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Evidence Theory and VPRS model

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Abstract

The Rough Set Theory (RST) was proposed by Pawlak [4] as a new mathematical approach to deal with uncertain knowledge in expert systems. In 1991 Ziarko [11] proposed the Variable Precision Rough Set Model (VPRSM) as a certain extension of the rough set theory. VPRSM approach makes it possible to use a certain level of misclassification.

The aim of this paper is to introduce belief and plausibility functions defined by the β–approximation regions. On the basis of the β–approximation regions, the β–basic probability assignment is defined and the Dempster's combination rule for product of two decision tables is constructed. This entire approach is illustrated by examples.

1 Introduction

The Evidence Theory (ET) or Dempster-Shafer Theory was proposed by Dempster in 1967 [2] as a statistical methodology for approximation of probability and developed by Shafer in 1976 [7] as an autonomic mathematical theory. The evidence theory approach is based on the idea of placing a number from the interval [0,1], to indicate a degree of belief for a given proposition on the basis of a given evidence [8].

In this paper we define basic numerical functions of evidence theory using the main concepts related to the β–approximation. We also define Dempster's combination rule for the product of decision tables. It gives us ability to:

- extract some information from sub-tables,
- join this information and create a new decision table.

At the end of the paper we show that our assumptions can be used to real data, which are stored in decision table.

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2 β–approximation

In RST [5] vague concepts are replaced by a pair of precise concepts of the lower and upper approximations. The lower approximation of a given set of objects (given concept) is a set of objects with certainty belonging to the concept. The upper approximation of a given set of objects (a given concept) is the set of objects probably belonging to the concept. According to β–approximation an object could be classified to a giv enset X as:

- certainly belonging to X, or
- with high probability belonging to X, or
- weakly belonging to X, or
- with high probability belonging to the complement of X, or
- certainly belonging to the complement of X.

According to Ziarko [11] the β–approximations of sets can be defined as follows.
Let $\mathbf{A} = (U, A)$ be an information system, where U is a nonempty , finite set of objects called the universe and A is a nonempty , finite set of attributes, i.e., $a : U \to V_a$ for $a \in A$, where V_a is called the value set of a, the indiscernibility relation $IND(B)$ for $B \subseteq A$ is defined b y

$$IND(B) = \{(x, y) \in U \times U : \underset{a\in B}{\forall} a(x) = a(y)\}.$$

By $[x]_B$ we denote the equivalence class of $IND(B)$, i.e., the set

$$[x]_B = \{y \in U : x\, IND(B)\, y\}.$$

Let $\emptyset \neq X \subseteq U$ and $\beta \in [0, 0.5)$. F oursets, called *β–approximation regions*, can be defined from X in the information system $\mathbf{A}$:

(i) $\underline{A}_\beta X = \{x \in U : \frac{|[x]_A \cap X|}{|[x]_A|} \geq 1 - \beta\}$ – *lower β–approximation* of X in $\mathbf{A}$;

(ii) $\overline{A}_\beta X = \{x \in U : \frac{|[x]_A \cap X|}{|[x]_A|} > \beta\}$ –*upper β–approximation* of X in $\mathbf{A}$;

(iii) $BN_{\mathbf{A},\beta} X = \{x \in U : \beta \leq \frac{|[x]_A \cap X|}{|[x]_A|} \leq 1 - \beta\}$ – *boundary region* of β–approximation of X in $\mathbf{A}$;

(iv) $NEG_{\mathbf{A},\beta} X = \{x \in U : 0 \leq \frac{|[x]_A \cap X|}{|[x]_A|} \leq \beta\}$ – *negative region* of β–approximation of X in $\mathbf{A}$.

For $\beta = 0$ we obtain approximation regions considered in rough set theory [5] and related to approximation of X.

3 Properties of β–approximation

An information system $\mathbf{A} = (U, A \cup \{d\})$, where $d \notin A$ is the decision attribute is called the *de cision table*. We assume the set of v alues of decision d to be equal to $\{1, \ldots, r(d)\}$.

