# Relating the proton, neutron and deuteron structure functions in the covariant Bethe-Salpeter formalism

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**Abstract**

The neutron structure function $F_2^{\mathrm{n}}(x)$ is evaluated within the kinematic range $10^{-3} < x < 1$ from the deuteron and proton data by employing relativistic theoretical description of $F_2^{\mathrm{D}}(x)$ and several assumptions on the high-$x$ asymptotics of $F_2^{\mathrm{n}}(x)/F_2^{\mathrm{p}}(x)$. It is shown that new measurements of $F_2^{\mathrm{D}}(x)$ in the range $0.6 < x \leq 0.8$ would substantially improve understanding of the relation between $d$ and $u$ valence quarks in the limit $x \to 1$.

# 1 Introduction

The valence quark structure of the proton and neutron has for some time been assumed to be understood. A number of experiments have provided a detailed representation of the nucleon's quark distributions over a wide range of kinematics with some exceptions: the range of Bjorken $x$ close to a single nucleon kinematic limit, $x = 1$ , remains to be inaccessible. Actually, the situation is even more unfortunate because the range beyond $x = 0.7$ has been poorly explored experimentally. Recent reviews of the neutron and proton structure, which still do not pretend to be complete, are presented in Refs. [1, 2]. The valence $u$ and $d$ quark distributions are generally obtained from measurements of the proton and neutron structure functions, $F_2^{\mathrm{p}}(x)$ and $F_2^{\mathrm{n}}(x)$, respectively. The $u$ quark distribution is relatively well constrained by the $F_2^{\mathrm{p}}(x)$ data for $x \leq 0.8$ but the absence of free neutron targets has resulted in large uncertainties in the $d$ quark distribution beyond $x > 0.6$. Major uncertainties arise from model considerations of the deuteron structure function $F_2^{\mathrm{D}}(x)$, from which $F_2^{\mathrm{n}}(x)$ is evaluated. They are normally represented as a spectrum of different ratios $R^{\mathrm{n/p}} \equiv F_2^{\mathrm{n}}/F_2^{\mathrm{p}}$ as a function of $x$. There is no theoretical constraint on the $x$ dependence of $R^{\mathrm{n/p}}$. Numerous very different constraints on $R^{\mathrm{n/p}}$ at the kinematic boundary $x = 1$ have been suggested in quark models [3, 4] and QCD inspired models [5, 6]. They rely on model considerations of the $d/u$ ratio in the limit $x \to 1$ and neglect the contribution from sea quarks. Such predictions offer a nice testing ground for our understanding of the role which valence quarks play in the nucleon wave function. However, it is extremely difficult to confront the predictions made for the limit $x \to 1$ with measured values which have to be extrapolated to this limit. This explains coexistence of numerous models of $F_2^{\mathrm{D}}(x)$ used in practice of measurements of $F_2^{\mathrm{n}}(x)$ as well as for motivating new experimental research into the valence quark structure.
As it is demonstrated in Ref. [7] by using examples of evaluating the $d/u$ ratio, improvement of the knowledge of $F_2^{\mathrm{n}}(x)$ in the region $x \geq 0.75$ is very important for many applications in hadron physics. The procedure of extraction of $F_2^{\mathrm{n}}(x)$ from the $F_2^{\mathrm{D}}(x)$ data and its theoretical justification was a subject of many publications (see e.g. Refs. [8]),

which defended a rigorous consideration of the role of nuclear effects. From the intense discussion in Ref. [9] one learns that compromises and simplifications distort considerably $F_2^{\rm n}(x)$ and, therefore, $d/u$ in the high $x$ range. Theoretical understanding of nuclear effects is further discussed in Ref. [10] by presenting model independent relations which follow from the concept of quark-hadron duality.

The present paper continues the series of publications [7] – [12] by suggesting an alternative approach to extraction of the $F_2^{\rm n}(x)$ from the data collected in deep inelastic scattering (DIS) experiments which relies on relativistic theoretical description of $F_2^{\rm D}(x)$ [13, 14] and well defined assumptions on the high-$x$ asymptotics for $R^{\rm n/p}$. We forward our criticism against unjustified simplifications frequently made in consideration of the "nuclear" effects in $F_2^{\rm D}$. Most common misapprehension shows up in attempts to find an analogy (and even extrapolation rule) between the EMC effect and a modification of the nucleon structure inside the deuteron. There is *no such theoretical concept* as the EMC effect, which is just a bare observation that $F_2^A/F_2^{\rm D} < 1$ in a certain range of $x$. Therefore, there are no grounds to relate directly the difference between $F_2^A$ and $F_2^{\rm D}$ with that of $F_2^{\rm D}$ and the free nucleon structure function $F_2^{\rm N}$, where $F_2^{\rm N} \equiv (F_2^{\rm n} + F_2^{\rm p})/2$. The difference between $F_2^{\rm N}$ and $F_2^{\rm D}$ can be conceptually original [15].

