Probability of winning and match length in Tiebreak Ten tennis

Peter O’Donoghue and Emma Simmonds

Cardiff School of Sport, Cardiff Metropolitan University, Cyncoed Campus, Cardiff, CF23 6XD.

Abstract

New formats of tennis have been developed to make matches more exciting and unpredictable than the traditional format of the game. The purpose of the current investigation was to compare and the probability of winning between Tiebreak Ten matches and two other formats of the game; Fast4 tennis and traditional tennis. A probabilistic model of winning Tiebreak Ten tennis matches was created and compared with existing models of Fast4 and traditional tennis matches. This analysis was done for a full range of probabilities of players winning points when they are serving. This involved 1000 simulations for each pair of probabilities for two players serving for multiple set matches in Fast4 tennis and traditional tennis. The probability of players beating higher ranked opponents was found to be higher in Tiebreak Ten matches than in Fast4 and traditional tennis matches. This confirms the claim that Tiebreak Ten matches are less predictable and hence more exciting than Fast4 and traditional tennis matches.

Key words: simulation, racket sports, rule changes, probability, modelling.

Introduction

Many rules for sports were developed in the late 1800’s and early 1900’s. Many of these rules and regulations have gradually changed as sports have evolved in the years since. There are various motives for changing rules such as maintaining competitiveness, improving the safety of athletes, accommodating improving player performance, attracting participants and spectators, adapting sports to children as well as commercial and media pressures (Williams, 2008; Arias, Argudo, & Alonso, 2011). Rule changes during the development of tennis include the introduction of tiebreaks between the 1950s and 1970s in different levels of the game as well as electronic systems to assist with line calls. “Cyclops” was an electronic system introduced in 1980 that alerted when serves, played close to the back line of the service box, were out. Cyclops was replaced by the Hawk-Eye review system in 2006 which adjudicates on a full range of line calls.

Traditional tennis matches are structured into sets, games and points with some matches being played to the best of 3 sets and some being played to the best of 5 sets. A set is won by a player who wins 6 games and being at least 2 games ahead of the opponent, or by a score of 7-5 in games or, if the set reaches 6-6 in games, by the player who wins a tiebreak. Some tournaments do not play tiebreaks in the final set, meaning that sets continue after a score of 6-6 in games until one player has won two more games than the opponent. Games in traditional tennis are won by winning at least 4 points and being at least 2 points ahead of the opponent. Tiebreaks in traditional tennis are won by winning at least 7 points and being 2 points ahead of the opponent.

In recent years, alternative formats of tennis matches have been introduced. Fast4 tennis matches are the best of 5 sets played to 4 games with tiebreaks played in any sets that reach a score of 3-3. The games are played to 4 points even where a game reaches a score of 3-3. Thus there are no repeated deuce scores. Tiebreaks in Fast4 tennis are played to 5 points even if the score within the tiebreak reaches 4-4. A coin is tossed to decide which player serves the decisive point in the tiebreak if the score does reach 4-4. Tiebreak Ten is another recently developed format of tennis where matches are made up of a single tiebreak without any games or sets. There are similarities and differences between Tiebreak Ten matches and traditional tiebreaks (Pryke, 2015). Tiebreak Tens are different to the tiebreaks played in Grand Slam tournaments in that a player needs to win at least 10 points and have won at least two more points than the opponent to win. As with traditional tiebreaks, one player serves the first point and then the opponent serves the next two points and the players continue to serve pairs of points each.

All formats of tennis have implications for the importance of points and critical situations faced by players. The triple nested scoring system used in traditional tennis matches makes it possible for players to win matches having won fewer points (Wright, Rodenberg, & Sackmann, 2013) and / or fewer games (Lisi, Grigoletto, & Cannesso, In Press) than the opponent. This is referred to as Quasi-Simpson's Paradox and it is also possible in Fast4 tennis which is also structured into sets, games and points. However, the winning player in a Tiebreak Ten match always wins more points than the opponent. Morris (1977) defined the importance of a point in tennis as being the difference between the conditional probability of winning the game if the point was won and the conditional probability of winning the game if the point was lost. Given the dominance of serve, Morris (1977) showed that 30-40 was the most important point in tennis games where the probability of the server winning a point was 0.6. Players do win a majority of points on serve in professional tennis (O'Donoghue, & Brown, 2008; Gerchak, & Kilgour, 2017). However, the probability of winning a point on serve varies from match to match depending on court surface as well as players' ability on first serve, second serve and when receiving (Barnett, 2009). In Fast4 tennis, the score of 3-3 (equivalent to Deuce in traditional tennis) is a much more important point then 2-3 (equivalent to 30-40 in traditional tennis) as the game is won or lost on this point. Points within tiebreaks have an increased importance because tiebreaks determine who wins the set. Where a tiebreak reaches a score of 4-4 in Fast4 tennis, the whole set will either be won or lost on the next point. The importance of points may be associated with psychological pressure that some players deal with better than others. How tennis players respond when facing critical situations, such as important points and break points, is related to career success (González-Díaz, Gossner, & Rogers, 2012). A specific psychological effect that may occur within tennis is psychological momentum. This could lead to players' performances in later parts of the game being influenced by success in earlier parts. For example, the outcome of a close tiebreak in the first set of traditional tennis matches has been found to influence the outcome of the second set. Winning such a tiebreak has resulted in players winning one more game in the second set than if they had lost the tiebreak (Page, 2009). A further study found that winning a close tiebreak in the first set elevated the chance of winning the second set to 60% in men's singles matches but had no effect in women's singles matches (Page, & Coates, 2017).

