Double deflation: theory and practice

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Abstract

Real GDP measured from the output side, GDP(O), should equal real GDP measured from the expenditure side, GDP(E), just as corresponding two approaches to measuring GDP in current prices are necessarily equal. But this is only the case even in theory if real value added in each industry is measured by double deflation. We set out the theory of double deflation using a matrix algebra treatment based on the framework of the Supply and Use Tables. The context is the UK's national accounts which measures volume growth by chained Laspeyres indices and which currently use single not double deflation. Initially we use simplified assumptions about prices. Later we introduce more realistic assumptions. We analyse the conditions on prices under which real GDP(O) equals real GDP(E). We consider three alternative methods of implementing double deflation. The preferred method makes use of all the price indices which the Office for National Statistics currently collects: Producer Price Indices, Services Producer Price Indices, Consumer Price Indices, Export Price Indices and Import Price Indices. We implement a simplified version of double deflation, using the same data as in the latest vintage of the national accounts, and compare our estimates with the official ones. In this version the same price index is used for each product regardless of whether the product is an output or an input. We find that double-deflated industry growth rates are consistently lower than the official single-deflated ones and also considerably more variable year-to-year. We interpret this finding as reinforcing the case for careful selection of the set of deflators to use for double deflation.

Keywords Double deflation, supply and use tables, value added, national accounts

JEL codes E01, O11, O40, O47, C67

1. Introduction[1]

In this paper we consider different ways in which the ONS's policy of adopting double deflation in the national accounts can be implemented and the empirical implications. Double deflation can be considered as a type of balancing, applied to real rather than nominal GDP. How to balance the various estimates of national income is an issue with a long history (Sefton and Weale 1995). It is well known that in current prices there are three ways of estimating GDP: from the expenditure, income and output sides. From the expenditure side GDP is the sum of all final expenditures less imports. From the income side it is the sum of returns to labour and capital. And from the output side it is the sum over all industries of value added (gross output[2] minus intermediate consumption). By definition, each of these approaches should yield the same answer. In practice they do not owing to errors and omissions in the data. Nowadays the ONS balances the three estimates at a detailed level by means of the Supply and Use Tables (SUTs). The method is a mixture of judgement and automatic balancing algorithms.[3]

If the different estimates of GDP in current prices are balanced, it would seem odd if the different estimates of *real* GDP were not balanced too. Real value added is the building block from which real GDP is constructed when the latter is measured from the output side. Real value added can be measured in either of two ways, by single or double deflation. Under single deflation as currently used in the UK the growth rate of real value added in a given industry is taken to be equal to the growth rate of real gross output in that industry. Single deflation only requires a price index or deflator for gross output, not for the inputs as well. Under double deflation, the growth rate of real value added is measured (roughly) by the growth rate of real gross output minus the weighted average growth rate of real input; real

[1] This research has been financed by the UK's Economic Statistics Centre of Excellence (ESCoE). Earlier versions have been presented at seminars to the Office for National Statistics in Newport, at the National Institute in London, and at the ESCoE Conference on Economic Measurement at the Bank of England, May16-17, 2018. We are grateful to participants for helpful comments, also to Jagjit Chadha. Thanks are also owed to Khanh Hoang for a most illuminating discussion on the methods of double deflation used by the Australian Bureau of Statistics. Finally, we thank the ONS and two anonymous referees for comments and suggestions. This paper is also available at https://www.escoe.ac.uk/research/discussion-papers.

[2] The term "output" is now preferred by national accountants to "gross output" (see the 2008 SNA) but we have retained the older term as less likely to be confusing.

[3] The simplest of the automatic methods is the RAS algorithm first developed by Richard Stone. This has the disadvantage that a zero entry in a table will always remain zero. More complicated methods not subject to this limitation are also available, either developed in-house as by the Australian Bureau of Statistics (ABS) or sold commercially. Sefton and Weale (1995) developed a method which automatically gives greater weight to the estimates considered more reliable.

input is a suitably weighted average of real energy, materials and bought-in services. So double deflation requires a price index for each of the inputs together with knowledge of the value of the purchases by each industry of energy, materials and services. As has been known for a long time, the main disadvantage of single deflation is that the estimate of GDP from the output side, real GDP(O), which it yields does not equal the estimate of real GDP from the expenditure side, real GDP(E), even when the underlying data are identical. The "solution" to this problem in the UK is to apply "coherence adjustments" to GDP(O) so that its growth matches that of real GDP(E) to a close tolerance. This is clearly not satisfactory if only because in practice the whole burden of adjustment is thrown onto the private service industries. This casts doubt on any stories about the economy which rely on developments in these industries.

It has also been known for a long time that under double deflation the growth of real GDP(O) equals in principle that of real GDP(E). The advantages of double deflation in compiling the national accounts are therefore threefold. First, to reiterate, it enables the growth of real GDP when measured from the output side to be equal in principle (i.e. in the absence of errors or omissions in the data) to the growth of real GDP measured from the expenditure side. Second, it ensures that (suitably weighted) the growth rates of real value added at the industry level aggregate up to the growth of real GDP. Third, it generates conceptually correct measures of total factor productivity (TFP) growth at the industry level. These in turn can be consistently aggregated to generate a whole-economy measure of TFP growth (Oulton 2017). The first advantage is self-evident. What is the use of two different measures of the growth of real GDP if these are generated by exactly the same data? Just as nominal GDP is conceptually the same whether measured by income, output or expenditure, so real GDP should be conceptually the same whether measured by output or expenditure.[4] The second advantage stems from the first. One could tell a story about which industries did well or badly during the Great Recession. But what credence could be placed on this if collectively the performance of individual industries does not add up to the generally-recognised total?

[4] There is no independent measure of real GDP from the income side. We are ignoring here the fact that on the expenditure side GDP is normally measured at purchasers' prices while on the output side it is normally measured at basic prices. This complication is discussed below.

Despite these advantages, double deflation has yet to be implemented in the UK's national accounts.[5] This is despite the fact that double deflation has long been recognised as conceptually superior in the System of National Accounts (SNA): see Commission of the European Communities et al. (1993) and European Commission *et al*. (2009). This has led to Eurostat recommending it as the preferred method (Eurostat 2013). Other countries, such as the US and around half of EU countries, have already adopted it. Currently, about half the G20 employ double deflation (Alexander et al. 2017). The UK is now in the process of implementing it with first estimates expected in Blue Book 2019 (Daniel et al. 2017).[6]

In the UK, the preferred measure of real GDP for annual data comes from the expenditure side: total final expenditure less imports, deflated by appropriate expenditure side deflators such as elements of the Consumer Price Index (CPI). On the output side real value added is measured by *single* deflation. The growth rate of real GDP from the output side is then estimated by aggregating the single-deflated real value added growth rates up to the whole-economy level, using nominal value added in the base year as weights. (Since this is a chained volume measure the base year shifts every year.) The resulting estimate will not be the same as when real GDP is measured from the expenditure side, whether due to inconsistencies in the data employed in the two estimates or to the conceptual issue or both. So the output side estimate is adjusted so that the growth rates of the two estimates match, to a close approximation, nowadays to within 0.1 per cent per year. In aligning the two estimates industrial production and government output are left unchanged so the whole burden of these "coherence adjustment" is thrown onto private service industries (Lee 2013).[7]

[5] There are exceptions. Double deflation has long been used to measure real value added in agriculture and energy. Outside of official statistics, Stoneman and Francis (1994) estimated double deflated real value added in UK manufacturing over 1979-1989.

[6] The UK's implementation of double deflation will be based on the so-called "H-approach" outlined in chapter 9 of the recent UN handbook on supply, use and input-output tables (United Nations 2018). This handbook appeared too late for us to take into account in the present paper. However, we believe our approach is broadly consistent with the H-approach, even though the latter is not set out explicitly in mathematical terms and does not discuss directly the crucial issue of consistency of price indices (see below).

[7] For quarterly estimates of the growth of real GDP output-side estimates are preferred since they are more timely. But once annual data from the Blue Book become available the quarterly output-side estimates are adjusted so that within each year they aggregate up to the annual growth rate determined from the expenditure side. So the output side estimates only influence the pattern of movement within each year, not the year-to-year growth of real GDP which as stated is determined from the expenditure side. The only qualification to this statement is that the output-side estimates prevail in the "tail", the most recent quarters for which Blue Book estimates are not yet available. When the Blue Book does become available the estimates for what were the tail quarters are revised to incorporate annual estimates of GDP(E). Of course by this time a new tail has appeared.

The reasons for preferring the expenditure side estimate have never been officially explained. But is quite easy to guess them. The government's main price programme is the CPI (or its predecessor the RPI). Far more is spent on this than on the output-side price programmes, the Producer Price Indices (PPIs) and the newer Services Producer Price Indices (SPPIs). In the case of the SPPIs, many are quite recently established and their coverage is still poor (Bean 2016). So on the expenditure side the ONS can benefit from the CPI which covers about two thirds of GDP. The remainder is covered mostly by PPIs and export price indices (EPIs) which may be of lower quality but will also have to be used for the output-side estimates anyway. The same goes for the price indices used to deflate health and education. These may be of debatable quality but they are the same on the output side as on the expenditure side.

The primacy of the expenditure side estimates of real GDP means that in an important sense GDP is already double deflated in the UK. The expenditure side nets out imports and all intermediate inputs, to leave only final expenditure on UK outputs. These final expenditures are deflated by what are (at the moment anyway) the best available deflators. So any move to double deflation on the industry side should not by itself change the headline figures for GDP, though it can be expected to change the industry composition of GDP. Of course, it is possible that better deflators for final expenditure might be developed in future and if carried back in time these might legitimately change our view of past GDP growth. But this is a different issue from the effect of double deflation per se.

The rest of this paper is divided into nine sections. In section 2 we compare single and double deflation at the industry level. Section 3 discusses double deflation in the context of the SUTs, the framework within which nominal balancing is done. The discussion here also reflects the general approach of the ONS to estimating real GDP. The UK uses annually-chained Laspeyres indices for real GDP in accordance with Eurostat requirements. This contrasts with the US which uses annually-chained Fisher indices. Here we set out the framework of the Supply and Use Tables (SUTs) which the ONS is planning to use for its own double deflation project. The analysis here is simplified in that some of the issues of consistency of price indices are assumed away. In section 4 we set out the (simplified) theory of double deflation in matrix algebra terms which allows simple proofs of the main propositions. Section 5 discusses the effect of relaxing some of the assumptions about prices made in the previous sections. It suggests three alternative ways in which double deflation could be implemented in practice. Section 6 presents a matrix algebra treatment of the preferred alternative. Section 7 extends the results established for chained Laspeyres indices to Paasche, Fisher and Törnqvist indices; this is relevant because after Brexit the ONS might choose to adopt the US approach and use Fisher indices and also because the ONS already uses Törnqvist indices in its *Productivity Bulletin*. Section 8 discusses the materials available to the ONS to actually implement double deflation. In Section 9 we present a preliminary empirical analysis of the impact of double deflation on industry outputs and GDP using publicly-available data on deflators at the level of the SUTs. Finally, section 10 concludes.

2. Single versus double deflation at the industry level

2.1 The ONS methodology for estimating real value added by single deflation

Under the ONS's current methodology, real single-deflated value added in each industry (V_{it}^{SD}) is estimated by assuming that it grows at the same rate as the real (gross) output of that industry:

$$\frac{V_{it}^{SD}}{V_{i,t-1}^{SD}} = \frac{Y_{it}}{Y_{i,t-1}}$$

where

$$\frac{Y_{it}}{Y_{i,t-1}} = \left(\frac{GO_{it}}{GO_{i,t-1}}\right) / \left(\frac{P_{it}}{P_{i,t-1}}\right)$$

Here V is real value added, Y is real gross output, GO is nominal gross output, and P is the price of output.[8]

2.2 Calculating real value added by double deflation

To derive real value added by double deflation (V_{it}^{DD}), note first the accounting identity that for the i-th industry in year t the value of output equals nominal value added plus the value of the inputs (intermediate consumption):

[8] This way of doing single deflation is called "single extrapolation" by Alexander at al. (2017) and is employed by seven of the G20 countries. The other way is to deflate nominal value added by the price index for output but amongst the G20 countries this method is currently used only by China and India (see their Table 1). We follow common usage by continuing to refer to the UK method as single deflation.

$$P_{it}Y_{it} = P_{it}^{V}V_{it}^{DD} + \sum_j \tilde{P}_{jt}X_{ijt} \tag{1}$$

We don't observe the price and quantity of value added separately, only the product of the two, nominal value added. The purpose of the index we seek, V_{it}^{DD}, is to enable us to separate price and quantity. Here X_{ijt} is the real amount of the *j*-th input purchased by the *i*-th industry in year *t*. Note that this now includes imports as well as domestic production. Consequently for this and other reasons we allow the price of a product as an input to differ potentially from its price as an output. Hence P_{it} is the price of the *i*-th industry's *output* while $\tilde{P}_{it}$ is the price of the *i*-th product as an *input*. At previous year's prices (PYP) equation (1) becomes

$$P_{i,t-1}Y_{it} = P_{i,t-1}^{V}V_{it}^{DD} + \sum_j \tilde{P}_{j,t-1}X_{ijt} \tag{2}$$

Lagging equation (1) one period

$$P_{i,t-1}Y_{i,t-1} = P_{i,t-1}^{V}V_{i,t-1}^{DD} + \sum_j \tilde{P}_{j,t-1}X_{ij,t-1} \tag{3}$$

Rearranging (2) to get value added on the left hand side and then dividing by $P_{i,t-1}^{V}V_{i,t-1}^{DD}$:

$$\left(\frac{V_{it}^{DD}}{V_{i,t-1}^{DD}}\right) = \frac{P_{i,t-1}Y_{it} - \sum_j \tilde{P}_{j,t-1}X_{ijt}}{P_{i,t-1}^{V}V_{i,t-1}^{DD}} \tag{4}$$

(Note that the denominator on the right hand side is nominal value added in year *t*-1, given implicitly by equation (3)). This formula draws attention to a possible problem with the use of a Laspeyres index, namely that value added at PYP, the numerator on the right hand side, might be negative, even though in practice at the industry level value added at current prices is never observed to be negative. Negative real value added makes the Laspeyres index hard to interpret.

