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Theory of a Strongly Interacting Electroweak Symmetry Breaking Sector

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ABSTRACT

In this review we discuss theories of the electroweak symmetry breaking sector in which the W and Z interactions become strong at an energy scale not larger than a few TeV.

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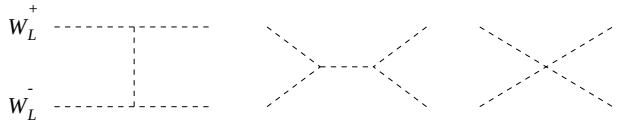
The standard $SU(2)_W \times U(1)_Y$ gauge theory of the electroweak interactions is in good agreement with all current experimental data [1]. Nonetheless, there is no direct evidence that shows which mechanism is responsible for the breakdown of this symmetry to the $U(1)$ of electromagnetism. However, it is clear that additional clues to the physics of symmetry breaking must appear at energies of order a TeV or lower. Consider a thought experiment [2], the scattering of longitudinally polarized W^+ and W^-:



(1)

Using the Feynman-rules of the electroweak gauge theory we can calculate $W_L^+ W_L^-$ scattering at tree level. We find that this amplitude grows like E_{cm}^2:

$$\mathcal{A} = \frac{g^2 s}{8 M_W^2} \left(1 + \cos\theta^*\right) \quad , \tag{2}$$

plus terms that do not grow with s. Projecting onto the s-wave state, we find

$$\mathcal{A}^{l=0} = \frac{g^2 s}{128\pi M_W^2} \sim \left(\frac{\sqrt{s}}{2.5 \text{ TeV}}\right)^2 \quad . \tag{3}$$

Unitarity implies that some new physics has to enter to cut off the growth of this amplitude before an energy of around 2.5 TeV[2][3]. That is, the dynamics associated with EWSB has to appear before that energy scale. There are three possibilities:

- There may be additional particles with masses less than or of order of a TeV, or
- the W and Z interactions may become strong at energies of order a TeV, or
- both of the above.

This review discusses the theory of a symmetry breaking sector in which the W and Z interactions become strong at or below an energy scale of order a TeV. For an introduction to the phenomenology of a strongly-interacting symmetry breaking sector, we refer the reader to the review of Chanowitz [4]. For a more detailed review of the phenomenological situation at specific proposed colliders, such as the LHC or NLC, we refer the reader to the sections on strongly coupled electroweak symmetry breaking in [5].

In the next section we discuss theories of electroweak symmetry breaking. In the second section, we discuss the use of effective Lagrangians to describe the phenomenology of a strongly-interacting symmetry breaking sector. In the third section, we discuss the limitations of the effective Lagrangian framework. Our conclusions are presented in the final section.

1. Theories of Electroweak Symmetry Breaking

Theories of electroweak symmetry breaking may be classified by the energy scale of the dynamics responsible for the symmetry breaking. There are theories, such as technicolor, in which the physics responsible for symmetry breaking occurs at an energy of order a TeV, and there are theories, such as the top mode standard model, in which the physics is at a much higher energy.

We begin our discussion of theories of symmetry breaking with a description of the successes and shortcomings of theories with fundamental scalars, in particular the standard one-doublet Higgs model. We argue that, because of triviality, any theory with "fundamental" scalars can only be regarded as a low-energy effective theory for some more fundamental dynamics at a higher energy scale which is ultimately responsible for electroweak symmetry breaking. We further argue that when the scale of new physics is high, the low-energy effective scalar theory is weakly-coupled and cannot give rise to strong W and Z interactions at energies of order a TeV.

Next, we discuss technicolor, the prototypical theory of dynamical electroweak symmetry breaking. In technicolor theories the scale of the physics responsible for electroweak symmetry breaking is of order a TeV. In contrast to theories with fundamental scalars, these theories can give rise to strong W and Z interactions at energies of order a TeV.

We conclude with a discussion of theories in which the scale of the physics of electroweak symmetry breaking may be adjusted to a value of order a TeV, in which case the theory is technicolor-like, or to a much higher value, in which case the theory generally contains light scalar particles which appear to be fundamental. As the scale of symmetry-breaking physics is varied, the behavior of the W and Z scattering amplitudes interpolates between the two extremes discussed above: when the scale of symmetry-breaking physics is of order a TeV, the W and Z interactions can become strong; if the scale is much higher they cannot.

