MASTER (MA)

Modelling and Analysis of Switched Physical Systems Using Mixed Boolean Singular Representation

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Award date:
2019

Awarding institution:
University of Bath

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A thesis submitted for the degree of Master of Philosophy
University of Bath
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June 2018

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Summary

Traditionally engineering systems have been analysed as linear systems. However, nowadays most engineering systems presents discrete behaviours (e.g. switches, clutches, contacts, to name some), necessitating a different approach for the analysis of such systems. For this reason, a hybrid systems approach is used. Hybrid systems contain continuous and discrete behaviours. These systems are also known as switched systems.

Hybrid systems behave in a continuous mode most of the time with discrete behaviour only appearing when a switching element commutes. These behaviours can occur depending on the commutation conditions (either controlled commutations or arbitrary commutation).

Bond graph models are useful to represent multidomain engineering systems. This is due to the unified representation of the elements for the different engineering domains involved (electrical, mechanical, hydraulics, thermal), meaning that elements from different domains have equivalent representations in the model. Therefore, their interactions can be observed in the entire model instead of being analysed as independent subsystems, which later will be placed under specific constraints in order to comply with some physical laws that allows the analysis of their interaction as a whole system.

Hybrid bond graphs are introduced in order to represent continuous and discrete behaviours. This is usually made by having different models, some for the continuous cases, and different ones for the discontinuous cases. In order to differentiate hybrid systems from linear time invariant systems a representation for commuting elements is introduced.

There are several approaches to represent commuting elements, either for the purpose of structural analysis or efficient numerical simulation of the systems. In this case switched junctions are used to introduce a standard notation for bond graph hybrid systems. The use of switched junctions affects the behaviour of the causality in some elements. This is known as *Dynamic causality.*

Dynamic causality represents the changes of power transfer on the system when a commuting element changes its state (either from ON configuration to OFF configuration or the other way around). These changes are usually ignored in traditional approaches in order to simplify the analysis of the systems for simulation purposes but at the cost of loss of information from the system.

Another consequence of the use of switched junctions is the introduction of Boolean parameters on the model equations. Boolean parameters help to-

wards the continuous analysis of the system, meaning that the analysis of the system is from a single general equation containing all the available configurations, rather than analysing all of the available configurations independently in order to obtain a particular implicit equation for each configuration.

The focus of this thesis is to propose a generalized notation for the hybrid bond graph systems, and a set of rules to obtain the hybrid bond graph model. This generalized notation is based on Boolean parameters in order to simplify the analysis and show all the available configurations in one general equation.

The main contribution of this thesis is the generalization of the notation of hybrid bond graph models, and the steps that needs to be followed in order to obtain the system's implicit equation. Also, the formulation of the general implicit equation, and the necessary conditions to identify the valid configurations are presented. The results are also used for the analysis of implicit equations that were obtained using any traditional mathematical approach.

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Acknowledgements

I would like to express appreciation towards my supervisor Dr Roger Ngwompo, who gave me helpful advice and guidance towards the research of the topic and writing of the thesis.

This research was funded via a CONACYT scholarship in conjunction with SICDET.

I would like to acknowledge my family who gave me their unconditional support all these years that my PhD last, and the people at the genomics laboratory under the supervision of Dr Araxi Urrutia for making my last months at university brighter.

Nomenclature

A	The A-matrix in the standard Linear Time Invariant system equations
B	The B-matrix in the standard Linear Time Invariant system equations
BGD	Bond graph model with preferred derivative causality assignment
BGI	Bond graph model with preferred integral causality assignment
C	The C-matrix in the standard Linear Time Invariant system equations
D	The D-matrix in the standard Linear Time Invariant system equations
D_{in}	Input vector from the system to the resistance field not affected by dynamic causality
$\tilde{D}_{in}$	Input vector from the system to the resistance field affected by dynamic causality
D_{out}	Output vector from the system to the resistance field not affected by dynamic causality
$\tilde{D}_{out}$	Output vector from the system to the resistance field affected by dynamic causality
E	The E-matrix in the standard Linear Time Invariant system equations
e	Generalised effort variable on a bond
F	The matrix characterising the storage field. In the LTI case, this is a diagonal matrix of the linear coefficients for storage elements (relating the states to their complements)
f	Generalised flow variable on a bond
G	Ground
in	Subscript denoting input
int	Subindex denoting internal variables in the junction structure
L	The matrix relating the outputs to inputs of the resistive field. In the LTI case, this is a diagonal matrix of the linear coefficients for resistive elements
$\mathbf{L}$	The linear coefficient for a single Inertia (I-element)
M	Mass
$\mathbf{M}$	Concatenation of matrices $\begin{bmatrix} E & A & B \end{bmatrix}$
$\bar{M}$	Reduced matrix resulting from the ommision of the ith row and the ith column of the matrix $\mathbf{M}$
$\hat{M}$	Reduced matrix resulting from the ommision of multiple rows and multiple columns of the matrix $\mathbf{M}$
MGY	Modulated Gyrator element
MTF	Modulated Transformer element
n	Model order. Number of storage elements in independent configuration
out	Subscript denoting output
p	Generalised Momentum
q	Generalised Displacement. Also used to denote bond graph rank in structural analysis.
R	The linear coefficient for a single Resistance (R-element)

S The junction structure matrix

U The input vector of the system

V The vector of complementary variables to the inputs

X The vector of all state variables

$\dot{x}$ The input vector to the system from storage elements in static integral causality (composed of $\dot{p}$ and $\dot{q}$)

$\dot{x}_i$ The input vector to the system from storage elements in dynamic integral causality (composed of $\dot{p}$ and $\dot{q}$)

$\dot{x}_d$ The output vector to the system from storage elements in dynamic derivative causality (composed of $\dot{p}$ and $\dot{q}$)

Y The output vector from the system

Z The vector of all complementary variables to the time-derivatives of the states

z_i The input vector from the system to storage elements in dynamic derivative causality (composed of f and e)

z_d The output vector from the system to storage elements in dynamic derivative causality (composed of f and e)

λ Boolean parameter indicating the state of a single switched junction

$\boldsymbol{\lambda}$ The vector of eigenvalues for a system. Also a vector of Boolean parameters

Λ Switching law. In this thesis, it is a diagonal matrix of Boolean functions that indicates if a mode of operation is active

$\tilde{}$ Denotes dynamic causality

Chapter 1

Introduction

1.1 Modelling

A model is a simplification of a real system. Depending upon the application there are several different types, including physical, mathematical, graphical and verbal. As physical models can be expensive to produce, they are not practical to use for the analysis of new design systems. This is because there can be some unknown dynamics in the system, making the modelling and simulation of mathematical models useful and needed.

Modelling does not only involve a simplified representation of a physical system, but also the analysis of the model behaviour and the study of the components contained in it. Properly modelling a system can help to simulate the response of the system under different circumstances, therefore avoiding the production of costly prototypes; as well as increasing the safety before its production and decreasing time of development. This therefore reduces the final price of the device.

The traditional approach to modelling dynamic systems is the mathematical approach. A 'mathematical model' is the description of a system using mathematical expressions. This description facilitates the study of the different elements contained within the system and allows the analysis of the behaviour of the system using mathematical equations. Usually these equations are generated from a single domain or discipline (Euler, Newton, Lagrange, Bernoulli, and Kirchhoff, to name some type of equations/domains). Therefore, for multidomain systems, it is necessary to generate separate sets of equations in order to represent the system behaviour. This leads to impose constraints on the system equations in order to allow the interaction between the different domains.

This approach is mostly done by hand, meaning that for large systems it takes longer to obtain the system equations. Not only is a large amount of time consumed by using a mathematical model, it is also more susceptible to errors introduced by whoever is analysing the model.

In order to face the challenge in modelling multidomain systems with the potential to contain different kinds of subsystems, a more generalized modelling technique is required, in this case bond graph.

1.1.1 Classical bond graph

Bond graph modelling represents a unified approach to the modelling and simulation of dynamic systems from a graphic representation of the systems. It is defined by Henry Paynter [1] as a graphical modelling technique that displays the physical behaviour of the system on a mathematical model. The interaction between elements is done through energy ports which describe the flow of power through the system.

It is common to find multidomain systems, which are usually analysed by identifying and dividing the different domains as subsystems and, at the end, through physical laws and the set of constraints, the mathematical model of the whole system is obtained. With bond graph approach, this is not necessary due to the unified representation of the elements from different domains. As the analysis is focussed on the transfer of power through the element ports, there is no need to identify the different subsystems, unless there is an interest in obtaining a particular response of a specific set of elements on the system. This allows to link sub-models from different domains more intuitively.

The concept of power transmitted through the system is described in terms of effort and flow (time derivatives of generalised momentum and displacement). These terms are related between the elements by using generalised inertia, capacitor and resistive elements, modulation elements, and ideal sources, which all have the same basic form irrespective of the domain.

1.1.2 Hybrid bond graph

Hybrid models are those which include both continuous and discontinuous equations. These equations are generated by the presence of commuting elements in the model.

The behaviour of all systems can be described by continuous equations. This is exploited by software packages in the form of differential algebraic equations (DAE). Nevertheless, usually rapidly-changing behaviours are considered as discontinuities in order to achieve a high level of accuracy.

Hybrid bond graphs models group the potential of modelling multi-disciplinary non-linear systems (from a bond graph perspective) and hybrid modelling.

This is done by introducing a representation of the commuting elements.

A hybrid bond graph model is achieved using elements that represents ideal switches with some kind of Boolean modulator or control.

There are different methods that have been tried to set a standard notation, such as:

- Modulated transformers [2, 3] that can inhibit the transfer of power between elements. This is done by changing the modulation ratio for each of the configurations (1 for ON and 0 for OFF).
- Flow or effort sources [3, 4, 5, 6, 7] that change its value to null in order to impose an OFF configuration.
- Energy storage elements as switching elements [8] that behaves as a regular energy storage element (with a non-intrusive value), and behaves as a null source during the OFF configuration.
- Switched power junction [9] that behaves as a regular junction during the ON configuration, and during the OFF configuration sever the power in the output by "deactivating" the bond.
- The latest method is the use of controlled junctions [10] that behaves as a regular junction during the ON configuration and as a null source during the OFF configuration.

Nevertheless, no single method has reached a common usage or inclusion as a standard element in bond graph modelling. This is due to the fact that the elements used create a separate model for each possible configuration, or that it needs to include extra elements to the model in order to avoid causal conflicts, therefore making their use optimal just for modelling or just for simulation, at the expense of increasing the complexity of the model. Only the last approach addresses these problems, however, the notation needs further development in order to be used as a general notation, which is the goal of this work.

1.1.3 Singular systems

Hybrid systems are described by dynamic equations, which might represent positions, velocities, or accelerations in Newtonian systems, prices or quantities in an economic system, etc. [11]. In order to select a minimal set of variables, they must be represented in a *singular* form or *descriptor* form.

This singular representation allows the analysis of static and dynamic behaviours in a single equation.

Using the notation of singular systems is helpful for modelling hybrid systems in a compact form.

1.2 Problem formulation

There are previous proposed approaches that are used to analyse hybrid systems, however, most of them avoids change in the number of state elements, which does not always represent the behaviour of the physical system [2, 3, 4, 5, 6, 7, 8, 9, 12, 13].

Most of these approaches set constraints into the model as an aid in order to simplify the equations obtained from the model, or to avoid the use of complex mathematics to represent the interaction between parts of the system that have different nature (e.g. mechanical, electrical, hydraulics, to name some).

Some of the approaches were designed to address the proper representation of the physical behaviour, such as the one proposed by Bonilla in [14, 15] and Margetts in [10]. However, none of these approaches have reached a standard usage due to the complexity of the mathematical notation for large systems or their recent development.

The goal of this research is to propose a standard notation for hybrid bond graph models by using switched junctions and dynamic causality, which will decrease the time of analysis and the number of equations that represent its behaviour. Another goal is to propose a general standard methodology to obtain the valid configurations from the implicit equation; this could be either from a bond graph model or a mathematical model, therefore, an analysis of the matrices can be done (either in singular systems, hybrid systems or hybrid bond graph systems). This is due to the inclusion of previously avoided behaviours caused by the constraints set on the models.
The following objectives were therefore defined:

- To propose a standard notation for hybrid bond graph models, which can be used for the analysis of any hybrid system, regardless of the number of commuting elements or valid configurations.
- To propose a methodology to find the valid configurations of hybrid systems, which can be used regardless of the methodology used to obtain the implicit equation of the system, meaning that can be used with existing mathematical models.
- To analyse the properties of hybrid systems exploiting the mixed Boolean model formulation.
- To apply the method to a selection of case studies.

1.3 Outline

This thesis is organized in a chronological structure: Background, Construction & Analysis of hybrid bond graph systems, Simulation of hybrid systems, and Case studies.

The literature review is presented in chapter 2 in order to explore the previous works on bond graph, singular and hybrid systems, and to allow a comparison with the work done in this research.

An introduction to hybrid dynamic bond graph systems is given in chapter 3, which also includes the different representations of the commuting elements and properties of the systems.

The analysis of these systems using the proposed approach is performed in chapter 4, including analysis of the properties of the systems.

In chapter 5 an introduction to the analysis and simulation of hybrid systems using software is done, where the necessary considerations for simulation are mentioned and explained.

Some case studies are presented and analysed in chapter 6, such as the commonly used buck converter in order to make a comparison between the different approaches.

Finally, in chapter 7 the discussion and conclusions of this work are presented, and some future work based on this research are discussed.

1.4 Novelty and contributions

To address the limitations imposed by previous approaches in the area of singular systems, hybrid systems, and hybrid bond graph systems, an expanded representation of the switched junctions is developed in this work.

Some of the novel aspects presented in this thesis are:

- The introduction of a standard notation for hybrid bond graph models, which will contain all the available configurations on the implicit equation of the system.
- The introduction of the representation of multiswitch (multiple selections in one switch element) and multiway switches.

- The set of rules to obtain a hybrid bond graph model, which are a modified version of the hybrid SCAP (Sequential Causality Assignment Procedure).
- The set of conditions that determines the validity of any configuration.
- The ability to extend the obtained results to other areas of modelling (e.g. singular systems and hybrid systems).

1.5 Publications

Madrigal Salas, J. and Ngwompo, R.F., Modelling and Analysis of Hybrid Physical Systems Using Bond Graph and Dynamic Causality. *Eurosis ESM'2015*, pp. 16-21, Leicester, UK, November 2015. (ESM'2015 Conference Proceedings, 2015)

Madrigal Salas, J. and Ngwompo, R.F., Analysis of switched systems using mixed Boolean singular representation. Submitted to SCS Simulation: Transactions of the Society for Modeling and Simulation International, under revision.

Chapter 2

Literature review

2.1 Preliminaries

In this literature review several topics are involved due to the complexity of hybrid bond graph systems. This is due to the proposed approach combining bond graph methodology with singular systems and traditional hybrid systems. Therefore, it is necessary to have an insight of the different topics involved and the analysis of the control properties.

Bond graph methodology is briefly reviewed as the chosen technique for modelling and analysis of hybrid systems, as this modelling technique allows a simplified representation of multidomain systems.

In order to understand the behaviours and elements involved in hybrid bond graphs it is necessary to introduce hybrid modelling and singular systems, this will set a precedent and also introduce the fundamentals of hybrid bond graph modelling.

Singular systems theory is reviewed as this is a useful representation of dynamic systems, which have a similar representation to hybrid systems.

Previously proposed notations for hybrid bond graph elements are reviewed in some detail to show the development on hybrid bond graph modelling.

The basics of the analysis of hybrid systems are presented, including some of the approaches that intend to address the proper representation of physical phenomena in a mathematical model, such as rectangular variable structure systems.

After the introduction of the basics of hybrid modelling, the analysis of the models is addressed. Different methods from classical control theory are reviewed, before demonstrating how control properties have been applied to both standard and hybrid bond graphs.

An introduction to dynamic causality in bond graph models is presented. Dynamic causality is studied in order to allow the representation of physical phenomena (change in the behaviour of the variable states) during commutation, which is not reflected using traditional mathematical approaches.

Based on these topics, the results are expected to be expanded in all of the involved domains (e.g. singular systems, hybrid systems, hybrid bond graph systems).

2.1.1 Classical modelling of hybrid systems

The classical modelling of hybrid systems is a variation of the linear time invariant (LTI) systems. However, hybrid systems contain continuous and discontinuous behaviours.

A problem arising from the proposed approach for the modelling of such systems, is that the system is considered to always have the same order. This means that the elements of the system behave the same in all of the configurations, which does not always represent the behaviour of the physical system. This can be seen in works such as [16, 17, 18, 19], where all of the analysed systems maintain the same order for all of the configurations, which is achieved by mathematical manipulation of the equations to simplify the analysis and simulation of the systems.

Singular systems are usually described by:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2.1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{q\times n}$.

As mentioned before, the analysis of the system (2.1) using the approaches introduced in the previous works does not allow changes in the order of the system, which does not represent the physical behaviour of some of the configurations.

Traditional switched systems are described in the form:

$$\dot{x}_\sigma = A_\sigma x + B_\sigma u \qquad \text{where } \sigma = 1, 2, \ldots, n$$

where $A_\sigma \in \mathbb{R}^{n\times n}$ and $B_\sigma \in \mathbb{R}^{n\times m}$.

It can be seen that the E matrix does not appears in this case, as they do not need to be singular to be solvable.

In order to allow a proper representation of the physical phenomena of the systems, a new representation of the systems is needed.

2.1.2 Variable structure systems

Variable structure systems are those that have either variable order, variable input sign or variable parameters [14].

Switched systems are a type of variable structure system, where the number of states could change during commutation. This change in the number of states represents the physical behaviour of the system in the model.

An approach to the representation of the physical phenomena into a mathematical model was introduced by Bonilla in [14], where variable structure systems are used to represent many behaviours of different nature.

In order to properly represent the physical phenomena, Bonilla [14] used rectangular implicit descriptions.

This is done when there are more state components than state equations, which is usual in systems with switches.

To illustrate this, the following example is used.

For systems containing two (or more) switches, such as the one displayed in Figure 2.1,

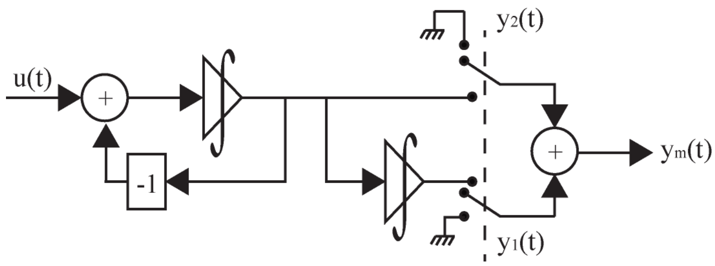

Figure 2.1: Example of a variable model [14]

the variable structure that describes the behaviour is represented by the following implicit differential equation:

$$\begin{aligned} \ddot{y}_1(t) + \dot{y}_1(t) + \dot{y}_2(t) + y_2(t) &= u(t) \\ y_m(t) &= y_1(t) + y_2(t) \end{aligned} \tag{2.2}$$

under the additional constraint:

$$y_1(t) \times y_2(t) \equiv 0 \tag{2.3}$$

where y_1 and y_2 are the non measurable outputs (switches), y_m is the measured point, and u the input of the system.
This behaviour can also be described by the following implicit description:

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{x}(t) &= \begin{bmatrix} -1 & -1 & 1 \\ 0 & -1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) \\ y_m(t) &= \begin{bmatrix} 1 & 1 \end{bmatrix} y(t) \end{aligned} \tag{2.4}$$

with for all $t \geq 0,\ x_1(t) = 0$ or $x_2(t) = 0$.

It can be seen that the representation of hybrid systems using variable structure equations allows a simplified representation of the available configurations. However, this rectangular representation complicates the analysis of the control properties of the system.

Nonetheless, this approach is useful to determine that the equations describing hybrid systems can be simplified into a general equation, which is an alternative to the classical approach to switched systems.

However, this is not as simple to implement as the previous approaches, leading to the development of a new representation which will be useful, and is proposed in this work, as it allows the proper representation of the physical phenomena of the systems.

2.2 Bond graph modelling

Henry Paynter developed bond graph modelling in the late 1950's and this was later formalised as a methodology by Karnopp, Margolis and Rosenberg in their textbook 'System Dynamics: Modeling, Simulation, and Control of Mechatronic Systems' [20]. In short, bond graph models can be defined as follow,

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Definition 1 ***Bond graph model [20]:*** *"A bond graph model is an abstract representation of a system where a collection of components interact with each other through energy ports and are placed in a system where energy is exchanged".*

Bond graph main element is the 'bond' (represented as a half arrow), which represents the power transfer between standard elements. These standard elements are the masses, compliances and resistances that are used for modelling in mechanical engineering. The power transferred between the elements is given as a generalised effort and flow, which allows the method to be extended to any engineering domain. Different elements are required to construct a bond graph model. These elements are ideal descriptions of physical phenomena and represent the principal energy processes, such as energy sources (in the form of effort and flow sources), power dissipative elements (resistances), power storage elements (capacitances and inertias), and power transformer elements (transformers and gyrators).

Causality establishes the orientation of effort and flow in a bond. The causal stroke (vertical line at the end of a bond) indicates the direction in which the effort signal is directed. This is displayed in Figure 2.2.

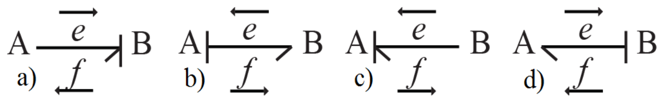


Figure 2.2: Causality on a bond

For computational models, the causality assignment is done following SCAP (Sequential Causality Assignment Procedure) ([21, 22]). SCAP assigns computational causality so that the maximum possible number of elements is set in *integral causality* (independent variables for energy storage elements). If there is a causal conflict, a change in causality is made in one or more elements by changing the integral causality for *derivative causality* (dependent variables for energy storage elements).

If the flow f is considered as an independent variable in the constitutive equations of the element A in the Figure 2.2, then there is an equation among the constitutive equations of the element A that enables the computation of the conjugate effort e. However, the flow f can only be an independent variable in the equations of the element A if one of the constitutive equations of B allows for computation of flow f, which means that the effort e is previously known. This can also be on the other way around.

Making a choice between the obtaining of effort e and flow f is called an assignment of *computational causality.* Causality assignment, in bond graph modelling, means that it has been decided which one of the two conjugate variables at a power port is the external one, meaning that this variable is considered as independent in the constitutive equations of the multiport element. Therefore, the power conjugate variable can be obtained in that element. This is further described by Borutzky in [23].

Most of the previous work in hybrid systems is based on results obtained in traditional bond graph modelling where causality is static. This is because some approaches to the modelling of hybrid systems attempt to preserve the fixed causality. Not to mention that preserving fixed causality simplifies the analysis and avoids undesired behaviour of the system. Nevertheless, for a proper representation of hybrid systems the causality must change depending on the different configurations of the switched elements; therefore dynamic causality is needed in order to properly represent the changes during commutation. This is discussed in the following section.

2.3 Hybrid systems modelling

2.3.1 Hybrid systems

A hybrid system is a system that contains both continuous and discontinuous behaviour as defined in [10]. In this type of system, the discontinuities are "caused" or controlled by a commuting element or a series of commuting elements. Commutation creates 2^n (where n is the number of commuting elements) number of models, which are called operational modes or configurations.

As an introduction to the analysis of hybrid systems, it is recommendable to analyse the change from continuous to discrete configuration. An example of this is the work of Mosterman [24], in which the analysis of the change of configuration occurs in a small lapse of time; therefore, the principle of divergence of time is used to ensure that the model does not falls into an infinite loop of instantaneous changes without moving in time.

The continuous events are modelled as a set of differential equations. These differential equations are supplemented by algebraic constraints that reflect underlying physical principles (such as Kirchhoff's laws, or Ohm's law to name some). Meanwhile, the discrete events are modelled using discrete states and switching functions. The discrete states correspond to real modes, where the behaviour of the system is governed by energy principles; and mythical modes, where the system behaviour transitions are instantaneous (there is no loss of energy). The interaction between the continuous and discrete parts consists of (a) discrete events generated by the continuous signal, and (b) a change of operational mode by the discrete model, requiring a consistent mapping of the continuous state vector.

There are other approaches to hybrid system analysis, such as the one proposed by Bornot [25] where the systems were defined by two types of rules: synchronization rules, that specify how an action is the result of the simultaneous processes that occur at the same time; and the interleaving rules, that specify how an action of a process is the result of the action of another process from a previous time. The combination of these two rules is essential for the proper analysis of the systems; however, the combination of them must satisfy two conflicting requirements, deadlock-freedom and maximal progress. Deadlocks may appear when there is a conflict between two states and can only be avoided by using interleaving rules. Maximal progress refers to the preference of synchronization over the interleaving when both are possible. The main difference between this approach and others is that this does not rely on a time basis, but on a parallel composition of the system.

A model of a hybrid system can be described as *hybrid automaton* i.e. one that contains both finite and continuous state spaces.

The terms *hybrid* and *switched systems* are used indistinctly in the bond graph literature, although, those are different type of systems, as explained in the following definitions.

Definition 2 ***Switched systems [26]***: *"Switched systems comprise a family of dynamical subsystems together with a switching signal determining the active system at a current time".*

Those are a subset of hybrid systems, where the discontinuous behaviours are introduced by on/off switches or another binary signal. Having this in consideration, any bond graph switched model always gives a hybrid bond graph model.

Hybrid models are categorised in [27] into *Switching* and *Impulse* models, which can be *Controlled* or *Autonomous*. Switching models are those that presents discontinuous changes when a state reaches a boundary. Meanwhile, impulse models are those that presents discontinuous changes when a state changes impulsively on specific regions of state space (e.g. when a collision occurs).

Definition 3 ***Switched Model [10]***: *"A Switched Model is a subset of Hybrid Model, which contains continuous equations and binary switching variables. The 'switches' select the active continuous equation(s) or behaviours at a given time."*

Definition 4 ***Impulse Model:*** *An impulse model is a subset of hybrid model where the state changes impulsively, which means that the changes in the model does not depends on switching signals, rather depends on impulse behaviours that are caused by physical phenomena.* [1]

[1] All the original contribution resulting from this research is indicated in boxes.

It must be taken into consideration that hybrid models are usually analysed as systems under controlled commutation in order to avoid instabilities or undesired behaviours. This is because some changes of configuration in short periods of time can create impulse modes, which in some cases can lead to instabilities or undesired behaviours.

As most of the physical systems contain discontinuous changes, more studies focused on hybrid systems have been done recently.

Most of the work have been done in the control area, which is important for the safety during the operation of physical systems.

These works can be divided into general control techniques and control techniques for specific purposes. The general approaches try to set a standard control technique for hybrid systems, this is the case of the work done by Branicky in [28], where an optimal hybrid control is presented. In this work, some initial steps towards a unified state space representation for hybrid control are presented, however, this could not be achieved because the used approach could not properly represent some physical behaviours (viscosity in this case). Some constraints are set to the model in order to use the proposed approach. Also, the computational power at the time of the development of the approach was limited, which decreased the impact of the approach at that moment. Some of those problems where addressed by Hedlund and Rantzer in [29], where dynamic programming and convex optimization where used.

As the control techniques in previous approaches were based on a synchronous switching, new approaches were proposed to address the asynchronous switching. That is the case of the work done by Hu *et al.* in [30], where a conic switching law is proposed, which not only stabilises the entire system, but also possesses robustness properties. This approach is limited by setting a constraint in the switching conditions, which are limited to a region of commuting to avoid instabilities. This is illustrated in Figure 2.3 to show how the conic switching occurs.

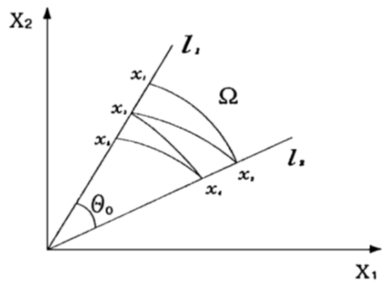


Figure 2.3: Switching happens on the boundary of the safe region Ω : $l1$ and $l2$.

One of the most recent approaches to control of hybrid systems was done by Jing *et al.* in [31], where a sliding mode control is proposed. This approach makes hybrid systems sturdier to uncertainties and perturbations, however, the switching signals must be controlled.

While the previous approaches were designed as general techniques, there are some proposed approaches that were done to fit certain type of hybrid systems.

That is the case of the proposed supervisory control technique introduced by Koutsoukos *et al.* in [32], where the approach is developed based on the process done in the chemical industry. This approach was modified in [33] to expand the technique to mechanical and electrical systems.

A predictive control for DC-DC power converters is proposed by Patino *et al.* in [34], where the proposed approach reaches an optimal periodic cycle to remain in a steady state behaviour.

There is a recent approach proposed by Chen *et al.* in [35] using neural networks. This approach uses a neural network to introduce an adaptive law in order to improve the performance of the system.

These approaches could not reach a broader use because the techniques need to be modified for different engineering domains, or even different systems from the same domain.

As hybrid systems contains several configurations, in order to simplify the representation of the system behaviour into a single general equation it is necessary to use a different notation. Boolean algebra allows the representation of several combinations into a single equation in the form of logic arithmetic.

2.3.2 Boolean algebra

Boolean algebra can be a helpful tool in the analysis of switched systems. This is because the behaviour of some systems contains abrupt changes in a small lapse of time. Systems containing abrupt changes in its behaviour are difficult to analyse as continuous systems if similar behaviours appear on several occasions. These behaviours can then be considered as discontinuities, which can be represented as different configurations of the system. Therefore, the desired lapse of time can be analysed independently to simplify the analysis of the system. In those cases, Boolean terms are used to represent the changes at different lapses of time as discontinuities in a continuous-time notation. This allows to have simpler equations that describe the system's behaviour, instead of having a series of differential equations that represents the evolution of the system behaviour. Also, Boolean expressions can be used to simplify the notation of the different configurations into a single equation.

An example of the simplification on notation is presented in the work of Lewis [36], where it can be seen that previous approaches to singular systems were done using differential equations containing Jacobians. The use of Jacobians allows to write the equations of the system as separated differential equations, in which a single equation represents a determined lapse of time in which a commutation occurs. This approach usually led to set algebraic constraints into the system equations, this is because the set of differential and algebraic equations that represents the system's behaviour are treated as a single entity, resulting in their elimination by introducing a set of admissible initial conditions with nonzero input, which causes loss of information from the system.

Using Boolean algebra and singular systems notation, simplifies the analysis of continuous and discrete behaviours on systems by decreasing the algebraic constraints set, allowing a better representation of the singular systems and a closer behaviour response to the physical response of the system. The constraints in this case are caused by physical relations between the different domains involved in a system, rather than constraints imposed to avoid instabilities in the system.

Boolean algebra has been used in the design of switching systems. This is due to the fact that the switching conditions for large systems can be reduced to a single equation when Boolean algebra is introduced in its notation.

The use of Boolean algebra in the design of switching circuits is introduced by Staehler in [37]. In this case the use of Boolean algebra does not only allow the reduction of switching elements, but also improves the controllability and observability analysis of such systems.

It is shown that previously all the control techniques applied to switching systems was made for sequential switching, which is no longer the case when Boolean algebra is applied to the model. This does not only allow better control techniques, but also the reduction in costs of deriving mathematical models.

Another use of Boolean algebra is in control. An example of this is found in [38], where Boolean algebra is used to avoid using Pulse Width Modulation (PWM) for control purposes. While using PWM, the system may lose some information of the control law. This is because the control strategies (PI, PD, or PID) are adjusted using PWM, which does not always fit to the desired parameters. To avoid this, Boolean algebra is used along sliding mode control (SMC). This allows the design of a control based on PID techniques, having as result similar behaviour as the ones obtained with a PWM control.

These previous works are used as base for the design of hybrid bond graph model's notation and the analysis techniques.

2.4 Hybrid bond graph

2.4.1 Hybrid bond graph modelling

There are different proposed methods to represent model discontinuities in bond graph without reaching a standard notation. This is due to each representation providing an advantage depending on the application.

The first approach to represent the behaviour of a switched element in bond graph was to use modulated transformers (MTF). This can be seen in [2] and [3], nevertheless, the use of MTF elements increased the complexity of the systems and increased the number of elements in the model, not to mention that in some models, this introduces changes on causality, which also increased the time of analysis of the system.

In the former work, it can be seen that the analysis of the systems using bond graphs was done differentiating the configurations, and then analysing one configuration at a time, not taking into consideration any behaviours introduced by the commutation. For these reason, the MTF were used adding a resistance that has a different value for the ON and OFF configuration of the switch.

The problem using MTF elements to represent switched elements is that for each MTF element added to the model, the complexity of the system was increased. This also increases the time of analysis for systems containing large number of switched elements, as the number of configurations increases exponentially, thus, making them more susceptible to errors from the modeller due to the increasing number of equations on the system.

Meanwhile in [3] a relation between R, C and I elements is used to help in reducing the complexity of the equations created by the MTF or MGY (modulated gyrators), at the cost of increasing the order or the number of equations in the system.

The first and most used representation for switched elements (as mentioned in [3, 4]) is the use of flow or effort sources as a commuting element or ideal switch.

For some models, resistor elements are used to help maintaining the integral causality of the elements. Examples of this are shown in [5, 6]. The resistor element is called *causality resistor*.

When this method is used, three new vectors need to be introduced to the system in order to represent the commuting element. These vectors are S^m_{out} composed of the sources acting as switch elements, D^m_{in} and D^m_{out} to describe the variables going into and out of the causality resistor.

The problem using causality resistors is that some of the variable eigenvalues can result in a high value, which creates numerical difficulties during simulations. This is due to the causality resistor selection being arbitrary, creating undesired behaviour and thus the amplitude of some variables has no physical meaning.

Sources as commuting elements can be used with the help of Karnaugh maps and Boolean functions for a less complicated system, as shown in [39] where there are several switches connected into the system. The problem with the implementation of Karnaugh maps and Boolean functions is the need of auxiliary switches in order to maintain the causality assignment for some models, which defeat the purpose of using this notation.

A less used approach to the modelling of switched bond graph systems is introduced by Gawthrop in [8]. This approach combines the concept of the switch element (effort- or flow-source) with an I or C element. This proves to be a great help for simulations at the cost of increasing the number of energy storage elements, which increase the complexity of equations.

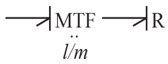
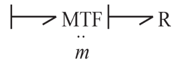
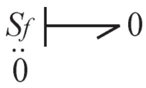
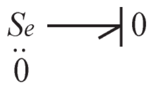
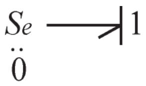
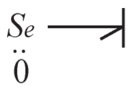
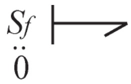
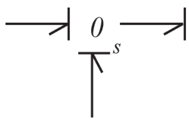
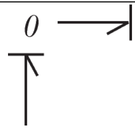
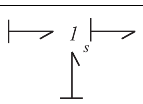
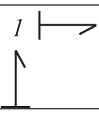
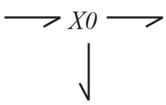
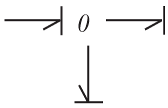
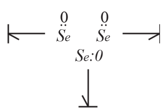
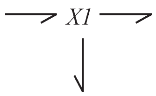
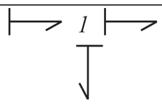
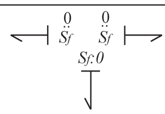
There is also a switched power junction proposed in [9]. This power junction has two or more mutually exclusive effort- or flow-deciding bonds (instead of having an ON or OFF configuration), allowing the rest of the system to remain with the same causality.

The introduction of these junctions allows a traditional analysis of the systems due to the lack of any new element that could affect the system equations. The inconvenience of using this switched power junction comes when there are more than two switched elements connected to the same power junction. This is due to the complexity of the equations that rule the switching of the elements which could create confusion to the user.

One of the latest approaches to the switched systems can be found in [10], where the controlled junctions are proposed as a two-port element associated to Boolean parameters. These Boolean parameters show the state of the junction (ON or OFF configuration of the switch element). These junctions cut the flow or effort according to the Boolean parameter, changing the causality of the port, therefore propagating dynamic causality in the bond graph model. The analysis made using these junctions is simple and avoids the increase in size or complexity of the system.

A graphical comparison and a description of the representations of switches and commuting elements in bond graph can be seen in table 1. These representations were previously described, mentioning the disadvantages in their use in bond graph models.

Table 1: Representations of switching elements in Bond Graph

Method	Bond Graph Representation			Description
	Element	Equivalent ON	Equivalent OFF	
Modulated transformer and resistor combination [2, 3]				m is Boolean (1 when ON and 0 when OFF), resistor commutes its causality assignment.
Switched source (Switched element) [3, 4, 5, 6, 7]				Commutes between a source of null flow and null effort, imposing a zero flow or effort according to the connecting junction when OFF.
				
Controlled storage element [8]				This is a compound element, acting as a storage element when ON, and as a null source when OFF. The values of the storage elements are selected to be of no consequence to the analysis of the model.
				
Switched power junction [9]				There are two mutually exclusive causal inputs to the junctions, s denotes the active input bond.
				
Controlled junction [10]				Is a regular 0- or 1- junction when ON, and act as null source on each bond when OFF.
				

2.4.2 Causality assignment

As previously mentioned, causality assignment establishes the orientation of effort and flow in a bond, where the flow or effort is selected as an independent variable in order to compute the conjugate variable. The causality is assigned on all the elements of the system in order to compute all the unknown variables to obtain the system behaviour. Some of the assignments are named (fixed causality, constrained causality, preferred causality, indifferent causality) by Broenink in [48] where dynamic causality is introduced.

Different procedures to assign causality have been proposed since the beginning of the development of bond graph, with SCAP (Sequential Causality Assignment Procedure [20]) being the most used.

There are several examples that allow to show how the causality assignment is helpful towards the analysis of the model. Some of the most useful are those made by Margolis and Rosenberg ([40, 41]), which demonstrate how the bond graph causal assignment (using SCAP) can be exploited to aid its simulation. This is done by systematically deriving state equations and gain insight into the system.

The assignment does not need to be done by hand, it can also be done using simulation software like Enport [23, 42, 43], AMESIM [102, 103], SYMBOLS2000 [102, 104], MS1 [23, 44], CAMP-G [23, 45], or 20-SIM [23, 46, 47].

Causality assignment can be used to reveal some of the control properties of the system such as observability and controllability on a graphic representation ([7, 23]).

Dynamic causality appears when there are switched elements. This type of causality assignment depends on the configuration of the commuting element which can be ON configuration or OFF configuration. The causality propagates to the nearest elements until it reaches an energy storage element or a resistive element.

2.4.3 Dynamic causality

From the previous representations of switched elements, it is clear that the causal assignment of the bond graphs can change during a commutation. This phenomena is called *dynamic causality*, and was addressed separately by Asher [49], Cellier *et al.* [50], Stromberg *et al.* [51] and Bidard *et al.* [52].

Dynamic causality is difficult to simulate, and for this reason some procedures to avoid it have been proposed. Low *et al* [53] proposed a Hybrid-SCAP for use with their proposed version of the controlled junction, which gives causally static bond graph models.

Dynamic causality is a consequence of the commutation of a switch element. Asher [49] proposed a *causality resistance* in order to minimize and control it. This resistance is useful for simulation; nevertheless, the use of this element can create a model without resemblance to the physical system.

The use of this causality resistance facilitates the modelling of hybrid bond graph systems in commercial software packages. Some examples of this are the ones produced at University of Twente by Breedveld and Van Kampen who have been working in the commercial bond graph simulation package 20sim [46, 47]. For these examples, controlled junctions are used including causality resistances to model friction [46], a Newton's Cradle [47], and a copier machine [54]. These are some examples where this causality resistance is helpful to simulate those systems, nevertheless there are other examples like the coupling of two DC motors using a clutch (presented in [55]) where the model is overly complex and does not relates completely to the physical system.

There is a discussion on whether dynamic causality must be used or not in hybrid bond graph models. One of the objectives of this work is to set a standard representation of switching elements and its notation, which have as consequence dynamic causality. Also, a generalized mixed Boolean formulation of switched systems will be proposed, in contrast with current studies that do not account for the changes in the order of the system.

As it is later shown, the effects of dynamic causality are minimised if the assignment is properly done following the proposed procedure.

2.5 Model properties of LTI systems

It is of interest for the modelers to not only run a simulation of a model (e.g. response in time of the state variables), but also to obtain all the model properties (e.g. controllability, observability, stability).

These properties can be obtained either from the information stored in the implicit state matrix, due to the interaction between state variables and inputs/outputs being contained in this matrix; or by visual analysis of the bond graph model (in some cases), which is done by following the causal paths between the elements and inputs/outputs.

The analysis of the model properties, using the approach proposed in this work, will allow to obtain some missing properties (such as general controllability and general observability of the system) if the analysis were done using a different approach.

It is also expected to expand its application to singular systems, hybrid systems, and hybrid bond graph systems, which are some of the model representations this approach is based on.

2.5.1 State and Implicit models

The state space representation of a system allows to find control properties. Kalman's General Theory of Control Systems [56] introduces this theory. It is common to find the representations of the state space equation in the context of the explicit linear time-invariant (LTI) Equation 2.5:

$$\begin{aligned} \dot{X} &= AX + BU \\ Y &= CX + DU \end{aligned} \tag{2.5}$$

where $X \in \mathbb{R}^n$ is a vector of state variables, $U \in \mathbb{R}^m$ is the vector of inputs, $Y \in \mathbb{R}^p$ is the vector of outputs, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$. The method used in this thesis is in the form of an implicit system model (2.6):

$$E\dot{X} = AX + BU \tag{2.6}$$

where $E \in \mathbb{R}^{n\times n}$ is an additional matrix that may be singular.
This formulation is used because this is the form that is naturally derived from the bond graph junction structure and element constitutive equations.

Singular systems, semistate systems and descriptor systems are specific implicit forms which have been investigated in detail. These forms were developed for specific cases, nevertheless, the terms are used interchangeably in the literature.

