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Structural Reliability Calculations Considering Concrete Tensile Membrane Action Using the Probability Density Evolution Method

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ABSTRACT: Based on experimental research on a real-scale one-way reinforced concrete slab with two spans for which the internal support was removed, a numerical model in ABAQUS was developed and validated. The model is subsequently used to investigate the uncertainty of the load bearing capacity of the damaged slab considering four key random variables. Further, the probability density evolution method (PDEM) is applied for the reliability assessment. Although PDEM has in general been applied to solve dynamic reliability problems, in this contribution this method was adapted and applied to a static problem. The PDF of the load bearing capacity is calculated considering the ultimate load bearing capacity of the slab in its damaged state, i.e. the load corresponding with the rupture of the reinforcement bars at the inner support due to the developed tensile membrane action (TMA). Taking into account the obtained PDF for the capacity of the damaged system, the reliability index of the damaged slab is assessed.

1. INTRODUCTION

The robustness of structural systems has received a wide-ranging attention after various structural failures, such as the progressive collapse of the apartment building resulting from a gas explosion at Ronan Point (UK) in 1968 and the collapse of the World Trade Centre resulting from terrorist attacks in New York in 2001 (Adam et al., 2018). The residual bearing capacity which is provided by an alternative load path is one of the measures to increase the structural robustness.

Small deformation theories, in which the resistance for bending and shear are the main design criteria, are traditionally applied to design reinforced concrete members. However, when an accidental situation occurs, e.g. characterized by the notional removal of a support column, large deformations need to be considered and the bending behavior is shifted towards membrane behavior. This membrane action can enable the development of an alternative load path and significantly increases the residual load-carrying capacity of reinforced concrete elements.

The membrane action, including tensile membrane action (TMA) and compressive membrane action (CMA), is obviously affected by parameters such as the span length, reinforcement properties and concrete properties (Botte et al., 2015). Hence, it is important to investigate how the structural reliability considering membrane action is affected by these parameters. Several techniques such as Monte-Carlo simulations

(MCS) have been proposed to perform such reliability analyses. But in case of complex systems these techniques are often extremely time-consuming or expensive in terms of computer resources due to the need of a large set of simulations. Alternative methodologies such as Latin Hypercube sampling (LHS) can be applied to effectively assess the mean value and standard deviation of the response, and to get rough information about the probability density function (PDF) of the response (Botte, 2017).

Aiming to obtain better insight into the full PDF while requiring only a limited number of simulations, the Probability Density Evolution Method (PDEM) has been presented in literature (Li & Chen, 2009) as an efficient method to capture the instantaneous probability density function and its evolution for high-dimensional linear and nonlinear stochastic systems. In this contribution, the application of the PDEM for evaluating the structural reliability of a RC slab considering tensile membrane action (TMA) was investigated.

2. TENSILE MEMBRANE ACTION IN REINFORCED CONCRETE SLAB

Based on the experimental research on a full-scale one-way reinforced concrete slab, a numerical model was developed in this section in order to validate and investigate the influence of different design parameters. The model is made in the FE software ABAQUS.

2.1. Experimental Investigation of Tensile Membrane Action

During the experiment three full-scale one-way slabs with originally 2 spans and a central support were exposed to the removal of the central support and loaded till failure (Gouverneur et al., 2013). For the considered slab in this contribution both the top and bottom longitudinal reinforcement are continuous over the entire length of the slab. In Figure 1 the test set-up is illustrated.

The total length and the width of the test specimen were 14.3 m and 1.8 m, respectively. The slab consisted of two inner spans of 4 m and two outer spans of 3.15 m. The concrete is of class C30/37 as defined in EN 1992-1-1 (2004). The flexural reinforcement included 16 bars of S500 with a nominal diameter of 10 mm for both top and bottom reinforcement. The concrete cover was 20 mm. The material properties of both concrete and steel were determined by testing.

