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PAPER

Multi-path interferometry using single photons

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Abstract

In most experiments which search for violations of Born's rule using light the diffractive elements are formed from material slits, or waveguides, which are treated classically. In this article we propose an alternative approach where the internal energy levels of particles are used instead of slits to test the superposition principle, thus removing the effect of the finite width of the slits. By quantising both the light field and the diffraction elements we propose a new way to probe Born's rule for light in a fully quantum mechanical way.

1. Introduction

Non-relativistic quantum theory uses the Schrödinger equation,

$$i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t} = \hat{H}\Psi(\vec{r}, t), \tag{1}$$

to describe the dynamics of quantum systems. Here, $\hbar$ is the reduced Planck constant, $\vec{r}$ is a position vector, t is time, and $\hat{H}$ and Ψ are the Hamiltonian and the wavefunction of the system concerned. Despite its prevalence in modern physics the philosophical interpretation of the wavefunction is something still debated [1, 2]. During the early days of quantum theory, while considering the scattering of electrons, Born realised that the modulus-squared value of solutions to the Schrödinger equation must always be a positive, real and normalisable quantity. He asserted that Ψ represents the probability amplitude to measure a quantum particle to be at a given potision [3], assigning physical meaning to Ψ, such that the probability density for an event to occur is described by the relation,

$$P(\vec{r}, t) = |\Psi(\vec{r}, t)|^2. \tag{2}$$

Commonly referred to as Born's rule, equation (2) is now firmly embedded within the modern formalism of quantum mechanics, and it can be shown to arise quite naturally from the unitary, linear structure of the dynamics of the Schrödinger equation [4–6]. Despite this fundamental role within quantum theory, there are relatively few experiments which seek to directly verify this relation [7–13]. This in itself is reason enough to stimulate further experimental investigations. Additionally, the difficulty in reconciling quantum mechanics and general relativity raises the question of whether our dynamical description of quantum particles is incomplete.

A direct consequence of Born's rule, equation (2), is that the probability of an event occurring can always be decomposed into a linear combination of terms which contain information only relating to at most two-paths — the super position principle. One can therefore probe the validity of Born's rule by looking for higher-order interference terms, or contributions to the interference pattern beyond Born's predictions and conventional quantum theory. A relativly simple way to search for multi-path interference is to look for deviations from the classical additivity of probabilities for mutually exclusive events where only three paths are available [14]. Experiments using light have already looked at the diffraction of laser beams and single-photons at a material mask [7] or using optical waveguides [12]. These methods rely on the classical diffraction of light—although the light itself is quantised the diffractive elements are not. This can have subtle consequences which can effect the interpretation of experiments [15–17].

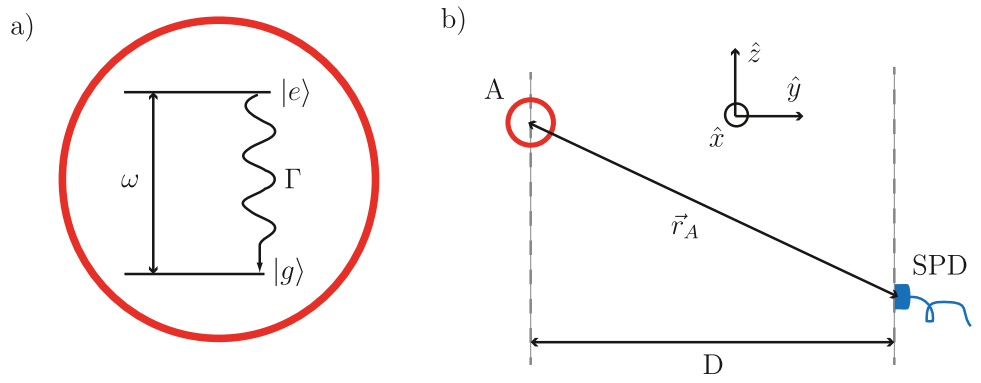

Figure 1. (a) A particle consisting of two internal energy levels $|g\rangle$ and $|e\rangle$ separated in energy by $E = \hbar\omega$. The excited state $|e\rangle$ decays to the ground state $|g\rangle$ through spontaneous emission at a rate of Γ. (b) A single, two-level particle is positioned a distance $\vec{r}_A$ from a single photon detector (SPD) which can detect photons scattered by particle A.

