9	0.166	-0.124	0.532	-0.222	0.070	-0.223
High Mom.	0.023	-0.204	0.049	-0.348	0.032	-0.353
Non-Dur.	0.475	-0.074	0.506	-0.047	0.071	-0.050
Durables	0.827	-0.026	0.423	-0.139	0.834	-0.120
Manuf.	0.598	-0.051	0.445	-0.148	0.256	-0.140
Energy	0.413	-0.100	0.839	-0.109	0.183	-0.130
High-Tec.	0.780	0.027	0.305	-0.153	0.009	-0.120
Telecom	0.943	-0.009	0.668	0.101	0.741	0.076
Retail	0.853	-0.019	0.859	0.023	0.561	0.025
Health	0.967	0.004	0.730	0.244	0.003	0.222
Utilities	0.588	0.075	0.313	0.249	0.082	0.256

Table 3.9: Asymmetric Betas – $\beta_+(c)$ and $\beta_-(c)$

This Table reports the p-values of the statistic $J_\beta = T(\beta_+ - \beta_-)' \hat{\Psi}^{-1} (\beta_+ - \beta_-)$, where $\beta_+(c) = \frac{cov(\tilde{r}_{i,t}, \tilde{r}_{m,t} | \tilde{r}_{m,t} > c)}{var(\tilde{r}_{m,t} | \tilde{r}_{m,t} > c)}$ and $\beta_-(c) = \frac{cov(\tilde{r}_{i,t}, \tilde{r}_{m,t} | \tilde{r}_{m,t} < -c)}{var(\tilde{r}_{m,t} | \tilde{r}_{m,t} < -c)}$, with $\tilde{r}$ denoting standardized returns and c thresholds. The null hypothesis is $H_0 : \beta_+(c) = \beta_-(c)\,,\ for\, all\, c \geq 0$ versus $H_a : \beta_+(c) \neq \beta_-(c)\,,\ for\, some\, c \geq 0$. $\overline{\beta_+ - \beta_-}$ refers to the average of the vector $\beta_+ - \beta_- = [\beta_+(c_1) - \beta_-(c_1), ..., \beta_+(c_n) - \beta_-(c_n)]'$.

	$c = [0]$		$c = [0, 0.5, 1, 1.5]$		$c = [0, 0.1, ..., 1.5]$	
Portfolios	**p-value**	$\overline{\beta_+ - \beta_-}$	**p-value**	$\overline{\beta_+ - \beta_-}$	**p-value**	$\overline{\beta_+ - \beta_-}$
Low. BEME	0.575	0.042	0.589	0.054	0.004	0.037
2	0.463	0.055	0.582	0.084	0.451	0.086
3	0.494	0.050	0.605	0.081	0.080	0.082
4	0.718	0.029	0.965	0.052	0.852	0.054
5	0.932	0.007	0.218	0.024	0.564	0.030
6	0.800	-0.021	0.970	-0.057	0.261	-0.050
7	0.444	0.061	0.618	0.115	0.990	0.113
8	0.836	-0.021	0.889	-0.054	0.480	-0.045
9	0.557	-0.046	0.893	-0.057	0.102	-0.049
High. BEME	0.326	-0.091	0.419	-0.104	0.049	-0.092
Low. ME	0.000	-0.374	0.000	-0.420	0.000	-0.416
2	0.003	-0.227	0.000	-0.229	0.000	-0.227
3	0.018	-0.181	0.001	-0.207	0.000	-0.206
4	0.029	-0.169	0.001	-0.196	0.000	-0.199
5	0.113	-0.121	0.027	-0.138	0.008	-0.136
6	0.332	-0.071	0.232	-0.052	0.090	-0.054
7	0.272	-0.087	0.165	-0.088	0.124	-0.090
8	0.671	-0.033	0.897	-0.052	0.158	-0.053
9	0.746	0.026	0.932	0.005	0.957	0.009
High. ME	0.255	0.084	0.092	0.110	0.207	0.105
Low Mom.	0.530	0.059	0.423	0.124	0.125	0.150
2	0.133	0.130	0.283	0.177	0.032	0.192
3	0.060	0.178	0.036	0.198	0.227	0.223
4	0.106	0.140	0.270	0.240	0.490	0.251
5	0.168	0.107	0.018	0.196	0.015	0.201
6	0.212	0.097	0.082	0.119	0.009	0.128
7	0.773	-0.022	0.856	-0.017	0.945	-0.020
8	0.859	0.013	0.788	0.049	0.825	0.044
9	0.144	-0.106	0.007	-0.134	0.002	-0.145
High Mom.	0.006	-0.193	0.001	-0.283	0.000	-0.301
Non-Dur.	0.893	0.011	0.215	0.040	0.215	0.040
Durables	0.813	0.022	0.177	-0.048	0.177	-0.048
Manuf.	0.677	-0.034	0.582	-0.099	0.582	-0.099
Energy	0.704	-0.033	0.335	-0.023	0.335	-0.023
High-Tec.	0.627	0.035	0.172	-0.037	0.172	-0.037
Telecom	0.198	0.112	0.472	0.133	0.472	0.133
Retail	0.868	0.014	0.616	0.055	0.616	0.055
Health	0.418	0.065	0.002	0.167	0.002	0.167
Utilities	0.302	0.102	0.045	0.205	0.045	0.205

Table 3.10: Asymmetric Betas – $\beta^+(c)$ and $\beta^-(c)$

This Table reports the p-values of the statistic $J_\beta = T\left(\beta^+ - \beta^-\right)'\hat{\Psi}^{-1}\left(\beta^+ - \beta^-\right)$, where $\beta^+(c) = \frac{cov\left(\tilde{r}_{i,t},\tilde{r}_{m,t}|\tilde{r}_{i,t}>c\right)}{var\left(\tilde{r}_{m,t}|\tilde{r}_{i,t}>c\right)}$ and $\beta^-(c) = \frac{cov\left(\tilde{r}_{i,t},\tilde{r}_{m,t}|\tilde{r}_{i,t}<-c\right)}{var\left(\tilde{r}_{m,t}|\tilde{r}_{i,t}<-c\right)}$, with $\tilde{r}$ denoting standardized returns and c thresholds. The null hypothesis is $H_0 : \beta^+(c) = \beta^-(c)\,,\ for\,all\,c \geq 0$ versus $H_a : \beta^+(c) \neq \beta^-(c)\,,\ for\,some\,c \geq 0$. $\overline{\beta^+ - \beta^-}$ refers to the average of the vector $\beta^+ - \beta^- = \left[\beta^+(c_1) - \beta^-(c_1), ..., \beta^+(c_n) - \beta^-(c_n)\right]'$.

	$c = [0]$		$c = [0, 0.5, 1, 1.5]$		$c = [0, 0.1, ..., 1.5]$	
Portfolios	**p-value**	$\overline{\beta^+ - \beta^-}$	**p-value**	$\overline{\beta^+ - \beta^-}$	**p-value**	$\overline{\beta^+ - \beta^-}$
Low. BEME	0.764	0.022	0.918	0.001	0.881	0.010
2	0.639	0.034	0.371	0.018	0.035	0.018
3	0.890	0.010	0.589	-0.021	0.578	-0.031
4	0.592	-0.042	0.863	-0.053	0.011	-0.061
5	0.440	-0.059	0.926	-0.138	0.456	-0.140
6	0.562	-0.048	0.900	-0.123	0.699	-0.122
7	0.957	-0.004	0.016	-0.071	0.028	-0.064
8	0.212	-0.124	0.499	-0.255	0.011	-0.264
9	0.046	-0.152	0.074	-0.255	0.137	-0.249
High. BEME	0.117	-0.140	0.324	-0.283	0.092	-0.271
Low. ME	0.000	-0.369	0.000	-0.523	0.000	-0.531
2	0.005	-0.219	0.004	-0.331	0.060	-0.337
3	0.021	-0.178	0.040	-0.270	0.007	-0.278
4	0.038	-0.163	0.044	-0.265	0.386	-0.260
5	0.114	-0.122	0.189	-0.201	0.755	-0.195
6	0.161	-0.104	0.105	-0.132	0.438	-0.130
7	0.178	-0.109	0.116	-0.162	0.020	-0.161
8	0.469	-0.055	0.886	-0.110	0.631	-0.112
9	0.848	0.015	0.873	-0.034	0.799	-0.024
High. ME	0.308	0.073	0.288	0.081	0.021	0.077
Low Mom.	0.841	-0.018	0.954	-0.087	0.032	-0.070
2	0.676	0.034	0.228	-0.087	0.006	-0.094
3	0.259	0.096	0.013	0.000	0.019	0.007
4	0.845	0.016	0.951	-0.014	0.043	-0.007
5	0.875	0.012	0.035	-0.111	0.346	-0.092
6	0.966	0.003	0.763	-0.022	0.386	-0.017
7	0.479	-0.053	0.304	-0.096	0.225	-0.094
8	0.937	-0.006	0.588	-0.007	0.005	-0.023
9	0.176	-0.098	0.664	-0.218	0.582	-0.217
High Mom.	0.015	-0.175	0.016	-0.295	0.016	-0.288
Non-Dur.	0.010	-0.196	0.001	-0.276	0.000	-0.273
Durables	0.472	-0.062	0.494	-0.180	0.039	-0.159
Manuf.	0.306	-0.083	0.728	-0.182	0.078	-0.181
Energy	0.021	-0.187	0.001	-0.190	0.013	-0.209
High-Tec.	0.830	0.015	0.391	-0.091	0.488	-0.083
Telecom	0.820	-0.019	0.954	-0.045	0.078	-0.050
Retail	0.307	-0.078	0.359	-0.160	0.346	-0.153
Health	0.487	-0.053	0.274	0.061	0.126	0.032
Utilities	0.162	-0.126	0.177	-0.143	0.000	-0.130

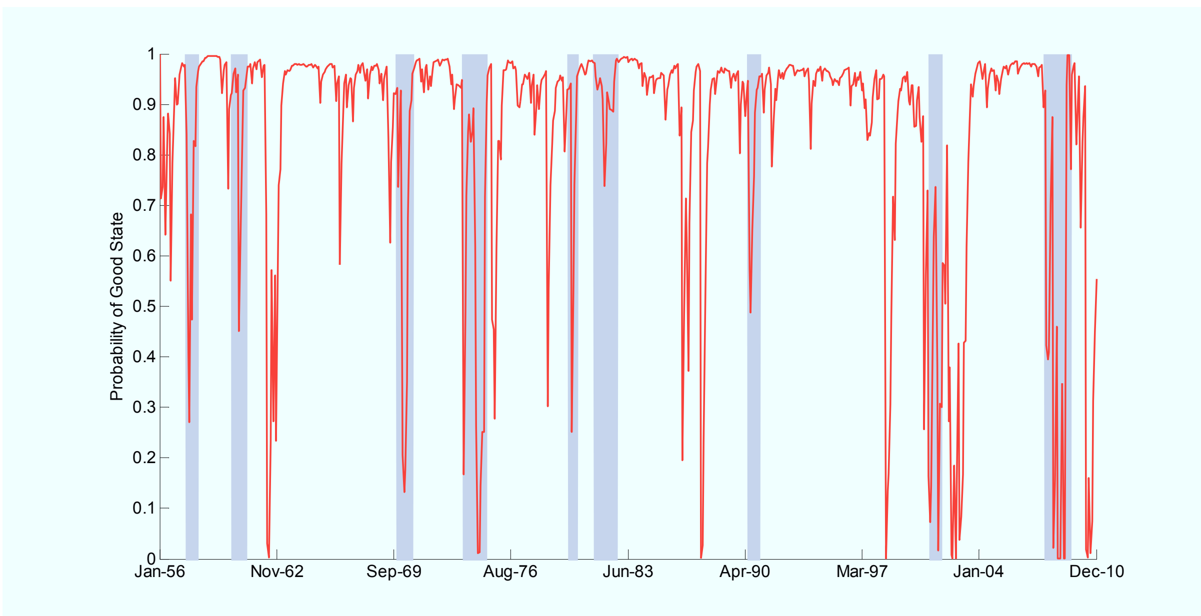

Figure 3.1: Markov-Switching Implied Beliefs $\hat{\pi}_t$

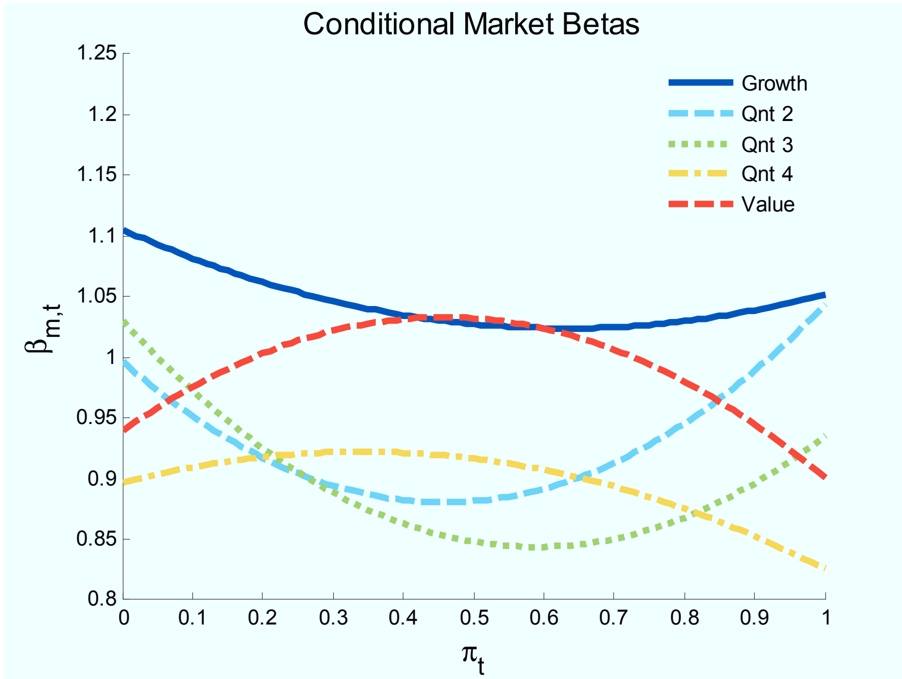

Figure 3.2: Estimates of Model (M1) of Conditional Betas (1956-2010)

