Impulsive Control Realization Applied to Non-Dissipative First Order Plants: An Academic Electronic Implementation

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Abstract: - Impulsive control design is a control strategy mainly based on the impulsive differential model of the plant to be stabilized. Hence, this system should have at least one state variable able to be driven by impulsive commands. On the other hand, analog electronic realization of an impulsive control algorithm may be an important competition for engineering students. Therefore, the main objective of this paper is to pose an impulsive control method to non-dissipative first order systems along with an electronic circuit design to validate a contribution on impulsive control aim. According to our experimental results, our electronic architecture to reproduce an impulsive control law results effective to accomplished asymptotic stability of the closed-loop system in the Lyapunov's sense.

Key-Words: - Impulsive control, Electronic implementation, First order systems, Lyapunov stability.

1 Introduction

Impulsive control is an efficient control methodology to systems whose state variables (at least one) can be changed instantaneously for a control target, such as the population control of some louses, intake supply control to chemical reactors, impulse investment strategies in financial systems, vibration control, synchronization of chaotic systems, and so on [1-4]. Even more, in some systems, impulsive control provides a control solution that cannot endure by continuous control design [5]. Additionally, and from the mathematical point of view, impulsive differential equations, the plant modelling with impulsive actions, is richer to analyze than the standard ordinary differential equations [6].

On the other hand, analog electronic realization of a control algorithm is an important maturation for control engineering students, for instance, see [7-9]. In the case of impulsive control implementation, the challenger is mayor because of the impulsive actions, if implemented as control commands instead of system resetting-times programming, requires a sample realization of the *Dirac* delta generalized function [10] to electronic capture these effects. Hence, the main objective of this work is to present an impulsive control design to non-dissipative first order systems along with an electronic realization of the obtained impulsive controller. According to our experimental results, our approach satisfies our objective inquires.

On the other hand, and from the educational point of view, the development of cheap experimental platforms to test new control laws is an important academic challenger [11-14]. Additionally, by using low cost digital electronics, new control algorithms can be also essayed [15]. Finally, impulsive control can effectively attenuated an external disturbance [4,16-17].

The rest of this presentation is outlined as follows. Section 2 presents the impulsive control design applied to our previous cited system, whereas the electronic realization of our impulsive control algorithm is described in Section 3. Finally, the conclusions are summarized in Section 4.

2 Impulsive Control Design

Basically, closed-loop impulsive systems can be composed by three main elements:

- A *continuous* set of differential equations governing the dynamical motion of the closed-loop system between impulsive events.

- A *difference equation* scheme manipulating the closed-loop system state transitions at the impulsive moments.

- A *rule* to determine the impulsive time-scheduling.

Mathematically, the above items may be captured by using the following linear model:

$$\dot{x} = Ax, t \neq \tau_k \quad (1)$$
$$\Delta x|_{t=\tau_k} = Kx, k = 1,2,3,\cdots, t = \tau_k \quad (2)$$
$$x(t_0^+) = x_0, t_0 \geq 0 \quad (3)$$

where $x \in R^n$ is the state variable, and the impulsive control instants $\{\tau_k\}$ satisfying:

$$0 < \tau_1 < \tau_2 < \cdots < \tau_k < \cdots, \tau_k \to \infty \text{ as } k \to \infty.$$

Additionally, $A \in R^{n\times n}$ is a constant matrix and the matrix K (of appropriate dimensions) is the gain for the impulsive control events. To note, the over impulsive system is governed by ordinary differential equation stated in (1) between the impulsive actions ($t \neq \tau_k$). On the other hand, the state variable is instantaneity changed from $x(\tau_k^-)$ to $x(\tau_k^+) = x(\tau_k^-) + \Delta x|_{t=\tau_k}$. If $A = 0$, then the system is *non-dissipative* between impulsive commands but may be stabilizable by an appropriate control impulses.

Now consider the next first-order system:

$$m\dot{v} = u \quad (4)$$

where m can represent a mass for a linear system for velocity control, and u is the impulsive control input. Then, the impulsive control objective consists in finding a control law given by:

$$u \coloneqq u(t) = K \textstyle\sum_k e(t)\delta(t - \tau_k) \quad (5)$$

and

$$e(t) = v(t) - v_{ref} \quad (6)$$

such that

$$\lim_{t\to\infty} e(t) = 0 \quad (7)$$

Finally, v_{ref} is the constant set-point given by the user, and $\delta(\cdot)$ is the Dirac delta generalized function. Mathematically, the closed-loop system (4)-(6) can be stated as:

$$m\dot{e} = 0, t \neq \tau_k \quad (8)$$
$$\Delta e|_{t=\tau_k} = Ke, t = \tau_k, k = 1,2,3,\cdots \quad (9)$$

