Fermion masses and mixings from heterotic orbifold models¹

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Abstract. We search for a possibility of getting realistic fermion mass ratios and mixing angles from renormalizable couplings on the Z_6 -I heterotic orbifold with one pair of Higgs doublets. In the quark sector, we find cases with reasonable $\overline{m}_c/\overline{m}_t, \overline{m}_s/\overline{m}_b$, and V_{cb} , if we ignore the first family. In the lepton sector, we can fit the charged lepton mass ratios, the neutrino mass squared difference ratio, and the lepton mixing angles, considering all three families.

In heterotic string theory, there are 26 bosonic leftmoving degrees of freedom and 10 supersymmetric right-moving degrees of freedom. Among these, four left-movers and four right-movers are the observed spacetime. The rest are compactified. Among the leftmovers, 16 of them are responsible for the internal gauge symmetry. Remaining six left-movers and six rightmovers can serve as an origin of the flavor structure of the Yukawa couplings. Orbifold is commonly used for the geometry of these internal six dimensions. On an orbifold, there are fixed points, and a twisted closed string ground state is attached to each of these fixed points. A trilinear string scattering amplitude of three of these states is written as

$$\int \mathscr{D}X \ e^{-S} \sigma_1(z_1) \sigma_2(z_2) \sigma_3(z_3) = Z_{qu} \sum_{\langle X_{cl} \rangle} e^{-S_{cl}}$$

where σ_i represents a twist field creating the appropriate twisted ground state. The quantum part Z_{qu} in the righthand side is a global factor for all the couplings with a given twist structure, and the flavor structure essentially comes from the classical part. The sum is over all the classical string configurations, and the classical action S_{cl} is given by the world sheet area. Therefore this amplitude depends on the distances among the three fixed points which are determined by the relative positions of the internal dimensions. Since the amplitude has an exponential suppression factor depending on the volumes, one can hope to use this to account for the hierarchical structure of fermion masses and mixings.

The overall picture of this work is illustrated in Fig. 1. We associate each of the matter fields such as quarks, leptons, and Higgses, to one or more of the fixed points. This association is done by hand, which means that we do not specify how a Standard-like model is obtained by



FIGURE 1. As an example, the up-type Higgs doublet H_u , the third generation SU(2) doublet quark Q_3 , and the SU(2) singlet charm quark are assigned to three different twisted close string ground states. The Yukawa matrix element $(Y_u)_{32}$ given by their trilinear string scattering amplitude, is proportional to the area of the world sheet shown in the drawing.

fixing gauge shifts and Wilson lines [1]. Therefore, this work is a sort of model-independent analysis performed on a specific type of orbifold, Z_6 –I in the present case. Once the field assignment is fixed, we can compute the Yukawa couplings of the quarks or leptons as functions of the moduli $R_{1,2,3}^2$, describing the sizes of the three two-dimensional tori. Then, we fit quark or lepton mass ratios and mixing angles varying $R_{1,2,3}^2$. We repeat this procedure for each possible assignment, looking for a case which can reproduce the observed mass ratios and mixings for well-chosen values of the moduli.

All the six-dimensional orbifolds that give rise to N = 1 four-dimensional supersymmetry have been classified. Among these, prime orbifolds such as Z_3 and Z_7 are not useful for our purpose. Suppose that three states attached to the three fixed points $f_{1,2,3}$ form a Yukawa coupling. Given f_1 and f_2 , the space group selection rule on a prime orbifold uniquely determines f_3 which can couple to f_1 and f_2 . Because of this property, a Yukawa matrix from renormalizable couplings either is diagonal, or has a zero eigenvalue. If we avoid a massless quark, we are lead to have a trivial CKM matrix equal to identity. Therefore, we do not consider a prime orbifold. On a

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non-prime orbifold, f_3 is not uniquely determined, and nontrivial mixing is possible. We consider the Z_6 –I orbifold because the other orbifolds have smaller number of available twisted states or smaller number of parameters which can be tuned to adjust fermion mass ratios and mixing angles. However, this does not necessarily mean that all the other non-prime orbifolds are not phenomenologically viable. Also, one should keep in mind that nonrenormalizable couplings may always contribute to Yukawa couplings. This is the reason why string phenomenology on prime orbifolds is not useless.