The impact of changing the format of tennis matches has been researched using probabilistic models since the 1970s (Schutz, 1970); it is possible to identify the probabilities that a player has of winning a game, set or match from these models. Carter Jr and Crews (1974) investigated how the introduction of tiebreaks affects the duration of a set or match, as well as the probabilities of winning. The probability of winning a tiebreak and duration of a match including a tiebreak have been analysed (Croucher, 1982; Pollard, 1983). Further work by Croucher (1986) presented the conditional probabilities of a player winning a game from any scoreline within a game. Simulations based on these underlying models can be used to aid the decision making process in tournament design (O'Donoghue, 2005). Fast4 tennis has now been introduced at the Next Generation Finals by the Association of Tennis Professionals (ATP) and International Tennis Federation (ITF). The probability of winning and the duration of Fast4 tennis matches has been investigated (Simmonds, & O'Donoghue, 2018). It is important to continue this research as further developments take place in tennis. Tiebreaks to 10 points are not only being used in their standalone form (Tiebreak Ten) but were introduced into final set of the Australian Open in 2019 for matches where the final set reached a score of 6-6 in games (Australian Open, no date). All other sets in Australian Open tennis have tiebreaks to 7 points if they reach a score of 6-6 in games. Therefore, knowledge is needed about the probability of winning the different forms of tiebreak and different types of match where tiebreaks are used. The duration of Tiebreak Ten matches will certainly be lower than that of traditional tennis matches and Fast4 tennis matches. Therefore, there is no value in comparing the durations of these different forms of tennis within the current investigation. Furthermore, the duration of traditional tennis matches and Fast4 tennis matches has already been researched (Simmonds, & O'Donoghue, 2018). However, there is still a need for information on the duration of Tiebreak Ten matches. Such information could be useful to tournament organisers and broadcasters. Therefore, a purpose of the current investigation is to determine the duration of Tiebreak Ten matches for a range of probabilities of players winning points on serve. The main purpose of the current investigation is to compare the probability of winning traditional tiebreaks, tiebreaks within Fast4 tennis and Tiebreak Tens. The study is a theoretical study undertaken by creating and analysing a probabilistic models to determine the outcome probabilities and the duration of matches.

Methods

Traditional Tennis Games

Equation (1) was specified by Croucher (1986) and shows the different ways in which the serving player (Player A) can win a game in traditional tennis, PA_{Game}, against the receiving player (Player B). The probability of Player A winning and losing a point on serve are represented by pA and qA respectively (qA = 1 – pA) while the probability of Player B winning and losing a point on serve are represented by pB and qB respectively (qB = 1 – pB). Changing pA and qA to pB and qB within equation (1) gives the probability of Player B winning their service game, PB_{Game}.

$$PA_{Game} = pA^4(1 + 4qA + 10qA^2) + 20pA^5qA^3/(1 - 2pAqA) \quad (1)$$

Traditional tiebreaks

In Grand Slam singles tennis, traditional tiebreaks are played at the end of sets if the score is 6-6 including the final set at the US Open. At Wimbledon, a traditional tiebreak is played in the final set if it reaches a score of 12-12. There is no tiebreak at the final set of the French Open and the Australian Open uses a tiebreak to 10 points if the final set reaches a score of 6-6. Equation (2) shows the different ways that Player A can win a traditional tiebreak where they serves first (O'Donoghue, 2013). Both players serve during the tiebreak which means that pA, qA, pB and qB are all used in the equation. Exchanging pA and qA with pB and qB respectively gives the probability of Player B winning a traditional tiebreak where they serve first.

$$\begin{aligned}PA_{Tiebreak} = {} & pA^3qB^4 \\ & + 4pA^4qB^3pB + 3pA^3qAqB^4 \\ & + 6pA^5qB^2pB^2 + 16pA^4qAqB^3pB + 6pA^3qA^2qB^4 \\ & + 4pA^5qB^2pB^3 + 30pA^4qAqB^3pB^2 + 40pA^3qA^2qB^4pB + 10pA^2qA^3qB^5 \\ & + 5pA^5qB^2pB^4 + 50pA^4qAqB^3pB^3 + 100pA^3qA^2qB^4pB^2 + 50pA^2qA^3qB^5pB + \\ & 5pAqA^4qB^6 \\ & + 6pA^6qBpB^5 + 75pA^5qAqB^2pB^4 + 200pA^4qA^2qB^3pB^3 + 150pA^3qA^3qB^4pB^2 + \\ & 30pA^2qA^4qB^5pB + pAqA^5qB^6 \\ & + (pA^6pB^6 + 36pA^5qApB^5qB + 225pA^4qA^2pB^4qB^2 + 400pA^3qA^3pB^3qB^3 \\ & +225pA^2qA^4pB^2qB^4 + 36pAqA^5pBqB^5 + qA^6qB^6)\ pAqB / (1 - pApB - qAqB) \quad (2)\end{aligned}$$

Fast4 Tennis Games

Equation (3) represents the probability that Player A wins their own service game in Fast4 tennis (Simmonds, & O'Donoghue, 2018). Equation (3) only differs from equation (1) for traditional tennis by replacing $pA^2 / (1 - 2pAqA)$ with pA because only one point will be played after the score reaches 3-3.

$$PA_{Game} = pA^4(1 + 4qA + 10qA^2) + 20pA^4qA^3 \quad (3)$$

Tiebreaks in Fast4 tennis

Equation (4) represents the probability of Player A winning a tiebreak in Fast4 tennis where they serve first in the tiebreak (Simmonds, & O'Donoghue, 2018). As in traditional tennis, both players serve during a Fast4 tiebreak. Therefore, pA, qA, pB and qB are all included within the model. However, one difference between a Fast4 tiebreak and a traditional tiebreak is that the player who serves first in a Fast4 tiebreak will serve the first two points rather than just the first point. After this, players alternate serving pairs of points each. The two probabilities of 0.5 at the score of 4-4 represent the coin toss where we assume that the probabilities of Player A and Player B serving the 9th point are equal.

$$
\begin{aligned}
PA_{Tiebreak} = {} & pA^3qB^2 \\
& + 2pA^4qBpB + 3pA^3qAqB^2 \\
& + pA^4qBpB^2 + 8pA^3qAqB^2pB + 6pA^2qA^2qB^3 \\
& + pA^4qBpB^3 + 12pA^3qAqB^2pB^2 + 18pA^2qA^2qB^3pB + 4pAqA^3qB^4 \\
& + (pA^4pB^4 + 16pA^3qApB^3qB + 36pA^2qA^2pB^2qB^2 + 16pAqA^3pBqB^3 + qA^4qB^4)(0.5pA \\
& + 0.5qB) \qquad (4)
\end{aligned}
$$

Tiebreak Ten

Table 1 shows the number of different ways of Player A winning the Tiebreak Ten with 10 points as well as the different ways of reaching a score of 9-9. As before, Player A is the player who serves first in the tiebreak and the opponent is Player B with pA, qA, pB and qB representing the same probabilities as they did in equations (1) to (4). A score of 9-9 in Tiebreak Ten is like a score of 6-6 in a traditional tiebreak. From this point on, the conditional probability of Player A winning the tiebreak is the same as the conditional probability of Player A winning a traditional tiebreak given the score has reached 6-6. This is given by expression (5)