In [57], Yip and Sincovec established the different properties of the descriptor systems, while Verghese *et al* presented a generalised theory for singular systems in [58]. Both works present control properties for implicit systems that are similar to each other, one from a singular systems approach, the other from a descriptor systems approach, nevertheless the results are similar, making both approaches to be considered the same for general purposes.

Another interesting approach for the analysis of singular systems is done by Dai in [59], where the analysis is done from a matrix-rank criteria only, which demonstrate the matrix-rank criteria is a sufficient condition to determine the controllability and observability. These results are useful in the hybrid bond graph area, therefore are used in the generation of equations that describe the states of hybrid bond graph systems.

2.5.2 Solvability, Order & Rank

Solvability of a system refers to the existence of a unique solution for any given input and any given admissible initial condition corresponding to the given input.

For singular systems described by Equation 2.6, this is translated as:

Definition 5 ***Solvability of singular systems [56]:*** *A singular system is solvable if* $\det(sE - A) \neq 0$.

Wlasowski and Lorenz determines the solvability of Bond Graph model in [60], where they establish the condition necessary for this:

Consider that the bond graph junction structure, displayed in Figure 2.4, have the following flow equation:

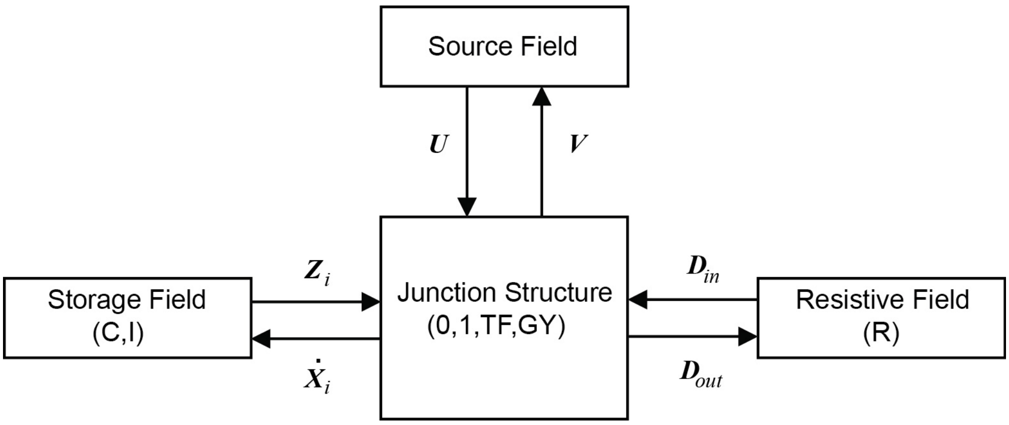


Figure 2.4: Junction Structure

$$\begin{bmatrix} f_{out} \\ f_{int} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} f_{in} \\ f_{int} \end{bmatrix} \tag{2.7}$$

where f_{out} is a vector of all the flow variables coming out of the junction structure, f_{in} is a vector of all the flow variables going into the junction structure, f_{int} is a vector of all the junction structure internal flow variables. Therefore, submatrices A_1, A_2, A_3 and A_4 describe the flow transfer that occurs in the junction elements. Also, matrix A_4 is necessarily a square matrix. The values of these submatrices depend on the elements that are connected to the junctions (0 or 1 junctions).

If the internal variables f_{int} are eliminated from (2.7), the resulting equation is:

$$f_{out} = F f_{in} \tag{2.8}$$

with

$$F = A_1 + A_2 \left(I - A_4\right)^{-1} A_3 \tag{2.9}$$

Leading to the condition for the solvability of the junction structure:

– The matrix F exist if $(I - A_4)$ is invertible, which means that there are no causal loops with a unity gain in the model.

This shows that singular systems and bond graph systems are only solvable if there are no causal loops with a unity gain in the system, also the notation is the same as bond graph are dynamic systems, which could lead to have compatible analysis techniques.

2.5.3 Controllability of LTI systems

Controllability is one of the most important results in the analysis of linear systems. It depends on the matrices A and B from the LTI state space Equation 2.5. The necessary controllability condition is given by:

$$rank \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix} = n \tag{2.10}$$

where n is the order of the model.

Kalman in [61] makes a differentiation between controllability and reachability, which led him to define the controllability of the system state variables. In here, it is explained that an event τ can be controllable if and only if it can be transferred to 0 in a finite time by an appropriate choice of an input function ω, while an event τ can be reachable if and only if there are an event s and an input function ω that depend on the event τ. This means that an event is controllable if an input can modify its behaviour at any time, however, it is reachable if it can only be controllable during determined periods time.

The study of hybrid systems in the control area is essential in designing sequential supervisory controllers for continuous systems, and it is central in designing intelligent control systems with a high degree of autonomy.

For traditional hybrid system, Mixed Logical Dynamic (MLD) notation is used for its analysis. This is because MLD allow to model nonlinearities, interdependent physical laws, logic rules, and operating constraints. Wolff and Murray introduce its use in [62]. where MLD tends to depend on Linear Temporal Logic (LTL) to be able to analyse the model of the system. This dependence is due to the LTL being the conditions that need to be presented at a determined lapse of time in order to obtain a desired behaviour. Therefore, by using MLD and LTL, there is more computational effective method to analyse hybrid systems. However, this approach allows to control hybrid systems by planning signal 'paths' (desired behaviour) at the cost of setting constraints on the model, which is something undesired to obtain a more physical related analysis.

As most of the analysis theories are based on previous notions of inputs/outputs, a different approach was proposed by Willems in [63], where the controllability of the system is obtained without the need of obtaining the state space equation. This approach is called behavioural, and is defined as:

Definition 6 ***Behavioural Controllability [63]:*** *"A behaviour is defined to be controllable if it is possible to transfer from any past trajectory to any future trajectory, while obeying the dynamical laws of the system ... This definition is applicable to nonlinear, discrete event and delay differential systems without having to introduce a state representation."*

Controllability is closely related to reachability (a state is reachable from any initial condition), stabilizability (controlling states to reach a steady state), observability (dual of controllability) and stability.

2.5.4 Observability of LTI systems

Observability is the dual property of controllability, therefore, the conditions to determine the observability of a system are equivalent from those of controllability.

Definition 7 ***Duality Principle* [61]:** *"Every problem of controllability in a real, (continuous-time, or discrete-time), finite dimensional, constant, linear dynamical system is equivalent to an observability problem in a dual system."*

The observability matrix depends on the matrices A and C, due to observability being established by solving the output, hence the observability condition:

$$rank \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} = n \tag{2.11}$$

For hybrid systems, the process of designing an observer consists of two parts: a location observer and a continuous observer. Location observer refers to identify the current location of the hybrid plant; the continuous observer produces an estimate of the evolution of the continuous state. This is explained in detail by Balluchi *et al* in [64].

2.5.5 Stability of LTI systems

Stability is the property of a system by which its behaviour will reach a steady state condition within enough time.

This area is the one in which most advances had been made for the switched systems, even though the studies on these systems are relatively new ([30, 65, 66, 67, 68, 69, 70, 71]). Most of the first stability studies were adaptations of continuous systems techniques, leading to a great number of errors and unstable modes in high-order systems.

For Lyapunov's stability theory in non-switched systems, there is a globally asymptotically stability (system is globally asymptotically stable (G.A.S.) if for every trajectory $x(t)$, there is $x(t) \to x_e$ as $t \to \infty$, where x_e is the equilibrium point).

There is also a globally exponentially stability (system is globally exponentially stable (G.E.S) for $\dot{x} = f(x)$ there is a M that satisfies $\|x(t)\| \leq Me^{-\alpha t/2} \|x(0)\|$, where α is a constant with value bigger than 0).

In order to implement this theory to switched systems, instead of aiming to obtain stability for each particular signal, a somewhat stronger property is needed, in this case, named uniform asymptotic or exponential stability (GUAS and GUES, respectively). This is because the stability property need to be uniform over the set of all switching signals, which must be independent of the choice of the switching signal. Liberzon showed this in [65] where this approach is used for arbitrary and constrained switching.

Some of the issues for the stability analysis of switched systems are caused by the switching signal. Even if all of the switching subsystems are stable, unconstrained switching may destabilise a switched system. It may be possible to stabilise a switched system by means of suitably constrained switching, even if some or all switching subsystems are unstable.

For hybrid systems, stability can also be defined as:

Definition 8 ***Asymptotic Stability of Switching Systems [65]****: A state is asymptotic stable if it asymptotically approaches the origin (i.e.* $x_0 = 0$*) through a series of switching.*

2.6 Equations generated from bond graph

Before analysing any of the properties of the bond graph model, it is necessary to establish how equations are obtained from the bond graph. In Karnopp et al [20] the state space equations are obtained from a bond graph by hand procedure, establishing that energy variables associated with storage elements are the state variables. To determine which of the effort and flow signals at a port is the input, a perpendicular line to the bond graph is added. This line is called *causal stroke*, which indicates the direction in which the effort signal is directed. For large models, the equations can be derived from the junction structure matrix and the constitutive equations of elements.

The junction structure is the relation between the inputs and the outputs of the different elements of the model. Bonds can be classified as internal and external. Internal bonds are those that represent the interaction between junctions (0 and 1 junctions, TF and GY). While the external bonds are those that represent the interaction between sources/detectors, storage elements, and dissipative elements. This relation is illustrated in Figure 2.4.

From this relation a junction structure matrix can be derived. This matrix contains 0's and 1's that represents the relation between the inputs and the outputs to the structure of each element. If there is a relation between elements that contains a transformer or a gyrator, a modulation term a is introduced instead of a 0 or a 1. These relations are displayed on Equation 2.12:

$$\begin{bmatrix} \dot{X}_i \\ Z_d \\ D_{out} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \end{bmatrix} \begin{bmatrix} Z_i \\ \dot{X}_d \\ D_{in} \\ U \end{bmatrix} \tag{2.12}$$

where X is the vector containing the state variables, the subscripts i and d denotes the integral and derivative causality respectively. Z are the coenergy variables, D is the inputs and outputs of the resistive elements, and U are the sources or external inputs.

2.7 Sequential Causality Assignment Procedure

There are several causality assignment procedures proposed in bond graph literature, but the most widely used is the Sequential Causality Assignment Procedure (SCAP).

SCAP consist in the computational assignment of causality so that the maximum possible number of elements is in integral causality, i.e. inertia and compliance elements are assigned an output so as to put underlying equations in terms of the integral of the input. This is known as *integral causality*, and the reverse case (where there is a derivative term in the constitutive equation) is known as *derivative causality*.

SCAP was proposed by Karnopp and Rosenberg in [22], and the steps to follow are:

1. Assign causality to a power port of one of the sources according to its type and propagate this causal information into the bond graph through its junction structure as far as possible by observing causality rules at element ports. Repeat this step until all ports of all sources are assigned an appropriate causality.

2. If there is a resistor port with a characteristic that does not have a unique inverse, assign the required causality to ensure correct formulation. For example, the Coulomb friction between two sliding bodies is most commonly assumed to be proportional to the sign if the velocity difference between two bodies. Propagate this causal information into the bond graph through its junction structure as far as possible by observing causality rules at element ports. Repeat this step until all such resistor ports have their correct causality.

3. Assign preferred integral causality to a port of energy store and propagate this causal information into the bond graph as far as possible. Propagation of the causality at a storage port may lead to derivative causality at power ports of other energy stores and often entails an assignment of causality at resistor ports. For instance, if an I element and a 1-port resistor representing Coulomb friction are attached to a 1-junctino, then the I element port must take derivative causality. Repeat this step until all storage ports are assigned a causality.

4. Finally, if there are still resistor ports or internal bonds without causality, one resistor port or an internal bond must be chosen. Causality is arbitrarily assigned and propagated through the junction structure. This step is repeated until no causally unassigned bonds are left.

2.8 Control Properties of Standard bond graph

2.8.1 Order & Rank

The order of the bond graph model is defined as:

Definition 9 ***Bond Graph Order*** (n) *[12]: "The number of storage elements in integral causality when a preferred integral causality is assigned to the bond graph model."*

It must be noted that the order (n) of ODE is given by the number of state variables. In a bond graph this order is defined by physical considerations, e.g. the inertia and compliance elements in integral causality.

Rank refers to the number of linearly independent columns (or rows; column and row ranks are equal) in a matrix. It is an indication of the number of dynamically independent state variables. Many of the standard control properties for descriptor systems are defined using matrix-rank criteria.

The rank of a bond graph model is helpful to obtain some of the model properties. In [72], Dauphin-Tanguy et al define a 'bond graph rank' which corresponds to the numerical rank of a matrix, this is due to the correlation between dependent elements on the mathematical representation and bond graph model.

2.8.2 Controllability of bond graph systems

In the field of bond graphs, the concept of structural controllability has been used as a more physically meaningful approach than classical state controllability.

Property 1. Structural Controllability of a bond graph [73]: "The model is structurally controllable if and only if:

1. There is a causal path between each dynamical element [in integral causality] and a modulated control input, i.e. all states (nodes) are input-reachable.
2. $Struct\_Rank[A] = n$, where the structural rank of $[A]$ is equal to
   (a) the rank of $(S_{11}\ S_{13}\ S_{14})$ or
   (b) $n - t_s$

   where t_s is "the number of storage elements remaining in integral causality when:

   (a) a derivative causality assignment is performed,
   (b) if the rank is not full, a dualization of the maximal number of input sources is performed in order to eliminate these integral causalities"."

2.8.3 Observability of bond graph systems

In order to analyse the observability of a bond graph model, a detector element must be added for every desired output. Observability is the dual property of controllability, therefore, the criteria to analyse it is similar to the one for controllability.

Property 2. Structural Observability of a Bond Graph [73]: "The model is structurally state observable if and only if:

1. There is a causal path between each dynamical element in integral causality and a detector.
2. $Struct\_Rank \begin{bmatrix} A & C \end{bmatrix} = n$. Where the structural rank of $\begin{bmatrix} A & C \end{bmatrix}$ is equal to
   (a) the rank of $\begin{pmatrix} S_{11}^T & S_{21}^T & S_{31}^T \end{pmatrix}^T$ or

(b) $n - t_d$

where t_d is "the number of storage elements remaining in derivative causality when:

(a) a derivative causality assignment is performed,

(b) a dualization of the maximal number of output detectors is performed in order to eliminate these integral causalities.""

It must be noted that the addition of detector elements does not affect the system equations, this is due to the fact that detector elements only detects effort or flow, which in real physical systems is equivalent to the sensor not drawing any power from the system.

2.8.4 Stability of bond graph systems

The analysis of stability of a bond graph model can be done from a Bond Graph with preferred Derivative causality assignment (BGD). Gonzalez et al propose this approach in [74]. This approach presented a new notation to differentiate the elements that change its causality in the BGD model, allowing the use of singular systems notation to simplify the analysis of the stability of the model. By using this notation, the structurally null modes (poles at the origin) can be obtained, and thus, determine the stability of the system.

Based on the previous results, the analysis of stability of switched bond graph systems will be done analysing the BGD of the model, because the graphical analysis can help to determine the stability of the model.

2.9 Singular Systems

Singular systems are also called *Descriptor* systems. The set of dynamic equations that describe a singular system are in the form

$$E\dot{x} = Ax + Bu \tag{2.13}$$

where $E,\ A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$.

Definition 10 ***Descriptor system* *[105]*:** *"Descriptor systems arise from formulating system equations in natural physical variables in the form* $E\dot{x}(t) = Ax(t) + g(t), x(t_0) = x_0$*, where* E *and* A *are matrices, possibly singular."*

Descriptor variables are those that have an inherent meaning or a natural interpretation within the context of the particular situation, i.e. position, velocity, acceleration, etc.

One of the interesting properties of singular systems is that its solvability depends on E and A matrices, as these are the matrices that describe the interaction between the energy storage elements. This also applies for hybrid systems because the matrices E and A represent the same interactions for both systems, which makes the techniques compatible for the analysis of their properties.

Also, Boolean algebra can be included in the notation to have a more robust representation of the model.

2.10 Singular systems properties

2.10.1 Controllability of singular systems

Singular systems controllability can be divided in two different types. Those are defined by Yip et al in [57] as *C-controllable* and *R-Controllable.*

Definition 11 ***C-controllable* *[57]*:** *"A system is completely controllable (C-controllable) if one can reach any state from any initial state."*

Definition 12 ***R-controllable* *[57]*:** *"The system* $E\dot{x} = Ax + Bu$ *is R-controllable within the set of reachable states if one can reach any state in the set of reachable states from any admissible initial state."*

2.10.2 Observability of singular systems

This property of the system is similar to the controllability due to being its dual property. The observability of a system is determined by the ability to reach a system's state from an output, meaning that the observability matrix is a function of the C matrix.

One of the observability of singular systems analysis used is the one proposed by Losse et al in [77]. Here the singular systems are represented as:

$$\begin{aligned} E\dot{x} &= Ax + B_1 u \\ y &= C_1 x \end{aligned} \tag{2.14}$$

where $y \in \mathbb{R}^p$ represents the outputs of the system.
In this work some conditions for the observability of singular systems are proposed, which are:

$$rank \begin{bmatrix} \alpha E - \beta A \\ C_1 \end{bmatrix} = n \text{ for all } (\alpha, \beta) \in \mathbb{C}^2 \tag{2.15}$$

$$rank \begin{bmatrix} \lambda E - A \\ C_1 \end{bmatrix} = n \text{ for all } \lambda \in \mathbb{C} \tag{2.16}$$

$$rank \begin{bmatrix} T_\infty^T (E) A \\ C_1 \end{bmatrix} = n \tag{2.17}$$

where α and β are generalized eigenvalues such that $\det(\alpha E - \beta A) = 0$, $\lambda = \frac{\alpha}{\beta}$ and T_∞ is a matrix with orthonormal columns spanning the left null-space of E associated with the eigenvalue ∞.

Then the following definitions are proposed:

C-observable [77]: "A singular system is completely observable if condition 2.15 holds."

R-observable [77]: "A singular system is observable in the reachable set if condition 2.17 holds."

S-observable [77]: "A singular system is strongly observable if conditions 2.16 and 2.17 hold."

Concluding that

Theorem 1 *A singular system is C-observable if:*

- *It is R-observable,*
- *and rank* $\begin{bmatrix} EA \\ C_1 \end{bmatrix} = n$ *(2.17bis)*

Proof. The first condition of **Theorem 1** determines that if a systems is not *R-observable*, a determined output of the system cannot be observable, therefore, if an output is not observable then the system cannot be *C-observable*. The singular system 2.14 is assumed to be in its standard decomposition form:

$$\begin{aligned} \dot{x}_1 &= A_1 x_1 + B_1 u \\ N\dot{x}_2 &= x_2 + B_2 u \\ y &= C_1 x_1 + C_2 x_2 \end{aligned} \tag{2.18}$$

where $x_1 \in \mathbb{R}^{n_1}, x_2 \in \mathbb{R}^{n_2}, n_1 + n_2 = n, N \in \mathbb{R}^{n_2+n_2}$ is nilpotent, the nilpotent index is denoted by h. Then, the system 2.18 is R-observable iff its slow subsystem is observable, i.e.,
$rank[sE - A/C] = n, \forall s \in C$,$s$ finite.

According to the state representation, any reachable state has the form:

$$\begin{aligned} x(t) &= \begin{bmatrix} \frac{x_1(t)}{x_2(t)} \end{bmatrix} \\ \dot{x}_1(t) &= A_1 x_1(t) + B_1 u(t) \\ x_2(t) &= -\sum_{i=0}^{h-1} N^i B_2 u^{(i)}(t), t > 0 \\ y &= y_1 + y_2 = C_1 x_1 + C_2 x_2. \end{aligned} \tag{2.19}$$

Note that in 2.19, $x_2(t)$ is uniquely determined by $u(t)$, and $y_1 \triangleq C_1 x_1(t) = y - C_2 x_2(t)$ is uniquely determined by $y(t)$ and $u(t)$. Thus, reconstructing the reachable state $x(t)$ from $y(t)$ and $u(t)$ is equivalent to that $x_1(t)$ may be uniquely determined by $y_1(t)$ and $u(t)$, which is, in turn, equivalent to the observability of (A_1, C_1).

For the second condition of **Theorem 1**, condition 2.17 proposed in [77] only complies when $C_1 \neq 0$. Therefore the system is not *R-observable*, making the system not *C-observable*.

■

This approach is useful for the research due to the use of the matrix E in the observability matrix, which in this case is the one that determines the configurations in which the storage elements behave as independent elements.

From this result it can be seen that the observability matrix for singular system is similar to the one proposed for linear systems, which will be helpful later on the research.

2.10.3 Stability of singular systems

Stability of singular systems is based on Lyapunov's stability theory, nevertheless, in order to use this approach, the model must be analysed as a continuous and discontinuous model separately.

Definition 13 ***Stability of singular systems [90]:*** *Singular system 2.14 is called stable if there exist scalars $\alpha, \beta > 0$ such that when $u(t) \equiv 0$ for $t > 0$ its state $x(t)$ satisfies:*
$\|x(t)\|_2 \leq \alpha e^{-\beta t}\|x(0)\|_2, t > 0.$
By definition, we stress stability under the Lyapunov sense, or asymptotic stability. When system 2.14 is stable and $u(t) \equiv 0, t > 0, lim_{t\to\infty} x(t) = 0$.

In order to determine the stability of a singular system, is necessary to find the poles of the system. In this case, the characteristic polynomial to find them is

$$f(s) = |sE - A| = a_{n_1}s^{n_1} + \cdots + a_1 s + a_0$$

The poles are those that satisfy $f(s) = |sE - A| = 0$. Since matrix E is singular, the number of poles is always smaller than n (number of storage elements).

2.11 Hybrid bond graph properties

2.11.1 Impulse modes

An impulse mode occurs when a commutation creates an instantaneous change on a system behaviour. This behaviour occurs when a storage element switches from integral to derivative causality, generating a step change in the value of the state.

For Buisson *et al* ([6, 79]), the impulse modes are only present after commutation. This behaviour is seen based on a reference mode, from which the maximum possible impulse modes (called impulsive modes in their work) are calculated assuming that all switched elements commute at the same time. This approach to impulse modes has been proven to be useful to find the switching conditions on which the system remains in a valid configuration.

Rahmani and Dauphin-Tanguy determined in [12] that the number of impulse modes can be obtained by analysing the bond graph model. They defined this as:

Definition 14 ***Number of impulse modes [12]:*** *"The number of impulse modes is given by the number of elements that appear in derivative causality after the commutation in the model with preferred integral causality."*

Therefore, the number of impulse modes can be determined by analysing the matrices E and A, where this can be seen even if the bond graph model is not available.

This result, if compared with the one obtained by Verghese *et al* in [58], allows to relate impulse modes with singular systems theory, which mean that impulse modes can be found in singular systems. Therefore, some of the results obtained in singular systems approach can be applied to hybrid bond graph systems.

2.11.2 Controllability of hybrid bond graph systems

Structural controllability can be determined by graphical analysis and rank criteria. Hihi proposed a set of conditions to determine the controllability of hybrid bond graph systems in [13], where the hybrid junction structure used to determine the conditions is:

$$\begin{bmatrix} \dot{x} \\ D_{out} \\ T_o \end{bmatrix} = \begin{bmatrix} S_{11}^i & S_{13}^i & S_{14}^i & S_{15}^i \\ -S_{13}^{iT} & S_{33}^i & S_{34}^i & S_{35}^i \\ -S_{14}^{iT} & -S_{34}^{iT} & S_{44}^i & S_{45}^i \end{bmatrix} \begin{bmatrix} z \\ D_{in} \\ T_{in} \\ u \end{bmatrix} \tag{2.20}$$

Which is derived from the junction structure displayed in Figure 2.5 where the switch field is added.

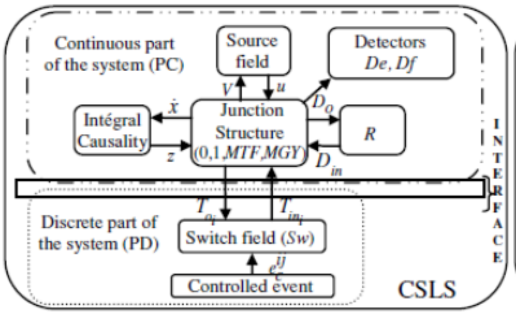


Figure 2.5: Junction structure including switch field.

By using the previous junction structure, the conditions are set as:

Proposition 1 *[13] "The hybrid bond graph system 2.20 is structurally state controllable if:*

1. *All dynamical elements in integral causality are causally connected with an input source.*
2. *$BG - rank \begin{bmatrix} A_i & B_{c_i} \end{bmatrix} = n$."*

where $A_i = \left(S_{11}^i - S_{13}^i H S_{13}^{iT}\right) F$, $B_{ci} = S_{15}^i + S_{13}^i H S_{35}^i$, $H = L_i \left(I - S_{33}^i L_i\right)^{-1}$, $D_{in} = LD_o$, and the subindex i denotes that all the elements have integral causality. If both conditions hold, the system is C-controllable, if only condition 1 holds the system is S-controllable, if only condition 2 holds the system is R-controllable.

Also in the same work the rank criteria for structural controllability is set as:

"If $rank \begin{bmatrix} B_1 & A_1 B_1 & \dots & A_1^{n-1} B_1 & \dots & B_q & \dots & A_q^{n-1} B_q \end{bmatrix} = n$, the system is structurally controllable." [13].

As hybrid systems contain impulse modes, Rahmani and Dauphin-Tanguy decompose structural controllability into *complete controllability, R-controllability* and *impulse controllability* in [12]. Those are defined as:

Definition 15 ***Non-impulsive inputs:*** *Non-impulsive inputs are inputs that does not create discontinuous changes in the states of the system.*

Definition 16 ***Complete controllability*** *[12]: A switched system is complete controllable if it is both R-controllable and impulse controllable at the same time.*

Proposition 2 ***R-controllability*** *[12]: "A switched system is structurally state R-controllable if and only if the two following conditions are verified:*

1. *All (I,C) elements are causally connected with a source in the BGI.*
2. $Bg\_rank \begin{bmatrix} sE - A & B \end{bmatrix} = n.$*"*

Definition 17 *[12]"*$bg\_rank \begin{bmatrix} sE - A & B \end{bmatrix} = n - t_c$ *where* t_c *is the number of dynamical elements remaining in integral causality when a dualization of the maximal number of input sources is performed in the BGD in order to cancel these integral causalities."*

Definition 18 ***Impulse controllability*** *[12]: "A switched system is impulse controllable if all the impulsive modes can be exited by suitably chosen non-impulsive inputs."*

2.11.3 Observability of hybrid bond graph systems

As observability is the dual property of controllability, it is logical that hybrid systems present *complete observability, R-observability* and *impulse observability*, which are related to the finite and impulse modes respectively. These are defined as:

Definition 19 ***Complete observability of a Hybrid bond graph*** *[12]: A switched system is completely observable if it is R-observable and impulse observable at the same time.*

Proposition 3 ***Structural R-Observability of a Hybrid Bond Graph*** *[12]: "A switched system is R-Observable if and only if the two following conditions are verified:*

1. *All (I,C) elements are causally connected with a detector.*
2. $Bg\_rank \begin{bmatrix} sE - A \\ C \end{bmatrix} = n.$*"*

The bond graph rank of the matrix $\begin{bmatrix} sE - A \\ C \end{bmatrix}$ *is given using causal manipulations in the bond graph model.*

Definition 20 ***Impulse Observability of a Hybrid Bond Graph [12]:*** *"A switching system is impulse observable if at each time* τ*, the knowledge of the output impulse and the input discontinuities at* τ *are sufficient to determine discontinuities of the state vector* x *at the same time."*

Proposition 4 ***Structural Impulse Observability of a Hybrid Bond Graph*** *[12]: "A switching system is structurally impulse observable if and only if, the number of impulse modes is equal to the number of disjoint causal paths between the switches and the output detectors passing through (I,C) elements in derivative causality in the BGI."*

It can be seen that the observability of hybrid bond graph systems can be determined from the bond graph model, as it is just necessary that the storage elements are connected to a detector, that they are independent (integral causality assigned), and that the impulse outputs are caused by the discontinuities from the input, which reduces the time of analysis compared to a more mathematical analysis.

2.12 Summary

Something that can be seen in the development in bond graph modelling is that most of the proposed techniques were designed initially for specific problems (e.g. LTI systems, second order descriptor systems, continuous descriptor systems, time-invariant singular system), which, when applied to different problems (e.g. switched singular systems, descriptor-variable systems, switched linear systems), some constraints were introduced in order to be able to use the desired approach.

In order to avoid these problems, it is necessary to propose a new model formulation, which will simplify the analysis of the systems by eliminating the introduction of constraints into the equations.

If this tendency continues, it is difficult to set a standard notation, which is not desirable. This problem does not only affect the analysis of hybrid and bond graph models, the simulation is affected as well. For hybrid systems these problems escalate due to the number of available representation of switching elements. Not to mention the common practice of avoiding the change of causality of the elements when a commutation occurs, which are usually representation of the change in behaviour of the model during commutation.

In order to reduce future problems, it is necessary to establish a standard notation and representation of the hybrid system and switching elements based on previous useful approaches. The benefits of this will help in the development of new analysis techniques and probably allows obtaining some missing properties of the models that were omitted when some constraints where set.

In the following chapter this will be addressed, and some new representations will be introduced, including a standard mathematical notation to simplify the analysis of hybrid systems.

Chapter 3

Hybrid Dynamic Bond Graph Systems

3.1 Preliminaries

A new method to construct hybrid bond graph models is proposed in this chapter. In order to do this, a new mathematical notation is introduced to make a distinction between the different elements involved and simplify the general notation of the models. This is due to the use of switched junctions and dynamic causality in the model. Procedure for the construction of the hybrid bond graph are given, along with a causality assignment procedure.

A differentiation between the previous representations and the proposed one is described. By doing this, it could be seen that the proposed approach allows a closer representation of the physical behaviour of the systems, not to mention the analysis of more properties that are usually not taken into consideration due to the constraints set on previous approaches, such as the general controllability and observability of hybrid systems.

The representation of physical switching elements is revised and expanded to include multi-way switches (which are used the control of a load from different points in the system).

Some properties of the new representation are obtained and enumerated to help with the analysis of the systems (e.g. change in the order of the system, new notation to simplify the analysis).

3.2 Procedure for the construction of a hybrid bond graph

In the following paragraphs the procedure for the construction of hybrid bond graph models is described. This is done by analysing the previous approaches and using the properties that prove to be useful for the approach that is being developed in this research.

Before the procedure is explained, the different parts involved are introduced and described in order to show how these are used in the construction of the model.

First, the representation of the commuting elements is described, followed by how this representation modifies the hybrid junction structure, ending with the new properties arising from the use of this representation.

3.2.1 Hybrid bond graphs

As previously explained, hybrid bond graphs are those that represent both continuous and discontinuous behaviour.

Discontinuities are abstractions made in order to simplify a model. This is usually done to simplify continuous behaviours that change rapidly in a small lapse of time. These discontinuities are introduced to improve simulations and simplify the analysis of the model behaviour.

Definition 21 ***Hybrid Bond Graph [10]:*** *"A hybrid bond graph is any bond graph that describes both continuous and discontinuous behaviour."*

There are different representations for hybrid bond graphs; most of them share the same construction procedure, while some of them have a different one.

In this case, all the representations that share the same procedure are those that contain the same number of variables for all of the configurations with fixed causality for all of the possible configurations. This is the case of the traditional representations of hybrid bond graphs such as MTF, controlled storage elements, and switched power junctions ([2, 3, 4, 5, 6, 8, 9]).

The ones that have different procedure are those which can have different number of variables for different configurations and variable causality. This change in causality depends on the representation of switched elements chosen. Some examples of this are the controlled junctions used in [10, 24, 53].

For the approach that keeps the same number of storage elements, the procedure for the construction is the same used in LTI systems. The main difference between modelling hybrid bond graph systems and LTI systems rest in the fact that hybrid systems have 2^n representations (where n is the number of switching elements), therefore, it is necessary to obtain the model of the system for each configuration that needs to be analysed.
With each state of the commuting elements, different behaviours are introduced in the final model (which was already presented in the previous chapter).

The main disadvantage of these modelling techniques is that some physical representations are not carried to the model. An example of this is the coupling of two masses, which is displayed in Figure 3.1.

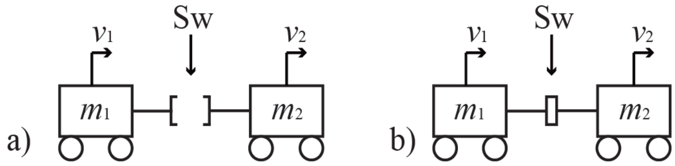


Figure 3.1: Two masses and a switch a) OFF configuration b) ON configuration

Using the traditional mathematical approach, each mass has an independent behaviour (in this case velocity) for all of the configurations when compliance the coupling is present, which is the case of the non-ideal switch. This is not the proper representation of the behaviour of the physical system, where the masses behave as one when they are coupled without friction. Therefore, in order to represent this physical behaviour, the switched junctions are proposed in this work.

The examples used in [5, 6, 9, 46] shows that by maintaining the causality assignment static the model does not represent the physical behaviour of the system for all the configurations of the commuting elements. This is caused by the use of non-ideal switches, which introduces parasitic elements, some representing frictions, other just as aid to maintain a static causality.

On the other hand, there are some representations that allow the model to change the number of variables on the system, i.e. controlled junctions. The procedure to construct the model is described by Margetts in [10], where the main difference with the traditional procedure lies in the fact that the use of controlled junctions allows the change of causality of the elements affected by the use of dynamic causality.

This change on the storage elements allows a closer representation of the physical behaviour of the system in the ideal case; nevertheless, the proposed approach still needs some development in order to be used as a standard notation, which is why this approach is used as a starting point for the research described in this work.

3.2.1.1 Switching source elements

Switching source elements were used to represent commuting elements and their behaviour. Most of these representations in bond graph are zero effort or zero flow sources connected to the system (according to the state of the switch).

These sources are inserted into the junction where the commuting element is connected in the model and are interchanged according to the commuting element state (ON or OFF configuration).

Some examples of this representation can be seen in [3, 4, 5, 6]. The behaviour of this element is shown in Figure 3.2.

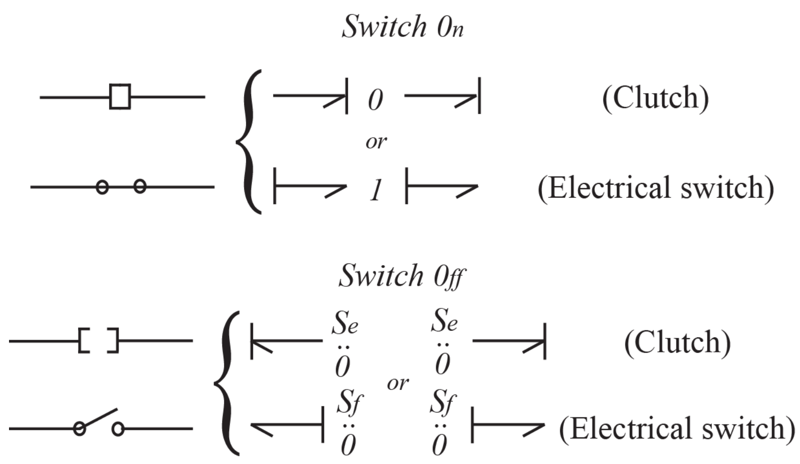


Figure 3.2: Ideal Switches

The use of the source representation is not considered in this work due to the introduction of constraints to the system, which will not allow a proper representation of the physical system.

3.2.1.2 Switched power junctions

Switched power junctions avoid the use of sources, instead, there is a "selecting" bond that decides which configuration the commuting element should be represented. They were introduced by Umarikar and Umanand in [9], and used for different applications in [4, 80, 81]. An example of this representation is shown in Figure 3.3.

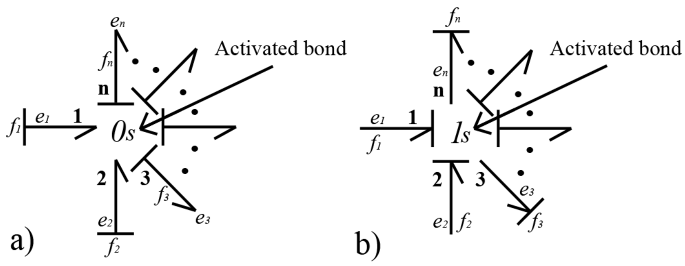


Figure 3.3: Switched power junctions a) 0- junction b) 1- junction

Using this representation could create confusion to the user. This is due to not having a static representation that allows a proper differentiation between the "activated" bond and the rest of the bonds. This usually happens when there are more than 3 elements connected to the same switched power junction, the *selecting* bond sends an *activation signal* to all of the bonds, which indicates the configuration selected, nonetheless the causality remains the same, this is why some confusion can be created. It can be seen in Figure 3.3 that the switching bonds are not clearly defined, as any of the connected bonds can switch at any moment, which is the part of the representation that could confuse the user as it needs to be more clearly displayed.

This representation allows a closer representation of the commuting elements, nonetheless, retain some of the constraints set on previous representations, making it unviable for a standard notation of hybrid bond graph.

3.2.1.3 Switched junctions

The representation used for the switched junctions is a modified version of the controlled junction previously introduced by Margetts in [10]. The behaviour of the switched junction is the same as the controlled junction, however, some changes where done to simplify the representation of hybrid systems and their analysis.

For the bond graph representation, the Boolean parameter is assigned to one of the ports instead to the element. Also, the notation in the junction structure matrix is modified to simplify the analysis by making a distinction between the elements affected by dynamic causality and those that are not affected, instead of separating the elements that have integral causality assigned and those that have derivative causality assigned.

The junction structure for both notations is displayed in Figure 3.4, where the simplification in the number of elements in the proposed approach allow a compact junction structure.

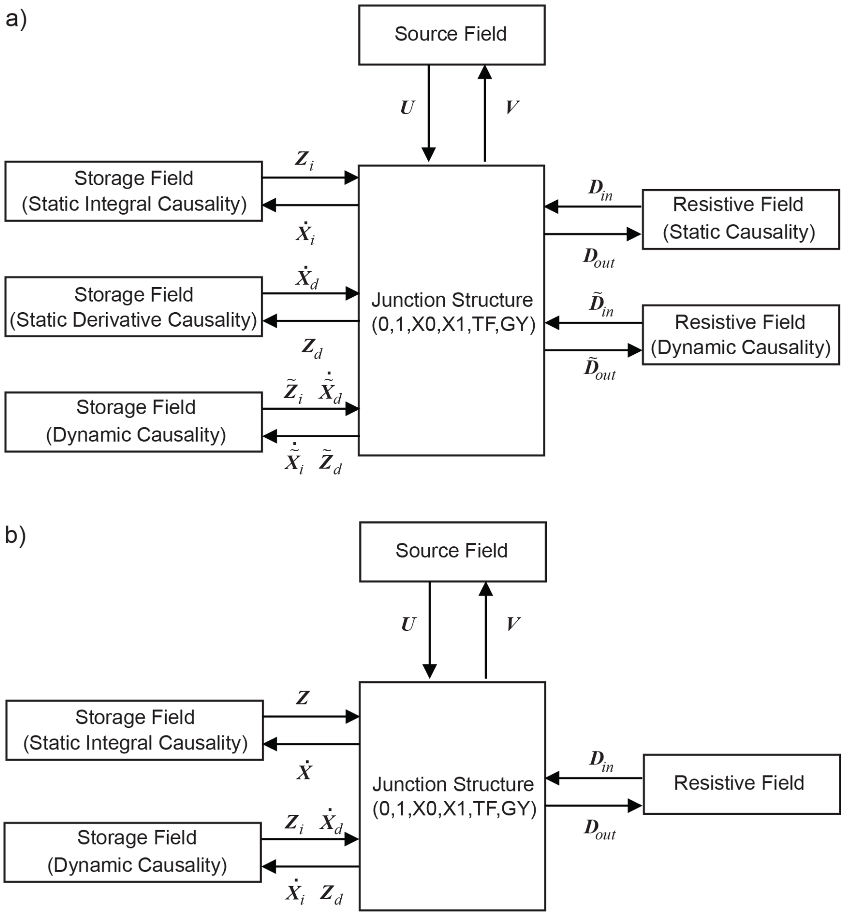


Figure 3.4: a) Junction structure of the controlled junctions, b) Junction structure of the switched junctions

From Figure 3.4, it can be observed that controlled junctions divide the storage field in three different elements,

1. Elements not affected by dynamic causality with static integral causality,
2. Elements not affected by dynamic causality with static derivative causality,
3. Elements affected by dynamic causality, which includes elements with preferred integral causality assignment and elements with preferred derivative causality assignment.

In the proposed approach, the junction structure using switched junctions, the storage field is only divided in two different elements,

1. Elements not affected by dynamic causality,
2. Elements affected by dynamic causality.

This simplification will be explained in detail in a later subsection. Meanwhile, a comparison between both notations is done in table 2 to show it is implemented for the analysis of the models.

