Time Series Predictive Analysis of Bitcoin with ARMA-GARCH model in Python and R

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Abstract:

Bitcoin arises recent years as a most successful and widely used cryptocurrency. Though it was first invented as a decentralized currency, Bitcoin has become more and more popular as a financial asset with a market capitalization of around $117 billion, as increasing number of investors and portfolio managers start to invest in and trade it. In this paper, we are interested in finding out the future course of Bitcoin prices and returns and examining the predictive power of the ARMA- GARCH model. The paper uses Python and R environment to analyze and model financial time series. The first part covers the preliminary analysis of the daily closing prices and returns of Bitcoin, and also the stationarity of the return series. The second part intends to fit an appropriate ARMA-GARCH model. The last part focuses on using fitted model to predict future returns and prices of Bitcoin and compare it to our validation dataset.

1. Background

Bitcoin is a cryptocurrency first introduced by Nakamoto in his paper "Bitcoin: A Peer-to-Peer Electronic Cash System" in 2008[1]. A cryptocurrency can be defined as "a virtual coinage system that functions much like a standard currency, enabling users to provide virtual payment for goods and services free of a central trusted authority" [2]. Three unique features of cryptocurrencies are that they are fully decentralized and depend on cryptographic protocols, or extremely complex code systems that encrypt sensitive data transfers to secure their units of exchange, and also control the finite supply. Bitcoin is the first widely used and traded cryptocurrency since 2009, when the Bitcoin software started to be available to the public and mining- the process of which new bitcoins can be created and transactions can be recorded and verified on the blockchain- begins. As Bitcoin becomes increasingly popular, and the idea of decentralized and encrypted currencies catch on, more rival, alternative cryptocurrencies appear. But Bitcoin remains the most successful and widely accepted cryptocurrency with a market cap at $117 billion, representing about 45% of the total estimated cryptocurrency capitalization at present (coinmarketcap.com accessed on Mar 30th 2018).

A few studies have already been conducted on the financial and statistical characteristics of Bitcoin. One group of economists has been focusing on price discovery in the Bitcoin market, for example, Brandvold et al. [3] and Bouoiyour et al. [4] reveal some lead-lag relationship between Bitcoin prices, transactions use, and investors' attractiveness. Other studies also show that Bitcoin price is subject to unique factors which are substantially different from those affecting conventional, financial assets, such as internet search [5], information on google trends, and word-of-mouth information on social media.[6] In fact, as Bitcoin is mainly used and viewed as an asset rather than a currency [7], and the Bitcoin market is currently highly speculative, and more volatile and susceptible to speculative bubbles than other currencies [8][9]. Moreover, the presence of long memory and persistent volatility [10] justifies the application of GARCH-type models.

The purpose of this paper is to utilize time series techniques to predict the future returns and prices of Bitcoin. At the same time, we want to examine the effectiveness of the popular ARMA-GARCH model in economics and financial world. As Bitcoin gradually has had a place in the financial markets and in portfolio management [11], time series analysis is a useful tool to study the characteristics of Bitcoin prices and returns, and extract meaningful statistics in order to predict future values of the series.

2. Data

The data used to fit the model are the daily closing prices of Bitcoin from July 18th, 2010 (as the earliest data available) to Jan 30th, 2018, which corresponds to a total of 2754 observations. We save the data from Jan 31st, 2018 to March 31st, 2018 to perform validation latter. The data is compiled from Bitstamp, the largest Bitcoin exchange, and covers a daily database denominated in US dollar, which is the main currency against which Bitcoin is the most traded. We calculate the log-returns by taking the natural logarithm of the ratio of two consecutive prices, as a good approximation of daily percentage changes in prices.

Fig. 1 -Fig.4 illustrate the Bitcoin daily closing prices and daily returns.

Fig.1: Daily Closing Prices from 2010-7-18 to 2018-1-30

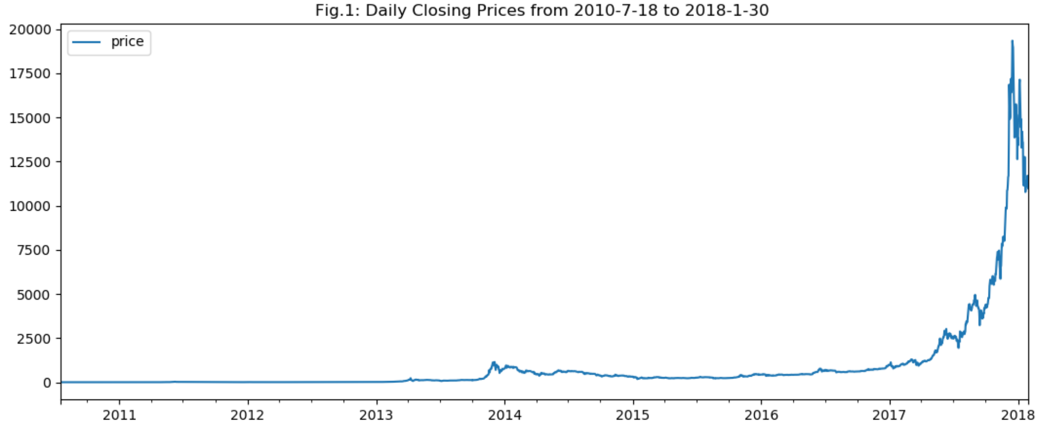


Fig.2: Daily Closing Prices from 2017-1-1 to 2018-1-30

Fig.3: Daily Log Return from 2010-7-18 to 2018-1-30

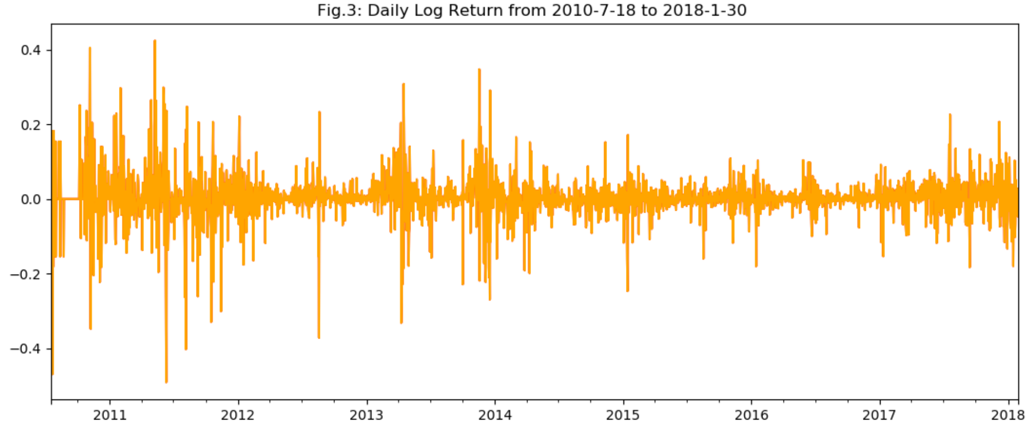
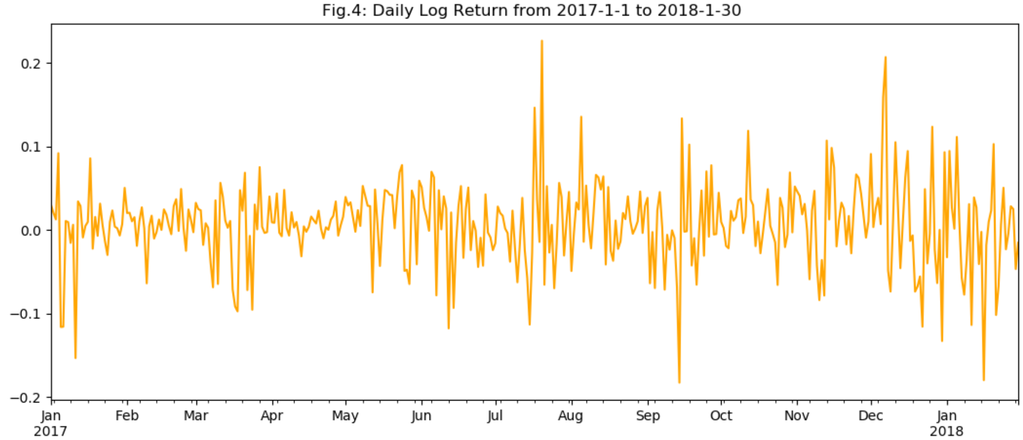