Equation (4) is not very intuitive so it is helpful to define the shares of nominal value added and of the inputs in nominal gross output:

$$v_{i,t-1} = \frac{P_{i,t-1}^{V}V_{i,t-1}}{P_{i,t-1}Y_{i,t-1}}$$

and

$$w_{ij,t-1} = \frac{\tilde{P}_{j,t-1}X_{ij,t-1}}{\tilde{P}_{i,t-1}Y_{i,t-1}}, \quad j = 1,...,N$$

Note that $v_{i,t-1} + \sum_j w_{ij,t-1} = 1$. Now substituting these into (4) we obtain:

$$\left(\frac{V_{it}^{DD}}{V_{i,t-1}^{DD}}\right)=\frac{1}{v_{i,t-1}}\left[\left(\frac{Y_{it}}{Y_{i,t-1}}\right)-\sum_j w_{ij,t-1}\left(\frac{X_{ijt}}{X_{ij,t-1}}\right)\right] \tag{5}$$

While (5) is more intuitive than (4), it suffers from the drawback that it cannot be calculated when any of the inputs are zero in the previous period (i.e. $X_{ij,t-1}=0,$ for some j), something which is seen quite frequently in the data. To cover this we adopt the convention (which is fully consistent with (4)), that $w_{ij,t-1}(X_{ijt} / X_{ij,t-1})=0$ in this case.

2.3 Reasons why double-deflated can differ from single-deflated value added

At this point we can note the reasons why double-deflated can differ from single-deflated value added at the industry level. Let us define the weighted average gross growth rate of the inputs in the *i*-th industry as

$$\frac{\bar{X}_{it}}{\bar{X}_{i,t-1}}=\frac{1}{\sum_j w_{ij,t-1}}\sum_j w_{ij,t-1}\left(\frac{X_{ijt}}{X_{ij,t-1}}\right)=\frac{1}{1-v_{i,t-1}}\sum_j w_{ij,t-1}\left(\frac{X_{ijt}}{X_{ij,t-1}}\right) \tag{6}$$

Plugging this into (5)

$$\left(\frac{V_{it}^{DD}}{V_{i,t-1}^{DD}}\right)=\frac{1}{v_{i,t-1}}\left[\left(\frac{Y_{it}}{Y_{i,t-1}}\right)-(1-v_{i,t-1})\left(\frac{\bar{X}_{it}}{\bar{X}_{i,t-1}}\right)\right] \tag{7}$$

Now if the weighted average gross growth rate of inputs is say g and this happens to be the same as the gross growth rate of real gross output then

$$\left(\frac{V_{it}^{DD}}{V_{i,t-1}^{DD}}\right)=\frac{1}{v_{i,t-1}}\left[g-(1-v_{i,t-1})g\right]=g=\left(\frac{V_{it}^{SD}}{V_{i,t-1}^{SD}}\right)$$

In other words double and single deflation produce the same answers in this case. In fact the growth rate under double deflation exceeds that under single if the weighted average growth rate of the inputs is lower than that of output. We can also say something about the effect of changes in the relative prices of inputs. Suppose there is a depreciation of the exchange rate which raises the home-currency price of imported inputs. Then this is very likely to slow the growth of these inputs. This could be offset by a faster growth of domestically-produced inputs, leading perhaps to no effect on real value added. Or it could lead to more production being carried out in-house in which case double deflated real value added will increase more rapidly than single-deflated.

2.4 Aggregating up from industry to GDP

Whether we use single or double deflation the growth of real GDP(O) using a chained Laspeyres approach is calculated by a base-weighted average of industry growth rates of real value added:

$$\frac{GDP_t}{GDP_{t-1}} = 1 + g_t^O = \sum_{j=1}^{N} s_{j,t-1}\left(\frac{V_{jt}}{V_{j,t-1}}\right) \tag{8}$$

Here g_t^O is the growth rate of real GDP(O) between year t and year t-1 and the weights are the shares of each industry in aggregate nominal value added (nominal GDP) in year t-1:

$$s_{j,t-1} = \left[\frac{P_{j,t-1}^V V_{j,t-1}}{\sum_{j=1}^{N} P_{j,t-1} V_{j,t-1}}\right]$$

In later sections we analyse the conditions under which this growth rate equals the corresponding growth rate from the expenditure side.

3. Double deflation and the SUTs: a simplified model

The SUTS are the framework within which the ONS does balancing in current prices. It is also the framework within which it plans to do balancing in real terms. So in this section we set out the framework and discuss a simplified model of double deflation which ignores some important real-world complications.

In supply-use analysis we distinguish between industries and products, even though there is the same number of each.[9] Each of the *N* industries is defined by its principal product, the product of which most though not necessarily all of its sales are composed. Thus the Agriculture industry sells Agriculture products but may sell other products too, e.g. accommodation services; equally, other industries may also sell Agriculture products. The supply table shows how much of each of the *N* products is produced by each of the *N* industries. Total supply is defined as supply from domestic output plus imports: see Table 1. Table 1 as shown here is fuller than the published SUTs which show only the total of

[9] The equality of products and industries is true for the UK though in the SUTs of some countries such as Australia there are more products than industries.

domestic output of each product (for each row, the sum of columns 1 to N in Table 1). In the form shown here Table 1 also incorporates the make matrix (the first N columns). But it is also simplified by comparison to the published tables since it omits two columns which are necessary to go from basic prices to purchasers' prices: margins and taxes less subsidies on products. That is, for purposes of exposition and for now we are ignoring the difference between basic prices and purchasers' prices; more on this below.

The row total of each row of the Supply table shows the total supply at current basic prices of each product. The column totals show the total gross output of each of the industries (the total of the sales of each of the products that an industry produces) at current basic prices.

The use table shows purchases of each product either for intermediate use or for final use: see Table 2. This is again an expanded version of the published combined use table. The latter combines purchases from domestic supply with imports; here these are shown separately. Each column total shows an industry's gross output since the column elements are the industry's costs with value added being the balancing item:

$$GO_j = \sum_{i=1}^{N} X_{ij} + \sum_{i=1}^{N} M_{ij} + VA_j \qquad j = 1, 2, ..., N$$

Value added can be further broken down into "taxes less subsidies on production" (not to be confused with "taxes less subsidies on *products*"), compensation of labour, and gross operating surplus (GOS), the latter now becoming the balancing item.

There are two links between the supply table and the use table. First, the column totals relating to industries are the same in both tables: gross output. This reflects the accounting identity that for each industry total purchases of inputs (intermediate, labour and capital services) must equal total sales (gross output). The second link is that total supply of each product must equal the total use of each product (intermediate or final). So for the i-th product:

$$\sum_{j=1}^{N} Y_{ij} + M_i = \sum_{j=1}^{N} X_{ij} + F_i + M_i$$

or

$$\sum_{j=1}^{N} Y_{ij} = \sum_{j=1}^{N} X_{ij} + F_i \qquad (9)$$

i.e. output of product i from domestic sources must equal purchases of domestic output (intermediate plus final). This equality is enforced through the balancing process.

GDP at current basic prices can be measured either from the output side, GDP(O), or from the expenditure side, GDP(E). (It can also be measured from the income side but we will not be concerned with this since only GDP(O) and GDP(E) can be used to measure real changes). GDP(O) is defined as the sum of value added across industries while GDP(E) at basic prices is defined as final expenditure on domestic output less imported intermediate goods and services:

$$\begin{aligned} GDP(O) &:= \sum\nolimits_{j=1}^{N} VA_j \\ GDP(E) &:= \sum\nolimits_{i=1}^{N} F - \sum\nolimits_{i=1}^{N} \sum\nolimits_{j=1}^{N} M_{ij} \end{aligned} \tag{10}$$

where ":=" indicates a definition. These two concepts of GDP must necessarily be equal if the tables are balanced. This is proved as Proposition 1 in the next section using matrix algebra (see below). For an alternative proof avoiding matrix algebra see the Annex.

In setting out these relationships we are implicitly using the prices of the current year, year *t*. But the same relationships would hold if we used the prices of any other year, e.g. year *t*-1. The prices of year *t*-1 are particularly appropriate if we are working within the framework of the UK national accounts which use chained Laspeyres indices to measure volumes. For example, if P_{t-1}^{GDP} is the price of GDP (the GDP deflator) in year *t*-1 relative to its price in year *t*, the index of real GDP in year *t* relative to year *t*-1, denoted by $Z_{t,t-1}^{GDP}$, is

$$Z_{t,t-1}^{GDP} = \frac{P_{t-1}^{GDP} GDP_t}{GDP_{t-1}}$$

The numerator, $P_{t-1}^{GDP} GDP_t$, is referred to as GDP at previous year's prices (PYP). These *Z* indices can be chained together for any number of years to give the chained volume measure (CVM) of real GDP. So the index for year *T* relative to year *R* is:

$$Z_{T,R}^{GDP} = Z_{R+1,R}^{GDP} \times Z_{R+2,R+1}^{GDP} \times \ldots \times Z_{T,T-1}^{GDP}$$

The key insight now is that we can revalue the supply table and the use table to previous year's prices and it will still be the case that the tables are balanced on the new price basis, at least on the assumption that the price indices are "consistent" in a sense to be defined below. Furthermore real GDP(E) will still equal real GDP(O) at PYP. This is illustrated in Tables 3 and 4 and will be proved more formally below. This is proved as Proposition 3 in the next section using matrix algebra (see below). For an alternative proof avoiding matrix algebra see the Annex.

In these tables it is assumed that there are two sets of price indices, one for domestic output (the P_i^Y) and one for imports (the P_i^M). These price indices are for *t*-1 relative to year *t*. E.g. $P_i^Y = p_{i,t-1}^Y / p_{i,t}^Y$ where $p_{i,t}^Y$ is the actual domestic price in year *t*. A given product, produced domestically, is sold at the same price whatever the use to which it is put. And a given import is sold at the same price whatever the use to which it is put. The consequences of relaxing these assumptions will be discussed later. In Table 3 each row of the supply table is multiplied by the appropriate domestic price index, except that imports are multiplied by the corresponding import price index.

The first *N* column sums in Table 3 define gross output at PYP for each industry. For industry *j*, gross output at PYP is

$$\overline{P}_j^Y GO_j = \sum_{i=1}^{N} P_i^Y Y_{ij}$$

The only initially unknown element in this equation is $\overline{P}_j^Y$, the weighted average price of the products sold by the *j*-th industry. This equation can therefore be taken as defining and measuring this average price (technically, a Paasche index).

The first *N* column sums of Table 4, the use table at PYP, are taken from Table 3. This then enables us to derive the initially unknown levels of value added at PYP for each industry. For the *j*-th industry

$$P_j^{VA} VA_j = \overline{P}_j^Y GO_j - \sum_{i=1}^{N} P_i^Y X_{ij} - \sum_{i=1}^{N} P_i^M M_{ij}$$

The only unknown in this equation is P_j^{VA} , the price of value added in the *j*-th industry, which we can now use this equation to determine. In other words, value added at PYP is determined from this equation as a residual.

4. Matrix algebra treatment of the SUTs

In this section we present a matrix algebra treatment of the SUTs. This is not just empty formalism. Proving the key propositions is much simpler using this approach. And just as important, implementing double deflation requires eventually an algorithm, expressed as a computer program, to do the calculations. And this program can be written in a matrix programming language such as Gauss, Matlab or Stata (or within Stata, Mata). So our matrix algebra can be translated later into a program to actually perform the necessary calculations.

The Supply and Use tables satisfy two accounting identities:

1. For each product, Supply equals Use (or sales equals purchases, or demand equals supply). In other words, each row sum of the supply table equals the corresponding row sum of the use table.
2. For each industry, Revenues (or sales) equal Costs. "Costs" includes a balancing item, value added (or more precisely, within value added, gross operating surplus), to make this identity hold. In other words each of the first *N* column sums of the use table equals the corresponding column sum of the supply table.

The analysis is greatly simplified by setting out the key relationships in matrix algebra terms. In what follows, all matrices are *N*x*N* and all vectors are *N*x1 column vectors. Consistent with Tables 1 and 2, we use the following notation:

$$\mathbf{1} = [1\ 1\ 1 ... 1]', \text{ a column vector of ones}$$

$$\mathbf{Y} = [Y_{ij}],\ \mathbf{M} = [M_{ij}],\ \mathbf{X} = [X_{ij}]$$

$$\mathbf{GO} = [GO_i],\ \mathbf{VA} = [VA_i],\ \mathbf{F} = [F_i],\ \mathbf{M}^{FD} = [M_i^{FD}],\ \mathbf{S} = [S_i],\ \mathbf{U} = [U_i]$$

where **S** is a vector of total supplies and **U** a vector of total uses. Then Tables 1 and 2 can be expressed as follows:

$$\begin{array}{ll}
\mathbf{S} = \mathbf{Y}\cdot\mathbf{1} + \mathbf{M}\cdot\mathbf{1} + \mathbf{M}^{FD} & \text{(supply = domestic output plus imports)} \\
\mathbf{U} = \mathbf{X}\cdot\mathbf{1} + \mathbf{M}\cdot\mathbf{1} + \mathbf{M}^{FD} + \mathbf{F} & \text{(use = intermediate plus final demand)} \\
\mathbf{S} = \mathbf{U} & \text{(supply = use)} \\
\mathbf{GO}' = \mathbf{1}'\cdot\mathbf{X} + \mathbf{1}'\cdot\mathbf{M} + \mathbf{VA}' & \text{(gross output = intermediate consumption} \\
 & \text{+ value added)} \\
\mathbf{GO}' = \mathbf{1}'\mathbf{Y} & \text{(gross output = sum of product sales)}
\end{array} \tag{11}$$

(Here a prime (′) denotes a transpose and a dot (·) denotes matrix multiplication.)[10] From the first accounting identity, $\mathbf{S} = \mathbf{U}$, we conclude that

$$\mathbf{Y} \cdot \mathbf{1} = \mathbf{X} \cdot \mathbf{1} + \mathbf{F} \tag{12}$$

From the second accounting identity (the last two equations of this system) we see that

$$\mathbf{VA}' = \mathbf{1}' \cdot \mathbf{Y} - \mathbf{1}' \cdot \mathbf{X} - \mathbf{1}' \cdot \mathbf{M} \tag{13}$$

By definition, GDP from the expenditure side is final expenditure on domestic output less imported intermediates:

$$GDP(E) := \mathbf{1}' \cdot \mathbf{F} - \mathbf{1}' \cdot \mathbf{M} \cdot \mathbf{1} \tag{14}$$

and from the output side it is the sum of value added across industries:

$$GDP(O) := \mathbf{VA}' \cdot \mathbf{1} \tag{15}$$

This enables us to state:

Proposition 1[11]

In current basic prices GDP(E) = GDP(O)

Proof

Inserting equation (13) into equation (15) and using equation (12):

$$\begin{aligned} GDP(O) := \mathbf{VA}' \cdot \mathbf{1} &= [\mathbf{1}'\mathbf{Y} - \mathbf{1}' \cdot \mathbf{X} - \mathbf{1}' \cdot \mathbf{M}] \cdot \mathbf{1} \\ &= \mathbf{1}'[\mathbf{Y} \cdot \mathbf{1} - \mathbf{X} \cdot \mathbf{1} - \mathbf{M} \cdot \mathbf{1}] \\ &= \mathbf{1}'[\mathbf{F} - \mathbf{M} \cdot \mathbf{1}] \qquad \text{(using (4))} \\ &= GDP(E) \end{aligned}$$

■

Now we wish to show that the same relationships hold at PYP. Initially we adopt a simplifying assumption: a given domestic product is sold at the same price whatever the use to which it is put, whether (intermediate or final) consumption, gross capital formation or exports.[12] This assumption is relaxed below. So in accordance with Tables 3 and 4 we define three price matrices, one for domestic output, one for imports, and one for value added:

[10] Note that for any *N*x*N* matrix **A**, **1′·A** is the row vector of the column sums of **A** and **A·1** is the column vector of the row sums of A.