The classification made by d is the set

$$CLASS_{\mathbf{A}}(d) = \{X_1, \ldots, X_r(d)\} \text{ where } X_i = \{x \in U : d(x) = i\}$$

and $r(d)$ is called *rank of* d.
The set $\Theta_{\mathbf{A}} = \{1, \ldots, r(d)\}$ is called *frame of discernment* defined by d.
A β*–boundary region* of $\theta \subseteq \Theta_{\mathbf{A}}$ is a set defined by:

$$Bd_{\mathbf{A},\beta}(\theta) = \bigcap_{i \in \theta} BN_{\mathbf{A},\beta} X_i \cap \bigcap_{i \notin \theta} NEG_{\mathbf{A},\beta} X_i.$$

Proposition 3.1 *All non-empty sets from the family*

$$\{\underline{A}_\beta X_1, \ldots, \underline{A}_\beta X_{r(d)}\} \cup \{Bd_{\mathbf{A},\beta}(\theta) : \theta \subseteq \Theta_{\mathbf{A}}\}$$

create a partition of the universe U.

4 Relationship between β–approximation and evidence theory

We extend $\Theta_{\mathbf{A}}$ to $\Theta_{\mathbf{A}} \cup \{0\}$ where 0 is a special element. It means that objects from $Bd_{\mathbf{A},\beta}(\emptyset)$ have a special decision $d = 0$.
Hence, $\Theta_{\mathbf{A}} = \{1, \ldots, r(d)\} \cup \{0\}$.
Now, we would like to find a function transforming subsets of $\Theta_{\mathbf{A}} = \{1, \ldots, r(d)\} \cup \{0\}$ into elements of the family

$$\{\underline{A}_\beta X_1, \ldots, \underline{A}_\beta X_{r(d)}\} \cup \{Bd_{\mathbf{A},\beta}(\theta) : \theta \subseteq \Theta_{\mathbf{A}}\}.$$

Such a function is defined by

$$\Phi_{\mathbf{A},\beta}(\theta) = \begin{cases} \underline{A}_\beta X_i \cup Bd_{\mathbf{A},\beta}(\{i\}) & \text{for } \theta = \{i\} \text{ where } i \in \{1, \ldots, r(d)\} \\ Bd_{\mathbf{A},\beta}(\emptyset) & \text{for } \theta = \{0\} \\ \emptyset & \text{for } \theta = \emptyset \\ Bd_{\mathbf{A},\beta}(\theta) & \text{for } |\theta| > 1 \text{ where } \theta \subseteq \{1, ..., r(d)\} \end{cases}$$

where $\theta \subseteq \Theta_{\mathbf{A}}$.

Let $\Theta = \{\theta_0, \theta_1, \theta_2, ..., \theta_k\}$ be a frame of discernment compatible with a given decision table and let $\chi : \Theta \to \Theta_{\mathbf{A}}$ be the standard bijection between Θ and $\Theta_{\mathbf{A}}$, i.e., $\chi(\theta_i) = $ i for i = 0, 1,. . . ,k.
Let us define a function $m_{\mathbf{A},\beta} : 2^\Theta \to R_+$ by

$$m_{\mathbf{A},\beta}(\theta) = \begin{cases} 0 & \text{for } \theta = \emptyset \\ \frac{|\Phi_{\mathbf{A},\beta}(\chi(\theta))|}{|U|} & \text{for } \theta \neq \emptyset \end{cases}$$

for any $\theta \subseteq \Theta$.

Proposition 4.1 *The function* $m_{\mathbf{A},\beta} : 2^\Theta \to R_+$ *defined above is a basic probability assignment (mass function).*

Proof. We have to show that:

$$m_{\mathbf{A},\beta}(\emptyset)=0 \quad \text{and} \quad \sum_{\Delta\subseteq\Theta} m_{\mathbf{A},\beta}(\Delta)=1.$$

The first condition in a simple consequence of $m_{\mathbf{A},\beta}$ definition. To prove the second condition let us observe that

$$\sum_{\Delta\subseteq\Theta} m_{\mathbf{A},\beta}(\Delta)=\sum_{\Delta\subseteq\Theta}\frac{|\Phi_{\mathbf{A},\beta}(\chi(\Delta))|}{|U|}=\frac{1}{|U|}\sum_{\Delta\subseteq\Theta}|\Phi_{\mathbf{A},\beta}(\chi(\Delta))|=$$