Phenomenological attempts to include gravity within a quantum theory typically introduce non-linearities [18] and alternatives to the Schrödinger equation can result in modifications to Born's rule [19, 20]. It therefore seems natural that experiments sensitive to violations of Born's rule can be used to search for physics beyond existing quantum theory. Non-linear extensions to quantum mechanics can be physically unrealistic, introducing problems ranging from entanglement from nothing and super luminal signaling [21] to the cloning of quantum states and polynomial-in-time operations for exponentially hard tasks [22]. However, as no established model for quantum gravity currently exists, and given the problems associated with many of the existing extensions, it remains interesting to consider how to probe quantum dynamics beyond those of the Schrödinger equation without assuming a particular functional form. Recently, tests of multi-path interference have been extended to massive particles [10]. The effects of gravity are more pronounced for massive particles, compared with photons, and therefore any gravitational modification to quantum dynamics may be significantly larger—although how to make a quantised grating for massive particles remains a challenge. An experiment using microwave photons [23], in a classical regime, has recently application of the superposition principle breaks down in a way consistent with classical electrodynamics when boundary conditions are included.

In this article we consider what happens when the internal states of spatially separated single particles are used instead of slits in an interference experiment. By quantising both the field and the diffraction element we propose a way to probe multi-path interference and therefore test Born's rule in a fully quantum mechanical framework.

2. Detecting photons after absorption and re-emission by a quantised particle

2.1. A single quantum absorber-emitter

Consider a single particle, A, with two internal energy levels $|g\rangle$ and $|e\rangle$ which represent the ground and excited states, respectively. This is shown schematically in figure 1(a). In practice this particle could be a trapped ion [24] or neutral atom [25–27], a molecule [28, 29] or quantum dot [30] on the surface of a substrate or an NV-centre in a diamond matrix [31].

Let us assume that particle A is initially prepared in $|g\rangle$. Illumination with a single photon $|\gamma_0\rangle$ results in the state,

$$|\phi\rangle = \alpha|e\rangle|0\rangle + \beta|g\rangle|\gamma_0\rangle, \tag{3}$$

where $|\alpha|^2$ and $|\beta|^2$ are the probabilities for the particle to absorb and not to absorb the photon respectively and $|0\rangle$ represents the vacuum state. Once excited A can spontaneously decay to the ground state at a rate Γ, emitting a photon as it does so. The re-emitted photon can be described by the state vector,

$$\begin{aligned}|\gamma\rangle &= \sum_{\vec{k}} \frac{g_k e^{i\vec{k}.\vec{r}}}{(\omega_{eg} - \omega_\gamma) + i\Gamma/2}|1_{\vec{k}}\rangle \\ &= E_0 \sum_{\vec{k}} e^{i\vec{k}.\vec{r}}|1_{\vec{k}}\rangle.\end{aligned} \tag{4}$$

Here, g_k is the particle-photon coupling constant, ω_{eg} is the frequency separation between the states $|e\rangle$ and $|g\rangle$, and $|1_{\mathbf{k}}\rangle$ is the state vector representing a single mode of electromagnetic radiation with wavenumber k. For

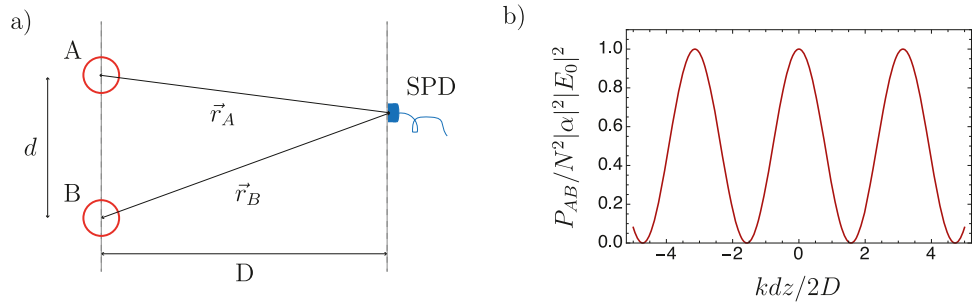

Figure 2. (a) Two identical particles, each containing two internal states, are separated from one another by a distance d. After illumination with a single photon $|\gamma_0\rangle$ one of the particles absorbs and eventually re-emits the photon which can then be measured using a single photon detector (SPD). (b) If there is no way if telling whether A or B absorbed and then scattered the photon then a superposition of different single photon modes are populated. This modifies the correlation function $G^{(1)}$, introducing a strong cosine dependence to the probability of detecting a photon at a given position z.

simplicity, the prefactors in $|\gamma\rangle$ can be gathered into a single coefficient E_0, which represents the electric field amplitude of the photon. We will also assume that the repetition rate of single photons is low compared with the decay lifetime of the excited state such that stimulated emission can be neglected.

