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Failure-Mode-Hierarchy Based Design for Reinforced Concrete Structures

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Notation

CFRP Carbon fibre reinforced polymer reinforcement,

DSP design and safety philosophy,

FMH Failure mode hierarchy,

FRP Fibre reinforced polymer reinforcement,

GFRP Glass fibre reinforced polymer reinforcement,

RC Reinforced concrete,

RCM Resistance-capacity margin between two failure modes,

C_{Cj} Concrete cost for each design configuration, j,

C_{Tj} Total cost of construction for a design configuration j,

C_{Fj} Cost of failure for a design configuration j,

C_{Rj} Reinforcement cost for each design configuration j,

C_{Tj} Total cost of construction for a design configuration j,

E_{FRP} Young's modulus for FRP reinforcement,

M_j Metric function for a design configuration j,

P_f Notional structural reliability level (taken as the probability of failure),

P_{fj} P_f determined for each design configuration,

PVL-ratio Ratio of permanent to variable load,

T Average measure of closeness,

X_i Primary design parameter,

ds Integration parameter,

f_i frequency of occurrence of a design parameter i,

f_c Concrete cylinder compressive strength,

x_i Secondary design parameter,

f_{FRP} Tensile strength of FRP reinforcement,

γ_{FRP-L} Material partial safety factor for longitudinal FRP reinforcement,

γ_{FRP-T} Material partial safety factor for transverse FRP reinforcement,

ε_c Concrete strain developed in the RC beam,

μ_{fcu} Mean value of concrete cube compressive strength,

$\mu_{flexure}$ Mean flexural resistance-capacity,

μ_{shear} Mean shear resistance-capacity,

ρ Ratio of longitudinal reinforcement,

$_i$ Subscript for design parameter,

$_k$ Subscript for characteristic value.

Synopsis

Innovations in concrete construction can be held back by the inability of codes of practice to accommodate new materials. The current design and safety philosophy (DSP) of reinforced concrete relies heavily on the properties of steel reinforcement. The need to embrace new materials, such as fibre reinforced polymer (FRP) reinforcement, led to an in-depth examination of the DSP of European concrete codes of practice and resulted in a new philosophy, presented in this paper. The basis of the new philosophy remains the limit-state design and achievement of target notional structural reliability levels, but aims at the attainment of a desired failure-mode-hierarchy. The implementation of the philosophy, through a proposed framework, utilises the concept of average measure of closeness for the determination of appropriate material partial safety factors. An example of the application of the proposed framework is presented for FRP reinforcement.

Introduction

Codes of practice based on the limit-state design approach[1] (safety level one of structural reliability theory) are currently used to design reinforced concrete (RC) structures. However, when novel techniques and innovative materials, such as FRP reinforcement[2], are developed, these codes of practice are often unable to allow their use in the design of structural elements. Consequently, new codes of practice and design guidelines are often developed specifically to accommodate such innovations. The development and statutory acceptance of new codes of practice is a lengthy process, and restraints the widespread use of innovations in construction.

The inability of concrete codes of practice to adopt construction innovation is deep routed and arises from the code's DSP. The current DSP aims to achieve a ductile flexural failure through yielding of the flexural reinforcement. It is implicit in the code of practice that, through the use of partial safety factors, the notional structural reliability (P_f) of RC elements will satisfy predefined target levels.

Research[3] into the DSP of the British (BS8110[4]) and European (Eurocode-2[5]) concrete codes of practice, demonstrated that design based on balanced RC sections achieves its objectives and leads to safe structures. However, it was found that this principle may occasionally result in RC elements that exhibit a brittle flexural behaviour, especially when using the new partial safety factor (1.05) adopted for steel reinforcement by BS8110 in 1997.