$$pAqB / (1 - pApB - qAqB) \qquad (5)$$

When determining the number of combinations of points possible for Player A to win the tiebreak, it is necessary to recognise that Player A must win the last point. Consider, for example, Player A winning the tiebreak 10-1. This can either be done by losing a single point on serve or by losing a single point when the opponent is serving. The first 11 points of a tiebreak include five where Player A serves and six where Player B serves. However, because Player A must win the 11th point (a point where Player B serves), we only consider combinations of the preceding five points where Player B serves. So where Player A does not lose any points on serve, there is only one way of winning all 5 service points but 5 ways of winning 4 of the first 5 points where Player B serves. Where Player A does lose a point on serve but wins the tiebreak 10-1, there are 5 ways of winning 4 of the 5 service points but only one way of winning all 5 of Player B's first 5 service points. Hence, the line "$5pA^5qB^5pB + 5pA^4qAqB^6$" in equation (6). Now consider the more complicated case where Player A wins the tiebreak 10-6 and, therefore, serves the last point played in the tiebreak. Table 1 shows that the combinations of Player A's service points are 1 to 7 points won out of the first 7 service points because Player A must win their eighth service point. The remaining points involve 2 to 8 points being won by Player A out of the 8 points where Player B serves. Consider the specific case where Player A wins the tiebreak 10-6 while losing 2 of their own service points. Player A wins 5 of their first 7 service points (there are 21 such combinations), their last service point and 4 of the 8 points where Player B served (there are 70 such combinations). Hence there are a total of 1470 ways in which Player A can win the tiebreak 10-6 having lost 2 service points. The probabilities of all of the different ways of Player A winning the tiebreak are summed to give the probability of Player A winning the tiebreak in equation (6).

Table 1. Ways of Player A winning a 10 point tiebreak up with 10 points or reaching a score of 9-9.

Score	Serve last point	Serve points lost	Combinations $\binom{N}{n}$ = N! / (n!(N-n)!)	Power pA	Power qA	Combinations $\binom{N}{n}$= N! / (n!(N-n)!)	Power pB	Power qB	Probability
10-0	B	0	$\binom{5}{5}$	5	0	$\binom{4}{4}$	0	5	pA^5qB^5
10-1	B	0	$\binom{5}{5}$	5	0	$\binom{5}{4}$	1	5	$5pA^5qB^5pB$
	B	1	$\binom{5}{4}$	4	1	$\binom{5}{5}$	0	6	$5pA^4qAqB^6$
10-2	A	0	$\binom{5}{5}$	6	0	$\binom{6}{4}$	2	4	$15pA^6qB^4pB^2$
	A	1	$\binom{5}{4}$	5	1	$\binom{6}{5}$	1	5	$30pA^5qAqB^5pB$
	A	2	$\binom{5}{3}$	4	2	$\binom{6}{6}$	0	6	$10pA^4qA^2qB^6$
10-3	A	0	$\binom{6}{6}$	7	0	$\binom{6}{3}$	3	3	$20pA^7qB^3pB^3$
	A	1	$\binom{6}{5}$	6	1	$\binom{6}{4}$	2	4	$90pA^6qAqB^4pB^2$
	A	2	$\binom{6}{4}$	5	2	$\binom{6}{5}$	1	5	$90pA^5qA^2qB^5pB$
	A	3	$\binom{6}{3}$	4	3	$\binom{6}{6}$	0	6	$20pA^4qA^3qB^6$
10-4	B	0	$\binom{7}{7}$	7	0	$\binom{6}{2}$	4	3	$15pA^7qB^3pB^4$
	B	1	$\binom{7}{6}$	6	1	$\binom{6}{3}$	3	4	$140pA^6qAqB^4pB^3$
	B	2	$\binom{7}{5}$	5	2	$\binom{6}{4}$	2	5	$315pA^5qA^2qB^5pB^2$
	B	3	$\binom{7}{4}$	4	3	$\binom{6}{5}$	1	6	$210pA^4qA^3qB^6pB$
	B	4	$\binom{7}{3}$	3	4	$\binom{6}{6}$	0	7	$35pA^3qA^4qB^7$
10-5	B	0	$\binom{7}{7}$	7	0	$\binom{7}{2}$	5	3	$21pA^7qB^3pB^5$
	B	1	$\binom{7}{6}$	6	1	$\binom{7}{3}$	4	4	$245pA^6qAqB^4pB^4$
	B	2	$\binom{7}{5}$	5	2	$\binom{7}{4}$	3	5	$735pA^5qA^2qB^5pB^3$
	B	3	$\binom{7}{4}$	4	3	$\binom{7}{5}$	2	6	$735pA^4qA^3qB^6pB^2$
	B	4	$\binom{7}{3}$	3	4	$\binom{7}{6}$	1	7	$245pA^3qA^4qB^7pB$
	B	5	$\binom{7}{2}$	2	5	$\binom{7}{7}$	0	8	$21pA^2qA^5qB^8$
10-6	A	0	$\binom{7}{7}$	8	0	$\binom{8}{2}$	6	2	$28pA^8qB^2pB^6$
	A	1	$\binom{7}{6}$	7	1	$\binom{8}{3}$	5	3	$392pA^7qAqB^3pB^5$
	A	2	$\binom{7}{5}$	6	2	$\binom{8}{4}$	4	4	$1470pA^6qA^2qB^4pB^4$
	A	3	$\binom{7}{4}$	5	3	$\binom{8}{5}$	3	5	$1960pA^5qA^3qB^5pB^3$
	A	4	$\binom{7}{3}$	4	4	$\binom{8}{6}$	2	6	$980pA^4qA^4qB^6pB^2$
	A	5	$\binom{7}{2}$	3	5	$\binom{8}{7}$	1	7	$168pA^3qA^5qB^7pB$
	A	6	$\binom{7}{1}$	2	6	$\binom{8}{8}$	0	8	$7pA^2qA^6qB^8$
10-7	A	0	$\binom{8}{8}$	9	0	$\binom{8}{1}$	7	1	$8pA^9qBpB^7$
	A	1	$\binom{8}{7}$	8	1	$\binom{8}{2}$	6	2	$224pA^8qAqB^2pB^6$
	A	2	$\binom{8}{6}$	7	2	$\binom{8}{3}$	5	3	$1568pA^7qA^2qB^3pB^5$
	A	3	$\binom{8}{5}$	6	3	$\binom{8}{4}$	4	4	$3920pA^6qA^3qB^4pB^4$
	A	4	$\binom{8}{4}$	5	4	$\binom{8}{5}$	3	5	$3920pA^5qA^4qB^5pB^3$
	A	5	$\binom{8}{3}$	4	5	$\binom{8}{6}$	2	6	$1568pA^4qA^5qB^6pB^2$
	A	6	$\binom{8}{2}$	3	6	$\binom{8}{7}$	1	7	$224pA^3qA^6qB^7pB$
	A	7	$\binom{8}{1}$	2	7	$\binom{8}{8}$	0	8	$8pA^2qA^7qB^8$
10-8	B	0	$\binom{9}{9}$	9	0	$\binom{8}{0}$	8	1	pA^9qBpB^8
	B	1	$\binom{9}{8}$	8	1	$\binom{8}{1}$	7	2	$72pA^8qAqB^2pB^7$
	B	2	$\binom{9}{7}$	7	2	$\binom{8}{2}$	6	3	$1008pA^7qA^2qB^3pB^6$
	B	3	$\binom{9}{6}$	6	3	$\binom{8}{3}$	5	4	$4704pA^6qA^3qB^4pB^5$
	B	4	$\binom{9}{5}$	5	4	$\binom{8}{4}$	4	5	$8820pA^5qA^4qB^5pB^4$
	B	5	$\binom{9}{4}$	4	5	$\binom{8}{5}$	3	6	$7056pA^4qA^5qB^6pB^3$
	B	6	$\binom{9}{3}$	3	6	$\binom{8}{6}$	2	7	$2352pA^3qA^6qB^7pB^2$
	B	7	$\binom{9}{2}$	2	7	$\binom{8}{7}$	1	8	$288pA^2qA^7qB^8pB$
	B	8	$\binom{9}{1}$	1	8	$\binom{8}{8}$	0	9	$9pAqA^8qB^9$
9-9		0	$\binom{9}{9}$	9	0	$\binom{9}{0}$	9	0	pA^9pB^9
		1	$\binom{9}{8}$	8	1	$\binom{9}{1}$	8	1	$81pA^8qAqBpB^8$
		2	$\binom{9}{7}$	7	2	$\binom{9}{2}$	7	2	$1296pA^7qA^2qB^2pB^7$
		3	$\binom{9}{6}$	6	3	$\binom{9}{3}$	6	3	$7056pA^6qA^3qB^3pB^6$
		4	$\binom{9}{5}$	5	4	$\binom{9}{4}$	5	4	$15876pA^5qA^4qB^4pB^5$
		5	$\binom{9}{4}$	4	5	$\binom{9}{5}$	4	5	$15876pA^4qA^5qB^5pB^4$
		6	$\binom{9}{3}$	3	6	$\binom{9}{6}$	3	6	$7056pA^3qA^6qB^6pB^3$
		7	$\binom{9}{2}$	2	7	$\binom{9}{7}$	2	7	$1296pA^2qA^7qB^7pB^2$
		8	$\binom{9}{1}$	1	8	$\binom{9}{8}$	1	8	$81pAqA^8qB^8pB$
		9	$\binom{9}{0}$	0	9	$\binom{9}{9}$	0	9	qA^9qB^9