A heavily reinforced edge beam (650 mm × 650 mm × 1300 mm) was built at each end of the slab. This edge beam was only used to restrict the inward movement of the slab edge; the outward movement was not restrained.

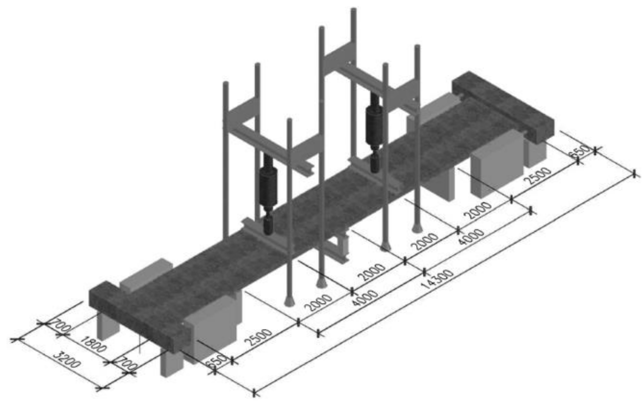


Figure 1: 3D model of the real-scale test set-up. (Gouverneur et al., 2013)

The experimental loading scheme of the slab was divided into three different loading phases (i.e. a service load of 60 kN was applied and removed in phase 1; the central support was removed in phase 2; and the line loads were increased until the collapse of the slab in phase 3). Only the third phase in which the line loads were increased until the collapse of the slab specimen is investigated in this contribution. As the second phase was the removal of the central support, the two inner spans of 4 m each were thus transformed into one span of 8 m. A detailed description of the test-up and corresponding results are presented in (Gouverneur et al., 2013).

2.2. Numerical Modelling of the RC Slab

A finite element model was created in this section to perform a stochastic analysis of the tensile membrane action (TMA) in the RC slab. In the present study, the finite element software ABAQUS was used. Only half of the specimen was modelled due to the symmetry of both its geometry and loading. Both geometrical nonlinearity and material nonlinearity were taken into account.

Plane stress elements (CPS8R) were used to model the concrete elements. And a relatively dense mesh was applied (8 elements though the slab depth). A Hordijk tension softening model was used (Hordijk, 1991) and a bi-linear stress-strain relationship was implemented for concrete in compression, see Figure 2. This simplified material model of concrete compression was found to be suitable since no concrete crushing was observed in the experiment. In addition, most parameters are related to the concrete compressive strength f_c. The Poisson ratio is taken as 0.15. It also should be noted that the concrete behavior of tension-stiffening is also included in the numerical model.

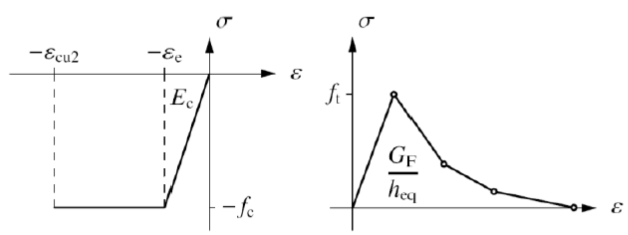


Figure 2: The stress-strain relationship: Concrete in compression (left); concrete in tension (right). (Hendriks et al., 2016)

For reinforcement, one dimensional fully embedded bar reinforcement (truss elements) were applied, considering perfect bond between the reinforcement and the neighboring concrete elements. A multi-linear stress-strain relationship in accordance with laboratory testing on reference specimens was applied for the reinforcement. i.e. the strain hardening of steel is considered as an elastic-plastic model which explicitly includes rupture of the reinforcement, and the ultimate strain ε_u is taken as 8.31 % (ductility Class C according to EN 1992-1-1). The sudden decrease in strength at rupture of the reinforcement bars was taken into account to consider the observed failure phenomenon of the slabs. In addition, the Poisson ratio of the reinforcing steel is taken as 0.30.

Lateral displacements were prohibited at the central support for the symmetry of the model. Vertical displacements were imposed to simulate a transverse line load on the slab.