Experimental information on the deuteron and proton structure functions is available from the experiments of BCDMS, EMC, SLAC, E665, NMC, H1 [16]. The data for $F_2^{\rm D}$ and $F_2^{\rm p}$ and their ratio are available in the range $10^{-3} < x < 0.6$. There is also data at $0.6 < x < 0.9$ from SLAC with relatively large errors. The SMC collaboration proposed overall fit of world data which gives $F_2^{\rm p}$ and $F_2^{\rm D}$ in the range $10^{-4} < x < 0.85$ [17]. We make use of this approximation and extend it up to $x = 1$ based on the quark counting rule for the $x \to 1$ limit of $F_2^{\rm p}$. Further we consider extraction of $F_2^{\rm n}(x)$ by assuming that the deuteron can be considered as the two-nucleon bound state. We show that the behavior of $F_2^{\rm n}(x)$ outside the $x$ range covered by measurements can be established by employing well known theoretical prescriptions. Possible ambiguities connected with these prescriptions are investigated.

# 2 Neutron structure function

Our consideration of the deuteron is based on the standard picture of the proton and neutron bound together into the simplest nucleus. It is then a favored source of information about neutron structure function $F_2^{\rm n}(x)$. The main obstacle in the quantitative evaluation of $F_2^{\rm n}(x)$ in this case comes from nuclear binding which is neglected in many analyses on the grounds that the deuteron binding energy is very small. Of course, one can assume that it will be sufficient to consider just Fermi motion which becomes particularly important at large Bjorken $x$. On the other hand the EMC effect [18] and nuclear shadowing [19] show us that even small binding can qualitatively change the observed nucleon structure. Its effect, as it is demonstrated by the analysis in Ref. [20], is clearly manifested in the entire $x$ range, including the range of $x < 0.1$. Up to now there is no well established and unified explanation of nuclear effects in whole range of $x$. Numerous models used to reproduce dynamics of the effects are not consistent. All these facts render the procedure of $F_2^{\rm n}(x)$ evaluation very ambiguous and model dependent.

To elucidate the problem of model dependence, one needs an approach which is less dependent on the details of dynamical nature of the effects and provides more general and unified picture. We use the approach based on the covariant Bethe-Salpeter formalism [13]. It yields a good description of the ratio of the nuclear to deuteron structure function which is shown to be universal for all nuclei [15, 20]. Within this approach the hadronic part of the nuclear deep inelastic amplitude $W^A_{\mu\nu}$ is expressed in terms of the off-mass-shell nucleon and antinucleon amplitudes, $W^{\rm N}_{\mu\nu}$ and $W^{\overline{\rm N}}_{\mu\nu}$, respectively, by the following expression:

$$W^A_{\mu\nu}(P,q) = \sum_i \int dk_i (W^{\rm N}_{\mu\nu}(k_i,q) f^{{\rm N}/A}(P,k_i) + W^{\overline{\rm N}}_{\mu\nu}(k_i,q) f^{\overline{\rm N}/A}(P,k_i)) \ , \qquad (1)$$

where the indices $\mu\nu$ denote the Lorentz components of the amplitude, index $i$ counts nucleons inside the nucleus, $P$ is the total momentum of the nucleus, $k_i$ is the relative momentum of the struck nucleon and $q$ is the transferred momentum from the photon. The distribution function $f^{{\rm N}/A}$ is expressed in terms of the $n$-nucleon Bethe-Salpeter vertex functions

$$f^{{\rm N}/A}(P,k_i) = \int dk_1 \dots dk_{i-1} dk_{i+1} \dots dk_n \overline{u}(\mathbf{k}_i) S_{(n)}(P,k_i) u(\mathbf{k}_i) \overline{\Gamma}(P,\mathcal{K}) S_{(n)}(P,\mathcal{K}) \Gamma(P,\mathcal{K}),$$
$$f^{\overline{\rm N}/A}(P,k_i) = \int dk_1 \dots dk_{i-1} dk_{i+1} \dots dk_n \overline{v}(\mathbf{k}_i) S_{(n)}(P,k_i) v(\mathbf{k}_i) \overline{\Gamma}(P,\mathcal{K}) S_{(n)}(P,\mathcal{K}) \Gamma(P,\mathcal{K}). \quad (2)$$