$$=\frac{1}{|U|}\left(\sum_{i\in\chi(\Theta)}|\Phi_{\mathbf{A},\beta}(\{i\})|+\sum_{\Delta\subseteq\Theta_{\mathbf{A}},|\Delta|>1}|\Phi_{\mathbf{A},\beta}(\Delta)|\right)=$$

$$=\frac{1}{|U|}\left(|Bd_{\mathbf{A},\beta}(\{0\})|+\sum_{i\in\chi(\Theta)}|\underline{A}_{\beta}(X_i)\cup Bd_{\mathbf{A},\beta}(\{i\})|+\sum_{\Delta\subseteq\Theta_{\mathbf{A}},|\Delta|>1}|Bd_{\mathbf{A},\beta}(\Delta)|\right)=$$

$$=\frac{1}{|U|}\left(|Bd_{\mathbf{A},\beta}(\{0\})|+\sum_{i\in\chi(\Theta)}\left|\underline{A}_{\beta}(X_i)\cup Bd_{\mathbf{A},\beta}(\{i\})\right|+\sum_{\Delta\subseteq\Theta_{\mathbf{A}},|\Delta|>1}|Bd_{\mathbf{A},\beta}(\Delta)|\right)=$$

$$=\frac{1}{|U|}\left(\sum_{i\in\chi(\Theta)}|\underline{A}_{\beta}(X_i)|+\sum_{\Delta\subseteq\Theta,|\Delta\geq 1|}|Bd_{\mathbf{A},\beta}(\Delta)|\right)=\frac{1}{|U|}|U|=1.$$

□

The *β–belief function* for a given **A** is defined by

$$Bel_{\mathbf{A},\beta}(\theta)=\sum_{\Delta\subseteq\theta} m_{\mathbf{A},\beta}(\Delta) \quad where\ \theta\subseteq\Theta.$$

Let Θ be a frame of discernment compatible with the decision table $\mathbf{A}=(U,A\cup\{d\})$ and let χ be a standard bijection between Θ and $\Theta_{\mathbf{A}}$. The following equalities hold:

$$Bel_{\mathbf{A},\beta}(\theta)=\sum_{\Delta\subseteq\theta} m_{\mathbf{A},\beta}(\Delta) \quad =\sum_{i\in\chi(\theta)}\frac{|\ \underline{A}_{\beta}X_i\ |}{|U|}+\sum_{\Delta\subseteq\theta,|\Delta|\geq 1}\frac{|Bd_{\mathbf{A},\beta}(\chi(\Delta))|}{|U|}$$

for any $\theta\subseteq\Theta$

The *β–plausibility function* for a given **A** is defined by

$$Pl_{\mathbf{A},\beta}(\theta)=\sum_{\Delta\cap\theta\neq\emptyset} m_{\mathbf{A},\beta}(\Delta) \qquad where\ \theta\subseteq\Theta.$$

The following equalities hold:

$$Pl_{\mathbf{A},\beta}(\theta)=\sum_{\Delta\cap\theta\neq\emptyset} m_{\mathbf{A},\beta}(\Delta)=1-\sum_{\Delta\cap\theta=\emptyset,\Delta\subseteq\Theta-\theta} m_{\mathbf{A},\beta}(\Delta)=1-Bel_{\mathbf{A},\beta}(\Theta-\theta)$$

for any $\theta\subseteq\Theta$.

Now we can define a new β–decision attribute $\partial^{\beta}_{\mathbf{A}}:U\to 2^{\Theta_{\mathbf{A}}}$, approximating the decision d in a following way:

$$\partial^{\beta}_{\mathbf{A}}(x)=\begin{cases}\{\mathrm{i}\} & \text{for } x\in\underline{A}_{\beta}X_i\cup Bd_{\mathbf{A},\beta}(\{i\})\\ \{0\} & \text{for } x\in\bigcap_{i\in\{1,\ldots,r(d)\}} NEG_{\mathbf{A},\beta}X_i\\ \theta & \text{for } x\in Bd_{\mathbf{A},\beta}(\theta),\theta\subseteq\{1,\ldots,r(d)\},|\theta|>1\end{cases}$$

where $\Theta_{\mathbf{A}} = \{0\} \cup \{1, \ldots, r(d)\}$ and $x \in U$.

5 Examples: β–approximation regions

Example 1.

Les us consider an example of decision table with 21 objects, three condition attributes a, b, c and one decision attribute D – Table 1.