$PA_{TiebreakTen}$ =

$$pA^5qB^5$$

$$+ 5pA^5qB^5pB + 5pA^4qAqB^6$$

$$+ 15pA^6qB^4pB^2 + 30pA^5qAqB^5pB + 10pA^4qA^2qB^6$$

$$+ 20pA^7qB^3pB^3 + 90pA^6qAqB^4pB^2 + 150pA^5qA^2qB^5pB + 20pA^4qA^3qB^6$$

$$+ 15pA^7qB^3pB^4 + 140pA^6qAqB^4pB^3 + 315pA^5qA^2qB^5pB^2 + 210pA^4qA^3qB^6pB + 35pA^3qA^4qB^7$$

$$+ 21pA^7qB^3pB^5 + 245pA^6qAqB^4pB^4 + 735pA^5qA^2qB^5pB^3 + 735pA^4qA^3qB^6pB^2 + 245pA^3qA^4qB^7pB + 21pA^2qA^5qB^8$$

$$+ 28pA^8qB^2pB^6 + 392pA^7qAqB^3pB^5 + 1470pA^6qA^2qB^4pB^4 + 1960pA^5qA^3qB^5pB^3 + 980pA^4qA^4qB^6pB^2 + 168pA^3qA^5qB^7pB + 7pA^2qA^6qB^8$$

$$+ 8pA^9qBpB^7 + 224pA^8qAqB^2pB^6 + 1568pA^7qA^2qB^3pB^5 + 3920pA^6qA^3qB^4pB^4 + 2920pA^5qA^4qB^5pB^3 + 1568pA^4qA^5qB^6pB^2 + 224pA^3qA^6qB^7pB + 8pA^2qA^7qB^8$$

$$+ pA^9qBpB^8 + 72pA^8qAqB^2pB^7 + 1008pA^7qA^2qB^3pB^6 + 4704pA^6qA^3qB^4pB^5 + 8820pA^5qA^4qB^5pB^4 + 7056pA^4qA^5qB^6pB^3 + 2352pA^3qA^6qB^7pB^2 + 288pA^2qA^7qB^8pB + 9pAqA^8qB^9$$

$$+(pA^9pB^9 + 81pA^8qAqBpB^8 + 1296pA^7qA^2qB^2pB^7 + 7056pA^6qA^3qB^3pB^6 + 15876pA^5qA^4qB^4pB^5 + 15876pA^4qA^5qB^5pB^4 + 7056pA^3qA^6qB^6pB^3 + 1296pA^2qA^7qB^7pB^2 + 81pAqA^8qB^8pB + qA^9qB^9)\ pAqB / (1 - pApB - qAqB) \quad (6)$$

Simulation

Players typically win a majority of points on serve in competitive professional tennis matches (Gerchak, & Kilgour, 2017). Therefore, probabilities of winning Tiebreak Ten matches were compared with probabilities of winning traditional tiebreaks, tiebreaks within Fast4 tennis, traditional tennis matches and Fast4 tennis matches for probabilities of winning points on serve of 0.5 or greater. Equation (6) was used to determine the probability of Player A winning the Tiebreak Ten match for a range of pA and pB values from 0.5 to 0.9 in steps of 0.1. The probability of Player A winning the Tiebreak Ten match is compared with the probability with which they win a best of three traditional sets match (with tiebreaks to 7 at the end of each set if required) and the probability of winning a best of five Fast4 sets match. This comparison used the models of Croucher (1986) for winning traditional games, Newton and Keller (2005) for winning a tiebreak to 7 points and Simmonds and O'Donoghue (2018) for winning Fast4 games and tiebreaks to 5. This was done for a range of pA values from 0.5 to 0.9 in steps of 0.01 and pB values of 0.5, 0.6 and 0.7.

The equations for winning games and tiebreakers were used as the underlying models of simulations for sets and matches in traditional and Fast4 tennis. Simulators of traditional and Fast4 tennis were programmed in Matlab 2017a (The MathWorks Inc., Natick, MA). Each simulator simulated 100,000 matches in the given format of tennis for each pair of values for pA and pB. The proportion of simulated matches won by Player A was used to determine the probability of winning a match.