[11] See the Annex for a proof using simple algebra.

[12] This assumption is the same as that made by Moyer et al. (2006) in their discussion of double deflation in the US context.

$$\mathbf{P}^{Y} = diag[P_{i,t-1}^{Y}]$$
$$\mathbf{P}^{M} = diag[P_{i,t-1}^{M}]$$
$$\mathbf{P}^{VA} = diag[P_{i,t-1}^{VA}]$$

E.g. the matrix $\mathbf{P}^{Y}$ has the prices $P_{i,t-1}^{Y}$ on the principal diagonal and zeros elsewhere, and similarly for the other two matrices. Then the system of equations (11) becomes

$$\begin{aligned}
&\mathbf{S}_{PYP} = \mathbf{P}^{Y}\cdot\mathbf{Y}\cdot\mathbf{1}+\mathbf{P}^{M}\cdot\mathbf{M}\cdot\mathbf{1}+\mathbf{P}^{M}\cdot\mathbf{M}^{FD}\\
&\mathbf{U}_{PYP} = \mathbf{P}^{Y}\cdot\mathbf{X}\cdot\mathbf{1}+\mathbf{P}^{M}\cdot\mathbf{M}\cdot\mathbf{1}+\mathbf{P}^{M}\cdot\mathbf{M}^{FD}+\mathbf{P}^{Y}\cdot\mathbf{F}\\
&\mathbf{S} = \mathbf{U}\\
&\mathbf{GO}'\cdot\mathbf{P}^{Y} = \mathbf{1}'\cdot\mathbf{P}^{Y}\cdot\mathbf{X}+\mathbf{1}'\cdot\mathbf{P}^{M}\cdot\mathbf{M}+\mathbf{VA}'\cdot\mathbf{P}^{VA}\\
&\mathbf{GO}'\cdot\mathbf{P}^{Y} = \mathbf{1}'\cdot\mathbf{P}^{Y}\cdot\mathbf{Y}
\end{aligned} \tag{16}$$

Here the vectors $\mathbf{S}_{PYP}$ $(\mathbf{U}_{PYP})$ denote supply (use) at PYP.

We need to show first that the system is balanced at PYP ($\mathbf{S}_{PYP} = \mathbf{U}_{PYP}$), given that it is balanced in current prices ($\mathbf{S} = \mathbf{U}$).

Proposition 2

If the system is balanced at current basic prices then it remains balanced at PYP.

Proof

The equality between the column totals of the supply and use matrices at PYP is ensured by making real value added the residual in the use table. It remains to prove that total supply and total use are equal for each product at PYP, given that they are equal at current prices.

Using the fact that $\mathbf{S} = \mathbf{U}$ and equation (12), we have

$$\begin{aligned}
\mathbf{S}_{PYP} &= \mathbf{P}^{Y}\cdot\mathbf{Y}\cdot\mathbf{1}+\mathbf{P}^{M}\cdot\mathbf{M}\cdot\mathbf{1}+\mathbf{P}^{M}\cdot\mathbf{M}^{FD}\\
&= \mathbf{P}^{Y}\cdot(\mathbf{X}\cdot\mathbf{1}+\mathbf{F})+\mathbf{P}^{M}\cdot(\mathbf{M}\cdot\mathbf{1}+\mathbf{M}^{FD})\\
&= \mathbf{U}_{PYP}
\end{aligned}$$

■

Next we wish to prove

Proposition 3

GFP(E) = GDP(O) at PYP as well as at current prices.

Proof[13]

The definitions of GDP at PYP are

$$GDP(E)_{PYP} := \mathbf{1}' \cdot \mathbf{P}^{Y} \cdot \mathbf{F} - \mathbf{1}' \cdot \mathbf{P}^{M} \cdot \mathbf{M} \cdot \mathbf{1} \quad (17)$$

and

$$GDP(O)_{PYP} := \mathbf{VA}' \cdot \mathbf{P}^{VA} \cdot \mathbf{1} \quad (18)$$

Putting the last two equations of (16) together:

$$\mathbf{VA}' \cdot \mathbf{P}^{VA} = \mathbf{1}' \cdot \mathbf{P}^{Y} \cdot \mathbf{Y} - \mathbf{1}' \cdot \mathbf{P}^{Y} \cdot \mathbf{X} - \mathbf{1}' \cdot \mathbf{P}^{M} \cdot \mathbf{M} \quad (19)$$

(Note that this last equation serves to define the price of value added, $\mathbf{P}^{VA}$, since all other terms are known). Inserting equations (12) and (19) into equation (18),

$$\begin{aligned} GDP(O)_{PYP} &:= \mathbf{VA}' \cdot \mathbf{P}^{VA} \cdot \mathbf{1} = [\mathbf{1}' \cdot \mathbf{P}^{Y} \cdot \mathbf{Y} - \mathbf{1}' \cdot \mathbf{P}^{Y} \cdot \mathbf{X} - \mathbf{1}' \cdot \mathbf{P}^{M} \cdot \mathbf{M}] \cdot \mathbf{1} \\ &= \mathbf{1}'[\mathbf{P}^{Y} \cdot \mathbf{Y} \cdot \mathbf{1} - \mathbf{P}^{Y} \cdot \mathbf{X} \cdot \mathbf{1} - \mathbf{P}^{M} \cdot \mathbf{M} \cdot \mathbf{1}] \\ &= \mathbf{1}'[\mathbf{P}^{Y} \cdot \mathbf{F} - \mathbf{P}^{M} \cdot \mathbf{M} \cdot \mathbf{1}] \\ &= GDP(E)_{PYP} \end{aligned}$$

■

4.1 The effect of data errors

Obviously if there are errors in the price indices, say due to inadequate treatment of quality change or new goods, or if an inappropriate price index is used (e.g. an import price index as a proxy for an export one), then the estimates of real GDP will be similarly affected. Likewise if any of the components of final demand are wrong, e.g. nominal consumers' expenditure. But are there any errors which could break the equality between real GDP(E) and real GDP(O)? The answer, within the strict framework of equations (16), is no.

For example, suppose that there are errors in the input-output relationships, the X_{ij} and M_{ij} (which is quite plausible in the UK given the absence of a Purchases Inquiry in recent years[14]). This will certainly change the estimates of real value added for individual industries.

[13] See the Annex for a proof using simple algebra.

[14] The Purchases Inquiry has now been reinstated and will inform the national accounts from Blue Book 2019 onwards.

But provided that the SUTs are still balanced in nominal terms (so that none of the row or column totals are changed), these errors will not affect the estimate of real GDP(O) which will still be equal to real GDP(E). In other words any errors in measuring the outputs of industries will be completely offsetting.

However, suppose that instead of using the same set of price indices in the supply table as in the use table, we use different sets, e.g. one set $\mathbf{P}^Y$ in the supply table and another set $\tilde{\mathbf{P}}^Y$ in the use table. Then the price indices are inconsistent and the equality between real GDP(O) and real GDP(E) will be broken.

4.2 Ensuring consistency of price indices

The crucial importance of consistency can be seen most easily if we just consider a single product. For the domestic output of each product, the SUTs enforce the identity Supply = Demand, in current prices (CP), through the balancing process. This identity must hold in previous year's prices (PYP) too. So for the domestic supply of and demand for the *i*-th product in current basic prices in year *t* we have

$$S_{it} = IC_{it} + C_{it} + I_{it} + G_{it} + EX_{it}, \quad i = 1, 2, ..., N$$

where *S* is supply (domestic output), *IC* is intermediate consumption, *C* is consumption expenditure by households and NPISH, *I* is gross capital formation, *G* is government consumption, and *EX* is exports. This identity has to hold at PYP also. So revaluing each item by its own price index

$$P^S_{i,t-1} \cdot S_{it} = P^{IC}_{i,t-1} \cdot IC_{it} + P^C_{i,t-1} \cdot C_{it} + P^I_{i,t-1} \cdot I_{it} + P^G_{i,t-1} \cdot G_{it} + P^{EX}_{i,t-1} \cdot EX_{it}, \quad i = 1, 2, ..., N \tag{20}$$

Here *P* with a superscript denotes each price and these price indices are for year *t*-1 relative to year *t*. The left hand side, real supply, essentially determines real value added after aggregating appropriately over products and industries.

Now the central issue is, how can we be sure that the second equation holds? After all, if we just pick six price indices at random there is no reason to expect the equation to hold. There are essentially two ways to ensure equality:

1. Pick just one price index and apply it to both sides of the equation. Let's say we decide to use the price at which this product is sold to other industries, $P^{IC}_{i,t-1}$. Then we have

$$P^{IC}_{i,t-1}S_{it} = P^{IC}_{i,t-1}\left[IC_{it} + C_{it} + I_{it} + G_{it} + EX_{it}\right], \quad i = 1, 2, ..., N$$

which obviously has to be true given the equality holds in CP. The problems here are: (a) The choice of a single price index is arbitrary — why not the price of consumption say? (b) The resulting estimate for real GDP may differ radically from the current estimates. (c) This method throws away the information in all the other price indices which are not used. (d) It makes the unrealistic assumption that the same price is charged to every user; we know already that a different price is charged to foreign buyers since the export price index tells us as much.

2. Pick whatever price indices seem most appropriate (and available) for each element of the right hand side. Then the supply price on the left hand side is determined as a weighted average of the prices on the right hand side. i.e. by solving equation (20) for the supply price we get

$$P^{S}_{i,t-1} = \left(\frac{IC_{it}}{S_{it}}\right)P^{IC}_{i,t-1} + \left(\frac{C_{it}}{S_{it}}\right)P^{C}_{i,t-1} + \left(\frac{I_{it}}{S_{it}}\right)P^{I}_{i,t-1} + \left(\frac{G_{it}}{S_{it}}\right)P^{G}_{i,t-1} + \left(\frac{EX_{it}}{S_{it}}\right)P^{EX}_{i,t-1}, \quad i = 1, 2, ..., N \tag{21}$$

This ensures that the equation holds as an identity. A consequence is that real gross output in each industry will now differ from the current official estimates which basically deflate just by a single price index (the PPI, SPPI or CPI as deemed appropriate); this might be considered an advantage. Under this option as under the first one real GDP(O) will automatically add up to real GDP(E).

Option 2 seems much better than option 1. It makes use of all the price indices which are collected. And it could be tailored, if desired, to match the current estimates of real GDP(E) in the national accounts.[15] The next section considers how it might be implemented in practice in the UK.

[15] The issue of consistency of price indices is mentioned by Moyer at al (2006), pages 271 and 279, but they do not discuss in detail any remedies.

5. Double deflation in practice: some complications

5.1 How is real GDP(E) currently measured?

In current ONS practice, the definition of real GDP from the expenditure side corresponds reasonably well to equations (14) and (17). There are two important differences. First, GDP(E) is usually estimated at purchasers' prices but here we have been estimating it in basic prices. Second, we have assumed just one category of final expenditure whereas the ONS uses four: Final consumption of households, Final consumption of government, Gross capital formation, and Exports.

5.2 Basic prices versus purchasers' prices

The published supply table takes the form shown in Table 5. This differs from Table 1 in that the latter's columns 1 to N are aggregated to a single total column while two extra columns are added to make the conversion from basic to purchasers' prices. The first additional column is margins, the amount added to the basic price to account for transport and wholesale and retail trade. The second is "taxes less subsidies on products" which in the UK consists mostly of non-refundable VAT and excise duties. Our expenditure-side price indices such as the CPI are measured at purchasers' prices. So to implement the framework of the previous section these would have to be converted to basic prices. This does not seem difficult in principle since we can use the information in Table 5 to back out basic prices from purchasers' prices.

Recognition of taxes less subsidies on products also leads to a change in the use table. Value added at basic prices is now defined as gross output at basic prices minus intermediate inputs at *purchasers'* prices. Now we need to eliminate taxes less subsidies from the value of intermediate inputs (the X_{ij} and M_{ij}) and add an additional row to the use table showing total taxes less subsidies paid by each industry. GDP(E) at basic prices must now be defined as total final expenditure minus imports minus taxes less subsidies on products.

5.3 More than one category of final expenditure

As just noted, the ONS uses more than one category of final demand. This would not matter if for a given product the same price index applied to each category. But this is not the case. For example, exports have their own set of price indices (the EPIs), different from the corresponding CPI, PPI or SPPI. The issue here is not any possible doubts over the quality of the EPI. Rather it is that recognition of different categories of final expenditure along with their different prices means that the prices in the supply table at PYP must now be averages over each of the price indices involved. In other words, in the simple case where there are only two price indices involved, P_i^X and P_i^Y, one for exports and one for everything else, the typical entry in the supply table at PYP is no longer $P_i^Y Y_{ij}$ but $\tilde{P}_i^Y Y_{ij}$ where

$$\tilde{P}_i^Y = \frac{P_i^Y [\sum_{j=1}^{N} Y_{ij} + C_i + I_i + G_i] + P_i^X X}{\sum_{j=1}^{N} Y_{ij} + C_i + I_i + G_i + X}$$

i.e. we need to use a weighted average of the two prices for each product. (This assumes implicitly that whichever industry supplies a given product, its sales are divided in the same proportion between exports and other uses.) See the discussion in the previous section on consistency of price indices.

5.4 Consistency of price indices

In the UK the ONS collects four sets of price indices: (1) PPIs (and SPPIs which for brevity we will include under PPIs); (2) EPIs; (3) IPIs and (4) CPIs. The first three sets are basic prices while the CPIs are purchasers' prices. Also the CPIs do not distinguish between imported and domestically produced products. The CPIs cover only the consumption part of final demand. Gross capital formation and government expenditure have their own price indices or deflators which are the same on the output as on the expenditure side.

So at least for consumption, there is still an issue over the consistency of the different sets of price indices to be employed in double-deflated national accounts. We could strip out margins and taxes to convert the CPI into a basic price. Then we could back out a CPI for expenditure on domestic output using the corresponding IPI. How well would that compare with the PPI for this product? We wouldn't expect the two to be identical if only because they come from different inquiries and samples (firms versus retail stores) and constructing the CPI side involves a degree of modelling (to strip out margins, taxes and imports).