In Table 1 we have three decision classes:

X_1={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}, X_2={13, 14, 15, 16, 17},

X_3={18, 19, 20, 21},

and four equivalence classes:

$[1]_A = \{1, 2, 19, 20, 21\}$, $[4]_A = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$,

$[3]_A = \{3\}$, $[14]_A = \{14, 15, 16, 17, 18\}$.

Let us observe that, lower β–approximation for $\beta = 0.1$ is a set of the following form:

$$\underline{A}_\beta(X) = \left\{ y \in U : \frac{|[y]_A \cap X|}{|[y]_A|} \geq 0.9 \right\}.$$

We have $\Theta_{\mathbf{A}} = \{1, 2, 3\}$. Let us observe that:

$\underline{A}_\beta(X_1) = [3]_A \cup [4]_A$, $\underline{A}_\beta(X_2) = \emptyset$, $\underline{A}_\beta(X_3) = \emptyset$,
$\overline{A}_\beta(X_1) = [1]_A \cup [3]_A \cup [4]_A$, $\overline{A}_\beta(X_2) = [14]_A$, $\overline{A}_\beta(X_3) = [1]_A \cup [14]_A$,
$BN_{\mathbf{A},\beta}(X_1) = [1]_A$, $BN_{\mathbf{A},\beta}(X_2) = [14]_A$, $BN_{\mathbf{A},\beta}(X_3) = [1]_A \cup [14]_A$,
$NEG_{\mathbf{A},\beta}(X_1) = [14]_A$, $NEG_{\mathbf{A},\beta}(X_2) = [1]_A \cup [3]_A \cup [4]_A$,
$NEG_{\mathbf{A},\beta}(X_3) = [3]_A \cup [4]_A$, $Bd_{\mathbf{A},\beta}(\emptyset) = \emptyset$, $Bd_{\mathbf{A},\beta}(\{1\}) = \emptyset$,
$Bd_{\mathbf{A},\beta}(\{2\}) = \emptyset$, $Bd_{\mathbf{A},\beta}(\{3\}) = \emptyset$, $Bd_{\mathbf{A},\beta}(\{1,2\}) = \emptyset$,
$Bd_{\mathbf{A},\beta}(\{1,3\}) = [1]_A$, $Bd_{\mathbf{A},\beta}(\{2,3\}) = [14]_A$, $Bd_{\mathbf{A},\beta}(\{1,2,3\}) = \emptyset$.

From above equations it follows that the equality

$$\bigcup_{i \in \{1,\ldots,r(d)\}} \underline{A}_\beta(X_i) \cup \bigcup_{\theta \subseteq \Theta_{\mathbf{A}}} Bd_{\mathbf{A},\beta}(\theta) = U$$

holds (see Proposition 3.1).

Table 1
Example 1 – Decision table

U	a	b	c	D		U	a	b	c	D
1	1	0	0	1		12	0	1	1	1
2	1	0	0	1		13	0	1	1	2
3	1	1	1	1		14	1	1	0	2
4	0	1	1	1		15	1	1	0	2
5	0	1	1	1		16	1	1	0	2
6	0	1	1	1		17	1	1	0	2
7	0	1	1	1		18	1	1	0	3
8	0	1	1	1		19	1	0	0	3
9	0	1	1	1		20	1	0	0	3
10	0	1	1	1		21	1	0	0	3
11	0	1	1	1						

Table 2
Example 1 – Table after transformation

U	a	b	c	$\partial^{\beta=0.2}$
1,2,19,20,21	1	0	0	{1,3}
3	1	1	1	{1}
4,5,6,7,8,9,10,11,12,13	0	1	1	{1}
14,15,16,17,18	1	1	0	{2}