In accordance with the real test set-up, the outward movements of the edge beams were allowed. However, the inward movements were restrained by transferring the occurring horizontal forces to spring elements (representing the anchor blocks). Therefore, two connector elements were employed to simulate springs that translate the relationship between the occurring horizontal forces and the corresponding horizontal displacement. This relationship was based on the measured load-displacement characteristic from the test. A mean value of 151.5 kN/mm was applied for each spring.

Table 1: Material properties for reinforcement and concrete. (Botte, 2017)

Reinf.	f_y (MPa)	f_t (MPa)	ε_y (%)	ε_u (%)	E_s (GPa)
	555	605	0.267	8.31	208.0
Conc.	$E_{c,c}$ (MPa)	f_c (MPa)	E_{ci} (GPa)		
	44.7	36.2	32.0		

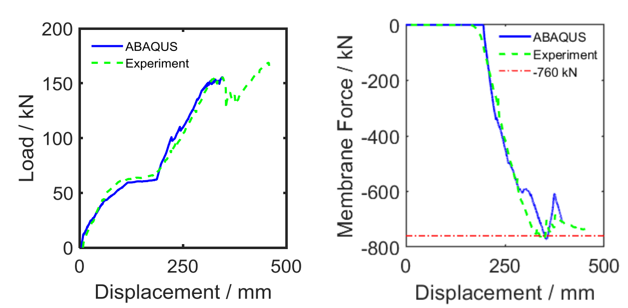


Figure 3: Comparison of numerical simulation and experimental result: Load – Displacement under load (left); Membrane force – Displacement under load (right).

The modelling is concerned with the third phase of the test. Initially, self-weight was applied. The obtained stresses and strains were then transmitted to the next phase. And a displacement controlled load was applied until the rupture of reinforcement bars over the support. Values of all the model parameters were taken in accordance to the measured values obtained from material testing, see Table 1.

Figure 3 illustrates the load-displacement curves (displacement under load) of both the numerical simulations (ABAQUS model) and experimental results. Comparing the laboratory tests and the numerical simulations, both the load-displacement curve and membrane force-displacement curve show good agreement with the laboratory tests. The experiment showed that tensile membrane actions, i.e. after 200 mm (see Figure 3 left), can significantly increase the residual load capacity in accidental situations, e.g. removal of the central support, when very large displacements occur. The rupture of the top reinforcement bars over the inner supports can be identified with a suddenly decrease of forces, it is easy to be found at the top of the numerical curve in Figure 3 left. The membrane forces, 775.5 kN and 772.1 kN for experimental result and numerical simulation respectively, see Figure 3 right, are both slightly larger than the theoretical ultimate tensile capacity of one reinforcement layer (total 16 bars), i.e. 760 kN, considering an ultimate tensile capacity $f_t = 605$ MPa. In addition, the rupture of the top reinforcement leads to a significant peak in the membrane force – displacement curve.

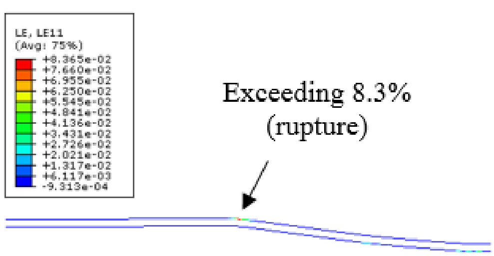


Figure 4: The strains of the reinforcement.

Figure 4 shows the strains of the reinforcement after rupture of the top reinforcement over the inner support (the strain at the maximum force is 8.3 %).

Comparing with the experimental results, the effective of the FE model is validated. The established model of the RC slab is used in the following to investigate the influence of the parameter uncertainties on the maximum generated tensile membrane force, i.e. the load bearing capacity of the damaged slab.