The mean duration of a Tiebreak Ten was determined by using the probabilities of a tiebreak requiring 10 through to 58 points. Each of these probabilities was multiplied by the given number of points to form a series of products. These products were then summed to determine the expected duration of the tiebreak. This was done for a range of pA values from 0.5 to 0.9 in steps of 0.01 and a range of pB values from 0.5 to 0.9 in steps of 0.05. The probability of a tiebreak exceeding 58 points was deemed negligible; the probability of this was 0.0014 when pA = pB = 0.9. Note that it is not possible to have an odd number of points in the tiebreak once it has reached a score of 9-9. The cumulative probability of a Tiebreak Ten requiring each number of points or fewer was also determined.

Results

Figure 1 shows the probability of Player A winning a Tiebreak Ten match for a range of probabilities of each player winning points on serve. Where pA = pB the probability of each player winning the match is 0.5. Furthermore, serving first does not have any impact on the probability of winning the match. For each of the 1681 pairs of pA and pB values from 0.5 to 0.9 in steps of 0.01, the probability of the superior player winning the match was the same no matter whether they served first or second.

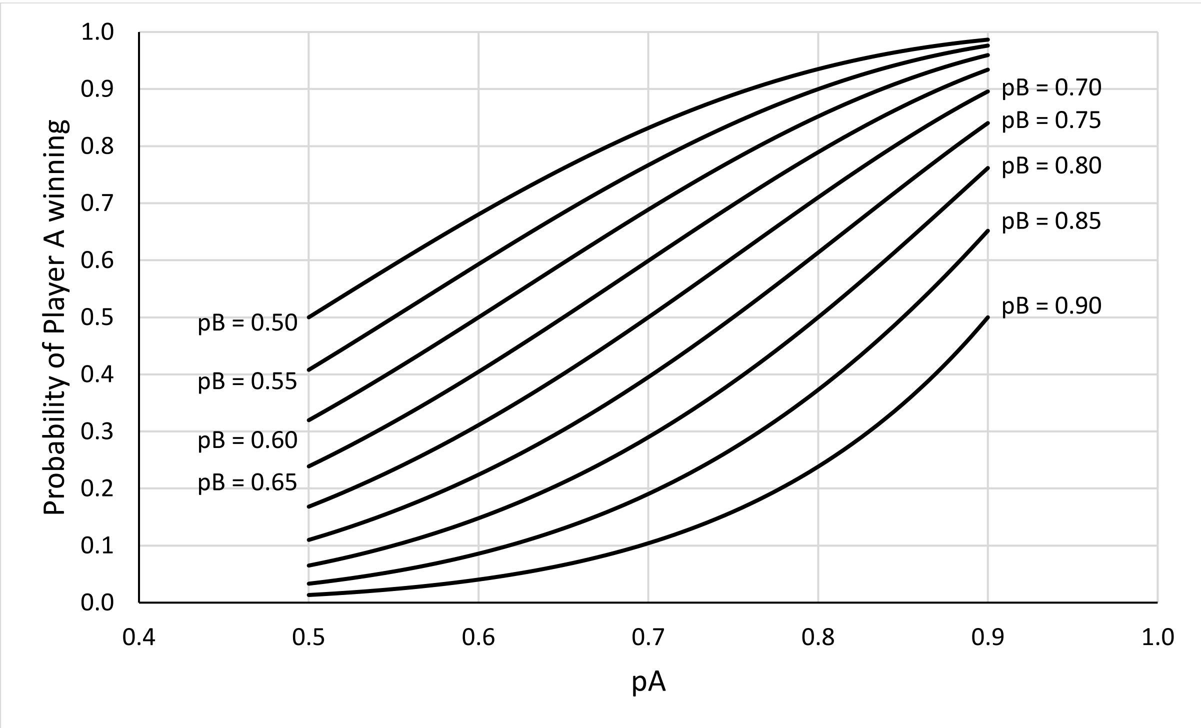

Figure 1. The probability of the player who serves first (Player A) winning a Tiebreak Ten match.

Figure 2 compares the probability of winning Tiebreak Ten matches with the probabilities of winning traditional tiebreaks and tiebreaks within Fast4 tennis matches. This shows that the superior player's chance of winning a tiebreak is elevated in a tiebreak to 10 points compared to the other two formats of tiebreak. Figure 3 compares the probability of winning Tiebreak Ten matches with matches that are the best of three traditional sets with a tiebreak at 6-6 and matches that are the best of 5 Fast4 sets. Let us consider matches where Player A is the favourite to win ($pA > pB$). Player A's probability of winning is 0.2 or more lower in a Tiebreak Ten match than in the best of three traditional sets for some pairs of values for pA and pB.

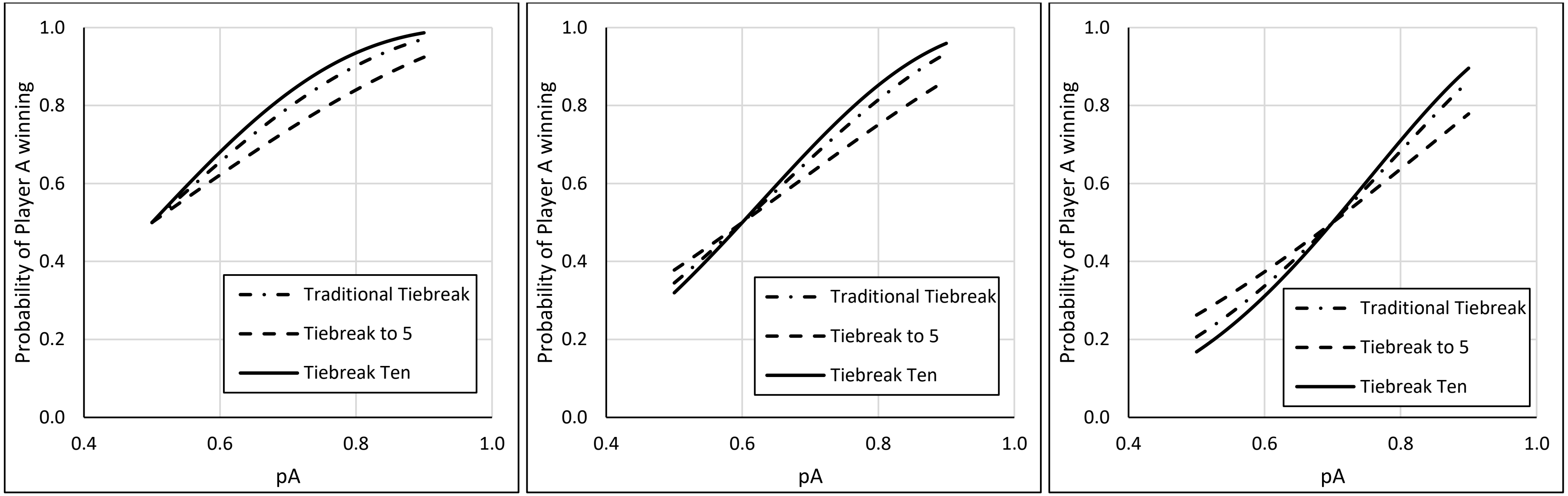

Figure 2. The probability of the player who serves first (Player A) winning different types of tiebreak: (a) pB = 0.5, (b) pB = 0.6, (c) pB = 0.7.