Let us take $\beta = 0.2$ and repeat our calculations:
$\underline{A}_\beta(X_1) = [3]_A \cup [4]_A$, $\underline{A}_\beta(X_2) = [14]_A$, $\underline{A}_\beta(X_3) = \emptyset$,
$\overline{A}_\beta(X_1) = [1]_A \cup [3]_A \cup [4]_A$, $\overline{A}_\beta(X_2) = [14]_A$, $\overline{A}_\beta(X_3) = [1]_A$,
$BN_{\mathbf{A},\beta}(X_1) = [1]_A$, $BN_{\mathbf{A},\beta}(X_2) = \emptyset$, $BN_{\mathbf{A},\beta}(X_3) = [1]_A$,
$NEG_{\mathbf{A},\beta}(X_1) = [14]_A$, $NEG_{\mathbf{A},\beta}(X_2) = [1]_A \cup [3]_A \cup [4]_A$,
$NEG_{\mathbf{A},\beta}(X_3) = [3]_A \cup [4]_A \cup [14]_A$, $Bd_{\mathbf{A},\beta}(\emptyset) = \emptyset$, $Bd_{\mathbf{A},\beta}(\{1\}) = \emptyset$,
$Bd_{\mathbf{A},\beta}(\{2\}) = \emptyset$, $Bd_{\mathbf{A},\beta}(\{3\}) = \emptyset$, $Bd_{\mathbf{A},\beta}(\{1,2\}) = \emptyset$,
$Bd_{\mathbf{A},\beta}(\{1,3\}) = [1]_A$, $Bd_{\mathbf{A},\beta}(\{2,3\}) = \emptyset$, $Bd_{\mathbf{A},\beta}(\{1,2,3\}) = \emptyset$.

We are looking for a new β-decision attribute.
Next transform Table 1 into Table 2 with this new attribute.

In our example we have $\Theta = \{\theta_0, \theta_1, \theta_2, \theta_3\}$. For all $\theta \subseteq \Theta$ we can present values of the basic numerical functions from evidence theory in Table 3.

Table 3
Example 1 – Basic numerical functions

θ	$\{\theta_1\}$	$\{\theta_2\}$	$\{\theta_3\}$	$\{\theta_1,\theta_2\}$	$\{\theta_2,\theta_3\}$	$\{\theta_1,\theta_3\}$	$\{\theta_1,\theta_2,\theta_3\}$	$\{\theta_0\}$
$\chi(\theta)$	{1}	{2}	{3}	{1,2}	{2,3}	{1,3}	{1,2,3}	{0}
$m_{\mathbf{A},\beta}(\theta)$	11/21	5/21	0	0	0	5/21	0	0
$Bel_{\mathbf{A},\beta}(\theta)$	11/21	5/21	0	16/21	5/21	16/21	1	0
$Pl_{\mathbf{A},\beta}(\theta)$	16/21	5/21	5/21	1	10/21	16/21	1	1

Table 4
Example 2 – Results

tab	β	certainty
1	0	0.72549
2	0.1	0.901961
3	0.2	0.901961
4	0.3	0.960784
5	0.4	0.960784

Example 2.

We have calculated the β–approximation regions for different β values for chosen decision table.

We consider a decision table with 51 objects, 7 condition attributes and one decision attribute with decisions 1, 2, 3. The table was without missing values and was consistent. First we calculated β–approximation regions for $\beta = \{0,\ 0.1,\ 0.2,\ 0.3,\ 0.4\}$. As the result we got five new decision tables with new decision attribute for any β. Next, we applied the Rosetta System for each table [3,6].

Initially we calculated dynamic reducts [1]. For these reducts we applied the Rosetta System for the rules generation and classification. The results are presented in the Table 4.

The coefficients of certainty are obtained from the confusion matrix which is computed during the classification.

A confusion matrix C is $V_d \times V_d$ matrix with integer entry $C(i,j)$ counts the number of objects that really belong to class i, but where classified as belonging to class j.

From confusion matrix presented in the Table 5, we are able to get an information that for $\beta = 0.3$ and $\beta = 0.4$ more than 96% of the objects from the decision table are properly classified. Observe that rules for $\beta = 0.4$ are more general.

Table 5
Example 2 – Confusion matrix

$\beta = 0.4$	1	2	3	
1	0	1	0	0.0
2	0	27	0	1.0
3	0	1	22	0.95652
		0.93103	1.0	0.96078

6 Dempster's combination rule

Let Θ be a frame of discernment compatible with two decision tables:

$$\mathbf{A_1} = (U_1, A_1 \cup \{d_1\}) \quad \text{and} \quad \mathbf{A_2} = (U_2, A_2 \cup \{d_2\}).$$

A decision table $\mathbf{A} = (U, A \cup \{d\})$ is called a *Θ–independent product of decision tables* $\mathbf{A_1}$ *and* $\mathbf{A_2}$ if the following properties hold [9]:

(i) $U = (U_1 \times U_2) \backslash (U_1 \otimes U_2)$,
where $U_1 \otimes U_2 = \left\{ (s_1, s_2) \in U_1 \times U_2 : \partial^{\beta}_{\mathbf{A_1}}(s_1) \cap \partial^{\beta}_{\mathbf{A_2}}(s_2) = \emptyset \right\}$,

(ii) If $(s_1, s_2) \in U_1 \times U_2$ then $\partial^{\beta}_{\mathbf{A_1}}(s_1) \cap \partial^{\beta}_{\mathbf{A_2}}(s_2) = d(s_1, s_2)$,

(iii) $\mathbf{A} = (\mathbf{A_1} \times \{1\}) \cup (\mathbf{A_2} \times \{2\})$,

(iv) If $(a, i) \in \mathbf{A}$ than for any $(s_1, s_2) \in U$ $(a, i)(s_1, s_2) = \begin{cases} a(s_1) & \text{for } i = 1 \\ a(s_2) & \text{for } i = 2. \end{cases}$

This Θ–independent product of decision tables is denoted by $\mathbf{A_1} \circ \mathbf{A_2}$.
The standard basic probability assignment for $\mathbf{A_1} \circ \mathbf{A_2}$ may be expressed in the following way: $m_{\mathbf{A_1} \circ \mathbf{A_2}, \beta} : 2^{2^{\Theta}} \to R_+$,

$$m_{\mathbf{A_1} \circ \mathbf{A_2}, \beta}(\Delta) = \begin{cases} 0 & \text{for } \Delta = \{\emptyset\} \\ 0 & \text{for } \Delta \subseteq 2^{\Theta}, |\Delta| > 1 \\ ? & \text{for } \Delta \subseteq 2^{\Theta}, |\Delta| = 1. \end{cases}$$

The next proposition explains, in a sense, the question mark in the above formula.

Theorem 6.1 *Let* $\mathbf{A_1} \circ \mathbf{A_2}$ *be Θ–independent product of decision tables. For any $\theta \subseteq \Theta$ the following equation, called Dempster's combination rule, holds:*

$$m_{\mathbf{A_1} \circ \mathbf{A_2}, \beta}(\theta) = \frac{\sum\limits_{\theta_1 \cap \theta_2 = \theta} m_{\mathbf{A_1}, \beta}(\theta_1) * m_{\mathbf{A_2}, \beta}(\theta_2)}{1 - \sum\limits_{\theta_1 \cap \theta_2 = \emptyset} m_{\mathbf{A_1}, \beta}(\theta_1) * m_{\mathbf{A_2}, \beta}(\theta_2)}$$

Example.

Let us consider two decision tables (Table 6) and $\beta = 0.4$.

Table 6
The decision tables – $\mathbf{A_1}$ and $\mathbf{A_2}$

U_1	A	B	D_1	$\partial^{\beta=0.4}$
1	0	1	1	{0}
2	0	1	2	{0}
3	0	1	3	{0}
4	1	1	1	{2}
5	1	1	2	{2}
6	1	1	2	{2}
7	1	0	1	{1,3}
8	1	0	3	{1,3}
9	0	0	2	{2}

U_2	C	E	D_2	$\partial^{\beta=0.4}$
10	0	1	1	{1}
11	0	1	1	{1}
12	0	1	2	{1}
13	0	1	3	{1}
14	1	1	1	{0}
15	1	1	2	{0}
16	1	1	3	{0}
17	0	0	2	{2}

Table 7
Basic probability assignment – $\mathbf{A_1}$ and $\mathbf{A_2}$

$\mathbf{A_1} = (U_1, \{a, b\} \cup \partial^{\beta=0.4})$

θ	{0}	{2}	{1,3}
$m_{\mathbf{A_1},\beta}(\theta)$	1/3	4/9	2/9

$\mathbf{A_2} = (U_2, \{c, e\} \cup \partial^{\beta=0.4})$

θ	{0}	{1}	{2}
$m_{\mathbf{A_2},\beta}(\theta)$	3/8	1/2	1/8

Instead of the two decision attributes D_1 and D_2 we put a new decision attribute. F or theabov etables we calculate the basic probability assignment. The decision table $\mathbf{A_1} \circ \mathbf{A_2}$ is presented in the T able8 .
For the abov etable the basic probability assignment is calculated.