Fig.4: Daily Log Return from 2017-1-1 to 2018-1-30



3. Preliminary analysis and results

3.1 Descriptive Statistics

Since its introduction in 2009, the value of Bitcoin grew rapidly and its price achieved all time high: $19, 340 at the end of 2017. However, it recently has been traded lower to $6,000 to $7,000 range in the past month. Table 1 reports the descriptive statistics of the returns of Bitcoin, with the maximum daily return in the sample being 0.4246, the minimum being -0.4915, an extremely high volatility for a financial asset. The mean return is 0.0043, with a quite high standard deviation of 0.0588. The result of the Jarque-Bera statistic indicates the departure from normality. The returns are also negatively skewed, while the excess kurtosis suggests evidence of a leptokurtic distribution.

Table 1: Descriptive statistics

Observations	2754
Mean	0.0042544
Median	0.0014238
Maximum	0.4245795
Minimum	-0.4915280
Std. Dev.	0.0587847
Skewness	-0.3561255
Kurtosis	11.9474458
JB	694554.402

3.2 Check Stationarity

Before modeling time index data, we need to check the stationarity, as a lot of statistical and econometric methods are based on stationarity. Based on the results of the Augmented Dickey–Fuller (ADF) tests as shown in Table 2, we fail to accept the null hypothesis of a unit root for the returns, and, hence, stationarity is guaranteed for the log-return series of Bitcoin.

Table 2: Results of Augmented Dickey–Fuller (ADF) test

Test Statistic	-11.69951
p-value	1.58108E-21
#Lags Used	13
#Observations Used	2739
Critical Value (10%)	-2.5673320
Critical Value (5%)	-2.862596
Critical Value (1%)	-3.43274

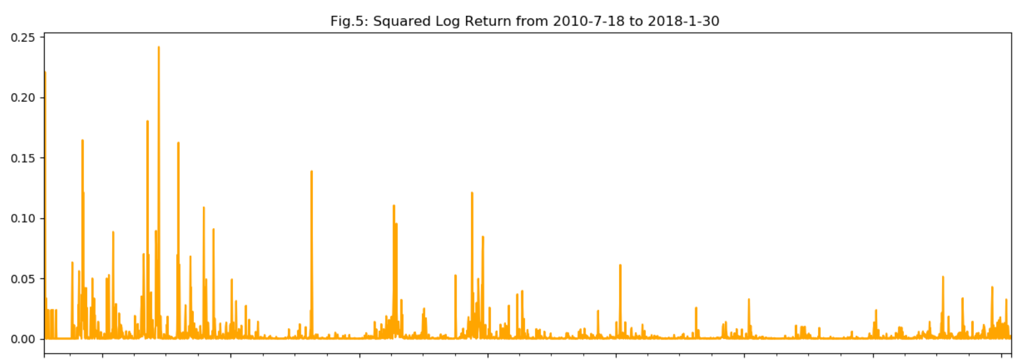
4. Methodology

4.1 Some Stylized Facts About Volatility in Financial Data

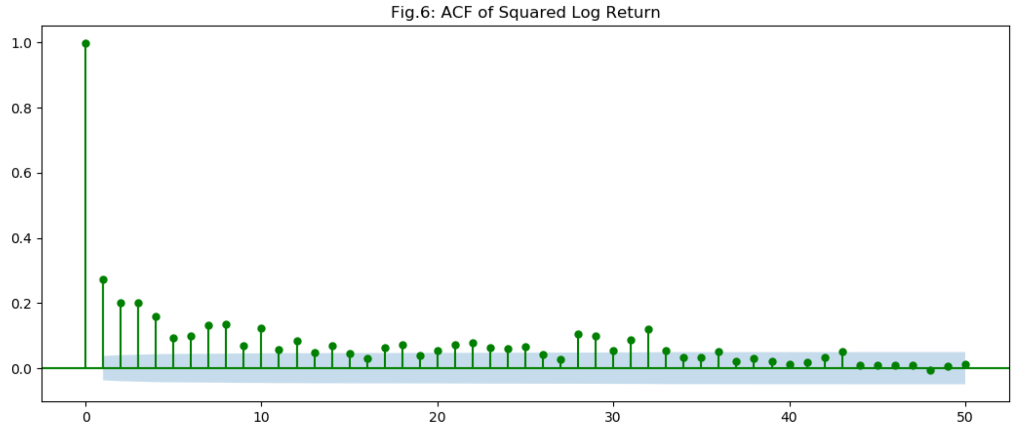
One common observation we get out of the economic and financial data is volatility clustering. Suppose we have noticed that recent daily returns have been unusually volatile. We might expect that tomorrow's return is also more variable than usual. We can also observe that the squared returns of an asset are usually positively auto-correlated, i.e. if an asset price made a big move yesterday, it is more likely to make a big move today. With economic and financial data, time-varying volatility is more common than constant volatility, and accurate modeling of time-varying volatility is of great importance.

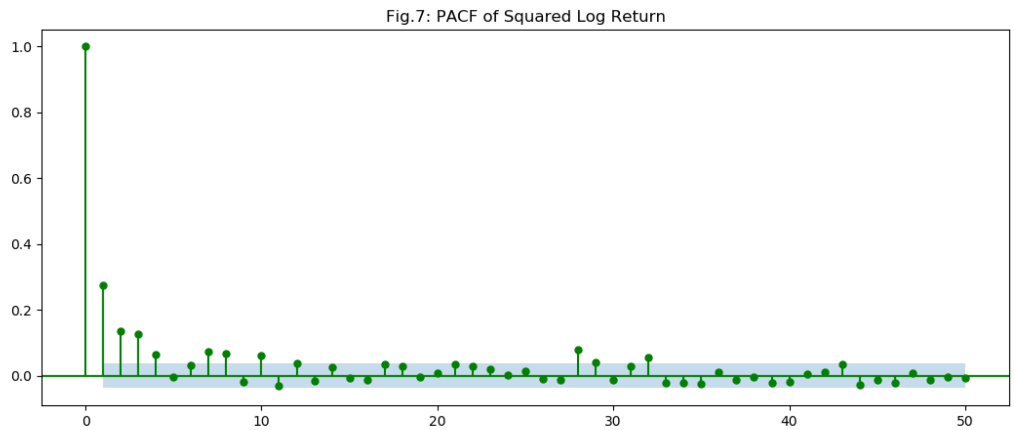
4.2 Justifications for ARMA-GARCH model

In our case, we have already known from the excess kurtosis that an obvious fat tails displayed in our series, a typical evidence of heteroskedastic effects as clustering of volatility. We can also observe from the squared log-return. Fig.5 illustrates that the squared returns appear to fluctuate around a constant level, but exhibit volatility clustering. Large changes in the squared returns tend to cluster together, and small changes tend to cluster together, which also indicates that the series exhibits conditional heteroscedasticity.