Figure 1(b) shows an ideal single-photon-detector (SPD) positioned a distance D away from particle A such that it can only detect photons which have been absorbed and then re-emitted through spontaneous emission. By translating the SPD and recording the number of detection events at different positions, the first order correlation function,

$$G^{(1)}(\vec{r},\, t;\, \vec{r}'t') = \langle \psi(t \to \infty)|\, E^{(-)}(\vec{r},\, t)E^{(+)}(\vec{r},\, t)|\psi(t \to \infty)\rangle, \tag{5}$$

can be measured. For brevity we shall write $G^{(1)}(\vec{r},\, t;\, \vec{r}'t') = G^{(1)}$ from here on.

Let us assign $|\psi(t \to \infty)\rangle$ as the state of our particle after it has decayed and write the electric field of the photon in terms of its positive and negative field operators,

$$\begin{aligned} E^{(+)}(\vec{r},\, t) &= \sum_{\vec{k}} \epsilon_{\vec{k}} \hat{a}_{\vec{k}} e^{i(\vec{k}.\vec{r}-\omega_\gamma t)}, \\ E^{(-)}(\vec{r},\, t) &= \sum_{\vec{k}} \epsilon_{\vec{k}} \hat{a}^\dagger_{\vec{k}} e^{-i(\vec{k}.\vec{r}-\omega_\gamma t)}. \end{aligned} \tag{6}$$

Here, $\epsilon_{\vec{k}}$ is the polarisation vector of our field and $\hat{a}_{\vec{k}}$ is the annihilation operator, whereby $\hat{a}_{\vec{k}}|1_{\vec{k}}\rangle = |0\rangle$. Let us also assume the same linear polarisation throughout and reduce $\epsilon_{\vec{k}} = 1$. At time $t = 0$ we have $|\psi(t = 0)\rangle = |e\rangle|0\rangle$. After a time $t \gg \Gamma$, the particle has decayed to $|\psi(t \to \infty)\rangle = |g\rangle|\gamma\rangle$ and the correlation function becomes,

$$\begin{aligned} G^{(1)}_A &= \langle 0|\langle g||E_0|^2|g\rangle|0\rangle \\ &= |E_0|^2. \end{aligned} \tag{7}$$

Recalling that the probability of the particle absorbing the initial photon $|\gamma_0\rangle$ is $|\alpha|^2$, then we find that the probabillity of detecting a photon is,

$$P_A(z) = |\alpha|^2|E_0|^2, \tag{8}$$

which is independent of the detector position. Here, we have assumed D to be large compared with the size of the detection region, such that the solid angle subtended by the screen relative to the particle is small.

2.2. Young's double slit experiment with single photons and quantised particles

Let us now consider the case of two identical particles, labelled A and B, separated by a distance d along z, illuminated by a single photon $|\gamma_0\rangle$ of wavelength $\lambda = 2\pi/k$. This system is shown in figure 2(a) and is analogous to a Young's double slit experiment where each narrow slit has been replaced by a two-level quantum system [32]. The positions of each particle relative to the SPD are described by the vectors $\vec{r}_A$ and $\vec{r}_B$. The absorption of $|\gamma_0\rangle$ by one of either A or B produces the superposition state,

$$|\psi(t = 0)\rangle = \frac{1}{\sqrt{2}}(|g_A,\, e_B\rangle + |g_B,\, e_A\rangle)|0\rangle. \tag{9}$$

If it is unknown which of the particles absorbed and then re-emitted the photon—i.e. there is no which-path information available—then a superposition of single modes of the electromagnetic spectrum are populated,