Table 2: Comparison between Controlled junctions and Switched junctions derived from Junction Structure Matrices

	Junction Structure Matrix	Notation
Controlled junctions	$\Lambda(\lambda)_{cj} \begin{bmatrix} \begin{bmatrix} \dot{\hat{x}}_i \\ \dot{\tilde{x}}_i \end{bmatrix} \\ \begin{bmatrix} \hat{z}_d \\ \tilde{z}_d \end{bmatrix} \\ \begin{bmatrix} \hat{D}_{out} \\ \tilde{D}_{out} \end{bmatrix} \end{bmatrix} = [S(\lambda)]_{cj} \begin{bmatrix} \begin{bmatrix} \hat{z}_i \\ \tilde{z}_i \end{bmatrix} \\ \begin{bmatrix} \dot{\hat{x}}_d \\ \dot{\tilde{x}}_d \end{bmatrix} \\ \begin{bmatrix} \hat{D}_{in} \\ \tilde{D}_{in} \end{bmatrix} \\ U \end{bmatrix}$	ˆ denotes the elements not affected by dynamic causality. ˜ denotes the elements affected by dynamic causality.
Switched junctions	$\Lambda(\lambda)_{sj} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ z_d \\ D_{out} \\ \tilde{D}_{out} \end{bmatrix} = [S(\lambda)]_{sj} \begin{bmatrix} z \\ z_i \\ \dot{x}_d \\ D_{in} \\ \tilde{D}_{in} \\ U \end{bmatrix}$	The elements affected by dynamic causality are denoted by the subindex i and d. i denotes integral causality. d denotes derivative causality.

where $$\Lambda(\lambda)_{cj} = \begin{bmatrix} \Lambda_{11}(\lambda) & 0 & 0 & 0 & 0 & 0 \\ 0 & \Lambda_{22}(\lambda) & 0 & 0 & 0 & 0 \\ 0 & 0 & \Lambda_{33}(\lambda) & 0 & 0 & 0 \\ 0 & 0 & 0 & \Lambda_{44}(\lambda) & 0 & 0 \\ 0 & 0 & 0 & 0 & \Lambda_{55}(\lambda) & 0 \\ 0 & 0 & 0 & 0 & 0 & \Lambda_{66}(\lambda) \end{bmatrix},$$

$$[S(\lambda)]_{cj} = \begin{bmatrix} S_{11}(\lambda) & S_{12}(\lambda) & S_{13}(\lambda) & S_{14}(\lambda) & S_{15}(\lambda) & S_{16}(\lambda) & S_{17}(\lambda) \\ S_{21}(\lambda) & S_{22}(\lambda) & S_{23}(\lambda) & S_{24}(\lambda) & S_{25}(\lambda) & S_{26}(\lambda) & S_{27}(\lambda) \\ S_{31}(\lambda) & S_{32}(\lambda) & 0 & 0 & 0 & 0 & S_{37}(\lambda) \\ S_{41}(\lambda) & S_{42}(\lambda) & 0 & 0 & 0 & 0 & S_{47}(\lambda) \\ S_{51}(\lambda) & S_{52}(\lambda) & 0 & 0 & S_{55}(\lambda) & S_{56}(\lambda) & S_{57}(\lambda) \\ S_{61}(\lambda) & S_{62}(\lambda) & 0 & 0 & S_{65}(\lambda) & S_{66}(\lambda) & S_{67}(\lambda) \end{bmatrix},$$

$$\Lambda(\lambda)_{sj} = \begin{bmatrix} \Lambda_{11}(\lambda) & 0 & 0 & 0 & 0 \\ 0 & \Lambda_{22}(\lambda) & 0 & 0 & 0 \\ 0 & 0 & \bar{\Lambda}^{*}_{22}(\lambda) & 0 & 0 \\ 0 & 0 & 0 & \Lambda_{44}(\lambda) & 0 \\ 0 & 0 & 0 & 0 & \bar{\Lambda}^{*}_{44}(\lambda) \end{bmatrix},$$

$$[S(\lambda)]_{sj} = \begin{bmatrix} S_{11}(\lambda) & S_{12}(\lambda) & S_{13}(\lambda) & S_{14}(\lambda) & S_{15}(\lambda) & S_{16}(\lambda) \\ -S^{T}_{12}(\lambda) & 0 & 0 & 0 & 0 & S_{26}(\lambda) \\ -S^{T}_{13}(\lambda) & 0 & S_{33}(\lambda) & S_{34}(\lambda) & S_{35}(\lambda) & S_{36}(\lambda) \\ -S^{T}_{14}(\lambda) & 0 & -S^{T}_{34}(\lambda) & S_{44}(\lambda) & S_{45}(\lambda) & S_{46}(\lambda) \\ -S^{T}_{15}(\lambda) & 0 & -S^{T}_{35}(\lambda) & -S^{T}_{45}(\lambda) & S_{55}(\lambda) & S_{56}(\lambda) \end{bmatrix}.$$

where $\hat{x}_i$ and $\tilde{x}_i$ are the energy storage elements with preferred integral configuration, $\hat{z}_d$ and $\tilde{z}_d$ are the energy storage elements with preferred derivative causality, $\hat{D}$ are the resistive elements not affected by dynamic causality (controlled junctions), $\tilde{D}$ are the resistive elements affected by dynamic causality, x are the energy storage elements not affected by dynamic causality, x_i are the energy storage elements affected by dynamic causality with preferred integral causality, x_d are the energy storage elements affected by dynamic causality with preferred derivative causality, Λ_{ij} are the submatrices containing Boolean parameters that indicates the configuration of the controlled/switched junction, S_{ij} are the submatrices in the state junction matrix that indicate the relationship between elements, (λ) indicates that the submatrices are dependent of the configuration of the controlled/switched junctions.

From the previous table it can be seen that the notation is simplified to avoid confusion to the modeller. For controlled junctions the elements are divided by their assigned causality, which creates a mixed junction structure as the elements affected by dynamic causality are mixed with those that are not.

While the switched junction's junction structure matrix separates the elements not affected by dynamic causality from those that are affected, which simplifies the notation and allow simpler identification of the elements in the different configurations.

A switched junction behaves as a normal 1- or 0- junction when it is in ON configuration and as a source of zero flow or effort when OFF configuration (respectively). Therefore, a switched 1-junction is used to break or inhibit flow (for example, an electrical switch which breaks the flow of current), and a switched 0-junction is used to inhibit effort (for example, a clutch or other physical non-contact in a mechanical system). This creates a dynamic causality on one of the attached bonds.

From this description, switched junctions $X1$ and $X0$ can be defined as 2-port elements with associated Boolean parameters λ. The bond graph representations of controlled junctions $X1$ and $X0$ are shown in 3.5, and their defining relationships are given by Equations 3.1 and 3.2, respectively.

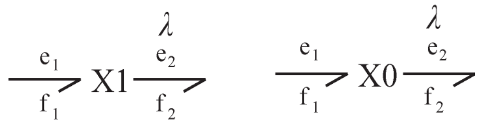


Figure 3.5: Bond graph representation of switched junctions $X1$ and $X0$

$$\begin{cases} \lambda f_1 = \lambda f_2 \\ \lambda (e_1 - e_2) = 0 \\ \bar{\lambda} f_1 = 0 \\ \bar{\lambda} f_2 = 0 \end{cases} \tag{3.1}$$

$$\begin{cases} \lambda e_1 = \lambda e_2 \\ \lambda (f_1 - f_2) = 0 \\ \bar{\lambda} e_1 = 0 \\ \bar{\lambda} e_2 = 0 \end{cases} \tag{3.2}$$

The assignment of Boolean values (configuration) to the parameter λ selects the set of equations that are valid given the state of the switch, $\lambda = 1$ when the switch is in ON configuration and $\lambda = 0$ when the switch is in OFF configuration.

For each switched junction, the defining Equations 3.1 and 3.2 lead to 3 possible causal configurations:

- 2 causal configurations when the switch is in ON configuration i.e. $\lambda = 1$ (first two equations equivalent to a normal 1- or 0- junction).
- a unique causal configuration when the switch is in OFF configuration i.e. $\lambda = 0$ (last two equations equivalent to null sources of flow or null sources of effort imposed by the element to both power ports with conjugate variables externally imposed to the element).

When a commutation occurs (switching from ON configuration to OFF configuration), the causal assignment from one of the ports must change. This is usually known as dynamic causality.

By using this representation of the commuting elements, a new procedure for modelling hybrid bond graphs is proposed. This procedure is explained later in this chapter; although, it is necessary to first introduce a new notation and the formulation of the junction structure matrix in order to fully exploit this approach.

3.2.1.4 Multiswitched junction

A new variant of the switched junction is introduced in order to represent commuting elements that could commute between two or more different subsystems. These types of switches are common in electrical circuits (e.g. buck converter), electronics (e.g. multiplexers), and mechanics (e.g. clutches), as displayed in Figure 3.6.

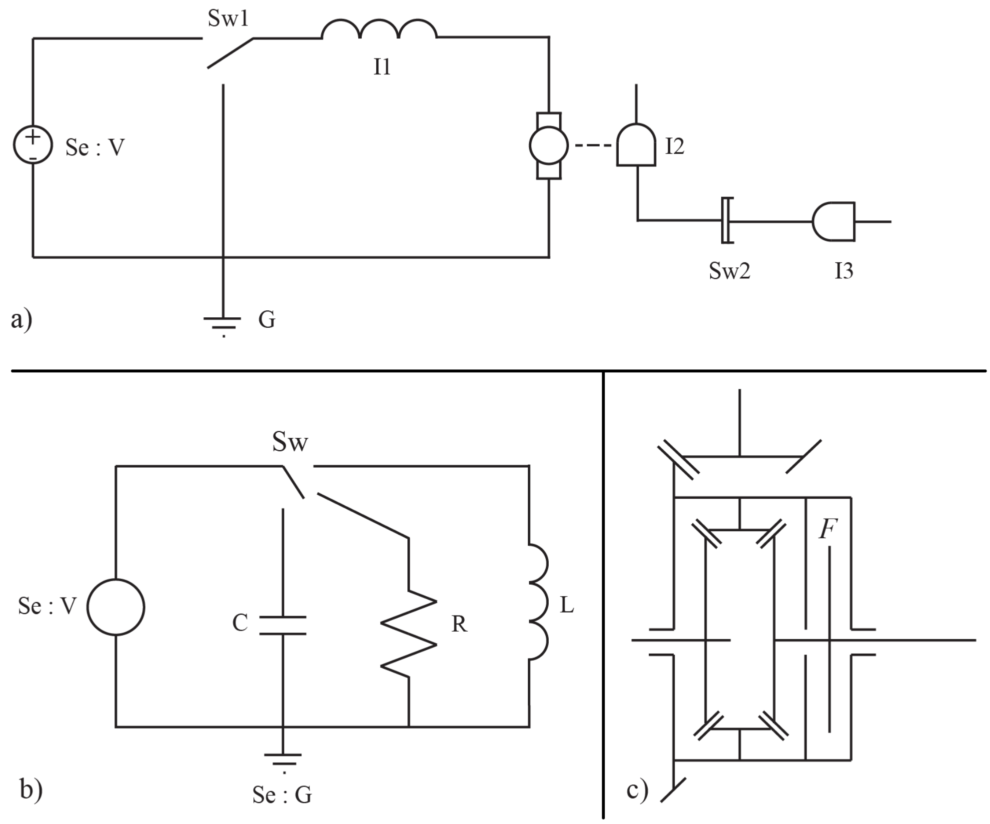


Figure 3.6: a) Buck converter, b) Electric multiplexor, c) Mechanical clutch

This junction behaves as a normal two port switched junction, the difference between these two is its use. While the two port switched junctions serves to connect and disconnect parts of a system, the multiswitched junction allows to select between n different parts of the system (where n is the number of subsystems connected by the multiswitched junction).

While the representation changes by increasing the number of ports connected to the switched junction, the notation remains the same; only the internal equation ruling the commutation changes depending on the admissible configurations. The representation of the multiswitched junction is displayed in the Figure 3.7.

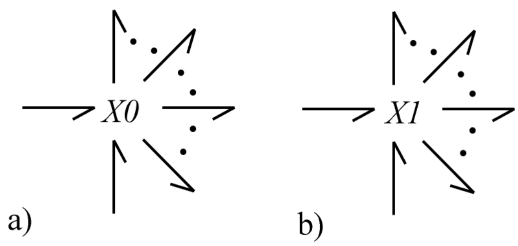


Figure 3.7: Multiswitched junction, a) X0 junction, b) X1 junction

A multiswitched junction is denoted as an $X0$ or $X1$ switched junction with two or more outputs, which is different of having a group of two port switched junctions connected to the same element or subsystem.

The multiswitched junction (msj) behaves as mode-switching tree (introduced by Margetts in [10] and illustrated in Figure 3.8), which is based on the multi-bond notation suggested by Mosterman and Biswas in [24]. Just as the mode-switching tree, the msj only allows one of the Boolean parameters to be in ON configuration at a time (mutually exclusive).

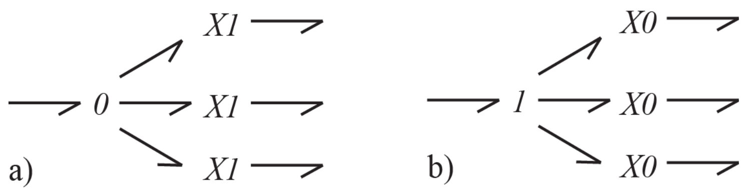


Figure 3.8: Bond graph 'Trees' a) 'Tree' of X1- junctions b) 'Tree' of X0-junctions

This representation (msj) allows all the bonds to be affected by dynamic causality.

To determine the causality on a multiswitched junctions it is necessary to follow the HSCAP mentioned in [10].

In order to differentiate the *input bond* from the rest of the bonds, it is necessary to keep that bond without a Boolean parameter assigned, while the rest of the ports will have a Boolean parameter assigned.

There are n possible internal configurations for the multiswitched junctions, nevertheless, there is only one subsystem connected at a time. If there are more than one subsystem connected at a time, a causal conflict is created, which will denote a non-admissible configuration. This causal conflict represents the physical constraint of a multiswitch that can only be connected to one output at a time. It must be noted that the configuration where all of the outputs are disconnected is considered valid using this approach, and only on this configuration the *input bond* will change its causality. This is in order to properly represent the behaviour of the physical system, which will impose a zero effort or flow on all the bonds.

Causality assignment in the msj is displayed in Figure 3.9,

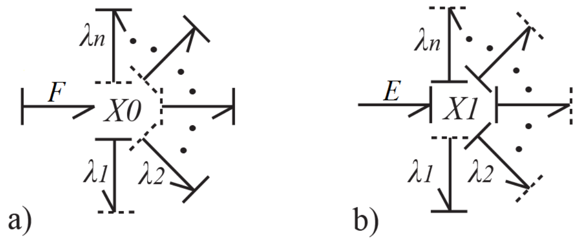


Figure 3.9: Causality assignment in: a) X0 msj b) X1 msj

On this notation, the flux F and effort E denotes the flow and effort (respectively) of the input bond which will remain in static causality.

From this causality assignment the equations that describe the msj behaviour can be obtained.

The flow and effort relation for the $X0$ msj are:

$$\begin{array}{c} e_0 = \lambda_1 e_1 + \lambda_2 e_2 + \cdots + \lambda_n e_n \\ (\lambda_1 + \lambda_2 + \cdots + \lambda_n) f_0 - \lambda_1 f_1 - \lambda_2 f_2 - \cdots - \lambda_n f_n = 0 \end{array} \tag{3.3}$$

where e_0 is the effort and f_0 is the flow of the input bond.

The flow and effort relation for the $X1$ msj are:

$$\begin{array}{c} f_0 = \lambda_1 f_1 + \lambda_2 f_2 + \cdots + \lambda_n f_n \\ (\lambda_1 + \lambda_2 + \cdots + \lambda_n) e_0 - \lambda_1 e_1 - \lambda 2 e_2 - \cdots - \lambda_n e_n = 0 \end{array} \tag{3.4}$$

Definition 22 *A multiswitched junction represents the connection of a group of subsystems that are connected between them, but only two can be connected at a time*

It must be noted that there is power transfer between only two ports at a time (or the lack of power transfer), which means that one bond will be considered as an "input" bond (which will not have a Boolean parameter λ assigned) and an "output" bond in ON configuration.

Even though just one bond will have a Boolean parameter assigned in this configuration, the previous relationships remain true. Therefore, the number of Boolean parameters that determine the configuration in the mathematical equation in the model is equal to the number of ports minus one. This is only done for the analysis of the behaviour of the system.

Definition 23 *The number of Boolean parameters assigned to a multiswitched junction is equal to the number of bonds minus one.*

For the rest of the bonds in the multiswitched junctions, the causality assignment could be interpreted as the existence of power transfer between the bonds with the assigned OFF configuration, but the modeller must have in consideration that there is transfer of power between only two bonds at a time.

A description of a multiswitched junction with two outputs is done as example of its behaviour.

The behaviour (and mathematical representation) of the multiswitched junction is described in the next example.

Example 2 *Figure 3.10 presents a system where the valid configurations of the switch element are the previously mentioned (connected to one subsystem at a time or disconnected from both).*

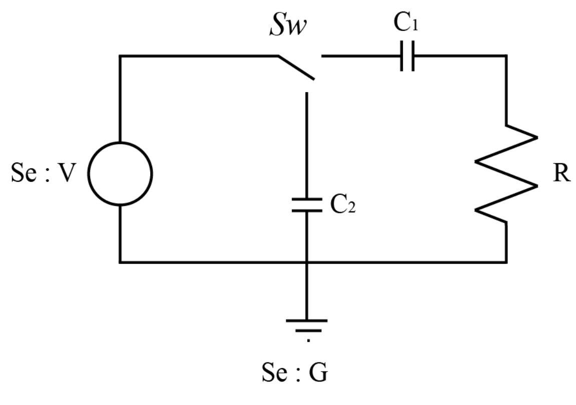


Figure 3.10: System with three valid operational modes

The next step consists in obtaining the bond graph model (displayed in Figure 3.11).

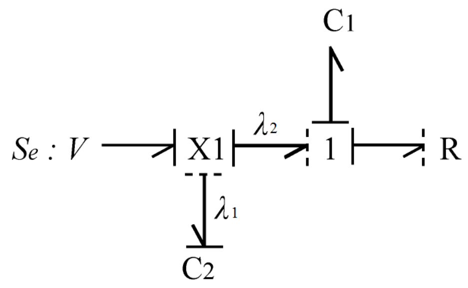


Figure 3.11: Bond graph model containing a multiswitched junction with three valid configurations

From the bond graph model, the configurations are analysed by obtaining a table of truth from the different internal configurations of the multiswitched junction.

Switched junction configuration	C_2 *causality*
$\lambda_1\bar{\lambda}_2$	*Derivative*
$\bar{\lambda}_1\lambda_2$	*Integral*
$\bar{\lambda}_1\bar{\lambda}_2$	*Integral*
$\lambda_1\lambda_2$	*Non − valid*

From this example, it can be seen why the multiswitched junction can have three admissible configurations in the case where there are no causal conflicts on any of the ports. Therefore, one Boolean term will represent one configuration, while another Boolean term will represent the other two configurations, because, as seen on the previous table of truth, there should be just one output in ON configuration at a time, or all of the outputs in OFF configuration. It is physically impossible to have two or more outputs connected at a time, as this configuration could create a behaviour of fast commutation, therefore, an unstable behaviour could appear, which is a valid reason to set a constraint in the multiswitched junction.

It must be noted that the configuration when both outputs are in OFF configuration is not considered as non-admissible, because it represents the moment of commutation in the multiswitched junction. This can be observed in Figure 3.12 where it is compared to an arrangement of two switched junctions.

This configuration does not create any causal conflicts on the output bonds, therefore it is not considered a non-admissible configuration.

An example of this is an electrical switch that is in a configuration between the two available outputs (displayed in Figure 3.12(a)), could cause a short circuit or an undesired behaviour caused by a fast commuting behaviour in the switch, therefore it is necessary to analyse this configuration.

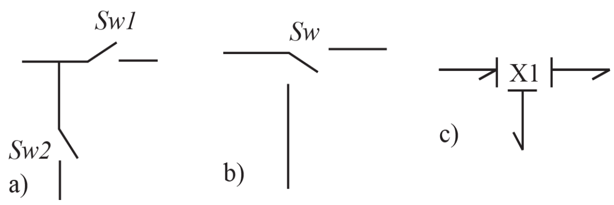


Figure 3.12: Comparison of a) two switched junctions, with b) a multiswitched junction, with c) simplified bond graph representation

In some systems there can be non-admissible configurations caused by causal conflicts, an example of this is displayed next.

Example 3 *A system with two valid configurations is displayed in Figure 3.13, and the correspondent bond graph model is displayed in Figure 3.14.*

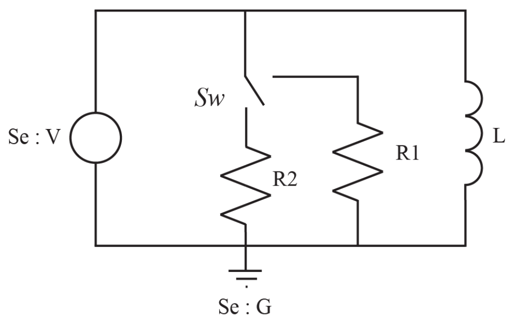


Figure 3.13: System with two valid configurations

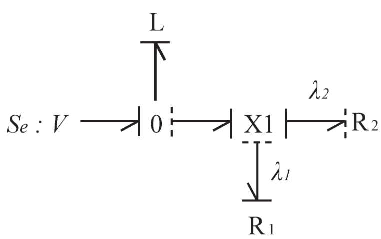


Figure 3.14: Bond graph model containing a multiswitched junction with two valid configurations

From the bond graph model, it can be seen that there is a causal conflict for the configuration $\bar{\lambda}_1\lambda_2$. Meaning that $\lambda_1\bar{\lambda}_2$ and $\bar{\lambda}_1\bar{\lambda}_2$ are the valid configuration for the system, as $\lambda_1\lambda_2$ is a non-valid configuration to avoid fast commutations *that could create instabilities.*

From these previous examples, it is deduced that for multiswitched junctions there cannot be complementary Boolean parameters to show the commuting of the element, this is due to the constraint set to the element and possible causal conflicts.

It must be noted that there is always a causal path between the elements where the commuting occurs (λ_1 and λ_2 output bonds), this does not mean that there is a flow of power between the elements on the output bonds, this is a consequence of the representation of the multiport switches, the only flow of power occurs between the input bond and the output bond that is in ON configuration.

3.2.1.5 Multiswitched junctions and switch power junctions

Both representations look similar, nonetheless, there are some differences between them.

One of the differences is the way a specific configuration is determined. While the switch power junctions (spj) have an indifferent representation (the configuration is determined by a Boolean parameter assigned to each bond), the multiswitched junctions (msj) indicate the configuration by modifying the causality of the bonds. An example of this can be seen in Figure 3.15, where a $0s$ and a $X0$ junctions are used.

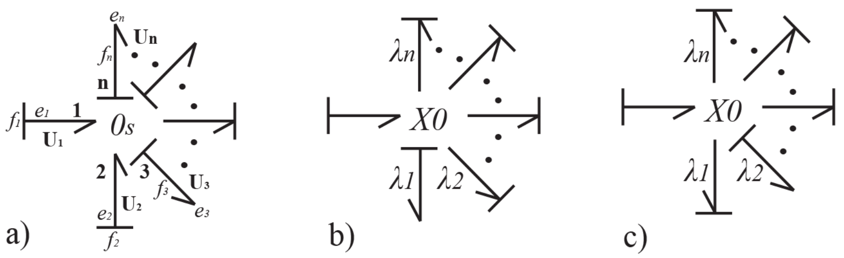


Figure 3.15: a) Spj with static causality, b) Msj with $\lambda_1 = 1$, c) Msj with $\lambda_2 = 1$.

It can be seen that using spj could create confusion for the modelers as the representation for all configurations is the same, while using msj each configuration has its own causality assignment, which makes it easier to identify the configuration used at a time.

Another difference is the use of dynamic causality. Spj use the Boolean parameters to denote the configuration of each bond, however the causality of the model remains static, which is a constraint that does not represent the physical behaviour of most of the systems.

This constraint is avoided with the use of msj, which allows the use of dynamic causality, that helps on the change of causality and its propagation through the model, leading to a more physically related behaviour of the system. Also, there are some configurations that are not being taken into consideration using spj, this is because the Boolean parameters assigned to the bonds are considered to be mutually exclusive, which is not the case for msj, where all of the possible configurations can be analysed and then determine which ones are not admissible. This is illustrated in Figure 3.16, where a modified boost converter is used to represent this behaviour.

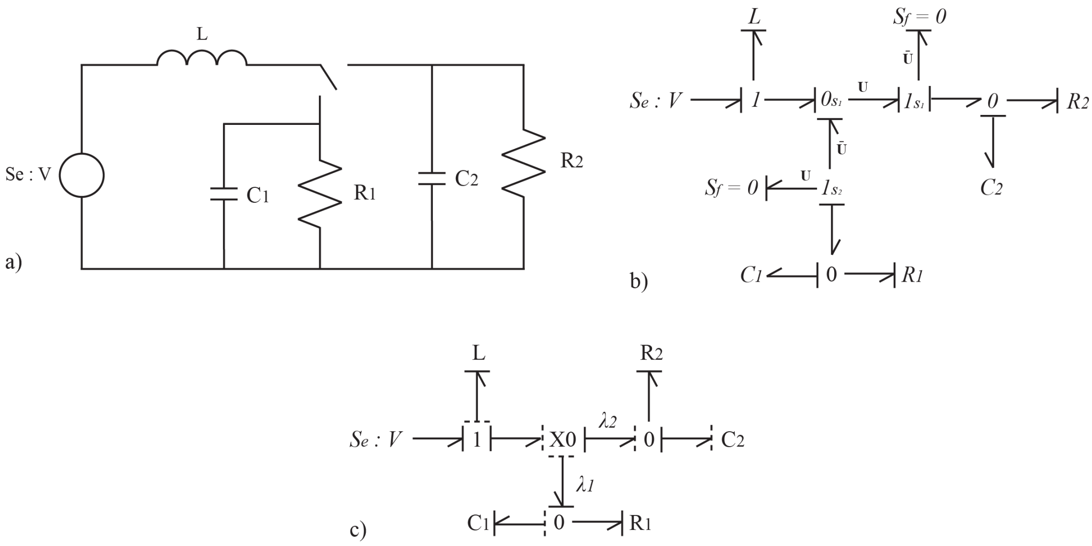


Figure 3.16: a) Modified boost converter, b) Model using spj, c) Model using msj

In this example it can be seen that in the model using spj, the causality is fixed and the only way to determine the configuration used is by determining the value of the Boolean parameter U, while the model using msj properly display the causality of each configuration. Not only that, also can be seen that the model using spj only allows two configurations (when $U = 1$ and when $U = 0$), which does not allow to analyse the configuration when the source and inductance are disconnected from the rest of the model. This configuration can however be analysed using msj.

As conclusion, the representations of spj and msj are similar, nonetheless their behaviour and notation are different.

3.2.1.6 Model simplification using switched junctions

It is well known that models constructed from the schematic diagram have potential to be simplified. The usual approach to simplification of bond graph models can also be applied to hybrid bond graph models using switched junctions.

The most common simplification is done by removing structures that does not provide any power to the system. This is the case of electrical and hydraulics circuits where there is a return line to a zero ground or open tank.

In some occasions ground parts are source elements which also act as a sink. For example, a mechanical ground is represented by a Sf-element (which has zero velocity and is a sink for force), and grounds in other domains are represented by Se-elements (e.g. an electrical ground, which is a source of 0V and a sink for current).

This simplification can be seen in Figure 3.17.

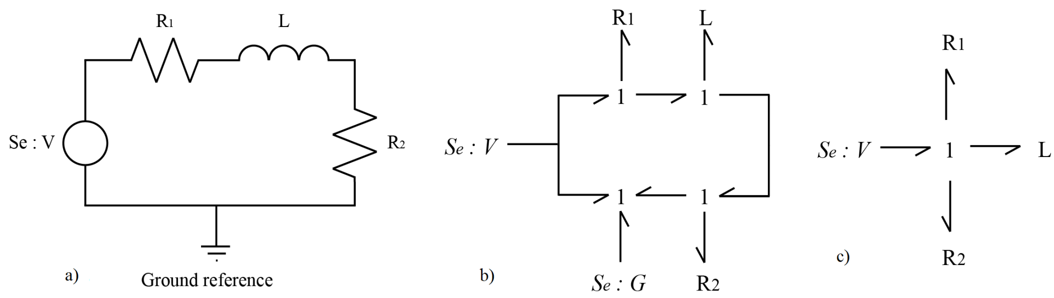


Figure 3.17: Example of simplification of a ground element

If there is a switched junction between a ground reference and the main structure of the model the previous procedure is not applicable. This is due to the behaviour of the system changing when the switched junction commutes, which is reflected on the change of the zero flow or effort imposed by the ground reference when it is disconnected from the system. This can be seen in Figure 3.18.

In the case where there are two neighbouring switched junctions these can be combined if there are no elements connected between them. If there are elements connected between them then these cannot be simplified. This is due to the change of power flow when one of the elements commutes, which causes a change of behaviour on both parts of the system.

This can be seen in Figure 3.19, where the configuration of Figure 3.19(a) can be simplified because the switched junctions are connected to each other, while the configuration of Figure 3.19(b) cannot be simplified because there is an element different than a switched junction between them.

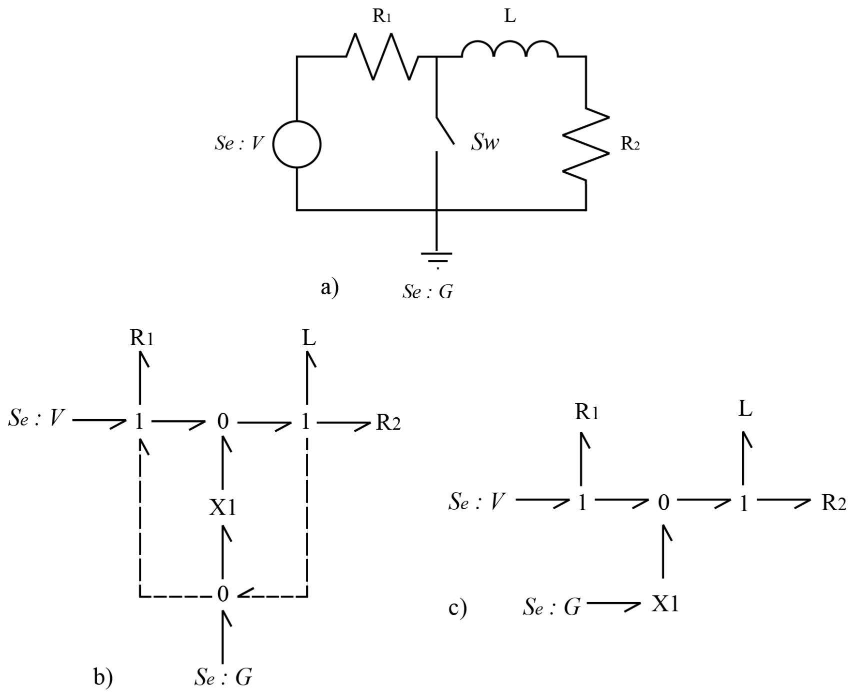


Figure 3.18: Example of simplification of a model with ground reference connected by a switched junction, a) diagram of an RL circuit, b) bond graph model (dashed lines represent connections to ground reference that can be eliminated), c) simplified bond graph model.

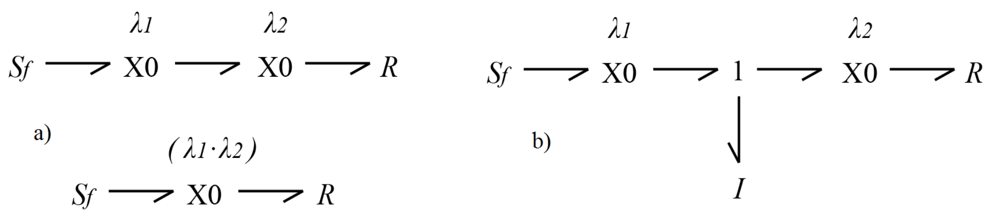


Figure 3.19: Example of neighbouring switched junctions, a) simplification of neighbouring switched junctions, b) arrange of neighbouring switched junctions that cannot be simplified

The neighbouring switched junctions displayed in Figure 3.19 are the possible combinations that could appear in a model. In this case, a flow source and a resistive element are connected to the ends of the Figure 3.19(a), this is in order to be able to use an $X0$ switched junction. The same principle is used in Figure 3.19(b), where an extra I element is added in order to show the non-simplification of neighbouring switched junctions.

3.3 Multiway switching

Multiway switching can be defined as an interconnection of two or more switches in order to control the behaviour of the system from different points in the system.

These loads that can be controlled can (but are not limited to) be lamps, electrical outlets, fans, pumps, heaters, motors, to name some. However, in this case the load is a generic one in order to avoid confusions on the use of the multiway switching using bond graph.

In order to have a closer representation of the multiway switching, multiswitched junctions are used, which at the same time will reduce the size of the model by reducing the number of elements used to represent the connection between the elements.

Multiway switching requires special switches (two-way and two-way four ports) which will have their own representation in bond graph.

3.3.1 Two-way switches in bond graph

Two-way switches are displayed in Figure 3.20, where it can be seen that this type of switches have one selector input/output that commutes between two output/input ports.

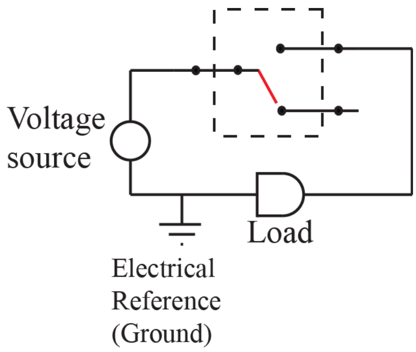


Figure 3.20: Two-way switch

These switches can be represented as a multiswitched junction (either $X1$ or $X0$), which remain with the same behaviour previously explained. This is displayed in 3.21.

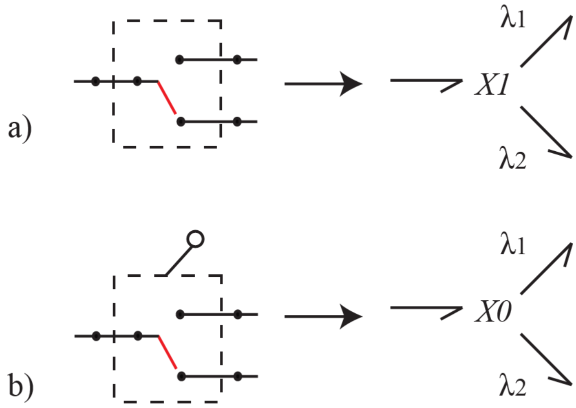


Figure 3.21: Two-way switch representation in bond graph, a) electrical two-way switch, b) mechanical two speed transmission

There is a common use of the two-way switch, in which two of these are used to control a load from two different places.

An example of this connection is explained next, where the two-way switches are connected to a load and a source of voltage or flow, this in order to assign causality to the elements and determine its behaviour.

The connection of two two-way switches is displayed in 3.22, where the internal configurations are independently represented. This configuration is commonly known as three-way switches.

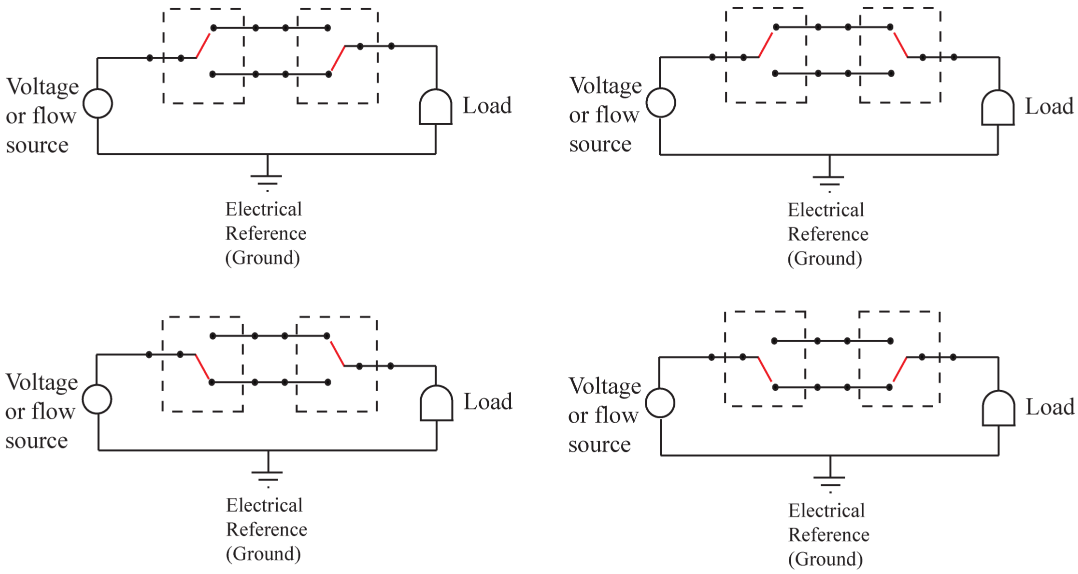


Figure 3.22: Configurations of two two-way switches (left side OFF configurations, right side ON configurations)

As previously mentioned, two-way switches can be represented by $X1$ junctions, the connection between two of these switches can be done using two of the same switched junction.

In Figure 3.23 the case for $X1$ multiswitched junctions is displayed, where the arrows show the flow of effort between ports, and the crosses show the lack of power transfer ($X1$ junction acting as a zero-flow source).

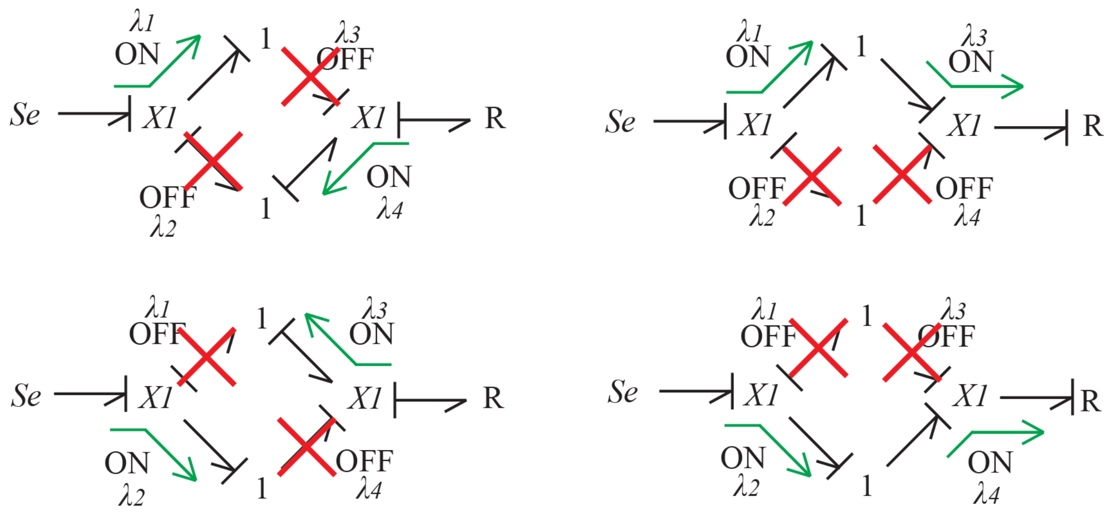


Figure 3.23: Connection of two two-way switches using X1 multiswitched junctions (left side OFF configurations, right side ON configurations)

It can be seen that these representations contain a causal conflict for the admissible configurations (the causality assignment in the 1-junction on the OFF bonds does not follow the proper causality assignment), however, these causal conflicts can be ignored for the analysis of the model as they represent zero flow or zero effort between bonds.

3.3.2 Two-way four port switches in bond graph

A two-way four port switch is displayed in Figure 3.24, where it can be seen that this type of switches has two input ports and two output ports.

In order to properly represent the admissible configurations of the two-way four port switch, a variant of the multiswitched junction is proposed. This variant is called $X10$ switched junction, where there are two input bonds and two output bonds (denoted by the Boolean parameters λ_1 and λ_2). The graphic representation is displayed in Figure 3.25.

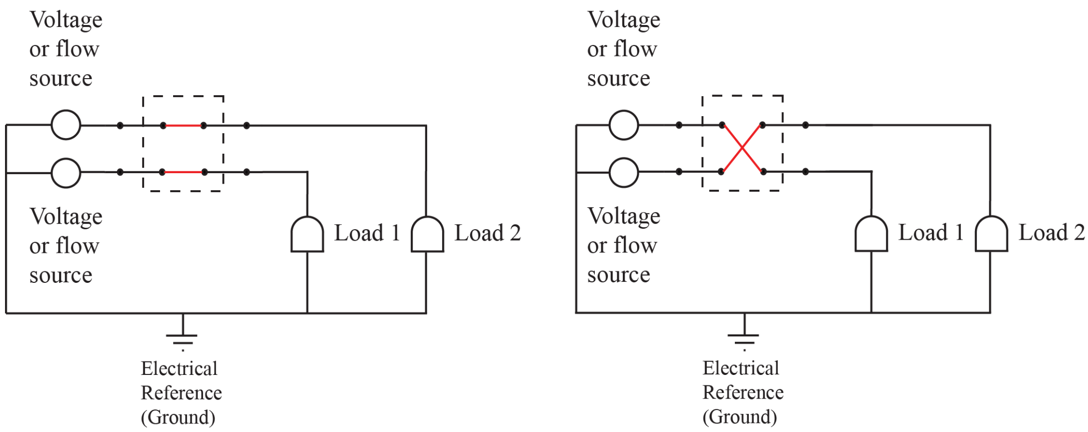


Figure 3.24: Internal configurations of the two-way four port switch, a) direct connection, b) crossed connection

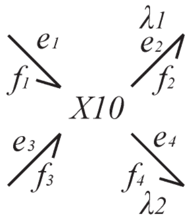


Figure 3.25: X10 multiswitched junction

The $X10$ multiswitched junction has this name because it shares the behaviour of both $X1$ and $X0$ switched junctions. When two causal strokes are on the same direction it indicates a general ON configuration, while the general OFF configurations are denoted by two causal strokes going into the junction ($X1$ switched junction behaviour) or two causal strokes going out of the junction ($X0$ switched junction). In order to differentiate the individual ON (λ) and OFF ($\bar{\lambda}$) configurations, it is decided to have as individual ON configuration when the causal stroke is going into the junction, while the individual OFF configuration is when the causal stroke is going out of the junction.

It must be noted that there is only power transfer between inputs and outputs, never between inputs or between outputs.