Results of the research also showed that the resistance-capacity margins (expressed as the ratio RCM), such as the one between the flexural and shear failure modes (equation 1, exemplified in figure 1), are not uniform for all flexural elements and hence, this can result in uneconomic structures. It was found that the flexural and shear resistance-capacities of RC elements depend on the value of concrete compressive strength (f_c) and ratio of longitudinal reinforcement (ρ). In particular, the mean flexural resistance-capacity increases proportionally with ρ, whereas the mean shear resistance-capacity increases at a lower rate. Thus, the RCMs increase as ρ decreases. The results also indicated that there is a small possibility of shear failure occurring prior to flexure.

$$RCM_{flex-shear} = \frac{\mu_{shear}}{\mu_{flexure}} \tag{1}$$

The above research also confirmed that the application of these codes of practice will, in general, satisfy the target P_f of 7×10^{-5}, adopted by Eurocode-1[6] for the design working life of a structure. However, it was shown that the structural reliability of flexural elements varies, as P_f is not only affected by the values adopted for load and partial safety factors, but it is also influenced by other design parameters. Primarily, P_f is affected by the ratio of permanent to variable load, improving as this ratio increases. In addition, ρ and f_c influence the flexural and shear P_f due to the effect of these parameters on the flexural and shear resistance-capacities. This indicates that uniform structural reliability can not be attained for all types of structures (e.g. buildings and bridges) and failure modes (limit states), unless limits are imposed on the most influential design parameters. Some reliability differentiation may be desirable for important structures, such as bridges and nuclear installations, and for undesirable failure modes (e.g. shear and bond).

A thorough consideration of the above issues has led to the development of a new DSP as a result of the need to develop new design guidelines for FRP RC elements. The basis of the philosophy remains the limit-state design approach, but it requires the attainment of a desired failure-mode-hierarchy (FMH) with the target P_f and RCMs being satisfied. After introducing the work on the DSP of FRP RC design guidelines, the paper presents a framework, based on the new DSP, that can be used to develop a general concrete code of practice. The second part of the paper gives an example of the application of the framework that leads to the determination of short-term partial safety factors (γ_{FRP}) for the design of FRP RC elements.

The determined γ_{FRP} are used for the design of FRP RC beams. The implications of the proposed DSP for designers, code-developers and manufacturers of FRP reinforcement are discussed at the end.

Work on design guidelines for FRP RC structures

Over the last decade, a number of design guidelines have been developed specifically for FRP RC structures. The majority of them, like the European[7] guideline, are primarily based on modifications of concrete codes of practice. As a result, these guidelines do not have an identifiable DSP. Examination[8,9] of the DSP of the European design guideline led to the general conclusion that the design of FRP RC elements can not be based on the philosophy developed for steel RC elements. The main findings are summarised below.

- For normal-strength concrete, the tensile strength of longitudinal FRP reinforcement is not fully utilised and hence, concrete crushing is the most probable type of flexural failure.
- The use of partial safety factors for longitudinal reinforcement (γ_{FRP-L}) may not be essential for the design of FRP RC beams, if the type of flexural failure intended at design is due to concrete crushing.
- The assumption that the use of a specific value of γ_{FRP-L} would always lead to the desired type of flexural failure is not valid for all design configurations, especially for the large values of γ_{FRP-L}.
- The effect of ρ and f_c on the flexural and shear P_f and on RCM is influenced by the type of failure for which the flexural design is performed.
- The ratio of permanent to variable load (PVL-ratio) has the greatest effect on P_f and thus, different partial safety factors could be used for different types of structures

Proposed design and safety philosophy:

The new DSP is still based on the limit-state design approach[1] and achievement of target P_f, but it aims at the attainment of a desired FMH and target RCMs. One of the main features of the philosophy is the differentiation of structural reliability for each type of structure covered by the code of practice. The philosophy can be utilised to develop new design guidelines for new materials that can be incorporated in a general concrete code of practice.

Design framework based on new DSP

The framework illustrated below (Fig. 2) is proposed for the implementation of the new DSP. This framework is proposed to be adopted as part of the overall code-development process presented by Nowak and Lind[10].

The first step (1.1) is carried out as part of the general definition of the scope and data space of the code of practice. In addition to defining the structural materials, types, elements and structural functions covered by the code of practice, this step involves the definition of all possible failure modes that can be predicted for each design configuration within the data space.

In the second step (1.2), the primary failure modes are evaluated for each structural element to be covered by the code of practice. This involves the classification of each failure mode in terms of the type of failure it represents and the seriousness of the structural damage sustained due to the occurrence of failure.

The selection of the primary failure modes is followed by the criteria for the formulation of FMHs for each type of element (step 1.3). A FMH should account for all the primary failure modes. The sequence of the failure modes in the hierarchy should follow the degree of undesirability of each failure mode. Thus, the most favourable failure mode is placed at the top of the hierarchy, whereas the least favourable one is placed at the bottom of the hierarchy. For example, Fig. 3 shows the FMH for an RC beam where the flexural failure is more desirable and bond failure is least desirable.