It gives $4D$ momentum distribution of the nucleon and antinucleon inside a nucleus. Thus, according to Eq. (1), all nuclear effects should follow from the $4D$ Fermi motion of the nucleon inside a nucleus [21]. The time component of the Fermi motion is exclusive feature of the relativistic approach. In the $3D$ limit this component results in the change of the nucleon structure [21]. Therefore, one can take explicit account of it by using a dynamical model for the interlinkage of the nucleon and nuclear structure. This problem is closely related to that of the off-mass-shell effects. Since the nucleon amplitude $W^{\rm N}_{\mu\nu}$ in Eq. (1) is off shell it cannot be connected with corresponding observable nucleon amplitude. This point makes Eq. (1) useless for practical applications. To solve this problem one can use analytic properties of the integrand in (1) and remove explicitly the Fermi motion along time axis by integrating over $k_0$. We do it assuming small relative momenta of bound nucleons. Preserving general form of Eq. (1) for the lightest nuclei one can transform it as follows:

$$W^A_{\mu\nu}(P,q) = \sum_{a,a'}^{A-1} \int \frac{d^3k_a}{(2\pi)^3} \left[ W^a_{\mu\nu}(k_a,q) + \Delta^A_{a,a'} \frac{dW^a_{\mu\nu}(k_a,q)}{dk_{a0}} \right]_{k_{a0}=\sqrt{\mathbf{k_a}^2+M_a^2}} \Phi^A_{a,a'}(\mathbf{k})^2, \qquad (3)$$

where $a$ and $a'$ denote the struck and spectator nuclear constituent, respectively. Depending on the mass of the considered nucleus, $a$ and $a'$ assume different symbols:

$$(a,a') = \begin{cases} ({\rm p},{\rm n}), ({\rm n},{\rm p}), \ \text{if } A=2 \ ({\rm D}) \\ ({\rm p},{\rm pn}), ({\rm n},{\rm pp}), ({\rm p},{\rm D}), ({\rm D},{\rm p}), \ \text{if } A=3 \ (^3{\rm He}) \\ ({\rm n},{\rm pn}), ({\rm n},{\rm pn}), ({\rm n},{\rm D}), ({\rm D},{\rm n}), \ \text{if } A=3 \ (^3{\rm H}) \\ \left.\begin{array}{l} ({\rm p},{\rm pnn}), ({\rm n},{\rm ppn}), ({\rm p},{\rm Dn}), ({\rm n},{\rm Dp}), \\ ({\rm D},{\rm pn}), ({\rm D},{\rm D}), ({\rm p},^3{\rm H}), ({\rm n},^3{\rm He}), \end{array}\right\} \text{if } A=4 \ (^4{\rm He}) \ , \end{cases} \qquad (4)$$

where $\Delta^A_{a,a'} = M_A - E_a - E_{a'}$ is the separation energy of the corresponding nuclear fragment. Now the nuclear effects are interpreted by the conventional $3D$ Fermi motion of nuclear fragments and the derivative with respect to $k_0$ of its DIS on-shell amplitudes.

The distribution function $\Phi^A_{a,a'}(\mathbf{k})^2$ is defined by the projection of the Eq. (2) onto $3D$ space

$$\Phi^A_{a,a'}(\mathbf{k})^2 = \left\{ f^{\widetilde{\mathrm{N}}/A}(P, k_i) \right\}_{p_i^2 = M_a^2} \tag{5}$$

and it is closely related to the nuclear spectral function. The term containing the derivative represents a dynamical modification of the bound nucleon structure observed in the $3D$ projection of the relativistic bound state as nuclear shadowing and the EMC effect. The spectrum of bound states presented by Eq. (4) is defined by analytic properties of the distribution function $f^{\mathrm{N}/A}$ and thereby ensures Pauli blocking of the forbidden states. Starting from Eq. (3) one can derive the $F_2^{\mathrm{D}}$ in the form [14]:

$$\begin{gathered} F_2^{\mathrm{D}}(x_{\mathrm{D}}) = \int \frac{d^3k}{(2\pi)^3} \frac{m^2}{4E^3(M_{\mathrm{D}} - 2E)^2} \left\{ F_2^{\mathrm{N}}(x_{\mathrm{N}}) \left( \frac{E - k_3}{M_{\mathrm{D}}} + \frac{M_{\mathrm{D}} - 2E}{M_{\mathrm{D}}} \right) f^{\mathrm{N/D}}(M_{\mathrm{D}}, k) - \right. \\ \left. - \frac{M_{\mathrm{D}} - 2E}{M_{\mathrm{D}}} \left( x_{\mathrm{N}} \frac{dF_2^{\mathrm{N}}(x_{\mathrm{N}})}{dx_{\mathrm{N}}} f^{\mathrm{N/D}}(M_{\mathrm{D}}, k) - F_2^{\mathrm{N}}(x_{\mathrm{N}})(E - k_3) \frac{\partial}{\partial k_0} f^{\mathrm{N/D}}(M_{\mathrm{D}}, k) \right) \right\}_{k_0 = E - \mathrm{M_D}/2}, \end{gathered} \tag{6}$$