Also, this proposed notation allows the analysis of the configuration where there is a commutation and the inputs are disconnected from the outputs, which allows a more robust analysis of the system.

Based on the previous considerations, the equations that rule the behaviour of the $X10$ multiswitched junction in direct connection are,

$$
\begin{aligned}
e_1 &= \bar{\lambda}_1 e_2, \quad e_3 = \lambda_2 e_4 \\
f_1 &= \bar{\lambda}_1 f_2, \quad f_3 = \lambda_2 f_4
\end{aligned}
\tag{3.5}
$$

or in the alternative configuration for the direct connection,

$$
\begin{aligned}
e_1 &= \lambda_1 e_2, \quad e_3 = \bar{\lambda}_2 e_4 \\
f_1 &= \lambda_1 f_2, \quad f_3 = \bar{\lambda}_2 f_4
\end{aligned}
\tag{3.6}
$$

And the equations that rule the behaviour of the $X10$ multiswitched junction in crossed connection are,

$$
\begin{aligned}
e_1 &= \lambda_2 e_4, \quad e_3 = \bar{\lambda}_1 e_2 \\
f_1 &= \lambda_2 f_4, \quad f_3 = \bar{\lambda}_1 f_2
\end{aligned}
\tag{3.7}
$$

or in the alternative configuration for the crossed connection,

$$
\begin{aligned}
e_1 &= \bar{\lambda}_2 e_4, \quad e_3 = \lambda_1 e_2 \\
f_1 &= \bar{\lambda}_2 f_4, \quad f_3 = \lambda_1 f_2
\end{aligned}
\tag{3.8}
$$

The previous behaviours are displayed in Figure 3.26, where they are shown as the possible configurations of the $X10$ junction, where the arrows represent the effort flow between ports.

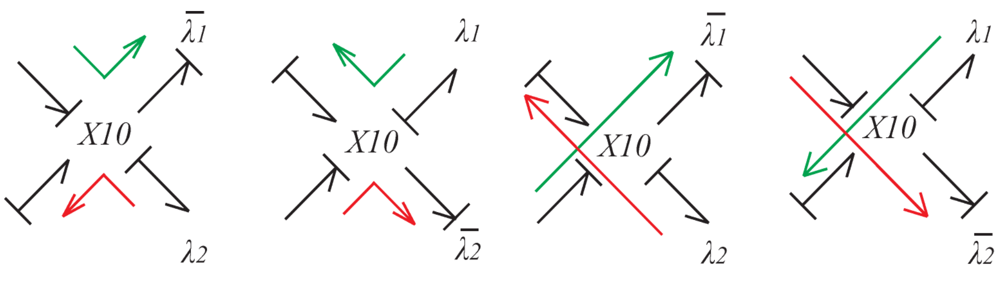


Figure 3.26: Configurations of the $X10$ multiswitched junction

As previously mentioned, there are two representations for each configuration based on the causality assignment.

The configurations displayed in Figure 3.27 are considered non-admissible for the $X10$ junction in order to avoid confusion to the user, the cases where there is a general OFF configuration are displayed in Figure 3.28.

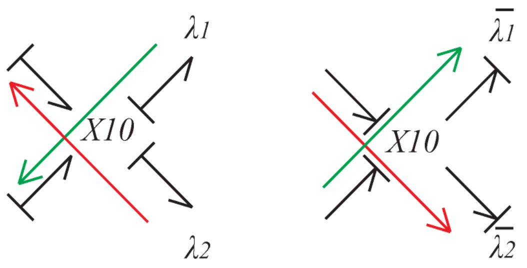


Figure 3.27: Non-admissible configurations for the $X10$ junction

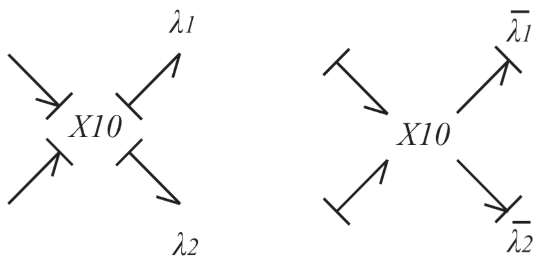


Figure 3.28: OFF configurations of the X10 switched junction

The general OFF configurations displayed in Figure 3.28 does not allow the transfer of flow or effort, which are based on the configurations of the $X0$ and $X1$ switched junctions. In configuration displayed in the left side, the junction does not allow transfer of effort, while on the configuration in the right side, the junction does not allow transfer of flow.

With a simplified representation displayed in Figure 3.29, where the different configurations are displayed using dashed causal strokes.

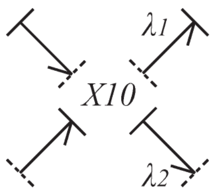


Figure 3.29: Simplified X10 representation

It can be seen that this variant of the multiswitch is a combination of the $X0$ and $X1$ switched junctions, where there is only power exchange between one input and one output at a time, never between one input and two outputs, two inputs and one output, between inputs or between outputs, which is the same behaviour as a four-way switch.

As previously shown, the causality assignment is the one that dictates the relation between inputs and outputs, which creates the two available internal configurations.

3.4 Implicit formulation of the hybrid model equations

3.4.1 Hybrid Junction structure

To expand the analysis of hybrid bond graph systems, dynamic causality and switched junctions are used in order to obtain a more physically related behaviour of the elements in the system. This introduces some changes to the general junction structure of the system.

The changes on the junction structure and the junction structure matrix for hybrid systems introduce a new matrix including Boolean parameters that displays the commutation of the switching elements.

These Boolean parameters denotes the available configurations of a switching element, nonetheless, as this being a general representation of the elements, the Boolean parameters do not have assigned values to them.

This matrix with Boolean parameters allows a simplified version of the junction structure matrix that represents all the available configurations contained in the model.

In order to properly represent all the interactions between the different elements on all available configurations, a general representation of the junction structure is introduced in Figure 3.30, with a junction structure matrix as:

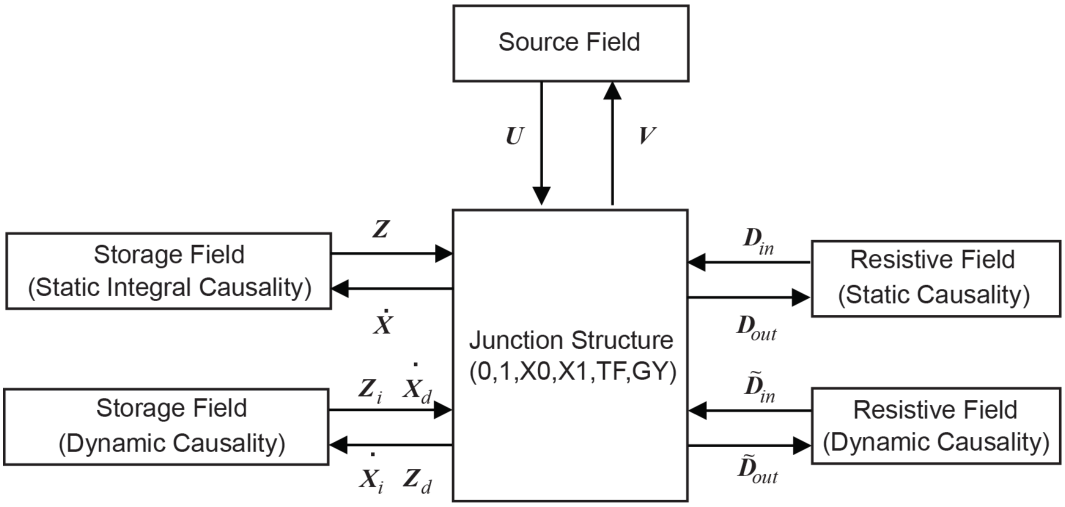


Figure 3.30: Hybrid Junction Structure

$$\begin{bmatrix} \Lambda_{11}(\lambda) & 0 & 0 & 0 & 0 \\ 0 & \Lambda_{22}(\lambda) & 0 & 0 & 0 \\ 0 & 0 & \bar{\Lambda}^*_{22}(\lambda) & 0 & 0 \\ 0 & 0 & 0 & \Lambda_{44}(\lambda) & 0 \\ 0 & 0 & 0 & 0 & \bar{\Lambda}^*_{44}(\lambda) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ z_d \\ D_{out} \\ \tilde{D}_{out} \end{bmatrix} =$$

$$\begin{bmatrix} S_{11}(\lambda) & S_{12}(\lambda) & S_{13}(\lambda) & S_{14}(\lambda) & S_{15}(\lambda) & S_{16}(\lambda) \\ -S^T_{12}(\lambda) & 0 & 0 & 0 & 0 & S_{26}(\lambda) \\ -S^T_{13}(\lambda) & 0 & S_{33}(\lambda) & S_{34}(\lambda) & S_{35}(\lambda) & S_{36}(\lambda) \\ -S^T_{14}(\lambda) & 0 & -S^T_{34}(\lambda) & S_{44}(\lambda) & S_{45}(\lambda) & S_{46}(\lambda) \\ -S^T_{14}(\lambda) & 0 & -S^T_{35}(\lambda) & -S^T_{45}(\lambda) & S_{55}(\lambda) & S_{46}(\lambda) \end{bmatrix} \begin{bmatrix} z \\ z_i \\ \dot{x}_d \\ D_{in} \\ \tilde{D}_{in} \\ U \end{bmatrix}$$

where $\dot{x}$ are the energy storage elements not affected by dynamic causality, $\dot{x}_i$ are energy storage elements affected by dynamic causality with integral causality assigned, $\dot{x}_d$ are energy storage elements affected by dynamic causality with derivative causality assigned, D are the resistive elements not affected by dynamic causality, and $\tilde{D}$ are the resistive elements affected by dynamic causality.

The junction structure displayed in Figure 3.30 is different from the on displayed in Figure 3.4, this to properly differentiate between the elements that affected by dynamic causality and those that are not. This is done to do a proper analysis of all of the possible behaviours in the systems.

It must be mentioned that there are no energy storage elements with fixed derivative causality in this notation. This is because it is considered that the elements with derivative causality are always affected by dynamic causality, which is done in order to simplify the notation and the analysis of hybrid dynamical systems.

3.4.2 Notation

It can be seen that there are new elements included in the junction structure matrix, these elements are introduced to explain the interaction between the different state variables when a commuting element is present in the system and are carried to the implicit matrix.

The relation between storage elements affected by dynamic causality and the rest of the elements is denoted by the submatrix Λ_{22}, which is the one that determines the available configurations for the commuting elements when the affected storage elements have integral causality.

In this proposed approach there is a distinction between the storage elements affected by dynamic causality and those which are not. In this case the elements that are not affected by dynamic causality are included in the state variables $\dot{x}$ for their analysis, while those affected by dynamic causality are included in the state variables $\dot{x}_i$ for their analysis.

Matrix $\bar{\Lambda}^*_{22}$ denotes the configurations of the commuting elements when the affected storage elements have derivative causality. In this case the storage elements affected by dynamic causality change from integral to derivative causality, resulting in z_d notation being used for this purpose.

The resistive elements can also be affected by dynamic causality, in this case the submatrices Λ_{44} and $\bar{\Lambda}^*_{44}$ are used to denote this behaviour.
In case where resistive elements are affected by dynamic causality the terms $\tilde{D}_{out}$ and $\tilde{D}_{in}$ are used to denote when these elements are affected, and the commuting elements are in OFF configuration.

As previously explained, if dynamic causality affects a resistive element, when the commuting element is in OFF configuration, the lack of power transferred allows to discard the previous submatrices and element's notation, which results in the traditional D_{out} and D_{in} elements. In this case, an identity matrix replaces the submatrix Λ_{44} to denote the presence of the resistive elements when those are not affected by dynamic causality.

The submatrices $S_{ij}(\lambda)$ (where $i = 1, 2, 3, 4, 5$ and $j = 1, 2, 3, 4$)) indicate the relations between the different elements (affected and non-affected by dynamic causality). It must be noted that these submatrices contain the element (λ) which denotes that the relation could depend on the configuration of one or more commuting elements. This does not mean that the elements are affected by dynamic causality, it just means that the causal path between the elements crosses a commuting element.

z remains as the conjugate power variable for the storage elements that are not affected by dynamic causality, U denotes the inputs, while z_i and $\dot{x}_d$ are the coenergy values of the storage elements affected by dynamic causality in ON and OFF configuration respectively.

In order to simplify the notation, in the following expressions the Boolean variables (λ) are omitted as the elements S_{ij}, Λ_{22} and $\bar{\Lambda}_{22}^{*}$ could contain a causal path with a commuting element.

During the analysis of the properties of the extended junction structure matrix, the behaviour of the resistive elements when affected by the dynamic causality is different from the one presented by energy storage elements.

While the switched junction is in ON configuration, the resistive element behaves normally, nevertheless, when the switched junction is in OFF configuration there is an absence of one of the power variables (either effort or flow), which impose a zero effort or zero flow into the resistive element. Therefore, a resistive element affected by the dynamic causality does not contribute to the dynamic of the system in the OFF configuration as no energy is generated, stored or dissipated.

This phenomenon leads to a simplified version of the junction structure, which can be seen in Figure 3.31, and a new junction structure matrix as:

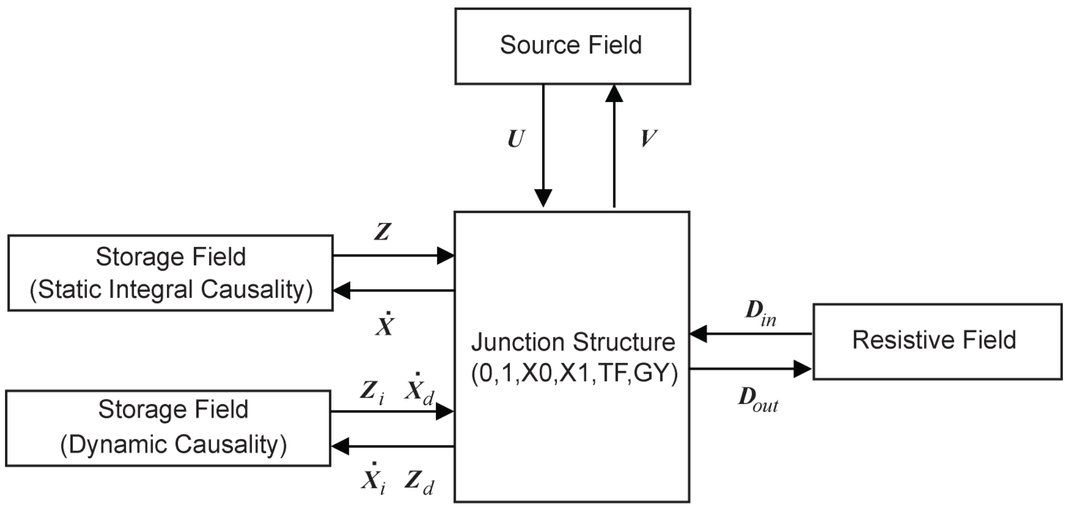


Figure 3.31: Simplified Hybrid Junction Structure

$$\begin{bmatrix} \Lambda_{11}(\lambda) & 0 & 0 & 0 \\ 0 & \Lambda_{22}(\lambda) & 0 & 0 \\ 0 & 0 & \bar{\Lambda}_{22}^{*}(\lambda) & 0 \\ 0 & 0 & 0 & I(\lambda) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ z_d \\ D_{out} \end{bmatrix} =$$

$$\begin{bmatrix} S_{11}(\lambda) & S_{12}(\lambda) & S_{13}(\lambda) & S_{14}(\lambda) & S_{15}(\lambda) \\ -S_{12}^T(\lambda) & S_{22}(\lambda) & 0 & S_{24}(\lambda) & S_{25}(\lambda) \\ -S_{13}^T(\lambda) & 0 & 0 & 0 & S_{35}(\lambda) \\ -S_{14}^T(\lambda) & -S_{24}^T(\lambda) & 0 & S_{44}(\lambda) & S_{45}(\lambda) \end{bmatrix} \begin{bmatrix} z \\ z_i \\ \dot{x}_d \\ D_{in} \\ U \end{bmatrix}$$

From this point onwards, the submatrices $S_{ij}(\lambda)$, which are functions of Boolean parameters λ indicating the state of the switches, will simply be denoted S_{ij}.

Also, the resistive field is reduced to a single block, this is the result of the analysis of the resistive field affected by dynamic causality, which does not allow power transfer during OFF configurations, therefore, there is power transfer only during the ON configurations, which includes the resistive elements that are not affected by dynamic causality.

3.4.3 Determination of the hybrid junction structure matrix

It is necessary to set a series of steps to obtain the hybrid junction structure matrix (HJSM) when using switched junctions. This is because an arbitrary assignment of causality can lead to undesired behaviours or the inclusion of non-admissible configurations to the HJSM. These steps are described in the following Dynamic Sequential Causal Assignment Procedure (DSCAP):

Procedure: DSCAP

1. Obtain the model of the system applying dynamic causality.
    (a) Draw the system's model without causality.
    (b) Assign causality to the elements with fixed causality (sources).
    (c) Assign integral causality to storage elements as much as possible while avoiding causal conflicts.
    (d) Expand causality following SCAP procedure to the rest of the model.
        - Switched junctions behaves as normal 0 or 1 junctions.
        - Dynamic causality is assigned following HSCAP, as described in the following steps.

  - A switched junction without causality assigned must be chosen. The causality must be assigned with ON configuration and propagate as far as possible. This is repeated until all of the switched junctions have their causality assigned. If a causal conflict appears during the assignment, the configuration of the switched junction that create the conflict must be changed.
  - The propagation of causality is finished by assigning it to the resistive elements and or remaining bonds and propagate as far as possible.
  - Taking each switched junction in turn, the causality assignment must be considered in the alternate configuration corresponding to the reference configuration. The new causality will be assigned with a dashed causal stroke, and propagate throughout the model. If a causal conflict occurs during this stage, then the alternate configuration of the switched junction is not allowed.
- Dynamic causality must be assigned giving priority to the elements that allow to maximise the assignment of integral causality to the energy storage elements.
- If more than one storage elements can be affected by dynamic causality in a single junction, dynamic causality is assigned to the element that has derivative causality in fewer configurations. In the case where dynamic causality created by multiple switched junctions affect several elements, the assignment is done for the combination that result in fewer elements changing causality. This is in order to maximize the assignment of integral causality.
- The assignment of causality on resistive elements must be done after the expansion of dynamic causality from the storage elements to the switched junctions that are affecting its behaviour. This is made to maximise the number of storage elements with integral causality during the assignment of dynamic causality.
- Dynamic causality is assigned to resistive elements only in the case where they allow to maximise the number of energy storage elements with integral causality.

2. Obtain a truth table from the switch junction elements interactions.

   (a) This truth table must show the causality assignment of the storage elements that are affected by dynamic causality for the different configurations of the switched junctions.

   (b) In the case where dynamic causality does not propagate to another switched junction, there is no need to obtain a table of truth of all of the available configurations.

There are cases were the dynamic causality created by a switch junction does not affect another switch junction. Also, there are other cases where elements are only affected by the dynamic causality created by one switch junction. This is done in order to have a reduced table of truth.

(c) Notation of this truth table must be done based on Boolean algebra notation.

(d) From this truth table the Λ matrix is obtained.

3. Obtain the equations for the system's elements.

   (a) The equation must contain all of the configurations in which an element has an interaction with another element.

   (b) It must be noted that matrices Λ_{22} and $\bar{\Lambda}^*_{22}$ are not complement of each other when there are non-valid configurations.

4. Generate junction structure equation.

   - It must be possible to analyse all of the possible configurations in the junction structure. If there are missing configurations, this mean that the causality assignment was not properly done.

5. Obtain the implicit equation.

   - The final implicit equation must comply with the established notation.

An example of the DSCAP procedure is illustrated for the diagram of an electric RC circuit shown in Figure 3.32:

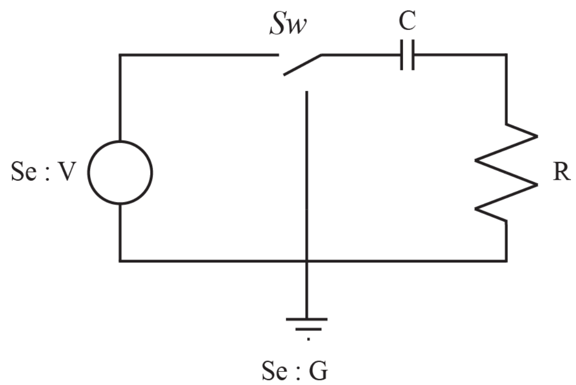


Figure 3.32: Diagram of a RC circuit

with a bond graph model displayed in Figure 3.33

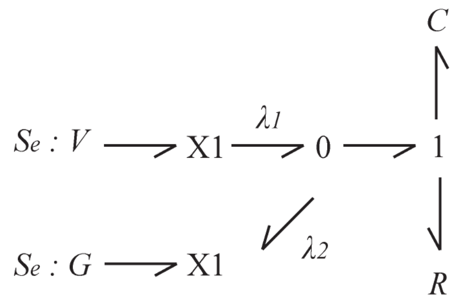


Figure 3.33: Model without causality

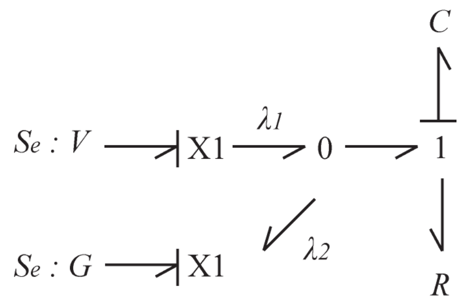


Figure 3.34: Model with fixed causality on sources and integral causality on energy storage elements

Then fixed causality is assigned to sources and integral causality to energy storage elements (Figure 3.34).

Expand causality following SCAP (Figure 3.35).

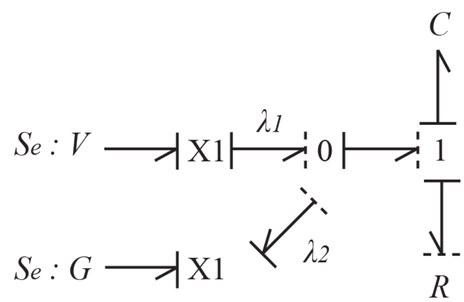


Figure 3.35: Model with complete causality assignment

The next step is the obtention of a truth table for the switched junctions interaction.

Switched junctions configuration	Causality of element C
$\bar{\lambda}_1\bar{\lambda}_2$	Integral
$\lambda_1\bar{\lambda}_2$	Integral
$\bar{\lambda}_1\lambda_2$	Integral
$\lambda_1\lambda_2$	Causal conflict

Using Boolean arithmetic, a simplified relation of configurations can be obtained,

Element C	$\bar{\lambda}_1\bar{\lambda}_2 + \lambda_1\bar{\lambda}_2 + \bar{\lambda}_1\lambda_2 = \overline{(\lambda_1\lambda_2)}$
Element R (before change in causality)	$\bar{\lambda}_1\bar{\lambda}_2$
Element R (after change in causality)	$\lambda_1\bar{\lambda}_2 + \bar{\lambda}_1\lambda_2 = \left(\bar{\lambda}_1 + \bar{\lambda}_2\right)$

From the previous relations, the Λ matrix is obtained,

$$\begin{bmatrix} \overline{(\lambda_1\lambda_2)} & 0 & 0 \\ 0 & \bar{\lambda}_1\bar{\lambda}_2 & 0 \\ 0 & 0 & \left(\bar{\lambda}_1 + \bar{\lambda}_2\right) \end{bmatrix} \begin{bmatrix} \dot{q} \\ f_R \\ e_R \end{bmatrix}$$

As mentioned in the procedure, the notation is Boolean algebra based, therefore, the symbol '+' denotes an 'OR' operation and $\cdot$ denotes 'AND' operation.

For the following step it is necessary to obtain the equations of the elements.

$$\begin{aligned} \overline{(\lambda_1\lambda_2)}\dot{q} &= \left(\bar{\lambda}_1 + \bar{\lambda}_2\right) f_R \\ \bar{\lambda}_1\bar{\lambda}_2 f_R &= 0 \\ \left(\bar{\lambda}_1 + \bar{\lambda}_2\right) e_R &= \lambda_1\bar{\lambda}_2 V + \bar{\lambda}_1\lambda_2 G - \left(\bar{\lambda}_1 + \bar{\lambda}_2\right) q \end{aligned}$$

With these equations the HJSM is generated.

$$\begin{bmatrix} \overline{(\lambda_1\lambda_2)} & 0 & 0 \\ 0 & \bar{\lambda}_1\bar{\lambda}_2 & 0 \\ 0 & 0 & \left(\bar{\lambda}_1 + \bar{\lambda}_2\right) \end{bmatrix} \begin{bmatrix} \dot{q} \\ f_R \\ e_R \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & \left(\bar{\lambda}_1 + \bar{\lambda}_2\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\left(\bar{\lambda}_1 + \bar{\lambda}_2\right) & 0 & 0 & \lambda_1\bar{\lambda}_2 & \bar{\lambda}_1\lambda_2 \end{bmatrix} \begin{bmatrix} q \\ e_R \\ f_R \\ V \\ G \end{bmatrix}$$

The last step consist in the obtention of the hybrid implicit equation.

$$\overline{(\lambda_1\lambda_2)}\dot{q} = \frac{(\bar{\lambda}_1 + \bar{\lambda}_2)}{R}q \tag{3.9}$$

For the models that contains msj the procedure is similar.

The diagram of an electric circuit with a multiswitch is displayed in Figure 3.36.

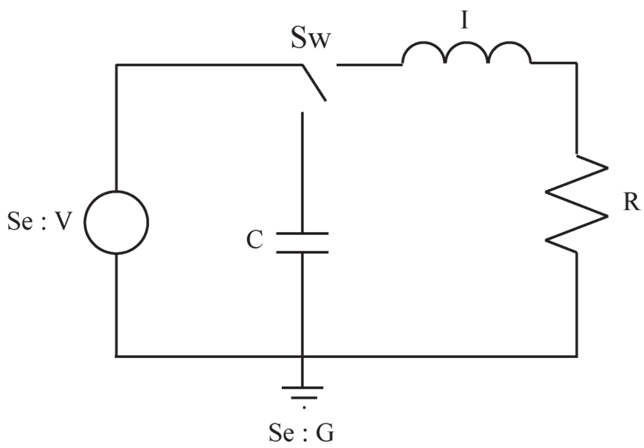


Figure 3.36: Electric circuit containing a multiswitch

First a model without causality is needed (Figure 3.37).

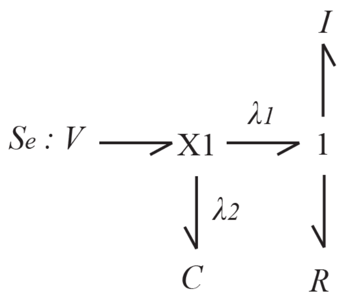


Figure 3.37: Model without causality

Then fixed causality is assigned to sources and integral causality to energy storage elements (Figure 3.38).

Expand causality following SCAP (Figure 3.39), then obtaining a truth table for the msj internal configurations.

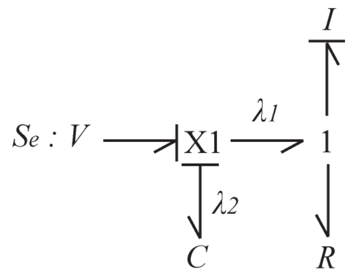


Figure 3.38: Model with causality on

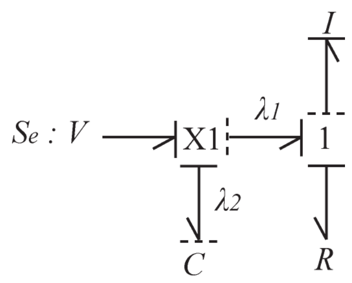


Figure 3.39: Model with complete causality assigned

msj configuration	Causality of element I	Causality of element C
$\bar{\lambda}_1\bar{\lambda}_2$	Derivative	Integral
$\lambda_1\bar{\lambda}_2$	Integral	Integral
$\bar{\lambda}_1\lambda_2$	Derivative	Derivative

Using Boolean arithmetic, a simplified relation of configurations can be obtained,

Element I (before change in causality)	$\lambda_1\bar{\lambda}_2$
Element I (after change in causality)	$\bar{\lambda}_1\bar{\lambda}_2 + \bar{\lambda}_1\lambda_2$
Element C (before change in causality)	$\bar{\lambda}_1\bar{\lambda}_2 + \lambda_1\bar{\lambda}_2$
Element C (after change in causality)	$\bar{\lambda}_1\lambda_2$

From the previous relations it can be seen that both storage elements are affected by both of the bonds in the msj, this is because using only the Boolean parameter connected to one of the storage elements causal path will not reflect the behaviour of the msj, therefore all of the Boolean parameters must be used for all of the outputs. Then the Λ matrix is

$$\begin{bmatrix} \lambda_1\bar{\lambda}_2 & 0 & 0 & 0 & 0 \\ 0 & \bar{\lambda}_1\bar{\lambda}_2 + \bar{\lambda}_1\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \bar{\lambda}_1\bar{\lambda}_2 + \lambda_1\bar{\lambda}_2 & 0 & 0 \\ 0 & 0 & 0 & \bar{\lambda}_1\lambda_2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_i \\ \dot{p}_i \\ f_{Id} \\ e_{Cd} \\ f_R \end{bmatrix}$$

The rest of the procedure remains the same as the previous example.

There is a special case where there are sources affected by dynamic causality, as sources have static causality it is necessary to use switched junction instead of the multiswitched junction to eliminate the forced 0 flow or 0 effort imposed to the source. This is displayed in the next example.

From the RL circuit displayed in Figure 3.40, first a model without causality is needed (Figure 3.41), then fixed causality is assigned to sources and integral causality to energy storage elements (Figure 3.42), and expand causality following SCAP (Figure 3.43).

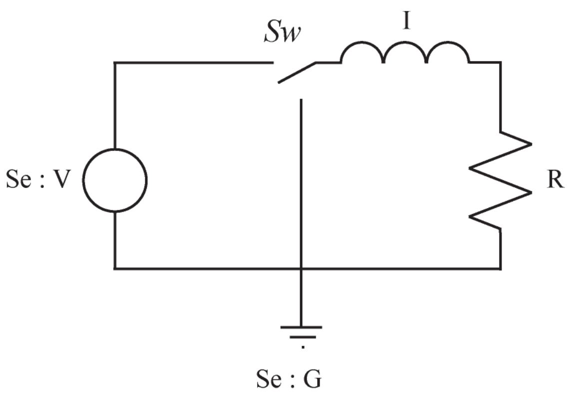


Figure 3.40: Diagram of a RL circuit

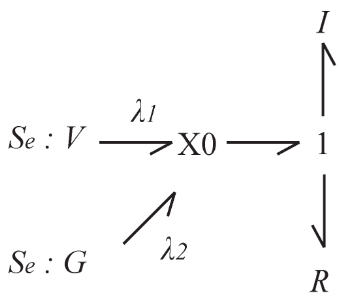


Figure 3.41: Model without causality

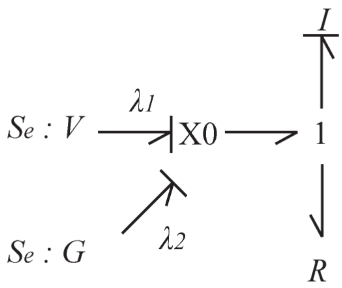


Figure 3.42: Model with causality on

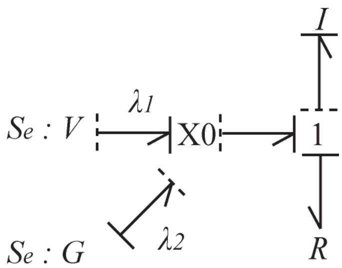


Figure 3.43: Model with complete causality assigned

As previously mentioned, if there are sources affected by dynamic causality, switched junction are used to connect each source (Figure 3.44).

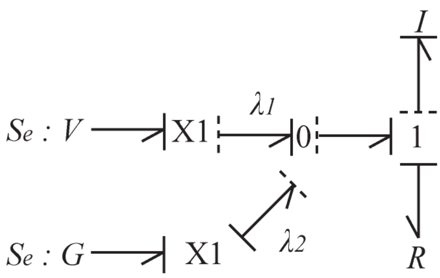


Figure 3.44: Model with auxiliary switched junctions

The next step is the obtention of a truth table for the auxiliary switched junctions interaction with the msj.

msj configuration	Causality of element I
$\bar{\lambda}_1\bar{\lambda}_2$	Derivative
$\lambda_1\bar{\lambda}_2$	Integral
$\bar{\lambda}_1\lambda_2$	Integral
$\lambda_1\lambda_2$	Causal conflict

Using Boolean arithmetic, a simplified relation of configurations can be obtained,

Element I (before change in causality)	$\lambda_1\bar{\lambda}_2 + \bar{\lambda}_1\lambda_2 = (\lambda_1 \oplus \lambda_2)$
Element I (after change in causality)	$\bar{\lambda}_1\bar{\lambda}_2$

As the final model use normal switched junction, the properties from those junctions are applied to the model. Therefore, it can be seen that there is one non-admissible mode, which gives a Λ matrix

$$\begin{bmatrix} (\lambda_1 \oplus \lambda_2) & 0 & 0 \\ 0 & \bar{\lambda}_1\bar{\lambda}_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{p}_i \\ f_{Id} \\ f_R \end{bmatrix}$$

It can be seen that it is not possible to use msj when there are sources affected by dynamic causality, this due to imposed flux or effort on a fixed causality element, therefore it is necessary to expand the msj element into its individual components, which in this case are individual switched junctions.

The procedure for the obtention of the hybrid implicit equation is described in the next section.

3.5 Hybrid Implicit model

The implicit equation that describes the model behaviour is obtained from the junction structure matrix,

$$\begin{bmatrix} \Lambda_{11} & 0 & 0 & 0 \\ 0 & \Lambda_{22} & 0 & 0 \\ 0 & 0 & \bar{\Lambda}^*_{22} & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ z_d \\ D_{out} \end{bmatrix} = \tag{3.10}$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ -S_{12}^T & S_{22} & 0 & S_{24} & S_{25} \\ -S_{13}^T & 0 & 0 & 0 & S_{35} \\ -S_{14}^T & -S_{24}^T & 0 & S_{44} & S_{45} \end{bmatrix} \begin{bmatrix} z \\ z_i \\ \dot{x}_d \\ D_{in} \\ U \end{bmatrix}$$

having the following equations,

$$\begin{aligned}
\Lambda_{11}\dot{x} - S_{13}\dot{x}_d &= S_{11}z + S_{12}z_i + S_{14}D_{in} + S_{15}U \\
\Lambda_{22}\dot{x}_i &= -S_{12}^T z + S_{22}z_i + S_{24}D_{in} + S_{25}U \\
0 &= -\bar{\Lambda}_{22}^* z_d - S_{13}^T z + S_{35}U \\
0 &= -D_{out} - S_{14}^T z - S_{24}^T z_i + S_{44}D_{in} + S_{45}U
\end{aligned} \tag{3.11}$$

From line 4 of 3.11,

$$D_{out} = -S_{14}^T z - S_{24}^T z_i + S_{44}D_{in} + S_{45}U \tag{3.12}$$

having the constitutive relations of the resistive elements as

$$D_{out} = LD_{in} \tag{3.13}$$

By substituting 3.13 in 3.12,

$$D_{in} = \left(L\left(I - S_{44}L^{-1}\right)\right)^{-1}\left(-S_{14}^T z - S_{24}^T z_i + S_{45}U\right) \tag{3.14}$$

In order to simplify, the next consideration is made

$$H = \left(L\left(I - S_{44}L^{-1}\right)\right)^{-1}$$

Now, by substituting 3.14 in rows 1 and 2 of 3.11

$$\begin{aligned}
\Lambda_{11}\dot{x} - S_{13}\dot{x}_d \quad = \quad & \left(S_{11} - S_{14}HS_{14}^T\right)z + \\
& \left(S_{12} - S_{14}HS_{24}^T\right)z_i + \\
& \left(S_{15} + S_{14}HS_{45}\right)U
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
\Lambda_{22}\dot{x}_i \quad = \quad & \left(-S_{12}^T - S_{24}HS_{14}^T\right)z + \\
& \left(S_{22} - S_{24}HS_{24}^T\right)z_i + \\
& \left(S_{25} + S_{24}HS_{45}\right)U
\end{aligned} \tag{3.16}$$

The complementary state variables are related to the state by the constitutive law for the storage elements. In this case the matrix representing the constitutive law of the storage elements is:

$$\begin{bmatrix} z \\ z_i \\ z_d \end{bmatrix} = \begin{bmatrix} F & 0 & 0 \\ 0 & F_i & 0 \\ 0 & 0 & F_d \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix} \tag{3.17}$$

Where matrix F is a diagonal matrix of linear coefficients C^{-1} or L^{-1} for elements that are not affected by dynamic causality.

For elements affected by dynamic causality during integral causality assignment, matrix F_i is a diagonal matrix with terms $\sum \lambda C^{-1}$ or $\sum \lambda L^{-1}$.

For elements affected by dynamic causality during derivative causality assignment, matrix F_d is a diagonal matrix with terms $\sum \lambda C$ or $\sum \lambda L$.

Using the constitutive relations for the storage elements 3.17 with 3.15, 3.16 and row 3 of 3.11, the implicit state equation is obtained

$$\begin{bmatrix} \Lambda_{11} & 0 & -S_{13} \\ 0 & \Lambda_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} =$$
$$\begin{bmatrix} \left(S_{11} - S_{14}HS_{14}^T\right)F & \left(S_{12} - S_{14}HS_{24}^T\right)F_i & 0 \\ \left(-S_{12}^T - S_{24}HS_{14}^T\right)F & \left(S_{22} - S_{24}HS_{24}^T\right)F_i & 0 \\ -S_{13}^T F & 0 & -\bar{\Lambda}_{22}^* F_d \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix}$$
$$+ \begin{bmatrix} S_{15} + S_{14}HS_{45} \\ S_{25} + S_{24}HS_{45} \\ S_{35} \end{bmatrix} [U] \tag{3.18}$$

3.6 Properties of the implicit model

3.6.1 Properties of the hybrid junction structure matrix

Assumption 1. Assuming that the initial configuration of the causal hybrid bond graph (i.e. before considering alternative switches states) has no elements in derivative causality, variables used in hybrid junction structure are shown in 3.31.

With the variables used in Fig. 3.31, the junction structure equations may be written as

$$\begin{bmatrix} \Lambda_{11} & 0 & 0 & 0 \\ 0 & \Lambda_{22} & 0 & 0 \\ 0 & 0 & \bar{\Lambda}_{22}^* & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ z_d \\ D_{out} \end{bmatrix} =$$
$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ -S_{12}^T & S_{22} & 0 & S_{24} & S_{25} \\ -S_{13}^T & 0 & 0 & 0 & S_{35} \\ -S_{14}^T & -S_{24}^T & 0 & S_{44} & S_{45} \end{bmatrix} \begin{bmatrix} z \\ z_i \\ \dot{x}_d \\ D_{in} \\ U \end{bmatrix}$$

Property 3. The submatrices S_{11}, S_{22} and S_{44} are skew-symmetric.

Property 4. The submatrices $S_{12} = S_{12}^T$, $S_{13} = S_{13}^T$, $S_{14} = S_{14}^T$, and $S_{24} = S_{24}^T$.

Properties 3 and 4 are due to the continuity of power through the junction structure.

Property 5. Λ_{22} and $\bar{\Lambda}_{22}^*$ are so that Λ_{22}ů$\bar{\Lambda}_{22}^*$=[0].

Matrices Λ_{22} and $\bar{\Lambda}_{22}^*$ are diagonal matrices. Elements of matrices Λ_{22} and $\bar{\Lambda}_{22}^*$ are Boolean expressions representing the combination of switches for which individual energy-storage elements affected by dynamic causality are in integral or in derivative causality, respectively. These expressions are derived from the tables of truth of a group of switches whose dynamic causality interact with each other.

This implies that for each combination of switches with Boolean variables (1 or 0) assigned Λ_{22}^* is the complement of Λ_{22} (i.e. $\Lambda_{22}^* = \Lambda_{22}$ if the value of λ in $\bar{\Lambda}_{22}^*$ is the complement of λ in Λ_{22}) when all modes of the switched system are valid.

Property 6. If $rank \left[\ \Lambda_{22} + \bar{\Lambda}_{22}^* \ \right] < n$, where n is the number of storage elements, then a invalid mode exist.

For such configurations, the associated Boolean terms in Λ_{22} and $\bar{\Lambda}_{22}^*$ are simultaneously 0.

The combinations of switches leading to invalid modes are excluded from Λ_{22} and $\bar{\Lambda}_{22}^*$ as these lead to causal conflicts or non-solvable equations that cannot be part of the junction structure equations.

For valid modes, the matrix $\Lambda_{22} + \bar{\Lambda}_{22}^*$ is an identity matrix.

Property 7. Matrices are so that $\Lambda_{22}S_{2i} = S_{2i}$ for i=1,2,4,5.

This relation is caused by the fact that matrix Λ_{22} determines the integral causality assignment in the energy storage elements, whose interactions are described by submatrices S_{2i}.

As a result of the above property, if a combination of switches is so that a specific row of $S_{2i} \neq 0$ for $i = 1, 2, 4, 5$, then the corresponding element in Λ_{22} has a value of 1 for the same combination of switches.

On the contrary, if a combination of switches is so that an element in Λ_{22} has a value of 0, then the corresponding row of $S_{2i} = 0$ for $i = 1, 2, 4, 5$ for the same combination of switches.

The same applies to $\bar{\Lambda}_{22}^{*} S_{3i} = S_{3i}$ for $i = 1, 5$. Where $\bar{\Lambda}_{22}^{*} = I(\lambda)$.