3. PROBABILITY DENSITY EVOLUTION METHOD (PDEM) BASED RELIABILITY ASSESSMENT

Considering the Probability Density Evolution Method (Li & Chen, 2009), the load-displacement relationship of the stochastic slab system can be denoted as

$$\mathbf{R} = \mathbf{H}(\boldsymbol{\Theta}, \delta) \tag{1}$$

where, $\mathbf{R}$ is the applied vertical load. δ is the vertical deformation under the applied load. $\boldsymbol{\Theta}$ is a random vector including all random parameters involved with a known joint PDF $p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$.

As the calculation is displacement controlled, the slab may exhibit its maximum capacity at a displacement smaller than the ultimate displacement. Therefore, the equivalent extreme value W of $\mathbf{R}$ is of interest. This can be denoted as:

$$W(\boldsymbol{\Theta}) = \max|\mathbf{R}(\boldsymbol{\Theta})| \tag{2}$$

the PDF of the equivalent extreme values defined by Eq. (2) can be obtained following the recommendations by (Li & Chen, 2007), i.e. by introducing the concept of a virtual stochastic process. This implies that a virtual stochastic process is constructed as

$$Z(\tau) = \psi(W(\boldsymbol{\Theta}), \tau) = \phi(\boldsymbol{\Theta}, \tau) \tag{3}$$

which must satisfy the conditions

$$Z(0) = 0, Z(\tau)|_{\tau=\tau_c} = W(\boldsymbol{\Theta}) \tag{4}$$

where τ_c is a chosen value.

Differentiating Eq. (3) with respect to τ yields Eq. (5) which is in form similar to the generalized density evolution equation (GDEE) (Li & Chen, 2008).

$$\dot{Z}(\tau) = \frac{\partial \phi(\boldsymbol{\Theta}, \tau)}{\partial \tau} = \dot{\phi}(\boldsymbol{\Theta}, \tau) \tag{5}$$

therefore, the GDEE can be employed to obtain the PDF of $Z(\tau)$

$$\frac{\partial p_{\boldsymbol{Z\Theta}}(z,\boldsymbol{\theta},\tau)}{\partial \tau} + \dot{\phi}(\boldsymbol{\theta},\tau)\frac{\partial p_{\boldsymbol{Z\Theta}}(z,\boldsymbol{\theta},\tau)}{\partial z} = 0 \quad (6)$$

$$p_{\boldsymbol{Z}}(z,\tau) = \int_{\Omega_{\boldsymbol{\Theta}}} p_{\boldsymbol{Z\Theta}}(z,\boldsymbol{\theta},\tau)\, d\boldsymbol{\theta} \quad (7)$$

and the PDF of W is then obtained as

$$p_W(w) = p_{\boldsymbol{Z}}(z,\tau)\,|_{\tau=\tau_c} \quad (8)$$

Considering Eq. (8) and a specified safe domain Ω_s for the resistance effect, the reliability of the damaged system can be evaluated by Eq. (9). In such situations, the reliability evaluation becomes a problem of one-dimensional integration of the extreme value distribution. When also the load effect is stochastic, the convolution of the resistance effect W and the stochastic load effect should be considered.

$$P_s = \mathrm{P}\{W(\boldsymbol{\Theta}) \in \Omega_s\} = \int_{\Omega_s} p_W(w)dw \quad (9)$$

4. RELIABILITY QUANTIFICATION OF THE SLAB CONSIDERING TENSILE MEMBRANE ACTION

4.1. Probabilistic models for the most important variables

Four key stochastic variables were selected, for which their uncertainty is considered explicitly when determining the ultimate load bearing capacity considering tensile membrane action of the damaged slab system. These key variables are based on the research by Botte (2017). These key variables are the concrete compressive strength f_c, the reinforcement yield stress f_y, the reinforcement tensile strength f_t and the strain ε_u at the maximum load. Hence, a total of four key random variables $\boldsymbol{\theta} = (\theta_{f_c}, \theta_{f_y}, \theta_{f_t}, \theta_{\varepsilon_u})$ were considered in this analysis. It should be noted that most of the properties of concrete are derived from the concrete compressive strength (Hendriks et al., 2016).