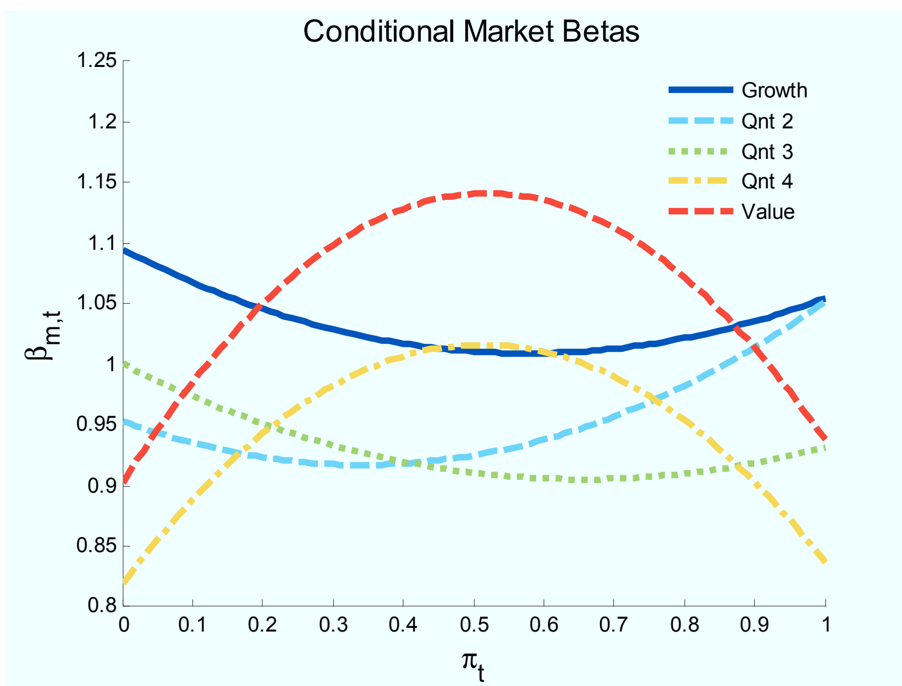

Figure 3.3: Estimates of Model (M1) of Conditional Betas (1956-2010 excl. years 1997-2001)