As a result of the above property, if a combination of switches is so that an specific row of $S_{3i} = 1$ for $i = 1, 5$ then the corresponding row of $\bar{\Lambda}_{22}^{*} = 1$ for the same combination of switches.

On the contrary, if a combination of switches is so that the correspondent row of $\bar{\Lambda}_{22}^{*} = 0$, then the correspondent row of $S_{3i} = 0$ for $i = 1, 5$ for the same combination of switches.

3.6.2 Properties of the hybrid implicit singular state matrix

Continuing with **assumption 1**, the resulting hybrid implicit singular state matrix is

$$\begin{bmatrix} \Lambda_{11} & 0 & -S_{13} \\ 0 & \Lambda_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} =$$

$$\begin{bmatrix} \left(S_{11} - S_{14} H S_{14}^T\right) F & \left(S_{12} - S_{14} H S_{24}^T\right) F_i & 0 \\ \left(-S_{12}^T - S_{24} H S_{14}^T\right) F & \left(S_{22} - S_{24} H S_{24}^T\right) F_i & 0 \\ -S_{13}^T F & 0 & -\bar{\Lambda}_{22}^{*} F_d \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix}$$

$$+ \begin{bmatrix} S_{15} + S_{14} H S_{45} \\ S_{25} + S_{24} H S_{45} \\ S_{35} \end{bmatrix} [U]$$

To simplify notation, this matrix can be rewritten as

$$\begin{bmatrix} E_{11} & 0 & E_{13} \\ 0 & E_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & 0 & A_{33} \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} [U] \tag{3.19}$$

Based on the previous properties for the hybrid junction structure matrix, the following properties are obtained

Property 8. If there are no storage elements with static integral causality,

$$E_{13} = A_{11} = A_{12} = B_1 = 0$$

Property 9. E_{22} is so that $E_{22}A_{2i} = A_{2i}$ for $i = 1, 2$.

This is due to $E_{22} = \Lambda_{22}$ and its properties remains.

As a result of the above property, if a combination of switches is so that $A_{22} = I$ for $i = 1, 2$, then $E_{22} = I$ for the same combination of switches. On the contrary, if a combination of switches is so that $E_{22} = 0$, then $A_{2i} = 0$ for $i = 1, 2$ for the same combination of switches.

3.6.3 Properties of a single mode equation

The hybrid implicit singular state matrix can also be analysed for individual configurations. In this case, there are three possible resulting equations,

- when there are only storage elements affected by dynamic causality with integral causality.
- when there are only storage elements affected by dynamic causality with derivative causality.
- when there are storage elements affected by dynamic causality in both integral and derivative causality (previous cases are present at the same time).

For the case when all the storage elements have integral causality the implicit state matrix is

$$\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} \left(S_{11} - S_{14}HS_{14}^T\right)F & \left(S_{12} - S_{14}HS_{24}^T\right)F_i \\ \left(-S_{12}^T - S_{24}HS_{14}^T\right)F & \left(S_{22} - S_{24}HS_{24}^T\right)F_i \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} + \begin{bmatrix} S_{15} + S_{14}HS_{45} \\ S_{25} + S_{24}HS_{45} \end{bmatrix} [U] \tag{3.20}$$

This case only contains storage elements in integral causality, meaning that the system could be considered as a linear time invariant system, which means that the properties for that type of systems can be used as well.

The next case is when all storage elements affected by dynamic causality have derivative causality, having as implicit state matrix the following

$$
\begin{bmatrix} I & -S_{13} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_d \end{bmatrix} =
\begin{bmatrix} \left(S_{11} - S_{14} H S_{14}^T\right) F & 0 \\ -S_{13}^T F & -F_d \end{bmatrix} \begin{bmatrix} x \\ x_d \end{bmatrix}
+ \begin{bmatrix} S_{15} + S_{14} H S_{45} \\ S_{35} \end{bmatrix} [U] \tag{3.21}
$$

In this case the system behaviour is similar to a linear time-invariant system with dependant elements, therefore, it shares the same properties of this kind of systems.

And for the last case it is the configuration where there are storage elements affected by dynamic causality in both integral and derivative causality.

$$
\begin{bmatrix} \Lambda_{11} & 0 & -S_{13} \\ 0 & \Lambda_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} =
\begin{bmatrix} \left(S_{11} - S_{14} H S_{14}^T\right) F & \left(S_{12} - S_{14} H S_{24}^T\right) F_i & 0 \\ \left(-S_{12}^T - S_{24} H S_{14}^T\right) F & \left(S_{22} - S_{24} H S_{24}^T\right) F_i & 0 \\ -S_{13}^T F & 0 & -\bar{\Lambda}_{22}^* F_d \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix}
+ \begin{bmatrix} S_{15} + S_{14} H S_{45} \\ S_{25} + S_{24} H S_{45} \\ S_{35} \end{bmatrix} [U] \tag{3.22}
$$

In this last case, the properties remain the same as with the hybrid implicit singular state matrix due to be the same matrix.

3.6.4 Properties from the hybrid dynamic junction structure matrix

There are some properties that can be obtained from the junction structure matrix (jsm) that allows to simplify the analysis of the system. This general analysis allows to spend less time looking at individual configurations that could not be admissible. It is easier to observe the non-admissible configurations from the jsm.

In this case the parameters determining the non-admissible configurations are usually found in lines containing Boolean parameters non-complement in the

Λ matrix.

An example of this can be seen in the buck converter used on several literature reviews.

In this case the non-admissible configuration is determined by the lines that contain the Boolean parameters that are not complement in the Λ matrix, which are $(\lambda_1 \oplus \lambda_2)$ and $\left(\overline{\lambda_1 \lambda_2}\right)$.

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (\lambda_1 \oplus \lambda_2) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \overline{\lambda_1 \lambda_2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \bar{\lambda}_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3} \\ p_{L1d} \\ p_{L3d} \\ f_{R1} \\ f_{R2}
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
0 & a(\lambda_1 \oplus \lambda_2) & 0 & 0 & -\lambda_3 & 0 & -\lambda_3 & 0 & 0 \\
-a(\lambda_1 \oplus \lambda_2) & 0 & 0 & 0 & 0 & -(\lambda_1 \oplus \lambda_2) & 0 & \lambda_1 \bar{\lambda}_2 & \bar{\lambda}_1 \lambda_2 \\
0 & 0 & 0 & 0 & 0 & 0 & -\bar{\lambda}_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (\lambda_1 \oplus \lambda_2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_3 & 0 & \bar{\lambda}_3 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
f_{L2} \\ f_{L1} \\ f_{L3} \\ \dot{p}_{L1d} \\ \dot{p}_{L3d} \\ e_{R1} \\ e_{R2} \\ V \\ G
\end{bmatrix}
$$

This analysis is simple to do and save time to the user by avoiding the analysis of all of the available configurations, as it is simpler to identify the non-admissible configurations. This is of great help in more complex cases, where a greater amount of time is needed for its analysis.

As there is only one general equation, the it is only necessary to change the values of the Boolean parameters to obtain the configuration to analyse. This procedure simplifies the analysis of bigger and more complex systems, as the general equation remain the same, thus, each individual configuration can be obtained in a single substitution of Boolean parameters.

3.7 Summary

A new notation and some new representations for the different elements involved in the modelling of hybrid systems and their relations were introduced.

This allowed the introduction of a new junction structure and implicit equation that could be used as a standard notation for hybrid bond graph systems.

This notation can be used in non-bond graph singular systems and hybrid systems. This is due to the properties of the systems being carried from some of the approaches on these systems.

From these results, it can be seen that, with a proper integration of the previous approaches, a solid general representation of hybrid systems can be achieved.

A new DSCAP is proposed to fit with the new notation and avoid the exclusion of some configuration into the junction structure matrix.

Some properties of the hybrid junction structure and the implicit matrix for the different cases were presented, which are useful in the analysis of the models.

Using this notation and representation, the analysis of the behaviour of the models can be determined, which will be the main topic in the next chapter.

It must be noted that the analysis of switched junctions is mostly done using electrical switches, this is because the main focus is done on the behaviour during commutation, which is assumed to be the same for physical contact, frictions, diodes, and other elements that represent commuting behaviours.

Chapter 4

Analysis of Hybrid Dynamic Systems

4.1 Preliminaries

The structural properties of the hybrid dynamic bond graph are identified in this chapter based on previous works done on singular systems and hybrid bond graphs.

Structural properties can be obtained from the structure of the mathematical model through matrix-rank criteria and the structure of the bond graph and its causal assignment. In the case of the proposed approach introduced in this research, the structural properties obtained are not limited to hybrid dynamical bond graph systems, they can also be used on the analysis of hybrid bond graph systems, hybrid systems, and singular systems due to being based on approaches from these areas that shares common results.

Exploiting dynamic causality and its assignment leads to a new notation of the resulting structure matrices, which is useful for further analysis of the systems. Some of the results are based on the inspection of causal paths between elements that are affected by dynamic causality.

Model properties are reviewed by using a new approach based on singular systems techniques and hybrid bond graph techniques. Most of the techniques are based on LTI systems analysis, although some can be used on non-LTI systems, such as input-state stability and observers design for nonlinear singular systems, ([106]), observer design problem for a large class of nonlinear singular systems with multi outputs ([107]), an invertible non-linear map which transforms a non-linear singular system into a regular one that preserves local properties under the application of specific feedback control laws ([108]), an adaptative control for nonlinear singular systems ([109]), control design of an induction machine ([110]), which increase the application of the approach proposed in this research.

This research is meant to be used for further development of the analysis techniques of hybrid systems, allowing the obtention of more information from the model compared to previous approaches.

4.2 Boolean matrix

As previously mentioned, a Boolean matrix is introduced in the analysis of hybrid systems to represent the several configurations of the system in a single equation. The Boolean parameters appear when the switched junctions are used.

This Boolean matrix is an extension of the traditional analysis of hybrid systems by implementing the principles and techniques of singular systems theory into the hybrid notation (i.e. simplification of continuous and discrete modes, control properties techniques). This allows a more compact representation of the system and a simple representation of all the available configurations in a single equation.

The Boolean matrix takes the place of the E matrix on the hybrid equation $E\dot{x} = Ax + Bu$, where Boolean parameters also appear in matrices A and B.

This E matrix is a mixture of the identity matrix and a matrix containing Boolean parameters. The identity matrix part corresponds to the storage elements that are not affected by dynamic causality, even though the elements can be connected to an element affected by dynamic causality, or a causal path could exist between them. While the Boolean parameters in the matrix represent the behaviour of the elements affected by dynamic causality. This can be seen in equation

$$\begin{bmatrix} \Lambda_{11} & 0 & -S_{13} \\ 0 & \Lambda_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} = \tag{4.1}$$

$$\begin{bmatrix} \left(S_{11} - S_{14}HS_{14}^T\right)F & \left(S_{12} - S_{14}HS_{24}^T\right)F_i & 0 \\ \left(-S_{12}^T - S_{24}HS_{14}^T\right)F & \left(S_{22} - S_{24}HS_{24}^T\right)F_i & 0 \\ -S_{13}^T F & 0 & -\bar{\Lambda}_{22}^* F_d \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix}$$

$$+ \begin{bmatrix} S_{15} + S_{14}HS_{45} \\ S_{25} + S_{24}HS_{45} \\ S_{35} \end{bmatrix} [U]$$

which has a simplified representation,

$$\begin{bmatrix} E_{11} & 0 & E_{13} \\ 0 & E_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & 0 & A_{33} \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} [U] \quad (4.2)$$

where the general implicit form is introduced.

Here the matrix E_{11} indicates that the elements not affected by dynamic causality can be determined in all of the admissible configurations, while the matrices E_{22} and E_{13} represent the storage elements affected by dynamic causality (elements with integral causality assigned and elements with derivative causality assigned respectively), which are dependent of the configuration of the switching elements.

By identifying the different parts that are contained in the Boolean matrix E, it is easier to do the structural analysis of the system from the general implicit state equation.

4.3 Hybrid dynamic junction structure matrix

The hybrid junction structure matrix was introduced in the previous chapter where the properties were explained.

For structural analysis it is important to understand the interactions contained in the junction structure matrix, since those interactions reveal some of the properties that were not used in previous approaches or that could not be obtained before.

The junction structure obtained using the proposed approach in this research simplifies the calculation of the implicit state equation with the use of the Boolean algebra generated by switched junctions.

Some of the structural properties can be obtained by observing the junction structure matrix, nevertheless it is recommended to follow the procedure proposed later in this chapter to avoid errors.

4.4 Dynamic causality

The notation for elements affected by dynamic causality is modified to be used as a general notation rather than an alternative to hybrid bond graph model analysis.

The changes are made on how the affected elements are allocated in the matrices to simplify their identification, rather than introducing a different notation. This is made to clearly differentiate the affected elements from those that are not affected, not to mention to simplify further analysis of the matrices involved.

Nevertheless, there are some changes for the causality assignment for the different elements affected by it. This is due to the fact that the previous approach proposed by Margetts in [10] could generate confusion to the user at the moment of causality assignment to the model, where there was no preference on energy storage elements over resistive elements when causal conflict occurs.

In the previous approaches there was a need to set some constraints in order to avoid undesired behaviours (set static causality), in this case, with the revised procedure this is not necessary anymore. Those constraints were set to avoid causal conflicts or non-valid configurations while assigning causality to the model.

While using dynamic causality, there are cases where a kinematic constraint is created between rigid bodies (i.e. a causal path between two I-elements, one of which will be in derivative causality) when the switched junction is ON. This is displayed in Figure 4.1.

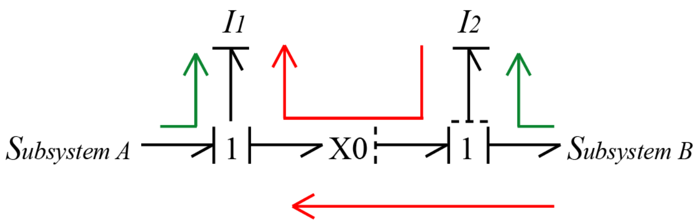


Figure 4.1: Behavior of elements affected by dynamic causality

Figure 4.1 shows how the dynamic causality affects the different elements on the system. In the initial configuration (solid causal strokes) it can be seen that the interaction between the elements does not exist because the commuting element is in OFF state, therefore both elements are in integral causality. As soon as the commuting element is activated and change to an ON state (dotted causal strokes) the element I_2 change its causality and it is now dependent of the element I_1.

This "forced" change in causality allows to represent the physical behaviour of the elements, which is one of the advantages of using dynamic causality in the proposed approach.

If the causality remained static during the change of configuration, a causal conflict will be created, which represent a ‘forbidden mode’. This behaviour is explained next.

Property 10. Causal conflict in the Dynamic causality assignment . When a causal conflict occurs during the dynamic causality assignment, this indicates that the configuration is a ‘forbidden mode’.

Forbidden modes are a reflection of a real case such as short-circuit or destructive behaviour (e.g. two masses colliding or gears coupling while rotating in contrary direction).

4.5 Valid configurations

As previously stated, the commuting elements have two different configurations, ON and OFF. This means that for every commuting element present in the system there are two configurations, which could be simplified as a 2^i configurations, where i is the number of commuting elements.

The solvability of the hybrid systems determines if there are non-admissible modes. In order to determine the solvability, the following considerations are made using Equations 4.1 and 4.2,

Property 11. If matrices Λ_{22} and $\bar{\Lambda}^*_{22}$ are complement of each other, all available modes are valid.

This can be proved using $rank(\begin{array}{cc} E_{22} & A_{33} \end{array}) = n$, where n is the number of storage elements.

Property 12. If matrices Λ_{22} and $\bar{\Lambda}^*_{22}$ are not complement of each other, then $rank\left(\begin{array}{cc} E_{22} & A_{33} \end{array}\right) < n$, this is because $E_{22} = \Lambda_{22}$ and $A_{33} = \bar{\Lambda}^*_{22}$.

When this happens, at least one state variable cannot be determined for that specific configuration, which is caused by a causal conflict.

These considerations are based on the properties of the matrices Λ_{22} and $\bar{\Lambda}^*_{22}$, and the fact that the non-admissible configurations can be observed with the interaction of these matrices.

To prove the previous property, the theory of the minor of a matrix is used. This is due to the submatrices E_{22} and A_{33} being square matrixes of n order.

Definition 24 ***Minor of a square matrix [82].*** *A minor (i, j) of a square matrix is the determinant of the submatrix obtained by deleting the i-th row and the j-th column. This number is denoted as M_{ij}.*

The elimination of redundant lines is the same procedure that is done for the analysis of hybrid systems, for each configuration the values for λ are assigned, then if there are rows that only contain zeros, those rows and their respective columns are deleted.

Then, for **property 11**, if matrices E_{22} and A_{33} (Equation 4.2) are complement of each other, the resulting $rank(\begin{array}{cc} E_{22} & A_{33} \end{array})$ is of full rank, as those are diagonal matrices. This can be proved by substituting the values of λ for each operational, and the result, after substitution, of the matrix $(E_{22} \cdot A_{33})$ must be the identity matrix.

For **property 12**, if matrices E_{22} and A_{33} are not complement of each other, that means that at least one configuration will contain two rows full of zeros, this is because during these configurations there is no transfer of power between the elements affected by the switched junction, which will create the rows containing zero values, therefore, this will cause a drop of the rank.

This analysis leads to the introduction of a general matrix M, which is based on the previous analysis and allows to simplify the analysis of the different configurations.

Let a matrix M be the concatenation of matrices E and A.

$$M = \left[\begin{array}{cc} E & A \end{array} \right]$$

To simplify the analysis of hybrid systems, a reduced matrix is proposed in order to omit redundant rows and columns that appear during commutation.

Notation 4 ***Reduced matrix $\bar{M}_{ij}$.*** *The reduced matrix $\bar{M}_{ij}$ is the result of omitting the ith row and the jth column.*

Notation 5 ***Reduced matrix $\hat{M}_{r_{1,\ldots,k}c_{1,\ldots,k,1+k,\ldots,2k}}$.*** *The reduced matrix $\hat{M}_{r_{1,\ldots,k}c_{1,\ldots,k,1+k,\ldots,2k}}$ is the reslt of omitting more than one row and more than one column.*

The reduced matrix $\bar{M}$ is used in hybrid systems where there is only one commuting element, which leads to the omission of one of the rows and one of the columns in the general state equation matrix to represent a determined configuration.

This approach is proposed for hybrid systems; however, the notation needs to be modified for models that contains more than one switched junctions, therefore the reduced matrix $\hat{M}$ is introduced.

As the proposed notation differentiate the elements affected by dynamic causality from the ones that are not affected, this is done by using Boolean parameters, the omitted lines would not be consecutive for most of the configurations, therefore the proposed notation for the hybrid reduced matrix is $\hat{M}_{r_{1,\ldots,k}c_{1,\ldots,k,1+k,\ldots,2k}}$, where $\hat{M}$ is the reduced notation of the hybrid matrices E and A, subindex $r_{1,\ldots,k}$ denotes the redundant rows to be omitted, and the subindexes $c_{1,\ldots,k,1+k,\ldots,2k}$ denotes the redundant columns to be omitted.

In this case, for the columns it is necessary to delete the columns in matrices E and A, which is the reason why the subindex contains the terms $1+k,\ldots,2k$. This means that the columns to be deleted will be in the range of $1,\ldots,k$ for the E matrix, while the columns deleted in the range of $1+k,\ldots,2k$ are for A matrix.

The omitted rows and columns are those that contain only elements of value 0 in the analysed configuration, which could change to value 1 in another configuration.

An example of this is displayed next where two lines and two rows are omitted,

$$c = 1,3$$

$$r = 2,3 \quad \begin{bmatrix} e_{11} & e_{12} & e_{13} & \cdots & e_{1k} \\ e_{21} & e_{22} & e_{23} & \cdots & e_{2k} \\ e_{31} & e_{32} & e_{33} & \cdots & e_{3k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e_{k1} & e_{k2} & e_{k3} & \cdots & e_{kk} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1k} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2k} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{k1} & a_{k2} & a_{k3} & \cdots & a_{kk} \end{bmatrix} \begin{bmatrix} b_{11} & \cdots & b_{u} \\ b_{21} & \cdots & b_{2u} \\ b_{31} & \cdots & b_{3u} \\ \vdots & \ddots & \vdots \\ b_{k1} & \cdots & b_{ku} \end{bmatrix}$$

having as a resulting reduced matrix,

$$\hat{M}_{r_{2,3}c_{1,3}} = \begin{bmatrix} e_{12} & e_{14} & \cdots & e_{1n} \\ e_{42} & e_{44} & \cdots & e_{4n} \\ \vdots & \vdots & \ddots & \vdots \\ e_{k2} & e_{k4} & \cdots & e_{kn} \end{bmatrix} \begin{bmatrix} a_{12} & a_{14} & \cdots & a_{1n} \\ a_{42} & a_{44} & \cdots & a_{4n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{k2} & a_{k4} & \cdots & a_{kn} \end{bmatrix} \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ b_{41} & \cdots & b_{4n} \\ \vdots & \ddots & \vdots \\ b_{k1} & \cdots & b_{kn} \end{bmatrix}$$

This is necessary because each configuration will have lines and rows containing only zero elements, which will lead to having redundant lines that could interfere with a proper analysis.

Proposition 5 ***Admissible configurations.*** *The configurations are determined as admissible or non admissible based on the following criteria:*

$$\textit{if rank}\left(\hat{M}_{r_{1,\ldots,n}c_{1,\ldots,k}}\right) = n \textit{ then the configuration is admissible}$$

$$\textit{if rank}\left(\hat{M}_{r_{1,\ldots,n}c_{1,\ldots,k}}\right) < n \textit{ then the configuration is non admissible}$$

where n is the number of energy store elements.

This means that the configuration needs to be of full rank in order to be considered as admissible. In the case that the configuration is not of full rank, then the behaviour of at least one of the energy store elements in the configuration cannot be obtained, rendering the configuration as non-admissible.

4.6 Properties of hybrid dynamic bond graph systems

The analysis of structural properties is usually done by using matrix-rank criteria, which in this case is the most appropriate approach due to the fact that the matrices that are obtained, can be exploited to obtain more information from the system's model.

In order to use this analysis, it is necessary to generate an input-output model, i.e. a model with causality assigned.

Previous approaches are not highly recommended because most of them introduces constraints on the model, which does not represent the physical behaviour for all the configurations of a hybrid system. The use of dynamic causality and switched junctions allows a closer representation of the physical behaviour of the system and the approach can be as reliable as geometric analysis such as proposed by Lewis [83] and Willems [84].

4.6.1 Controllability of hybrid dynamic bond graph systems

Controllability of a system depends on the existence of a connection between a source and a storage element. Therefore, the controllability matrix is a function of the A and B matrices, which at the same time are comprised from the submatrices of S.

It is established that a system is controllable if the controllability matrix has a rank equal to the model order. This subsection shows the different types of controllability and how it is determined for the different type of systems before obtaining the controllability matrix for hybrid dynamical bond graphs.

4.6.1.1 Singular systems controllability

The controllability of a system is determined by the ability to control a determined variable from an input. There are different types of controllability depending on the number of states that can be reached from the input.

To analyse the controllability of hybrid systems, an approach based on singular systems and bond graph is introduced. This is due to singular systems and bond graph controllability sharing the same principles. This approach also uses switched junctions and dynamic causality in order to obtain more information from the model.

Controllability of singular systems has been extensively analysed in [57, 77, 85, 86, 87, 88] and [89], having different approaches based on the type of singular system, or are a variation from a traditional control technique.

Singular systems are described in the form

$$E\dot{x}\left(t\right) = Ax\left(t\right) + Bu\left(t\right) \tag{4.3}$$

where $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $x \in \mathbb{R}^{n}$, $u \in \mathbb{R}^{m}$.

In order to differentiate the hybrid singular systems (singular systems containing Boolean notation used during this research) from regular singular systems, a new notation is proposed.

Definition 25 *Hybrid singular switched systems are described in the form*

$$E_{\lambda}\dot{x} = A_{\lambda}x\left(t\right) + B_{\lambda}u\left(t\right)$$

where $E_{\lambda}, A_{\lambda} \in R^{(n_1+2n_2)\times(n_1+2n_2)}$ and $B_{\lambda} \in R^{n_1+2n_2\times m}$ are constant matrices containing Boolean parameters, where n_1 is the number of energy storage elements not affected by dynamic causality, n_2 is the number of energy storage elements affected by dynamic causality, and m is the number of inputs.

In this case the dimension of the matrices is n_1+2n_2 in order to differentiate the storage elements not affected by dynamic causality and the storage elements affected by dynamic causality, that have two lines for each storage element (one for each configuration). This is because the commutation of switching elements can change the behaviour of the storage elements from independent element to dependent element or the other way around.

It must be noted that only one line for each element is present at a given time, which represents the configuration at that given time.

All singular systems must be solvable before they can be analysed.

Definition 26 ***Solvability [57]***: *"The system (4.3) is solvable if* $\det(sE - A) \neq 0$, *meaning that there is a unique solution for any given admissible initial condition for any given* $u(t)$*."*

In [87], Yamada and Luenberger approach is based on the descriptor systems in the form

$$Ex(t+1) = Ax(t) + Bu(t) \qquad t = 0, 1, \cdots .$$

described in the canonical form

$$E = \underbrace{\left[\begin{array}{c|c} E_{11} & E_{12} \\ \hline E_{21} & E_{22} \end{array}\right]}_{\leftarrow n \rightarrow \quad \leftarrow d \rightarrow} \begin{array}{c} \uparrow \\ n \\ \downarrow \\ \uparrow \\ d \\ \downarrow \end{array}$$

$$rank(E) = rank(E_{11}) = n$$

$$\mathfrak{n} = n + d$$

this rank condition implies that $E_{22} - E_{21}E_{11}^{-1}E_{12} = 0$.

Then matrices A and B are partitioned as well,

$$\left[\begin{array}{c|c} F_{11} & F_{12} \\ \hline F_{21} & F_{22} \end{array}\right] \triangleq \left[\begin{array}{c|c} I & 0 \\ \hline -E_{21}E_{11}^{-1} & I \end{array}\right] \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \left[\begin{array}{c|c} I & -E_{11}^{-1}E_{12} \\ \hline 0 & I \end{array}\right]$$

With this, they proceed to transform 4.3,based on graph theory, with the assumption of causality, the resulting canonical form is:

$$\left\{ \begin{array}{c} y(t+1) = \bar{A}y(t) + \bar{B}_1 u(t) \\ z(t) = -\bar{B}_2 u(t) \end{array} \right.$$

where

$$\begin{array}{c} \bar{A} \triangleq D\left[\frac{A_{11}}{A_{21}}\right], \bar{B}_1 \triangleq D\left[\frac{B_1}{B_2}\right], \bar{B}_2 \triangleq F_{22}^{-1}\left[\begin{array}{c|c} -E_{21}E_{11}^{-1} & I \end{array}\right]\left[\frac{B_1}{B_2}\right] \\ D \triangleq E_{11}^{-1}\left[\begin{array}{c|c} I & -F_{12}F_{22}^{-1} \end{array}\right]\left[\begin{array}{c|c} I & 0 \\ \hline -E_{21}E_{11}^{-1} & I \end{array}\right], \left[\frac{y(t)}{z(t)}\right] \triangleq W^{-1}x(t) \\ W \triangleq \left[\begin{array}{c|c} I & -E_{11}^{-1}E_{12} \\ \hline 0 & I \end{array}\right]\left[\begin{array}{c|c} I & 0 \\ \hline -F_{22}^{-1}F_{21} & I \end{array}\right] \end{array}$$

Then, the causal descriptor system 4.3 is

i) R-controllable if and only if

$$rank\left(\bar{B}_1, \bar{A}\bar{B}_1, \bar{A}^2\bar{B}_1, \cdots, \bar{A}^{n-1}\bar{B}_1\right) = n \quad \text{(full)} \tag{4.4}$$

ii) C-controllable if and only if, in addition to (4.4),

$\bar{B}_2$ has full row rank.

Controllability of singular systems with feedback and derivative feedback was not addressed at the time; that is why Garcia-Plana and Tarragona decided to propose an approach to this in [89]. They used the triplets of matrices (E, A, B) to do this.

Using the results introduced in [90], a system $(E, A, B) \in \mathbb{R}$ is controllable if and only if all of the following conditions are verified

$$\begin{array}{c} rank\left(\begin{array}{cc} sE - A & B \end{array}\right) = n, \text{ for all } s \in C, \\ rank\left(\begin{array}{cc} E & B \end{array}\right) = n. \\ rank\left[\begin{array}{cccc} B & AB & \cdots & A^{n-1}B \end{array}\right] = n \end{array}$$

where n is the number of state variables.

This approach is useful to consolidate that the controllability of the systems is determined by the matrices E, A and B, making the use of those matrices the main approach for this research.

Based on these previous approaches, it can be seen that controllability can be determined from the matrices E, A and B, which could change for each configuration.

4.6.1.2 Hybrid systems controllability

Controllability of hybrid systems is usually analysed with linear systems techniques, such as [30, 91, 92], which are based on Lyapunov functions or set several constraints into the system (controlled switching, limited interval of time, to name some), and this is not optimal or accurate. For that reason other approaches were proposed.

In [93], Hihi propose an observability condition for the hybrid system,

$$E_i\dot{x} = A_i x + B_i u \tag{4.5}$$

where E_i, $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, are the matrices of the ith mode, and the subindex i denotes the mode ($i = 1, 2, \ldots, n$, where n is the number of modes), $x \in R^n, u \in R^m$.

As controllability and observability are dual properties, based on the results obtained in [93], the controllability condition is:

$$C = \begin{bmatrix} C_1 & C_2 & \cdots & C_q \end{bmatrix}$$

where,

$$C_i = \begin{bmatrix} B_i & A_iB_i & \cdots & (A_i)^{n-1}B_i \end{bmatrix}$$

From there some conditions for the controllability are defined,

Remark 6 *"1) The system (4.5) can be controllable if there is only one controllable subsystem (mode)*
2) It is possible that no subsystem is controllable but that the system

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + B_1 u(t) \\ y(t) &= C_1 x(t) \end{aligned}$$

is controllable".

Campbell et *al* [85] and Losse [77] reached a more common approach for the controllability that is being more commonly used.

Based on the results from these previous works, it can be concluded that the conditions for the controllability of a singular system are:

Proposition 6 ***Controllability of a singular system****: The model is controllable if and only if:*
The system

$$E\dot{x} = Ax + Bu, \tag{4.6}$$

complies with the controllability condition

$$rank \begin{bmatrix} B & EAB & (EA)^2 B & \dots & (EA)^{n-1} B \end{bmatrix} = n \tag{4.7}$$

where n is the number of storage elements.

Proof. In the case of singular systems, matrix E is singular, A represents the interactions of the elements on the system, and B represents the interaction of the storage elements with the inputs. Therefore,

- If $rank(B) < n$, then at least one energy storage element cannot be reached from any input, then the system is not C-controllable.
- If $rank(B) = 0$, then the system is not controllable
- If $rank[E \times AB] < n$, then at least one storage element is disconnected from the system and cannot be controlled.

■

4.6.1.3 Hybrid bond graph systems controllability

Controllability in bond graph is dependent on the existence of a causal path between a source element and a storage element with integral causality. This is the reason why the matrices A and B are commonly used to determine the controllability of a system by checking the rank of:

$$rank \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix} = n \tag{4.8}$$

where n is the number of storage elements with integral causality. This remains valid for hybrid systems for all of the different approaches proposed so far. The reason for this was that the hybrid systems were considered to have the same number of states for all the possible configurations, which is clearly not true for all the systems when a commutation occurs.

Hihi proved that the use of the $rank \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix} = n$ is valid for hybrid systems in [13], where the system (4.3) is considered to have instant changes and all the elements are in integral causality before and after the commutation.
Based on a previous work ([94]) the conditions for controllability are mentioned:

Proposition 7 *[94] "The hybrid system is structurally state controllable if:*
1. All dynamical elements in integral causality are causally connected with an input source.
2. $BG - rank \begin{bmatrix} A_i & B_{c_i} \end{bmatrix} = n$*".*

These conditions are from the classical bond graph analysis, and that is the reason why the elements need to remain with the same causality.

For hybrid systems using switched junctions, in [10] the conditions for a *completely* controllable (*C-controllable*) system are mentioned, which are:

Property 13. Structural C-controllability of a Hybrid Bond Graph.

1. There must be a causal path between each storage element with integral causality an a source element for all of the system's configurations. This means that if a causal path crosses a switched junction the system is not *C-controllable* but can be *reachable* controllable (*R-controllable*).

2. The rank of the controllability matrix $rank \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix} = n$ is equal to the model order.

In this work a new formulation is introduced, therefore some changes to the controllability matrix are made.

In order to show these changes, the new notation developed in chapter 3 is displayed next:

$$
\begin{bmatrix} \Lambda_{11} & 0 & -S_{13} \\ 0 & \Lambda_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} =
$$

$$
\begin{bmatrix} \left(S_{11} - S_{14}HS_{14}^T\right)F & \left(S_{12} - S_{14}HS_{24}^T\right)F_i & 0 \\ \left(-S_{12}^T - S_{24}HS_{14}^T(\lambda)\right)F & \left(S_{22} - S_{24}HS_{24}^T\right)F_i & 0 \\ -S_{13}^T F & 0 & -\bar{\Lambda}_{22}^* F_d \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix}
$$

$$
+ \begin{bmatrix} S_{15} + S_{14}HS_{45} \\ S_{25} + S_{24}HS_{45} \\ S_{35} \end{bmatrix} [U]
$$

Having as a general notation the following equation:

$$
\begin{bmatrix} E_{11} & 0 & E_{13} \\ 0 & E_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & 0 & A_{33} \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} [U]
$$

Having in consideration the previous conditions **property 11**, $det(sE-A) \neq 0$, and $rank[\ EA \quad B\ ] = n$ and results for singular systems and hybrid bond graph systems ($rank \begin{pmatrix} sE - A & B \end{pmatrix} = n, rank[\ B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B\ ] = n$), by using the new suggested notation, the following conditions must be presented in order to determine the controllability of a system.

Proposition 8 *A hybrid system is C-controllable if and only if the following conditions are met:*

1. *The switched junction does not affect any storage element causality.*

2. *The submatrices B_1 and B_2 have a value different of 0.*

3. *Due to the submatrix B_2 being dependent on the submatrix E_{22}, and the submatrices A_{3k} (where $k = 1, 3$) and B_3 being used for the configurations when the prefered causality assignment is derivative, the equation to determine the controllability of the system changes to*

$$
rank \begin{bmatrix} B_1 & E_{11}A_{11}A_{12}B_1 & (E_{11}A_{11}A_{12})^2B_1 & \dots & (E_{11}A_{11}A_{12})^{n-1}B_1 \\ B_2 & E_{22}A_{21}A_{22}B_2 & (E_{22}A_{21}A_{22})^2B_2 & \dots & (E_{22}A_{21}A_{22})^{n-1}B_2 \end{bmatrix} = n \tag{4.9}
$$

where n is the number of storage elements.
If condition 1 is not met but conditions 2 and 3 hold, then the hybrid system is R-controllable.

Proof. Note that E_{11} is the submatrix that determines the presence of independent storage elements not affected by dynamic causality, A_{11} is the submatrix that represent the interaction of the storage elements not affected by dynamic causality with the rest of the elements, A_{12} is the submatrix that represent the interaction of the storage elements affected by dynamic causality with the rest of the elements, and B_1 is the submatrix that relates the inputs with the independent storage elements. If the elements in B_1 are equal to 0 when the elements in E_{22} change its value, this means that there is a causal path between the storage elements and the input. Nevertheless, if in a given configuration the elements contained in E_{11} are equal to 0, this means that there are no independent elements not affected by dynamic causality, therefore, B_1 does not exist.

Also, the submatrix E_{22} must always contains $n - m$ rows (where m is the number of non-commuting elements), if this is not true, this means that there is a causal conflict for that specific configuration.

If any of the previous conditions (or both) are present, the rank of the controllability matrix will drop, meaning that the system is not controllable because there are not causal paths between the storage elements and the inputs, otherwise the system remains controllable. ■

Proposition 9 *The non-controllability of a configuration is determined by the causal paths that cross a switched junction when a switched junction is in OFF configuration.*

Example 7 *This can be seen in the model illustrated in Figure 4.2,*

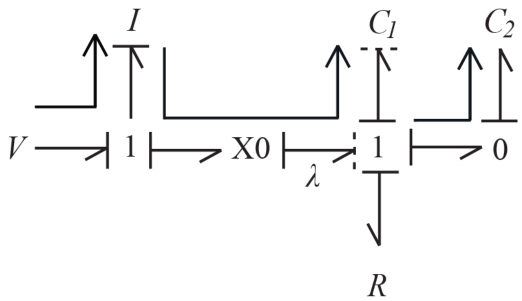


Figure 4.2: Determination of controllability using causal paths

In this case the element I_1 can be controlled in all of the configurations because there is a direct causal path with the source element, however, the elements C and I_2 are only controllable when the switched junction is in ON configuration, this is because the causal path is severed when the switched junction is in OFF configuration.

Proposition 10 *Due to the fact that x and x_i could have different order, and by using the singular systems approach, the controllability matrix can be reduced to the expression 4.9, were the rank drops when a causal path crosses a switched junction in OFF configuration.*

In this case E_{11} is used to determine if there are elements with integral causality not affected by dynamic causality for any configuration, while B_1 is used to determine if there is a causal path between an input and a storage element crossing a switched junction. E_{22} is used due to being the submatrix that determines the actual configuration, therefore the values on the submatrix B_2 depends on the value assigned to E_{22} (i.e. the correspondent B_2 value is not considered if its correspondent E_{22} value is equal to 0).

This can be explained using the previous example. The implicit equation that describe its behaviour is,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda & 0 & -\bar{\lambda} \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{p}_2 \\ \dot{p}_1 \\ \dot{p}_{1d} \end{bmatrix} = \begin{bmatrix} \frac{-\lambda R}{L} & \frac{-\lambda}{C_2} & \frac{-\lambda}{C_1} & 0 \\ \frac{\lambda}{L} & 0 & 0 & 0 \\ \frac{\lambda}{L} & 0 & 0 & 0 \\ 0 & \frac{-\bar{\lambda}}{C_2} & 0 & \frac{-\bar{\lambda}}{C_1} \end{bmatrix} \begin{bmatrix} q \\ p_2 \\ p_1 \\ p_{1d} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} [V]$$

where $E_{11} = \begin{bmatrix} 1 & 0 \\ 0 & \lambda \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $E_{22} = [\lambda]$, $B_2 = [0]$.
If the Boolean parameters are substituted, the $rank \begin{bmatrix} E_{11} & B_1 \end{bmatrix}$ does not change for the ON configuration, however, it drops during the OFF configuration, which means that the system is non-controllable for that configuration.

4.7 Controllability conditions of hybrid dynamical systems

Hybrid dynamical systems have a general implicit equation as follows,

$$\begin{bmatrix} E_{11} & 0 & E_{13} \\ 0 & E_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & 0 & A_{33} \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} U \quad (4.10)$$

From this general implicit state equation the controllability conditions for the different configurations can be obtained.

A state variable can be controlled only if there is a direct connection between a source and the state variable. However, it can be reachable controlled if the state variable is dependent of a state variable that can be controlled.

The following conditions are necessary and sufficient to determine the controllability of the admissible configurations from 4.10.

Proposition 11 *Hybrid dynamical systems are controllable if and only if:*

- *rank* $\begin{bmatrix} E_{11} & B_1 \end{bmatrix} = n_1$ *(where* n_1 *is the number of storage elements not affected by dynamic causality)*
  - *the configurations are C-controllable when rank* $\begin{bmatrix} E_{22} & B_2 \end{bmatrix} = n_2$. *(where* n_2 *is the number of storage elements affected by dynamic causality)*
  - *the configurations are R-controllable when* $E_{22} \cdot B_2 \neq 0$ *and* $A_{33} \cdot B_3 \neq 0$*, and also rank* $\begin{bmatrix} B_2 \\ B_3 \end{bmatrix} = n$*, where* n *is the number of energy storage elements; otherwise the configurations are not controllable.*

When rank $\begin{bmatrix} E_{11} & B_1 \end{bmatrix} < n_1$ *all the configurations are not controllable. The configurations are also not controllable when rank* $\begin{bmatrix} A_{33} & B_3 \end{bmatrix} \neq n$.

Proof. From the non simplified general implicit equation

$$\begin{bmatrix} \Lambda_{11} & 0 & -S_{13} \\ 0 & \Lambda_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} =$$

$$\begin{bmatrix} \left(S_{11} - S_{14}HS_{14}^T\right)F & \left(S_{12} - S_{14}HS_{24}^T\right)F_i & 0 \\ \left(-S_{12}^T - S_{24}HS_{14}^T(\lambda)\right)F & \left(S_{22} - S_{24}HS_{24}^T\right)F_i & 0 \\ -S_{13}^T F & 0 & -\bar{\Lambda}_{22}^* F_d \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix}$$

$$+ \begin{bmatrix} S_{15} + S_{14}HS_{45} \\ S_{25} + S_{24}HS_{45} \\ S_{35} \end{bmatrix} [U]$$

It can be seen that $E_{11} = \Lambda_{11}$, therefore,

- if $\Lambda_{11} \neq I$ then at least one of the elements with static causality is in a path affected by a switching element, and cannot be reached when the switched junction(s) is in OFF configuration,

  making those configurations non controllable. This is because the elements with static causality are not affected by the behaviour of dynamic causality, but a switched junction could be between a source and another element that share a causal path with the element with static causality, then making it unreachable during the OFF configuration even though it does not change its causality.

- if *rank* $[B_1] \neq n$ then at least one of the state variables cannot be reached from a source element, therefore that configuration is non controllable.

This is because B_1 represent the interaction between the source elements and the storage elements, therefore, if the rank of B_1 is not of full rank, then there is at least one storage element that cannot be controlled from a source element for the configurations where $rank\,[B_1] \neq n_1$.