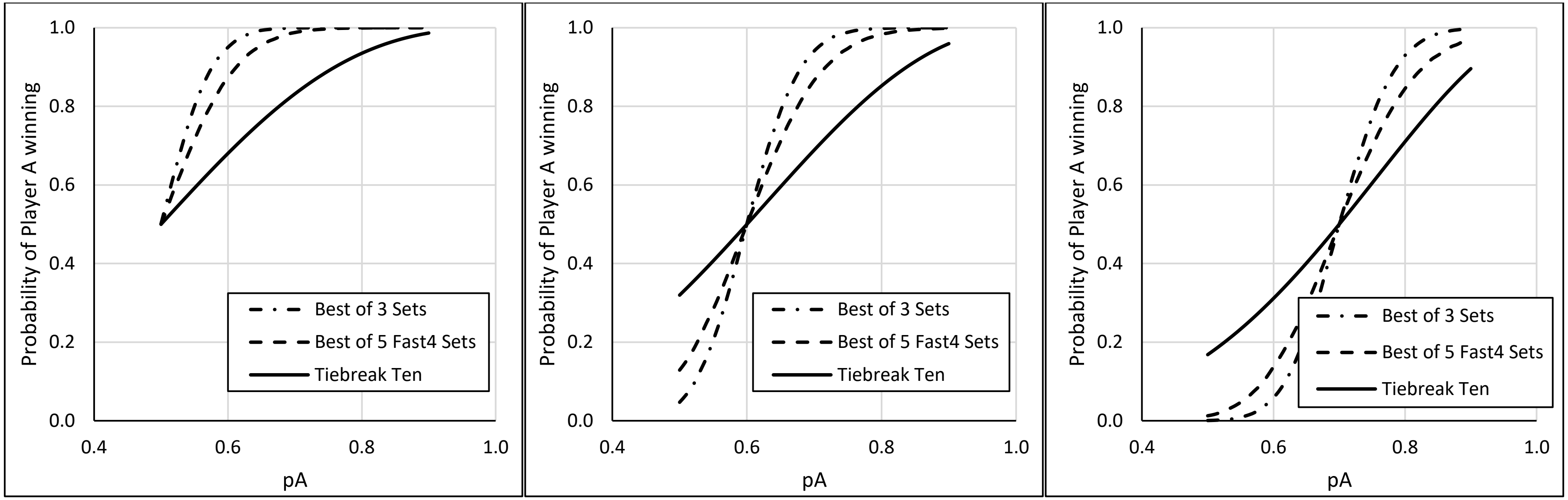

Figure 3. The probability of the player who serves first (Player A) winning different types of tennis match: (a) pB = 0.5, (b) pB = 0.6, (c) pB = 0.7.

Figure 4 shows the duration of Tiebreak Ten matches for a range of probabilities of each player winning service points. The shortest matches shown are the most one sided matches where one player has a 0.9 probability of winning a service point while the opponent's probability of winning a service point is 0.5. The longest matches are where two equally matched players have a very high probability of winning points on serve and a low probability of winning points when receiving serve. Figure 4 shows that the mean duration of matches is 20.51 points when pA = pB = 0.9. However, it is worth noting that the median duration for such matches is 18 points with 75% of such matches being 22 points or fewer and only 5% of such matches exceeding 38 points. Where both players have a 0.5 probability of winning a point on serve, the median duration is 17 points with 95% of such matches requiring 22 points or fewer.

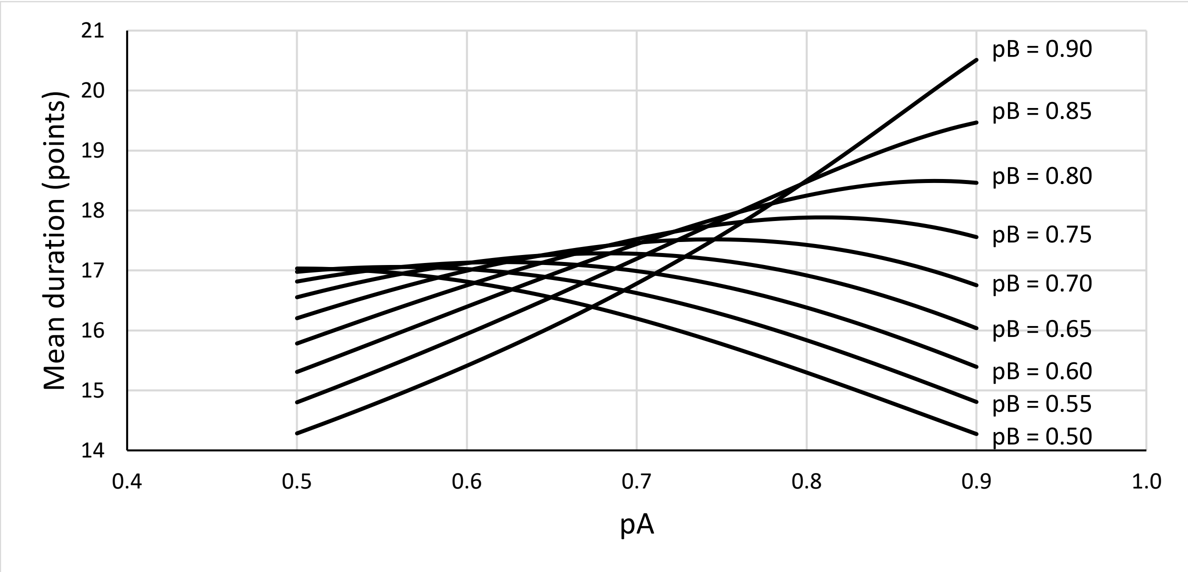

Figure 4. The duration of Tiebreak Ten matches.