Before describing the Z_6 –I orbifold, let us first look at two-dimensional orbifolds which will be used to construct it. A 2D Z_3 orbifold looks like Fig. 2 (a). It is a torus modded by 120° of rotation. It has the following fixed points and their respective twisted ground states:

$$g_{Z_{3,1}}^{(0)} = (0,0) \qquad \rightarrow \qquad |g_{Z_{3,1}}^{(0)}\rangle,$$

$$g_{Z_{3,1}}^{(1)} = (1/3,2/3) \qquad \rightarrow \qquad |g_{Z_{3,1}}^{(1)}\rangle, \qquad (1)$$

$$g_{Z_{3,1}}^{(2)} = (2/3,1/3) \qquad \rightarrow \qquad |g_{Z_{3,1}}^{(2)}\rangle.$$

The parenthesized coordinates are in the units of the basis vectors e_1 and e_2 . Another relevant 2D orbifold is the Z_6 orbifold, shown in Fig. 2 (b). It is a torus modded by 60° of rotation. Let θ denote this rotation. The structure of fixed points and the twisted states on this orbifold is more involved since it has θ^2 -twisted and θ^3 -twisted sectors as well as θ -twisted sector. They are summarized in Table 1. A physical state should be a θ eigenstate. Taking linear combinations of the states attached to the fixed points [2], one can get the θ eigenstates,

$$\begin{split} |g^{(0)}_{Z_{6,1}}\rangle, \\ |g^{(0)}_{Z_{6,2}}\rangle, \ |g^{(1)}_{Z_{6,2}}; \pm 1\rangle \equiv |g^{(1)}_{Z_{6,2}}\rangle \pm |g^{(2)}_{Z_{6,2}}\rangle, \\ |g^{(0)}_{Z_{6,3}}\rangle, \ |g^{(1)}_{Z_{6,3}}; \gamma\rangle \equiv |g^{(1)}_{Z_{6,3}}\rangle + \gamma |g^{(2)}_{Z_{6,3}}\rangle + \gamma^{2} |g^{(3)}_{Z_{6,3}}\rangle, \end{split}$$
(2)

where $\gamma = 1, \omega, \omega^2$ with $\omega = e^{2\pi i/3}$. The 6D Z₆–I orbifold is a direct product of two 2D Z₆ orbifolds and one 2D Z₃ orbifold. The fixed points are given as direct products of those on the 2D orbifolds, and therefore the corresponding twisted states follow in the same manner. Due to the point group selection rule and *H*-momentum conservation, one has two types of possible Yukawa couplings,

$$\widehat{T}_1 \widehat{T}_2 \widehat{T}_3, \quad \widehat{T}_2 \widehat{T}_2 \widehat{T}_2,$$

where \hat{T}_1 , \hat{T}_2 , and \hat{T}_3 are states from the θ -, θ^2 -, and θ^3 twisted sectors, respectively. Each of these states is a direct product of two states from (2) with the corresponding twist, and their concrete expressions can be found in [3, 4]. One can show that the 2D Z₃ orbifold contributes only to the overall factor of a Yukawa matrix, thus being irrelevant to fermion mass ratios and mixings.



FIGURE 2. Two-dimensional Z_3 and Z_6 orbifolds. A cross marks a fixed point. The region inside dashed lines is the fundamental domain of the orbifold.

However, this part can be used to scale a Yukawa matrix to a desirable order of magnitude. For example, $\tan\beta$ can be changed by scaling either Y_u or Y_d .

In this work, we assume that there exists a model based on the Z_6 -I heterotic orbifold realizing the following points:

- $SU(3) \times SU(2) \times U(1)_Y$ observable gauge group.
- · Three families of quarks and leptons.
- One family of Higgs doublets, H_{μ} and H_{d} .
- All matter fields come from twisted sectors.

Among these, the last point is due to the fact that it is very hard to get hierarchical fermion masses using untwisted sector fields.