It can be seen that $E_{22} = \Lambda_{22}$ and $A_{33} = -\bar{\Lambda}_{22}^{*} F_d$, therefore,

– the behaviour of these matrices is ruled by **Property 4**, which was introduced in Chapter 3 and all of their interactions are explained.

– if $rank\,[B_2] \neq n$ or $rank\,[B_3] \neq n$ then at least one of the state variables with dynamic causality cannot be reached from a source element. This is because B_2 and B_3 represent the interaction between the source elements and the storage elements affected by dynamic causality in their different configurations, therefore, if the rank of B_2 or B_3 are not of full rank, then there is at least one storage element affected by dynamic causality that cannot be controlled from a source element for the configurations where $rank\,[B_2] \neq n_2$ or $rank\,[B_3] \neq n_3$.

■

4.7.1 Observability of hybrid dynamic bond graph systems

Observability remains as the dual of controllability for singular systems, therefore the properties are similar.

Proposition 12 ***Observability of singular systems***: *The model is observable if and only if:*

1. *It is C-observable and R-observable*
2. *and* $rank \begin{bmatrix} EA \\ C \end{bmatrix} = n.$

4.7.1.1 Observability of hybrid systems

One of the most commonly used approaches to the analysis of hybrid systems was introduced by Hihi in [93],where the observability of the system (2.14) for hybrid systems is given by the equation:

$$O = \begin{bmatrix} O_1^T & O_2^T & \cdots & O_q^T \end{bmatrix}^T$$

where

$$O_i = \left[\begin{array}{cccc} C_i^T & A_i^T C_i^T & \cdots & \left(A_i^T\right)^{n-1} C_i^T \end{array} \right]^T$$

where i is the ith mode.
From there some conditions for the observability are defined,

---

Remark 8 *[95]"There are algebraic sufficient conditions, and necessary and sufficient algebraic conditions, which are:*
Algebraic sufficient conditions:
1) The system (4.5) can be observable if there is only one observable subsystem (mode).
2) It is possible that no subsystem is observable but that the system (4.5) is observable.
Necessary and sufficient condition:
In [17] and [93] the following subspace sequence is defined as
$G_1 = O_1 + \cdots + O_q = \sum_{i=1}^{q} \langle A_i^t | C_i^t \rangle$,
where $\sum_{i=1}^{q} \langle A_i^t | C_i^t \rangle = C_1^t + A_1^t C_1^t + \cdots + \left(A_1^t\right)^{n-1} C_1^t \cdots C_q^t + A_q^t C_q^t + \cdots + \left(A_1^t\right)^{n-1} C_q^t$
$G_{j+1} = \sum_{i=1}^{q} \langle A_i^t | G_j \rangle$, $j = 1, 2, \ldots$ *and*
$G = \sum_{k=1} G_k$.
The system 4.5 is observable, if and only if rank $(G) = n$*, where* n *is the number of storage elements."*

As can be seen, the observability matrix remains the same for hybrid systems, but it is also proved that the observability of the whole system does not depends on a single configuration, which is useful for the development of the observability matrix in this research.

4.7.1.2 Observability of hybrid bond graphs

Structural observability of bond graph models is dependent on the existence of a causal path between a detector element and all storage elements. This is the reason why the matrices A and C are used to determine the observability of a system by checking the rank of:

$$rank \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} = n$$

where n is the number of storage elements. This remains valid for hybrid systems for all of the different approaches.

For hybrid bond graph systems, one of the most used approaches is the one proposed by Hihi in [95]. This approach is based on the results of structural observability of LTI bond graph systems, from where the following definition results:

Theorem 9 ***Hybrid bond graph observability [95]:*** *"The Controlled Switching Linear System is structurally state observable if:*

- On the BGI all dynamical elements in integral causality are causally connected with a continuous output De or Df associated to $y(t)$ *or a discrete output Sw (commuting element) associated to* $T_{O_i}(t)$ *(output of the commuting element), where* $T_{O_i}(t)$ *is composed of the complementary variables in the switches.*

- $BG - rank \begin{bmatrix} A_i \\ \mathbf{C}_i \end{bmatrix} = n$*. With* $\mathbf{C}_i = \begin{bmatrix} C_i \\ C_{di} \end{bmatrix}, i \in \{1, \ldots, q\}$*, where* C_{di} *are the storage elements directly connected to a commuting element."*

This result proves that the observability matrix for hybrid bond graph systems is similar to the one for singular systems, which will allows the implementation of both approaches for the approach developed in this research.

With both singular systems observability and bond graph observability having similar principles, this can also be used to determine the observability of hybrid systems using switched junctions and dynamic causality.

For hybrid systems using switched junctions, in [10] the conditions for an observable system are mentioned, which are:

1. There must be a causal path between each storage element with integral causality and a detector element for all of the system's configurations. This means that if a causal path crosses a switched junction in OFF configuration the system is not reachable observable (R-observable).

2. The rank of the observability matrix is equal to the model order.

In this work a new notation is introduced, therefore some changes to the observability matrix are made.

In order to show this changes, the new notation is displayed next:

$$\begin{bmatrix} \Lambda_{11} & 0 & -S_{13} \\ 0 & \Lambda_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} =$$

$$\begin{bmatrix} \left(S_{11} - S_{14}HS_{14}^T\right)F & \left(S_{12} - S_{14}HS_{24}^T\right)F_i & 0 \\ \left(-S_{12}^T - S_{24}HS_{14}^T\right)F & \left(S_{22} - S_{24}HS_{24}^T\right)F_i & 0 \\ -S_{13}^T F & 0 & -\bar{\Lambda}_{22}^* F_d \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix}$$

$$+ \begin{bmatrix} S_{15} + S_{14}HS_{45} \\ S_{25} + S_{24}HS_{45} \\ S_{35} \end{bmatrix} [U]$$

$$Y = \begin{bmatrix} \left(S_{51} - S_{54}HS_{14}^T\right)F & \left(S_{52} - S_{54}HS_{24}^T\right)F_i \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} + S_{53}\dot{x}_d + [S_{55} + S_{54}HS_{45}][U]$$

Having as a simplified notation the following equation:

$$\begin{bmatrix} E_{11} & 0 & E_{13} \\ 0 & E_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_i \\ \dot{x}_d \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & 0 & A_{33} \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} [U]$$

$$Y = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} + S_{53}\dot{x}_d + [D][U]$$

Remark 10 *The notation for the output of the system does not match the classical notation of the output $Y = CX + DU$ due to the commuting elements.*

Proposition 13 *Having in consideration the previous conditions that*

- *the system must be R-observable,*
- *rank* $\begin{bmatrix} E \\ C_1 \end{bmatrix} = n$
- *there must be a causal path between each storage element with integral causality a detector element for configurations with all switched junction in ON configuration, and the observability equation* $O = \begin{bmatrix} O_1^T & O_2^T & \cdots & O_q^T \end{bmatrix}^T$*, by using the new suggested notation, the following conditions must be presented in order to determine the observability of a system.*

1. *A hybrid system is completely observable when the switched junctions are in ON configuration, and there is a casual path between a designated detector element and a storage element with integral causality.*

2. *Due to the observability being dependant on the change of causality on the output, the resulting observability amtrix of the system is*

$$rank \begin{bmatrix} C_1 A_{11} A_{12} \\ C_2 A_{21} A_{22} \end{bmatrix}^T = n \tag{4.11}$$

where n is the number of storage elements with integral causality.

Proof. Note that C_1 is the submatrix that determines the presence of storage elements not affected by dynamic causality that are reachable form the output, and C_2 is the submatrix that relates the storage elements affected by dynamic causality that can be reached from the output. If the elements in C_1 or C_2 are equal to 0, this means that these elements cannot be reached from the output.

It can be seen that during commutations these matrices can change its rank due to the causal path between the output and a storage element crosses a switched junction in OFF configuration.

In this notation the submatrices $E_{11}$and E_{22} are not included in the observability matrix due to be implicit included, this means that C_1 depends on E_{11} and C_2 on E_{22}, nevertheless, when the submatrices E_{11} and E_{22} have value of 1 does not implies that all the storage elements not affected by dynamic causality can be reached from an output. ■

Proposition 14 *The non-observability of a configuration is determined by the causal paths that cross a switched junction in OFF configuration, meaning that the elements that have a causal path crossing a switched junction are the ones that can become non-observable when a commutation occurs.*

As observability being the dual property of controllability, the same principle applies to the observability of the elements using causal paths. Using the model illustrated in Figure 4.3, the non-observable configurations can be obtained,

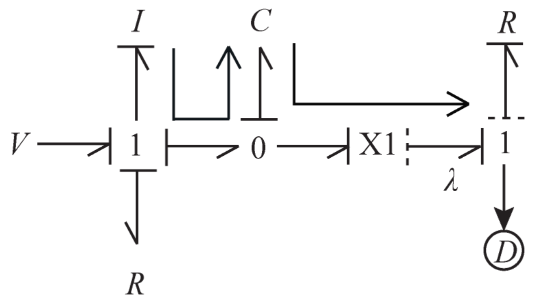


Figure 4.3: Example of non-observable configurations

It can be seen that during the ON configuration the system is observable from the detector element, however its non-observable during the OFF configuration because the causal path between the detector element and the storage element is severed.

Proposition 15 *The system is considered C-observable if the ranks of the matrices A and C are equal to the number of storage elements, and R-observable if at least one of the storage elements can be reached from the output.*

Similar to the non-observability of the system, if the storage elements cannot be reached using a causal path, the elements are non-observable. In Figure 4.4 an example of this is illustrated.

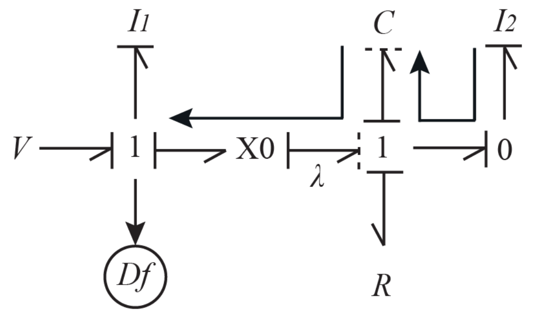


Figure 4.4: Example of *C-observable* and *R-observable* configurations

In this case, during the ON configuration all the storage elements can be reached from the output (detector), while during the OFF configuration only one storage elements can be reached.

4.7.2 Stability of hybrid dynamic bond graph systems

The stability of the hybrid systems is usually established by finding the eigenvalues of the characteristic polynomial. In this case roots with positive real parts indicate unstable behaviours. This a numerical approach rather than a structural one.

Definition 27 ***Structural stability of hybrid systems [75]:*** *"Where asymptotic stability does not exist, it indicates the presence of 'zero modes' (eigenvectors with vanishing eigenvalues). Recall that 'structurally null modes' (i.e. eigenvalues which are zero or the poles at the origin) are given by the storage elements which are in integral causality when preferred derivative causality is assigned".*

To obtain the number of structural null modes, two approaches can be used in hybrid bond graph analysis. One is introduced by Margetts in [10], where it is described that the stability of hybrid bond graph systems depends on the number of storage elements that remains with integral causality plus the storage elements affected by dynamic causality when a preferred derivative causality is assigned to the bond graph model. The resulting stability equation is:

$$D_0 = (\dim[X] + \dim[X_i])_{BGD}$$

where X are the storage elements not affected by dynamic causality, and X_i are the storage elements affected by dynamic causality with integral causality assigned.

The other approach is the traditional mathematical approach, which can be expanded to identify k structurally null modes. These structurally null modes correspond to the elements that remains in integral causality when a preferred derivative causality is assigned. Having as a characteristic polynomial for the hybrid systems:

$$P(s) = |s\mathbf{E}(\lambda) - \mathbf{A}(\lambda)| = s^k \left(s^q + a_{q-1}s^{q-1} + \ldots + a_1 s + a_0\right)$$

where q is the bond graph rank. The k structurally null modes have no steady state and therefore not asymptotically stable.

4.8 Summary

The purpose of this chapter was to revise the previous results and approaches for the analysis of structural analysis. In this case the main difference with the previous approaches is the use of dynamic causality and switched junctions, which allows the exploitation of previous unused properties of the systems.

There is a new notation that it is intended to be proposed as a standard for hybrid bond graphs and singular systems. It is expected to be used in a more robust analysis of the systems in a future due to have been proven that it retains the properties of previous approaches while having improvements from different approaches that allows a more complete representation of the system behaviour.

The results are not only useful for the analysis of hybrid bond graphs but also for hybrid and singular systems, this is due to the implementation of Boolean algebra into the general notation.

After obtaining these results for the structural analysis, the necessary considerations for a proper software simulation of the models will be introduced in the following chapter.

Chapter 5

Introduction to the use of software tools for the analysis and simulation of Hybrid systems

5.1 Preliminaries

The use of computational tools has proven to be helpful during the analysis of physical systems. However, it is necessary to obtain the mathematical model of the system before it can be analysed. This mathematical model needs to be obtained in a set of assigned statements. If the procedure to obtain the mathematical model is standardized, it can be implemented into a computational tool, which in this case can be a simulation software.

In the case of bond graph models, there is a standard procedure to obtain the mathematical model called SCAP. This approach is used for LTI systems in various software such as 20-Sim, Dymola, MATLAB (with the use of Simulink add-ons), CAMP-G, MS1, to name some.

However, this approach is not suitable for hybrid systems, which leads to the use of the HSCAP that deals with the change in the number of independent variables during commutation.

To implement the HSCAP into software it is necessary to introduce a set of rules to deal with causality assignment problems.

For hybrid systems most of these problems arise from the use of dynamic causality, which allows the change in causality in the model.

This change of causality in the model could lead to causal conflict. In this case, most of the causal conflicts or causal loops can be avoided by using the modified HSCAP proposed in this work. There are occasions where causal conflicts are unavoidable, which means that the configuration containing it is considered non-admissible for the analysis of its behaviour.

The procedure to adapt HSCAP into software is described in this chapter along with some examples.

5.2 HSCAP implementation in software tools

As previously mentioned, there are some considerations to be made for the implementation of HSCAP to a simulation software. Some are done to avoid causal conflicts such as the reassignment of causality after a commutation, while others are necessary rules to identify some admissible modes that could be considered as non-admissible, such as configurations that created causal conflicts while assigning static causality.

Most of the procedure remain the same as with the use of traditional implementations, which are based on SCAP. However, for hybrid systems, the focus is in the change of causality due to the use of dynamic causality in the bond graph model, or in a traditional mathematical approach, the proper representation of a physical behaviour. This requires further analysis of each of the configurations, which in previous implementations had static causality, which mean that all the elements had the same behaviour for all of the configurations.

There is a problem arising from the simulation model of the switched junctions, which is the change of causality during commutation, which was previously avoided in order to simplify the simulation of hybrid systems.

This can be seen in the work done by Van Kampen in [96], where a switching junction is designed in 20-Sim. In here, the analysis is done for a copying machine, to be more specific, in the paper path. This is displayed in 5.1.

The main focus for developing the model was to have an accurate timing and accurate velocity of the rollers and sheets. This model is displayed in 5.2.

In here, to obtain a proper transition between a freely rotating pinch and the pinch in contact with the paper, an extended inertia element was developed. This extended inertia consists in a switching junction connected to an internal inertia, which is a representation of the external inertia connected to the switching junction with a different causality assignment. This is displayed in Figure 5.3.

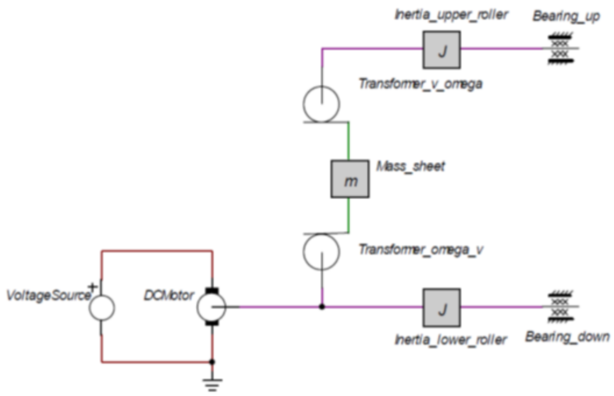


Figure 5.1: Photocopying machine diagram.

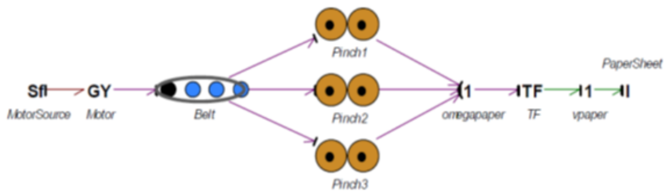


Figure 5.2: Bond graph model of the paper path.

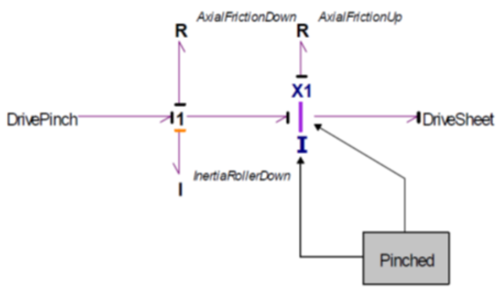


Figure 5.3: Pinch bond graph in the dynamic causality model. This model has a switching inertia element (shown as X1-I).

The internal inertia remains disconnected from the junction in the ON configuration and it is connected during the OFF configuration. This is done to represent the change in causality during commutation. This representation is illustrated in 5.4.

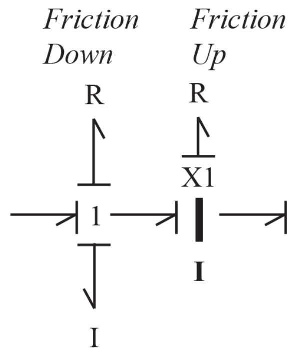


Figure 5.4: Representation of a switching junction with an internal inertia

In this case, when the system is in ON configuration (Figure 5.5(a)), the internal inertia and the resistance representing the friction up are disconnected from the model, while the switching junction behaves as a 1 junction.

During the OFF configuration (Figure 5.5(b)), the switched junction behaves as a zero flow source, while the internal inertia and the resistance representing the friction up are connected to represent the behaviour of the subsystem (inertia and resistance) when the source is disconnected.

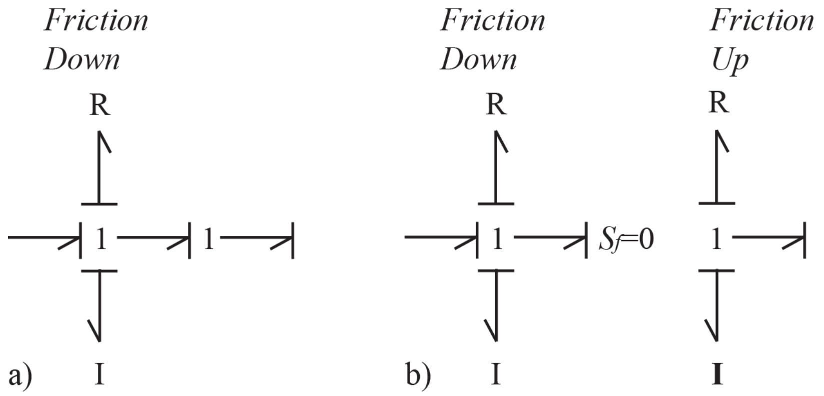


Figure 5.5: Behaviour of switching junction with internal inertia. a)ON configuration b)OFF configuration

However, this interaction limits the use of the switching element to a single element in a determined causal path, which will need to be programmed to that specific switching junction. In the case of a multiswitch element the number of additional internal elements increase, therefore, increasing the complexity in the analysis of the system.

To avoid increasing the complexity of the model, a proper representation of the change in causality during commutation is needed, therefore, it is necessary to introduce energy storage and resistive elements that can change causality, rather than introduce extra elements inside the switching junctions (as shown in the previous example).

This approach requires to properly define the behaviour of the different elements during commutation, which will also require defining the condition in which the change of behaviour of the elements is presented.

Definition 28 ***Dynamic elements.*** *A dynamic element (in simulation) is an element that can change its behaviour during commutation (from independent to dependent or the other way around).*

In hybrid systems, standard simulation elements have a static causality once assigned, which does not always allow the proper simulation of a physical element, therefore dynamic elements are introduced.

They must have to behave just as the existing elements in simulation software, however, dynamic elements will represent the change in the causality during simulation, this will allow the proper representation of some physical phenomena previously not considered.

The change in causality must comply with the proposed HSCAP procedure, where if there are changes in causality caused by dynamic causality during commutation, these changes must result in the maximum number of energy storage elements in integral causality. Therefore, it is necessary to introduce these rules into the HSCAP procedure of the simulation software by either introducing new elements to the software or adding an algorithm to set these rules.

An approach to this was discussed by Roychoudhury *et al* in [97], where the system was divided in block diagrams where the causality remains static for elements not affected by the switching elements, while the elements affected by the switching elements change their causality. This is possible because the approach is based on the analysis of transfer of energy, instead of using mathematical equations all the time, just as bond graph.

In Figure 5.6 a representation of the Bond Graph model in block diagrams is illustrated.

Figure 5.6(a) is the bond graph model with both switching elements (1a and 1b junctions) in ON configuration, Figure 5.6(b) is the block diagram equivalent of the previous bond graph model, Figure 5.6(c) presents the bond graph model with the switch element 1a in OFF configuration, Figure 5.6(d) is the diagram block equivalent of the previous bond graph model.

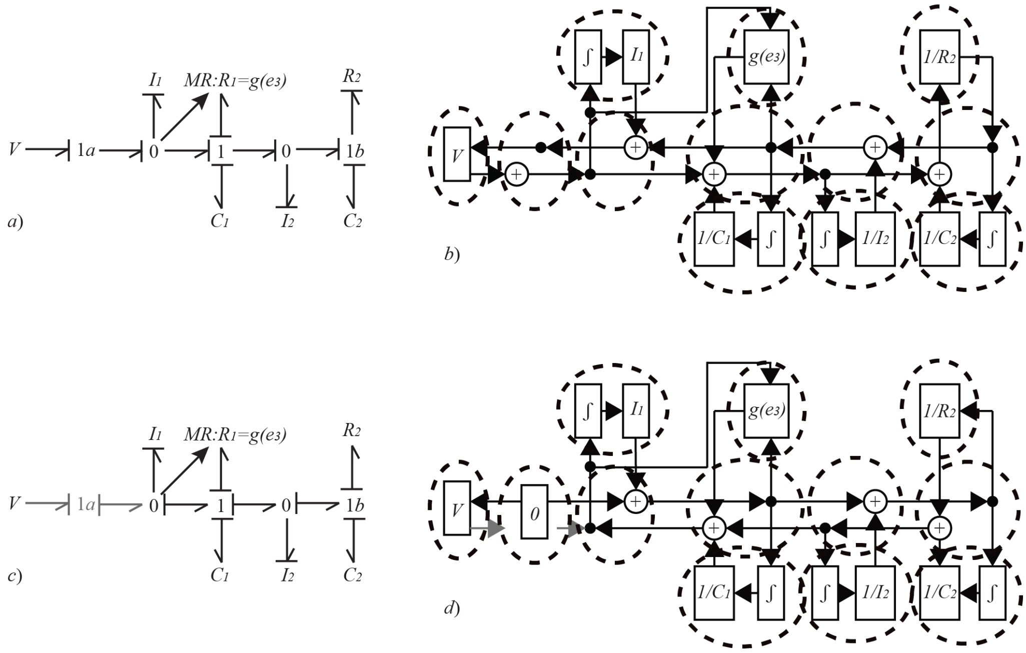


Figure 5.6: Representation of the bond graph model in diagram blocks

It can be seen that the block diagram simplifies the analysis of the OFF configurations by eliminating the power transfer, and the rest of the system remains unaltered. In this case the causality assignment cannot be seen in the bond graph model, however, the change in causality is carried in the equations obtained from the block diagram.

This approach allows a simplified analysis of the models by analysing the blocks after generating the model for the new configuration. However, this only allows the simulation of a single configuration at a given time, rather than a continuous simulation of the available admissible configurations, not to mention that the change in causality is not done in the Bond Graph model. Nonetheless, this approach leads to the proposition of static causality blocks, which will contain the subsystems not affected by dynamic causality.

Definition 29 ***Static causality blocks.*** *The model can be divided in subsystems affected and subsystems not affected by dynamic causality. Static causality blocks are the subsystems not affected and remaining with static causality in the model, which can be modeled as a block.*

If static causal blocks are used in the modelling, the assignment of dynamic causality can be easily determined and assigned to the correspondent elements. This is achieved by dividing the subsystems into those that are not affected by dynamic causality and those that are affected by it, by doing this, the subsystems with static causality will be contained inside static causal blocks. Therefore, only the elements affected by dynamic causality will be displayed on a general representation, where the changes in causality can be observed. By doing this, the maximum number of elements with integral causality is ensured.

This approach allows a proper representation of the physical phenomena, as it allows the change in causality in energy storage elements if needed.

There are some causal conflicts that cannot be avoided due to the use of some commuting elements, such as two-way switches (Figure 3.23 in chapter 3), which create a valid causal conflict during the ON configuration. Therefore, these causal conflicts must be considered as exemptions in order to avoid considering as non-admissible the configurations that contains any of these elements in the mentioned configuration.
However, it is necessary to clearly define this exemption, which will only exist in the connection between two two-way switches. This is in order to avoid having causal conflicts in neighbouring junctions considered as exemptions. If a causal conflict appears outside the connection of two two-way switches the configuration is not admissible.

Definition 30 ***Causal conflict in two-way switches.*** *A connection between two two-way switches creates a causal conflict during the ON configuration, which will not considered as such to avoid the oimission of a valid configuration due to the lack of power transfer.*

In Figure 5.7 the valid causal conflict (there is no effort coming into the 1-junctions in OFF configuration) is illustrated during the ON configuration of the two-way switch.

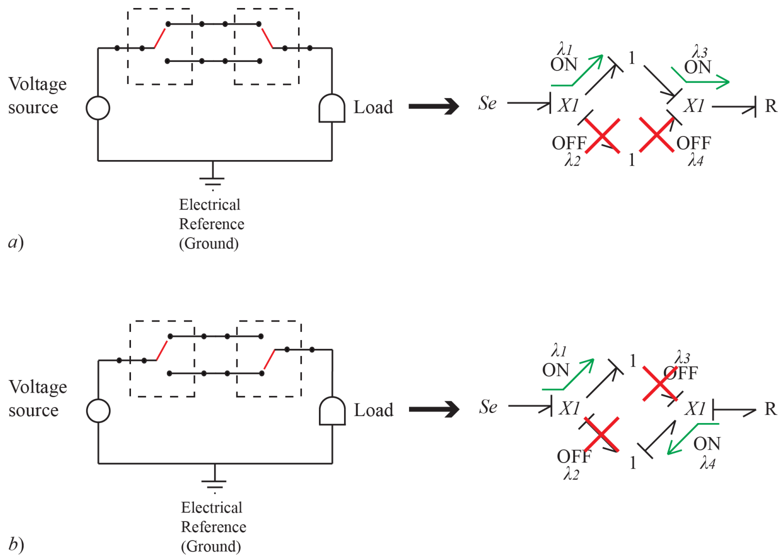


Figure 5.7: Causal conflict in two-way switches a) Valid causal conflic during ON configuration b) Configuration without causal conflict during OFF configuration

In the case where there are unavoidable causal conflicts, the configuration is considered as non admissible and is then discarded from the analysis and simulation of the model. An example of this is displayed in Figure 5.8.

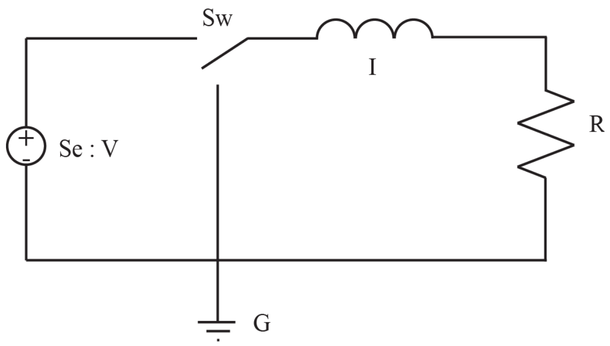


Figure 5.8: RL circuit with a switch

In Figure 5.9.the bond graph model is displayed.

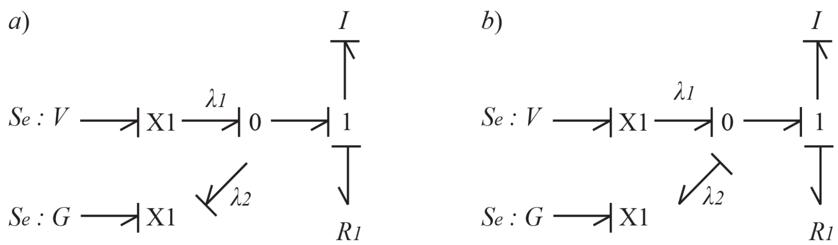


Figure 5.9: Example of unavoidable causal conflicts a)Valid configuration when only one switched element is in ON configuration b)Non-valid configuration caused by causal conflict when both switched elements are in ON configuration

Before the analysis of the response in time of the model, it is necessary to create a list of the admissible configurations. This is in order to generate warnings to the user to avoid including non-admissible configurations into the simulation sequence.

The response in time of the system, is then determined by using the admissible configurations which were previously obtained. The order of the configurations is determined by the user as well as the time the configuration is simulated.

As the model has a general representation of the available configurations, an "initial configuration" needs to be set. This configuration is considered "initial" as it is the configuration where there are more storage elements with integral causality, which is the configuration that results from the use of DSCAP. The configuration can be either selected by the user or by an algorithm.

Once all the parameters for the elements are set, and the configuration sequence and time if simulation as well (for each configuration and the general simulation time), the behaviour of this configuration is obtained.

Before there is a change of configuration, the behaviour of the elements must be saved. This is done in case any of the elements change its causality during commutation, therefore, the last value of the previous behaviour can be used as an initial value for the configuration at the moment. This is necessary to avoid having behaviours that do not have any physical meaning or representation.

5.3 Software algorithms

In order to help in the analysis of hybrid systems, it is helpful to have an algorithm that analyses the structure of the implicit state equation. This algorithm will determine if the configurations are admissible or not, also if they are controllable, all of this based on the results previously obtained; this in order to decrease the time on the analysis of the systems and focus on the analysis of the properties.

The software used to test this algorithm is MATLAB due to its characteristics. MATLAB is a matrix-based software which can be used to write and test algorithms, simulations based on physical systems using existing tools, analysis of complex systems based on mathematical equations, to name some of its uses in this field. By being matrix -based software, MATLAB is the most suitable option to develop an algorithm to automatize and simplify the analysis of the hybrid bond graph systems. Besides, the matrices can be directly obtained/feed via Simulink, which can be used for simulation purposes.

A comparison between the actual representation of switched bond graph elements on simulation programs with future representations is addressed as well.

As most of the previous approaches are based on the premises that the systems remain with the same number of storage elements for all of its configurations, all the software-based developments are a novelty.

An algorithm based on the reduced matrix notation can be developed. Reduced matrices are obtained by deleting rows and columns in order to obtain a smaller square matrix that represents the current configuration.

The redundant elements in this case are the lines and columns containing only 0 values when the Boolean parameters λ are assigned for an specific configuration. For non-admissible configurations the matrices A, B and E will have extra lines and columns removed, the number depends on the number of states involved in causal conflicts in the bond graph model.

The algorithm does not only allow to simplify the implicit equation matrix for the different configurations, it also allows to determine if a specific configuration is admissible. It must be noted that it can also be implemented the analysis of controllability of the model.

Algorithm 1 *Admissible configurations*

- *Matrices E, A and B are introduced*
- *The Boolean parameters are substituted*
- *The matrices E, A and B are obtained for an specific configuration*
- *Checks if there are any lines containing only* 0 *elements*
  - *If there are lines containing only* 0 *elements, the line is deleted from the matrices. Also the corresponding column is deleted from the matrices.*
  - *Else the lines contains values different of* 0*, the algorithm proceed to the next step*
- *The rank of the simplified matrices E, A and B is analysed by the algorithm*
  - *If the rank is the same as the number of storage elements, the configuration is considered admissible*
  - *Else there rank is not equal to the number of storage elements, the configuration is considered non-admissible*
- *A new configuration for the system is analysed using the original E, A and B matrices*
- *After all the configurations are analysed the algorithm displays which configurations are admissible and non-admissible. It also displays the reduced matrices E, A and B.*

In order to properly explain the procedure of the algorithm, an example is necessary, in this case, the buck converter previously analysed will be used.

The model of the buck converter is displayed in Figure 5.10,

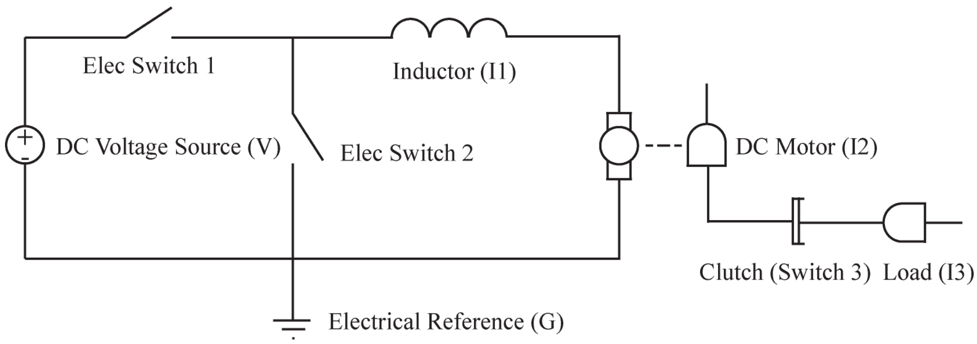


Figure 5.10: Buck converter connected to a DC motor with load

with a bond graph model displayed in Figure 5.11,

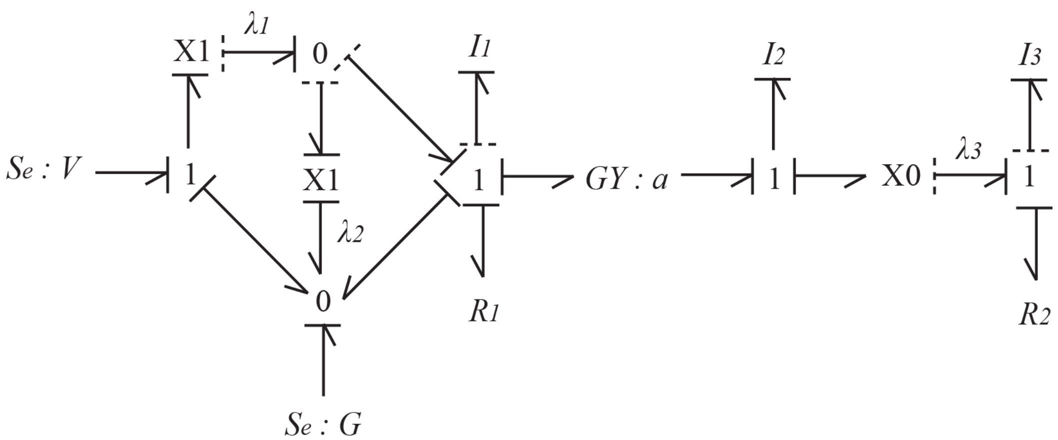


Figure 5.11: Bond graph model of the buck converter

First an admissible configuration will be used to illustrate the procedure, which latter will be compared with a non-admissible configuration; this in order to show how the algorithm simplifies the analysis of the model for the user.

The general implicit equation of the buck converter is,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \lambda_3 \\ 0 & (\lambda_1 \oplus \lambda_2) & 0 & 0 & 0 \\ 0 & 0 & \bar{\lambda}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3} \\ \dot{p}_{L1d} \\ \dot{p}_{L3d} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{-\lambda_3 R_2}{L_2} & \frac{a(\lambda_1 \oplus \lambda_2)}{L_1} & 0 & 0 & 0 \\ \frac{-a(\lambda_1 \oplus \lambda_2) R_2}{L_2} & \frac{-(\lambda_1 \oplus \lambda_2) R_1}{L_1} & 0 & 0 & 0 \\ 0 & 0 & \frac{-\bar{\lambda}_3 R_2}{L_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{-\left(\overline{\lambda_1 \lambda_2}\right)}{L_1} & 0 \\ \frac{\lambda_3}{L_2} & 0 & 0 & 0 & \frac{-\lambda_3}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L1} \\ p_{L3} \\ p_{L1d} \\ p_{L3d} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \lambda_1 \bar{\lambda}_2 & \bar{\lambda}_1 \lambda_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ G \end{bmatrix}$$

From where an admissible mode is obtained,

$$\lambda_1 = 1, \lambda_2 = 0, \lambda_3 = 1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3} \\ \dot{p}_{L1d} \\ \dot{p}_{L3d} \end{bmatrix} = \begin{bmatrix} \frac{-R_2}{L_2} & \frac{a}{L_1} & 0 & 0 & 0 \\ \frac{-aR_2}{L_2} & \frac{-R_1}{L_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{L_2} & 0 & 0 & 0 & \frac{-1}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L1} \\ p_{L3} \\ p_{L1d} \\ p_{L3d} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ G \end{bmatrix}$$

from where the redundant lines and columns (those that only contain 0 value) are removed by the algorithm,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3} \\ \dot{p}_{L1d} \\ \dot{p}_{L3d} \end{bmatrix} = \begin{bmatrix} \frac{-R_2}{L_2} & \frac{a}{L_1} & 0 & 0 & 0 \\ \frac{-aR_2}{L_2} & \frac{-R_1}{L_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{L_2} & 0 & 0 & 0 & \frac{-1}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L1} \\ p_{L3} \\ p_{L1d} \\ p_{L3d} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ G \end{bmatrix}$$

which simplified is,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3d} \end{bmatrix} = \begin{bmatrix} \frac{-R_2}{L_2} & \frac{a}{L_1} & 0 \\ \frac{-aR_2}{L_2} & \frac{-R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & \frac{-1}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L1} \\ p_{L3d} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ G \end{bmatrix}$$

In this case the reduced matrices are determined by the redundant lines and columns, rather than being determined by the user, which decrease the time in the analysis of the state equation.

A non-admissible configuration is obtained to be compared with the previous admissible mode,

$$\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3} \\ \dot{p}_{L1d} \\ \dot{p}_{L3d} \end{bmatrix} = \begin{bmatrix} \frac{-R_2}{L_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{L_2} & 0 & 0 & 0 & \frac{-1}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L1} \\ p_{L3} \\ p_{L1d} \\ p_{L3d} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ G \end{bmatrix}$$

where the redundant lines and columns are removed by the algorithm,

$$\begin{bmatrix} 1 & \cancel{0} & \cancel{0} & \cancel{0} & 1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & \cancel{0} & \cancel{0} & \cancel{0} & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \cancel{\dot{p}_{L1}} \\ \cancel{\dot{p}_{L3}} \\ \cancel{\dot{p}_{L1d}} \\ \dot{p}_{L3d} \end{bmatrix} = \begin{bmatrix} \frac{-R_2}{L_2} & \cancel{0} & \cancel{0} & \cancel{0} & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \frac{1}{L_2} & \cancel{0} & \cancel{0} & \cancel{0} & \frac{-1}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ \cancel{p_{L1}} \\ \cancel{p_{L3}} \\ \cancel{p_{L1d}} \\ p_{L3d} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ G \end{bmatrix}$$

which simplified is,

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L3d} \end{bmatrix} = \begin{bmatrix} \frac{-R_2}{L_2} & 0 \\ \frac{1}{L_2} & \frac{-1}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L3d} \end{bmatrix}$$

It can be seen that there is a drop in the order of the matrices, which compared to the admissible configuration, it can be seen that one of the state variables cannot be defined, and that determines why the configuration is non-admissible.

As previously mentioned, the algorithm can contain (if it is useful to the user) the controllability condition equation, which could provide extra information of the analysed configurations and reduce the time of analysis. This is done by adding an extra line at the end of the algorithm (but before the new configuration is analysed), where the implicit state equation is analysed using the controllability condition equation, which could save time on the analysis of the model. This was not implemented in the algorithm used in this chapter as the main objective was to obtain the admissible configurations.

5.4 Simulation of hybrid bond graph

There are some attempts to represent hybrid bond graphs in simulation programs. Some of these representations can be found in Dymola, 20-Sim, and MATLAB.

The representation of switched elements in Dymola is the ideal switch. An example of the use of this representation can be seen in [98] where several systems from aircraft are modelled and analysed. This representation does not allow a change of causality on state variable elements, only on resistive elements, which in occasions are added to reduce the impulse modes during commutation. The addition of the extra resistive elements could introduce some undesired behaviours into the system.

An example of the 20-Sim representation are the $X0$ and $X1$ junctions, which behave has a regular 0- and 1- junction (respectively) when the switched element is in ON configuration, while it behave as an 0 effort or 0 flow source when the element is in OFF configuration. In this representation the commuting is indicated by an external signal which can have value of 0 or 1 depending on the configuration. It must be noted that this representation of switching elements in 20-Sim does not allow a change on the causality during commutation, which mean that some configurations remain unrelated to the physical representation.

For MATLAB (and 20-Sim also), there is a representation of switched elements as a programmable diagram block introduced by Roychoudhury *et al* in [99].

Although this algorithm does allow change in the causality, this change is only applied to the resistive elements, leaving the storage energy elements with fixed causality, which mean that the results are similar to representations that does not allow change in the causality of the model.

5.5 Future in simulation

The proper representation of the switched junctions can be developed based on the considerations made in this chapter, which includes multiswitched junctions and multi-way junctions.

An introduction to the implementation to dynamic causality and an approach is proposed, which is expected to be useful in the implementation in a simulation software.

The algorithm described is expected to be used as a starting point in simulation software, which can be modified to include all of the possible configurations to determine the admissible configurations with a single iteration rather than one for each configuration. This is planned to be done by optimizing the computational resources within the algorithm as previously proposed.

It is necessary to mention that this can be done in one or more simulation software, as the description of the procedure is general, rather than be focused on an specific language/software.