Behind the consistency issue there are different economic assumptions which one could make. One could assume first that there is no price discrimination and firms sell a given product at the same price on the domestic market whoever is the buyer (we already know that they sell at different prices in export markets since there is a separate EPI). Second and alternatively one could assume that firms sell at the appropriate PPI when selling to other industries and at a different price (the modified CPI) when selling to consumers. Of course once one admits the possibility of price discrimination one could assume that firms in one industry sell the same product at different prices to different industries, or that firms located in different industries but selling the same product charge different prices. This may be the case in practice but we have no way to handle this empirically since there is only one PPI for each product, irrespective of the buyer.[16]

This suggests that double deflation could be implemented under different assumptions about consistency and price discrimination:

Alternative 1. No price discrimination except for exports. For consumption and domestic intermediate sales of domestic products, adjusted CPIs for each product apply to all domestic buyers. EPIs apply to exports; PPIs are used to deflate gross capital formation expenditure; the usual deflators are applied to government output. I.e. no use is made of PPIs except where CPIs are lacking.

Alternative 2. No price discrimination except for exports. PPIs for each product applied to all buyers. I.e. for each product the appropriate PPI is applied to all uses except exports. No use is made of CPIs.

Alternative 3. Price discrimination allowed for exports, sales to domestic industry, and consumption. Adjusted CPIs used for consumption; PPIs used for intermediate sales to

[16] What we have been calling price discrimination could equally well be differences in the mix of products sold to different buyers. After all, the most recent SUTs distinguish only 81 “products” so there is clearly room for differences in the product mix sold to different buyers. Empirically this comes to the same thing as price discrimination.

domestic industries; EPIs used for exports; PPIs used for gross capital formation; usual deflators used for government output.[17]

Alternatives 1 and 3 should produce the same estimates of real GDP(E) as the current official method since they employ the same price indices as the official ones to deflate final demand. Since Alternative 2 uses PPIs to deflate final demand, it will produce different estimates of real GDP(E) from the current method. Alternatives 1 and 2 maximise the use of respectively CPIs and PPIs. Alternative 3 allows the use of both.

When estimating industry-level real gross output, researchers often deflate nominal gross output by the appropriate PPI. But this is not the approach used under any of these three alternatives. Even under Alternative 2 which maximises the use of PPIs the industry-level price index is a weighted average of the PPI and the EPI. Under Alternative 1 it is a weighted average of the CPI and the EPI. Under Alternative 3 it is a weighted average of the PPI, the EPI and the CPI.

Whichever of these three alternatives is adopted, real GDP(O) will necessarily be equal to real GDP(E). But each alternative will generate different estimates of real gross output and real value added at the industry level. If we believe that the CPIs are better quality than the PPIs, then we should adopt Alternatives 1 or 3. If we think that there is also some useful information embodied in the PPIs then Alternative 3 should be preferred. More complicated combinations of the price indices are also possible. We could for example apply reliability weights to the various indices, though this would come at the cost of changing the official estimates of real GDP(E).

6. The preferred Alternative 3: a matrix algebra treatment

The equality of real GDP(E) and real GDP(O) under any of the three alternatives may not be obvious and requires some further demonstration. we now show more formally that this is the case for Alternative 3. (Proofs for the other alternatives are simpler). For this purpose we

[17] A variant of this method would use a weighted average of PPIs and adjusted CPIs for the prices of products sold for intermediate use. The reason is that PPIs are meant to cover all domestic sales not just intermediate sales.

make the simplifying assumption that there are only two categories of final expenditure on domestic output, consumption (C) and exports (EX); the analysis could easily be extended to incorporate additional categories.

Let CPI_i^d be the basic price CPI for the *i*-th domestically produced product, i.e. the CPI for domestic product *i* after stripping out import prices, margins and taxes from the regular CPI. We can find this by solving the following system for CPI_i^d:

$$\begin{aligned} CPI_i^{bp} &= \left[\frac{C_{i,t-1} + M_{i,t-1}^{FD}}{C_{i,t-1} + M_{i,t-1}^{FD} + MARG_{i,t-1} + T_{i,t-1}}\right]\left[\frac{C_{it} + M_{it}^{FD}}{C_{it} + M_{it}^{FD} + MARG_{it} + T_{it}}\right]^{-1} CPI_i \\ CPI_i^{bp} &= \frac{C_{it}}{C_{it} + M_{it}^{FD}} CPI_i^d + \frac{M_{it}^{FD}}{C_{it} + M_{it}^{FD}} P_i^M \end{aligned} \tag{22}$$

Here CPI_i^{bp} is the CPI for product *i* at basic prices and we use the notation of Table 5, adding time subscripts where necessary; recall that all prices are for year *t*-1 relative to year *t*.[18] Then at PYP the typical entry in the Supply table is $\tilde{P}_{ij}^Y Y_{ij}$, where $\tilde{P}_{ij}^Y$ is the weighted average price of domestic output of product *i*.

$$\begin{aligned} \tilde{P}_i^Y &= \frac{X_{it}}{X_{it} + EX_{it} + C_{it}} PPI_i + \frac{EX_{it}}{X_{it} + EX_{it} + C_{it}} P_i^{EX} + \frac{C_{it}}{X_{it} + EX_{it} + C_{it}} CPI_i^d \\ &= w_{it}^X PPI_i + w_{it}^{EX} P_i^{EX} + w_{it}^C CPI_i^d \end{aligned}$$

Then the gross output at PYP of the *j*-th industry over all the products it produces (the column sum of the Supply Table) is:

$$\bar{\tilde{P}}_j^Y GO_j = \sum_{i=1}^N \tilde{P}_i^Y Y_{ij}$$

which serves as a definition of the average price of the *j*-th industry's output, $\bar{\tilde{P}}_j^Y$. This column total then serves also as the corresponding column total in the use table.

The typical entry in the intermediate section of the use table at PYP is now $PPI_i X_{ij}$ and the final demands for the *i*-th product now become $CPI_i^d C_{it} + P_i^{EX} EX_{it}$.

In matrix algebra terms the system under Alternative 3 at PYP can now be written as follows:

---

[18] For simplicity, we are assuming here that taxes and margins apply only to final, not intermediate, expenditure
,

$$
\begin{aligned}
&\mathbf{S}_{PYP} = \tilde{\mathbf{P}}^{Y} \cdot \mathbf{Y} \cdot \mathbf{1} + \mathbf{P}^{M} \cdot \mathbf{M} \cdot \mathbf{1} + \mathbf{P}^{M} \cdot \mathbf{M}^{FD} \\
&\mathbf{U}_{PYP} = \mathbf{PPI} \cdot \mathbf{X} \cdot \mathbf{1} + \mathbf{P}^{M} \cdot \mathbf{M} \cdot \mathbf{1} + \mathbf{P}^{M} \cdot \mathbf{M}^{FD} + \mathbf{CPI}^{d} \cdot \mathbf{C} + \mathbf{P}^{EX} \cdot \mathbf{EX} \\
&\mathbf{S} = \mathbf{U} \\
&\mathbf{GO}' \cdot \tilde{\mathbf{P}}^{Y} = \mathbf{1}' \cdot \mathbf{PPI} \cdot \mathbf{X} + \mathbf{1}' \cdot \mathbf{P}^{M} \cdot \mathbf{M} + \mathbf{VA}' \cdot \mathbf{P}^{VA} \\
&\mathbf{GO}' \cdot \tilde{\mathbf{P}}^{Y} = \mathbf{1}' \cdot \tilde{\mathbf{P}}^{Y} \cdot \mathbf{Y} \\
&\tilde{\mathbf{P}}^{Y} = \mathbf{W}^{X} \cdot \mathbf{PPI} + \mathbf{W}^{C} \cdot \mathbf{CPI}^{d} + \mathbf{W}^{EX} \cdot \mathbf{P}^{EX} \\
&\mathbf{W}^{X} + \mathbf{W}^{C} + \mathbf{W}^{EX} = \mathbf{I}
\end{aligned}
\tag{23}
$$

Here we employ the additional notation

$$
\begin{aligned}
&\mathbf{C} = [C_{it}], \text{ an } Nx1 \text{ column vector} \\
&\mathbf{EX} = [EX_{it}], \text{ an } Nx1 \text{ column vector} \\
&\tilde{\mathbf{P}}^{Y} = diag[\tilde{P}_{i}^{Y}] \\
&\mathbf{PPI} = diag[PPI_{i}] \\
&\mathbf{CPI}^{d} = diag[CPI_{i}^{d}] \\
&\mathbf{W}^{Z} = diag[w_{it}^{Z}] = diag\left[\frac{Z_{it}}{X_{it} + C_{it} + EX_{it}}\right],\ Z = X,\ C,\ EX \\
&\mathbf{I}: NxN \text{ identity matrix}
\end{aligned}
\tag{24}
$$

It remains to verify first that the system is still balanced at PYP if it is balanced in current prices and second that GDP(O) at PYP equals GDP(E) at PYP.

Proposition 4

If the system (24) is balanced at current basic prices then it remains balanced at PYP under Alternative 3, i.e. $\mathbf{S}_{PYP} = \mathbf{U}_{PYP}$.

Proof

The equality between the column totals of the supply and use matrices at PYP is ensured by making real value added the residual in the use table. It remains to prove that total supply and total use are equal for each product at PYP, given that they are equal at current prices.

From the fact that $\mathbf{S} = \mathbf{U}$ it follows that (analogously to equation (12) of the simpler system)

$$\mathbf{Y} \cdot \mathbf{1} = \mathbf{X} \cdot \mathbf{1} + \mathbf{C} + \mathbf{EX}$$

i.e. for the *i*-th product

$$Y_{it} = X_{it} + C_{it} + EX_{it}$$

where $Y_{it} = \sum_{j=1}^{N} Y_{ij}$ and $X_{it} = \sum_{j=1}^{N} X_{ij}$. Hence the matrices of weights in the price index $\tilde{P}_i^Y$ can be written

$$\mathbf{W}^Z = diag\left[\frac{Z_{it}}{Y_{it}}\right],\ Z = X,\ C,\ EX$$

Then from the first equation of (23) the supply of the *i*-th product at PYP from domestic sources is

$$\begin{aligned}\tilde{P}_i^Y Y_{it} &= \left[\frac{X_{it}}{Y_{it}} PPI_i + \frac{C_{it}}{Y_{it}} CPI_i^d + \frac{EX_{it}}{Y_{it}} P_i^{EX}\right] Y_{it} \\ &= PPI_i X_{it} + CPI_i^d C_{it} + P_i^{EX} EX_{it}\end{aligned} \tag{25}$$

which is equal to the demand for domestic output of this product (from the second equation of (23)). In matrix terms

$$\tilde{\mathbf{P}}^{\mathbf{Y}} \cdot \mathbf{Y} = \mathbf{PPI} \cdot \mathbf{X} + \mathbf{CPI}^d \cdot \mathbf{C} + \mathbf{P}^{EX} \cdot \mathbf{EX} \tag{26}$$

Since equation (25) holds for every product we have

$$\mathbf{S}_{PYP} = \mathbf{U}_{PYP}$$

■

Proposition 5

Under Alternative 3, GDP(E) = GDP(O) at PYP as well as at current prices.

Proof

The definitions of GDP at PYP are

$$GDP(E)_{PYP} := \mathbf{1}' \cdot \mathbf{CPI}^d \cdot \mathbf{C} + \mathbf{1}' \cdot \mathbf{P}^{EX} \cdot \mathbf{EX} - \mathbf{1}' \cdot \mathbf{P}^M \cdot \mathbf{M} \cdot \mathbf{1} \tag{27}$$

and

$$GDP(O)_{PYP} := \mathbf{VA}' \cdot \mathbf{P}^{VA} \cdot \mathbf{1} \tag{28}$$

Combining the fourth and fifth equations of (23):

$$\begin{aligned}GDP(O)_{PYP} &:= \mathbf{VA}' \cdot \mathbf{P}^{VA} \cdot \mathbf{1} = \mathbf{1}' \cdot \tilde{\mathbf{P}}^Y \cdot \mathbf{Y} - \mathbf{1}' \cdot \mathbf{PPI} \cdot \mathbf{X} - \mathbf{1}' \cdot \mathbf{P}^M \cdot \mathbf{M} \\ &= \mathbf{1}' \cdot \mathbf{CPI}^d \cdot \mathbf{C} + \mathbf{1}' \cdot \mathbf{P}^{EX} \cdot \mathbf{EX} - \mathbf{1}' \cdot \mathbf{P}^M \cdot \mathbf{M} \cdot \mathbf{1} \\ &= GDP(E)_{PYP}\end{aligned} \tag{29}$$

where use is made of (26). ■

Note that as before the first and second lines of equation (29) serves to define the price of value added, $\mathbf{P}^{VA}$, since all other terms are known.

7. Other volume indices: Fisher and Törnqvist

So far the argument has been developed in terms of chained Laspeyres volume indices because these are the indices used in the UK's national accounts and also in the rest of the EU. But the argument would apply just as much to alternative indices such as the chained Fisher as used in the US national accounts. The Fisher volume index is the geometric mean of a Laspeyres and a Paasche volume index. So to show that all the arguments above apply to the Fisher we just need to show that they apply to the Paasche as well as to the Laspeyres.

The Paasche volume index uses the prices of the next period to value the outputs of the current period. So it involves calculating supply and use tables at next year's prices (NYP) rather than previous year's prices (PYP). (Instead of revaluing the outputs and inputs of year t to the prices of year t-1, we revalue the outputs and inputs of year t-1 to the prices of year t.) But the matrix algebra of previous sections is formally the same whether we use previous year's prices or next year's prices. So the same arguments that show that the SUTs are balanced at PYP (provided the price indices are consistent with each other) also shows that they are balanced at NYP. Likewise, if GDP(E) equals GDP(O) at PYP then the two are also equal at NYP. This must be the case if the same (consistent) price indices are being used for both the PYP and the NYP calculations. In other words Propositions 2-5 hold at NYP as well as at PYP. It follows that under these conditions GDP(O) and GDP(E) are equal under double deflation when a Fisher index is employed to measure growth.

Though chained Laspeyres volume indices are used in the UK national accounts, chained Törnqvist indices are often used for productivity analysis, including by the ONS in its *Productivity Bulletin*. The chained Törnqvist can be viewed as a discrete approximation to a continuous Divisia index which has many desirable properties. For example, the product of a Divisia quantity index and a Divisia price index is the value index. And a Divisia index is consistent in aggregation. It has been shown that real GDP(O) and real GDP(E) are equal when prices and volumes are measured by Divisia indices and real value added is measured by double deflation: see Oulton (2004), Annex A, for a proof. Also chained Törnqvist and chained Fisher indices usually yield very similar estimates in time series. So we can expect that the equality of GDP(E) and GDP(O) will hold under double deflation when Törnqvist indices are used, at least approximately.