where $E$ is the on-mass-shell nucleon energy. We consider this expression as an integral equation with the unknown function $F_2^{\mathrm{n}}$. Functions $F_2^{\mathrm{p}}$ and $F_2^{\mathrm{D}}$ are assumed to be known from experiments. Since the available data does not cover the entire kinematic range, the behaviour of these functions at boundaries ($x = 0$, $x = 1$) can not be defined from experiments. The latter makes definition of the boundary conditions for the solution of Eq. (6) model dependent. Basically, the boundary behavior of the ratio $F_2^{\mathrm{n}}/F_2^{\mathrm{p}}$ can be found from an iteration procedure. Experience of using such procedure shows that the final result strongly depends on the zero iteration [12], which is hard to define on the basis of physical conditions. Another way of solving this problem is to fit the right hand side of Eq. (6) to experimental data on $F_2^{\mathrm{D}}(x)$. The boundary conditions are introduced explicitly as asymptotics of the nucleon structure functions at $x = 0$ and $x = 1$ and their influence on $F_2^{\mathrm{D}}(x)$ at medium $x$ is studied.
As an initial condition we use the following anzats for $F_2^{\mathrm{n}}(x)$:

$$F_2^{\mathrm{n}}(x) = R^{\mathrm{n/p}}(x) F_2^{\mathrm{p}}(x) \tag{7}$$

with the extended SMC fit of $F_2^{\mathrm{p}}$ and the function $R^{\mathrm{n/p}}(x)$ in the form

$$R^{\mathrm{n/p}}(x) = a_1(1-x)^{\alpha_1} + a_2 x^{\alpha_2} + b_1 x^{\beta_1}(1-x)^{\beta_2}(1 + c_1 x^{\gamma_1}) \ . \tag{8}$$

The parameters $a_1$, $a_2$, $\alpha_1$, $b_1$ are introduced in order to satisfy the asymptotics of the nucleon structure functions: $a_1 = 1$ (corresponds to the limit $F_2^{\mathrm{p}}(0) = F_2^{\mathrm{n}}(0)$); the parameter $a_2$ should be fixed according to $lim_{x\to 1} F_2^{\mathrm{n}}(x)/F_2^{\mathrm{p}}(x)$. We use three values for this limit in order to study possibility to extract it from the experimental data. The simplest quark model with SU(6) symmetry gives $R^{\mathrm{n/p}}_{x=1} = 2/3$. On the other hand, the kinematical limit $x = 1$ corresponds to the elastic scattering off the nucleon. Therefore, the ratio of the nucleon structure functions becomes equivalent to the ratio of the elastic cross sections — $R^{\mathrm{n/p}}_{x=1} = \sigma^{\mathrm{n}}_{\mathrm{elastic}}/\sigma^{\mathrm{p}}_{\mathrm{elastic}}$. It gives another value for the ratio in this limit, $R^{\mathrm{n/p}}_{x=1} = 0.47$. The minimal value of the ratio has been provided by the model with SU(6) symmetry breaking with scalar diquark dominance — $R^{\mathrm{n/p}}_{x=1} = 1/4$. Thus one has a wide range of possible values for the structure functions ratio at $x = 1$. In the next section we show