1.1. The Standard One-Doublet Higgs Model and Generalizations Thereof

In the standard one-doublet Higgs model one introduces a fundamental scalar doublet of $SU(2)_W$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad , \tag{1.1}$$

which has a potential of the form

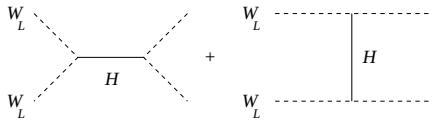
$$V(\phi) = \lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2 \quad . \tag{1.2}$$

In the potential (1.2), v^2 is assumed to be positive in order to favor the generation of a non-zero vacuum expectation value for ϕ. This vacuum expectation value breaks the electroweak symmetry, giving mass to the W and Z. When symmetry breaking takes place, the four degrees of freedom in ϕ divide up. Three of them become the longitudinal components, W_L and Z_L, of the gauge bosons, and the fourth, commonly called H (for Higgs particle), is left over

$$\phi = \Omega \begin{pmatrix} 0 \\ \frac{H+v}{\sqrt{2}} \end{pmatrix} \quad . \tag{1.3}$$

In (1.3), Ω is an $SU(2)$ matrix. If we make an $SU(2)_W$ gauge transformation until Ω is the identity, we arrive at unitary gauge.

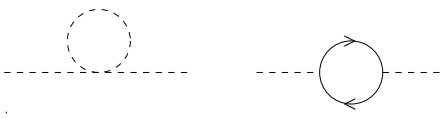
The exchange of the Higgs boson contributes to $W_L W_L$ scattering. In the limit in which E_{cm} is large compared to the masses of the particles in the process, the leading contribution (in energy) from Higgs boson exchange exactly cancels the bad high-energy behavior displayed in eqn. (2)



$$\rightarrow \mathcal{A} = -\frac{g^2 s}{8 M_W^2} \left(1 + \cos\theta^*\right) \quad , \tag{1.4}$$

plus terms which do not grow with energy. At tree-level the Higgs boson has a mass given by $m_H^2 = 2\lambda v^2$. In order for this theory to give rise to strong W and Z interactions, it would be necessary that the Higgs boson be heavy and, therefore, that λ be large.

This explanation of electroweak symmetry breaking is unsatisfactory for a number of reasons. For one thing, this model does not give a dynamical explanation of electroweak symmetry breaking: one simply assumes that the potential is adjusted to produce the desired result. In addition, when embedded in theories with additional dynamics at higher energy scales, these theories are technically unnatural [6] in the following sense: radiative effects (*e.g.* one-loop contributions to the Higgs mass), are typically proportional to whatever cutoff is put on the theory



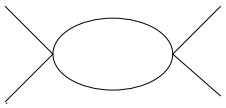
$$\rightarrow \delta m_H^2 \propto \Lambda^2 \quad . \tag{1.5}$$

More precisely, there is no ordinary[1] symmetry protecting the mass of the Higgs. When a fermion mass goes to zero, there is a chiral symmetry that protects the fermion mass from getting large radiative corrections; the Higgs mass has no such protection in the standard model. Therefore, the parameters of the theory must be carefully adjusted in order to keep the weak scale of order 250 GeV. In particular, in a theory with a higher scale, such as a Grand Unified Theory, there is no explanation for why the Higgs mass is not equal to the GUT scale.

Perhaps most unsatisfactory, however, is that theories of fundamental scalars are probably "trivial" [8], *i.e.*, it is not possible to construct an interacting theory of scalars in four dimensions that is valid to arbitrarily short distance scales. In quantum field theories, fluctuations in the vacuum screen charge – the vacuum acts as a dielectric medium. Therefore there is an effective coupling constant which depends on the energy scale (μ) at which it is measured. The variation of the coupling with scale is summarized by the β–function of the theory

$$\beta(\lambda) = \mu \frac{d\lambda}{d\mu} \quad . \tag{1.6}$$

The only coupling in the Higgs sector of the standard model is the Higgs self-coupling λ. In perturbation theory, the β-function is calculated to be



$$\rightarrow \beta = \frac{3\lambda^2}{2\pi^2} \quad . \tag{1.7}$$

Using this β–function and the differential equation eq. (1.6), one can compute the behavior of the coupling constant as a function of the scale[2]. One finds that the coupling at a scale μ is related to the coupling at some higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log \frac{\Lambda}{\mu} \quad . \tag{1.8}$$

In order for the Higgs potential to be stable, $\lambda(\Lambda)$ has to be positive. This implies that

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log \frac{\Lambda}{\mu} \quad . \tag{1.9}$$

[1] In supersymmetric theories, the mass of the Higgs particle(s) are protected by the chiral symmetry of their fermionic partners. In such theories, however, the scalar self-couplings are related to the gauge coupling constants and, therefore, these theories do not give rise to strong W and Z interactions [7].