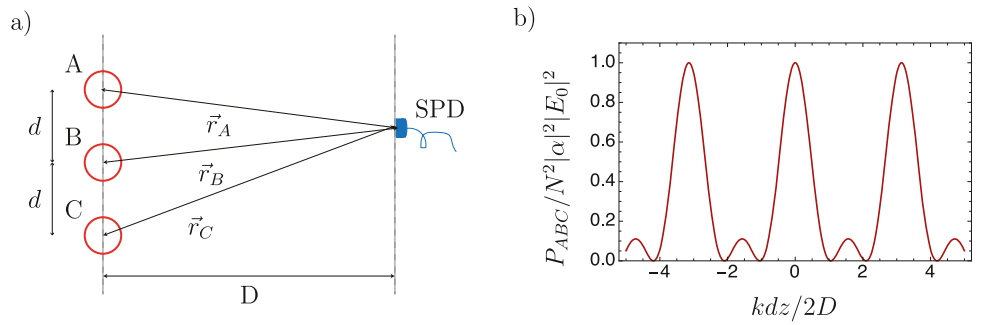

Figure 3. (a) Three particles each separated by a distance d can scatter single photos which can be detected by SPD. (b) The probability of detecting scattered photons as a function of the detector position along z.

$$|\psi(t \to \infty)\rangle = \frac{1}{\sqrt{2}}|g_A, g_B\rangle(|\gamma_A\rangle + |\gamma_B\rangle), \tag{10}$$

and the total state vector can be written as,

$$|\psi(t \to \infty)\rangle = \frac{E_0}{\sqrt{2}}|g_A, g_B\rangle\left[\sum_{\vec{k}_A} e^{i\vec{k}_A.\vec{r}_A}|1_{\vec{k}_A}\rangle + \sum_{\vec{k}_B} e^{i\vec{k}_B.\vec{r}_B}|1_{\vec{k}_B}\rangle\right]. \tag{11}$$

Throughout this article we shall consider the case where emission by one particle and absorption by another before re-emission and then detection can be neglected.

By combining the state vector from equations (11) with (5) we find the first order correlation function to be $G^{(1)}_{AB} = |E_0|^2(1 + \cos[k(r_A - r_B)])$. However, one must also account for the increased probability that one of the particles, A or B, absorb $|\gamma_0\rangle$ after illumination—two particles are twice as likely to absorb than one. This increased probability of absorption is $2|\alpha|^2$ and results in a probability for the SPD detecting a scattered photon of,

$$P_{AB}(z) = 2|\alpha|^2|E_0|^2(1 + \cos[k(r_A - r_B)]). \tag{12}$$

figure 2b) shows how the number of scattered photons varies as the detector is translated laterally with respect to the particles A and B. As one might expect, this result takes the same form as the classical description of two interfering plane waves after propagating through a Young's double slit apparatus where the single slit contribution to the diffraction pattern can be ignored.

2.3. Three path interference using single photons

Let us now extend our system from two particles to three. Following on from the previous section one finds that if a single photon is absorbed by one of three particles A, B, or C, as shown in figure 3, the system initially resides in the state,

$$|\psi(t = 0)\rangle = \frac{1}{\sqrt{2}}(|g_A, g_B, e_C\rangle + |g_A, e_B, g_C\rangle + |e_A, g_B, g_C\rangle)|0\rangle. \tag{13}$$

After a time $t \gg 1/\Gamma$ this decays to,

$$|\psi(t \to \infty)\rangle = \frac{E_0}{\sqrt{3}}|g_A, g_B, g_C\rangle\left[\sum_{\vec{k}_A} e^{i\vec{k}_A.\vec{r}_A}|1_{\vec{k}_A}\rangle + \sum_{\vec{k}_B} e^{i\vec{k}_B.\vec{r}_B}|1_{\vec{k}_B}\rangle + \sum_{\vec{k}_C} e^{i\vec{k}_C.\vec{r}_C}|1_{\vec{k}_C}\rangle\right], \tag{14}$$

and therefore,

$$G^{(1)}_{ABC} = \frac{|E_0|^2}{3}[3 + 2\cos k(r_A - r_B) + 2\cos k(r_A - r_C) + 2\cos k(r_B - r_C)] \tag{15}$$

Once the factor of $3|\alpha|^2$ is included in order to account for the increased probability of one particle absorbing, the total probability of detecting a photon at a given position reduces to

$$\begin{aligned} P_{ABC}(z) = |\alpha|^2|E_0|^2[3 &+ 2\cos k(r_A - r_B) + 2\cos k(r_A - r_C) \\ &+ 2\cos k(r_B - r_C)]. \end{aligned} \tag{16}$$