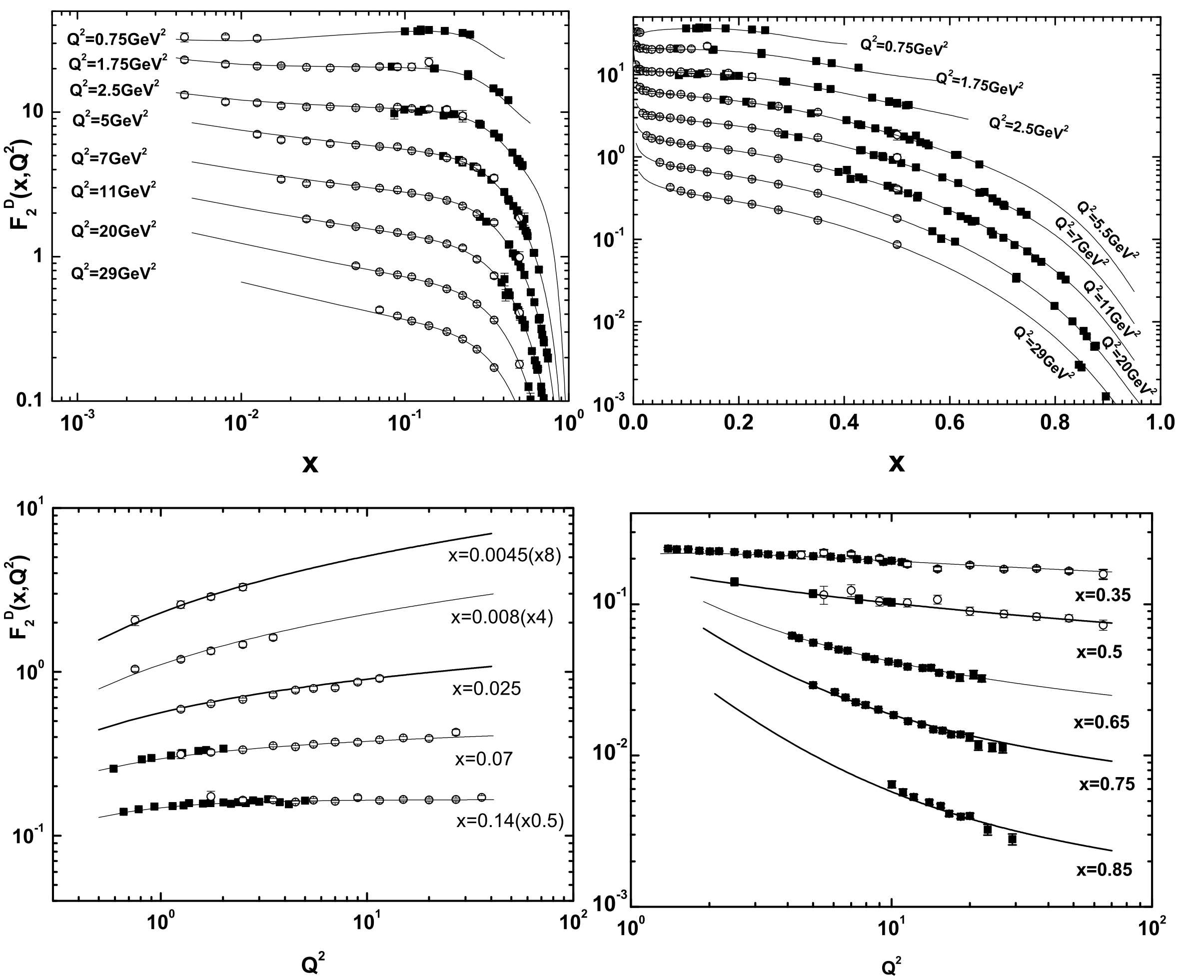


Figure 1: The structure function of the deuteron $F_2^{\mathrm{D}}$ measured in the SLAC (filled squares) and NMC (empty circles) experiments, shown as a function of $x$ for bins of fixed $Q^2$ (left upper panel — on a log scale, and right upper panel — on a linear scale), and as a function of $Q^2$ for bins of fixed $x$ (two lower panels). It is approximated with Eq. (6) in the range $10^{-3} < x < 0.6$ with the constraints as explained in the text.

how measurements of $F_2^{\rm D}/F_2^{\rm p}$ are sensitive to these assumptions. The last constraint for the parameters can be obtained from the asymptotics of the proton and neutron structure functions by assuming validity of the quark counting rule in the range $x \to 1$. This results in similar asymptotic behavior for $F_2^{\rm p}(x)$ and $F_2^{\rm n}(x)$:

$$\lim_{x\to 1} F_2^{\rm p}(x) \simeq C_{\rm p}(1-x)^3, \tag{9}$$
$$\lim_{x\to 1} F_2^{\rm n}(x) \simeq C_{\rm n}(1-x)^3. \tag{10}$$

Accordingly, the derivative of $R^{\rm n/p}$ at $x = 1$ is zero because

$$\lim_{x\to 1} R^{\rm n/p}(x) = \frac{C_{\rm n}}{C_{\rm p}} = Const. \tag{11}$$

This gives following constraints on the parameters of Eq. (8): $\alpha_1 = 1$, $\beta_2 = 1$, $b_1 = (\alpha_2 a_2 - 1)/(1 + c_1)$. All other parameters are considered as free and used to fit Eq. (6) to the deuteron data in the range $10^{-3} < x < 0.6$.