Once the criteria to establish FMHs for the entire data space are established, rules for establishing appropriate RCMs between each critical failure mode are specified (step 1.4). This step is performed for all FMHs. In the example of Fig. 3, a lower RCM was chosen for the shear-bond than for the flexure-shear failure mode. Although, the definition of appropriate RCMs is likely to be heavily influenced by the relative cost of the excess resistance-capacity, it is logical to assume that a substantial RCM will be required between the most favourable failure mode and the least favourable one in the hierarchy.

The last step (1.5) in the procedure involves the definition of a target P_f, which can be determined by considering social, economic and socio-economic constraints that are relevant to structures. Alternatively, the target P_f can be determined by calibration to existing design practices and experience.

Application of framework to specific materials

The proposed framework, if adopted by codes of practice, could enable (through the following procedure) the determination of appropriate partial safety factors for specific new reinforcing materials, as illustrated in Fig. 4.

The first step (2.1) in the procedure involves the definition of appropriate FMHs and RCMs for each type of structural elements using the specific reinforcing materials. At this step, appropriate models are developed to predict the elements' resistance-capacity for each failure-mode (contained in the selected FMHs).

In step 2.2, different design configurations are chosen for each type of element from the entire data space of the code, and their structural reliability is probabilistically assessed for the failure modes contained in each FMH. This is performed for different partial safety factors. Each design configuration is checked to establish whether it satisfies the target RCMs and P_f.

Based on the above, a set of partial safety factors is determined for each FMH (step 2.3) by utilising the concept of average measure of closeness between the code and the target P_f, as presented by Nowak and Lind[10]. The objective of this concept is to optimise the structural design by comparing the structural reliability with the corresponding expected total cost of construction.

It must be emphasised that, to attain the desired RCMs, it may be necessary to impose limits on the design parameters considered by each limit-state prediction model. This would result in the attainment of the chosen FMH and the satisfaction of the target P_f. Since the new philosophy aims at the differentiation of structural reliability for various types of structures, individual partial safety factors should be specified for each type of structure (e.g. buildings, bridges) covered by the code of practice.

Application of framework for CFRP and GFRP reinforcement:

The validity of the proposed framework needs to be demonstrated for a number of structural elements (e.g. beams, columns, walls) and reinforcing materials (e.g. steel, FRP and stainless steel). For simplicity, in the current study, the framework is demonstrated only for the case of concrete beams, reinforced with either carbon (CFRP) or glass (GFRP) FRP reinforcement (both longitudinal and transverse). As in the previous section, the first stage involves the establishment of the design framework. The second stage deals with the determination of partial safety factors for the selected reinforcing materials.

Establishment of design framework

The first step (1.1) in this procedure involves the definition of all possible failure modes for each type of element covered by the code of practice. The following failure modes are defined as possible for the ultimate limit-state design of CFRP and GFRP RC beams.

1.A Flexure due to concrete crushing.
1.B Flexure due to fracture of longitudinal FRP reinforcement.
2.A Shear due to fracture (or lack) of transverse FRP reinforcement.
2.B Shear due to concrete failure.
3.A Torsion
4.A Bond due to splitting.
4.B Bond due to splicing.
4.C Bond due to anchorage.

The definition of the possible failure modes is followed by the identification of the primary modes and classification according to their seriousness (step 1.2). Flexural failure due to reinforcement fracture is unlikely due to the unrealistically low ρ needed to achieve it. The most likely type of flexural failure to be sustained by FRP RC beams is concrete crushing. Shear failure due to concrete failure is only likely in un-reinforced or over-reinforced in shear RC beams. Both situations are normally covered by using lower and upper limits for transverse reinforcement. Hence, the most likely shear failure mode is failure due to fracture of the transverse reinforcement. Since torsional stresses are normally small in RC beams, this mode is not considered in this study. Bond failure may be desirable, if a pseudo-ductile bond behaviour can be ensured, however this is not the case for the reinforcing bars under examination. Though the bond characteristics of FRP re-bars are generally good[11], the models for design are still being developed and as a result, this mode of failure is not considered in the present study. Flexural failure due to concrete crushing and shear failure due to fracture of transverse reinforcement are therefore selected as the primary modes of failure.