8. Data and analytical requirements for implementing double deflation

In this section we start by discussing the availability of SUTs and IOATs for implementing double deflation in the periods 1948-1996 and 1997 to the present. Next we turn to the availability of price indices.

8.1 1997 up to the present

The starting point for this period has to be the published SUTs which currently run from 1997 to 2015 on the basis of consistent industrial and product classifications (SIC07 and CPA 2008). These are compiled on a 114 by 114 basis which is then collapsed down to 105 products and industries for publication. These tables are consistent in current prices with the most recent Blue Book published in 2017. We can anticipate that for a few years at least each new Blue Book will lead to an additional set of SUTs which will be consistent with all the previous ones back to 1997, the latter being revised in accordance with the new Blue Book. This situation will persist at least until there is a change in the product or industry classification systems.

However the published SUTs need to be augmented in several ways. First, in the use table, total use has to be split between domestic supply and imports. This split is done in the input-output tables (now officially known as Input Output Analytical Tables or IOATs) which until recently were constructed only once every five years and unlike the SUTs are not revised when a new Blue Book appears or when the classification system changes. Second, we need to augment the supply table by constructing a make matrix for each year, which requires an expansion of the published supply table to show a breakdown of how much of each product is supplied by each industry.

Table 6 lists the various IOATs which have been published covering the years from 1968 to 2013 (ten in all). Over the period 1997-2015 we have three IOATs, for 2005, 2010 and 2013, consistent with respectively the 2009, 2013 and 2016 Blue Books. The SIC changed from the 2003 SIC to the 2007 SIC. Also the number of industries varied from 108 to 114 and back to 105. Each IOAT includes a table splitting total use into domestic supply and imports. It would be possible to use the IOATs for 2005, 2010 and 2013 to split the use table in the SUTs for 1997-2015 into domestic and imports, though this would require a considerable amount of estimation.

A sketch of the general approach for using an IOAT to split total use into domestic and imports is as follows. First, convert the IOAT to the same basis as the SUT of the same year, adjusting for differences in Blue Book year and classification systems. Here we could use the 2013 IOAT which is consistent with the 2016 Blue Book and uses the same classification systems, so we would just have to adjust from the 2016 to the 2017 Blue Book. The simplest approach would be to use the RAS procedure to do this. This would then yield a split between domestic and imported use for 2013. The same split for the years 1997-2012 and 2014 and 2015 could then be made by applying the RAS procedure to the SUTs of these years. It would of course be better to make use of the IOATs for earlier years too, particularly 2005 and 2010, but this would require much more estimation since these tables use different classification systems.

None of these three IOATs includes a make matrix. However, an earlier version of the SUTs for 1992-2014 (available in electronic form), which used the 2003 SIC and is consistent with the 2006 Blue Book, published a table entitled "Supply of products". This latter table is a highly condensed version of the make matrix. But it does contain the following important bits of information: (a) the percentage of sales of each product supplied by the industry whose principal product it is; and (b) the percentage of each industry's output comprised of sales of its principal product. This information could be combined with the make matrix from the 1990 IOAT (see below) to generate make matrices for each of the years 1997-2015. Again this involves a considerable amount of estimation.

8.2 1948-1996

The foundation for double deflation from 1997 onwards is the SUTs. For the period before 1997 these are largely missing, except as mentioned above we have SUTs covering 1992 to 1996. We must therefore rely much more on the IOATs. Table 6 lists all the IOATs which are likely to be useful for the earlier period, seven in all from 1968 to 1995. Electronic versions of the 1984 and later tables exist but no electronic versions for earlier years are to be found in the National Archives. Tables for years prior to 1968 do exist, e.g. for 1954, but this latter covers only manufacturing, so is of very limited relevance in the present context. The tables listed here employ three different SICs: 1968, 1980 and 1992. The number of industry groups also varies but the lowest, 91 in the 1968 table, is still quite high. On the other hand the 1968 table aggregates the service sector into only seven industries. The seven IOATs for 1968 to 1990 are superior to their successors in that as mentioned above each includes a make matrix.

Carrying back double deflation to the period 1948-1996 therefore appears to be very challenging. It might be possible to push back from 1997 to 1992 since SUTs exist for 1992-2004 though on a different basis to the later ones.

8.3 Price indices for double deflation

As emphasised earlier, good quality estimates under double deflation require all the information on prices gathered by the ONS in its various programmes —— PPIs, SPPIs, EPIs, IPIs, and CPIs —— aggregated to the level of the SUTs. As discussed above, there are two published sources for deflators at the level of the SUTs. The first source is the implicit deflators which can be constructed by dividing nominal by real value added; these are the deflators which we used in the empirical work reported earlier. The second source is the "experimental industry-level deflators" (ONS 2017). These two sources differ mainly in that the latter excludes the coherence adjustments. Both these sources give an average of the PPI (or similar) and the EPI, whereas for double deflation we need each index separately. The "experimental deflators" certainly, and very likely the implicit deflators too, are Laspeyres indices built up from lower level price indices at the product level. For double deflation Paasche indices are preferable so that deflation of nominal values generates Laspeyres indices of the volume of industry output.

Apart from these two sets, price indices at the level of aggregation of the SUTs have never been published by the ONS. These indices certainly exist on the GDP(O) side, at least for 1997-2015, since they are described in one of the documents published alongside each Blue Book: see Office for National Statistics (2017) for the most recent one. This gives in effect the recipe for each industry-level deflator but without publishing the data series themselves.

E.g. for “Accounting, bookkeeping and auditing services; tax consultancy services” (industry 69.2) the Average Weekly Earnings Index and the CPIY each gets a 50% weight. For manufacturing the deflator is an average of the PPI and the EPI though the weights are not shown. Presumably on the GDP(E) side there is a comparable set of recipes covering CPIs and IPIs, and also PPIs for some categories of expenditure like capital formation.

For the period prior to 1997 the situation is less clear. It is not clear how much material on deflators exists in ONS’s own archives for years before 1997. PPIs (in an earlier incarnation, wholesale price indices) certainly exist and were also published though at a lower level of aggregation than the IOATs. Aggregating these up to the level of the SUTs and the IOATs and reclassifying them to the current product basis would require a considerable data-gathering and analytical effort. And these would of course only cover the production sector (mining, manufacturing, construction and the utilities). They could no doubt be fairly easily augmented by deflators for transport and wholesale and retail trade. But finding deflators for private services and for government would be problematic.

The conclusion is that implementing double deflation on the basis we recommend here is quite feasible for the period 1997-2015, and possibly back to 1992. It would however require a considerable amount of analytical work in expanding the SUTs. However carrying back double deflation to 1948 or even just to 1968 appears much more challenging. SUTs and IOATs at the necessary level of detail are lacking. And developing deflators particularly for private services would present major difficulties.

9. Double deflation: a preliminary empirical analysis for the UK

9.1 Methodology

In this section we present preliminary estimates of industry-level real value added (at roughly the level of the SUTs) together with estimates of real GDP(O). The industry estimates are compared with the official ones. We also compare our GDP(O) estimates with the ones which appear in the Blue Book. Our estimates are preliminary since we use a simplified treatment derived from the published SUTs which aggregate imported and domestic inputs. So we are in effect ignoring the distinction between import prices and prices of domestically-produced products. The deflator we use for each industry is the one which can be derived from the series for nominal and real value added by industry published alongside the Blue Book, together with nominal gross output from the SUTs. These deflators are in principle averages of PPIs (or SPPIs) and export price indices but have also been subject to "coherence adjustments" designed to make the growth of real GDP(O) match that of GDP(E) to a close tolerance, nowadays 0.1% pa. These coherence adjustments are applied only to the private service industries (Lee 2011).[19] Our data come from the latest vintage available at the time of writing and are all consistent with the 2017 Blue Book. They cover the years 1997 to 2015.

As discussed in section 6, our preferred methodology would use a much more comprehensive set of price indices: CPIs, EPIs, and IPIs, as well as PPIs and SPPIs. But implementing this methodology is challenging since it requires expanding the SUTs to split total use between domestic and imported and estimating a make matrix for each year. It also requires all price indices to be at the level of the SUTs and such indices are not published. So this approach is beyond the scope of the current paper.

As we saw above, under the ONS's current methodology, real single-deflated value added in each industry (V_{it}^{SD}) is estimated by assuming that it grows at the same rate as real (gross) output:

$$\frac{V_{it}^{SD}}{V_{i,t-1}^{SD}} = \frac{Y_{it}}{Y_{i,t-1}}$$

where

$$\frac{Y_{it}}{Y_{i,t-1}} = \left(\frac{GO_{it}}{GO_{i,t-1}}\right) / \left(\frac{P_{it}}{P_{i,t-1}}\right)$$

Here V is real value added, Y is real gross output, GO is nominal gross output, and P is the price of output. From our point of view the unknowns here are the price of output P and real output Y since these are not published. But from the SUTs we know nominal gross output and

[19] There is an alternative source of industry-level price indices ("Experimental industry-level deflators") but these have the coherence adjustments stripped out. So we cannot use these to reproduce the official estimates of GDP(O); see https://www.ons.gov.uk/economy/inflationandpriceindices/adhocs/006718industryleveldeflatorsexperimentaluk1997to2015.

from the low-level aggregates workbook (see below) we know single-deflated real value added. So we can back out the output price by

$$\left(\frac{P_{it}}{P_{i,t-1}}\right)=\left(\frac{GO_{it}}{GO_{i,t-1}}\right)/\left(\frac{V_{it}^{SD}}{V_{i,t-1}^{SD}}\right)$$

We then fix price levels by setting the price equal to 1 in the reference year.

These deflators are for industry output. Our simplifying assumption is that an industry deflator is the same as the deflator for the corresponding product when it is used as an input. So our equation for estimating double deflated real value added is a simplified form of equation (4), namely

$$\left(\frac{V_{it}^{DD}}{V_{i,t-1}^{DD}}\right)=\frac{P_{i,t-1}Y_{it}-\sum_{j}P_{j,t-1}X_{ijt}}{P_{i,t-1}^{V}V_{i,t-1}^{DD}} \tag{30}$$

(Note that the denominator on the right hand side is nominal value added in year t-1). Since we have data on prices (derived just above) and nominal gross output, nominal value added and nominal inputs, these formulas can be readily implemented. We aggregate the industry-level growth rates to give the growth rate of GDP using equation (8).

Below we also present results using the Törnqvist formula for the growth of real value added. The latter is given by

$$\Delta\ln V_{it}^{DD}=\left(\frac{1}{\bar{s}_{it}}\right)\Delta\ln Y_{it}-\left(\frac{1}{\bar{s}_{it}}\right)\sum_{j}\bar{w}_{ijt}\Delta\ln X_{ijt} \tag{31}$$

where

$$\bar{s}_{it}=\frac{1}{2}\left[\frac{P_{it}^{V}V_{it}}{\sum_{i}P_{it}^{V}V_{it}}+\frac{P_{i,t-1}^{V}V_{i,t-1}}{\sum_{i}P_{i,t-1}^{V}V_{i,t-1}}\right]\text{ and}$$

$$\bar{w}_{ijt}=\frac{1}{2}\left[\frac{P_{jt}X_{ijt}}{GO_{it}}+\frac{P_{j,t-1}X_{ij,t-1}}{GO_{i,t-1}}\right]$$

The Törnqvist growth rates are aggregated up to give GDP growth using nominal value added as weights by the following formula (an analogue of equation (8)):

$$\Delta\ln GDP_{t}=\sum_{i}\bar{s}_{it}\Delta\ln V_{it}^{DD} \tag{32}$$

In summary, we are using exactly the same data as did the ONS for the 2017 Blue Book. So any differences between our results and the official ones are entirely due to the method. We are not claiming that our estimates are better than the official ones, if only because we are using a simplified methodology, not our preferred one. The interest is in seeing how sensitive the estimates are to the method.

9.2 Sources

The data sources used in this exercise are taken from the latest vintage at the time of the exercise and are consistent with the 2017 Blue Book. They cover the period 1997 to 2015. Nominal gross output and nominal inputs (intermediate consumption), which are both published in £ million, are taken from the SUTs[20]. Nominal and real value added, which are also both published in £ million, are obtained from the low level aggregates dataset, where nominal value added is consistent with the estimates published in the SUTs.[21]

Our estimates are mostly at the two digit industry level, matching the industry aggregation provided in the low level aggregates data. However, the industry level provided in the SUTs does not match the aggregation level in the low level aggregates data as it sometimes provides industries at a more aggregated level and sometimes goes down to even three digits for other industries. We cannot exploit this level of detail at the three digit industry level because we need information on real value added to construct the output deflators and this information is only published in the low level aggregates data at the two digit level. To arrive at a consistent aggregation level of industries across both data sources, we use the following industries in the low level data, which are already published at a more aggregated level, to match the level provided in the SUTs:

- 06 and 07 (crude petroleum and natural gas and metal ores)
- 41, 42 and 43 (construction)
- 59 and 60 (motion picture, video and TV programme production, sound recording and music publishing activities and programming and broadcasting activities)
- 87 and 88 (residential care and social work activities)

[20] Our source for nominal data is the 31st October 2017 release, which is available at https://www.ons.gov.uk/economy/nationalaccounts/supplyandusetables/datasets/inputoutputsupplyandusetables.

[21] Our source for nominal and real value added is the 26th January 2018 release, available at https://www.ons.gov.uk/economy/grossdomesticproductgdp/datasets/ukgdpolowlevelaggregates/current.

Furthermore, the SUTs only provides output as well as input information for industries 11.01-06 and 12 (alcoholic beverages and tobacco products) as a whole while the low level aggregates data provides value added in nominal and real terms for industry 10, 11 and 12 separately or for all three industries aggregated. Therefore, the aggregated estimates for industries 10 to 12, as published in the low level aggregates data, is used and the information on nominal gross output and nominal inputs from the SUTs are aggregated to match this level. We arrive finally at 79 industries.

9.3 Results

We now compare single deflation with double deflation measures of real value added over 1997-2015 for 79 industries which aggregate to the whole economy. The single-deflated measure is the official one and the two double-deflated ones are our own. For each industry we calculate the mean growth rate of real value added over the whole period 1997-2015, and two sub-periods, 1997-2007 and 2007-2015, the standard deviation of the industry growth rate in these periods, and also correlations between the three different estimates of these growth rates. We do this for the official, single deflation method and for two versions of double deflation, using respectively the Laspeyres and the Törnqvist formulas. These detailed figures appear in Tables 9-11 and are summarised in Tables 7and 8.[22] For Table 7 we calculate unweighted means across the 79 industries of these statistics (the time-series mean growth rates, the standard deviation of these growth rates and the correlations between the three different estimates of these growth rates).