[2] Since these expressions were computed in perturbation theory, they are only valid when $\lambda(\mu)$ is sufficiently small. We will return to the issue of strong coupling below.

Thus, we have the bound

$$\lambda(\mu) \leq \frac{2\pi^2}{3\log\left(\frac{\Lambda}{\mu}\right)} \quad . \tag{1.10}$$

If this theory is to make sense to arbitrarily short distances, and hence arbitrarily high energies, we should take Λ to ∞ while holding μ fixed at about 1 TeV. In this limit we see that the bound on λ goes to zero. In the continuum limit, this theory is trivial; it is free field theory.

The inequality above can be translated into an upper bound on the mass of the Higgs boson[9]. From eq. (1.10) we have

$$\frac{\Lambda}{\mu} \leq \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right) \quad , \tag{1.11}$$

but

$$m_H^2 \sim 2v^2\lambda(m_H) \quad , \tag{1.12}$$

thus

$$\Lambda \leq m_H \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right) \quad . \tag{1.13}$$

For a given Higgs boson mass, there is a *finite* cutoff energy at which the description of the theory as a fundamental scalar doublet stops making sense. This means that the standard one-doublet Higgs model can only be regarded as an *effective* theory valid below this cutoff.

The theory of a relatively light weakly coupled Higgs boson, can be self-consistent to a very high energy. For example, if the theory is to make sense up to a typical GUT scale energy, 10^{16} GeV, then the Higgs boson mass has to be less than about 170 GeV [10]. In this sense, although a theory with a light Higgs boson does not really answer any of the interesting questions (*e.g.*, it does not explain *why* $SU(2)_W \times U(1)_Y$ breaking occurs), the theory does manage to postpone the issue up to higher energies.

The theory of a heavy Higgs boson (*i.e.* with a mass of about 1 TeV), however, does not really make sense. Since we have computed the β-function in perturbation theory, this answer is only reliable at energy scales at which $\lambda(\mu)$ (as well as the Higgs boson mass) is small. Fortunately, non-perturbative lattice calculations are available. Early estimates [11] indicated that if the theory was to make sense up to 4 TeV, the mass of the Higgs boson had to be less than about 640 GeV. More recent results [12] imply that this bound may be relaxed somewhat; one might be able to get away with an 800 GeV Higgs boson, but the Higgs boson mass is certainly bounded by a value of this order of magnitude. The

triviality limits on the mass of the Higgs boson imply that it is not possible for the W_L and Z_L scattering amplitudes in the standard model to truly become large at energies well below the cutoff. This result is especially interesting because it implies that if nothing shows up below energies of the order 700–800 GeV, then something truly "non-trivial" is going on. We just have to find it.

It is straightforward to generalize the one-doublet Higgs model to models with more than one fundamental scalar doublet, or to models with scalars in other representations of the $SU(2)_W$ [7]. In such theories, one or more particles with the quantum numbers of the standard-model Higgs boson (as well as, potentially, particles of weak-isospin 2 [7][13]) contribute to $W_L W_L$ scattering. However, all such models[3] suffer from the problems described above for the one-doublet standard model. In fact, because these theories involve more scalar degrees of freedom, they typically have β-functions which are *larger* (more positive) then the standard model. For this reason, the corresponding triviality constraints on the masses of particles are typically *stronger* [14][13].