The establishment of criteria for the formulation of the desired FMHs is the next step (1.3) in the procedure. Both primary modes of failure selected are brittle in nature, even though the flexural mode dissipates some inelastic energy through concrete. From the two modes, the flexural mode has the most reliable prediction models and as result, is selected as the predominant mode of failure for design purposes.

The formulation of the FMH is followed by the definition of target values for the RCMs (step 1.4). It must be emphasised that such targets have never been discussed by code developers, standardisation committees and researchers. Therefore, the RCMs, evaluated for steel RC beams in previous studies[3,8] are presumed to be sufficient. Similarly, the value of 7 10^{-5} (set as a target P_f by Eurocode-1[6]) is considered to be appropriate.

Determination of appropriate γ_{FRP}

The establishment of the design framework is followed by the determination of appropriate short-term partial safety factors for the chosen materials. Table 1 summarises the tensile strength and Young's Modulus of the CFRP and GFRP reinforcement used in this study, which was developed during the Eurocrete project[12,13].

The first step (2.1) in this procedure, which has to be followed for any new material, involves the definition of appropriate FMHs and RCMs for the FRP RC beams. A single FMH is adopted here, since only two primary failure modes are considered, one of which has already been selected as the most desirable mode of failure. Hence, flexural failure due to concrete crushing is located at the top of the hierarchy and is followed by shear failure due to fracture of the transverse FRP reinforcement. In the general code development stage, it was decided to adopt target RCMs that reflect the current steel RC practice and hence, the average value of 1.4 is adopted as target for the flexure-shear RCM. In addition, the resistance-capacity prediction models[9,14], developed as part of the ConFibreCrete[15] research network and outlined in Appendix A, are adopted in the present study.

The next step (2.2) is to perform structural reliability analyses to determine the flexural P_f, shear P_f and flexural-shear RCMs for the values of γ_{FRP} tabulated in Table 2. The values for the $\gamma_{FRP\text{-}L}$ and partial safety factor for transverse reinforcement ($\gamma_{FRP\text{-}T}$) were selected by considering the findings of previous investigations[8]. The assessment was carried out for 48 different design configurations (presented in Neocleous[8]). The analyses were performed by utilising the Monte-Carlo[16] Simulation method in conjunction with the Latin-Hypercube[17] and Conditional-Expectation[16] variance reduction techniques.

Results obtained for the flexural P_f indicated that, for the selected γ_{FRP-L}, the target P_f was attained by all design configurations. Whereas in the case of shear P_f (Fig. 5 and 6), the target P_f was achieved by all design configurations, only if the value of γ_{CFRP-T} and γ_{GFRP-T} was 1.8 and 2 respectively.

The RCMs obtained for each γ_{FRP-T} are shown in figure 7 and 8 for CFRP and GFRP RC beams respectively. The target value of 1.4 was attained by all design configurations only if the values of γ_{CFRP-T} and γ_{GFRP-T} were 1.8 and 2, respectively. It is noted that the value chosen for γ_{FRP-L} does not affect the flexure-shear RCMs, as the design aimed to achieve flexural failure due to concrete crushing.

The use of γ_{FRP-L} also does not influence the flexural P_f (provided that flexural failure occurs due to concrete crushing). Therefore, it may be possible to eliminate the use of γ_{FRP-L} and incorporate the uncertainties relevant to the longitudinal FRP reinforcement in the partial safety factor (γ_c) adopted for f_c. However, this will require the modification of γ_c used currently in flexural limit state design. Since, the long-term mechanical behaviour of FRP reinforcement is not fully understood, it is not prudent to abolish the use of γ_{FRP-L} based on existing knowledge. To be conservative, it is decided to use the values of γ_{FRP-L} currently examined (1.15 and 1.3 for CFRP and GFRP reinforcement respectively) for short-term loading conditions, since they reflect uncertainties in material characterisation. A limit is also imposed on ρ (equation 2) to eliminate the possibility of flexural failure occurring due to reinforcement fracture.

$$\rho_{min} = \frac{0.81(f_{ck}+8)\,\varepsilon_c}{f_{FPRk}\left(\frac{f_{FRPk}}{E_{FRPk}}+\varepsilon_c\right)} \qquad (2)$$