It is even more clear, if we plot the sample autocorrelation function (ACF) and partial autocorrelation function (PACF) of squared log-return, as shown in Fig.6 and Fig.7.





The sample ACF and PACF show significant autocorrelation in the squared log-return series.

Then, a Ljung-Box Q-test can more formally assess autocorrelation.

Table 3. Results of Ljung-Box Q-test

lag	p-value	Q	c-value	rejectH0
1	0.0000	2756.729	2.706	TRUE
2	0.0000	5513.13	4.605	TRUE
3	0.0000	8267.938	6.251	TRUE
4	0.0000	11021.165	7.779	TRUE
5	0.0000	13232.577	9.236	TRUE
6	0.0000	15435.093	10.645	TRUE
7	0.0000	17628.810	12.017	TRUE
8	0.0000	19823.069	13.362	TRUE
9	0.0000	22008.538	14.684	TRUE
10	0.0000	24194.156	15.987	TRUE

11	0.0000	26379.902	17.275	TRUE
12	0.0000	28561.907	18.549	TRUE
13	0.0000	30740.143	19.812	TRUE
14	0.0000	32914.629	21.064	TRUE
15	0.0000	35076.471	22.307	TRUE
16	0.0000	37238.845	23.542	TRUE

As illustrated in the ACF and PACF plot of squared returns and the results of Ljung-Box test, there is clearly autocorrelation present. The model we are going to look at will attempt to capture the autocorrelation of squared returns, clustering volatility, as well as the heteroscedasticity. The significance of the lags in both the ACF and PACF indicate we need both AR and MA components for our model. As we know that ARMA models are used to model the conditional mean of the process given past information, which however, assumes the conditional variance given the past is constant. ARMA model alone fails to capture the volatility clustering behavior. Thus, we will use GARCH process that has become widely used in econometrics and finance, to correct the heavy tails and model the randomly varying volatility in Bitcoin's return.

We describe the mean equation of the log-return series r_t by the process

$$r_t = E(r_t|r_{t-1}, r_{t-2}, \dots) + \varepsilon_t \ ,$$

where $E(r_t|r_{t-1}, r_{t-2}, \dots)$ denotes the conditional expectation operator. ε_t denotes the innovations or residuals of the log-return series with zero mean, and plays the role of the unpredictable part of the time series, generated from a GARCH process. We first model the mean equation as an ARMA process.

4.3 ARMA process

Recall that the ARMA(m, n) process of autoregressive order of m, and moving average order n can be described as:

$$r_t = \mu + \sum_{i=1}^{m} a_i r_{t-i} + \sum_{j=1}^{n} b_j \varepsilon_{t-j} + \varepsilon_t$$

With mean μ, autoregressive coefficients a_i, and moving average coefficients b_j.

To choose the best order(m, n) , we try out different combinations and select the one with the lowest AIC and BIC in Python. Table 4 illustrates ARMA(2,1) performs best, and shows the estimated coefficients results for ARMA(2,1).

Table 4. AIC and BIC of ARMA (m,n)