Then, by conceiving the next quadratic Lyapunov function:

$$V(e)(t) = V(e) = \tfrac{1}{2}e^2 \quad (10)$$

we can obtain, between impulsive events ($t \neq \tau_k$), that:

$$\dot{V} = e\dot{e} = 0 \quad (11)$$

and at the impulsive times ($t = \tau_k$), we have

$$V(e + Ke) = \tfrac{1}{2}(e + Ke)^2 = (1 + K)^2V(e) \quad (12)$$

Therefore, in order to satisfy our control objective, we require that: $-2 < K < 0$. Summarizing, we have the following result.

Theorem 1.- *The closed-loop system (4)-(6) is Lyapunov asymptotically stable if* $-2 < K < 0$.

Remark 1.- *Theorem 1 states a sufficient condition for asymptotic stability of our closed-loop system (4)-(6) if there exist impulsive instants satisfying (12) with* $-2 < K < 0$. *Hence, the time-impulsive scheduling events is open, and, obviously, the convergence-time will depend on this time-scheduling.*

3 Electronic Realization and Experimental results

This section describes the analog electronic circuit realization of our control impulse command stated in (5) with K satisfying the condition given in Theorem 1. Due the Dirac delta generalized function is unrealistic for implementation, we require a sample of it. According to [10], a sample of this Dirac delta expression can be any square signal where its area under its form should be equal to one. See Figure 1. This square signal is obtainable by using, for instance, the timer IC-NE555. See Figure 2. On this way, we are realizing a periodic Dirac delta function such that $\tau_{k+1} - \tau_k = T$. Therefore, we require $V_{cc} \times w \approx 1$.

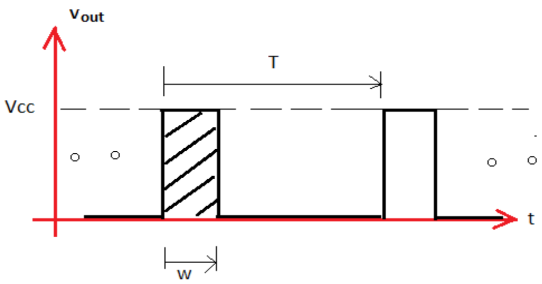

Fig. 1 Periodic sample Dirac-delta wave generator.

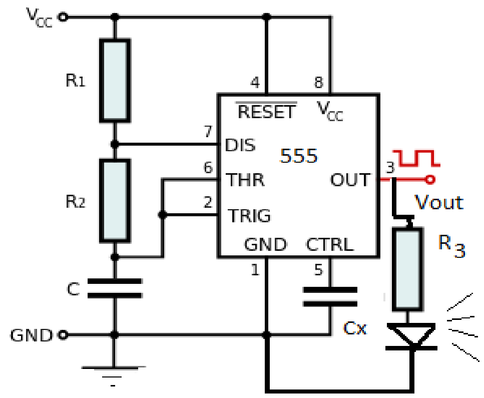

Fig. 2 Schematic representation of the timer NE555 operating in astable operation. We use: $R_1 = 2.2K\Omega$, $R_2 = 2.0K\Omega$, $R_3 = 1K\Omega$, $C = 470\mu F$, $C_x = 0.1\mu F$, and $V_{cc} = 9V$.

The circuit to realize the error signal $V_{o_1} = K\sum_k e(t)\,\delta(t-\tau_k)$ is depicted in Figure 3. In this figure, a relay device is also drawn where its input signal comes from the circuit given in Figure 2. Hence, the relay and the operational amplifier output is realizing the control signal stated in (5). It is important to note that our relay is employing the Normally Connected (NC) pin option. Furthermore, in V_2, we introduce the v_{ref} by using a function generator. Finally, in V_1 we enter the state variable $v(t)$ from our plant. We modelled our plant by employing analog electronic too. See Figure 4. This corresponds to an integrator plant model:

$$\dot{v} = u,$$

where $V_{o_2} = v$ and $V_{in} = V_{o_1} (= u)$. The corresponding experimental results are shown in Figures 5 and 6.