Let us first discuss the quark sector [3]. We assign each of $Q_{1,2,3}$, $u_{1,2,3}^c$, $d_{1,2,3}^c$, H_u , and H_d to one of the twisted states. In order to get nontrivial quark mass ratios and mixings, we are lead to consider one of the five classes of assignments shown in Table 2. In each of

TABLE 1. Two-dimensional Z_6 orbifold has three different twisted sectors. The fixed points which are invariant under each twist are shown here. The coordinates in the parentheses are with respect to e_1 and e_2 in Fig. 2 (b).

Under	Fixed points are			
$\theta (= 60^{\circ} \text{ rot.})$	$g_{Z_6,1}^{(0)} = (0,0)$			
θ^2	$g_{Z_{6},2}^{(0)} = (0,0), g_{Z_{6},2}^{(1)} = (0,1/3),$ $g_{Z_{6},2}^{(2)} = (0,2/3)$			
θ^3	$g_{Z_{6},3}^{(0)} = (0,0), \qquad g_{Z_{6},3}^{(1)} = (0,1/2),$ $g_{Z_{6},3}^{(2)} = (1/2,0), \qquad g_{Z_{6},3}^{(3)} = (1/2,1/2)$			

TABLE 3. An example field assignment from each class. Values of the moduli R_1^2 and R_2^2 which lead to the best fit of the quark mass ratios and V_{cb} are also shown. Central mass ratio values in the last row are from the running quark masses at m_W scale. Meaning of each symbol denoting a state can be found in [3].

Class	$Q_2 \ Q_3 \ u_2^c \ u_3^c \ d_2^c \ d_3^c \ H_u \ H_d$	R_1^2	R_2^2	$\overline{m}_c/\overline{m}_t$	$\overline{m}_s/\overline{m}_b$	V_{cb}
1	$\widehat{T}_{2}^{(2)}\widehat{T}_{2}^{(4)}\widehat{T}_{3}^{(3)}\widehat{T}_{3}^{(2)}\widehat{T}_{3}^{(1)}\widehat{T}_{3}^{(3)}\widehat{T}_{1} \widehat{T}_{1}$	27.8	107	0.0038	0.029	0.041
2	$\tilde{T}_{3}^{(2)} \tilde{T}_{3}^{(4)} \tilde{T}_{2}^{(3)} \tilde{T}_{2}^{(2)} \tilde{T}_{2}^{(1)} \tilde{T}_{2}^{(3)} \tilde{T}_{1} \hat{T}_{1}$	24.0	150	0.0038	0.032	0.041
3	$\tilde{T}_{2}^{(1)}\tilde{T}_{2}^{(4)}\tilde{T}_{3}^{(2)}\tilde{T}_{3}^{(4)}\tilde{T}_{2}^{(2)}\tilde{T}_{2}^{(4)}\tilde{T}_{1}$ $\tilde{T}_{2}^{(4)}$	196	316	0.0038	0.019	0.042
4	$\widehat{T}_{2}^{(2)}\widehat{T}_{2}^{(4)}\widehat{T}_{2}^{(2)}\widehat{T}_{2}^{(3)}\widehat{T}_{3}^{(1)}\widehat{T}_{3}^{(4)}\widehat{T}_{2}^{(4)}\widehat{T}_{1}^{(4)}$	416	226	0.0040	0.035	0.035
5	$\widehat{T}_{2}^{(2)}\widehat{T}_{2}^{(4)}\widehat{T}_{2}^{(2)}\widehat{T}_{2}^{(4)}\widehat{T}_{2}^{(3)}\widehat{T}_{2}^{(2)}\widehat{T}_{2}^{(4)}\widehat{T}_{2}^{(4)}$	368	400	0.0038	0.029	0.041
Central values from measurements [5]				0.0038	0.025	0.041

TABLE 2. Five classes of assignments.