Order	AIC
(0,1)	-7791.96
(0,2)	-7796.68
(1,0)	-7792.15
(1,1)	-7798.64
(1,2)	-7843.69
(2,0)	-7795.67
(2,1)	-7836.96
(2,2)	-7807.82

```
ARMA(1,2)


Residuals:
      Min         1Q     Median         3Q        Max
-0.487415 -0.016283 -0.002524  0.018147  0.422052


Coefficient(s):
           Estimate  Std. Error  t value   Pr(>|t|)
ar1       -0.914004    0.019341  -47.257  < 2e-16 ***
ma1        0.962524    0.027655   34.805  < 2e-16 ***
ma2        0.040547    0.019782    2.050  0.04039 *
intercept  0.008478    0.002216    3.826  0.00013 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Next, we need to check the residuals after fitting ARMA(2,1), which should be align with our previous discussion in 4.2, and displays heteroscedasticity, as illustrated in Fig.8-11.

Fig 8. Residuals of ARMA(1,2) Model Fit

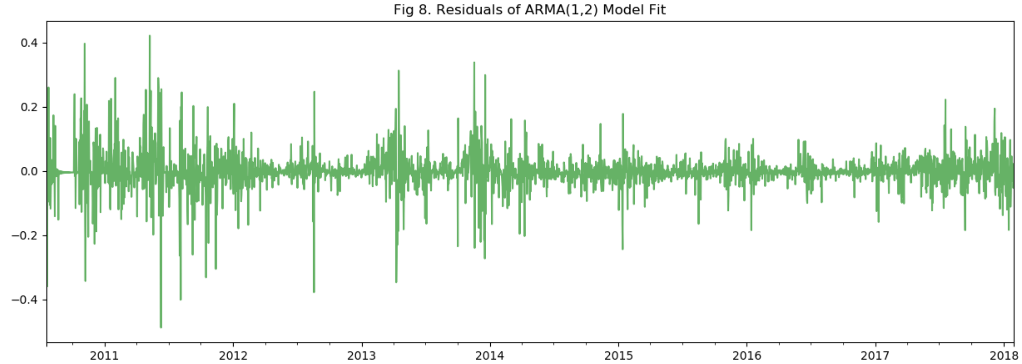


Fig 9. Squared Residuals of ARMA(1,2) Model Fit

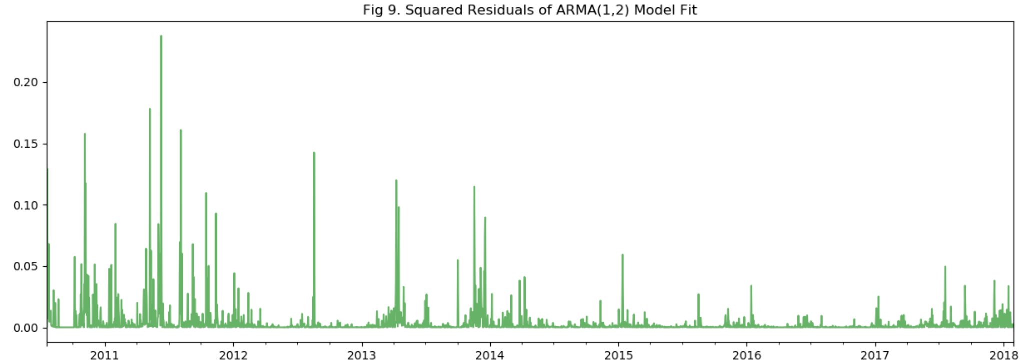
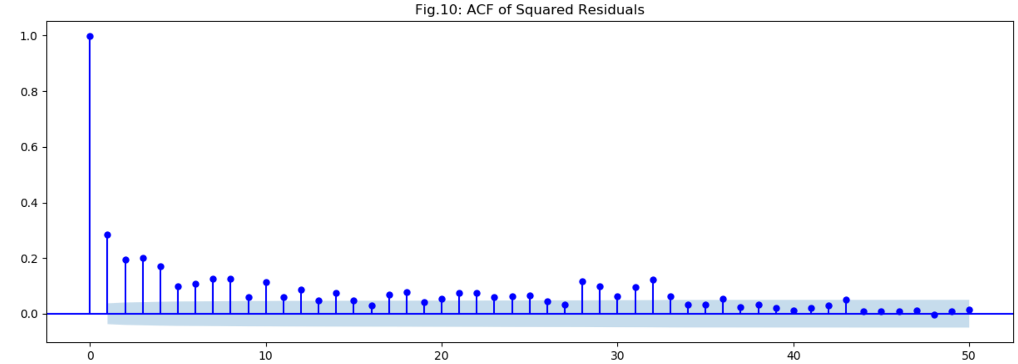
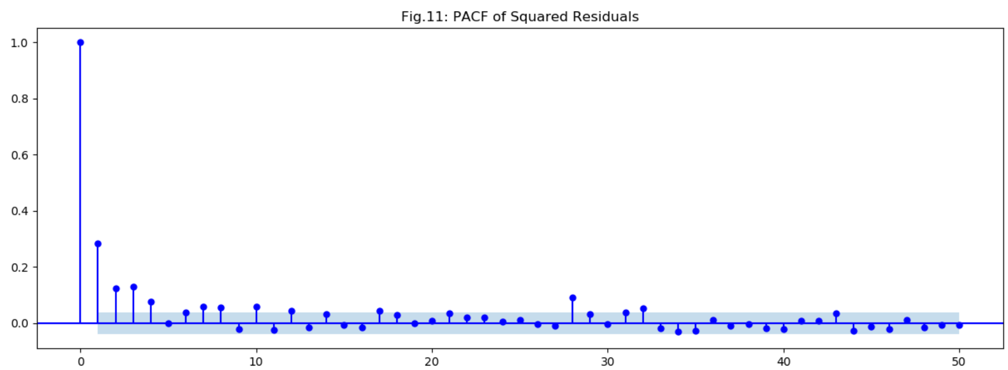


Fig.10: ACF of Squared Residuals





4.4 GARCH Process

As we already detect the autocorrelation effects in our residual/innovation series, we now need to apply GARCH (p,q) model in order to estimate the conditional variance going forward, using:

$$\varepsilon_t = z_t \sigma_t$$

$$\sigma_t^2 = \gamma V_L + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i \, \sigma_{t-j}^2$$

where

$$\sigma_t^2 = Var(\varepsilon_t | \varepsilon_{t-1}, \varepsilon_{t-2}, \dots)$$

denotes the conditional variance and z_t is a noise term iid ~(0,1) .

Three parameters (γ, α, β) must satisfy that

$$\gamma + \alpha + \beta = 1$$

If we introduce a new parameter ω to denote the weighted average long-term variance, where

$$\omega = \gamma V_L$$

GARCH(p,q) model becomes:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i \, \sigma_{t-j}^2$$

The model tells us that tomorrow's variance is a function of today's squared innovations, today's variance, and the weighted average long-term variance.

The estimation of (ω, α, β) can be conducted utilizing the maximum likelihood method, which is an iterative process by looking for the maximum value of the sum among all sums defined as:

$$\sum\nolimits_{i=3}^{N} [-\ln(\sigma_i^2) - \frac{\varepsilon_i^2}{\sigma_i^2}]$$

where N denotes the length of the innovation or residuals series $\{\varepsilon_j\}$ (j=2,…, N)

To choose the best order(p, q) , we try out different combinations and make the decision based on Log-Likelihood, and Information Criteria. Fig.12 gives us ARMA(1,2)-GARCH(1,2) as the best model.

Table 5. Log Likelihood and Information Criteria of ARMA(1,2)-GARCH(p,q)

Model (p,q)	Log-Likelihood	AIC	BIC	HQIC
(1,1)	5245.095	-3.8039	-3.7846	-3.7969
(1,2)	5248.175	-3.8054	-3.7839	-3.7977
(1,3)	5248.156	-3.8047	-3.7810	-3.7961
(2,1)	5245.124	-3.8032	-3.7817	-3.7954
(2,2)	5248.159	-3.8047	-3.7810	-3.7961
(2,3)	5248.150	-3.8040	-3.7782	-3.7946
(3,1)	5244.112	-3.8018	-3.7781	-3.7932
(3,2)	5247.063	-3.8032	-3.7774	-3.7938
(3,3)	5248.108	-3.8032	-3.7752	-3.7931

Fig 12. Full model parameters

```
Mean Model: ARMA(1,2)

Variance Model: GARCH(1,2)

Optimal Parameters

------------------------------------

        Estimate  Std. Error    t value Pr(>|t|)