Discussion

The fairness of Tiebreak Ten tennis can be considered in terms of players' chances of winning and the importance of points. Where the players have an equal probability of winning points on serve, the probability of winning a Tiebreak Ten match is 0.5, as it is in any other format of tennis. The probability of winning is not influenced by who serves first. The reduced chance of higher quality players winning Tiebreak Ten matches greatly reduces their chances of progressing through tournaments. Consider a player who wins 70% of their own service points against an opponent who wins 60% of points on serve. The probability of the player winning a Tiebreak Ten match is 0.689 compared to 0.941 in the best of three traditional sets and 0.866 in the best of five Fast4 tennis sets. The probability of the player winning three consecutive matches against this quality of opponent are 0.327 which is very much lower than the 0.833 in the best of three traditional sets and 0.650 in the best of five Fast4 tennis sets. Tournament organisers need to balance the need for unpredictability and excitement with the need for having high profile players reaching the latter stages of tournaments. Playing a match which is the best of three Tiebreak Tens is a possible match format that would reduce the chance of upsets while still being more unpredictable than the other two formats discussed. In the case of the player who wins 70% of points of serve against an opponent who wins 60% of points on serve, the probability of the player winning the best of three Tiebreak Tens is 0.770; winning three consecutive matches against this quality of opponent is 0.457. Players competing in such a match format may need to develop the psychological skills to take advantage of positive psychological momentum and minimise the effect of negative psychological momentum. This is because close tiebreaks have been shown to impact on success in subsequent sets of traditional tennis (Page, 2009; Page, & Coates, 2017). Tiebreaks played to 10 or more points can be closer than traditional tiebreaks and may, therefore, impact on performance in subsequent sets played using the Tiebreak Ten format.

Higher quality players who have greater probabilities of winning points on serve and when receiving than their opponents, are more likely to win matches than their opponents (Wright et al., 2013; Lisi et al., In Press). This assumes that the probability of winning a point is independent of the score within the tiebreak. However, some points are more important than others, especially match points. At a normal game level, Morris (1977) defined the importance of a point as the difference between the conditional probability of winning the game if the point is won and if the point is lost. Applying this to Tiebreak Ten tennis reveals that match points are the most important. If the two players have an equal chance of winning points then the importance of the scores 8-9, 9-10, 10-11, etc is 0.5. This is because the conditional probability of winning the Tiebreak Ten match is 0.5 when these points are won but 0 if they are lost. Player B could be disadvantaged if they under-perform when serving on important points. This is because Player B would be serving at the score 8-9 while Player A would not be in such a situation until the score 9-10. Professional tennis players with higher mentally toughness perform better during critical points than those with lower mental toughness (Cowden, 2016). This also helps these players have more successful professional tennis careers than those who do not cope with such pressure as well (González-Díaz et al., 2012). Therefore, players with higher mental toughness may perform better in Tiebreak Ten tennis than those with lower mental toughness.

The duration of Tiebreak Ten matches is shorter than the durations of other formats of tennis. The 95th percentile for match duration is 38 in an extreme case where the match is contested between two evenly matched players who each win 90% of their own service points. This is shorter than the relatively one sided matches in Fast4 and traditional tennis. For example, if one player has a probability of 0.9 of winning a point on serve and the opponent has a probability of 0.5 of winning a point on serve, the lower quartile of the duration of best of three traditional set matches is 79 points and the lower quartile of the duration of best of five Fast4 set matches is 74 points based on the model of Simmonds and O'Donoghue (2018).

Before 2019, tiebreaks were only used in the final set at the US Open; a tiebreak to 7 points is used in any set of US Open matches that reach a score of 6-6. In 2019 tiebreaks were introduced in the final sets of the Australian Open and Wimbledon but not the French Open. At the Australian Open a tiebreak to 10 points is used where the final set reaches a score of 6-6 while at Wimbledon a tiebreak to 7 points has been introduced where the final set reaches a score of 12-12. The French Open has longer rallies than other Grand Slam tournaments (Unierzyski, & Wieczorek, 2004; Brown, & O'Donoghue, 2008) which might suggest a final set tiebreak may be required in this tournament more than in other tournaments. During the

2018 French Open, 3 retirements occurred, there was a walkover in one women's singles match and 5 matches saw the final set going beyond 6-6. Competing on clay courts involves higher energy expenditure than on hard courts (Chapelle, Clarys, Meulemans, & Aerenhouts, 2017). The tiebreak to 10 points could improve player welfare at this tournament while also being more representative of player ability than a tiebreak to 7 points. However, the serve is less dominant at the French Open than at other Grand Slam tournaments (Filipcic, Zecic, Reid, Crespo, Panjan, & Nejc, 2015) meaning that more service breaks, fewer tiebreaks and lower numbers of points are played in French Open matches than in other Grand Slam tournaments.

By contrast, the greatest serve dominance in Grand Slam tournaments occurs at Wimbledon (Brown, & O'Donoghue, 2008). The faster grass surface at Wimbledon encourages players to serve faster than they do at other Grand Slam tournaments (Vaverka, Nykodym, Hendl, Zhanel, & Zahradnik, 2018). This makes breaks of serve less frequent at Wimbledon and has resulted in some very long final sets at this tournaments (O'Donoghue, 2013). This, in turn, can result in match congestion which is associated with decreasing serve accuracy (Marage, Duffield, Gescheit, Perri, & Reid, 2018), increased pain ratings (Marage et al., 2018), increased error rates (Gescheit et al., 2016) and fatigue (Fernandez-Fernandez, Sanz-Rivas, & Mendez-Villanueva, 2009). This leads to the question as to why final set tiebreaks at Wimbledon occur later than they do at the Australian Open. A good reason for introducing a tiebreak at 6 games all in the final set of Australian Open matches is due to the greater prevalence of heat stress at this tournament (Smith, Reid, Kovalchik, Woods, & Duffield, 2018). In the 2018 Australian Open, four men's singles matches were not completed due to player retirements compared to 2 at the French Open and none at Wimbledon. There is an opportunity for future research to compare serve accuracy, error rates and retirements before and after the introduction of different types of tiebreaks at different stages in the final set of Australian Open and Wimbledon.

In conclusion, this study confirms the claim that Tiebreak Ten tennis is more unpredictable than other traditional tennis and Fast4 tennis. While Tiebreak Ten matches can theoretically be infinite, there is a drastically reduced chance of matches being very long in this format of the game compared to traditional formats used in professional tennis.

References

Arias, J.L., Argudo, F.M., & Alonso, J.I. (2011). Review of rule modification in sport. *Journal of Sports Science and Medicine, 10*, 1-8.

ATP World Tour (2017). ATP announces trial of rule changes and innovation for Next Gen ATP Finals in Milan ATP World Tour Tennis. https://www.atpworldtour.com/en/news/rule-changes-innovation-for-next-gen-atp-finals-2017, accessed 28th September 2018.