Class	Q or L	u^c or N	d^c or e^c	H_u	H_d
1	\widehat{T}_2	\widehat{T}_3	\widehat{T}_3	\widehat{T}_1	\widehat{T}_1
2	\widehat{T}_3	\widehat{T}_2	\widehat{T}_2	\widehat{T}_1	\widehat{T}_1
3	\widehat{T}_2	\widehat{T}_3	\widehat{T}_2	\widehat{T}_1	\widehat{T}_2
4	\widehat{T}_2	\widehat{T}_2	\widehat{T}_3	\widehat{T}_2	\widehat{T}_1
5	\widehat{T}_2	\widehat{T}_2	\widehat{T}_2	\widehat{T}_2	\widehat{T}_2

these classes, we examine every possibility of field assignment, for which we perform fitting of $\overline{m}_u/\overline{m}_t, \overline{m}_c/\overline{m}_t$, $\overline{m}_d/\overline{m}_b, \overline{m}_s/\overline{m}_b, V_{us}, V_{cb}, \text{ and } V_{ub}, \text{ varying } R_1^2 \text{ and } R_2^2. \text{ Re-}$ call that R_3^2 is irrelevant to mass ratios and mixings. We ignore quark mass running between the string scale and the weak scale. If we try to fit all of the mass ratios and the CKM matrix elements above, we do not find a satisfactory result. Ignoring the first family quarks, however, we can get values of $\overline{m}_c/\overline{m}_t$, $\overline{m}_s/\overline{m}_b$, and V_{cb} , which are fairly close to the central values from measurements. We quote one instance of fit from each assignment class in Table 3. There are a number of other instances in addition to the one shown in the table for each class. To our knowledge, this is the first work that showed a possibility of getting realistic mixing angles from renormalizable couplings in string models with one pair of Higgs fields.

Now we turn to the lepton sector [4]. The analysis procedure here almost parallels that for the quark sector. Computation of the lepton Yukawa couplings is performed in the same way except that we should replace (Q, u^c, d^c) by (L, N, e^c) . A complication is that the neutrino masses may be of Majorana type, in addition to Dirac type which is a direct analogy of quark masses. One customarily incorporates the seesaw mechanism for Majorana neutrino masses to explain lightness of neutrinos. In this work, we consider two neutrino mass generation mechanisms: Dirac mass scenario, and seesaw scenario with the right-handed neutrino mass matrix proportional to a unit matrix. For each of these scenarios, we assign lepton and Higgs fields to the twisted states and fit m_e/m_{τ} , m_{μ}/m_{τ} , $\Delta m_{31}^2/\Delta m_{21}^2$, $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$, and $\sin^2 \theta_{13}$.

The result in the Dirac scenario is summarized in Table 4. In Classes 1, 2, 3, and 5, we do not find a good fit of the neutrino mass squared difference ratio and mixing angles. Approximate (in)equalities in these classes show the typical behavior of each quantity for assignments with relatively low χ^2 . In contrast to the other classes, Class 4 leads to promising results. It is notable that we can fit all the above six observables tuning only two parameters R_1^2 and R_2^2 , in this class. One example assignment is as follows:

$$\begin{split} (L_1, L_2, L_3) &= (\widehat{T}_2^{(2)}, \widehat{T}_2^{(3)}, \widehat{T}_2^{(4,-1)}), \\ (N_1, N_2, N_3) &= (\widehat{T}_2^{(2)}, \widehat{T}_2^{(4,1)}, \widehat{T}_2^{(4,-1)}), \\ (e_1^c, e_2^c, e_3^c) &= (\widehat{T}_3^{(1)}, \widehat{T}_3^{(2)}, \widehat{T}_3^{(4,1)}), \\ (H_u, H_d) &= (\widehat{T}_2^{(2)}, \widehat{T}_1). \end{split}$$

Meaning of each symbol on the right-hand sides is available in [4]. This assignment results in the fit shown in Table 4 for $(R_1^2, R_2^2) = (26.0, 20.6)$. In this scenario, smallness of neutrino masses should be accounted for by small Yukawa couplings. For this, one can use the 2D Z_3 orbifold. For example, *L*, *N*, and H_u can be put at three different fixed points on the Z_3 orbifold, with three families of *L* or *N* gathered at a single point. If the size of this orbifold is taken to be big enough so that $R_3^2 \sim 1000$, one can have a sufficient suppression factor for the neutrino Yukawa couplings.