Three facts stand out from Table 7. First, the cross-industry mean growth rate of single-deflated value added is on average much larger than that of double-deflated: 1.23% pa compared to 0.52% pa (Laspeyres) or -0.19% pa (Törnqvist) over the whole period. A similar pattern is found in the first sub-period, 1997-2007, though in the second sub-period Laspeyres and single deflation are close. The Laspeyres measure is always on average significantly higher than the Törnqvist one. This remains the case if we break the time period into halves, 1997-2007 and 2007-2015. Second, the volatility of growth, as measured by the

[22] We found one instance of negative value added when calculating the Laspeyres measure: coal mining (industry 05) in 2015. We dealt with this by setting the growth rate of the Laspeyres measure equal to that of the single deflation measure in 2015. There are instances when an input is recorded as zero in one year and then as positive in an adjacent year which makes calculating the Törnqvist measure impossible. We dealt with this by setting a zero input equal to £0.4 million which is within the rounding error in the SUTs.

standard deviation of the growth rate, is much higher under double deflation than under single, about 2.5 times as high in fact. These two conclusions, lower mean and higher volatility, survive the exclusion of outliers such as Coal mining (industry 05). Third, the cross-industry mean correlation coefficient between the growth of single-deflated and that of the two measures of double-deflated real value added is quite low, 0.77 over the whole period. The correlation between the two double deflation measures is by contrast high (0.99) as we would expect. In summary, the industry growth rates under either version of double deflation are much more volatile than under single deflation. Furthermore the time series pattern of these growth rates is not very similar.

We ran tests to see whether volatility of double deflation growth rates was due to volatility of the cost shares or of the value added/gross output ratios (see equation (5)). We calculated the double deflation growth rates first of all holding the cost shares constant at their mean levels in each industry and second holding the value added/gross output ratios constant at their mean levels. If anything this tended to increase the measured volatility. So volatility of growth of real value added must be due to volatility in the growth of relative prices.

The figures in Table 7 are unweighted means. It is possible that any differences between single and double deflation might tend to cancel out at the aggregate level when we calculate weighted means. To see if this is the case, Table 8 shows weighted mean growth rates for three broad sectors: Production plus utilities (gas , electricity and water); Private services; and Public services, plus the whole economy (GDP). The weights here are value added in the previous period (i.e. growth rates between years t-1 and t are weighted by value added in year t-1).

Note first that the figure for the whole economy shown as “Single (Official)”, 1.34 % pa, is the official estimate of the growth of GDP from the output side over this period as published in the 2017 Blue Book. This figure is substantially higher than the double deflation Laspeyres measure (0.52 % pa) and still more so than the Törnqvist measure (0.16 % pa). Recall that all measures employ the same set of deflators and that the underlying data used for weighting is again the same for all three measures. Only in market services are the official and the Laspeyres measures quite close. Table 2 illustrates that the adoption of double deflation has the potential to change the past substantially. Recall however that the deflators we have used for this exercise are not the ones which we would ideally like to employ. To repeat, our preferred method, Alternative 3, would leave past GDP growth unchanged.

10. Conclusions

Three alternative ways of implementing double deflation in the national accounts have been set out in matrix algebra form. Each alternative follows the ONS methodology of measuring volumes by a chained Laspeyres index All are characterised by two properties: first, if the SUTs are balanced at current prices then they continue to be balanced at PYP; second, as a corollary GDP(E) = GDP(O) at PYP just as it does in current prices. They differ however in the assumptions they make about prices. At the moment Alternative 3 is the preferred one for two reasons. First, it uses all five sets of price indices which the ONS collects: PPIs, SPPIs, CPIs, EPIs and IPIs. Second, the estimates of GDP(E) should be the same as under the current methodology. A consequence of adopting Alternative 3 is that the methodology for estimating real gross output at the industry level will differ from the current one. The current methodology deflates each industry's gross output in current prices by a weighted average of the appropriate PPI (or SPPI) and the EPI. Under Alternative 3, the deflator will be a weighted average of the PPI (or SPPI), the CPI (adjusted for imports, margins and taxes) and the EPI.

Only Alternative 3 (or something like it) is capable of delivering GDP growth rates which (given the same data) are the same as the current official ones. The deflators for final expenditure used for GDP(E), including as they do the CPIs, even though far from perfect are of higher quality than the PPIs and SPPIs used for estimating GDP(O). So real GDP should continue to rely on the former unless and until some other deflators can be shown to be superior.

This conclusion is reinforced by our empirical analysis of the effects of double deflation where we have used the published implicit deflators from the ONS's GDP(O) programme. These are averages of PPIs and EPIs, with added "coherence adjustments". Our findings are as follows. First, on average the growth rate of industry-level real value added was substantially lower under double deflation than under single. Second, the year-to-year volatility of growth rates was some two and a half times higher under double deflation. Third, if we use our estimates to estimate GDP using a chained Laspeyres approach, similar in spirit to the single deflation measures in the National Accounts which also use chained Laspeyres, we find that GDP would have grown substantially more slowly than the official estimate for 1997-2015. Our estimate grows at only 0.52 % pa compared to the official one which grows at 1.34 % pa.

Summing up, our theoretical findings suggests that it is possible to implement double deflation in a different way such that there is no effect on the headline growth rate of GDP. This requires using a different, more comprehensive, set of deflators which are currently not publicly available. The results reported here strongly suggest that this approach should be seriously considered. The current official estimate of real GDP relies on the expenditure-side deflators which are considered to be generally more reliable than the output side deflators. Hence it would be rash to move from the present system to one which relies much less on the more reliable deflators. It is possible however that even using this different approach the industry-level estimates of real value added may still be quite different from the current official ones.

Tables

Table 1
Supply table in year *t*, product by industry (*N*x*N*), at current (year *t*) basic prices

	Product/ industry	Domestic output (1)	(2)	…	(*N*)	Imports	Total supply (row total)
Sales	(1)	Y_{11}	Y_{12}	…	Y_{1N}	M_1^{all}	$\sum_{j=1}^{N} Y_{1j} + M_1^{all}$
	(2)	Y_{21}	Y_{22}	…	Y_{2N}	M_2^{all}	$\sum_{j=1}^{N} Y_{2j} + M_2^{all}$
	…	…	…	⋱	…	…	…
	(*N*)	Y_{N1}	Y_{N2}	…	Y_{NN}	M_N^{all}	$\sum_{j=1}^{N} Y_{Nj} + M_N^{all}$
Column total	Gross output	$GO_1\left[=\sum_i Y_{i1}\right]$	$GO_2\left[=\sum_i Y_{i2}\right]$	…	$GO_N\left[=\sum_i Y_{iN}\right]$	$M^{all}\left[=\sum_i M_i^{all}\right]$	$\sum_{j=1}^{N} GO_j + M^{all}$

Key Y_{ij} : sales of product *i* by industry *j* ; M_i^{all} : total imports of product *i*, intermediate and final ; GO_j : gross output of industry *j*.
Note: the bottom right hand element is both the sum of the row totals and the sum of the column totals.

Table 2
Domestic and imports use table in year *t*, product by industry (*N*x*N*), at current (year *t*) basic prices

Product/ industry		Intermediate purchases (1)	(2)	$\cdots$	(N)	*Total intermediate*	Final demand	*Total demand for products (row total)*
Sales by domestic industries	(1)	X_{11}	X_{12}	$\cdots$	X_{1N}	$\sum_{j=1}^{N} X_{1j}$	F_1	$\sum_{j=1}^{N} X_{1j} + F_1$
	(2)	X_{21}	X_{22}	$\cdots$	X_{2N}	$\sum_{j=1}^{N} X_{2j}$	F_2	$\sum_{j=1}^{N} X_{2j} + F_2$
	$\cdots$	$\cdots$	$\cdots$	$\ddots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	(N)	X_{N1}	X_{N2}	$\cdots$	X_{NN}	$\sum_{j=1}^{N} X_{Nj}$	F_N	$\sum_{j=1}^{N} X_{Nj} + F_N$
Sales of imports	(1)	M_{11}	M_{12}	$\cdots$	M_{1N}	$\sum_{j=1}^{N} M_{1j}$	M_1^{FD}	M_1^{all}
	(2)	M_{21}	M_{22}	$\cdots$	M_{2N}	$\sum_{j=1}^{N} M_{2j}$	M_2^{FD}	M_2^{all}
	$\cdots$	$\cdots$	$\cdots$	$\ddots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	(N)	M_{N1}	M_{N2}	$\cdots$	M_{NN}	$\sum_{j=1}^{N} M_{Nj}$	M_N^{FD}	M_N^{all}
IC		$\sum_{i=1}^{N}[X_{i1}+M_{i1}]$	$\sum_{i=1}^{N}[X_{i1}+M_{i1}]$	$\cdots$	$\sum_{i=1}^{N}[X_{i1}+M_{i1}]$	$\sum_{i=1}^{N}\sum_{j=1}^{N}\left[X_{ij}+M_{ij}\right]$	$\sum_{i=1}^{N}\left[M_i^{FD}+F_i\right]$	$\sum_{i=1}^{N}\left[M_i^{all}+F_i+\sum_{j=1}^{N} X_{ij}\right]$
	Value added	VA_1	VA_2	$\cdots$	VA_N	$\sum_{j=1}^{N} VA_j$	–	–
Column total	Gross output	GO_1	GO_2	$\cdots$	GO_N	$\sum_{j=1}^{N} GO_j$	–	–

Key See Table 1. Also

X_{ij}: purchases of domestic output of product i by industry j ; M_{ij} : purchases of imports of product i by industry j

M_i^{FD} : purchases of imports of product i for final demand ; M_i^{all} : total imports of product i ;

F_i : purchases of domestic output of product i for final demand ; IC : intermediate consumption.

Table 3
Supply table in year *t*, product by industry (*N*x*N*), at PYP, i.e. at basic prices of year *t*-1

	Product/ industry	Domestic output (1)	(2)	…	(*N*)	Imports	Total supply (row total)
Sales	(1)	$P_1^Y Y_{11}$	$P_1^Y Y_{12}$	…	$P_1^Y Y_{1N}$	$P_1^M M_1^{all}$	$P_1^Y \sum_{j=1}^N Y_{1j} + P_1^M M_1^{all}$
	(2)	$P_2^Y Y_{21}$	$P_2^Y Y_{22}$	…	$P_2^Y Y_{2N}$	$P_2^M M_2^{all}$	$P_2^Y \sum_{j=1}^N Y_{2j} + P_2^M M_2^{all}$
	…	…	…	⋱	…	…	…
	(*N*)	$P_N^Y Y_{N1}$	$P_N^Y Y_{N2}$	…	$P_N^Y Y_{NN}$	$P_N^M M_N^{all}$	$P_N^Y \sum_{j=1}^N Y_{Nj} + P_N^M M_N^{all}$
Column total	Gross output	$\bar{P}_1^Y GO_1 \left[= \sum_i P_i^Y Y_{i1} \right]$	$\bar{P}_2^Y GO_2 \left[= \sum_i P_i^Y Y_{i2} \right]$	…	$\bar{P}_N^Y GO_N \left[= \sum_i P_i^Y Y_{iN} \right]$	$\bar{P}^M M^{all} \left[= \sum_i P_i^M M_i^{all} \right]$	$\sum_{j=1}^N \bar{P}_j^Y GO_j + \bar{P}^M M^{all}$

Key See Tables 1 and 2. Also:
P_j^Y : price at which domestic producers sell product j ; P_j^M : price at which foreign producers sell product j ;
$\bar{P}_j^Y$: weighted average price of industry j's sales; $\bar{P}^M$: weighted average price of imports.
Note: All prices are those of year *t*-1 relative to those of year *t*. The bottom right hand element is both the sum of the row totals and the sum of the column totals.

Table 4
Domestic and imports use table in year *t*, product by industry (*N*x*N*), at PYP, i.e. at basic prices of year *t*-1

		Intermediate purchases				*Total intermediate*	Final demand	*Total demand for products (row total)*
	Product/ industry	(1)	(2)	…	(*N*)			
Sales by domestic industries	(1)	$P_1^Y X_{11}$	$P_1^Y X_{12}$	…	$P_1^Y X_{1N}$	$P_1^Y \sum_{j=1}^N X_{1j}$	$P_1^Y F_1$	$P_1^Y \left[\sum_{j=1}^N X_{1j} + F_1\right]$
	(2)	$P_2^Y X_{21}$	$P_2^Y X_{22}$	…	$P_2^Y X_{2N}$	$P_2^Y \sum_{j=1}^N X_{2j}$	$P_2^Y F_2$	$P_2^Y \left[\sum_{j=1}^N X_{2j} + F_2\right]$
	…	…	…	⋱	…	…	…	…
	(*N*)	$P_N^Y X_{N1}$	$P_N^Y X_{N2}$	…	$P_N^Y X_{NN}$	$P_N^Y \sum_{j=1}^N X_{Nj}$	$P_N^Y F_N$	$P_N^Y \left[\sum_{j=1}^N X_{Nj} + F_N\right]$
Sales of imports	(1)	$P_1^M M_{11}$	$P_1^M M_{12}$	…	$P_1^M M_{1N}$	$P_1^M \sum_{j=1}^N M_{1j}$	$P_1^M M_1^{FD}$	$P_1^M M_1^{all}$
	(2)	$P_2^M M_{21}$	$P_2^M M_{22}$	…	$P_2^M M_{2N}$	$P_2^M \sum_{j=1}^N M_{2j}$	$P_2^M M_2^{FD}$	$P_2^M M_2^{all}$
	…	…	…	⋱	…	…	…	…
	(*N*)	$P_N^M M_{N1}$	$P_N^M M_{N2}$	…	$P_N^M M_{NN}$	$P_N^M \sum_{j=1}^N M_{Nj}$	$P_N^M M_N^{FD}$	$P_N^M M_N^{all}$
Intermediate consumption		$\sum_{i=1}^N [X_{i1} + M_{i1}]$	$\sum_{i=1}^N [X_{i1} + M_{i1}]$	…	…	$\sum_{i=1}^N \sum_{j=1}^N \left[X_{ij} + M_{ij}\right]$	$\sum_{i=1}^N \left[M_i^{FD} + F_i\right]$	$\sum_{i=1}^N \left[M_i + F_i + \sum_{j=1}^N X_{ij}\right]$
Column total	Value added	$P_1^{VA} VA_1$	$P_2^{VA} VA_2$	…	$P_N^{VA} VA_N$	$\sum_{j=1}^N P_j^{VA} VA_j$	–	–
	Gross output	$\bar{P}_1^Y GO_1$	$\bar{P}_2^Y GO_2$	…	$\bar{P}_N^Y GO_N$	$\sum_{j=1}^N \bar{P}_j^Y GO_j$	–	–

Key See Tables 1-3

Table 5
Supply table in year *t*, product by industry (*N*x*N*), at current (year *t*) basic and purchasers' prices

	Product	Total domestic output at basic prices	Imports	Margins	Taxes less subsidies on products	Total supply at purchasers' prices (row total)
Sales	(1)	$\sum_{j=1}^{N} Y_{1j}$	M_1^{all}	$MARG_1$	T_1	S_1
	(2)	$\sum_{j=1}^{N} Y_{2j}$	M_2^{all}	$MARG_2$	T_2	S_2
	…	…	…	…	…	…
	(*N*)	$\sum_{j=1}^{N} Y_{Nj}$	M_N^{all}	$MARG_N$	T_N	S_N
Column total	Gross output	$\sum_{j=1}^{N} GO_j$	$M^{all}\left[=\sum_i M_i^{all}\right]$	$0\left[=\sum_{i=1}^{N} MARG_i\right]$	$\sum_{i=1}^{N} T_i$	$\sum_{i=1}^{N} S_i$

Key See Tables 1-4. Also

T_i : taxes less subsidies on product i ;. $MARG_i$: transport and trade margins on product i .