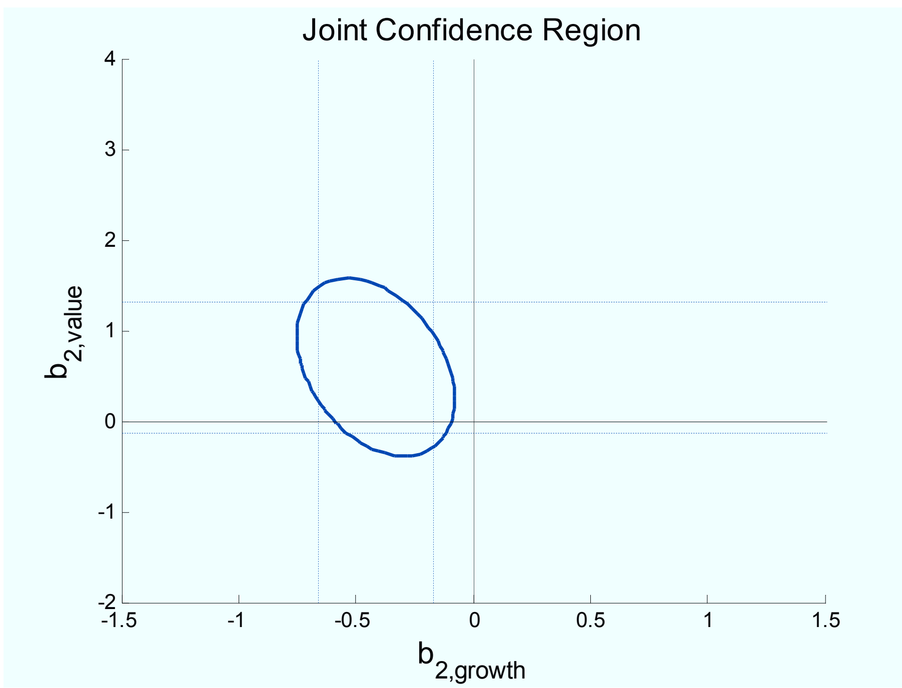

Figure 3.4: Joint Confidence Region for Model (M2) with $UC_t = VXI_t$

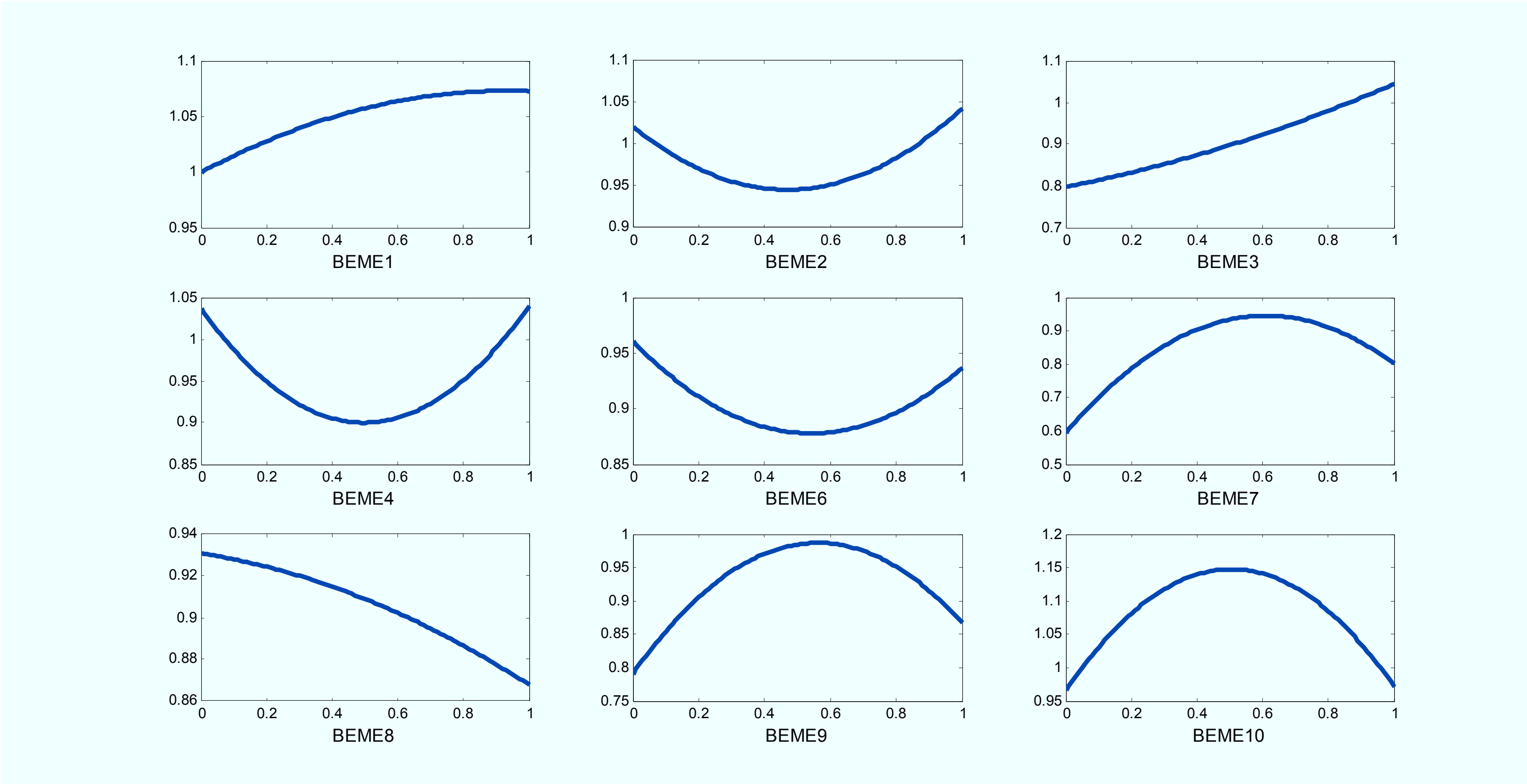

Figure 3.5: Conditional Market Betas of Book-to-Market Sorted Portfolios

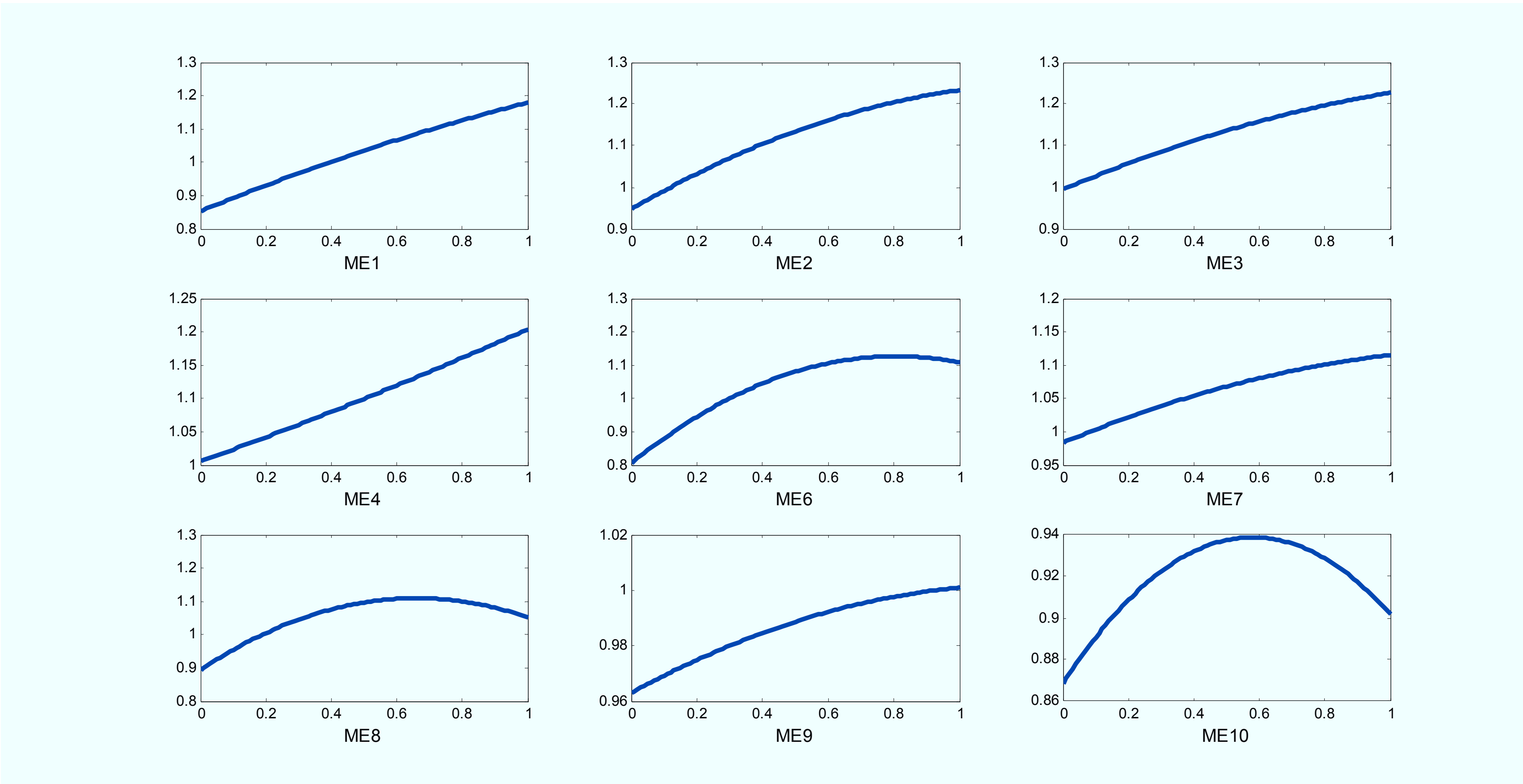

Figure 3.6: Conditional Market Betas of Size Sorted Portfolios

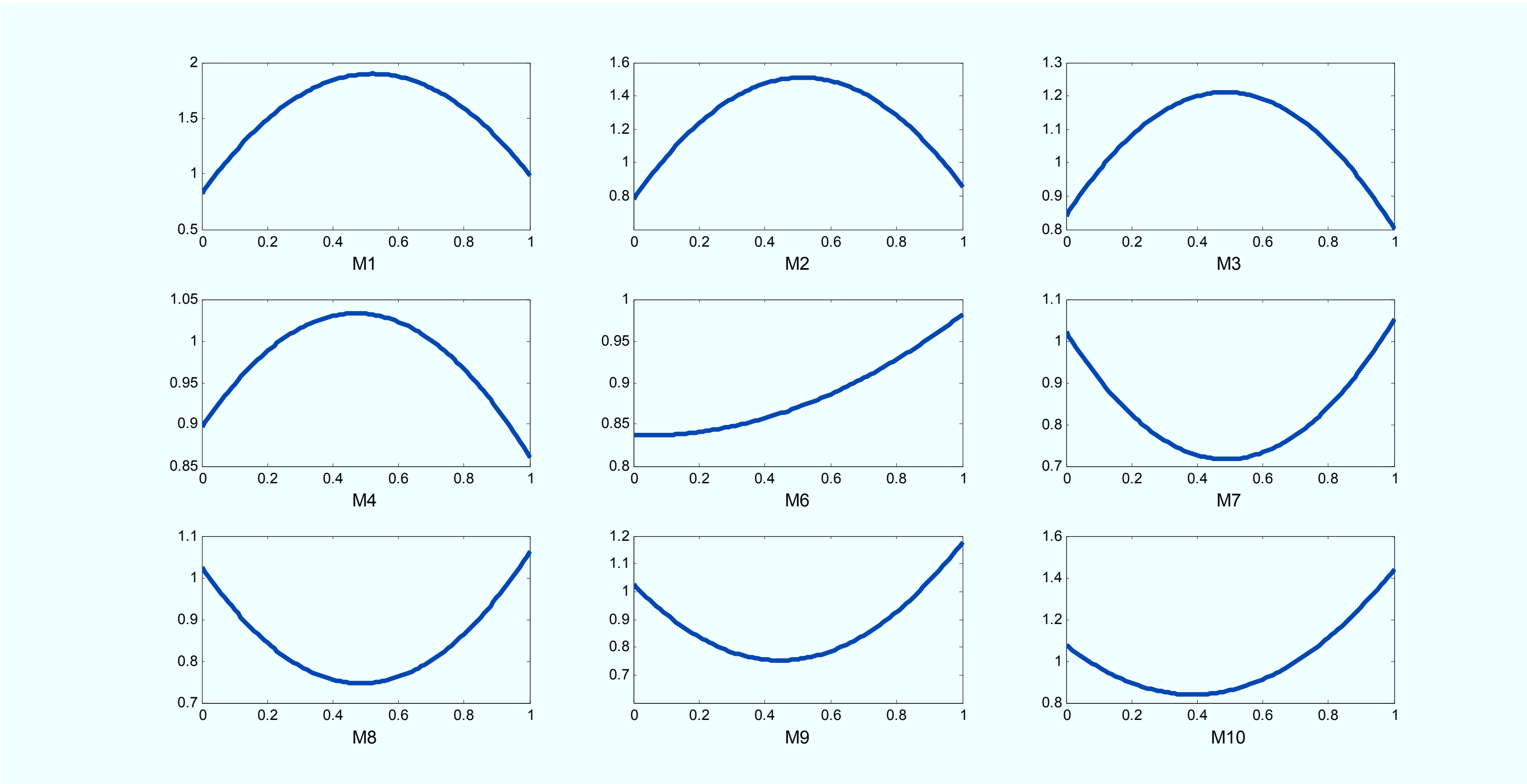

Figure 3.7: Conditional Market Betas of Momentum Sorted Portfolios

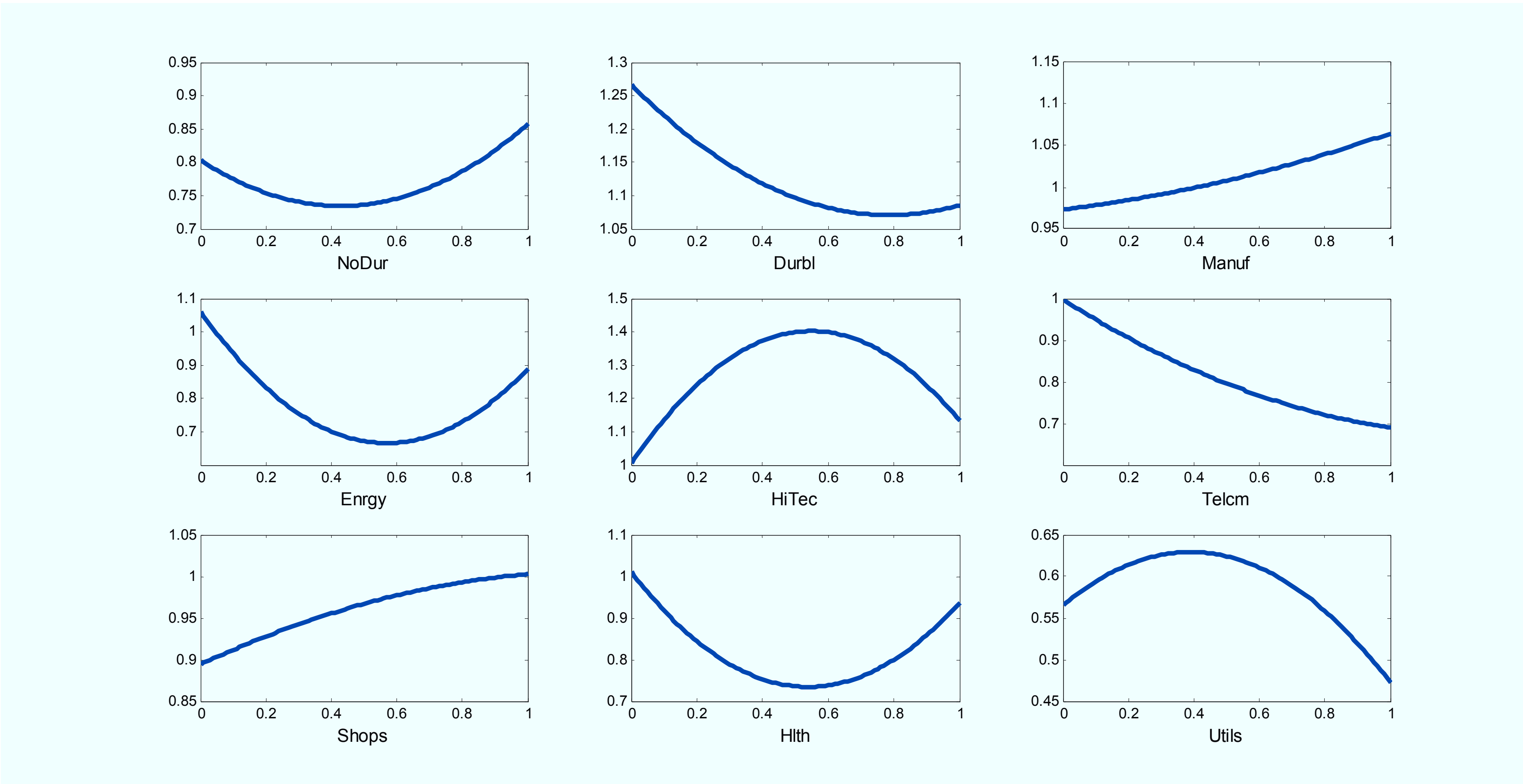

Figure 3.8: Conditional Market Betas of Industry Portfolios

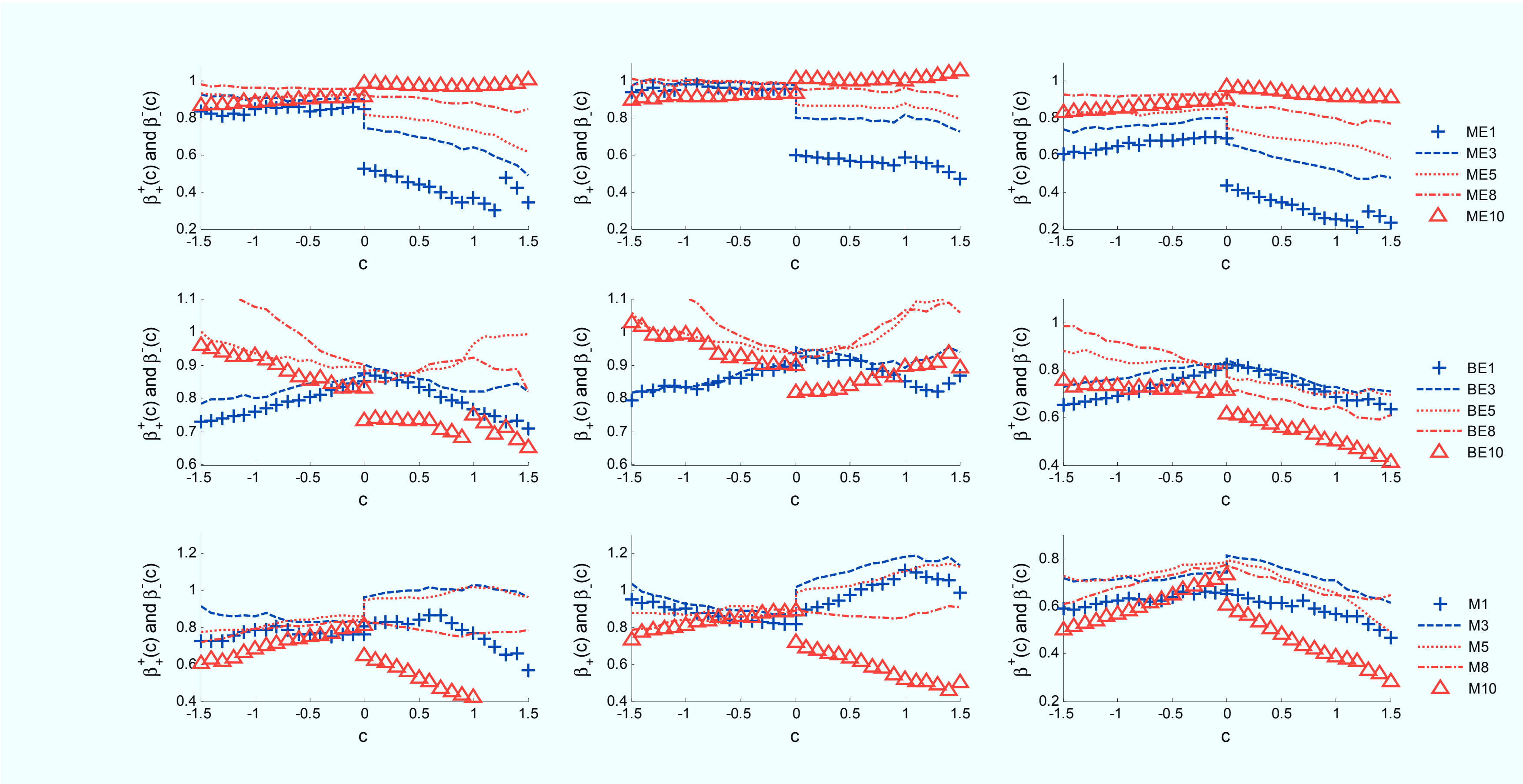

Figure 3.9: Upside and Downside Betas of Size, Book-to-Market and Momentum Portfolios