In the seesaw scenario, we assume that the righthanded neutrino mass matrix is proportional to an identity matrix. Therefore, the lepton mixing angles are essentially determined by the Yukawa couplings as in the Dirac scenario. One difference is that a physical neutrino mass eigenvalue is proportional to the square of a Yukawa matrix eigenvalue. This enhances the mass squared difference ratio relative to that in the Dirac scenario. Indeed, $\Delta m_{31}^2/\Delta m_{21}^2$ in Table 5 for an example assignment in Class 4 is larger and hence is closer to the

TABLE 4. Characteristics of each class in the Dirac neutrino case. Typical behavior of each quantity is described for combinations resulting in relatively good fits in a given class except Class 4. The row corresponding to Class 4 shows the best fit. We omit m_e/m_{τ} and m_{μ}/m_{τ} because they can be fit in all the classes.

Class	$\Delta m_{31}^2/\Delta m_{21}^2$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$\sin^2 \theta_{13}$
1	~ 100	$\lesssim 10^{-5}$	$\lesssim 10^{-2}$	$\lesssim 10^{-7}$
2	~ 100	$\lesssim 10^{-5}$	$\lesssim 10^{-2}$	$\lesssim 10^{-7}$
3	$\gtrsim 1.4$			
4	14	0.38	0.70	$6.3 imes 10^{-6}$
5	~ 28	≤ 0.09		$\lesssim 10^{-2}$
Central values [6]	27	0.30	0.50	0.000

TABLE 5. Characteristics of each class in the seesaw case. Typical behavior of each quantity is described for combinations resulting in relatively good fits in a given class. The row corresponding to Class 4 shows the best fit. We omit m_e/m_{τ} and m_{μ}/m_{τ} because they can be fit in all the classes.

Class	$\Delta m_{31}^2/\Delta m_{21}^2$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$\sin^2 \theta_{13}$
1	~ 6000	$\lesssim 10^{-5}$	$\lesssim 10^{-2}$	$\lesssim 10^{-7}$
2	~ 7000	$\lesssim 10^{-5}$	$\lesssim 10^{-2}$	$\lesssim 10^{-7}$
3	$\gtrsim 2$			
4	29	0.32	0.48	$3.6 imes 10^{-6}$
5	~ 28	≤ 0.09		$\lesssim 10^{-2}$
Central values [6]	27	0.30	0.50	0.000

central value than in Table 4. This fit was obtained using the following assignment:

$$\begin{split} (L_1, L_2, L_3) &= (\widehat{T}_2^{(1)}, \widehat{T}_2^{(2)}, \widehat{T}_2^{(4,1)}), \\ (N_1, N_2, N_3) &= (\widehat{T}_2^{(2)}, \widehat{T}_2^{(3)}, \widehat{T}_2^{(4,1)}), \\ (e_1^c, e_2^c, e_3^c) &= (\widehat{T}_3^{(1)}, \widehat{T}_3^{(2)}, \widehat{T}_3^{(4,1)}), \\ (H_u, H_d) &= (\widehat{T}_2^{(2)}, \widehat{T}_1), \end{split}$$

for $(R_1^2, R_2^2) = (22.5, 26.0)$. In this case, we need the scale of the right-handed neutrino mass $M_N \sim 10^{15}$ GeV for the neutrino masses to be of the right order of magnitude. As in the Dirac scenario, the other classes do not lead to an acceptable fit of the observables. Let us remark that the predicted value of θ_{13} is vanishingly small and the neutrino mass spectrum shows normal hierarchy both in the seesaw scenario and in the Dirac scenario.

In conclusion, we systematically searched for possibilities to get realistic fermion mass ratios and mixings from the Z₆–I heterotic orbifold. We assumed that Yukawa matrices of quarks and leptons arise from renormalizable couplings with one family of Higgses. In the quark sector, we could obtain reasonable values of $\overline{m}_c/\overline{m}_t$, $\overline{m}_s/\overline{m}_b$, and V_{cb} ignoring the first family, although we failed to get an acceptable fit in the three family analysis. In the lepton sector, we could fit the six observables of m_e/m_{τ} , m_{μ}/m_{τ} , $\Delta m_{31}^2/\Delta m_{21}^2$, θ_{12} , θ_{23} , and θ_{13} , by adjusting only two moduli parameters in either the Dirac or the seesaw scenario.

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