The concept of average measure of closeness[10] (expressed in terms of structural utility), T (equation 3), is utilised to select appropriate values for γ_{FRP-T}. The selection criteria comprise the attainment of the target RCMs and P_f and minimisation of the resulting cost of construction. To perform such an assessment, it is necessary to estimate the demand function, D_{fj}, for each configuration. D_{fj} is expressed as the product of the frequency of occurrence estimated for each of the main design parameters, considered by each configuration (equation

4). The main design parameters are ρ, f_c and PVL-ratio. The values of the frequency of occurrence, estimated for each of these variables, are summarised in Table 3.

$$T = \int M D_f d_s \qquad (3)$$

$$D_{fj} = f_{fc} \, f_{\frac{G_k}{Q_k}} \, f_{\rho} \qquad (4)$$

The metric function, M_j, for each design configuration is taken as the total cost of construction, C_{Tj} (equation 5). The C_{Tj} and initial cost of construction for each design configuration, C_{Ij}, are determined by equations 6 and 7, respectively. The cost of failure for each design configuration, $C_{Fj,}$ (such as the cost of loss of use and cost of fatalities) is determined by considering a number of scenarios as indicated in Table 4. Table 5 shows the unit prices adopted for the cost of concrete and cost of CFRP and GFRP reinforcement. It is noted that the same prices are used for longitudinal and transverse reinforcement.

$$M_j = C_{Tj} \qquad (5)$$

$$C_{Tj} = C_{Ij} + C_{Fj} P_{fj} \qquad (6)$$

$$C_{Ij} = C_{C_j} + C_{R_j} \qquad (7)$$

The values of T obtained for the shear failure mode (Tables 6 and 7) indicated that T increases slightly with the value of γ_{FRP-T}. This is due to the increase in the C_{Rj} for transverse reinforcement; more transverse reinforcement is required as γ_{FRP-T} increases. Furthermore, the results of the cost-optimisation indicate that, for the small values of γ_{FRP-T}, the average measure of closeness increases significantly, as C_{Fj} becomes relatively large. This is due to the increased influence of P_f on C_{Tj}. It is also observed that the shear average measure of closeness is significantly higher for PVL-ratio equal to 1. This is due to the relatively longer beam spans used for this particular PVL-ratio.

It was decided to select the highest value of γ_{FRP-T} considered in the assessment (table 8), as the application of these values seemed to be the most economical (for scenarios 5 and 6) and it also satisfied both the target RCMs and P_f.

Design example

The γ_{FRP}, recommended for load ratio equal to 0.5, are utilised for the limit state design and structural reliability assessment of two FRP RC beams (Fig. 9) that were tested during the Eurocrete project[12,13] and failed in flexure due to concrete crushing. This is to verify that the application of the chosen γ_{FRP} leads to the desired FMH and satisfies the target P_f and RCMs. The material properties of the beams are tabulated in Table 1 and the stress-strain models adopted for concrete compressive strength and FRP reinforcement are illustrated in Fig. A.1 and A.2 (Appendix A).The average concrete cube compressive strength of the beams is shown in Fig. 9.

Table 9 summarises the results obtained from the design and the structural reliability assessment of the two beams. It is clear that the application of the proposed γ_{FPR} leads to the desired FMH, since the shear resistance-capacity is higher than the flexural resistance-capacity. In addition, it is observed that the target RCMs are satisfied. It must be noted that there is a good correlation, in particular for beam CB17, between the flexural resistance-capacity and the experimental resistance-capacity. The values obtained for the flexural and shear P_f indicate that the P_{ft} of 7 10^{-05} is satisfied for both failure modes.

Discussion and conclusions

A new design and safety philosophy for reinforced concrete structures has been developed. A design framework, based on the new philosophy, is proposed for adoption during the development of a general concrete code of practice. The implications of the new design and safety philosophy for code developers and manufacturers of construction materials are quite significant.

A comprehensive application of the proposed philosophy would require the analysis of a greater amount of failure modes than considered here and hence, it is necessary that reliable resistance-capacity prediction models are developed for each failure mode under consideration. Furthermore, the concept of the failure-mode-hierarchy would minimise the necessity of developing additional design guidelines and codes of practice each time a construction innovation becomes available.