# 3 Discussion of results

The procedure described in the previous section is used to approximate the SLAC and NMC data on $F_2^{\rm D}(x, Q^2)$ in the range $0.6$ GeV$^2 \le Q^2 \le 65$ GeV$^2$ and $10^{-3} \le x \le 0.6$. This is done by fixing three different limits of $F_2^{\rm n}/F_2^{\rm p}$ at $x = 1$ and by varying four parameters in Eq. (8), namely $\alpha_2$, $\beta_1$, $c_1$ and $\gamma_1$. In the considered kinematic range all three limits provide equally good approximation, which means that the solution for $F_2^{\rm n}(x)/F_2^{\rm p}(x)$ converges in the range $x < 0.6$ to the virtually unique function described by Eq. (8). The result of the fit, which corresponds to the case $F_2^{\rm n}(1)/F_2^{\rm p}(1) = 0.47$ is displayed in Fig. 1. The three alternative solutions are shown in Fig. 2 with three lines, which virtually coincide if $x < 0.4$. Outside this range, the obtained ratio $F_2^{\rm n}(x)/F_2^{\rm p}(x)$ is represented by three lines, which exactly reproduce the constraints imposed at $x = 1$. The convergence to three different solutions is apparently provided by two constraints: available data on $F_2^{\rm D}$ and by three considered boundary conditions. Therefore, the solution is always model independent in the range of $x$, in which the constraint from $F_2^{\rm D}$ measurement is stronger than the one from boundary conditions.
The solutions, which we find for $F_2^{\rm n}(x)$ are compared in Fig. 2 with the results of NMC experiment [22] and SLAC analysis [23]. The NMC results are obtained assuming a naive approach $F_2^{\rm n} = 2F_2^{\rm D} - F_2^{\rm p}$ that is equivalent to the assumption $F_2^{\rm D} = F_2^{\rm N}$. The points corresponding to SLAC results were transformed by us from $F_2^{\rm D}(x)/F_2^{\rm p}(x)$ to $F_2^{\rm n}(x)/F_2^{\rm p}(x)$ in the same naive way as it was done by NMC, namely by neglecting nuclear effects in the deuteron. In the range $x < 0.6$ all three lines are in good agreement with the results of NMC and SLAC. We explain the success of the naive approach in the range $x < 0.6$ by the effect of cancellation of the modifications of the free nucleon structure function due to the nuclear (deuteron) binding and nucleon Fermi motion in this very kinematic range. Of course, the cancellation is not complete, which is seen from small (2–4 %) overestimation of the data at medium values of $x$. The difference between the naive approximation and the exact result becomes dramatic at $0.62 < x < 1$, which is a well known problem. This is well illustrated in Fig. 2 by inconsistency between data points at $x < 0.8$ and $x > 0.8$.

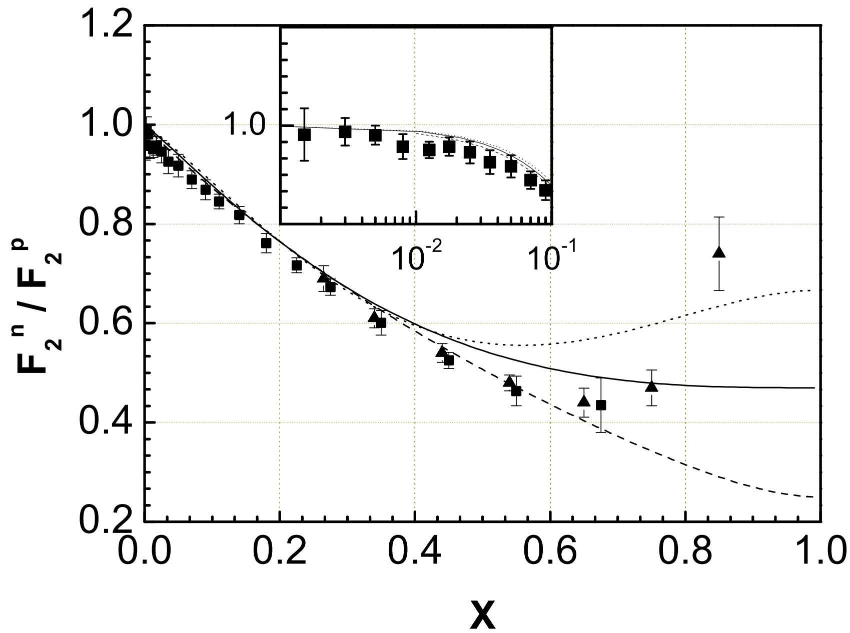


Figure 2: The ratio of the neutron to proton structure functions evaluated in the framework of the presented approximation by setting three different values for the parameter $R^{\mathrm{n/p}}_{x=1}$: (1) — 2/3 (short-dashed line), (2) — 0.47 (solid line) and (3) — 1/4 (long-dashed line). Points with error bars depict the results of the NMC experiment [22] (solid squares) and SLAC analysis [23] (solid triangles) obtained on the basis of the naive approach. Additionally, the small $x$ range ($x < 0.1$) is displayed on a log scale in the inset.