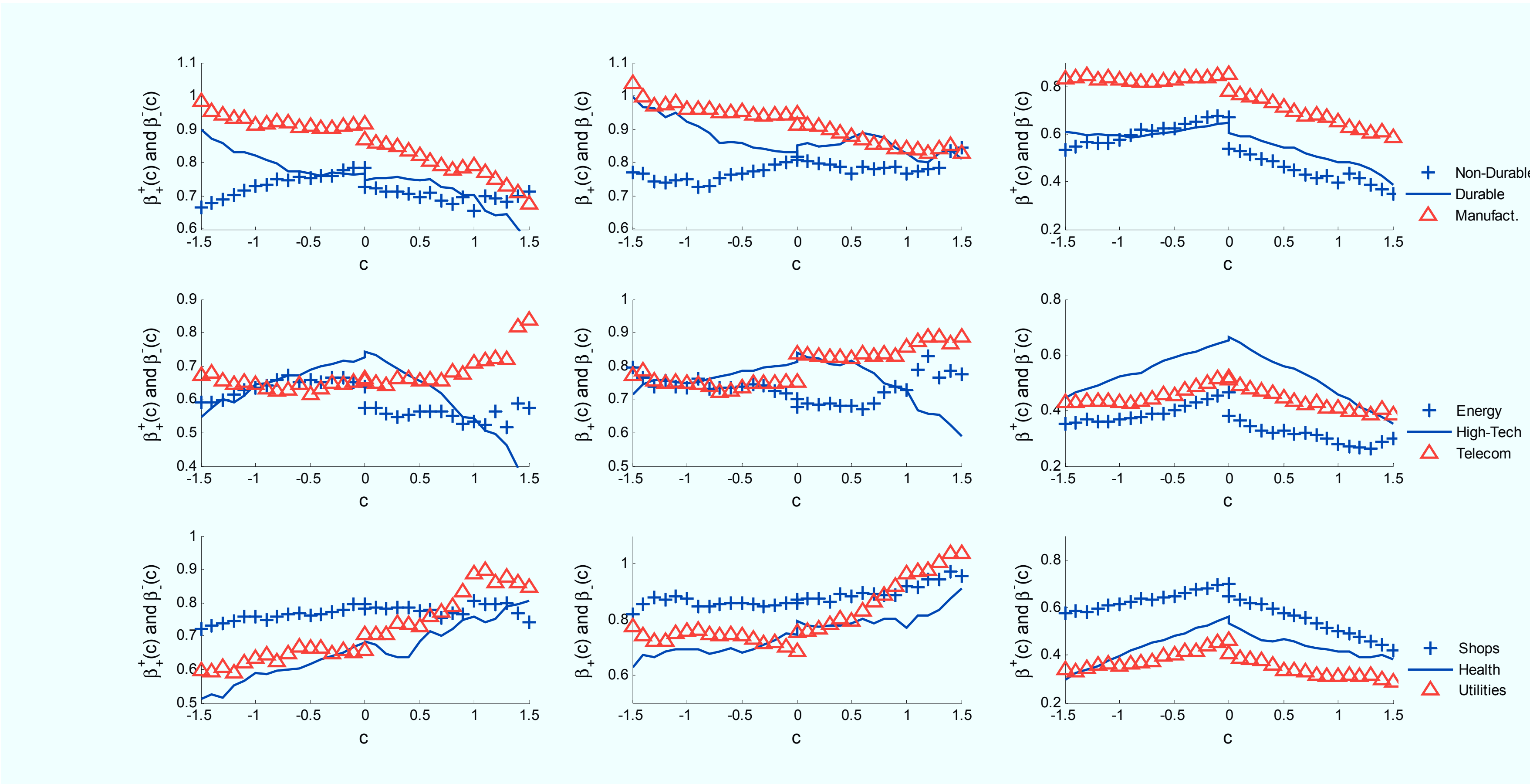

Figure 3.10: Upside and Downside Betas of Industry Portfolios

Chapter 4

Decomposing Betas

4.1 Introduction

It is natural to presume that positive and negative returns have different implications to risk. Despite the popularity of the mean-variance characterization of investor risky choices, which imposes positive and negative returns to have the same risk implications, asymmetric measures of risk have long been advocated. Indeed, we can go back as far as Markowitz (1959), who proposed a mean-semivariance characterization of investor risky choices. Measures of asymmetric risk, such as Value at Risk (VaR), are also widely used by practitioners. The implications of asymmetric measure of portfolio risk to asset pricing have also long been investigated; Hogan and Warren (1974) derived a semi-variance version of the CAPM, and Bawa and Lindenberg (1977) derived a lower partial moments CAPM. Recently, empirical implications of asymmetric risk to asset pricing have also been uncovered. Ang, Chen, and Xing (2006), henceforth ACX, find that exposures to downside risk, captured by the downside beta, are priced in the cross-section of U.S. returns and carry a premium of 6% per year. Harvey and Siddique (2000) also find that exposures to systematic skewness, captured by the coskewness, help explain the cross-section of stock returns.

In this chapter, we dissect betas and analyze the sources of its asymmetries. We propose a decomposition that partitions betas into four "signed-betas", each according to one of the four market conditions: the stock is up and the market is up (β_{+}^{+}), the stock is down and the market is up (β_{+}^{-}), the stock is up and the market is down (β_{-}^{+}), and the stock is down and the market is down (β_{-}^{-}).

The theoretical justification of a premium for downside risk is usually based on asymmetric preferences. For instance, Hogan and Warren (1974) assumes investor choices can be represented by a mean versus semivariance trade-off; ACX assume investor has the disappointment-aversion (DA) preferences of Gul (1991); Harvey and Siddique (2000) conjectured that investors, given mean and variance, prefer positive skewness.

In the model discussed in Chapter 2, asymmetries arise in a different context. The particular signs of returns matter because it can signal a potential change in the economic conditions. In particular, the average negative news, weighted by a signal-to-noise ratio, increases uncertainty, whereas the average positive news decreases uncertainty. Because betas and prices of risk depend on the level of uncertainty, asymmetries also arise in expected returns. An appealing feature of this approach is that it also indicates which assets are more prone to asymmetries, namely those more susceptible to change in the economic conditions. In the model, economic conditions refer to macroeconomic conditions, such as business cycles. Furthermore, it is also possible that asset related news to impact its systematic risk, as it can also be informative about the general economic conditions. However, not much is known about this direction of the causality. Such insights have motivated the sign decomposition of betas and the exploratory investigation of this chapter.

The main results of this chapter are the following. First, using data on common stocks traded on the NYSE, Amex and Nasdaq, during the years 1963 and 2009, the proposed decomposition shows that equally signed betas contributed the most for the overall betas. On average, the contributions are: 0.62 from the β_{-}^{-}, 0.67 from the β_{+}^{+}, -0.12 from the β_{-}^{+}, and -0.15 from the β_{-}^{+}.

Second, we find that the cross-signed betas and the downside betas, β_{+}^{-},β_{-}^{+} and β_{-}^{-}, are all associated with higher returns in the cross-section of stocks. We also find that the two betas related to downside markets, β_{-}^{+} and β_{-}^{-}, are not relevant when controlling for coskewness and cokurtosis. However, the higher return associated with β_{+}^{-}, the beta when the stock is down and market is up, remains positive and significant after controlling for the sources of risk commonly studied in the literature.

The chapter proceeds as follows. In Section 4.2 we describe the decomposition. Then, in Section 4.3, we present the empirical results. Finally, in Section 4.4 we conclude with some final remarks.

4.2 Decomposing Market Betas

With the availability of high-frequency data, estimators of return variance and covariance based on realized measures have become increasingly popular. Following such developments, Andersen, Bollerslev, Diebold, and Wu (2006) have proposed an estimator of the market betas using high-frequency data, the so-called realized beta:

$$\beta_{i,t,t+1} = \frac{\sum\limits_{j=1,\ldots,[1/\Delta]} r_{m,t_j} r_{i,t_j}}{\sum\limits_{j=1,\ldots,[1/\Delta]} r_{m,t_j}^2}$$

where the market return r_m and stock return r_i are in excess of the risk-free rate, Δ is the number of partitions in the period between t and $t+1$. Under this framework, upside and downside realized betas are defined as:

$$\beta_+ = \frac{\sum\limits_{j=1,\ldots,[1/\Delta]} r_{m,t_j} r_{i,t_j} \mathbf{I}_{[r_{m,t_j} \geq \tau]}}{\sum\limits_{j=1,\ldots,[1/\Delta]} r_{m,t_j}^2 \mathbf{I}_{[r_{m,t_j} \geq \tau]}}$$

$$\beta_- = \frac{\sum\limits_{j=1,\ldots,[1/\Delta]} r_{m,t_j} r_{i,t_j} \mathbf{I}_{[r_{m,t_j} < \tau]}}{\sum\limits_{j=1,\ldots,[1/\Delta]} r_{m,t_j}^2 \mathbf{I}_{[r_{m,t_j} < \tau]}}$$

where τ is a threshold, that will be set to $\tau = 0$, and $\mathbf{I}_{[.]}$ the indicator function. These upside and downside realized betas can be regarded as the high-frequency estimates of the upside and downside betas of Bawa and Lindenberg (1977), who defined them as $\beta_-^{BL} = E\left(r_i r_m | r_m < 0\right) / E\left(r_m^2 | r_m < 0\right)$ and $\beta_+^{BL} = E\left(r_i r_m | r_m \geq 0\right) / E\left(r_m^2 | r_m \geq 0\right)$, and of ACX, who defined them as $\beta_-^{ACX} = cov\left(r_i r_m | r_m < 0\right) / var\left(r_m | r_m < 0\right)$ and $\beta_+^{ACX} = cov\left(r_i r_m | r_m \geq 0\right) / var\left(r_m | r_m \geq 0\right)$.

$\beta_+^{ACX} = cov\left(r_i r_m | r_m \geq 0\right) / var\left(r_m | r_m \geq 0\right)$.

This notation reveals a simple but interesting decomposition of the realized beta: as the sum of upside β_+ and downside β_- betas, with appropriately chosen weights:

$$\beta = \omega_+\beta_+ + \omega_-\beta_-$$

$$\omega_+ = \frac{\sum_{j=1,\ldots,[1/\Delta]} r_{m,t_j}^2 \mathbf{I}_{[r_{m,t_j}>0]}}{\sum_{j=1,\ldots,[1/\Delta]} r_{m,t_j}^2}$$

$$\omega_- = \frac{\sum_{j=1,\ldots,[1/\Delta]} r_{m,t_j}^2 \mathbf{I}_{[r_{m,t_j}<0]}}{\sum_{j=1,\ldots,[1/\Delta]} r_{m,t_j}^2}$$

where it follows directly that $\omega_+ + \omega_- = 1$. The decomposition of betas into market news is a natural one because of the long tradition of downside betas in the literature, but other decompositions can be computed, as long as the conditioning events are disjoints. For example, betas could be decomposed according to industry news, country news and according to the size of news.

As Chapter 2 revealed within an imperfect information model, signs of returns can contain additional information about the underlying riskiness of an asset. The sign on market return may be the most informative, but firms specific news can also contain relevant information about the systematic risk of the stock. In order to shed new light on the importance of the signs of returns to systematic risk, we take the above decomposition one step further. We decompose downside and upside betas according to firm specific news. This would give us a four-fold decomposition, each resulting from all the possible combinations of up and down markets returns with up and down stocks returns:

$$\beta = \omega_1\beta_+^+ + \omega_2\beta_+^- + \omega_3\beta_-^+ + \omega_4\beta_-^- \tag{4.1}$$

where

$$\beta_+^+ = \frac{\sum_{j=1,\ldots,[1/\Delta]} r_{m,t_j} r_{i,t_j} \mathbf{I}_{\left[r_{i,t_j}>0, r_{m,t_j}>0\right]}}{\sum_{j=1,\ldots,[1/\Delta]} r_{m,t_j}^2 \mathbf{I}_{\left[r_{i,t_j}>0, r_{m,t_j}>0\right]}}$$

$$\omega_1 = \frac{\sum_{j=1,\ldots,[1/\Delta]} r_{m,t_j}^2 \mathbf{I}_{\left[r_{i,t_j}>0, r_{m,t_j}>0\right]}}{\sum_{j=1,\ldots,[1/\Delta]} r_{m,t_j}^2}$$

and likewise for the other betas β_-^+, β_+^- and β_-^- and weights ω_2, ω_3 and ω_4. As before, $1 = \sum_{i=1}^4 \omega_i$. We will refer β_+^+ as the beta on PP-markets (positive asset return and positive market return), and analogously for the other cases. This four-fold decomposition includes as a special case the downside and upside betas of ACX and Bawa and Lindenberg (1977), as, for instance, the downside beta can be recovered from it with appropriate weights, $\beta_- = (\omega_3/\omega_-)\,\beta_-^+ + (\omega_4/\omega_-)\,\beta_-^-$.

4.3 Empirical Results

For this empirical study, we use all the common stocks (share-codes 10 and 11) available on the Center for Research in Security Prices (CRSP) dataset that are, or were, traded on the NYSE, Amex and Nasdaq stock markets during the years of 1963 and 2009. We use log-excess returns at the daily frequency.

Table 4.1 shows some descriptive statistics of the four-fold decomposition (4.1). The averages across all assets are aggregated in two different ways, one with equal-weights and windsorizing, and the other with weights based on the market size of the firm. Only stocks with more than 150 trading days in the year were included. Consider the panel with the value-weighted statistics. The first column shows that, on average, a stock has 221 valid observations per year and, of those, in 80 the stock and market were up, in 32 the stock was up and the market down, in 40 the stock was down and the market up, and in 69 both the market and stock were down. Despite the PP case is the most frequent, the NN case is given higher weights, $\omega_4 = 39\%$ while $\omega_1 = 37\%$, a result of down markets being more volatile. However, betas are higher during PP markets, $\beta_+^+ = 1.85$ versus $\beta_-^- = 1.66$, and the overall contribution of PP market is actually higher, $\omega_1\beta_+^+ = 0.67$ versus $\omega_4\beta_-^- = 0.62$. The last column of the table shows the cumulative returns on conditional on each event.

Figure 4.1 and 4.2 show the evolution of such statistics over the years. First, we observe an increase in the dispersion of the decomposed log-excess returns; the gap on the negative returns during the NN and NP markets versus PP and PN markets has being widened. Second, the four decomposed betas, in absolute value, have been decreasing over past 20 years. Third, the PN markets are the least likely over the years, followed by NP, NN and PP markets. Fourth, among the cross-signed betas, the NP beta contributes slightly more than the PN beta.

We now turn to the question of how each of these signed betas are related to the dispersion of stock returns. To do so, we follow the approach suggested by ACX. In the paper, the authors investigate if a downside beta is priced on the cross-section of returns. They define downside beta as $\beta_-^{ACX} = cov\left(r_i r_m | r_m < 0\right)/var\left(r_m | r_m < 0\right)$ and compute them from 12 months of rolling-windows with daily data. They proceed by sorting stocks into portfolios according to their downside betas. The authors find that portfolios with highest downside betas earn on average 6% per year more than portfolios with lowest downside betas. Because the procedure requires computing returns contemporaneously to the calculation of betas, the moments have to be centralized in order to avoid implicitly selecting stocks with lower returns. In other to avoid introducing such biases in the sorting, we centralize betas by computing them from centralized sample moments: $\beta_-^- = cov\left(r_i r_m | r_m < 0, r_i < 0\right)/var\left(r_m | r_m < 0, r_i < 0\right)$, and likewise for other betas in the decomposition. Note that now these centered betas can take positive and negative values on every quadrant.