2.4. N-path interference using single photons

Recently tests of multi-path interference using more than three slits have been demonstrated [13]. Let us therefore now extend our chain of particles from 3 to N, each separated from their nearest neighbour by a

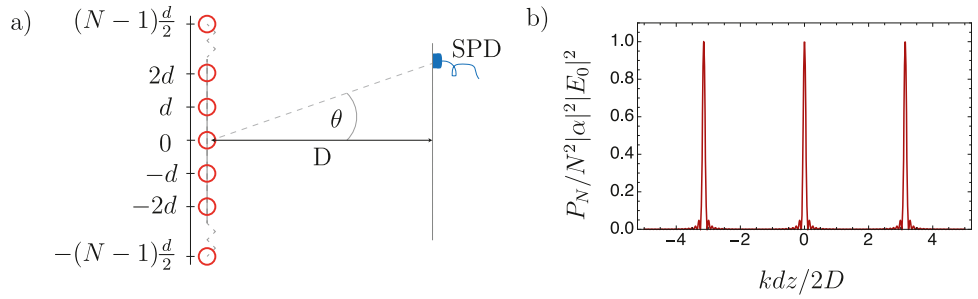

Figure 4. (a) N particles each separated by a distance d from their nearest neighbour. Each particle can scatter single photons which can be detected by SPD. Here we have shown an array containing an odd number of particles. We find identical results when using an even number. (b) The probability of detecting scattered photons as a function of the detector position along z for a linear array of $N = 30$ particles. We find the probability of detecting a scattered photon peaks sharply at values constant with the Bragg relation $2\pi m = kd \sin\theta$, where m is an integer describing the diffraction order.

distance d. This is shown schematically in figure 4. Following the same reasoning as in section 2 we find the probability of detecting a photon at position z for N different emitters can be written as,

$$\begin{aligned} P_N = |\alpha|^2|E_0|^2[N &+ 2\cos k(r_A - r_B) + 2\cos k(r_A - r_C) + 2\cos k(r_A - r_N)... \\ &+ 2\cos k(r_B - r_C) + 2\cos k(r_B - r_D)...+2\cos k(r_B - r_N) \\ &+ 2\cos k(r_C - r_D) + 2\cos k(r_C - r_E)...+2\cos k(r_C - r_N) + ...] \end{aligned} \tag{17}$$

When $D \gg d$ this equation can be simplified. In this limit, the separation between two particles is well approximated by the relation $r_{j+n} - r_j = n(r_{j+1} - r_j) = n\delta r$, where $\delta r = d\sin\theta$.

This enables equation (17) to be expressed in terms of the finite series,

$$P_N = |\alpha|^2|E_0|^2\sum_{j=0}^{N} 2(N-j)\cos(jkd\sin\theta) - N, \tag{18}$$

which after some trigonometry reduces to,

$$\begin{aligned} P_N &= |\alpha|^2|E_0|^2\frac{\cos(Nkd\sin\theta) - 1}{\cos(kd\sin\theta) - 1} \\ &= |\alpha|^2|E_0|^2\frac{\sin^2(Nkd\sin\theta/2)}{\sin^2(kd\sin\theta/2)}. \end{aligned} \tag{19}$$

Reassuringly, this resembles the familiar formula for a linear array of in phase dipole emitters [33].

3. In search of multi-path interference

Born's rule dictates that the probability of detecting any event can always be decomposed into a series of terms which contain at most information relating to two paths. Let us consider the simplest scenario where higher order contributions to the interference pattern may become apparent, namely a three slit type experiment. Using equations (8), (12) and (16) from the previous section one arrives at the following equality,

$$P_{ABC} = P_{AB} + P_{AC} + P_{BC} - P_A - P_B - P_C, \tag{20}$$

which can be used to define the parameter

$$\epsilon = P_{ABC} - (P_{AB} + P_{AC} + P_{BC}) + (P_A + P_B + P_C), \tag{21}$$

first introduced by Sorkin [14]. According to conventional quantum theory $\epsilon = 0$, with finite values corresponding to some degree of multi-path interference—illustrative of dynamics beyond the Schrödinger equation, or in experiments using material slits it can arise from the change of boundary conditions.