Note $S_i := \sum_{j=1}^{N} Y_{ij} + M_i^{all} + MARG_i + T_i, \quad i = 1,..., N.$

The bottom right hand element is both the sum of the row totals and the sum of the column totals.

Table 6
Input-output tables, 1968 to the present

Table year	*Consistent with Blue Book of*	*SIC*	*No. of industry groups*	*Make matrix?*
1968	1972	1968	91	Yes
1974	1980	1968	102	Yes
1979	1982	1980	100	Yes
1984	?*	1980	100	Yes
1985**	1988	1980	102	Yes
1990	1993	1980	123	Yes
1995	2001	1992	123	No
2005	2009	2003 (Nace Rev 1.1)	108	No
2010	2013	2007 (Nace Rev 2)	114	No
2013	2016	2007 (Nace Rev 2)	105	No

* No longer downloadable from the National Archives owing to broken links.
** An update of the 1984 benchmark tables.

Source: ONS.

Table 7
Growth rates under single and double deflation, 1997-2015: unweighted means across 79 industries

		1997-2007	*2007-2015*	*1997-2015*
Mean growth rate, % pa	Single (official)	1.84	0.46	1.23
	Double (Laspeyres)	0.62	0.40	0.52
	Double (Törnqvist)	0.13	-0.59	-0.19
S.d. of growth rate, % pa	Single (official)	5.05	7.02	6.32
	Double (Laspeyres)	13.00	15.70	14.85
	Double (Törnqvist)	12.76	16.09	14.93
Correlation coefficients between different measures of growth	Single with double (Laspeyres)	0.72	0.78	0.77
	Single with double (Törnqvist)	0.72	0.78	0.77
	Törnqvist with Laspeyres	1.00	1.00	0.99

Source: SUTs, low level aggregates spreadsheet, 2017, and own calculations. Growth rates are calculated as 100 x log differences.

Table 8
Weighted mean growth rates by broad sector, 1997-2015, % pa
(nominal value added weights)

	Single (Official)	Double (Laspeyres)	Double (Törnqvist)
Production + utilities	-0.59	-1.59	-2.28
Market services	2.46	2.32	1.65
Public services	1.86	0.03	-0.12
Whole economy (GDP)	1.34	0.52	0.16

Note Industry growth rates of real value added are weighted together using nominal value added shares in total value added as the weights. The Single (Official) and Double (Laspeyres) use the chained Laspeyres formula of equation (8), converted to 100 times the log change, i.e. $100 \times \log(1 + g_t^O)$ The Double (Törnqvist) uses the chained Törnqvist formula of equation (32).

Source Industry growth rates from Table 8; value added from the SUTs.

Table 9
Mean growth rates, % pa, 1997-2015

SIC07	1997-2007 Single	1997-2007 Törnqvist	1997-2007 Laspeyres	2007-2015 Single	2007-2015 Törnqvist	2007-2015 Laspeyres	1997-2015 Single	1997-2015 Törnqvist	1997-2015 Laspeyres
1	1.17	3.31	2.09	1.94	3.60	2.53	1.51	3.44	2.29
2	-1.19	-5.83	-6.20	5.36	7.33	4.45	1.72	0.02	-1.47
3	-6.45	-16.38	-16.78	1.17	-8.35	-13.46	-3.06	-12.81	-15.30
5	-12.17	-34.80	-29.65	-9.21	-45.76	-43.42	-10.85	-39.67	-35.32
06-07	-4.31	-6.26	-6.60	-6.93	-9.89	-11.31	-5.48	-7.87	-8.69
8	4.78	6.53	6.75	-2.68	-2.64	-4.66	1.46	2.46	1.68
9	0.56	-0.87	-0.74	4.37	5.36	4.86	2.25	1.90	1.75
10-11	0.09	-0.02	-0.43	0.73	0.49	0.12	0.37	0.21	-0.18
13	-4.97	-2.61	-2.89	-3.94	-4.93	-5.46	-4.51	-3.64	-4.03
14	-5.98	-7.12	-7.44	-0.49	3.77	2.16	-3.54	-2.28	-3.17
15	-9.10	-10.59	-13.27	-0.49	-1.08	-1.76	-5.27	-6.37	-8.15
16	0.28	0.73	0.54	-3.78	-8.98	-9.44	-1.52	-3.59	-3.89
17	-0.41	-0.30	-0.60	-0.85	1.72	1.27	-0.61	0.60	0.23
18	-0.77	-1.95	-2.15	-2.73	-4.61	-5.47	-1.64	-3.13	-3.62
19	-1.87	2.64	-0.26	-3.63	13.33	-14.89	-2.65	7.39	-6.76
20	0.87	-0.81	-1.05	-0.42	4.38	3.40	0.30	1.50	0.93
21	5.04	7.65	7.53	-3.34	-6.13	-6.30	1.31	1.52	1.39
22	-0.37	-0.25	-0.48	-2.25	-3.20	-3.74	-1.21	-1.56	-1.93
23	1.64	-0.32	-0.78	-2.14	-4.31	-6.18	-0.04	-2.09	-3.18
24	-0.83	-2.38	-2.84	-2.51	-1.33	-5.15	-1.58	-1.91	-3.87
25	1.10	0.78	0.57	-1.54	0.01	-0.22	-0.07	0.44	0.22
26	-1.38	3.10	2.50	-2.05	0.88	0.52	-1.68	2.11	1.62
27	-0.58	0.07	-0.39	-2.24	-4.48	-6.28	-1.32	-1.95	-3.01
28	0.58	2.96	2.57	-2.30	-7.27	-8.82	-0.70	-1.59	-2.49
29	0.61	-3.73	-4.38	2.15	8.20	2.44	1.29	1.58	-1.35
30	2.08	0.20	-0.22	5.83	10.44	8.54	3.75	4.75	3.67
31	-0.00	-0.02	-0.76	-0.60	2.39	1.82	-0.27	1.05	0.39
32	-0.60	-1.43	-1.77	-0.60	1.22	0.76	-0.60	-0.25	-0.64
33	0.73	1.76	0.76	1.74	2.72	2.21	1.18	2.19	1.41
35	2.00	3.59	3.17	-1.11	-4.37	-4.83	0.62	0.05	-0.38
36	-1.08	-1.99	-2.12	-0.57	-1.67	-1.76	-0.86	-1.84	-1.96
37	4.16	4.63	4.68	1.56	1.40	1.28	3.01	3.19	3.16
38	4.27	3.61	2.70	1.26	1.35	0.66	2.93	2.61	1.80
39	3.65	2.14	8.05	2.44	6.99	11.24	3.11	4.30	9.47
41-43	2.13	-0.92	-1.12	0.10	-0.79	-1.13	1.22	-0.86	-1.13
45	2.03	0.73	0.30	2.54	4.02	3.64	2.26	2.19	1.78
46	0.73	-0.73	-1.34	0.31	1.06	0.75	0.54	0.06	-0.41
47	3.38	2.29	2.19	1.08	0.50	0.40	2.35	1.50	1.40
49	1.90	1.81	1.52	0.27	1.16	0.60	1.18	1.52	1.11
50	-1.08	-10.08	-11.97	-2.60	-10.63	-12.10	-1.76	-10.33	-12.03
51	4.77	10.02	9.23	1.25	6.46	5.21	3.20	8.44	7.44

52	4.35	3.88	3.65	-0.42	-0.96	-1.22	2.23	1.73	1.49
53	4.73	4.97	4.61	-2.17	-7.72	-8.85	1.66	-0.67	-1.37
55	2.76	2.08	1.71	2.96	5.64	5.39	2.85	3.67	3.35
56	3.24	3.05	2.76	-0.06	0.17	-0.10	1.77	1.77	1.49
58	0.37	0.71	0.56	-3.13	-3.00	-3.17	-1.18	-0.94	-1.10
59-60	3.85	2.77	2.48	4.71	6.75	6.30	4.23	4.54	4.18
61	12.61	16.77	16.14	1.02	1.78	1.50	7.46	10.11	9.63
62	8.21	7.77	7.53	3.69	4.63	4.50	6.20	6.38	6.19
63	9.29	11.69	11.15	2.60	2.24	2.06	6.32	7.49	7.11
64	7.00	11.17	10.57	-2.12	-5.63	-5.89	2.95	3.70	3.25
65	-0.02	-26.59	-24.14	-1.10	4.55	2.47	-0.50	-12.75	-12.31
66	3.24	0.61	-0.03	-0.20	0.60	0.13	1.71	0.61	0.04
68	2.53	2.27	2.25	2.06	1.43	1.92	2.32	1.90	2.10
69	6.33	6.68	6.59	2.01	2.56	2.50	4.41	4.85	4.77
70	12.25	13.17	12.47	3.92	5.02	4.56	8.55	9.55	8.95
71	6.05	5.22	5.07	2.61	4.74	4.55	4.52	5.00	4.84
72	7.70	7.55	7.29	2.52	0.19	0.02	5.40	4.28	4.06
73	1.67	-3.07	-3.74	2.89	2.72	2.06	2.21	-0.50	-1.16
74	2.72	0.95	0.54	3.20	3.33	2.73	2.93	2.01	1.51
75	4.37	6.13	6.17	3.78	2.33	2.21	4.11	4.44	4.41
77	4.04	4.54	4.34	4.82	7.62	7.34	4.39	5.91	5.67
78	8.91	10.74	10.00	7.01	7.74	7.59	8.07	9.41	8.93
79	-0.67	-4.90	-5.87	-0.96	4.24	4.07	-0.80	-0.84	-1.45
80	5.30	4.01	2.67	1.29	2.93	2.41	3.52	3.53	2.55
81	6.39	7.50	7.15	0.66	-2.36	-2.57	3.84	3.12	2.83
82	3.73	3.34	2.84	4.81	5.19	4.94	4.21	4.16	3.77
84	1.14	-3.39	-3.78	-1.26	-1.30	-1.51	0.07	-2.46	-2.77
85	0.60	-0.64	-0.72	0.71	0.63	0.56	0.65	-0.08	-0.15
86	4.00	3.09	2.99	3.30	1.74	1.67	3.69	2.49	2.40
87-88	3.24	1.31	1.18	1.28	-1.53	-1.69	2.37	0.04	-0.10
90	2.50	1.61	0.42	2.48	4.16	3.77	2.49	2.74	1.91
91	1.96	-0.85	-1.36	1.07	2.89	2.40	1.56	0.82	0.31
92	1.80	-0.24	-0.39	-1.92	-2.54	-2.62	0.15	-1.26	-1.38
93	2.72	2.03	1.12	-1.61	-2.03	-2.74	0.79	0.23	-0.60
94	1.89	-0.49	-1.02	3.57	3.96	3.82	2.64	1.49	1.13
95	-1.48	-5.21	-21.68	6.30	8.34	7.62	1.98	0.81	-8.66
96	1.44	0.75	0.51	1.12	2.10	2.00	1.30	1.35	1.17
97	-0.88	-0.87	-0.88	-0.06	-0.05	-0.06	-0.52	-0.50	-0.52
AVER-AGE	1.84	0.62	0.13	0.46	0.40	-0.59	1.23	0.52	-0.19

Table 10
Standard deviations of mean growth rates, % pa, 1997-2015

	1997-2008			**2008-2015**			**1997-2015**		
SIC07	Single	Törnqvist	Laspeyres	Single	Törnqvist	Laspeyres	Single	Törnqvist	Laspeyres
1	6.97	16.91	17.35	7.82	21.53	21.46	7.15	18.50	18.68
2	6.91	13.95	14.01	10.77	33.60	30.12	9.18	24.76	22.52
3	6.77	17.29	17.62	5.47	34.65	36.79	7.20	25.88	26.92
5	7.97	67.48	57.42	13.79	80.83	87.97	10.69	71.64	69.32
06-07	5.59	8.11	8.35	9.37	19.29	19.24	7.38	13.84	13.97
8	4.84	15.63	14.41	24.17	34.13	35.16	16.36	25.12	25.55
9	7.03	7.51	7.43	5.26	11.09	11.15	6.43	9.52	9.42
10-11	1.46	3.07	3.34	3.63	5.37	5.32	2.58	4.12	4.20
13	4.52	9.29	9.02	4.59	13.59	13.88	4.45	11.10	11.14
14	6.20	12.52	12.19	7.74	15.94	15.88	7.27	14.79	14.37
15	9.60	23.28	29.10	11.19	22.73	22.45	10.94	22.88	26.28
16	2.96	9.46	9.36	8.17	10.27	10.32	6.04	10.74	10.78
17	2.19	9.73	9.72	4.42	9.75	9.76	3.26	9.50	9.49
18	2.11	5.57	5.57	3.76	11.25	10.37	3.03	8.39	7.98
19	4.43	36.63	40.47	4.71	82.77	106.29	4.51	59.67	74.66
20	2.08	5.04	4.83	6.34	18.61	18.13	4.39	12.77	12.36
21	4.63	7.55	7.41	6.38	8.63	8.82	6.82	10.51	10.54
22	2.33	6.47	6.40	7.35	12.28	12.11	5.10	9.30	9.21
23	2.77	8.99	9.44	9.50	21.57	22.18	6.70	15.45	16.04
24	5.50	11.63	11.74	12.90	46.35	44.76	9.24	30.93	29.99
25	2.23	7.52	7.33	8.06	13.82	13.90	5.59	10.43	10.40
26	8.85	16.83	16.09	3.23	5.69	5.71	6.77	12.83	12.31
27	4.55	13.13	12.89	11.21	26.65	29.95	7.96	19.73	21.60
28	3.34	6.97	6.93	13.29	29.20	29.84	8.99	20.10	20.64
29	4.70	11.15	11.45	16.26	47.08	54.42	11.01	31.87	36.07
30	7.23	13.31	13.03	7.59	18.06	17.63	7.42	15.98	15.42
31	3.38	11.97	13.03	7.86	12.78	12.97	5.62	12.03	12.68
32	1.99	6.78	6.85	5.81	8.89	8.76	4.00	7.66	7.62
33	8.67	10.99	10.76	4.16	11.12	11.18	6.87	10.73	10.65
35	1.81	9.00	8.37	3.64	15.44	16.16	3.12	12.56	12.70
36	1.38	3.85	3.80	4.75	7.76	7.88	3.22	5.71	5.77
37	3.41	3.14	3.18	7.51	9.53	9.54	5.58	6.73	6.77
38	6.37	16.88	16.85	6.61	17.18	17.64	6.47	16.54	16.72
39	5.87	59.40	40.99	3.83	33.12	24.05	4.96	48.23	33.62
41-43	2.44	3.62	3.75	7.79	9.42	9.58	5.40	6.59	6.72