Another possible approach, which is not pursued here, is suggested by Post, Van Vliet, and Lansdorp (2009). The authors investigate the (uncentered) version of downside beta suggested by Bawa and Lindenberg (1977), $\beta_-^{BL} = E\left(r_i r_m | r_m < 0\right)/E\left(r_m^2 | r_m < 0\right)$. They also construct portfolios by sorting stocks according to their downside betas, but instead of computing returns contemporaneously, returns are calculated one period ahead of the sorting, so no overlapping occurs.

Table 4.2 shows the returns of the portfolios following the sorting procedure adopted by ACX. We observe the following. First, we find that the sorting of stocks by downside beta, β_-, shows a significant premium. The ex-post average return on the portfolio that contains the stocks with highest downside betas is 9.5% per year, while that containing stocks with the lowest downside beta is 3.0%. This results in a premium of 6.6% per year, which is also statistically significant, with a heteroskedasticity and autocorrelation consistent (HAC) t-statistic of 1.94. This premium is similar to the one found by ACX, of 6%. In contrast, the sorting by the upside beta, β_{+}, does not result in a significant premium.

Second, despite the insignificant premium on the sorting by upside beta, the four-fold decomposition reveals that a positive premium arises on β_{+}^{-}, the NP-markets beta, when the stock is negative but the market is positive. Stocks with high β_{+}^{-} earn, on average, 3.2% more than stocks with low β_{+}^{-}. This premium is also statistically significant, with a HAC t-statistic of 3.24.

Third, both betas based on downside markets, β_{-}^{+} and β_{-}^{-}, show a statistically significant premia. Stocks with highest β_{-}^{+} earn on average 4.8% more, while stocks with highest β_{-}^{-} earn on average 6.8% more than stocks with lowest betas.

We observed similar results when we used a reduced sample, that included only NYSE stocks, when we weighted stocks equally, and when we extended the sample period to the years between 1927 and 2009. These result are not reported.

Since we have found a significant premium across the sortings, a natural question that arises is whether these betas are in fact capturing the variation on other sources of risk. In order to answer this question, we run the two-step Fama and MacBeth (1973) regressions of the cross-section returns on such betas and several other variables associated to various elements of risk. For this analysis, we restrict the sample to stocks traded on the NYSE only, although we perform one regression with our entire universe. The results that we show in Table 4.3 reveal the following. First, regression IV shows that the premium for downside beta, β_{-}, remains significant and large even after controlling for other characteristics usually associated with risk. These results confirm the main finding by ACX, and extend the evidence to the sample period between 1963 and 2009.

Second, when we regress the cross-section of returns on the sign decomposed betas, the estimated coefficients confirm what we observed on the sorting of stocks into portfolios. The

PP-markets betas, β_+^+, shows no premium but all the other three betas do show positive and significant premia. The highest coefficient is on the NN-markets beta, β_-^-, with a point estimate of 0.052. The other two significant coefficients are 0.011, for the β_+^- , and 0.014, for the β_-^+.

Third, when we control for the variables that contain relevant information about risk, and that is not captured by the symmetric market beta, such as log-size, book-to-market and part-returns, the coefficients of the decomposed betas remain significant (regressions VI through VIII). However, when we include the cokurtosis and coskewness variables, that were recently suggested by Harvey and Siddique (2000) and Dittmar (2002), the coefficients on the downside betas, β_-^- and β_-^+, turn insignificant. Surprisingly, the coefficient on the negative asset return and positive market return, β_+^-, remains significant and with a value of 0.013, consistent throughout the regressions (regressions IX through XII). The significance of this coefficient still remains when we expand the Fama-Macbeth regressions to our entire sample, which includes stocks traded on the NYSE, Nasdaq and Amex markets (regression XII).

Following the surprising results about the significance of the NP-markets betas, β_+^-, even after controlling for the common sources of risk usually considered by the literature, we further investigate which stock and firms characteristics may be associated with it. In order to do so, we run again several Fama-Macbeth regressions, but now the cross-section of β_+^- as the regressand. To control for industry-specific characteristics, we include 49 industry dummies in the regressions but do not report the estimated coefficients. We show the results in Table 4.4. First, we note that β_+^- is positively related to the standard deviation of returns, the cokurtorsis of returns with the market, the size of the firm; and negatively related to dividends.

Second, NP-markets beta, β_+^-, shows almost no persistence, as the coefficient on its lagged value is close to zero, and so could not be used as a stable predictor of future risk.

Third, the coefficients on the firm accounting ratios are not generally significant, and so do not show any clear pattern relating them to β_+^-. Only marginally, we could argue a positive relation of β_+^- with the leverage ratio Total Assets / Common Equity (TACE).

As a final exercise, we sort all stocks into deciles according to β_+^-, form value-weighted portfolios, and compute several characteristics of the resulting portfolios. The numbers calculated for each decile are shown in Table 4.5. We observe that β_+^- is non-trivially related to the firm and stock characteristics. Some of the patterns that emerge across deciles and the variables conforming to these are:

- Increasing from lowest to highest deciles: coskewness, cokurtosis and past returns;
- Higher on the mid-deciles and lower on extremities: dividend yield, size, price of the stock, activity ratio (inventory turnover (IT)), profitability ratios (return on equity (ROE); net profit margin (NPM)), leverage ratios (Interest Coverage Before Tax (ICBT); Long-Term Debt/Shareholder's Equity (LTBSE); Total Assets/Common Equity (TACE));
- Lower on mid-deciles and higher on extremities: standard deviation, default risk, volume, turnover, book-to-market ratio, activity ratio (Total Asset Turnover (TAT)), performance ratio (Sales (Net)/ Stockholder's Equity (SSE)), liquidity ratios (Current Ratio (CR); Quick Ratio (QR)) and default ratio (Total Debt/Total Capital (TBTC)).

As the various non-linear patterns and different variables suggest, it is not clear how we can relate β_{+}^{-} to firms and stocks characteristics.

4.4 Conclusion

We have shown in Chapter 2 *why* the signs of returns are important to betas. Because of the properties of the learning process discussed in that model, the average negative news increases investor uncertainty, while the average positive news decreases investor uncertainty. As a result, betas, which are (approximately) monotonic functions of uncertainty, are also asymmetrically related to positive and negative news. The direction of such asymmetry is determined by the asset's risk characteristics.

In this chapter, we took an exploratory approach and investigated *how* the signs of returns are important to betas.

The main contributions of this chapter are the following. First, we propose a decomposition of betas motivated by the definition of realized betas that resulted into four signed betas, one for each possible combination of positive and negative market and stock returns. We show that the four betas exactly add up to the full, symmetric beta, by using the appropriate weights. Using data on common stocks traded on the NYSE, Amex and Nasdaq, during the years 1963 and 2009, we observe that the same signed betas contributed the most for the overall betas. On average, the contributions are: 0.62 from the β^-_-, 0.67 from the β^+_+, -0.12 from the β^+_-, and -0.15 from the β^+_-.

Second, we analyzed how each of the four betas relate to the cross-section risk premium. Following the same procedure adopted by ACX, we find that cross signed betas and the downside betas, β^-_+,β^+_- and β^-_-, are all associated with higher returns in the cross-section of stocks. Furthermore, we find that the two betas related to downside markets, β^+_- and β^-_-, are not relevant when controlling for coskewness and cokurtosis. However, the higher returns associated with β^-_+, the beta when the stock is down and market is up, remain positive and significant after controlling for the sources of risk commonly studied in the literature.

While we cannot conclude, based solely on the results presented here, that β^-_+ actually captures exposures to a common factor of risk, we do take this as another evidence that asymmetries are an important feature of systematic risk.

4.5 Graphs and Tables

Table 4.1: Beta Decomposition – Descriptive Statistics

Realized betas are computed from daily log-excess returns of common stocks on the CRSP dataset, from 1963 to 2010. Only stocks with more than 150 trading days on a year are considered. The Table shows two aggregations across stocks, an equally-weighted (windsorized at the 5% and 95% levels) and a value-weighted by the market size of the firm. Weights, ω_i, betas, β, contributions, $\omega_i\beta$, are shown for all 4 cases: PP, $[r_i > 0, r_m > 0]$, PN, $[r_i > 0, r_m < 0]$, NP, $[r_i < 0, r_m > 0]$, and NN, $[r_i < 0, r_m < 0]$. Rets are the accumulated log-excess returns over the year conditional on each case.

	Equally Weighted					Value Weighted				
Case	**Days**	**Weights**	**Contr.**	**Betas**	**Rets.**	**Days**	**Weights**	**Contr.**	**Betas**	**Rets.**
ALL	169	74.96%		0.76	-0.063	221	92.65%		1.01	0.081
PP	53	24.91%	0.76	3.75	1.618	80	37.37%	0.67	1.85	1.335
PN	30	9.58%	-0.32	-3.76	0.865	32	7.49%	-0.12	-1.70	0.402
NP	39	12.22%	-0.41	-3.59	-1.082	40	8.81%	-0.15	-1.63	-0.483
NN	48	28.25%	0.74	3.23	-1.449	69	38.96%	0.62	1.66	-1.168

Table 4.2: Cross Section Sorting – NYSE, Amex and Nasdaq (1963-2009)

Stocks on CRSP, traded on NYSE, Amex and Nasdaq, are sorted every month based on (overlapping) 12-month-estimates of risk measures. Then, quintiles-portfolios are formed and value weighted returns calculated contemporaneously. Only common shares with more than 150 valid observations (i.e. days with trading activity on the 12-month span) are included. T-stats are based on HAC std. errors with 12 lags.

Beta	Quintile	Mean	Median	Std. Dev.	Skew.	Kurt.	t-stat
β	**Low**	0.040	0.038	0.127	0.365	1.293	
	Qnt 2	0.050	0.060	0.124	-0.035	0.125	
	Qnt 3	0.044	0.057	0.137	-0.231	0.540	
	Qnt 4	0.052	0.078	0.173	-0.531	0.163	
	High	0.070	0.114	0.280	-0.384	-0.178	
	High-Low	0.030	0.042	0.254	-0.046	0.826	0.940
β_+	**Low**	0.028	0.042	0.173	-0.146	1.630	
	Qnt 2	0.051	0.059	0.133	-0.088	0.717	
	Qnt 3	0.054	0.067	0.137	-0.208	0.330	
	Qnt 4	0.049	0.069	0.161	-0.411	0.043	
	High	0.047	0.091	0.252	-0.483	-0.185	
	High-Low	0.019	0.020	0.207	-0.199	1.325	0.780
β_-	**Low**	0.030	0.032	0.131	0.617	2.368	
	Qnt 2	0.039	0.051	0.122	-0.154	0.108	
	Qnt 3	0.057	0.078	0.154	-0.291	0.133	
	Qnt 4	0.074	0.104	0.200	-0.358	0.025	
	High	0.095	0.124	0.308	-0.212	-0.001	
	High-Low	0.066	0.061	0.276	0.188	1.756	1.940
β_+^+	**Low**	0.049	0.055	0.165	0.564	1.712	
	Qnt 2	0.049	0.057	0.133	-0.046	0.844	
	Qnt 3	0.050	0.061	0.140	-0.174	0.275	
	Qnt 4	0.054	0.077	0.166	-0.414	0.030	
	High	0.048	0.094	0.253	-0.520	-0.102	
	High-Low	-0.001	0.011	0.192	-0.207	1.751	-0.060
β_+^-	**Low**	0.023	0.051	0.191	-0.529	0.530	
	Qnt 2	0.052	0.070	0.154	-0.486	0.369	
	Qnt 3	0.057	0.069	0.151	-0.217	0.251	
	Qnt 4	0.060	0.080	0.159	-0.323	-0.073	
	High	0.056	0.073	0.210	-0.320	-0.018	
	High-Low	0.032	0.028	0.097	0.206	1.442	3.240
β_-^+	**Low**	0.022	0.050	0.198	-0.490	0.153	
	Qnt 2	0.045	0.062	0.147	-0.335	0.136	
	Qnt 3	0.059	0.078	0.149	-0.316	0.145	
	Qnt 4	0.070	0.092	0.172	-0.269	0.067	
	High	0.070	0.087	0.248	-0.148	0.289	
	High-Low	0.048	0.027	0.126	1.208	2.995	3.300
β_-^-	**Low**	0.009	0.008	0.134	0.565	1.498	
	Qnt 2	0.045	0.057	0.128	-0.084	0.133	
	Qnt 3	0.055	0.072	0.151	-0.319	0.353	
	Qnt 4	0.068	0.097	0.189	-0.399	0.099	
	High	0.077	0.108	0.290	-0.300	0.057	
	High-Low	0.068	0.069	0.249	0.227	1.687	2.240

Table 4.3: Fama-Macbeth Regressions – NYSE (1963-2009)

This Table shows the Fama-Macbeth 2-step regression results where the dependent variable is the 12-month compounded excess returns. Independent variables are also computed over a 12-month period. The regressions are performed monthly. Betas, coskewness, cokurtosis and standard deviation are contemporaneous to the dependent variable. Log-size, book-to-market ratio and past returns correspond to the previous year values. Standard errors are HAC with 12 lags. T-statistics are in brackets. Only stocks with more than 150 valid (i.e. days with transaction) in the 12-month span are included. Regression XII also includes stocks in the the Nasdaq and Amex markets.