Since innovation in the field of FRP reinforcement is expected to continue, it is believed that the values of the partial safety factor of FRP reinforcement (adopted for each failure-mode-hierarchy) should be provided by the FRP manufacturers according to the proposed framework and be subjected to appropriate independent verification. The manufacturers should provide the code developers with material characteristics and any information that is essential for the development of any failure-mode-hierarchy.

The effectiveness of the proposed framework was demonstrated successfully for the case of concrete beams reinforced with carbon and glass FRP reinforcement.

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Appendix A – Models for resistance-capacity of FRP RC beams

A.1 Flexural failure mode

The model adopted for the calculation of the flexural (moment) resistance-capacity of FRP RC beams is based on the design rules of Eurocode-2[5], and hence the same assumptions apply for the current model. The compression strength of FRP reinforcement is also ignored due to the anisotropic nature of the reinforcement[8]. Since the failure of FRP RC beams is brittle, the model is modified accordingly and the calculations are based on the control of the strain in the FRP reinforcement[18]. The following algorithm is applied for the evaluation of the moment resistance-capacity. Fig. A.1 and A.2 shows the stress-strain models adopted for concrete compressive strength and FRP reinforcement, respectively. It is noted that the concrete model is based on the model adopted by Eurocode-2 for design purposes.

Initially the effective depth of the RC is calculated based on an assumed bar diameter. Then, it is assumed that flexural failure occurs due to concrete crushing. Assuming that the concrete compressive strain at failure, ε_c, is equal to 0.0035, the design concrete compressive force, F_{Cd}, is derived (equation A.1.1). Since Eurocode 2 is the basis for the design rules, a specially derived equation[8] is used to determine the mean stress factor, $\alpha = -68711\,\varepsilon_c^2 + 464.79\,\varepsilon_c + 0.01$, which is used in the simplified stress block for concrete.

$$F_{Cd} = \frac{\alpha f_{ck}\ x\ b}{\gamma_c} \qquad \text{(A.1.1)}$$

Where x is the neutral axis depth, f_{ck} is the characteristic compressive strength, b is the width of the section and γ_c is the partial safety factor adopted by Eurocode-2 for the concrete compressive strength.

Since it is assumed that failure occurs due to concrete crushing, the actual stress in the reinforcement (equation A.1.2) is deemed to be less than the design stress (equation A.1.3) at which fracture of the reinforcement occurs. The design force of the reinforcement is derived based on this assumption (equation A.1.4).

$$f_{FRP} = \varepsilon_{FRP} E_{FRP} \tag{A.1.2}$$

$$f_{FRPd} = \frac{f_{FRPk}}{\gamma_{FRP}} \tag{A.1.3}$$

$$F_{Sd} = A_{FRP}\, f_{FRP} = A_{FRP}\, \varepsilon_{FRP}\, E_{FRP} \tag{A.1.4}$$

Where ε_{FRP} is the actual strain developed in FRP reinforcement, E_{FRP} and f_{FRPk} and A_{FRP} are the characteristic tensile strength, Youngs' modulus and area of longitudinal FRP reinforcement, respectively. γ_{FRP} is the partial safety factor adopted for longitudinal FRP reinforcement. By considering a simple strain diagram, the neutral axis depth is calculated by equation A.1.5 (ε_c is the concrete compressive strain and d is the effective depth of the reinforced concrete section):

$$x = \frac{\varepsilon_c\, d}{\varepsilon_{FRP} + \varepsilon_c} \tag{A.1.5}$$

Then by considering force equilibrium between F_{Cd} and F_{Sd}, equations A.1.1, A.1.4 and A.1.5 are solved simultaneously to determine ε_{FRP}:

$$\frac{\alpha\, f_{ck} \left(\dfrac{\varepsilon_c\, d}{\varepsilon_{FRP} + \varepsilon_c} \right) b}{\gamma_c} = A_{FRP}\, \varepsilon_{FRP}\, E_{FRP} \tag{A.1.6}$$

$$\varepsilon_{FRP}^2 + \varepsilon_c\, \varepsilon_{FRP} - \frac{\alpha\, f_{ck}\, b\, d\, \varepsilon_c}{\gamma_c\, A_{FRP}\, E_{FRP}} = 0 \tag{A.1.7}$$