5.6 Summary

An introduction to the steps needed to properly represent switched junctions where named, which are expected to be used in a later work in the implementation in a simulation software.

Some considerations for the proper simulation are named and addressed as well.

It was mentioned that MATLAB has been used to code a routine to simplify the analysis of the system's implicit equation for all of the available configurations.

This allowed to reduce the time of analysis and therefore more models were analysed in order to test the robustness of the results of this research.

Similar algorithms can be developed in different software, which will be useful to reduce time in the analysis of the behaviour of the model.

Some possible applications of the results are mentioned, in this case on the simulation on a different software package rather than just an implementation of a variation of the algorithm on MATLAB.

This is due to the implementation of the switched junctions on a bond graph software allows a closer representation of the systems to the physical behaviour, while the algorithm was just written to reduce the analysis time.

After this introduction of simulation of hybrid systems in software, some case studies will be analysed in order to compare the results obtained using the proposed approach in this work with previous approaches. At the moment this is being developed for a future implementation to expand its use.

Chapter 6

Case Studies

6.1 Preliminaries

Some case studies are presented in this chapter to demonstrate the procedure and differences of the proposed approach compared to previous approaches.

The buck converter is presented due to being widely-used as an example in the literature, making it ideal for comparison with previous approaches. It will be compared with the most recent approach done by Margetts [10] where it is expected to be a clearer differentiation of the commuting elements and how their behaviour affects the system.

There is also a two-phase bridge to show how the new notation is used to represent systems with large number of commuting elements while having more non-valid configurations than the previous case. This will allow demonstrating how the proposed approach decrease the time used on the analysis of large systems compared to previous approaches.

Having these cases allows demonstrating that all of the non-valid configurations are included in the general implicit equation, also how those modes are represented compared to the valid ones when the commuting elements have their values assigned.

In order to show the behaviour of the multiswitched junctions, the buck converter and a modified boost converter are analysed, and their properties are obtained, which are later compared to the previously obtained on the model with switched junctions.

Also, the controllability and observability of the analysed examples are obtained with the proposed procedures introduced in Chapter 4.

6.2 Buck converter

6.2.1 Overview of the buck converter

The buck converter shown in Figure 6.1 is an example that incorporates both electrical switches and a mechanical clutch. This example is used by Buisson et al [79], Margetts [10], and Edström et al [100] (on a simplified version), therefore, making it useful for comparison with the previous approaches.

The model used is a buck converter connected to a DC motor that can be connected or disconnected to a load using a clutch. This model is used to represent different subsystems connected through a commuting element to analyse the interaction between both subsystems in different configurations.

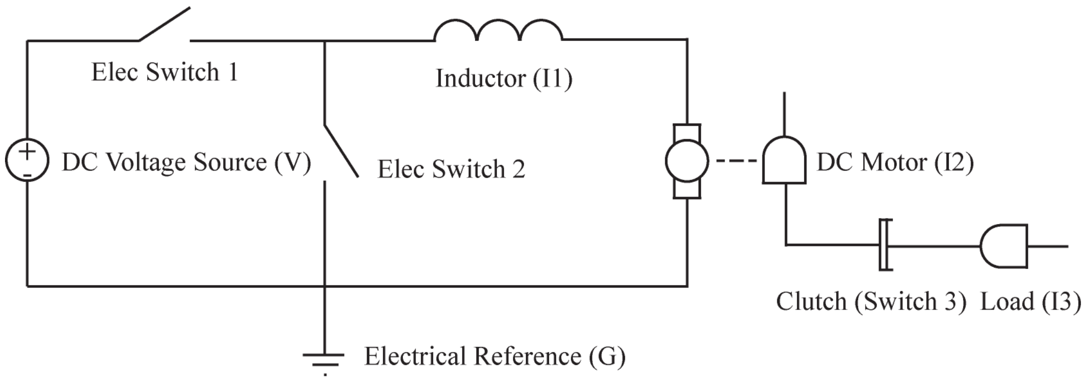


Figure 6.1: Buck converter connected to a DC motor with load

6.2.2 Hybrid bond graph of the buck converter

In order to obtain the hybrid bond graph model, it is necessary to follow the instructions of the modified HSCAP to determine the configurations. Therefore, the first step consists in determining the commutation sequences and its consequences. This is displayed on the following table,

λ_1	λ_2	λ_3	I_1	I_3
0	0	–	Derivative	–
0	1	–	Integral	–
1	0	–	Integral	–
1	1	–	Non-valid	–
–	–	0	–	Integral
–	–	1	–	Derivative

It can be seen that the commuting elements just affect a part of the system and that is why the table is divided in two sections that does not interfere with the other one.

The switches λ_1 and λ_2 create an interaction between the source, ground reference and I_1. This interaction creates different results (which are mentioned in the previous table). If only one switch is closed at a given time, the inertia store energy; if both are open at a given time, the inertia releases the stored energy; however, if both switches are closed at a given time, this creates a short circuit, making this configuration non-valid.

As previously explained, this is done in order to reduce the table size and to observe the behaviour of the different parts of the system.

The following step consists of the assignment of causality to the elements in the model. It is highly important to check that the initial assignment of the causality allows representing the highest number of energy storage elements on integral configuration (independent configuration).

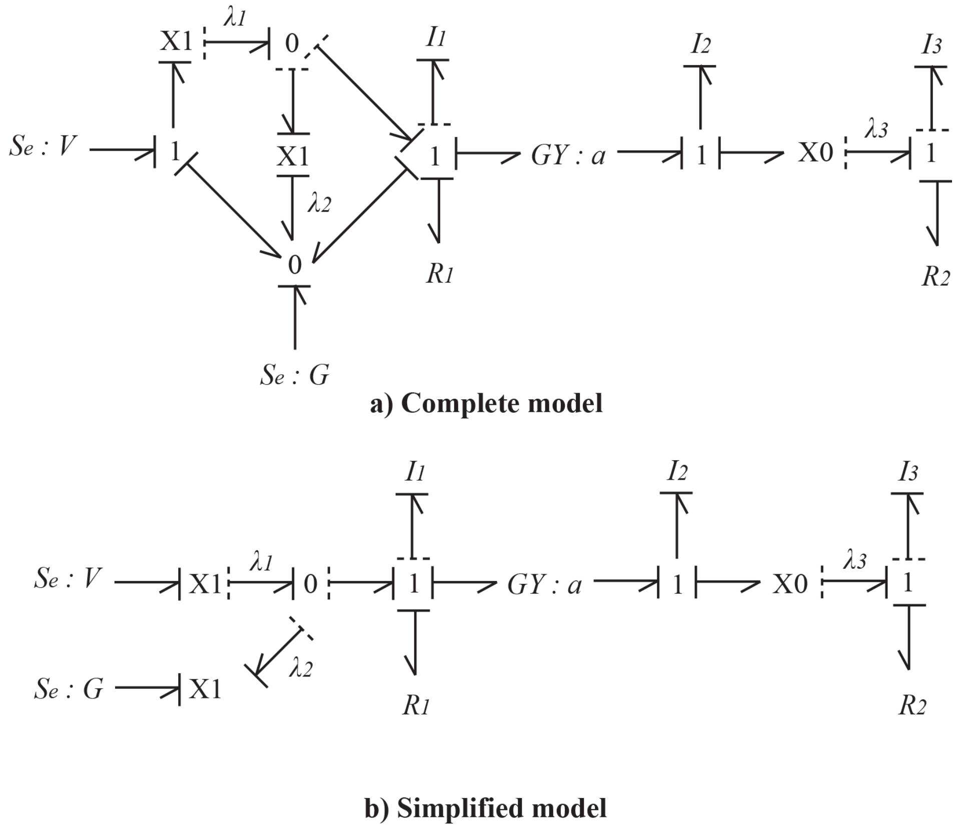


Figure 6.2: Bond graph model of the buck converter

The bond graph of the buck converter is shown in Figure 6.2. Note that some resistance elements have been added (R1 and R2) to model losses in the circuit and friction in the moving parts.

In this case the element I_2 cannot be selected as an element affected by dynamic causality due to the causality assignment in the GY and $X0$ elements next to it.

The full bond graph model is shown to display the interaction between the different elements connected to the ground (0V reference), that is then simplified by removing the bonds connected to the ground that are redundant (bonds that add a 0V to the junction).

6.2.3 Junction structure and Implicit state equations of the buck converter

In order to construct the Junction Structure Matrix, the modes of operation and any consequential dynamic causality must be identified. Therefore, it is necessary to obtain the equations that describe the energy exchange between the different elements interacting with each other.

The following equations show these interactions,

$$
\begin{aligned}
&\dot{p}_{L2} = a\left(\lambda_1 \oplus \lambda_2\right) f_{I1} - \lambda_3 \left(f_{I3} + e_{R2}\right) \\
&\left(\lambda_1 \oplus \lambda_2\right) \dot{p}_{L1} = \left(\lambda_1 \oplus \lambda_2\right)\left(-e_{R1} - a f_{I2}\right) + \lambda_1 \bar{\lambda}_2 V + \bar{\lambda}_1 \lambda_2 G \\
&\bar{\lambda}_3 \dot{p}_{L3} = -\bar{\lambda}_3 e_{R2} \\
&\overline{\lambda_1 \lambda_2} p_{L1d} = 0 \\
&\lambda_3 p_{L3d} = \lambda_3 f_{I2} \\
&f_{R1} = \left(\lambda_1 \oplus \lambda_2\right) f_{I1} \\
&f_{R2} = \bar{\lambda}_3 f_{I3} + \lambda_3 f_{I2}
\end{aligned}
$$

Using these equations the general junction structure matrix can be obtained,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\lambda_1 \oplus \lambda_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{\lambda}_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\lambda_1 \lambda_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3} \\ p_{L1d} \\ p_{L3d} \\ f_{R1} \\ f_{R2} \end{bmatrix} = \tag{6.1}$$

$$\begin{bmatrix} 0 & a\left(\lambda_1 \oplus \lambda_2\right) & 0 & 0 & -\lambda_3 & 0 & -\lambda_3 & 0 & 0 \\ -a\left(\lambda_1 \oplus \lambda_2\right) & 0 & 0 & 0 & 0 & -\left(\lambda_1 \oplus \lambda_2\right) & 0 & \lambda_1 \bar{\lambda}_2 & \bar{\lambda}_1 \lambda_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\bar{\lambda}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \left(\lambda_1 \oplus \lambda_2\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & \bar{\lambda}_3 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_{L2} \\ f_{L1} \\ f_{L3} \\ \dot{p}_{L1d} \\ \dot{p}_{L3d} \\ e_{R1} \\ e_{R2} \\ V \\ G \end{bmatrix}$$

The constitutive law of the R-field is:

$$D_{in} = L D_{out}$$

$$\begin{bmatrix} f_{R_1} \\ f_{R_2} \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} e_{R_1} \\ e_{R_2} \end{bmatrix} \tag{6.2}$$

The constitutive law for the storage elements:

$$\begin{bmatrix} Z \\ Z_i \\ Z_d \end{bmatrix} = \begin{bmatrix} F & 0 & 0 \\ 0 & F_i & 0 \\ 0 & 0 & F_d \end{bmatrix} \begin{bmatrix} X \\ X_i \\ X_d \end{bmatrix} \tag{6.3}$$

where: $F = \left[L_2^{-1}\right]$, $F_i = \begin{bmatrix} L_1^{-1} & 0 \\ 0 & L_3^{-1} \end{bmatrix}$, $F_d = \begin{bmatrix} L_1^{-1} & 0 \\ 0 & L_3^{-1} \end{bmatrix}$

from where the general implicit state equation is obtained,

$$
\begin{bmatrix} 1 & 0 & 0 & 0 & \lambda_3 \\ 0 & (\lambda_1 \oplus \lambda_2) & 0 & 0 & 0 \\ 0 & 0 & \bar{\lambda}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3} \\ \dot{p}_{L1d} \\ \dot{p}_{L3d} \end{bmatrix} = \tag{6.4}
$$

$$
\begin{bmatrix} \frac{-\lambda_3 R_2}{L_2} & \frac{a(\lambda_1 \oplus \lambda_2)}{L_1} & 0 & 0 & 0 \\ \frac{-a(\lambda_1 \oplus \lambda_2)}{L_2} & \frac{-(\lambda_1 \oplus \lambda_2) R_1}{L1} & 0 & 0 & 0 \\ 0 & 0 & \frac{-\bar{\lambda}_3 R_2}{L_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{-\left(\overline{\lambda_1 \lambda_2}\right)}{L_1} & 0 \\ \frac{\lambda_3}{L_2} & 0 & 0 & 0 & \frac{-\lambda_3}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L1} \\ p_{L3} \\ p_{L1d} \\ p_{L3d} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \lambda_1 \bar{\lambda}_2 & \bar{\lambda}_1 \lambda_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ G \end{bmatrix}
$$

If the implicit state equation is analysed for each configuration, it can be seen that there are two modes that only contains two rows. Based in the results from Proposition 10, these modes are considered non-valid. This is because the matrices $\Lambda_{22}\left(\begin{bmatrix} (\lambda_1 \oplus \lambda_2) & 0 \\ 0 & \lambda_3 \end{bmatrix}\right)$ and $\Lambda_{22}^{*}\left(\begin{bmatrix} \overline{\lambda_1 \lambda_2} & 0 \\ 0 & \bar{\lambda}_3 \end{bmatrix}\right)$ are not complement of each other during those configurations, therefore there are three rows containing only zero elements, which creates a drop in the rank of the model.

6.2.4 Simulation of the buck converter

The simulation and comparison between a previous approach (modulated transformers) and the proposed approach (switched junctions) of the buck converter, where it could be seen that one of the admissible configurations could not be analysed with the previous approach, which did not allow to observe the impulse mode when the source and ground reference are disconnected, which could lead to undesired behaviours in the physical system.

The model using modulated transformers is displayed in Figure 6.3, in which some resistive elements. These resistive elements represent electrical losses (R1) and friction (R2).

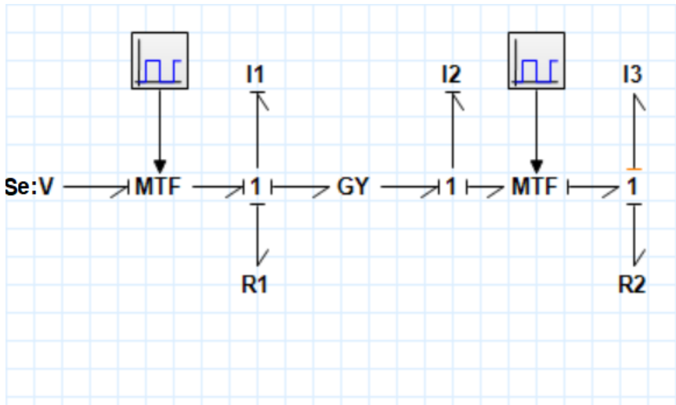


Figure 6.3: Buck converter model using modulated transformers.

As previously explained, for this model the causality remains the same for all of the configurations, which means that the elements will have the same behaviour for all of the configurations. This can be seen in Figure 6.4, where the commuting signals are displayed in the top graphs, and simulation results are displayed in the bottom graph. The ON and OFF configurations for the V effort source goes in 10s segments (from 0s to 10s ON, then OFF configuration from 10s to 20s, and so on), while the ON configuration for the MTF affecting the I_3 element stays in that configuration for 30s and then the OFF configuration goes in.

It can be seen that the behaviour is as expected, the elements store energy during the ON configurations, while in the OFF configurations the stored energy is consumed.

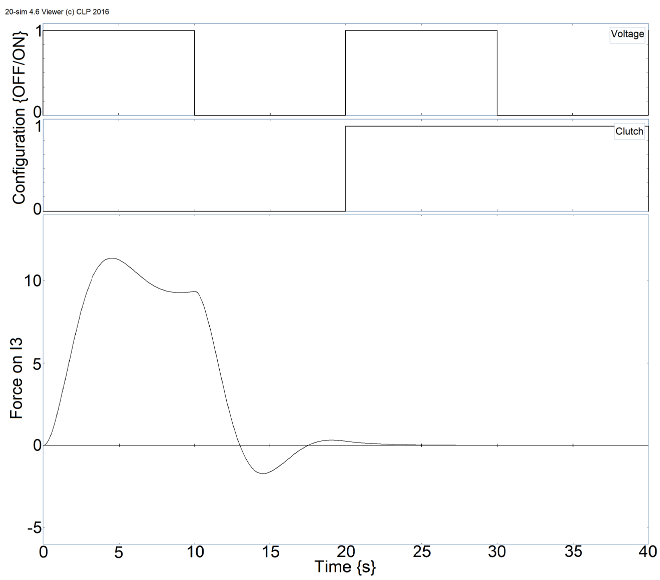


Figure 6.4: Switching signals and simulation results of buck converter.

The buck converter is also modeled using $X0$ and $X1$ switched junctions, this is displayed in Figure 6.5.

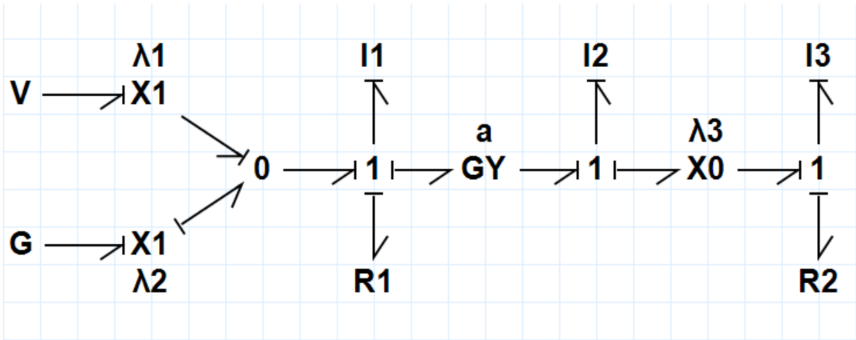


Figure 6.5: Model of the buck converter in 20 Sim.

For this simulation the values of the elements remain the same, also the commuting times remain the same in order to have a proper comparison.

The simulation of the first configurations (source V and ground reference G are connected to the system) are displayed in Figure 6.6.

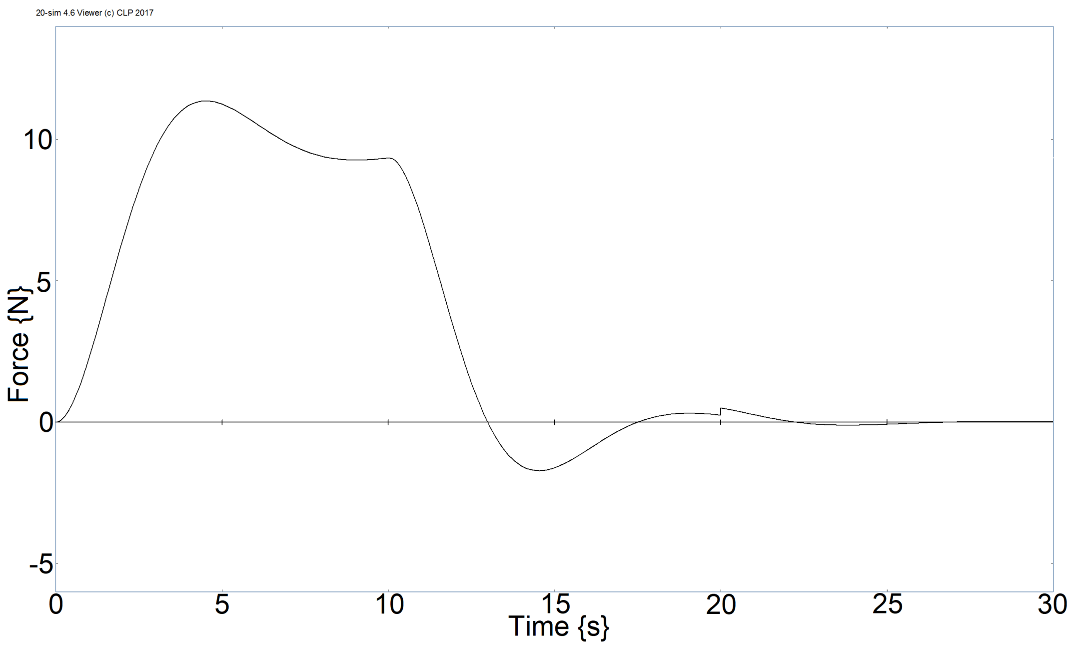


Figure 6.6: Simulation results using X0 and X1 switched junction while V (λ_1) and G (λ_2) commutes. λ_3 is in OFF configuration during the first 20s, then is in ON configuration for the last 10s.

It can be seen that the behaviour of the storage elements remains similar, this is because the system is the same as the previous approach. However, the behaviour of the system when both source and ground reference are disconnected from the system were not analysed previously, which in this case is one of the admissible configurations.

However, the impulse mode can be observed during the change of configuration.

This behaviour is displayed after 20 seconds of simulation in Figure 6.6, where it can be seen that as soon as the source V and ground reference G are disconnected an impulse response occurs and then there system goes to a zero response, this due to the absence of power input.

If the results are compared, it can be seen that both systems behave the same when the source V and the ground reference G are connected to the system and are commuting, nonetheless, the behaviour when these are disconnected from the system were not previously analysed, which can be done using switched junctions, therefore, more information can be obtained from simulation to have a proper representation of the physical system.

6.2.5 Change in order of the system at commutation of the buck converter

The changes in order for this system represent the interaction between the source and the ground reference which creates a non-valid mode. As it was previously mentioned in **Proposition 10**, there is a drop on the rank of the implicit equation matrix, meaning that the configuration is not valid.

An example of this is when the system changes from a valid configuration to a non-valid configuration.

Consider the case when the system is connected only to the source, and then it is connected to both the source and ground reference. The first configuration being:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3} \end{bmatrix} = \begin{bmatrix} 0 & \frac{a}{L_1} & 0 \\ \frac{-aR_2}{L_2} & \frac{-R_1}{L_1} & 0 \\ 0 & 0 & \frac{-R_2}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L1} \\ p_{L3} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ G \end{bmatrix} \tag{6.5}$$

After it is connected:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L3} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-R_2}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L3} \end{bmatrix} \tag{6.6}$$

The system changes from having three differential equations to having two differential equations. As previously stated in **Proposition 10**, when the rank of the matrices are determined, if there is a drop of the rank, then the configuration is not valid, which in this case are the modes which contains the interaction between the source and the ground reference.

6.2.6 Structural analysis of the power converter of the buck converter

The order of the model varies, since two of the storage elements are in dynamic causality. Any configuration that does not contains an interaction between the source and the ground reference gives the highest order, which in this case is 3.

For this model, the rank can be 2 or 3 depending on the configuration. It can be clearly seen that the presence of the elements B_{21} and B_{22} are the ones that determines the rank of the model in the reduced matrix $\hat{M}$.

The controllability of the configurations can obtained by using the controllability matrix 4.9 introduced in Chapter 4.

As example of this, the configuration $\lambda_1 = 1,\ \lambda_2 = 0,\ \lambda_3 = 1$ is used, which results in:

$$rank \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 1$$

This means that the system is not *completely controllable*, because only one storage element can be controlled through the input. However, the system is *reachable controllable*, which is illustrated in Figure 6.7. This reachable controllability is achieved by controlling the behaviour of elements I_2 and I_3 through element I_1.

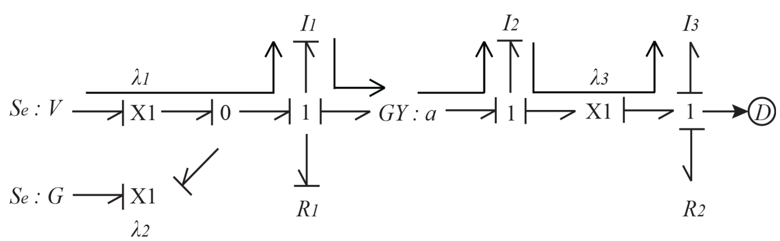


Figure 6.7: Controlling causal paths

It can be seen that the elements I_2 and I_3 can be reached using the element

I_1, however, this does not means that there is complete control over those elements. The observability of the system can be determined by using the observability matrix 4.11 introduced in Chapter 4. This matrix will have a different value depending on the placement of the output. An example of this is developed next.

By setting an output (current detector) in the 1- junction where the I_3 element is connected (illustrated in Figure 6.8), the resulting Y matrix is:

$$Y = \begin{bmatrix} \frac{\lambda_3}{L_2} & 0 & 0 & 0 & \frac{\bar{\lambda}_3}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L1} \\ p_{L3} \\ p_{L1d} \\ p_{L3d} \end{bmatrix}$$

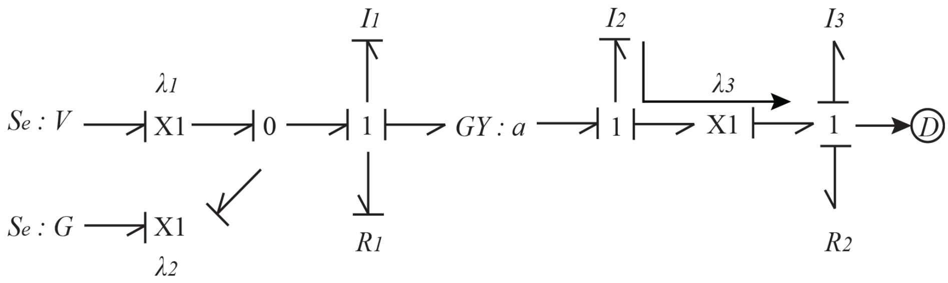


Figure 6.8: Buck converter with a flow detector

If the same configuration used in the previous example is analysed, the observability matrix is:

$$rank \begin{bmatrix} \frac{\lambda_3}{L_2} \\ 0 \end{bmatrix} = 1$$

which means that only one of the states is observable, which in this case is I_2.

The *stability* of the system can be determined from BGD model of the system (Figure 6.9). It can be seen that there are no storage elements in integral configuration, and as previously explained, if there are no elements with integral causality in the BGD model then there are no null modes, meaning that the stability of the system cannot be determined.

This means that the model is *structurally stable* in this configuration. Due to the dynamic behaviour of the elements I_1 and I_2, the system could contain a possible structural null mode, which render the system asymtotically unstable, nevertheless, if the model implicit equation is solvable even when there are storage elements in integral configuration, then the model is stable.

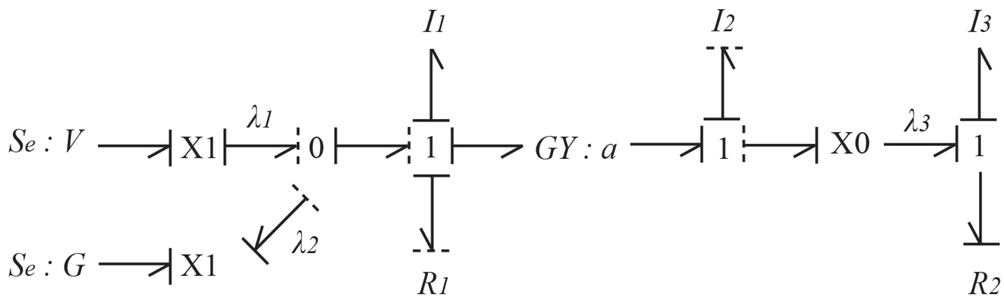


Figure 6.9: Hybrid bond graph model of the buck converter with preferred derivative causality

6.3 Two-phase bridge

6.3.1 Overview of the two-phase bridge

The two phase bridge (Figure 6.10) is a modified model of the Graetz bridge (Figure 6.11) analysed by Cormerais and Buisson on [101]. The changes on the model are made in order to show the behaviour of complex systems that contains a large number of non-valid configurations. The two-phase bridge has an extra inductance connected to the source V1 with the purpose to observe if the change in the behaviour of the system when its values changes. In this case a low value is selected to avoid instabilities during commutation.

This system behaviour is determined by the interaction between the commuting elements connected to the sources V_1 and V_2. This interaction is controlled by the sequenced commutation of the switches 1, 2, 3 and 4, which are set in a controlled sequence to avoid instabilities on the system. Nonetheless, for this study, the analysis of the model is done with arbitrary commuting sequence, which allows the analysis of all of the available configurations in order to compare its behaviour.

Valid configurations appear when there is at least one of the sources V_1 or V_2 connected to the rest of the circuit (both can be connected at the same time). The modes that allow both inputs being connected to the model correspond to the commuting sequence when both switches 1 and 2 are in ON configuration while both switches 3 and 4 are in OFF configuration; another possible configuration is when both switches 3 and 4 are in ON configuration while both switches 1 and 2 are in OFF configuration.; there are also the cases where both switches 1 and 4 are in ON configuration while both switches 2 and 3 are in OFF configuration; or when both switches 2 and 3 are in ON configuration while both switches 1 and 4 are in OFF configuration. While, the modes that allow only one input being connected to the model at a determined time, are those that only contain a single switch in ON configuration while the other 3 switches are in OFF configuration.

The previous configurations are obtained only in simulation, where the values of the threshold of the diodes can be modified. This is done to observe the different behaviours of the system under different operational conditions.

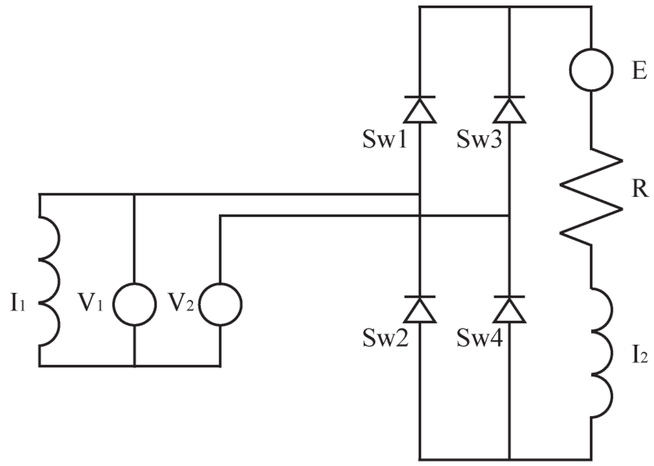


Figure 6.10: Two phase bridge

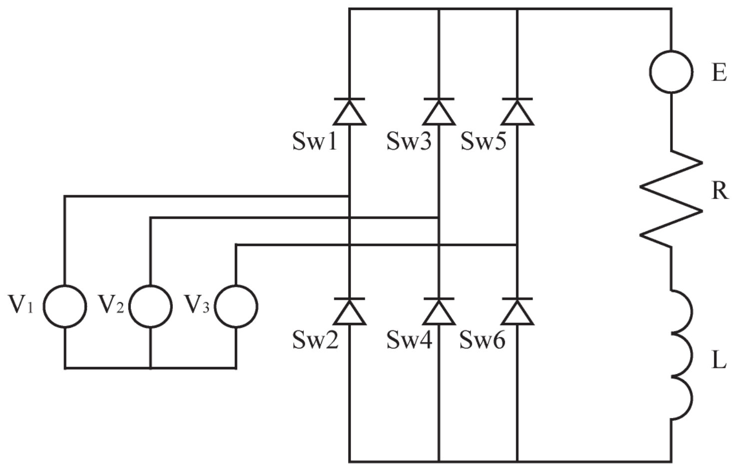


Figure 6.11: Graetz bridge

Non-valid configurations occur when both sources V_1 and V_2 are connected to the system at the same time (by having switches 1, 2, 3 and 4 in ON configuration at the same time), or neither of the sources are connected to the system. This is due to short circuits or open circuits created by the interactions of the commuting elements. These cases are caused by having at least two adjacent switches in ON or OFF configuration at the same time; this being both switches 1 and 3 in ON or OFF configuration at the same time, or both switches 2 and 4 in ON or OFF configuration at the same time.

6.3.2 Hybrid bond graph of the two-phase bridge

The bond graph model of a two-phase bridge is showed on 6.12.

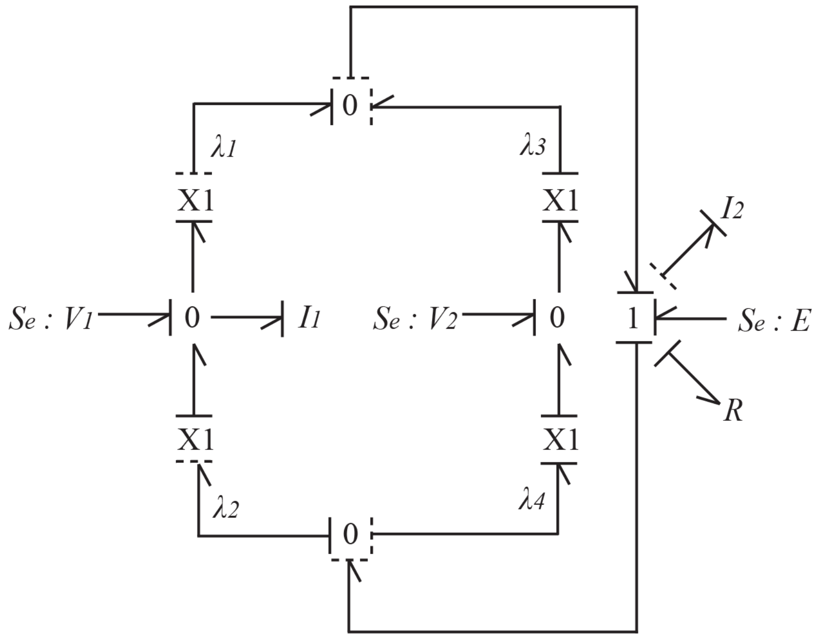


Figure 6.12: Bond graph model of the two phase bridge

The different configurations are easier to identify on this bond graph model compared to the previous model. As previously explained, the valid modes are those that allows one (or both) of the sources V_1 and V_2 are connected to the model. It can be seen that the cases that only allows to have one of the sources connected cause a change on the causality of the storage element I_2, while the configurations that allows both sources to be connected to the model does not change the causality of the element.

The non-valid configurations are represented by causal conflicts that can be seen on the junction where switches λ_1 and λ_3 are connected to each other, just as with the junction where the switches λ_2 and λ_4 interact with each other.

6.3.3 Junction structure and implicit state equations of the two-phase bridge

Based on the previous case, in order to obtain the junction structure of the model, it is necessary to obtain the equations that describe the system's behaviour for the different configurations, which are show next,

$\dot{p}_1 = V_1$
$\phi\dot{p}_2 = \phi\left(E - e_R\right) + aV_1 + b\left(V_1 + V_2\right) + c\left(V_1 + V_2\right) + dV_2$
$\varphi f_{L2} = 0$
$f_R = \phi f_{L2}$

where

$a = \lambda_1\lambda_2\bar{\lambda}_3\bar{\lambda}_4$
$b = \lambda_1\bar{\lambda}_2\bar{\lambda}_3\lambda_4$
$c = \bar{\lambda}_1\lambda_2\lambda_3\bar{\lambda}_4$
$d = \bar{\lambda}_1\bar{\lambda}_2\lambda_3\lambda_4$
$\phi = a + b + c + d$
$\varphi = \lambda_1\bar{\lambda}_2\bar{\lambda}_3\bar{\lambda}_4 + \bar{\lambda}_1\lambda_2\bar{\lambda}_3\bar{\lambda}_4 + \bar{\lambda}_1\bar{\lambda}_2\lambda_3\bar{\lambda}_4 + \bar{\lambda}_1\bar{\lambda}_2\bar{\lambda}_3\lambda_4$

The resulting junction structure matrix of the model is,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \phi & 0 & 0 \\ 0 & 0 & \varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ f_{L2} \\ f_R \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\phi & \phi & a+b+c & b+c+d \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_{L1} \\ f_{L2} \\ \dot{p}_{L2} \\ e_R \\ E \\ V_1 \\ V_2 \end{bmatrix}$$

Obtaining the following implicit equation,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_{2d} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\phi R}{L_2} & 0 \\ 0 & 0 & \frac{-\varphi R}{L_2} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_{2d} \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ \phi & a+b+c & b+c+d \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E \\ V_1 \\ V_2 \end{bmatrix}$$

If the implicit state equation is analysed for each configuration, it can be seen that the non-valid modes creates two lines full of zeros, due to only one storage element is affected by dynamic causality, which decrease the order of the system.

This is because the matrices $\Lambda_{22}\ (\phi)$ and $\Lambda_{22}^{*}\ (\varphi)$ are not complement of each other during those configurations, therefore there are three rows containing only zero elements, which creates a drop in the rank of the model.

In this case the non-valid modes are created by the interaction between the commuting elements, either creating a causal conflict (short circuit by having adjacent switches in ON configuration at the same time) or the lack of power flow through the system (having all the switches in OFF configuration) that is acting as an open circuit.

The rest of the configurations correspond to the system being connected to (at least) one of the sources V_1 or V_2 as previously explained.

6.3.4 Simulation of the two-phase bridge of the two-phase bridge

The simulation of the two-phase bridge using ideal switches is not possible. This is because the causal assignment cannot be done for the arrange of the four switches, therefore, an independent simulation of each configuration is done.

The behaviour of the configurations is displayed in Figure 6.13, where it can be seen that the elements behave the same for all of the configurations.

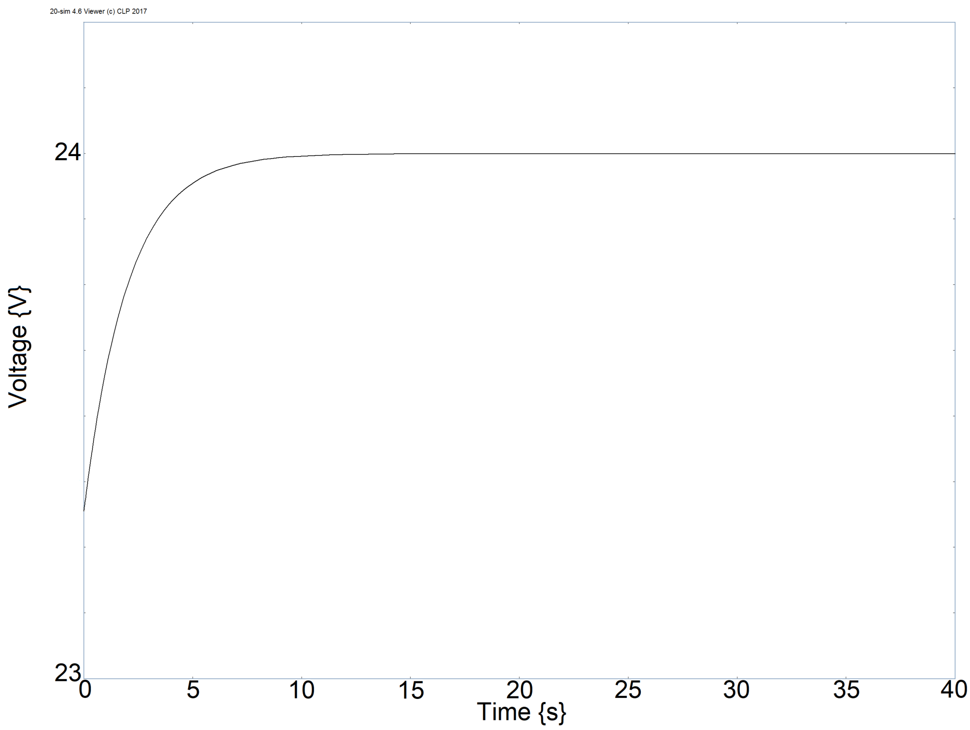


Figure 6.13: Simulation of the two-phase bridge

In this figure, the displayed behaviour indicates that the model using ideal switches is the same as an ideal ac to dc converter, which does not reflect the behaviour of the physical system.

The next step is the simulation of the model using switched junctions, which is displayed in Figure 6.14.

In this case all of the admissible configurations can be simulated on the same model, which simplifies the simulation. The results are displayed in Figure 6.15.

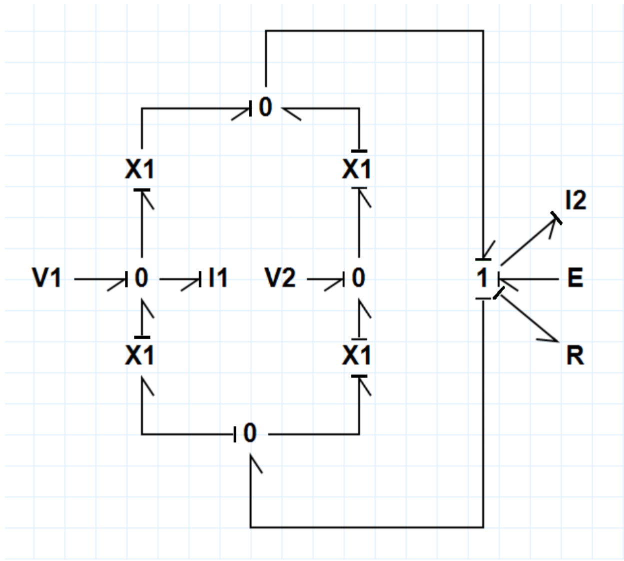


Figure 6.14: Two-phase bridge using switched junctions

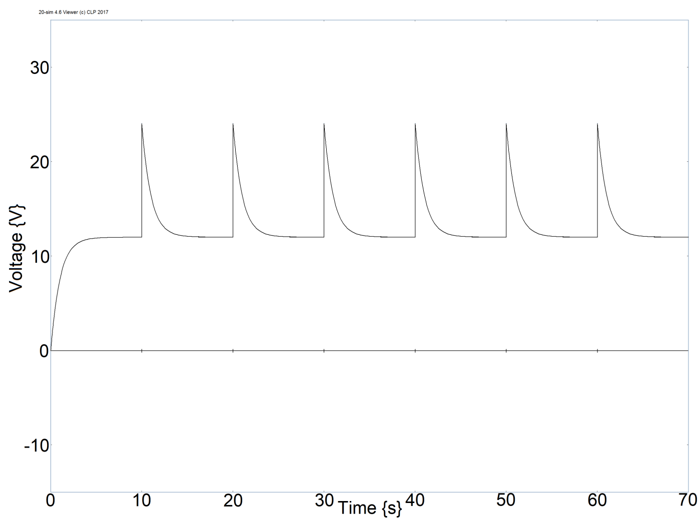


Figure 6.15: Simulation results of the two-phase bridge using switched junctions

It can be seen that the behaviour of the system is similar as analysing the model for each configuration, however, using switched junctions, the impulse modes can be determined, which are caused by the commutation of the switching elements (in this case a change of configuration occurs every 10 seconds). These impulse modes cannot be obtained using the traditional approach because the elements remain with the same causality during commutation.