One can seek to bound any multi-path contribution by measuring ϵ. In order to do this one must compare the interference pattern for three scattering paths with those of all linear combinations of two and one paths as described in equation (21). In experiments where physical slits are used to define the paths, the interference patterns corresponding to all different permutations of A, B and C are controlled by selectively covering individual slits [7]. An appealing alternative, when using particles containing two-internal states to scatter incoming single-photons, is to selectively detune the particles from resonance with $|\gamma_0\rangle$—for example by making $(\omega_{eg} - \omega_\gamma)$ in equation (4) very large. Figure 5 illustrates a way this can be achieved. By applying a local potential to only some of the particles in the chain, for example by Stark-shifting the internal states using an electric field, then the absorption of $|\gamma_0\rangle$ can be controlled and under appropriate condition switched on and off. This removes

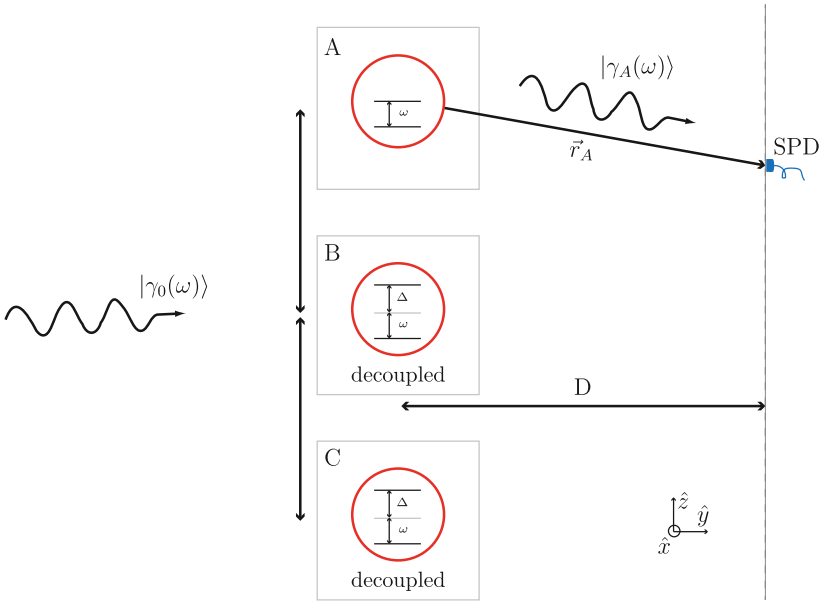

Figure 5. A local potential applied to particles individually—in this case *B* and *C*—modifies their internal energy levels, shifting them out of resonance with the incoming photon. Subsequently, the probability of these particles absorbing $|\gamma\rangle$ can be made vanishingly small, $|\alpha_B| \simeq |\alpha_C| \simeq 0$. Here, switching off the scattering probabilities is analogous to the covering of material slits, without the undesirable consequence of significantly changing the boundary conditions of the experiment. In this way the a controllable multi-slit experiment can be performed where the effective slits can be replaced by the internal states of particles.

the need to physically mask the individual scattering sites which could distort the trajectories of scattered photons and thereby introduce erroneous, finite vales of ϵ. By manipulating the internal energy levels of each scattering particle in this way a non-invasive measurement of ϵ can be performed, where each effective slit remains near-identical.

4. Discussion

We have derived the probability density of detecting single photons scattered by linear arrays of particles each containing two internal states. The case of one, two, three and *N* particle scattering is derived and we show how these can be used to place bounds on the multi-path contribution to the resulting interference pattern.

During the preparation of this manuscript a complementary perspective was published [34], which highlights the importance of the photon detector used when interpreting multi-path interference experiments, although it retains the the use of material slits. Here, we propose a method using single-particles to scatter single-photons which avoids the complex diffraction processes that occur for light at material slits [7, 8, 15], or waveguides [12], which complicate experimental realisations of optical tests of Born's rule. For particles with a scattering probability which can be sufficiently controlled without affecting their neighbours we have shown how they can be used to probe Born's rule and the underlying dynamics of the Schrödinger equation. Namely, how to perform a multi-path photon interference experiment where the effective slits can be treated in a fully quantum mechanical fashion.

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