To restore the consistency, complete accounting of both the nuclear binding and Fermi motion has to be done in evaluating $R^{\mathrm{n/p}}(x)$.
In order to understand theoretical uncertainties in description of the data one should investigate how the extracted $F_2^{\mathrm{n}}(x)$ can modify theoretical evaluation of the deuteron to proton ratio. The three different constraints on $R^{\mathrm{n/p}}_{x=1}$ considered in this paper change the approximation of $F_2^{\mathrm{D}}(x)$ in the range of high $x$ which is better seen in the ratio $F_2^{\mathrm{D}}/F_2^{\mathrm{p}}$ shown in Fig. 3, in which three alternative calculations are compared with data from Ref. [22] and the results of the analysis performed in Ref. [23]. We conclude, that the present status both of the data and theory does not allow to constrain the value of the function $R^{\mathrm{n/p}}(x)$ at $x = 1$: both 1/4 and 0.47 are in agreement with experiments, while the value 2/3 can be regarded as less preferable. Further understanding of the relation between $F_2^{\mathrm{p}}(x)$ and $F_2^{\mathrm{n}}(x)$ can not be achieved without improvement of experimental data on $F_2^{\mathrm{D}}(x)/F_2^{\mathrm{p}}(x)$ in the range above $x = 0.6$. The latter is also valid for points from Ref. [23] even if they have apparently smaller error bars. Contrary to the NMC data which have similar $x$ dependence in different $Q^2$ intervals, the data from SLAC [24] used for the analysis [23] scatter considerably, which is not consistent with the parton picture suggesting the valence quarks dominance in the high $x$ region. Our understanding of the problem is that the NMC data comes from the *single* experiment, whereas SLAC results are based on *eight* experiments on deep inelastic $e - p$ and $e - d$ scattering.

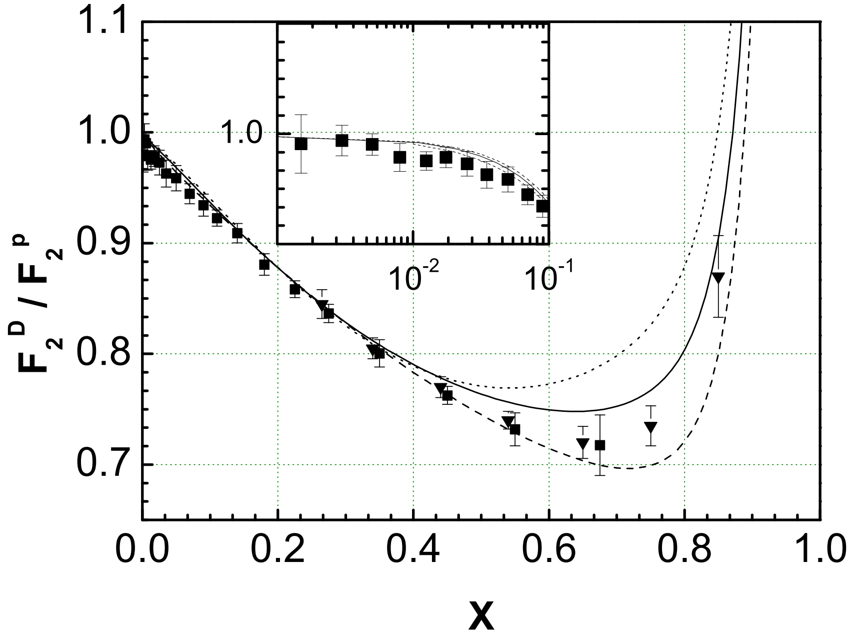


Figure 3: The result of modification of the ratio of the deuteron to proton structure functions evaluated for three values of the parameter $R^{\mathrm{n/p}}_{x=1}$ as in Fig. (2) compared to the data of Ref. [22] (solid squares) and results of the analysis performed in Ref. [23] (solid triangles). Additionally, the small $x$ range is displayed on a log scale in the inset.

We would like to note that the analysis performed in this paper by employing Eq. (3) is consistent with the data available for the ratio of $^4$He and deuteron structure functions in the range $10^{-3} < x \leq 0.8$ [13, 15]. Good agreement with experiment in two different cases observed in a wide $x$ range proves that Eq. (3) offers a general approach for accounting of the nuclear effects in the whole region of $x$.
Naturally, $F_2^{\mathrm{D}}(x)$ can be further modified in the high $x$ region if one assumes the presence of non-nucleonic degrees of freedom in a nucleus, which are not implied in the nucleon structure. Moreover, if there exist dibaryon states, like $6q$ states, which can not be excluded on theoretical grounds, $F_2^{\mathrm{D}}(x)$ can significantly change at $x \simeq 1$. Our calculations therefore provide reference lines for the search of such effects in forthcoming measurements proposed for the upgraded CEBAF facility [25].