Based on the experimental investigations as well as suggestions in the Probabilistic Model Code from the Joint Committee for Structural Safety (JCSS, 2001), a set of probabilistic models were employed for these key variables (see Table 2). It should be noted that the reinforcement yield stress f_y is modelled as perfectly correlated to the reinforcement tensile strength f_t.

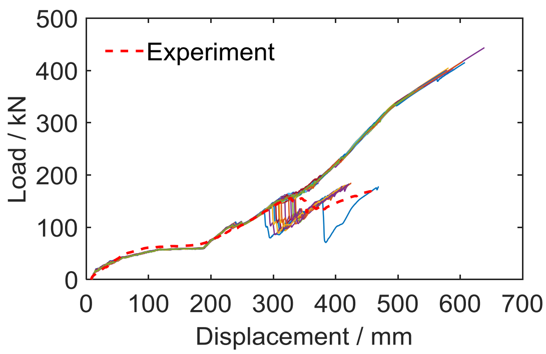
To solve the generalized density evolution equation (GDEE) of Eq. (6) numerically, a sample of model realizations is calculated first, each corresponding with a specific set of values of the parameters $\boldsymbol{\theta} = \left(\theta_{f_c}, \theta_{f_y}, \theta_{f_t}, \theta_{\varepsilon_u}\right)$. The Generalized F-discrepancy (GF-discrepancy) based optimal point selection strategy (Chen et al., 2016) was used. Based on the suggested probabilistic models, 47 sample sets and corresponding assigned probabilities were generated. Then every deterministic analysis was carried out with the described FE Model of the damaged RC slab. Further, the generalized density evolution equation was numerically solved to obtain the distribution of the maximum load bearing capacity, as described in Section 3 above.

Table 2: Probabilistic models for the most important variables involved in the stochastic analysis of the tensile membrane action.

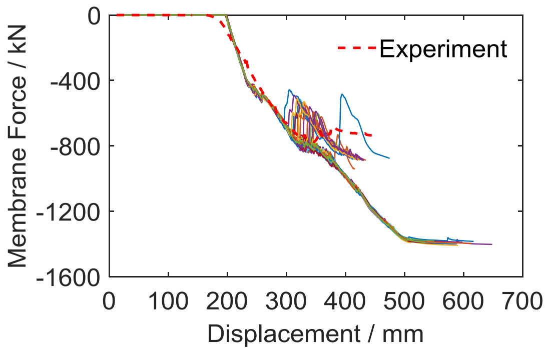
Property	*Distribution*	*μ*	*COV*
f_c	*LN*	*36.2 MPa*	*0.10*
f_y	*LN*	*555 MPa*	*0.03*
f_t	*LN*	*605 MPa*	*0.03*
ε_u	*LN*	*8.3 %*	*0.15*

4.2. Results of the stochastic analysis

The load-displacement results of the set of random FEM simulations are presented in Figure 5. In addition, the experimental results are also presented for comparison. Figure 5(a) shows the stochastic responses of the load against the displacements under load. At the elastic and the plastic stages the uncertainty on the response is small. However, the load-displacement diagram varies significantly in the tensile membrane stage. For several simulations, rupture of the top layer reinforcement bars is obtained at lower displacements than the experimental results. i.e. the rupture of the reinforcement layer establishes at lower displacements than the experimental curve. Furthermore, it should be noted that one curve increases again even after the second rupture of reinforcement, due to the fact that the latter rupture occurred under the load instead of at the support.



(a)



(b)

Figure 5: The load-displacement relationship (a); membrane force-displacement relationship (b). (Model realizations in full lines and experimental result in dashed lines)

Figure 5(b) shows the stochastic responses of the membrane forces against the displacements under load. Significant variability is observed for the responses of membrane forces. While 760 kN is the theoretical ultimate tensile capacity of one reinforcement layer (16 bars) at the inner support, considering an ultimate tensile capacity f_t = 605 MPa, the ultimate tensile capacities of some samples were lower than 760 kN when the rupture of top reinforcement bars at the inner support.