Model	I	II	III	IV	V	VI	VII	VIII	IX	XI	XII
Intercept	0.038	0.529	0.040	0.469	0.059	0.282	0.190	0.186	0.200	0.430	0.596
	[2.28]	[6.86]	[2.42]	[6.46]	[3.54]	[3.78]	[3.05]	[3.03]	[3.28]	[6.07]	[6.53]
β	0.056	0.140									
	[2.75]	[5.07]									
β_+			-0.015	0.011							
			[-2.22]	[1.19]							
β_-			0.061	0.064							
			[4.80]	[4.91]							
β_+^+					-0.009	0.000	0.007	0.007	0.019	0.018	0.008
					[-1.09]	[-0.04]	[0.92]	[0.95]	[1.84]	[2.10]	[0.73]
β_+^-					0.014	0.015	0.015	0.015	0.021	0.013	0.020
					[3.80]	[4.06]	[3.83]	[3.89]	[5.23]	[3.57]	[2.08]
β_-^+					0.011	0.011	0.012	0.010	0.006	0.002	0.007
					[2.98]	[2.79]	[2.61]	[2.50]	[1.52]	[0.60]	[1.20]
β_-^-					0.052	0.051	0.053	0.049	0.021	0.014	0.004
					[3.97]	[3.96]	[4.54]	[4.28]	[1.78]	[1.57]	[0.39]
Log-Size		-0.036		-0.033		-0.018	-0.013	-0.013	-0.015	-0.033	-0.055
		[-6.88]		[-6.53]		[-3.54]	[-3.06]	[-3.13]	[-3.71]	[-6.48]	[-8.31]
BEME		0.023		0.025			0.028	0.030	0.028	0.024	0.025
		[3.49]		[3.91]			[4.32]	[4.42]	[4.20]	[3.66]	[5.40]
Past Ret.		-0.010		-0.010				0.011	0.011	-0.003	-0.001
		[-0.72]		[-0.69]				[0.63]	[0.62]	[-0.22]	[-0.03]
Coskew.		-0.174		-0.090					-0.164	-0.199	-0.191
		[-5.97]		[-1.96]					[-4.08]	[-4.57]	[-4.80]
Cokurt.		0.018		0.042						0.071	0.138
		[1.89]		[4.14]						[5.99]	[7.54]
Std. Dev.		-6.347		-4.541						-2.753	0.122
		[-6.34]		[-4.64]						[-2.79]	[0.07]
Adj. R2	0.048	0.148	0.053	0.142	0.048	0.072	0.084	0.100	0.127	0.143	0.146

Table 4.4: Determinants of β_{+}^{-} – NYSE (1963-2009)

This Table shows the results of Fama-Macbeth regressions where the dependent variable is the cross-signed beta $\beta_{+,t}^{-}$. The independent variables are lagged financial or market variables. All regressions include 49 industry dummies. Standard errors are HAC with 12 lags. T-statistics are in brackets. Only stocks with more than 150 valid observations (i.e. days with transaction) in the 12-month span are included. The financial ratios are the following: Activity: Inventory Turnover (IT), Total Asset Turnover (TAT); Performance: Sales (Net) to Stockholders Equity (SSE); Liquidity: Current Ratio (CR), Quick Ratio (QR); Profitability: Return on Equity (ROE); Leverage and Default: Interest Coverage Before Tax (ICBT), Long-Term Debt/Shareholders Equity (LTBSE), Total Debt/Total Capital (TBTC), Total Assets/Common Equity (TACE).

Model	I	II	III	IV	V	VI	VII	VIII	IX	X
$\beta_{+,t-1}^{-}$	0.007	-0.005	0.007	0.042	0.023	0.076	0.008	-0.009	-0.020	0.015
	[1.76]	[-0.32]	[0.93]	[1.33]	[1.80]	[1.12]	[1.50]	[-0.52]	[-0.69]	[2.26]
Std. Dev.	1.266	1.078	1.532	1.552	2.173	2.079	1.441	1.630	1.165	1.402
	[4.31]	[3.02]	[5.00]	[3.81]	[2.73]	[2.99]	[4.31]	[3.71]	[3.57]	[4.42]
Log-Size	0.008	0.004	0.007	0.010	0.017	0.013	0.008	0.009	0.005	0.007
	[3.11]	[0.95]	[2.54]	[3.49]	[1.81]	[2.51]	[3.26]	[3.20]	[1.11]	[2.40]
Returns	0.001									
	[0.10]									
Coskewness	0.014	0.005	-0.007	-0.075	-0.104	-0.131	0.011	0.019	-0.004	-0.028
	[1.30]	[0.34]	[-0.25]	[-0.93]	[-1.17]	[-1.01]	[0.44]	[1.00]	[-0.15]	[-0.78]
Cokurtosis	0.021	0.021	0.018	0.011	0.009	0.004	0.019	0.023	0.014	0.018
	[6.51]	[5.43]	[4.22]	[1.01]	[1.01]	[0.23]	[4.83]	[5.91]	[2.54]	[3.59]
Dividends		-0.660	-0.320	-0.596	-2.228	-0.563	-0.470	-0.348	-0.263	-0.312
		[-2.30]	[-1.70]	[-2.07]	[-1.14]	[-1.85]	[-2.11]	[-2.85]	[-0.72]	[-2.00]
Book-to-Market		-2.465								
		[-0.57]								
IT			-0.001							
			[-0.28]							
TAT			0.009							
			[0.99]							
SSE				0.002						
				[1.38]						
CR					0.019					
					[0.83]					
QR					-0.017					
					[-0.75]					
ROE						-0.023				
						[-1.39]				
ICBT							0.000			
							[0.65]			
LTBSE								0.000		
								[-0.30]		
TBTC									-0.002	
									[-1.27]	
TACE										0.001
										[1.82]
Avg. R^2	0.096	0.091	0.103	0.099	0.103	0.102	0.101	0.096	0.117	0.102

Table 4.5: Sorting Stocks by β_{+}^{-} – NYSE, AMEX and Nasdaq (1963-2009)

The Table shows the characteristics of portfolios sorted by β_{+}^{-}. Only stocks with more than 150 days with transactions in the rolling 12-month period are included. Default Risk Index is obtained from Maria Vassalou's website, and spans from 1971 and 1999. The financial ratios are the following: activity: Inventory Turnover (IT), Total Asset Turnover (TAT); Performance: Sales (Net) to Stockholders Equity (SSE); Liquidity: Current Ratio (CR), Quick Ratio (QR); Profitability: Return on Equity (ROE), Net Profit Margin (NPM); Leverage and Default: Interest Coverage Before Tax (ICBT), Long-Term Debt/Shareholders Equity (LTBSE), Total Debt/Total Capital (TBTC), Total Assets/Common Equity (TACE).

	Performance									**Measures of Risk**							
		Returns		β	β_{+}	β_{-}	β_{+}^{+}	β_{+}^{-}	β_{-}^{+}	β_{-}^{-}	**MES**	σ	σ_{+}	σ_{-}	**Cosk.**	**Cokur.**	**D. Risk.**
Dec.	**t-12**	**t+1**	**t+12**	**t-12**	**t-12**	**t-12**	**t-12**	**t-12**	**t-12**	**t-12**	**t-12**	**t-12**	**t-12**	**t-12**	**t-12**	**t-12**	t
1	6.20%	1.07%	13.16%	0.77	0.20	1.05	1.13	-1.27	0.19	0.70	-0.05	0.05	0.03	0.03	-0.13	1.26	0.12
2	10.96%	1.48%	11.36%	0.75	0.47	0.94	0.48	-0.44	0.11	0.62	-0.05	0.03	0.02	0.02	-0.13	1.53	0.06
3	11.61%	1.54%	10.74%	0.72	0.51	0.87	0.44	-0.21	0.14	0.57	-0.05	0.03	0.02	0.02	-0.13	1.64	0.05
4	11.90%	1.25%	10.28%	0.70	0.54	0.83	0.45	-0.07	0.13	0.54	-0.05	0.03	0.02	0.02	-0.13	1.71	0.04
5	12.18%	1.23%	10.12%	0.71	0.59	0.82	0.41	0.04	0.14	0.54	-0.05	0.03	0.02	0.02	-0.13	1.78	0.05
6	12.61%	1.27%	10.03%	0.75	0.67	0.86	0.41	0.14	0.20	0.56	-0.04	0.03	0.02	0.02	-0.12	1.87	0.05
7	12.86%	1.22%	9.98%	0.82	0.77	0.93	0.46	0.25	0.20	0.60	-0.04	0.03	0.02	0.02	-0.11	1.96	0.03
8	13.21%	1.22%	9.94%	0.92	0.91	1.02	0.50	0.39	0.19	0.66	-0.04	0.03	0.02	0.02	-0.10	2.05	0.04
9	13.32%	1.28%	10.32%	1.04	1.10	1.14	0.55	0.59	0.22	0.73	-0.04	0.03	0.02	0.02	-0.09	2.08	0.06
10	11.35%	0.45%	11.11%	1.23	1.57	1.27	0.86	1.23	0.63	0.84	-0.04	0.05	0.03	0.03	-0.05	1.91	0.14

	Trade Characteristics			**Performance Charac.**								**Financial Ratios**					
	P	**Vol.**	**Turn.**	**DY**	**Size**	**BEME**	**IT**	**TAT**	**SSE**	**CR**	**QR**	**ROE**	**NOM**	**ICBT**	**LTBSE**	**TBTC**	**TACE**
	t	**t-12**	**t-12**	**t-12**	**t**	**t**	**t**	**t**	**t**	**t**	**t**	**t**	**t**	**t**	**t**	**t**	**t**
1	14.8	213,111	4.70	1.32%	519	1.00	3.83	0.39	2.94	2.82	2.28	-0.09	-0.43	3.29	56.08	3.02	2.91
2	21.5	168,610	3.47	2.01%	791	0.86	4.04	0.38	2.86	2.74	2.15	0.00	-0.21	9.19	56.68	2.35	3.33
3	22.9	162,720	3.09	2.38%	977	0.82	4.19	0.36	2.73	2.66	2.05	0.04	-0.15	12.28	58.44	2.28	3.61
4	27.3	164,752	2.93	2.57%	1,147	0.80	4.30	0.35	2.65	2.62	2.01	0.05	-0.11	13.93	59.18	2.09	3.75
5	34.0	175,417	2.83	2.64%	1,257	0.80	4.41	0.35	2.60	2.58	1.95	0.06	-0.09	13.62	59.19	2.10	3.84
6	38.2	180,651	2.86	2.60%	1,330	0.79	4.53	0.35	2.64	2.58	2.00	0.06	-0.09	13.36	59.78	2.19	3.86
7	33.4	193,228	3.05	2.40%	1,330	0.79	4.21	0.36	2.71	2.62	1.95	0.06	-0.11	13.89	57.43	2.28	3.72
8	24.8	205,521	3.37	2.14%	1,261	0.76	4.09	0.37	2.76	2.68	2.02	0.03	-0.15	11.93	57.77	2.61	3.51
9	21.3	208,825	3.86	1.74%	1,077	0.77	3.86	0.38	2.80	2.80	2.17	0.02	-0.23	9.22	54.63	2.85	3.18
10	16.5	226,435	5.09	1.09%	694	0.92	3.82	0.38	2.95	2.88	2.31	-0.09	-0.42	1.19	57.56	3.33	2.97