# 4 Conclusions

We have proposed theoretically justified and fully consistent procedure for extracting the neutron structure function $F_2^{\mathrm{n}}(x)$ in the kinematic range $10^{-3} < x \leq 1$ under three different assumptions on $F_2^{\mathrm{n}}/F_2^{\mathrm{p}}$ at $x = 1$. The procedure involves a numerical fit of the expression (6) to the deuteron data. The performed analysis indicates that the increase in experimental accuracy in measurements of $F_2^{\mathrm{D}}(x)/F_2^{\mathrm{p}}(x)$ in the range $0.6 < x < 0.8$ by factor of two will be sufficient for verification of models suggested for the evaluation of the $d/u$ ratio at $x = 1$. The developed procedure allows one to avoid appreciable theoretical ambiguities which are present in other analyses largely due to simplifications in the treatment of Fermi motion. This concerns rather wide interval of $x$ and not only high $x$ range as it is commonly believed. The procedure proved to be a robust one because it relies on a good approximation of $F_2^{\rm D}(x)$ which is not sensitive to different high $x$ limits of the neutron structure function. This also means that $F_2^{\rm D}(x)$ measured by already completed DIS experiments ($x \leq 0.9$) can be described without introducing nonbaryonic degrees of freedom. The interval which remains unmeasured can in principle accommodate dibaryon states or some other exotica. More data in the high $x$ region is required in order to obtain model independent information on hadronic structure of the deuteron and to find which physics is responsible for the $d/u$ ratio.

# 5 Acknowledgments

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# 6 Appendix

Here we present for completeness the parametrization for $F_2^{\rm p}(x,Q^2)$ as suggested in Ref. [17] and slightly modified in the present analysis:

$$F_2^{\rm p}(x,Q^2) = x^{\lambda_1}(1-x)^{\lambda_2} \sum_{n=1..5} C_n (1-x)^{n-1} \left( \frac{ln(\frac{Q^2}{\Lambda})}{ln(\frac{Q_0^2}{\Lambda})} \right)^{B(x)} \left( 1 + \frac{\sum\limits_{n=1..4} \kappa_n x^n}{Q^2} \right) \quad (A.1)$$

where

$$B(x) = \rho_1 + \rho_2 x + \frac{\rho_3}{\rho_4 + x}.$$

Fit parameters as obtained in Ref. [17] are presented in Table 1. The parametrization is restricted to the kinematic region $3.5 \cdot 10^{-5} < x < 0.85$. In order to extend it to the

| $i$ | $\lambda_i$ | $\rho_i$ | $\kappa_i$ | $C_i$ |
|---|---|---|---|---|
| 1 | −0.2499713175097 | 0.1141083888210 | −1.451744104784 | 0.2289630236346 |
| 2 | 2.396344728724 | −2.235597858569 | 8.474547402342 | 0.08498360257578 |
| 3 | – | 0.03115195484229 | −34.37914208393 | 3.860797992943 |
| 4 | – | 0.02135222381130 | 45.88805973036 | −7.414275585348 |
| 5 | – | – | – | 3.434223579597 |

Table 1: Values of the parameters for $F_2^{\rm p}$ given in Eq. (A.1).

region of high $x$ and satisfy the $(1-x)^3$ behavior at $x \to 1$ we modify the parameter $\lambda_2$ as follows:

$$\lambda_2 \to \tilde{\lambda}_2(x) = \lambda_2 + (3-\lambda_2)x^{15} \; .$$

This correction does not affect the values of $F_2^{\rm p}$ at $x < 0.6$ and affords an approximation of the proton data in a much wider kinematic region.
The neutron structure function is defined by Eqs. (7) and (8) in the main text. Taking into account the constraints on the parameters we arrive at the following expression for $R^{\rm n/p}$:

$$R^{\rm n/p}(x) = (1-x) + a_2 x^{\alpha_2} + \frac{\alpha_2 a_2 - 1}{1+c_1} x^{\beta_1}(1-x)(1+c_1 x^{\gamma_1}) \ . \qquad \text{(A.2)}$$

The fit parameters are presented in Table 2.

| | $a_2 = 2/3$ | $a_2 = 0.47$ | $a_2 = 1/4$ |
|---|---|---|---|
| $\alpha_2$ | 3.13971 | 2.2262 | 1.15416 |
| $\beta_1$ | 2.2129 | 1.61188 | 0.88126 |
| $c_1$ | −1.01176 | −1.00692 | 0.86217 |
| $\gamma_1$ | 0.01901 | 0.08483 | 5.65744 |

Table 2: Parameters of Eq. (A.2) which is the constrained form of Eq. (8) connecting the proton and neutron structure functions.

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