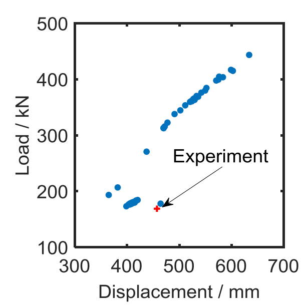
4.3. Reliability assessment

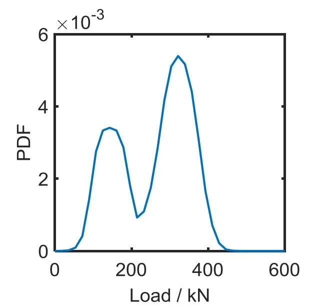
According to the simulated responses as illustrated in Figure 5, the ultimate loads were obtained corresponding to the rupture of the reinforcement bars at the inner support, see Figure 6 (a). And it should be noted that the TMA significantly increases the capacity of the slab. Most of these ultimate loads had values between 100 kN and 450 kN. These observed ultimate loads were regarded as extreme values to apply the virtual stochastic process and the PDEM as described in Section 3. Considering Eqs. (3) and (4), the virtual stochastic process is chosen as

$$Z(\tau) = \psi(W(\mathbf{\Theta}, T), x) = W(\mathbf{\Theta}, x)\sin(w\tau) \quad (10)$$

where $w = 2.5\pi$, $\tau_c = 1$. This satisfies the requirements in Eq. (4).

As one virtual stochastic process was introduced, the PDF of the extreme values of the ultimate loads was constructed by solving the PDEM. The PDF is shown in Figure 6 (b). The PDF is quite irregular and has two peaks. It varies between 42.8 kN (0.1 percentile) and 450.0 kN (99.9 percentile).





(a) (b)

Figure 6: The ultimate load-displacement relationship corresponding to the rupture of reinforcement bars (a); PDF of the ultimate load at the rupture of reinforcement bars (b).

In the following Eq. (11) is considered as the limit state equation,

$$Z = R - E = 0 \quad (11)$$

where R is the resistance with known PDF, see Figure 6 (b). As the original slab was designed considering a design value of 90 kN, which was modelled as a Gumbel distribution with mean value 45.7 kN and a COV of 0.25 by Botte (2017), the load effect E is modelled as the same Gumbel distribution. Further, Monte Carlo simulations are applied to assess the structural reliability.

Considering the limit state defined by Eq. (11), the failure probability is approximately 10^{-3}. This corresponds approximately to a reliability index of 3.09. This results corresponds well to the reliability index of 3.04 reported in (Botte, 2017) for the damaged state, confirming the good performance of the method.

5. CONCLUSIONS

According to the test of a real-scale one-way reinforced concrete slab with two spans for which the central support was removed, a numerical model was developed, validated and subsequently used to investigate the uncertainty of the membrane force considering four key random variables. The probability density evolution method (PDEM) was employed to perform the uncertainty assessment of the resistance.

Comparing the numerical simulations with the experimental results, both the predicted reaction forces and membrane forces agree well with the experimental values. The tensile membrane action (TMA) can really increase the load capacity of the slab. According to the results of the stochastic analysis, only minor random variability was observed at the elastic stage. However, both the loads and membrane forces exhibit a significantly uncertainty at the tensile membrane stage. For several simulations, rupture of the top layer reinforcement bars occurred much earlier than observed in the experimental results. Therefore, it is important and essential to consider the uncertainty when analyzing the tensile membrane action.

Considering both the ultimate load bearing capacity, i.e. the load for which the reinforcement bars were ruptured over the inner support, resulting from the PDEM calculation and a Gumbel distribution of the load effect, the failure probability was assessed as approximately 10^{-3}, corresponding with a reliability index of approximately 3.09. This value corresponds very well to the result obtained in the work of (Botte, 2017).

6. ACKNOWLEDGEMENTS

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