6.3.5 Change in order of the system at commutation

For this system, the change in the order of the system is caused by the lack of power flowing through the system, or by a causal conflict created by the interaction of two adjacent switched junctions. If the switched junctions create a loop, the system remains on the same order as the power keeps flowing. This can be seen on configurations where there is at least one switched junction in ON configuration, or two non-adjacent switched junctions in ON configuration.

Based on the procedure described in **Proposition 10,** the change in the order occurs when the matrices Λ_{22} and Λ_{22}^{*} are not complement of each other. This can be seen on the following equation corresponding to a configuration with just one switched junction in ON configuration,

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_1 \\ \dot{p}_{2d} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{L_2} \end{bmatrix} \begin{bmatrix} p_1 \\ p_{2d} \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E \\ V_1 \\ V_2 \end{bmatrix} \tag{6.7}$$

that later commutes, having all the switched junctions in OFF configuration,

$$[1]\,[\dot{p}_1] = [0]\,[p_1] + \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} E \\ V_1 \\ V_2 \end{bmatrix} \tag{6.8}$$

It can be seen that in the first configuration Λ_{22} and Λ_{22}^{*} are complements, while on the second configuration they are not.

6.3.6 Structural analysis of the bridge of the two-phase bridge

Controllability of the system depends on the configuration, on which it can be completely controllable, reachable controllable, or structural controllable.

Using the controllability condition matrix 4.9 introduced in Chapter 4, this can be determined for all of the available configurations.

Most of the configurations are controllable due to the storage element I_1 is not affected by dynamic causality and it is connected to the input V_1, which their controllability equation is:

$$rank \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$

The *completely controllable* configurations are those in which both storage elements I_1 and I_2 are in integral causality, which at the same time are the structural controllable configurations which have the controllability equations:

$$rank \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{R}{L_2} & \frac{R}{L_2} & 0 \end{bmatrix} = 2$$

or

$$rank \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{R}{L_2} & \frac{R}{L_2} & \frac{R}{L_2} \end{bmatrix} = 2$$

or

$$rank \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{R}{L_2} & 0 & \frac{R}{L_2} \end{bmatrix} = 2$$

The *observability* of the configurations is determined by the location of the output, which could lead to the observability of one of the states or both of them. This is determined using the observability condition Equation 4.11 introduced in Chapter 4.

The *asymptotical stability* of the system cannot be achieved under any of the commuting conditions, this is due to one of the storage elements (I_1) cannot be assigned with derivative causality. Although this does not means that the system cannot reach a stable state, in this case the stability of the system is determined by the solvability of the implicit equation.

6.4 Buck converter using multiswitched junction

6.4.1 Overview of the buck converter using multiswitched junction

The buck converter is analysed using a multiport switch (Figure 6.16) to compare its behaviour with the traditional representation. This in order to determine the viability of the use of the multiswitched junction instead of individual switched junctions.

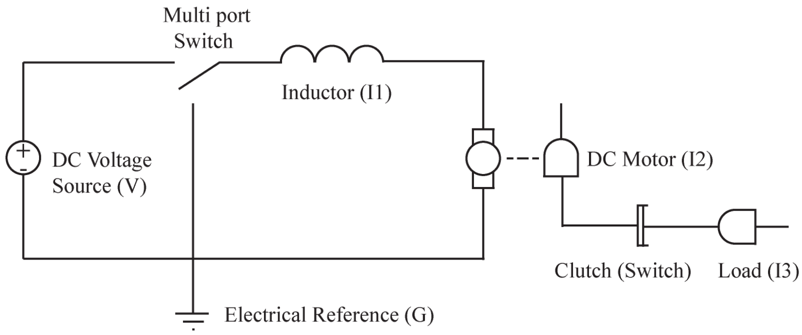


Figure 6.16: Buck converter using a multiswitch

6.4.2 Hybrid bond graph of the boost converter using multiswitched junction

Following the procedure of the HSCAP, first the truth table is obtained,

λ_1	λ_2	λ_3	I_1	I_3
0	0	–	Derivative	–
0	1	–	Integral	–
1	0	–	Integral	–
1	1	–	Non-valid	–
–	–	0	–	Integral
–	–	1	–	Derivative

It can be seen that the behaviour of the storage elements I_1 and I_3 remain the same as the model containing switched junctions. This is because the msj is a simplification of the interaction of two or more switched junctions.

The resulting bond graph model of the buck converter using the multiswitched junction it is displayed in Figure 6.17, which will be used to compare the resulting equations with the previous approach, and to look at the simplifications that this new representation brings.

As previously mentioned, the sources are affected by dynamic causality as result of using a newly introduced multiswitched junction, it is necessary to use traditional switched junctions, therefore the bond graph model used for the analysis is displayed next (Figure 6.18),

The use of the switched junctions leads to the same equations previously obtained.

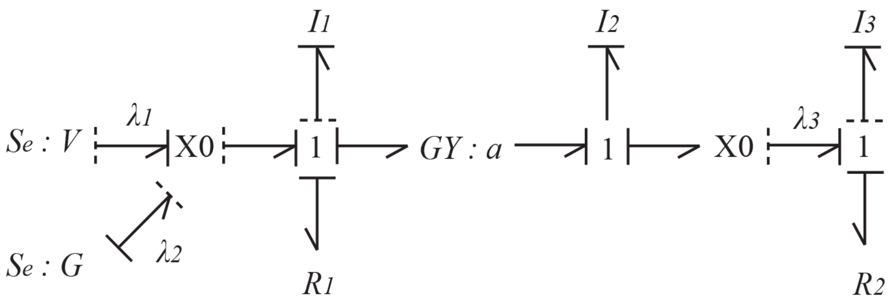


Figure 6.17: Model of the buck converter using a multiswitched junction

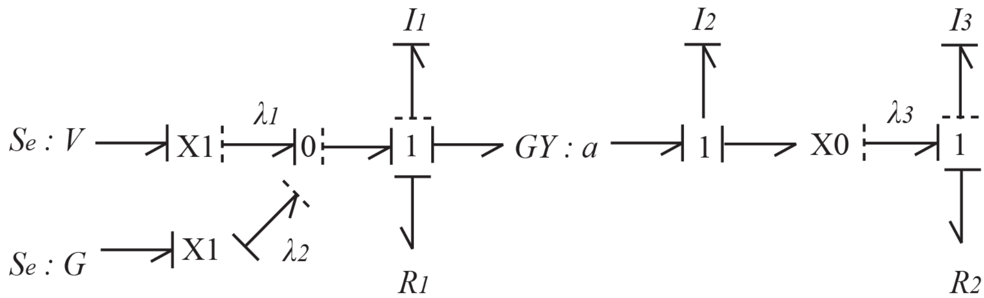


Figure 6.18: Model of buck converter using auxiliary switched junctions

6.4.3 Junction structure and implicit state equations of the buck converter using multiswitched junction

In order to construct the JSM, the equations that describe the interaction between elements in the system are obtained,

$$\dot{p}_{L2} = a\left(\lambda_1 \oplus \lambda_2\right) f_{I1} - \lambda_3\left(f_{I3} + e_{R2}\right)$$
$$\left(\lambda_1 \oplus \lambda_2\right) \dot{p}_{L1} = \left(\lambda_1 \oplus \lambda_2\right)\left(-e_{R1} - a f_{I2}\right) + \lambda_1 \bar{\lambda}_2 V + \bar{\lambda}_1 \lambda_2 G$$
$$\lambda_3 \dot{p}_{L3} = -\bar{\lambda}_3 e_{R2}$$
$$\overline{\lambda_1 \lambda_2} p_{L1d} = 0$$
$$\bar{\lambda}_3 p_{L3d} = \lambda_3 f_{I2}$$
$$f_{R1} = \left(\lambda_1 \oplus \lambda_2\right) f_{I1}$$
$$f_{R2} = \bar{\lambda}_3 f_{I3} + \lambda_3 f_{I2}$$

Using these equations the general JSM can be obtained,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\lambda_1 \oplus \lambda_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\lambda_1 \lambda_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{\lambda}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3} \\ p_{L1d} \\ p_{L3d} \\ f_{R1} \\ f_{R2} \end{bmatrix} = \tag{6.9}$$

$$\begin{bmatrix} 0 & a(\lambda_1 \oplus \lambda_2) & 0 & 0 & -\lambda_3 & 0 & -\lambda_3 & 0 & 0 \\ -a(\lambda_1 \oplus \lambda_2) & 0 & 0 & 0 & 0 & -(\lambda_1 \oplus \lambda_2) & 0 & \lambda_1 \bar{\lambda}_2 & \bar{\lambda}_1 \lambda_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\bar{\lambda}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\lambda_1 \oplus \lambda_2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & \bar{\lambda}_3 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_{L2} \\ f_{L1} \\ f_{L3} \\ \dot{p}_{L1d} \\ \dot{p}_{L3d} \\ e_{R1} \\ e_{R2} \\ V \\ G \end{bmatrix}$$

with constitutive relation between resistive elements as

$$\begin{bmatrix} f_{R1} \\ f_{R2} \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} e_{R1} \\ e_{R2} \end{bmatrix}$$

and constitutive relation between storage elements as

$$\begin{bmatrix} z \\ z_i \\ z_d \end{bmatrix} = \begin{bmatrix} F & 0 & 0 \\ 0 & F_i & 0 \\ 0 & 0 & F_d \end{bmatrix} \begin{bmatrix} x \\ x_i \\ x_d \end{bmatrix}$$

where $F = \left[L_2^{-1}\right]$, $F_i = \begin{bmatrix} L_1^{-1} & 0 \\ 0 & L_3^{-1} \end{bmatrix}$, $F_d = \begin{bmatrix} L_1^{-1} & 0 \\ 0 & L_3^{-1} \end{bmatrix}$.

By using the JSM and the constitutive relations between elements, the general state equation is obtained,

$$
\begin{bmatrix} 1 & 0 & 0 & 0 & \lambda_3 \\ 0 & (\lambda_1 \oplus \lambda_2) & 0 & 0 & 0 \\ 0 & 0 & \bar{\lambda}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_{L2} \\ \dot{p}_{L1} \\ \dot{p}_{L3} \\ \dot{p}_{L1d} \\ \dot{p}_{L3d} \end{bmatrix} = \tag{6.10}
$$

$$
\begin{bmatrix} \frac{-\lambda_3 R_2}{L_2} & \frac{a(\lambda_1 \oplus \lambda_2)}{L_1} & 0 & 0 & 0 \\ \frac{-a(\lambda_1 \oplus \lambda_2)}{L_2} & \frac{-(\lambda_1 \oplus \lambda_2) R_1}{L1} & 0 & 0 & 0 \\ 0 & 0 & \frac{-\bar{\lambda}_3 R_2}{L_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{-(\overline{\lambda_1 \lambda_2})}{L_1} & 0 \\ \frac{\lambda_3}{L_2} & 0 & 0 & 0 & \frac{-\lambda_3}{L_3} \end{bmatrix} \begin{bmatrix} p_{L2} \\ p_{L1} \\ p_{L3} \\ p_{L1d} \\ p_{L3d} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \lambda_1 \bar{\lambda}_2 & \bar{\lambda}_1 \lambda_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ G \end{bmatrix}
$$

It can be seen that the behaviour of the system remains the same due to the use of switched junctions instead of msj in order to maintain a proper causality assignment. Therefore, the simulation and properties of the system are proved to remain the same.

6.5 Modified Boost converter

6.5.1 Overview of the modified boost converter

Another example using the multiswitched junction is needed to demonstrate the behaviour of these switching elements when sources are not affected by dynamic causality. In this case a modified version of the boost converter is used (Figure 6.19). The modification of the boost converter allows to select between two different boost ratios by changing the configuration of the switching element. The model used behaves as a regular boost converter connected to a load, which in this case can be two different loads with different conversion ratio.

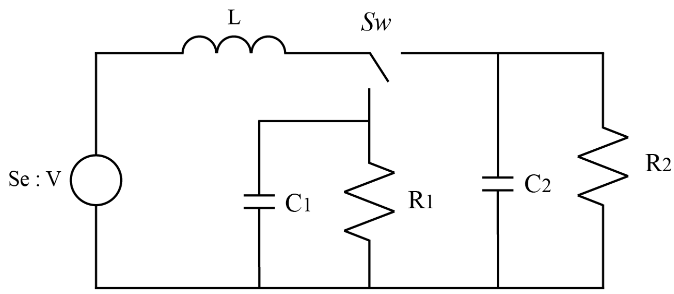


Figure 6.19: Modified boost converter.

6.5.2 Hybrid bond graph of the modified boost converter

Following the procedure of the HSCAP, first the truth table is obtained,

λ_1	λ_2	I	C_1	C_2
0	0	Derivative	Integral	Integral
0	1	Integral	Integral	Derivative
1	0	Integral	Derivative	Integral
1	1	Non-valid	Non-valid	Non-valid

In physical systems, if a multiswitch is connected to more than one output, the multiswitch could create a high-frequency switching, which usually leads to instabilities. Therefore, to avoid such behaviour, this configuration is considered non-admissible. This is introduced in bond graph models as causal conflicts.

The bond graph model of the modified boost converter can be seen in Figure 6.20.

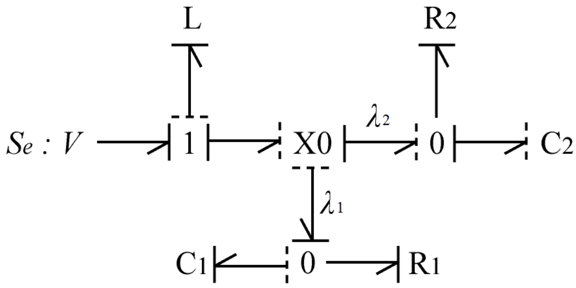


Figure 6.20: Bond graph model of the modified boost converter.

As previously mentioned, when multiswitched junctions are used there are three possible configurations, which will reduce the number of equations that describe the behavior of the system.

6.5.3 Junction structure and model equations of the modified boost converter

The equations describing the power exchange between elements are,

$$\lambda_1 \oplus \lambda_2 \dot{p}_L = \overline{\lambda_1 \lambda_2} V - \lambda_1 \bar{\lambda}_2 e_{C_1} - \bar{\lambda}_1 \lambda_2 e_{C_2}$$
$$\lambda_1 \bar{\lambda}_2 \dot{q}_{C_1} = \lambda_1 \bar{\lambda}_2 (f_L - f_{R_1})$$
$$\bar{\lambda}_1 \lambda_2 \dot{q}_{C_2} = \bar{\lambda}_1 \lambda_2 (f_L - f_{R_2})$$

$\bar{\lambda}_1\bar{\lambda}_2 p_{Ld} = 0$
$\bar{\lambda}_1\lambda_2 q_{C_1d} = 0$
$\lambda_1\bar{\lambda}_2 q_{C_2d} = 0$
$e_{R_1} = \lambda_1\bar{\lambda}_2 e_{C_1}$
$e_{R_2} = \bar{\lambda}_1\lambda_2 e_{C_2}$

The resulting junction structure matrix of the model is,

$$\begin{bmatrix} \lambda_1 \oplus \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1\bar{\lambda}_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{\lambda}_1\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{\lambda}_1\bar{\lambda}_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{\lambda}_1\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_1\bar{\lambda}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{p}_L \\ \dot{q}_{C_1} \\ \dot{q}_{C_2} \\ p_{Ld} \\ q_{C_1d} \\ q_{C_2d} \\ e_{R_1} \\ e_{R_2} \end{bmatrix} =$$

$$\begin{bmatrix} 0 & -\lambda_1\bar{\lambda}_2 & -\bar{\lambda}_1\lambda_2 & 0 & 0 & 0 & 0 & 0 & \lambda_1 \oplus \lambda_2 \\ \lambda_1\bar{\lambda}_2 & 0 & 0 & 0 & 0 & 0 & \lambda_1\bar{\lambda}_2 & 0 & 0 \\ \bar{\lambda}_1\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{\lambda}_1\lambda_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1\bar{\lambda}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{\lambda}_1\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_L \\ e_{C_1} \\ e_{C_2} \\ \dot{p}_{Ld} \\ \dot{q}_{C_1d} \\ \dot{q}_{C_2d} \\ f_{R_1} \\ f_{R_2} \\ V \end{bmatrix}$$

where the constitutive relations between resistive elements is,

$$\begin{bmatrix} e_{R_1} \\ e_{R_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} & 0 \\ 0 & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} f_{R_1} \\ f_{R_2} \end{bmatrix}$$

and the constitutive relations between the storage elements are,

$$F_i = F_d = \begin{bmatrix} \frac{1}{L} & 0 & 0 \\ 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & \frac{1}{C_2} \end{bmatrix}$$

By using the previous equations the implicit state equation is obtained.

$$
\begin{bmatrix}
\lambda_1 \oplus \lambda_2 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_1\bar{\lambda}_2 & 0 & 0 & 0 & 0 \\
0 & 0 & \bar{\lambda}_1\lambda_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{p}_L \\ \dot{q}_{C_1} \\ \dot{q}_{C_2} \\ \dot{p}_{Ld} \\ \dot{q}_{C_1 d} \\ \dot{q}_{C_2 d}
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
0 & \frac{-\lambda_1\bar{\lambda}_2}{C_1} & \frac{-\bar{\lambda}_1\lambda_2}{C_2} & 0 & 0 & 0 \\
\frac{\lambda_1\bar{\lambda}_2}{L} & \frac{\lambda_1\bar{\lambda}_2}{C_1 R_1} & 0 & 0 & 0 & 0 \\
\frac{\bar{\lambda}_1\lambda_2}{L} & 0 & \frac{\bar{\lambda}_1\lambda_2}{C_2 R_2} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\bar{\lambda}_1\bar{\lambda}_2}{L} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\bar{\lambda}_1\lambda_2}{C_1} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\lambda_1\bar{\lambda}_2}{C_2}
\end{bmatrix}
\begin{bmatrix}
p_L \\ q_{C_1} \\ q_{C_2} \\ p_{Ld} \\ q_{C_1 d} \\ q_{C_2 d}
\end{bmatrix}
+
\begin{bmatrix}
\lambda_1 \oplus \lambda_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
[V]
$$

If the implicit state equation is analysed for each configuration, it can be seen that there is only non-valid configuration, which fill all the matrices with zeros, because this configuration does not have a proper assignment of causality.

As the previous examples, this is because the matrices $\Lambda_{22} \left(\begin{bmatrix} \lambda_1 \oplus \lambda_2 & 0 & 0 \\ 0 & \lambda_1\bar{\lambda}_2 & 0 \\ 0 & 0 & \bar{\lambda}_1\lambda_2 \end{bmatrix} \right)$ and $\Lambda_{22}^* \left(\begin{bmatrix} \bar{\lambda}_1\bar{\lambda}_2 & 0 & 0 \\ 0 & \bar{\lambda}_1\lambda_2 & 0 \\ 0 & 0 & \lambda_1\bar{\lambda}_2 \end{bmatrix} \right)$ are not complement of each other during this configuration, therefore, there is a drop in the rank of the model.

6.5.4 Simulation of the modified boost converter

The modified boost converter is simulated using a classical approach (ideal switches represented by modulated transformers) where the elements remains with static causality after a commutation occurs, and the proposed approach (switched junctions) where dynamic causality affect some elements after a commutation occurs.

First, the classical approach is used for the simulation. The model is displayed in Figure 6.21.

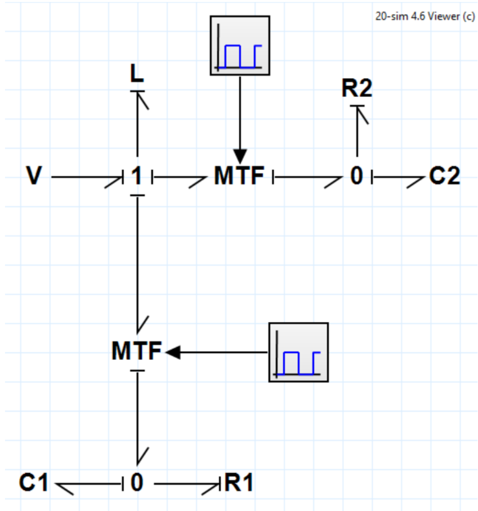


Figure 6.21: Model of the modified boost converter using ideal switches.

Its behaviour is displayed in Figure 6.22.

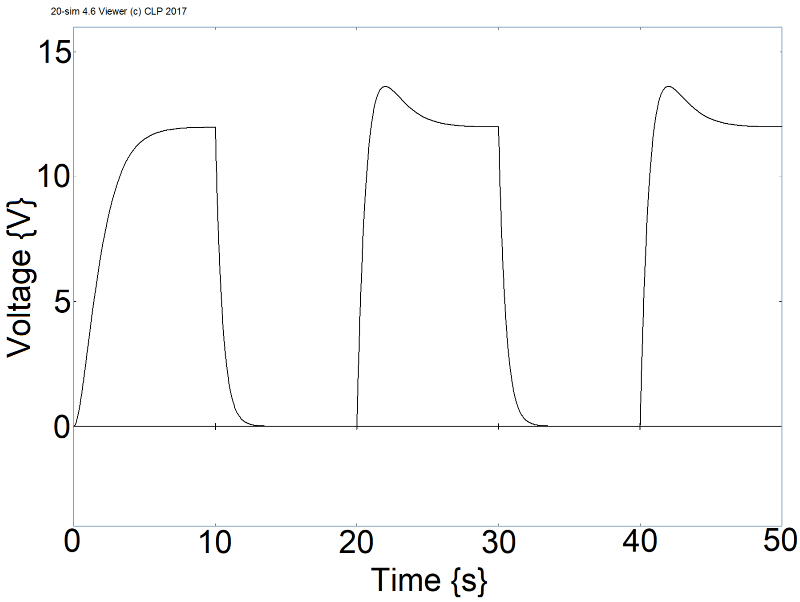


Figure 6.22: Behaviour of the system using modulated transformers.

In Figure 6.23 the model in 20Sim is displayed.

The behaviour of the model is displayed in Figure 6.24.

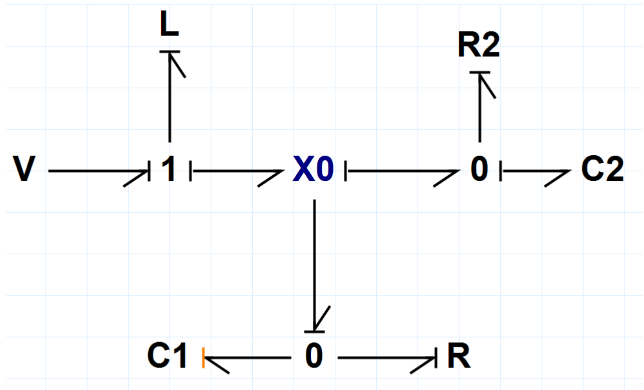


Figure 6.23: Model of the modified boost converter

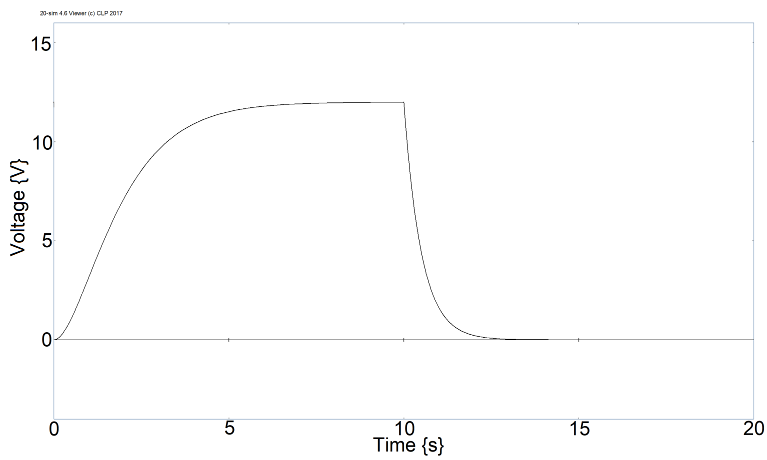


Figure 6.24: Simulation results of the modified boost converter

6.5.5 Structural analysis of the modified boost converter

The order of the model remains the same for the admissible configurations due to the absence of power flowing to the elements that are connected to the output bond in OFF configuration, in this case the storage element is giving the stored energy to the resistive element but does not affect the order of the model.

By using the controllability condition matrix 4.9 (Chapter 4) the order of the configurations is determined. For this model the order of the system can be 1 for most of the configurations, which have a controllability equation:

$$rank \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$

This is because the elements C_1 and C_2 are only *reachable controllable*, which is illustrated in Figure 6.25.

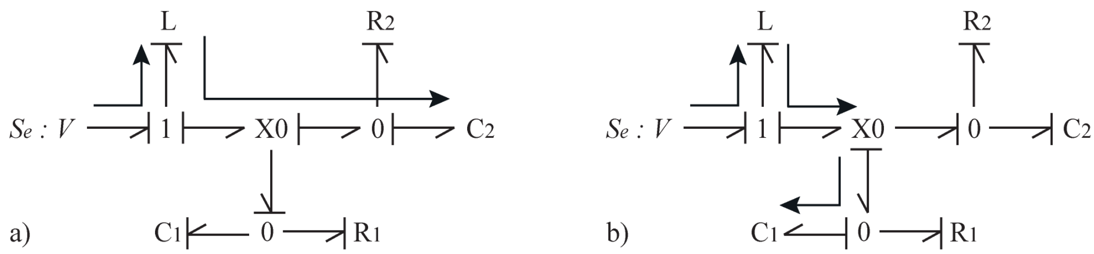


Figure 6.25: Controlling causal paths

The *observability* of the configurations is determined by the location of the output, which could lead to the observability of one of the states or both of them. This is determined using the observability condition 4.11 (Chapter 4).

The *asymptotical stability* of the system it cannot be achieved under any of the commuting conditions, this is due to one of the storage elements (C_1 or C_2) cannot be assigned with derivative causality. This can be seen in Figure 6.26, in this case the stability of the system is determined by the solvability of the implicit equation.

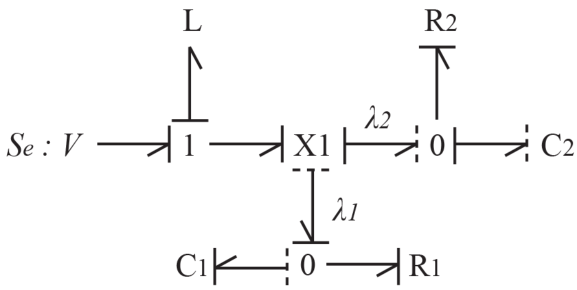


Figure 6.26: Hybrid bond graph of the modified boost converter with preferred derivative causality.

6.6 Multiway switching using three-way and four-way switches

6.6.1 Overview of the multiway switching system

A multiway switching system, as previously mentioned, are systems where a load is controlled from different parts of the system, this is done by using two-way and two-way four port switches, where the simpler system uses two

two-way and one two-way four port. An example of this type of systems (and the possible configurations) is displayed in Figure 6.27.

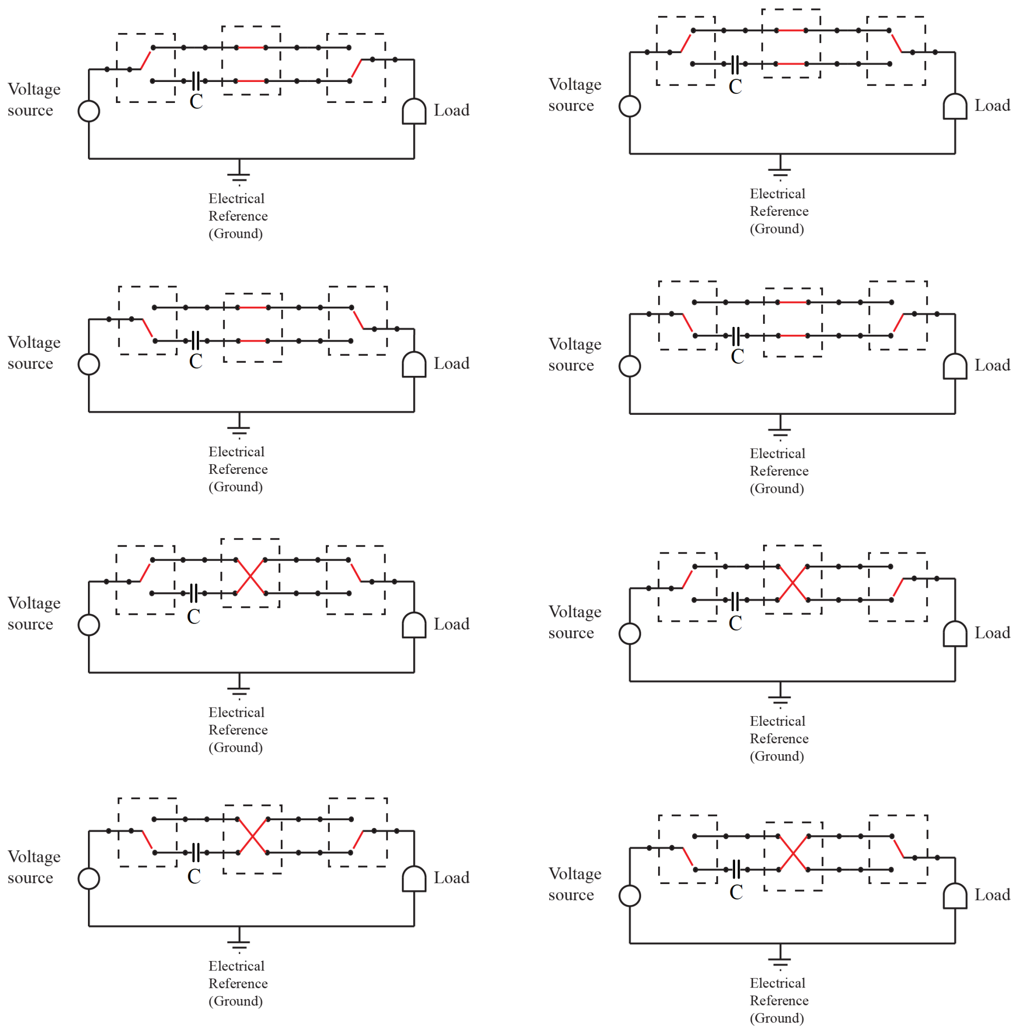


Figure 6.27: Left side OFF configurations, right side ON configurations

In this case a load can be controlled from different places in a circuit. These types of circuits are commonly found in electric installations in buildings (to turn ON and OFF lights) and some assembly lines in industries (to turn OFF assembly lines in emergencies).

6.6.2 Hybrid bond graph of the multiway switching system

The hybrid bond graph model is displayed in Figure 6.28.

In this model all the possible configurations are taken into consideration, in order to properly represent the behaviour of the physical system.

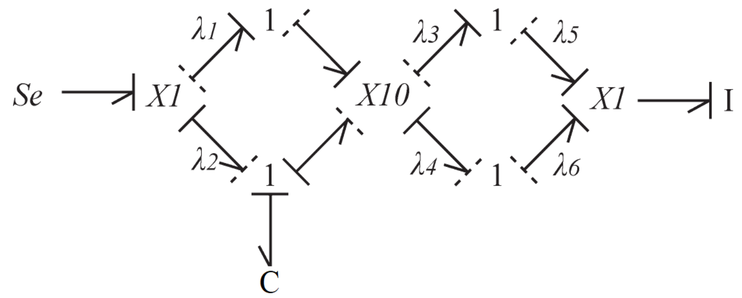


Figure 6.28: Hybrid bond graph model of the multi-way circuit

It must be noted that the ON configurations containing causal conflicts are included as these are admissible and are considered under special cases of admissible configurations. This also applies to the OFF configurations, because the causal conflicts represent the absence of flow of power.

6.6.3 Junction structure and model equations of the multiway switching system

In order to obtain the junction structure of the model, it is necessary to obtain the equations that describe the system's behaviour for the different configurations, which are shown next,

$a\dot{q} = -af_I$
$b\dot{p} = bS_e + ae_C$
$cf_I = 0$

where

$a = \bar{\lambda}_1\lambda_2\bar{\lambda}_3\lambda_4\lambda_5\bar{\lambda}_6\lambda_7\bar{\lambda}_8 + \bar{\lambda}_1\lambda_2\bar{\lambda}_3\bar{\lambda}_4\lambda_5\lambda_6\bar{\lambda}_7\lambda_8$
$b = \lambda_1\bar{\lambda}_2\lambda_3\bar{\lambda}_4\bar{\lambda}_5\lambda_6\bar{\lambda}_7\lambda_8+\bar{\lambda}_1\lambda_2\bar{\lambda}_3\lambda_4\lambda_5\bar{\lambda}_6\lambda_7\bar{\lambda}_8+\lambda_1\bar{\lambda}_2\lambda_3\lambda_4\bar{\lambda}_5\bar{\lambda}_6\lambda_7\bar{\lambda}_8+\bar{\lambda}_1\lambda_2\bar{\lambda}_3\bar{\lambda}_4\lambda_5\lambda_6\bar{\lambda}_7\lambda_8$
$c = \bar{\lambda}_1\bar{\lambda}_2\bar{\lambda}_3\bar{\lambda}_4\bar{\lambda}_5\bar{\lambda}_6\bar{\lambda}_7\bar{\lambda}_8+\bar{\lambda}_1\bar{\lambda}_2\lambda_3\lambda_4\lambda_5\lambda_6\bar{\lambda}_7\bar{\lambda}_8+\bar{\lambda}_1\lambda_2\lambda_3\lambda_4\lambda_5\lambda_6\lambda_7\bar{\lambda}_8+\lambda_1\bar{\lambda}_2\lambda_3\lambda_4\lambda_5\lambda_6\bar{\lambda}_7\lambda_8$

The resulting junction structure matrix of the model is,

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{p} \\ f_I \end{bmatrix} = \begin{bmatrix} 0 & -a & 0 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ p \\ e_I \end{bmatrix} + \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} [S_e]$$

Obtaining the following implicit equation,

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{p}_d \end{bmatrix} = \begin{bmatrix} 0 & \frac{-a}{I} & 0 \\ \frac{a}{C} & 0 & 0 \\ 0 & 0 & \frac{-b}{I} \end{bmatrix} \begin{bmatrix} q \\ p \\ p_d \end{bmatrix} + \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} [S_e]$$

In this example, the non-valid configurations are all that represent the two-way switches as open in both ends ($\lambda_1\lambda_2$ and $\lambda_7\lambda_8$), which is physically impossible. In this case the matrices Λ_{22} (a) and Λ_{22}^* (b) are not complement of each other, leading to the non-admissible configurations, as previously explained in Chapter 3.

6.6.4 Structural analysis of the multiway switching system

The controllability of the system is determined using the controllability condition Equation 4.9 introduced in Chapter 4, which depends on the configuration. It is completely controllable in the ON configurations, as there is a direct connection between the source and the motor (I), which have a controllability equation:

$$rank \begin{bmatrix} \frac{-ab}{C} \\ 0 \end{bmatrix} = 1$$

And as expected, is *not controllable* during the OFF configurations. Which have a controllability equation:

$$rank \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

The *observability* of the configurations is determined by the location of the output, which could lead to the observability of one of the states or both of them. This is determined using the observability condition Equation 4.11 introduced in Chapter 4.

The system is *asymptotical stable* in all admissible configurations as the motor (I) can be assigned with preferred derivative causality in the BGD.

6.7 Discussion

Based on the results shown in this chapter, it can be concluded that the proposed procedure for the analysis of hybrid bond graphs allows to represent all of the behaviours of the systems without setting constraints. This includes the representation of non-valid configurations on the implicit state equation, which allows to represent all of the available configurations.

The examples used in this chapter allow to compare previous approaches with the one proposed in this work, which is useful to validate the advantages of the latter.

All of the systems analysed in this chapter contains a large number of configurations, which shows how the proposed approach is useful for this type of systems by simplifying their representation in a general implicit state equation, rather than having to generate the state equation of each configuration.

This allows to simplify the analysis of the system only to the valid configuration, rather than making an independent analysis of each configuration, that usually included the non-valid configurations. It is then proven that the time spent doing the analysis is reduced, because the non-valid configurations can be detected in a simpler manner instead of being detected after the analysis of such configurations.

Also, as previously mentioned, a more physically related behaviour is obtained. This can be seen when a change in configuration occurs and the number of states changes as well, rather than having the same number of states for all of the configurations as it was on previous analysis techniques.

This is not a general behaviour of all of the hybrid systems, however, it is common to find systems that contain elements showing this behaviour, such as mechanical systems where a coupling is made during a commutation, when two or more masses behaves as one.

On the case of multiswitched junctions, it can be seen that models containing sources affected by dynamic causality, the multiswitch must be changed by an arrange of switched junctions. This is because the multiswitched junctions is a simplification of a group of switched junctions with an added constraint to avoid instabilities.

This constraint is useful on models that do not have sources affected by dynamic causality, because this allows to omit configurations that are not physically possible, nonetheless, the non-valid configurations are contained into the general implicit state equation.

On the case of the multiway switched system, the model cannot be simulated because there is a causal conflict in the model that should be considered as an admissible configuration, however, this behaviour could not be programmed into any of the available simulation software. This can be solved by modifying the source code of an open source simulation software, or by using a modified multiway switch. Both options are being analysed at the moment to observe the behaviour resulting by using this proposed element.

It can be concluded that switched junctions and multiswitched junctions are useful to properly model the physical behaviours of the systems, which could not be done using previous approaches, such as ideal switches in the case of the switched junctions or switched power junctions in the case of the multi-switched junctions. This is because the previous approaches where done with the purpose of simplify the simulation of the hybrid systems, and it was done at the cost of adding a constraint of static causality to the elements, which does not allow to represent the physical behaviour of most of the hybrid systems.

Chapter 7

Discussion and Conclusions

7.1 Discussion

The goal of this research was to propose a general notation for Hybrid Bond Graphs which could be used as a widespread notation and be suitable for simulation activities. In doing so, it was important to retain the graphical advantages of bond graph modelling and the principles of idealised physical, acausal model construction.

This notation is based on singular systems theory and hybrid bond graphs notation. Singular systems notation is used to simplify the notation of the configurations of the hybrid systems. As previously explained, the number of configurations are 2^n where n is the number of commuting elements. Therefore, the more commuting elements on the system, the larger number of configurations are to be analysed.

While with a traditional notation, it would be necessary to find the implicit equation for each configuration, which is time consuming (not only for systems with a large number of commuting elements). It is not the case of the proposed notation because it is easier to use in a computational environment, therefore the time consumed on the analysis is drastically reduced.

7.2 Conclusions

A general Hybrid Bond Graph notation and a method for its construction have been defined. This notation allows differentiating the elements that are not affected by dynamic causality from those that are affected by dynamic causality. By having this differentiation of the elements affected by dynamic causality, the analysis of the system is simplified compared to previous approaches, this is due to the implicit equations for the different configurations remains quite similar, compared to previous approaches where the implicit equations where needed to be obtained independently, or the elements affected by dynamic causality where mixed and therefore the equations took longer to be obtained or could create confusion to the user.

Also, a more physically related behaviour can be obtained from the model by using this notation and representation of the commuting elements, which mean that there is no need to set any constraints into the model, and the rules to properly assign the causality to the model and avoid mistakes on the equations that represent the system's behaviour.

It is possible to identify the valid configurations from the implicit equation, which reduce the time of analysis of the model. This is done by checking the rank of the implicit equation matrices, instead of doing the independent analysis of each of the available configurations, which exponentially as the number of commuting elements increase.

The results of this research can be used in singular systems theory and classic hybrid bond graph theory as the results obtained in this research are based in approaches proposed on these areas. This is due to the fact that the techniques on which this approach is based on are used without any alteration. Therefore, when this approach is applied on singular systems analysis, or hybrid bond graph analysis, the results could be similar to the ones that can be obtained by using previous analysis techniques, or in some cases the result are improved due to the lack of constraints set into the models, and therefore more information can be obtained compared to previous approaches.

7.3 Further Work

The determination of the stability of hybrid systems using switched junction is determined using traditional bond graph approaches. An approach to determine the steady state of the hybrid systems is being studied at the moment. This work is expected to be concluded in the following months, as it need to be tested using systems with large number of switched junctions to prove its usefulness.

An implementation of the switched junctions can be done in 20 Sim for simulation purpose, either represented as a traditional junction and a zero flow/effort source, or as a dynamic element. This lead to the design of dynamic elements, which is hard to implement, however it is not impossible.

Simulation of switched systems using traditional elements in 20 Sim is done by using traditional zero flow/effort sources, square signals, and modulated transformers. To obtain the proper behaviour of the system using this arrangement of elements, the system must be modelled for each configuration, with the correspondent causality assignment. These models will allow to obtain a closer representation of the physical phenomena of the system, however, the obtained behaviour it is not as accurate as it should. For this reason, the creation of dynamic elements is suggested in Chapter 5.

There are two types of dynamic elements, switched junctions and storage/resistive elements. Switched junctions are simpler to programme, as the governing energy equations between ports must be programmed accordingly, while the causality assignment is kept in consideration. However, the storage/resistive elements must be programmed independently, as each element behaves differently according to the configuration of the switched junction and the configuration of the surrounding elements. This can be simplified if the considerations mentioned in Chapter 5 are followed.

The algorithm previously used is being improved so it can be used to obtain all the information possible from the implicit equation. As is an algorithm made on MATLAB, it can be translated to different programming languages; this is due to MATLAB uses a programming language based on C, which is widely used.

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