mu      0.001368    0.000090    15.2490        0

ar1     0.965530    0.003229   299.0540        0

ma1    -0.983092    0.000887 -1107.7277        0

ma2     0.030457    0.000260   117.1919        0

omega   0.000057    0.000009     6.0399        0

alpha1  0.278171    0.019459    14.2952        0

beta1   0.461024    0.025163    18.3215        0

beta2   0.259804    0.020355    12.7637        0
```

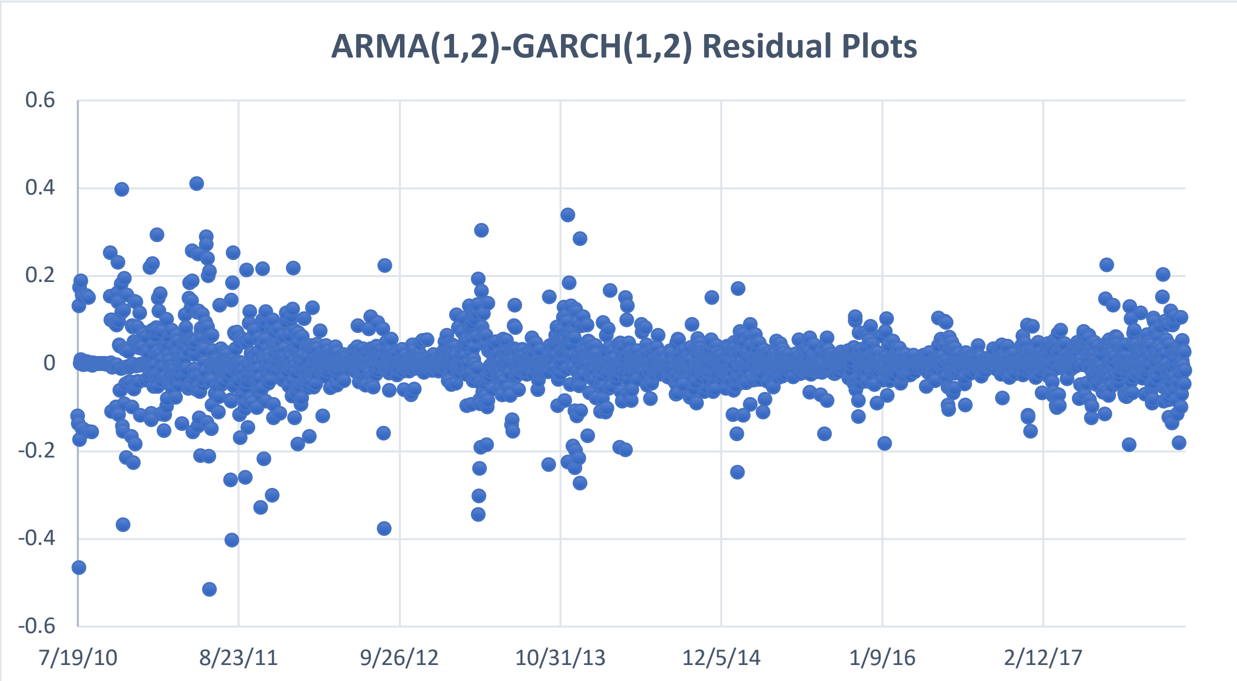
So far, we have successfully obtained the full ARMA(1,2)-GARCH(1,2) model:

$$r_t = 0.001368 + 0.965530 r_{t-1} + (-0.983092)\varepsilon_{t-1} + 0.030457\varepsilon_{t-2} + \varepsilon_t$$

$$\varepsilon_t = z_t \sigma_t, \qquad z_t \sim iid(0,1)$$

$$\sigma_t^2 = 0.000057 + 0.278171\, \varepsilon_{t-1}^2 + 0.461024\sigma_{t-1}^2 + 0.259804\sigma_{t-2}^2$$

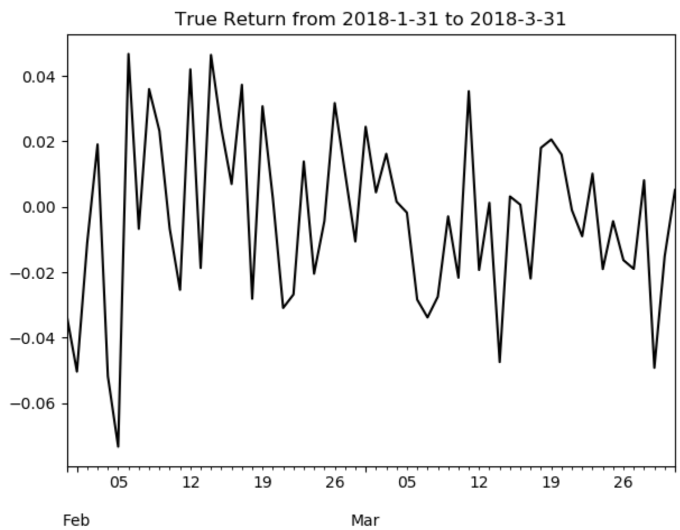
And here is the residual plot we get from our fitting our model:

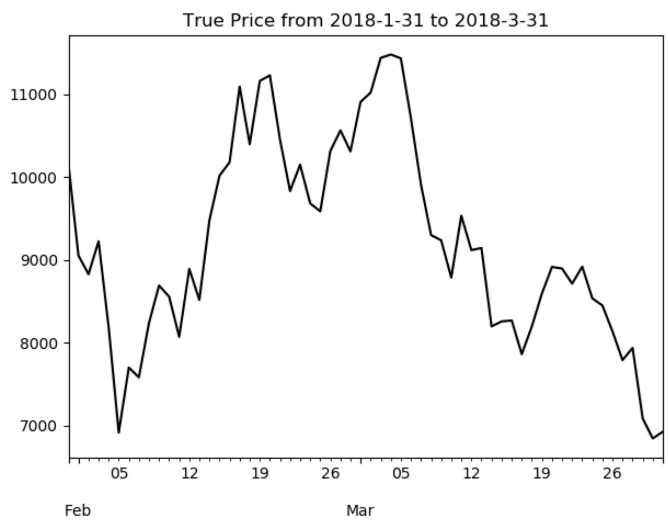


5. Forecasting

5.1 Validation Dataset

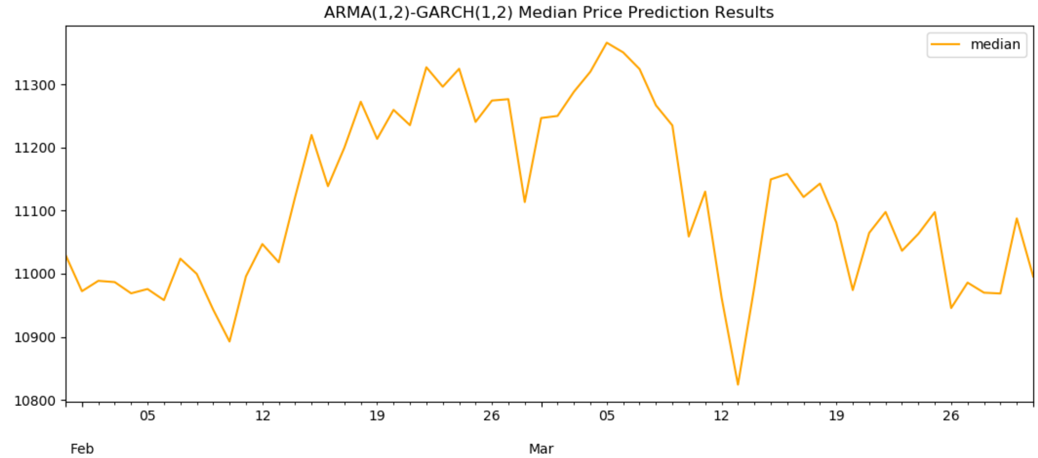
After obtaining the fitted ARMA-GARCH model, we are curious about how the model would perform in predicting the future prices of Bitcoin. Here we load the saved validation dataset, i.e. the daily closing prices for Bitcoin from Jan 31st, 2010 to March 31st, 2018, and calculate the daily log-returns during such period.

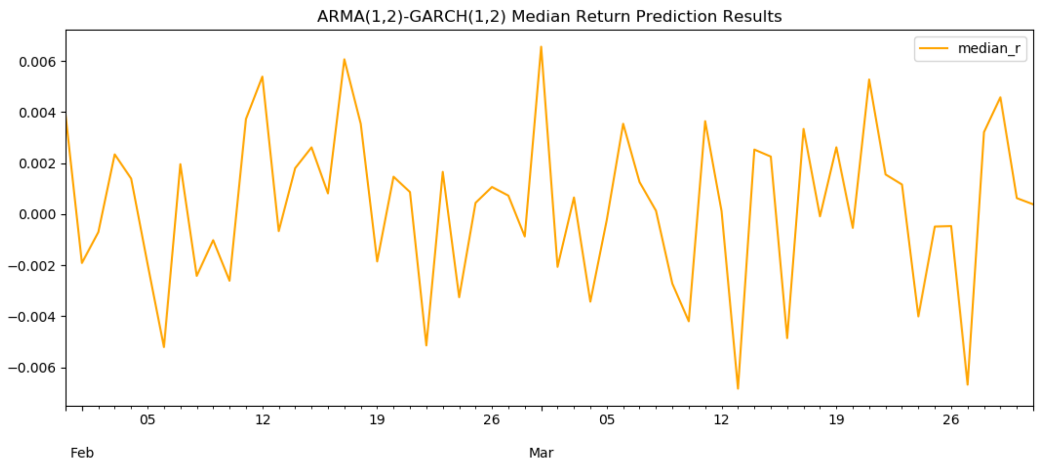


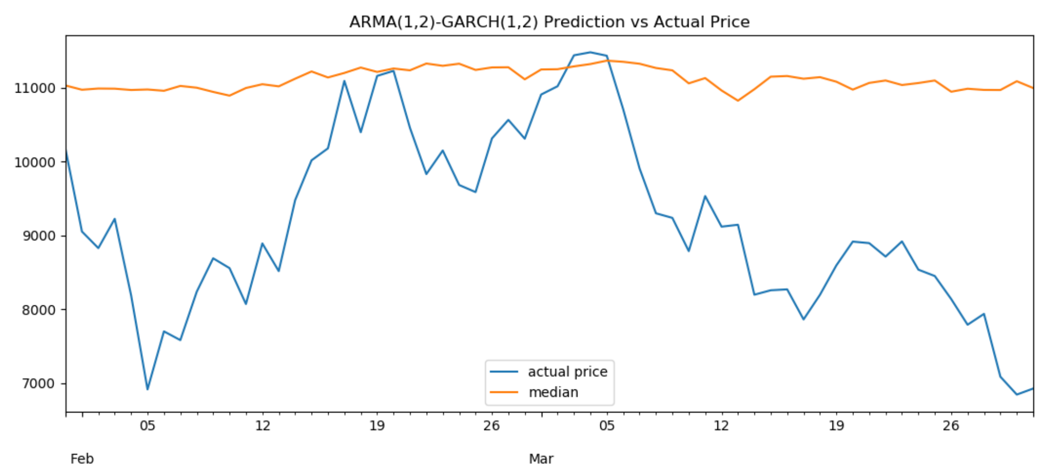


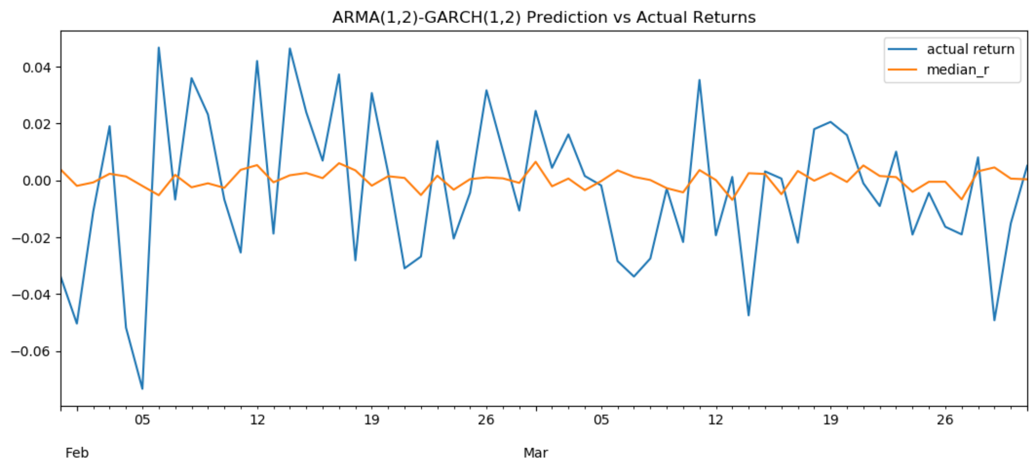
5.2 Forecasting results

Using the methodology we describe earlier and resulted ARMA(1,2)-GARCH(1,2) model, we are able to forecast the future prices (from 2010-07-18 to 2018-01-30) for Bitcoin. The graphs below illustrate the median value of predicted prices resulted from 500 simulations.