Australian Open (no date). https://ausopen.com/articles/news/final-set-tiebreaks-australian-open-2019, accessed 11th January 2019.

Barnett, T. (2009). Applying tennis match statistics to increase serving performance during a match in progress. *Journal of Medicine and Science in Tennis, 14*(3), 16-19.

Brown, E., & O'Donoghue, P. (2008). Gender and surface effect on elite tennis strategy. *ITF Coaching and Sport Science Review, 46*, 9-11.

Carter, W.H., & Crews, S.L. (1974). An analysis of the game of tennis. *American Statistician, 28*, 130-134.
Chapelle, L., Clarys, P. Meulemans, S., & Aerenhouts, D. (2017). Physiological responses, hitting accuracy and step count of a tennis drill in function of court surface: a randomised cross-over design. *International Journal of Performance Analysis in Sport, 17,* 622-629.
Cowden, R.G. (2016). Competitive performance correlates of mental toughness in tennis. *Perceptual and motor skills, 123,* 341-360. doi:10.1177/0031512516659902.
Croucher, J.S. (1982). The effect of the tennis tie-breaker. *Research Quarterly for Exercise and Sport, 53,* 336-339.
Croucher, J.S. (1986). The conditional probability of winning games of tennis. *Research Quarterly in Exercise and Sport, 57,* 23-26.
ESPN (2018). http://www.espn.co.uk/tennis/story/_/id/22585510/davis-cup-transformed-world-cup-tennis-2019, accessed 28th September 2018.
Fernandez-Fernandez, J., Sanz-Rivas, D., & Mendez-Villanueva, A. (2009). A review of the activity profile and physiological demands of tennis match play. *Strength and Conditioning Journal, 31*(4), 15-26.
Filipcic, A., Zecic, M., Reid, M., Crespo, M., Panjan, A., & Nejc, S. (2015). Differences in performance indicators of elite tennis players in the period 1991-2010. *Journal of Physical Education and Sport, 15,* 671. doi:10.7752/jpes.2015.04102.
Gerchak, Y., & Kilgour, D.M. (2017). Serving strategy in tennis: accuracy v power. *Mathematics Magazine, 90,* 188-196.
Gescheit, D.T., Duffield, R., Skein, M., Brydon, N., Cormack, S.J., & Reid, M. (2016). Effects of consecutive days of match play on technical performance in tennis. *Journal of Sports Sciences, 35,* 1988-1994.
González-Díaz, J., Gossner, O., & Rogers, B. W. (2012). Performing best when it matters most: Evidence from professional tennis. *Journal of Economic Behavior & Organization, 84,* 767-781.
Lisi,, F., Grigoletto, M., & Cannesso, T. (in press). Winning tennis matches with fewer points or games than the opponent. *Journal of Sports Analytics*.
Marage, N., Duffield, R.Gescheit, D., Perri, T., & Reid, M. (2018). Playing not once, not twice but three times in a day: the effect of fatigue on performance in junior tennis players. *International Journal of Performance Analysis in Sport, 18,* 104-114.
Meffert, D., O'Shannessy, C., Born, P., Grambow, R., & Vogt, T. (2018). Tennis serve performances at break points: Approaching practice patterns for coaching. *European journal of Sport Science, 18,* 1151-1157.
Morris, C. (1977). The most important points in tennis, In Ladany, S.P., & Machol, R.E. (eds.), *Optimal Strategies in Sport* (pp. 131-140), New York: North Holland.
Newton, P.K. and Keller, J.B. (2005), The probability of winning at tennis. *Theory and Data, Studies in Applied Mathematics, 114,* 214-269.
O’Donoghue, P.G. (2005). The Role of Simulation in Sports Tournament Design for Game Sport, *International Journal of Computer Science in Sport, 4*(2), 14-27.
O’Donoghue, P.G. (2013). Rare events in tennis. *International Journal of Performance Analysis in Sport, 13,* 535-552.
O'Donoghue, G. P., & Brown, E. (2008). The importance of service in Grand Slam singles tennis. *International Journal of Performance Analysis in Sport, 8*(3), 70-78.
Page, L. (2009, July). The momentum effect in competitions: field evidence from tennis matches. In Econometric Society Australasian Meeting.
Page, L., & Coates, J. (2017). Winner and loser effects in human competitions. Evidence from equally matched tennis players. *Evolution and Human Behavior, 38,* 530-535.

Pollard, G.H. (1983). An analysis of classical and tie-breaker tennis. *Australian Journal of Statistics, 25*, 496-505.

Pryke, M. (2015). Tie break tens: how to put together a successful tennis tournament. https://www.thesportreview.com/2015/12/tie-break-tens-how-to-put-together-a-successful-tennis-tournament, accessed 5th April 2019.

Rodenberg, R. (2014). What every pro tennis player does better than Roger Federer. https://www.theatlantic.com/entertainment/archive/2014/01/what-every-pro-tennis-player-does-better-than-roger-federer/283007/, accessed 13th June 2018.

Schutz, R.W. (1970). A mathematical model for evaluating scoring systems with specific reference to tennis. *Research Quarterly for Exercise and Sport, 41*, 552-561.

Simmonds, E., & O'Donoghue, P.G. (2018). Probabilistic models comparing Fast4 and traditional tennis. *International Journal of Computer Science in Sport, 17*, 141-162.

Smith, M.T., Reid, M., Kovalchik, S., Woods, T.O., & Duffield, R. (2018). Heat stress incident prevalence and tennis matchplay performance at the Australian Open. *Journal of Science and Medicine in Sport, 21*, 467-472.

Tiebreak Tens (No date). www.tiebreaktens.com, accessed 9th August 2018.

Unierzyski, P., & Wieczorek, A. (2004). Comparison of tactical solutions and game patterns in the finals of two grand slam tournaments in tennis. In Lees, A., Kahn, J., & Maynard, I. (eds.), *Science and Racket Sports III* (pp.200-205), Oxon: Routledge.

Vaverka, F., Nykodym, J., Hendl, J., Zhanel, J., & Zahradnik, D. (2018). Association between serve speed and court surface in tennis. *International Journal of Performance Analysis in Sport, 18*, 262-272.

Williams, J. (2008). Rule changes in sport and the role of notation. In Hughes, M., & Franks, I.M. (eds.), *The Essentials of Performance Analysis: an introduction* (pp.226-242), London: Routledge.

Wright, B., Rodenberg, M.R., & Sackmann, J. (2013). Incentives in best of N contests: quasi-Simpson's Paradox in tennis. *International Journal of Performance Analysis in Sport, 13*, 790-802.