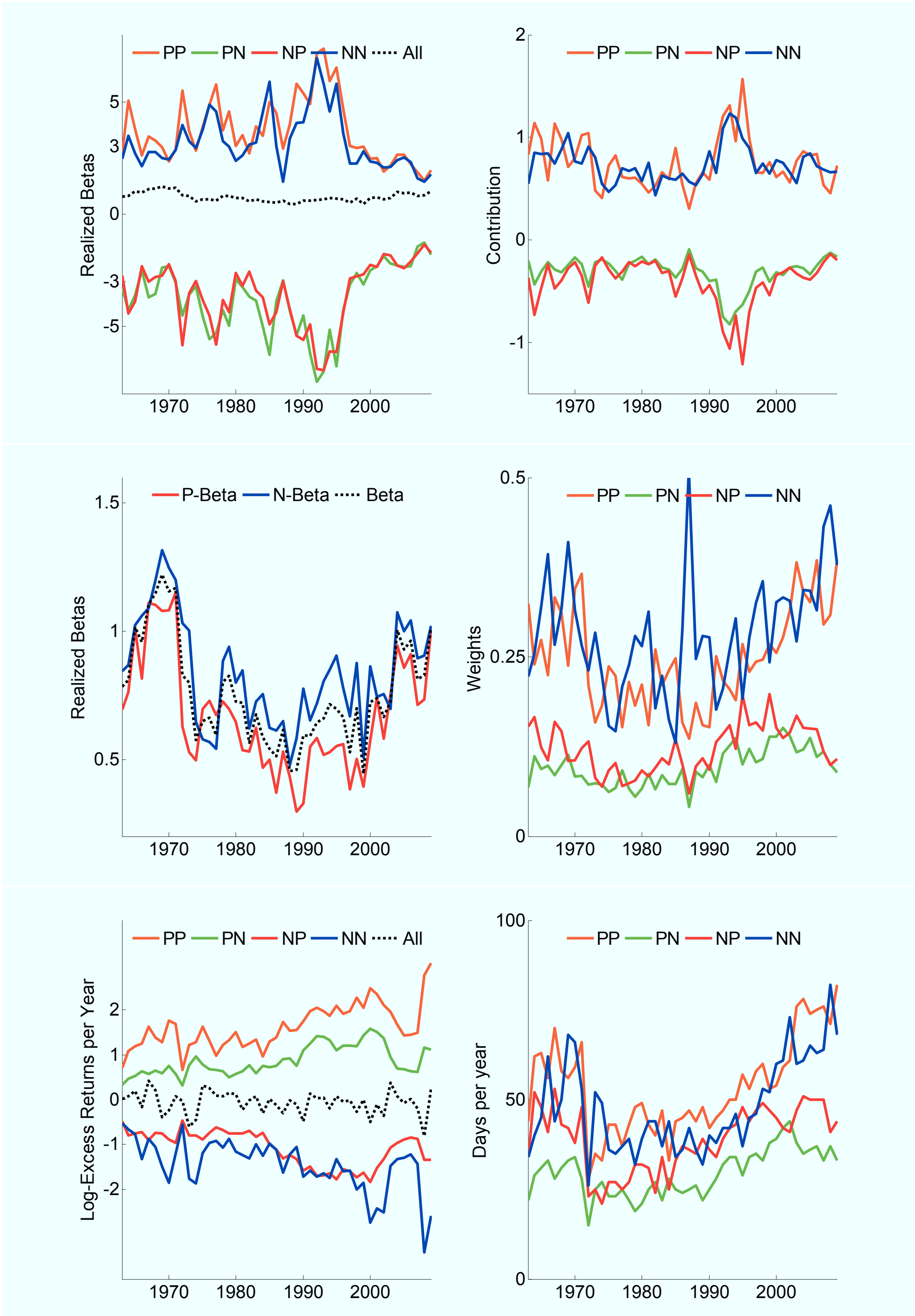

Figure 4.1: Four-Fold decomposition – Equally-Weighted

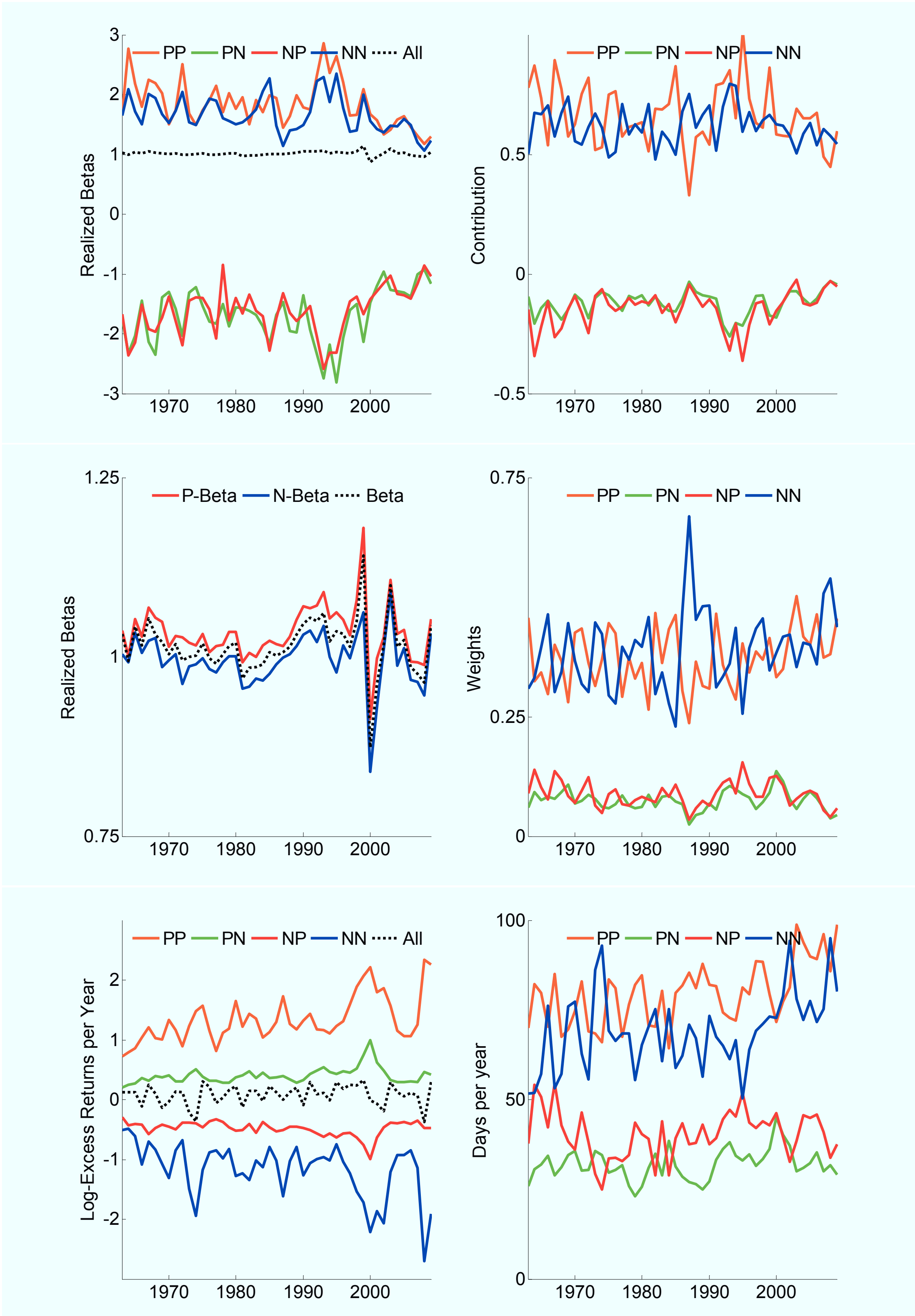

Figure 4.2: Four-Fold Decomposition – Value-Weighted

Chapter 5

Appendix to Chapter 2

In this appendix we solve the investor problem and derive the asset pricing equations. This model was also derived by Ribeiro and Veronesi (2002). The problem of the representative investor has two parts. In the first part, the investor optimally infer the conditional means of the cash-flow processes. In the second part, the investor maximize the utility function subject to the intertemporal budget constraint, with choice variables consumption, $\{c_t\}$, and demand for assets, $\{X_t\}$, $X_t = [x_{1t} \dots x_{nt}]'$. The maximization is solved using the Bellman-Hamilton-Jacobi equation with two state variables, wealth, W_t, and the belief π_t.

Recall the assumptions about the available assets in this economy. There are n risky assets in this economy that pay a continuous stream of cash-flows: $dD_t = \theta_t dt + \Phi d\xi_t$. The random vector θ_t, is not observed by the investor, who only knows the values it can take, $[\theta_G, \theta_B]$, and that it follows a 2 state Markov process with the following infinitesimal transition matrix:

$$M = \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$$

with $\lambda = Prob\left(\theta_{t+dt} = \theta_B | \theta_t = \theta_G\right)$ and $\mu = Prob\left(\theta_{t+dt} = \theta_G | \theta_t = \theta_B\right)$. The lemma (1) shows that the investor's optimal beliefs about the state of the economy conditional on $\mathcal{F}_t = \sigma\left(D_\tau, \tau < t\right)$ can be represented by the following stochastic differential equation:

$$d\pi_t = (\lambda + \mu)\left(\pi_s - \pi_t\right) dt + \pi_t \left(1 - \pi_t\right) \Delta\theta' \Phi'^{-1} dv_t$$

Under this incomplete information set, $\mathcal{F}_t$, cash-flows can be written as $dD_t = \alpha_{Dt}dt + \Phi d\mathbf{v}_t$, where $\alpha_{Dt} = [\alpha_{1D,t}, ..., \alpha_{nD,t}]'$ and $\alpha_{iD,t} \equiv \theta_{iG}\pi_t + \theta_{iB}(1-\pi_t)$.

With the optimal beliefs already defined, we now turn to the utility maximization problem. First, since the risk free is inelastically supplied, the budget constraint is given by:

$$\begin{aligned} dW_t &= X_t'\left(dP_t + D_t dt\right) + \left(W_t - X_t'P_t\right) r dt - c_t dt \\ &= X_t'\left(dP_t + D_t dt - rP_t dt\right) + \left(W_t r - c_t\right) dt \end{aligned}$$

where $X_t = [x_{1t} \ldots x_{nt}]'$ are the demand for asset shares and $P_t = [P_{1t} \ldots P_{nt}]$ the asset prices.

As in Veronesi (1999), first conjecture a functional form for prices and then find are parameters that solve the problem. The conjectured form is linear in D_t but possibly non-linear in π_t, through the function S_i:

$$P_{it} = p_{i0} + p_{i\pi}\pi_t + p_{iD}D_{it} + p_{i1} + S_i\left(\pi_t\right)$$

and by Ito's lemma we obtain:

$$dP_{it} = \alpha_{ip}dt + \sigma_{ip}dv_t \quad i = 1, ..., n$$

$$\begin{aligned} \alpha_{ip} &= \left(p_{i\pi} + S_i'\left(\pi_t\right)\right)\alpha_\pi + p_{iD}m_{it} + \frac{1}{2}S_i''\left(\pi_t\right)h\left(\pi_t\right)^2 H \\ \sigma_{ip} &= h\left(\pi_t\right)\left(p_{i\pi} + S_i'\left(\pi_t\right)\right)\Delta\theta'\Phi'^{-1} + p_{iD}\sigma_i \end{aligned}$$

with the simplifying notation $\alpha_\pi \equiv (\lambda + \mu)\left(\pi_s - \pi_t\right)$, $h\left(\pi_t\right) \equiv \pi_t\left(1 - \pi_t\right)$ and $H \equiv \Delta\theta'\Sigma^{-1}\Delta\theta$. Furthermore, denote the vector of price changes by: $dP_t = \alpha_p dt + \Phi_p dv_t$, where $\alpha_p = [\alpha_{1p}, ..., \alpha_{np}]'$ and Φ_p is a $n \times n$ matrix that stacks the row vectors σ_{ip}, and $\Sigma_p = \Phi_p\Phi_p'$. Substitute the conjecture prices into the budget constraint to obtain:

$$dW_t = \left[X_t'\left(\alpha_p + D_t - rP_t\right) + W_t r - c_t\right]dt + X_t'\Phi_p dv_t$$

Risk Neutral Prices

The parameters $p_0 = [p_{10}, ..., p_{n0}]'$, $p_\pi = [p_{1\pi}, ..., p_{n\pi}]'$ and $p_D = [p_{1D}, ..., p_{nD}]'$ are found by solving for risk neutral prices, $P_{i,t}^{RN}$:

$$P_{i,t}^{RN} \equiv E_t\left[\int_0^\infty e^{-rs} D_{i,t+s} ds\right] = \int_0^\infty e^{-rs} E_t\left[D_{i,t+s}\right] ds$$

where the equality follows from Foubini's theorem. Since, $D_{i,t+s} = D_{it} + \int_0^s \alpha_{iD,t+\tau} d\tau + \sigma_i \left(v_{t+s} - v_t\right)$, the only conditional expectation that matters is $\int_0^s E_t\left[\alpha_{iD,t+\tau}\right] d\tau$. For this, we need the eigendecomposition of the infinitesimal transition matrix M to compute the transition matrix over τ periods. The eigenvalues of M are 0 and $-(\lambda + \mu)$ with corresponding eigenvectors $[1\,1]'$ and $\left[-1\,\frac{\mu}{\lambda}\right]'$. The transition matrix over τ is:

$$\begin{aligned} P(\tau) &= \begin{bmatrix} 1 & -1 \\ 1 & \frac{\mu}{\lambda} \end{bmatrix} \begin{bmatrix} e^{0\tau} & 0 \\ 0 & e^{-(\lambda+\mu)\tau} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & \frac{\mu}{\lambda} \end{bmatrix}^{-1} \\ &= \frac{1}{(\lambda + \mu)} \begin{bmatrix} \mu + \lambda e^{-(\lambda+\mu)\tau} & \lambda - \lambda e^{-(\lambda+\mu)\tau} \\ \mu - \mu e^{-(\lambda+\mu)\tau} & \lambda + \mu e^{-(\lambda+\mu)\tau} \end{bmatrix} \end{aligned}$$

and so $E_t\left[\alpha_{iD,t+\tau}\right] = \left[\pi_t \, 1 - \pi_t\right] P(\tau) \left[\theta_{iG}\, \theta_{iB}\right]' = \theta_{is} + \Delta\theta_i \left(\pi_t - \pi_s\right) e^{-(\lambda+\mu)\tau}$, where $\pi_s = \mu / (\mu + \lambda)$ and $\theta_{is} = \theta_{iG}\pi_s + \theta_{iB}(1 - \pi_s)$. Now, the conditional expectation of cash-flows are:

$$\begin{aligned} E_t\left[D_{i,t+u}\right] &= D_{it} + \int_0^s \left[\theta_{is} + \Delta\theta_i \left(\pi_t - \pi_s\right) e^{-(\lambda+\mu)\tau}\right] d\tau \\ &= D_{it} + \theta_{is} s + \frac{\Delta\theta_i \left(\pi_s - \pi_t\right)}{\lambda + \mu} \left[e^{-(\lambda+\mu)s} - 1\right] \end{aligned}$$

and finally, risk neutral prices are found by continuously discounting expected dividends at the risk free rate:

$$\begin{aligned}
P_{i,t}^{RN} &= \int_0^\infty e^{-rs}\left[D_{it}+\theta_{is}s+\frac{\Delta\theta_i\left(\pi_s-\pi_t\right)}{\lambda+\mu}\left[e^{-(\lambda+\mu)s}-1\right]\right]ds \\
&= \frac{D_{it}}{r}+\frac{\theta_{is}}{r^2}-\frac{\Delta\theta_i\left(\pi_s-\pi_t\right)}{\lambda+\mu}\left[\frac{1}{r}-\frac{1}{(\lambda+\mu+r)}\right] \\
&= p_{i0}+p_{i\pi}\pi_t+p_{iD}D_{it}
\end{aligned}$$

where

$$\begin{aligned}
p_{i0} &= \frac{\theta_{iB}}{r^2}+\frac{\Delta\theta_i\mu}{r^2\left(\lambda+\mu+r\right)} \\
p_{i\pi} &= \frac{\Delta\theta_i}{r\left(\lambda+\mu+r\right)} \\
p_{iD} &= \frac{1}{r}
\end{aligned}$$