45	3.48	8.46	8.98	8.06	11.69	11.72	5.76	9.85	10.11
46	4.78	11.28	11.76	6.88	10.37	10.13	5.63	10.61	10.80
47	1.67	3.20	3.24	1.81	1.76	1.74	2.05	2.75	2.77
49	2.84	6.00	6.00	4.99	6.86	6.92	3.90	6.21	6.25
50	4.50	17.37	20.91	8.91	14.72	16.12	6.64	15.78	18.40
51	4.80	10.12	9.38	7.08	15.11	14.30	6.01	12.31	11.62
52	3.81	6.07	6.03	6.22	8.28	8.36	5.44	7.34	7.37
53	8.45	12.63	12.66	10.31	19.33	20.28	9.70	16.75	17.37
55	1.57	7.33	7.67	5.18	7.10	6.93	3.52	7.25	7.38
56	1.57	6.01	6.08	3.91	6.29	6.24	3.23	6.13	6.14
58	2.88	4.17	4.16	4.70	6.49	6.63	4.08	5.49	5.56
59-60	6.32	9.58	9.45	6.82	12.59	12.38	6.36	10.86	10.69
61	9.97	15.47	14.67	3.76	7.83	7.90	9.68	14.51	13.99
62	5.05	5.20	5.26	4.47	5.24	5.22	5.20	5.31	5.31
63	10.05	16.51	16.12	8.41	9.68	9.68	9.71	14.36	14.06
64	1.22	10.71	10.27	2.93	8.38	8.61	5.10	12.78	12.54
65	3.18	91.59	49.72	2.70	22.65	23.08	2.94	70.04	41.39
66	5.35	10.88	11.46	8.91	14.42	14.72	7.14	12.18	12.60
68	1.48	2.27	2.27	1.07	7.10	5.60	1.30	4.86	3.96
69	4.35	5.51	5.47	4.75	6.10	6.08	4.92	5.98	5.96
70	10.99	18.39	17.71	9.32	15.97	16.06	10.86	17.36	16.99
71	6.76	8.90	8.83	6.22	8.61	8.50	6.57	8.51	8.43
72	6.80	9.36	9.34	6.28	9.76	9.73	6.91	9.99	9.95
73	4.23	7.37	8.22	8.21	19.90	19.59	6.13	14.16	14.23
74	7.07	11.07	11.44	10.01	17.97	17.91	8.23	14.12	14.23
75	1.33	6.95	6.53	4.94	8.86	8.67	3.33	7.85	7.59
77	3.83	6.16	6.07	10.65	12.23	12.02	7.39	9.18	9.02
78	12.29	17.85	17.66	12.34	9.89	9.99	11.98	14.53	14.41
79	10.05	17.92	18.68	8.64	9.29	9.17	9.18	15.08	15.66
80	10.19	20.75	21.28	8.73	16.13	15.63	9.52	18.31	18.45
81	5.72	10.31	9.98	5.13	7.58	7.68	6.06	10.26	10.08
82	8.09	12.54	12.81	7.33	11.26	11.46	7.56	11.68	11.92
84	1.55	2.67	2.90	1.91	3.59	3.59	2.07	3.20	3.33
85	2.06	2.76	2.78	1.53	2.73	2.73	1.79	2.74	2.76
86	1.19	1.95	1.96	1.12	3.51	3.48	1.18	2.75	2.73
87-88	1.81	3.22	3.30	3.25	5.14	5.17	2.66	4.30	4.35
90	7.40	18.48	18.10	7.84	16.25	16.10	7.37	17.06	16.82
91	4.85	13.51	13.06	7.73	13.71	13.58	6.10	13.33	13.04
92	1.84	4.50	4.50	7.93	9.22	9.22	5.60	6.86	6.86
93	4.73	9.41	9.93	5.43	14.78	15.22	5.38	11.88	12.30

94	12.58	17.42	18.14	6.11	8.14	7.91	9.99	13.90	14.35
95	14.83	43.57	81.29	11.01	19.60	19.13	13.50	34.80	62.24
96	4.91	7.96	8.01	5.21	7.49	7.42	4.90	7.56	7.57
97	3.27	3.27	3.27	6.78	6.78	6.78	4.98	4.98	4.98
AVER-AGE	5.05	13.00	12.76	7.02	15.70	16.09	6.32	14.55	14.93

Table 11
Correlation coefficients between different measures of mean growth rates, 1997-2015

	1997-2007			2007-2015			1997-2015		
SIC07	Single & T.	Single & L.	T. & L.	Single & T.	Single & L.	T. & L.	Single & T.	Single & L.	T. & L.
1	0.87	0.85	1.00	0.90	0.89	1.00	0.88	0.87	1.00
2	0.92	0.92	1.00	0.82	0.87	0.99	0.85	0.88	0.99
3	0.75	0.79	0.98	0.50	0.53	0.99	0.54	0.52	0.98
5	0.53	0.56	0.96	0.87	0.53	0.86	0.71	0.49	0.88
06-07	0.96	0.96	1.00	0.98	0.97	1.00	0.96	0.96	1.00
8	0.01	0.03	1.00	0.99	0.99	1.00	0.86	0.88	1.00
9	0.93	0.94	1.00	0.43	0.46	1.00	0.70	0.71	1.00
10-11	0.48	0.42	0.99	0.83	0.82	1.00	0.74	0.71	1.00
13	0.90	0.90	1.00	0.98	0.98	1.00	0.90	0.90	1.00
14	0.60	0.61	1.00	0.39	0.44	0.99	0.56	0.58	1.00
15	0.84	0.83	0.99	0.98	0.98	1.00	0.89	0.87	0.99
16	0.67	0.67	1.00	0.93	0.93	1.00	0.81	0.81	1.00
17	0.46	0.43	1.00	0.73	0.74	1.00	0.58	0.57	1.00
18	0.68	0.68	1.00	0.47	0.54	1.00	0.55	0.60	0.99
19	0.58	0.62	0.97	0.20	0.14	0.94	0.29	0.28	0.93
20	0.47	0.48	1.00	0.73	0.75	1.00	0.65	0.67	1.00
21	0.71	0.71	1.00	0.89	0.88	1.00	0.89	0.89	1.00
22	0.77	0.76	1.00	0.90	0.89	1.00	0.86	0.85	1.00
23	0.49	0.49	1.00	0.93	0.92	1.00	0.86	0.85	1.00
24	0.73	0.74	1.00	0.97	0.96	1.00	0.92	0.92	0.99
25	0.65	0.66	1.00	0.94	0.94	1.00	0.85	0.86	1.00
26	0.71	0.72	1.00	0.15	0.19	1.00	0.66	0.68	1.00
27	0.77	0.78	1.00	0.97	0.96	1.00	0.92	0.93	1.00
28	0.56	0.56	1.00	0.92	0.92	1.00	0.90	0.89	1.00
29	0.74	0.74	1.00	0.89	0.93	0.99	0.88	0.91	0.99
30	0.71	0.71	1.00	0.81	0.76	0.99	0.78	0.75	0.99
31	0.90	0.87	1.00	0.90	0.90	1.00	0.83	0.81	1.00
32	0.71	0.69	1.00	0.80	0.81	1.00	0.72	0.72	1.00
33	0.20	0.28	0.99	0.56	0.58	1.00	0.28	0.34	0.99
35	0.34	0.32	1.00	-0.16	-0.08	0.99	0.14	0.18	0.99
36	0.70	0.71	1.00	0.96	0.97	1.00	0.90	0.91	1.00
37	0.72	0.72	1.00	0.93	0.92	1.00	0.89	0.89	1.00
38	0.98	0.99	1.00	0.95	0.96	1.00	0.96	0.96	1.00
39	0.47	0.42	0.96	0.13	0.06	0.96	0.39	0.33	0.96
41-43	0.42	0.42	1.00	0.88	0.88	1.00	0.80	0.80	1.00

45	0.71	0.71	1.00	0.90	0.90	1.00	0.82	0.81	1.00
46	0.83	0.83	1.00	0.90	0.90	1.00	0.84	0.83	1.00
47	0.92	0.92	1.00	0.67	0.67	1.00	0.81	0.81	1.00
49	0.93	0.92	1.00	0.63	0.63	1.00	0.72	0.73	1.00
50	0.58	0.49	0.98	0.81	0.79	1.00	0.65	0.58	0.98
51	0.58	0.62	1.00	0.78	0.79	1.00	0.71	0.74	1.00
52	0.89	0.89	1.00	0.82	0.82	1.00	0.86	0.86	1.00
53	0.96	0.96	1.00	0.90	0.90	1.00	0.93	0.93	1.00
55	0.85	0.85	1.00	0.89	0.90	1.00	0.74	0.73	1.00
56	0.83	0.83	1.00	0.76	0.76	1.00	0.72	0.72	1.00
58	0.88	0.88	1.00	0.92	0.91	1.00	0.91	0.91	1.00
59-60	0.99	0.99	1.00	0.90	0.89	1.00	0.93	0.93	1.00
61	0.97	0.97	1.00	0.81	0.82	1.00	0.96	0.96	1.00
62	0.39	0.36	1.00	0.97	0.97	1.00	0.67	0.65	1.00
63	0.98	0.98	1.00	0.92	0.92	1.00	0.96	0.96	1.00
64	0.42	0.44	1.00	0.74	0.75	1.00	0.77	0.78	1.00
65	-0.28	-0.07	0.95	-0.19	-0.18	0.99	-0.27	-0.15	0.94
66	0.95	0.95	1.00	0.94	0.94	1.00	0.91	0.90	1.00
68	0.98	0.98	1.00	0.31	0.32	1.00	0.44	0.50	0.99
69	0.94	0.94	1.00	0.97	0.97	1.00	0.96	0.96	1.00
70	0.96	0.96	1.00	0.98	0.97	1.00	0.95	0.95	1.00
71	0.96	0.97	1.00	0.96	0.97	1.00	0.93	0.94	1.00
72	0.87	0.88	1.00	0.74	0.75	1.00	0.84	0.85	1.00
73	0.60	0.58	1.00	0.95	0.96	1.00	0.87	0.87	1.00
74	0.98	0.97	1.00	0.98	0.98	1.00	0.97	0.98	1.00
75	-0.06	-0.04	1.00	0.69	0.71	1.00	0.49	0.51	1.00
77	0.93	0.93	1.00	0.98	0.99	1.00	0.96	0.96	1.00
78	0.96	0.95	1.00	0.91	0.91	1.00	0.91	0.91	1.00
79	0.97	0.97	1.00	0.85	0.85	1.00	0.87	0.86	1.00
80	0.96	0.95	1.00	0.98	0.98	1.00	0.95	0.93	1.00
81	0.95	0.95	1.00	0.88	0.88	1.00	0.94	0.94	1.00
82	0.95	0.95	1.00	0.86	0.86	1.00	0.92	0.91	1.00
84	0.33	0.27	1.00	0.61	0.61	1.00	0.17	0.14	1.00
85	0.82	0.82	1.00	0.93	0.93	1.00	0.84	0.83	1.00
86	0.63	0.62	1.00	0.26	0.28	1.00	0.44	0.45	1.00
87-88	0.74	0.74	1.00	0.73	0.72	1.00	0.76	0.76	1.00
90	0.98	0.98	1.00	0.70	0.71	1.00	0.85	0.86	1.00
91	0.73	0.73	1.00	0.98	0.98	1.00	0.82	0.83	1.00
92	0.09	0.08	1.00	0.90	0.91	1.00	0.78	0.78	1.00
93	0.74	0.75	1.00	0.88	0.88	1.00	0.80	0.80	1.00

94	1.00	1.00	1.00	0.87	0.88	1.00	0.98	0.98	1.00
95	0.95	0.86	0.94	0.87	0.88	1.00	0.91	0.82	0.93
96	0.96	0.96	1.00	0.87	0.87	1.00	0.91	0.91	1.00
97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
AVER-AGE	0.72	0.72	1.00	0.78	0.78	1.00	0.77	0.77	0.99

Annex Simple algebraic proofs of Propositions 1 and 3

Proposition 1

In current basic prices, GDP(O)=GDP(E) where

$$GDP(O) = \sum_{j=1}^{N} VA_j$$

$$GDP(E) = \sum_{i=1}^{N} F_i - \sum_{i=1}^{N} \sum_{j=1}^{N} M_{ij}$$

Proof

From the definition of GDP(E) and since total use must equal total supply,

$$GDP(E) = \sum_{i=1}^{N} F_i - \sum_{i=1}^{N} \sum_{j=1}^{N} M_{ij} = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[Y_{ij} - X_{ij} - M_{ij} \right]$$

$$= \sum_{j=1}^{N} \sum_{i=1}^{N} \left[Y_{ij} - X_{ij} - M_{ij} \right] \text{ (reversing the order of summation)}$$

$$= \sum_{j=1}^{N} VA_j = GDP(O)$$

Proposition 3

At PYP, GDP(O)=GDP(E) where

$$P_{t,t-1}^{GDP} GDP(O) = \sum_{j=1}^{N} P_j^V VA_j$$

$$P_{t,t-1}^{GDP} GDP(E) = \sum_{i=1}^{N} P_i^Y F_i - \sum_{i=1}^{N} \sum_{j=1}^{N} P_i^M M_{ij}$$

Proof

From the definition of GDP(E) and since total use must equal total supply,

$$P_{t,t-1}^{GDP} GDP(E) = \sum_{i=1}^{N} P_i^Y F_i - \sum_{i=1}^{N} \sum_{j=1}^{N} P_i^M M_{ij} = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[P_i^Y Y_{ij} - P_i^Y X_{ij} - P_i^M M_{ij} \right]$$

$$= \sum_{j=1}^{N} \sum_{i=1}^{N} \left[P_i^Y Y_{ij} - P_i^Y X_{ij} - P_i^M M_{ij} \right] \text{ (reversing the order of summation)}$$

$$= \sum_{j=1}^{N} P_i^{VA} VA_j = P_{t,t-1}^{GDP} GDP(O)$$

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