One can observe that for θ={0} the Dempster's combination rule has the following form:

$$m_{\mathbf{A_1}\circ\mathbf{A_2},\beta}(\{0\}) = \frac{\sum\limits_{\theta_1\cap\theta_2=\{0\}} m_{\mathbf{A_1},\beta}(\theta_1) * m_{\mathbf{A_2},\beta}(\theta_2)}{1 - \sum\limits_{\theta_1\cap\theta_2=\emptyset} m_{\mathbf{A_1},\beta}(\theta_1) * m_{\mathbf{A_2},\beta}(\theta_2)}$$

Let us observe that:

$$\sum_{\theta_1\cap\theta_2=\emptyset} m_{\mathbf{A_1},\beta}(\theta_1) * m_{\mathbf{A_2},\beta}(\theta_2) =$$

$$= m_{\mathbf{A_1},\beta}(\{0\}) * m_{\mathbf{A_2},\beta}(\{1\}) + m_{\mathbf{A_1},\beta}(\{0\}) * m_{\mathbf{A_2},\beta}(\{2\}) +$$

$$+ m_{\mathbf{A_1},\beta}(\{2\}) * m_{\mathbf{A_2},\beta}(\{1\}) + m_{\mathbf{A_1},\beta}(\{2\}) * m_{\mathbf{A_2},\beta}(\{0\}) +$$

$$+ m_{\mathbf{A_1},\beta}(\{1,3\}) * m_{\mathbf{A_2},\beta}(\{0\}) + m_{\mathbf{A_1},\beta}(\{1,3\}) * m_{\mathbf{A_2},\beta}(\{2\}) =$$

$$= 1/3 * 1/2 + 1/3 * 1/8 + 2/9 + 1/6 + 1/12 + 1/36 = 17/24$$

Table 8
The decision table $\mathbf{A_1} \circ \mathbf{A_2}$

$U_1 o U_2$	A_1	B_1	C_2	E_2	$\partial^{\beta}_{A_1 o A_2}$
(1,14)	0	1	1	1	{0}
(1,15)	0	1	1	1	{0}
(1,16)	0	1	1	1	{0}
(2,14)	0	1	1	1	{0}
(2,15)	0	1	1	1	{0}
(2,16)	0	1	1	1	{0}
(3,14)	0	1	1	1	{0}
(3,15)	0	1	1	1	{0}
(3,16)	0	1	1	1	{0}
(4,17)	1	1	0	0	{2}
(5,17)	1	1	0	0	{2}

$U_1 o U_2$	A_1	B_1	C_2	E_2	$\partial^{\beta}_{A_1 o A_2}$
(6,17)	1	1	0	0	{2}
(7,10)	1	0	0	1	{1}
(7,11)	1	0	0	1	{1}
(7,12)	1	0	0	1	{1}
(7,13)	1	0	0	1	{1}
(8,10)	1	0	0	1	{1}
(8,11)	1	0	0	1	{1}
(8,12)	1	0	0	1	{1}
(8,13)	1	0	0	1	{1}
(9,17)	0	0	0	0	{2}

Table 9
Basic probability assignment – $\mathbf{A_1} \circ \mathbf{A_2}$

θ	{0}	{1}	{2}
$m_{\mathbf{A_1}\circ\mathbf{A_2},\beta}(\theta)$	9/21	4/21	8/21

and

$$\sum_{\theta_1 \cap \theta_2 = \{0\}} m_{\mathbf{A_1},\beta}(\theta_1) * m_{\mathbf{A_2},\beta}(\theta_2) = m_{\mathbf{A_1},\beta}(\{0\}) * m_{\mathbf{A_2},\beta}(\{0\}) = 1/8.$$

Finally we obtain

$$m_{\mathbf{A_1}\circ\mathbf{A_2},\beta}(\{0\}) = \frac{1/8}{1 - 17/24} = 3/7.$$

7 Conclusions

We presented that in the VPRS model the basic numerical functions from evidence theory can be defined. It gives us a method for inducing decision rules. Their quality can be tuned by means of β.

Moreover, the Dempster's combination rule for product of two decision tables is constructed. This entire approach is illustrated by examples.

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