Before proceeding into the calculation of the lever arm, z, and design moment resistance, M_u, it is checked if ε_{FRP} has exceeded the design limit, ε_{FRPd}, which is defined by equation A.1.8.

$$\varepsilon_{FRPd} = \frac{f_{FRPd}}{E_{FRP}} \tag{A.1.8}$$

If ε_{FRP} is greater than ε_{FRPd}, then flexural failure occurs due to fracture of the FRP reinforcement. In this case, F_{Sd} and x are determined from equation A.1.9 and A.1.10 respectively. F_{Cd} is also re-derived (equation A.1.11) by substituting equation A.1.10 to A.1.1. The concrete compressive strain is iteratively reduced until the force equilibrium between F_{Cd} (equation A.1.11) and F_{Sd} (equation A.1.9) is satisfied.

$$F_{Sd} = A_{FRP}\, f_{FRPd} \qquad \text{(A.1.9)}$$

$$x = \frac{\varepsilon_c\, d}{\varepsilon_{FRPd} + \varepsilon_c} \qquad \text{(A.1.10)}$$

$$F_{Cd} = \frac{\alpha\, f_{ck} \left(\dfrac{\varepsilon_c\, d}{\varepsilon_{FRPd} + \varepsilon_c} \right) b}{\gamma_c} \qquad \text{(A.1.11)}$$

Using the appropriate value of x, the centroid factor, γ, and the lever arm, z, are then determined from equation A.1.12.

$$z = d - \gamma\, x \quad \text{where, } \gamma = 1962.6\, \varepsilon_c^2 + 17.89\, \varepsilon_c + 0.33 \qquad \text{(A.1.12)}$$

Finally, M_u is calculated, depending on the mode of flexural failure. If failure occurs due to concrete crushing, equation A.1.13 is applied. It should be noted that F_{Cd} is determined from equation A.1.1. Otherwise, if failure is due to fracture of the re-bar, equation A.1.14 is determined by using the appropriate value of F_{Sd}.

$$M_u = F_{Cd}\, z \qquad \text{(A.1.13)}$$

$$M_u = F_{Sd}\, z \qquad \text{(A.1.14)}$$

A.2 Shear failure mode

The model adopted by Eurocode-2[5] for the shear resistance-capacity of RC beams is adopted for the calculation of the shear resistance-capacity of FRP RC beams. The same assumptions and algorithms apply for the case of FRP RC beams. However, the design models are modified to account for the unique mechanical properties of FRP reinforcement.

The modifications, based on the strain approach[14], introduce the application of the ϕ factor, which is applied on the concrete shear strength $V_{Rd1(FRP)}$ and the shear contribution given by transverse FRP reinforcement (equations A.2.1 and A.2.2, respectively). The ϕ factor is the ratio of the maximum allowable strain in the shear FRP reinforcement, ε_{FRPv}, to the yield strain of steel shear reinforcement, ε_y. In this study, based on available experimental data for ε_{FRP}, a value of 1.8 was used for ϕ. The ϕ factor is also applied to modify the strength of the shear FRP reinforcement (equation A.2.2). It is noted that V_{Rd1} is calculated according to Eurocode-2.

$$V_{Rd1(FRP)} = V_{Rd1}\left(\frac{E_{FRP}}{E_s}\phi\right)^{1/3} \tag{A.2.1}$$

$$f_{FRPv} = 0.0025\, E_{FRPv}\, \phi \tag{A.2.2}$$

Where E_s is the Youngs modulus of steel reinforcement (value used in this study is 200,000 N/mm^2). E_{FRPv} is the Youngs modulus of transverse FRP reinforcement (values used in this study are the same as those used for the E_{FRP} of longitudinal reinforcement).