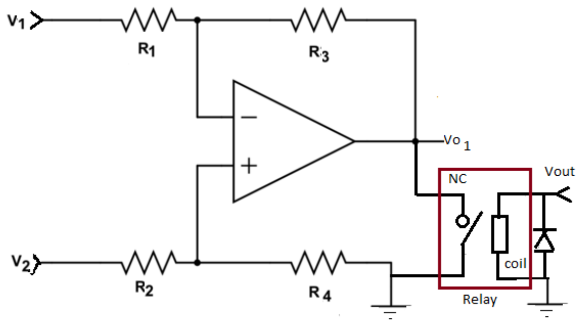

Fig. 3 Electronic circuit to realize our impulsive control. We set: $R_1 = R_2 = 2K\Omega$, $R_3 = R_4 = 1.8K\Omega$, and the supply voltage to the operational amplifier (LM741) is $\pm 15V$.

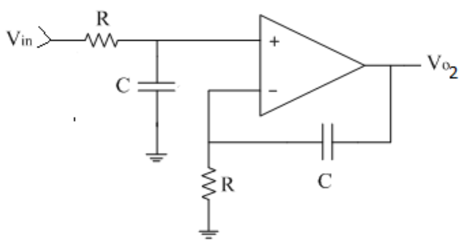

Fig. 4 Integrator plant system, where $R = 1M\Omega$, and $C = 1\mu F$. The operational amplifier (LM741) is supply by a $\pm 15V$ power supply.

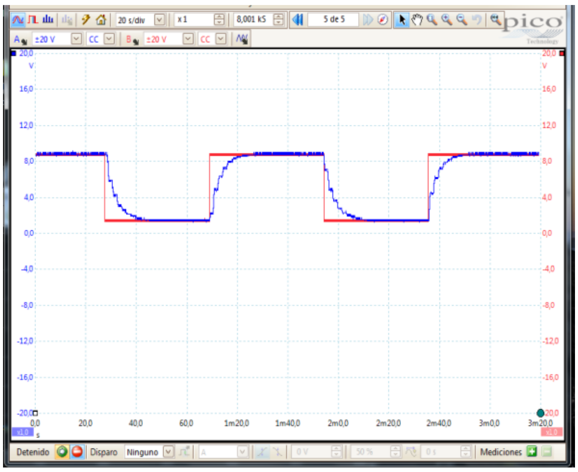

Fig. 5 Experimental results. In red is the reference command, or set point, generated by a generator function device. In blue is the plant's output.

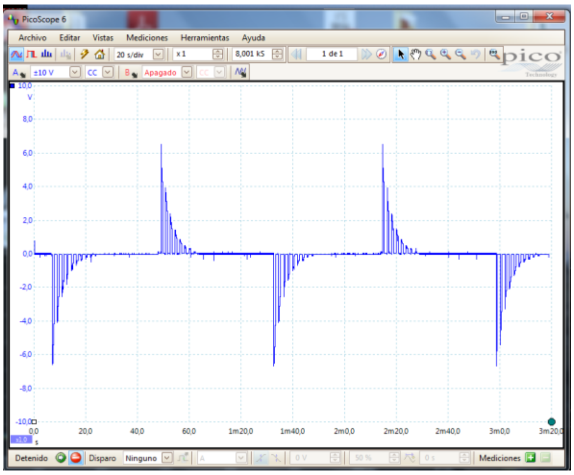

Fig. 6 Experimental result. The generated impulsive control signal.

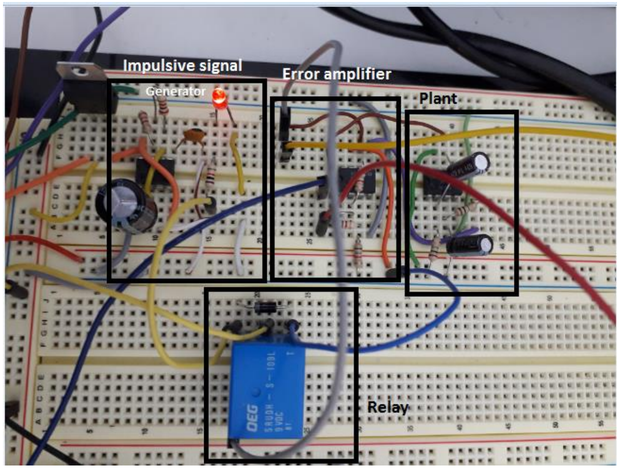

Fig. 7 A photo of our experimental platform.

On the other hand, Figure 7 shows a picture of our experimental platform. Finally, our experimental data were captured by using the digital PicoScope 2000 series device.

4 Conclusion

From the educational point of view, to provide low cost experimental platforms results effective to teach important engineering concepts. See, for instance [11-14]. At this point, our platform is affordable to teach impulsive control along with its important electronic theory on real realization of impulsive systems. In fact, the control law stated, for instance in [16], may be replaced by our control strategy. This is left as a future work.

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