ARMA(1,2)-GARCH(1,2) Median Price Prediction Results
median
11300
11200
11100
11000
10900
10800
05
12
19
26
05
12
19
26
Feb
Mar


ARMA(1,2)-GARCH(1,2) Median Return Prediction Results
median_r
0.006
0.004
0.002
0.000
−0.002
−0.004
−0.006
05
12
19
26
05
12
19
26
Feb
Mar

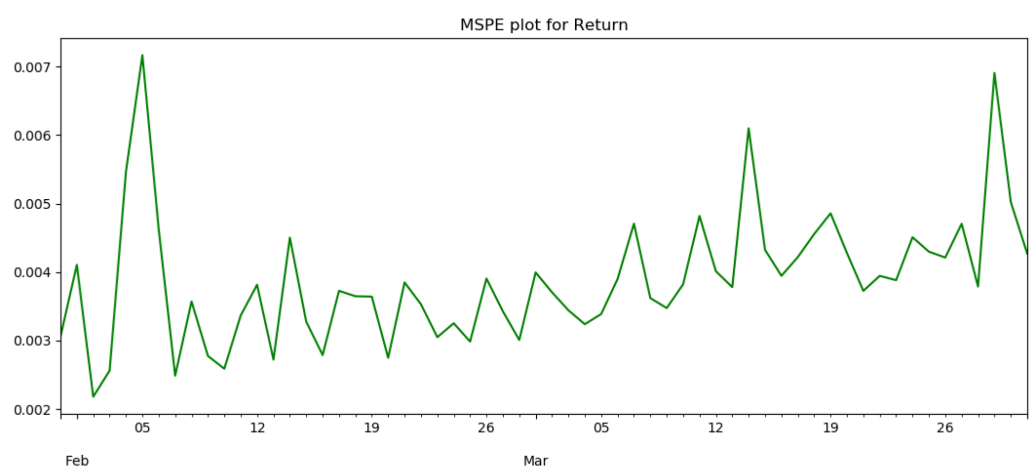

ARMA(1,2)-GARCH(1,2) Prediction vs Actual Price
11000
10000
9000
8000
7000
actual price
median
05
12
19
26
05
12
19
26
Feb
Mar

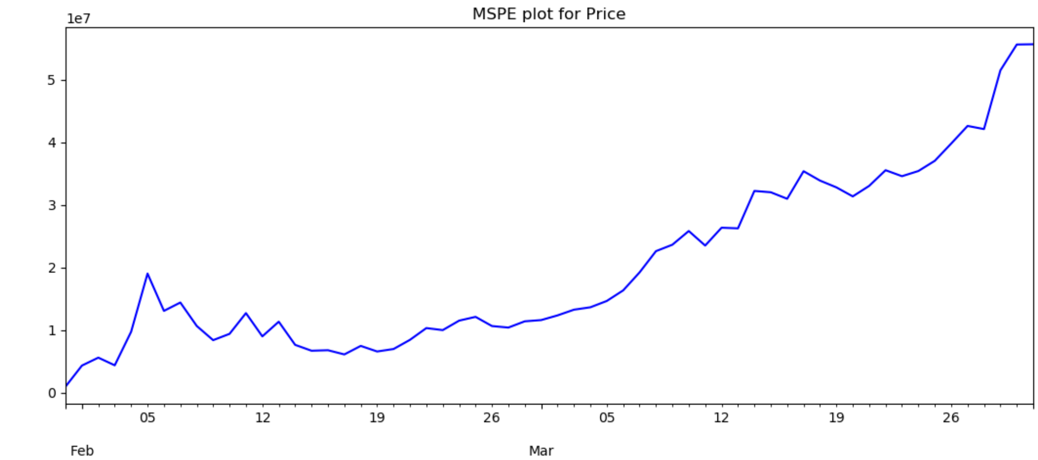


Compared with the actual prices, our model can roughly predict the ups and downs of Bitcoin's return/price movement. However, it fails to capture the high volatility of the daily return/price and thus gives a prediction around a rather constant level relative to the actual return/price. We can also use a numeric measurement to judge how well our model performs across time:

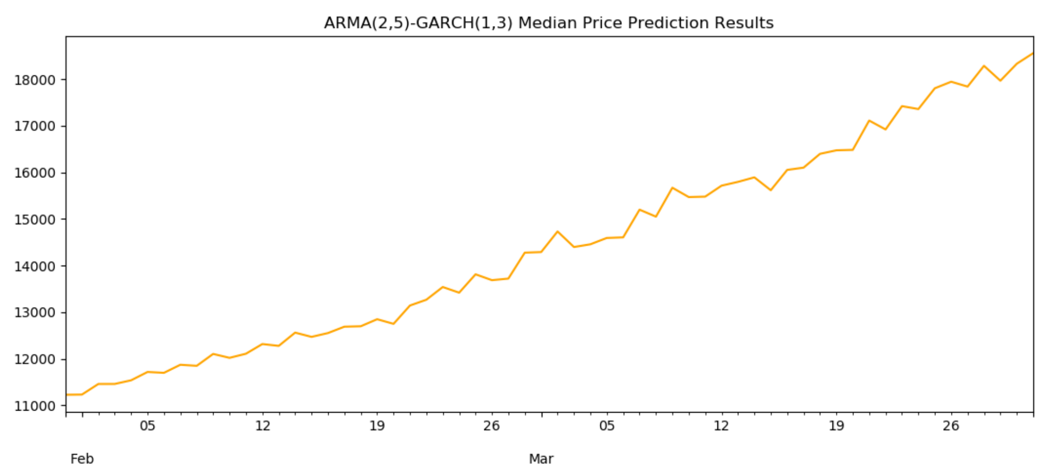
$$MSPE = \frac{1}{M}\sum_{m=1}^{M}(P_{mt} - X_t)^2$$

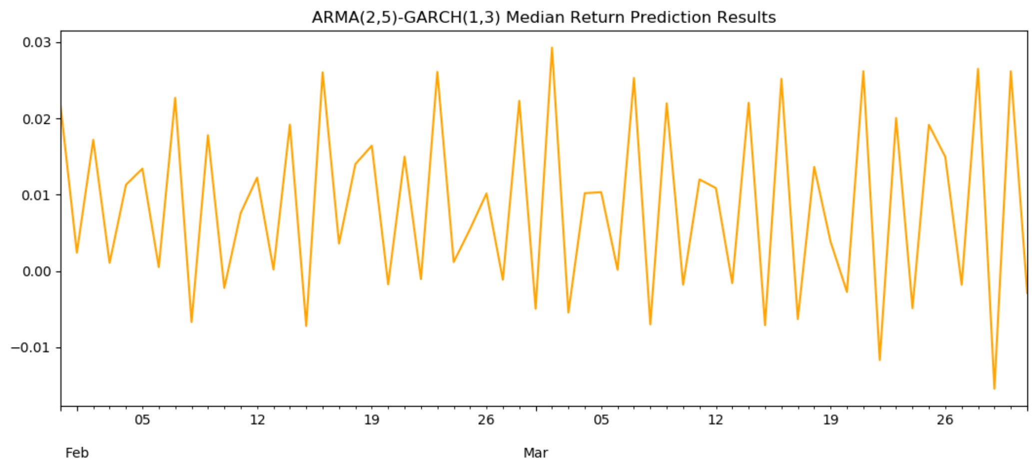
where MSPE stands for "Mean Squared Prediction Error"; M stands for the number of simulations; P_{mt} stands for the prediction return/price at time t for simulation m; X_t stands for the actual return/price at time t.

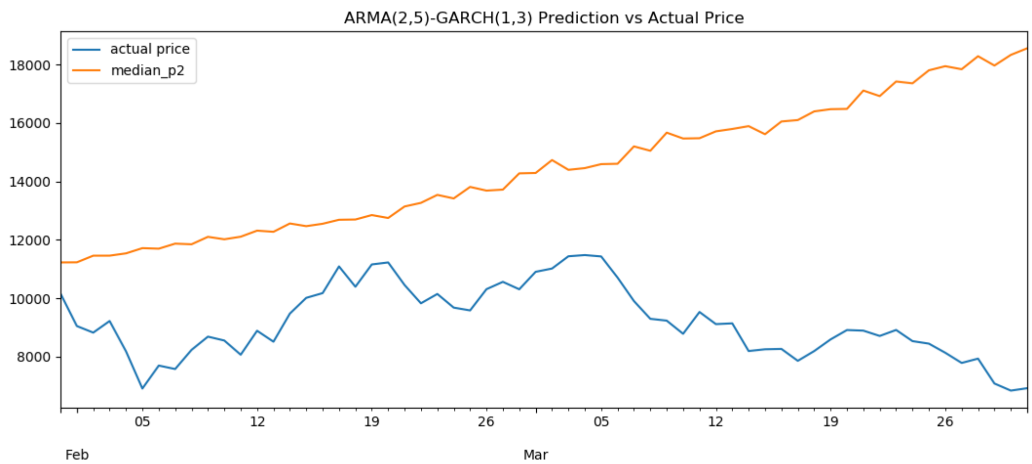


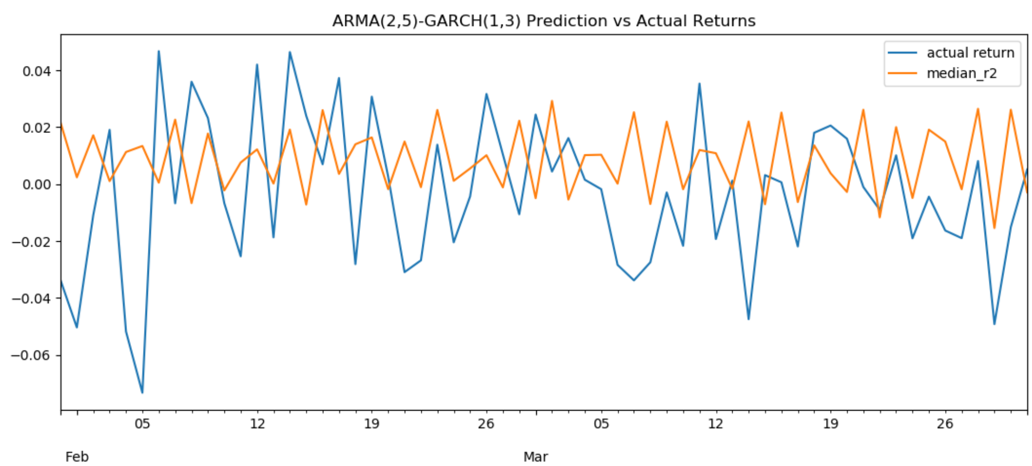


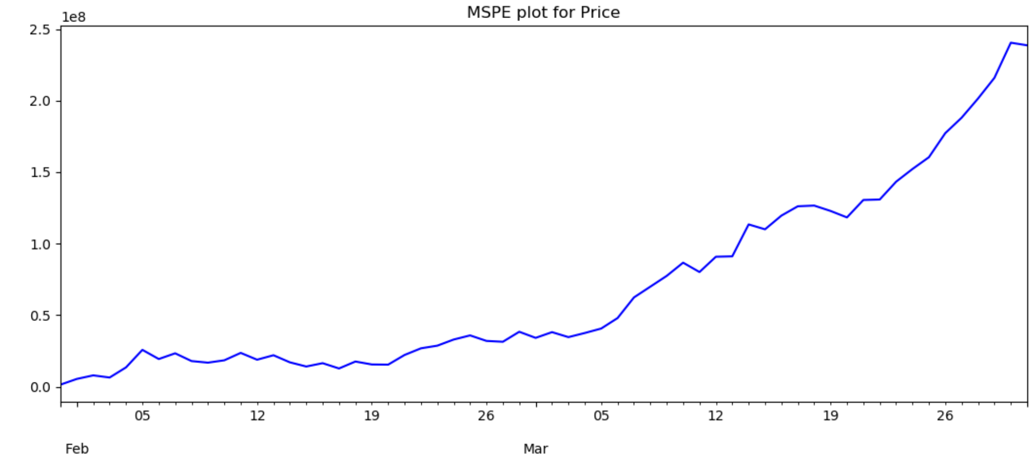
We can follow the same procedure to fit different models on different datasets. For example, the following two pairs of graphs report the best models resulting from different sets of data used. If we use the closing price of Bitcoin from 2017-1-1 to 2018-3-30, which corresponds to the period when Bitcoin starts to draw public attention and become popular among investors, the best possible model is ARMA(2,5)-GARCH(1,3). The following graphs show the prediction result and also the MSPE for returns and prices.