Risk Averse Prices

To solve for the risk aversion case, we need to solve the investor problem:

$$\begin{aligned}
J\left(W_t,\pi_t,t\right) &= \max_{\{c_t,X_t\}} E\left[\int_0^\infty U\left(c_s,s\right)ds\right] \\
\text{s.t. } dW_t &= \left[X_t'\left(\alpha_p+D_t-rP_t\right)+W_tr-c_t\right]dt+X_t'\Phi_p dv_t \text{ (Budget Constraint)} \\
X_t &= \left[\omega_1\ldots\omega_n\right]'\equiv\omega \qquad \text{(Market Clearing)}
\end{aligned}$$

This problem is solved using the Hamilton-Bellman-Jacobi equation:

$$\begin{aligned}
0 = \max_{c_t,X_t} &\left[U\left(c_t,t\right)+J_t+J_W\frac{E_t\left[dW_t\right]}{dt}+J_\pi\frac{E_t\left[d\pi_t\right]}{dt}+\frac{1}{2}J_{WW}\frac{E_t\left[dW_t^2\right]}{dt}\right. \\
&\left.+\frac{1}{2}J_{\pi\pi}\frac{E_t\left[d\pi_t^2\right]}{dt}+J_{W\pi}\frac{E_t\left[dW_td\pi_t\right]}{dt}\right]
\end{aligned}$$

where we have that:

$$
\begin{aligned}
E_t [dW_t] &= \left[X_t' (\alpha_p + D_t - rP_t) + W_t r - c_t\right] dt \\
E_t \left[dW_t^2\right] &= X_t' \Sigma_p X_t dt \\
E_t [d\pi_t] &= \alpha_\pi dt \\
E_t \left[d\pi_t^2\right] &= h (\pi_t)^2 H dt \\
E_t [dW_t d\pi_t] &= X' \Phi_p \sigma_\pi' dt
\end{aligned}
$$

A solution to problem, c_t^* and X_t^*, satisfy the first order conditions:

$$
\begin{aligned}
0 &= U_c (c_t^*, t) - J_W \\
0 &= J_W (\alpha_p + D_t - rP_t) + J_{WW} \Sigma_p X_t + J_{W\pi} \Phi_p \sigma_\pi' \qquad (5.1)
\end{aligned}
$$

In order to advance, we have to conjecture a functional form for the value function. Following the univariate model of Veronesi (1999), we set $J (W_t, \pi_t, t) = -exp (-\rho t - r\gamma W_t - g (\pi_t) - \beta)$ where $g (\pi_t)$ is a function to be determined and β a constant to be defined. Substituting the partial derivatives of the conjecture value function and of the utility function, $U (c_t, t) = -exp (-\rho t - \gamma c_t)$, on the first order conditions we obtain:

$$
\begin{aligned}
c_t^* &= \frac{1}{\gamma} (r\gamma W_t + g (\pi_t) + \beta - ln (r)) \qquad (5.2) \\
X_t^* &= \frac{1}{r\gamma} \Sigma_p^{-1} (\alpha_p + D_t - rP_t) - \frac{g' (\pi_t)}{r\gamma} \Sigma_p^{-1} \Phi_p \sigma_\pi' \qquad (5.3)
\end{aligned}
$$

We have an extra equation that will help to identify the problem. Evaluate the HJB equation at the maximum and set it equal to zero:

$$
\begin{aligned}
0 = & -exp (-\rho t - \gamma c_t^*) - \rho J - r\gamma J \left[X_t^{*\prime} (\alpha_p + D_t - rP_t) + W_t r - c_t\right] - g' (\pi_t) J \alpha_\pi + \qquad (5.4) \\
& \frac{1}{2} (r\gamma)^2 J X_t^{*\prime} \Sigma_p X_t^* + \frac{1}{2} \left(-g'' (\pi_t) + g (\pi_t)^2\right) J h (\pi_t)^2 H + r\gamma g' (\pi_t) J X^{*\prime} \Phi_p \sigma_\pi'
\end{aligned}
$$

Before we proceed, we can simplify the expression for $\alpha_p + D_t - rP_t$ by substituting the parameters that were obtained for the risk neutral price, p_{i0}, $p_{i\pi}$ and p_{iD}.

$$\begin{aligned}\alpha_{ip} + D_{it} - rP_{it} &= \left(p_{i\pi} + S_i'(\pi_t)\right)\alpha_\pi + p_{iD}D_{it} + \frac{1}{2}S_i''(\pi_t)\,h(\pi_t)^2 H + D_{it} \\ &\quad -r\left(p_{i0} + p_{i\pi}\pi_t + p_{iD}D_{it} + p_{i1} + S_i(\pi_t)\right) \\ &= -rp_{i1} - rS_i(\pi_t) + S_i'(\pi_t)\,\alpha_\pi + \frac{1}{2}S_i''(\pi_t)\,Hh(\pi_t)^2\end{aligned}$$

Take the above simplification, the expression for c_t^* from the first order condition (5.2) and the market clearing $X_t^* = \omega$ and substitute them in the equality (5.4) to get:

$$\begin{aligned}0 &= r - \rho - r\gamma\left[\omega'\left(-rp_1 - rS(\pi_t) + S'(\pi_t)\,\alpha_\pi + \frac{1}{2}S''(\pi_t)\,Hh(\pi_t)^2\right) - \frac{g(\pi_t)}{\gamma} - \frac{\beta}{\gamma} + \frac{ln(r)}{\gamma}\right] + \\ &\quad r\gamma g'(\pi_t)\left[\omega'\left(h(\pi_t)\left(p_\pi + S'(\pi_t)\right)\Delta\theta'\Phi'^{-1} + p_D\Phi\right)\left(\Phi^{-1}\Delta\theta h(\pi_t)\right)\right] + \\ &\quad \frac{1}{2}(r\gamma)^2\left[\omega'\left(h(\pi_t)\left(p_\pi + S'(\pi_t)\right)\Delta\theta'\Phi'^{-1} + p_D\Phi\right)\left(h(\pi_t)\left(p_\pi + S'(\pi_t)\right)\Delta\theta'\Phi'^{-1} + p_D\Phi\right)'\omega\right] + \\ &\quad \frac{1}{2}\left(-g''(\pi_t) + g(\pi_t)^2\right)h(\pi_t)^2 H - g'(\pi_t)\,\alpha_\pi\end{aligned}$$

where we have used the notation $p_{m1} \equiv \omega' p_1$, $p_{m\pi} \equiv \omega' p_\pi$, $\Delta\theta_m \equiv \omega'\Delta\theta$. Also, let $\sigma_\omega^2 \equiv \omega'\Sigma\omega$ and $\sigma_{i\omega} \equiv e_i'\Sigma\omega$ denote the variance of the market portfolio cash-flow and covariance of the market and asset i cash-flows, where e_i is a vector with zeros and one the ith position. Note that in the above equation the $S = [S_1, ..., S_n]'$ vector of functions is multiplied by the market clearing vector ω and so the equality only depends on $S_m \equiv \omega' S$. After some simplifications and substituting $f(\pi_t) = g(\pi_t) + r\gamma S_m(\pi_t)$ we get the following nonlinear differential equation for $f(\pi_t)$:

$$0 = -f''(\pi_t)\,Q_3(\pi_t) + \left(f'(\pi_t)\right)^2 Q_3(\pi_t) + f'(\pi_t)\,Q_2(\pi_t) + f'(\pi_t)\,r + Q_0(\pi_t)$$

where

$$
\begin{aligned}
Q_3(\pi_t) &= \frac{1}{2}h^2(\pi_t)H \\
Q_2(\pi_t) &= \gamma h(\pi_t)\Delta\theta_m + r\gamma h(\pi_t)^2 H\frac{\Delta\theta_m}{r(r+\mu+\lambda)} - \alpha_\pi \\
Q_0(\pi_t) &= \frac{1}{2}H\left(\frac{r\gamma h(\pi_t)\Delta\theta_m}{r(r+\mu+\lambda)}\right)^2 + r\gamma^2 h(\pi_t)\frac{\Delta\theta_m^2}{r(r+\mu+\lambda)}
\end{aligned}
$$

where some extra terms in $Q_0(\pi_t)$ were eliminated after choosing appropriately the parameters β and p_1:

$$
\begin{aligned}
\beta &= \frac{\rho}{r} + ln(r) + \frac{\gamma^2}{2r}\sigma_\omega^2 - 1 \\
p_{i1} &= -\frac{\gamma}{r^2}e_i'\Sigma\omega
\end{aligned}
$$

which in vector notation is $p_1 = -\frac{\gamma}{r^2}\Sigma\omega$. This non-linear differential equation f is the same one in Veronesi (1999) and it was shown there it has a unique solution on the relevant domain, $\pi_t \in (0,1)$.

Next, we have to find the individual discounting functions, S_i. In order to do so, we use the first order conditions (5.3) for asset demands, X_t^*, and the market clearing condition $X_t^* = \omega$ to get the equalities:

$$
\begin{aligned}
r\gamma\Sigma_p X_t^* &= (\alpha_p + D_t - rP_t) - g'(\pi_t)\Phi_p\sigma_\pi' \\
r\gamma\Sigma_p\omega &= \left(-r - \frac{\gamma}{r^2}\Sigma\omega - rS(\pi_t) + S'(\pi_t)\alpha_\pi + \frac{1}{2}S''(\pi_t)Hh(\pi_t)^2\right) \\
&\quad - \left(f'(\pi_t) - r\gamma S_m'(\pi_t)\right)\Phi_p\sigma_\pi'
\end{aligned}
$$

If we left multiply both sides of the above expression by e_i, $i = 1,...,n$, we get individual expression for S_i:

$$
\begin{aligned}
r\gamma\sigma_{im,p} &= \left(-r - \frac{\gamma}{r^2}\sigma_{im} - rS_i(\pi_t) + S_i'(\pi_t)\alpha_\pi + \frac{1}{2}S_i''(\pi_t)Hh(\pi_t)^2\right) \\
&\quad - \left(f'(\pi_t) - r\gamma S_m'(\pi_t)\right)\sigma_{ip}\sigma_\pi'
\end{aligned}
$$

that if we substitute for $\sigma_{im,p}$, σ_{ip}, σ_{im} and σ_{π} and rearrange the terms, we observe that the market discount function $S'_m(\pi_t)$ cancels out and a differential equations for each asset $i = 1, ...n$ is obtained:

$$0 = S''_i(\pi_t) P_3(\pi_t) + S'_i(\pi_t) P_2(\pi_t) + S'_i(\pi_t) r + P_{i0}(\pi_t)$$

where

$$\begin{aligned}
P_3(\pi_t) &= -\frac{1}{2} h^2(\pi_t) H \\
P_2(\pi_t) &= \gamma h(\pi_t) \Delta\theta_m + r\gamma h(\pi_t)^2 H \frac{\Delta\theta_m}{r(r+\mu+\lambda)} + h(\pi_t)^2 H f'(\pi_t) - \alpha_\pi \\
P_{i0}(\pi_t) &= \gamma h(\pi_t) \frac{\Delta\theta_i \Delta\theta_m}{r(r+\mu+\lambda)} \left(2 + \frac{h(\pi_t) H}{(r+\mu+\lambda)}\right) + f'(\pi_t) \Delta\theta_i h(\pi_t) \left(\frac{h(\pi_t) H}{r(r+\mu+\lambda)} + \frac{1}{r}\right)
\end{aligned}$$

This differential equation is essentially the same one in Veronesi (1999). We refer the reader to that paper for a proof that a solution exists on relevant domain, $\pi_t \in (0, 1)$. Note that only the last term, $P_{i0}(\pi_t)$, varies across assets. Furthermore, we observe that if two assets have the same $\Delta\theta_i$ they will share the same discounting function.

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