Tables

Table 1 Mechanical properties of CFRP and GFRP reinforcement

	Tensile Strength (N/mm^2)		Young's Modulus (N/mm^2)	
	GFRP	CFRP	GFRP	CFRP
Mean μ_i	810	1380	45000	115000
Standard Deviation σ_i	40.5	69	2250	5750
Characteristic i_k	747	1272	41500	106000

Tensile strain of CFRP and GFRP reinforcement: 1.2% and 1.8%, respectively

Table 2 γ_{FRP} examined in the structural reliability assessment

	CFRP reinforcement	GFRP reinforcement
γ_{FRP-L}	1.15	1.3
γ_{FRP-T}	1.15, 1.5, 1.8	1.5, 1.75, 2

Table 3 Frequency of occurrence for ρ, fc and PVL-ratio

ρ %	f_ρ	f_{cuk} N/mm^2	f_{fcuk}	PVL-ratio $\left(\frac{G_k}{Q_k}\right)$	$f_{\frac{G_k}{Q_k}}$
0.5 - 1.0	0.25	20 - 30	0.25	0.01 - 0.8	0.3
1.0 - 1.5	0.25	30 - 40	0.25	0.8 - 1.5	0.5
1.5 - 2.0	0.25	40 - 50	0.25	1.5 - 2.5	0.2
2.0 - 3.0	0.25	50 - 60	0.25	-	-

Table 4 Scenarios considered for cost of failure

Scenario	1	2	3	4	5	6
C_{Fj}	C_{Ij}	3 C_{Ij}	10 C_{Ij}	100 C_{Ij}	1000 C_{Ij}	10000 C_{Ij}

Table 5 Indicative unit price for ready mix concrete and FRP reinforcement

Ready Mix Concrete £/m^3	CFRP Reinforcing Bars £/m per ϕ13.5mm (per10x4mmlink)	GFRP Reinforcing Bars £/m per ϕ13.5mm (per 10x4mm link)
55	1.95	0.82

Table 6. Cost-optimisation results for CFRP RC beams (shear failure)

Load Ratio	$\gamma_{CFRP\text{-}T}$	Average P_f	Average Measure of Closeness for each Scenario, £ 1	2	3	4	5	6
0.5	1.15	1.7E-05	29.76	29.77	29.79	30.08	32.94	61.59
	1.5	2.3E-06	30.34	30.34	30.34	30.37	30.71	34.03
	1.8	4.4E-07	30.84	30.84	30.84	30.84	30.89	31.39
1	1.15	4.1E-05	58.68	58.70	58.76	59.57	67.64	148.31
	1.5	3.7E-06	59.70	59.70	59.71	59.77	60.45	67.24
	1.8	4.0E-07	60.62	60.62	60.62	60.63	60.70	61.39
2	1.15	3.5E-05	33.46	33.48	33.54	34.37	42.66	125.54
	1.5	2.1E-06	34.06	34.06	34.07	34.12	34.62	39.67
	1.8	1.5E-07	34.62	34.62	34.62	34.63	34.66	35.03

Table 7. Cost-optimisation results for GFRP RC beams (shear failure)

Load Ratio	$\gamma_{GFRP\text{-}T}$	Average P_f	Average Measure of Closeness for each Scenario, £ 1	2	3	4	5	6
0.5	1.5	5.2E-06	29.97	29.97	29.97	30.05	30.85	38.78
	1.75	1.1E-06	30.35	30.35	30.35	30.36	30.50	31.92
	2	2.3E-07	30.73	30.73	30.73	30.73	30.75	31.00
1	1.5	1.4E-05	58.98	58.99	59.01	59.28	61.99	89.02
	1.75	2.0E-06	59.66	59.66	59.67	59.70	60.08	63.84
	2	2.9E-07	60.34	60.34	60.34	60.35	60.40	60.88
2	1.5	1.6E-05	33.69	33.69	33.72	34.11	37.97	76.57
	1.75	2.1E-06	34.09	34.09	34.10	34.15	34.66	39.78
	2	2.1E-07	34.50	34.50	34.50	34.51	34.56	35.08

Table 8 γ_{FRP} selected for the shear failure mode

PVL-ratio	γ_{CFRP}	γ_{GFRP}
0.5	1.8	2
1	1.75	2
2	1.75	2

Table 9 Design results for beams GB9 and CB17

	GB9	CB17
Experimental Load, kN	103.6	127.6
Design Load Fd, kN	63.3	72.5
Flexural P_f	2.7E-07	1.0E-06
Flexural resistance-capacity		
Mean value, $\mu_{flexure}$, kN	100.6	127.2
Design value, kN	63.3	72.5
Shear P_f	1.7E-17	7.3E-16
Shear resistance-capacity		
Mean value, μ_{shear}, kN	164.5	175.4
Design value, kN	65.0	72.9
RCM	1.6	1.4

List of Captions