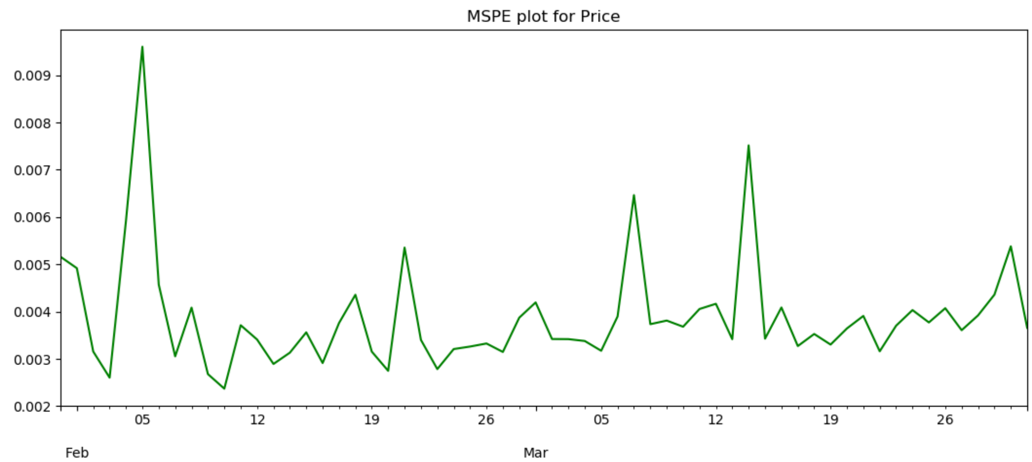



ARMA(2,5)-GARCH(1,3) Median Return Prediction Results
0.03
0.02
0.01
0.00
−0.01
05
12
19
26
05
12
19
26
Feb
Mar


ARMA(2,5)-GARCH(1,3) Prediction vs Actual Price
actual price
median_p2
18000
16000
14000
12000
10000
8000
05
12
19
26
05
12
19
26
Feb
Mar


ARMA(2,5)-GARCH(1,3) Prediction vs Actual Returns
actual return
median_r2
0.04
0.02
0.00
−0.02
−0.04
−0.06
05
12
19
26
05
12
19
26
Feb
Mar





We can tell that ARMA(2,5)-GARCH(1,3) fails to predict the both the level and trend of future Bitcoin price. That means for our case using a more recent dataset doesn't improve our results of prediction of Bitcoin price. However, it does better than ARMA(1,2)-GARCH(1,2) model in forecasting the future volatility in returns, especially for a longer period, by comparing the MSPE of ARMA(2,5)-GARCH(1,3) with that of ARMA(1,2)-GARCH(1,2).

6. Conclusions

GARCH modeling builds on advances in the understanding and modeling of volatility. It takes into account excess kurtosis (i.e. fat tail behavior) and volatility clustering, two important characteristics of financial time series, which are also observable in the Bitcoin case. It's theoretically able to provide accurate forecasts of variances and covariances of returns through modeling time-varying conditional variances. As a consequence, GARCH models have become quite popular in diverse fields as risk management, portfolio management and asset allocation, option pricing, foreign exchange, and the term structure of interest rates.[12]

In this paper, we intend to predict the future prices of Bitcoin, one of the most widely used and traded cryptocurrency, and study the predictive power of ARMA-GARCH model on the Bitcoin return/price series. From the predicted results, we have realized that although GARCH models are useful across a wide range of financial and economical applications, they are not a quite effective and suitable model candidate in studying the Bitcoin return/price series. One of the main reasons is that GARCH models are parametric specifications that operate best under relatively stable market conditions[13]. Although GARCH is explicitly designed to model time-varying conditional variances, GARCH models often fail to capture highly irregular phenomena, including wild market fluctuations (e.g., crashes and subsequent rebounds), and other highly unanticipated events that can lead to significant structural change, which are exactly what has been going on in the Bitcoin market recently. Events and factors such as the recent announcement by Google, that it will put a ban on all cryptocurrency related ads, the collapsed cryptocurrency exchange Mt. Gox, increasingly hostile regulatory climate in China, increased regulatory scrutiny of ICOs, banks are also putting more and more pressure on the market by barring their customers from purchasing cryptocurrencies using their plastic cards, all contributed towards uncertainty, geopolitical risks, and wild fluctuations in the Bitcoin market. For future studies, it is possible for us to further explore different types of GARCH-type models such as asymmetric GARCH, EGARCH, TGARCH, etc. hopefully to capture the characteristics of the Bitcoin series, and make a better prediction.

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