In addition, in models with more than one doublet of scalars, care must be taken to insure that the weak-interaction ρ-parameter

$$\rho = \frac{M_W}{M_Z \cos\theta_W} \quad , \tag{1.14}$$

does not deviate significantly from one. In the standard model, this parameter is (at tree-level) automatically equal to one. This is the result of an accidental symmetry [15]. While the potential eqn (1.2) has only a manifest $SU(2)_W \times U(1)_Y$ invariance, it is actually invariant under a global $O(4) \approx SU(2)_{L(W)} \times SU(2)_R$ symmetry. When symmetry breaking occurs, the symmetry breaking sector in the one-doublet Higgs model has a residual $SU(2)_{L+R}$ "custodial" symmetry which ensures that the relation $\rho = 1$ is satisfied.

Finally, we note that any theory of electroweak symmetry breaking must also allow for the symmetry breaking to be transmitted to the quarks and leptons, so that they can become massive as well. In the standard model, fermion masses are obtained by introducing Yukawa interactions that couple the Higgs doublet to the left- and right-handed fermions. After the Higgs field develops an expectation value, the fermions obtain a mass proportional to the Yukawa coupling. By choosing the Yukawa couplings appropriately,

[3] Supersymmetric models have a Higgs sector containing two scalar doublets. In principle, they are trivial as well. However, as noted above [7], the quartic couplings in such models are typically quite small and the physics of symmetry-breaking may arise at much higher scales.

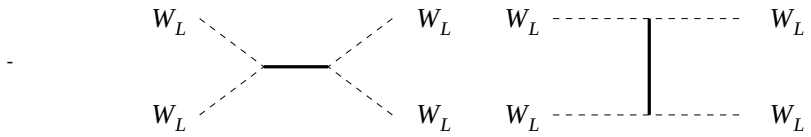
one can accommodate the observed masses (and mixing angles) of the quarks and leptons. Understanding the couplings of the fermions to the symmetry-breaking sector, therefore, generally involves understanding the physics of flavor symmetry breaking. As we will not be discussing the physics of flavor here, we will have little to say about the couplings of ordinary fermions to the symmetry-breaking sector in the current review.

1.2. Technicolor

In models with fundamental scalars, electroweak symmetry breaking can be accommodated if the parameters in the potential (which presumably arise from additional physics at higher energies) are suitably chosen. By contrast, technicolor theories strive to explain electroweak symmetry breaking in terms of physics operating at an energy scale of order a TeV. In technicolor theories, electroweak symmetry breaking is the result of chiral symmetry breaking in an asymptotically-free, strongly-interacting gauge theory with massless fermions. Unlike theories with fundamental scalars, these theories are technically natural: just as the scale Λ_{QCD} arises in QCD by dimensional transmutation, so too does the weak scale v in technicolor theories. Accordingly, it can be exponentially smaller than the GUT or Planck scales. Furthermore, asymptotically-free non-abelian gauge theories may be fully consistent quantum field theories.

In the simplest technicolor theory one introduces a (massless) left-handed weak-doublet of "technifermions", and the corresponding right-handed weak-singlets, which transform as N's of a strong $SU(N)_{TC}$ technicolor gauge group. In analogy to the (approximate) chiral $SU(2)_L \times SU(2)_R$ symmetry on quarks in QCD, the strong technicolor interactions respect an $SU(2)_L \times SU(2)_R$ global chiral symmetry on the technifermions. When the technicolor interactions become strong, the chiral symmetry is broken to the diagonal subgroup, $SU(2)_{L+R}$, producing three Nambu-Goldstone bosons which become, via the Higgs mechanism, the longitudinal degrees of freedom of the W_L and Z_L. Because the left-handed and right-handed techni-fermions carry different electroweak quantum numbers, the electroweak interactions break to electromagnetism. If the f-constant of the theory, the analog of f_π in QCD, is chosen to be 246 GeV, then the W mass has its observed value. Furthermore, since the symmetry structure of the theory is *precisely* the same as that of the standard one-Higgs-doublet model, the remaining $SU(2)_{L+R}$ custodial symmetry insures that, to lowest order in the hypercharge coupling, $M_W = M_Z \cos\theta_W$. As discussed in section 2, at low energies, the phenomenology of a model with an $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ symmetry can be described in terms of an effective chiral Lagrangian.

In addition to the "eaten" Nambu-Goldstone bosons, such a theory will give rise to various resonances, the analogs of the ρ, ω, and possibly the σ, in QCD. In general, the growth of the W_L and Z_L scattering amplitudes (eq. (2)) are cut off by exchange of these heavy resonances,



just as in QCD the growth of pion–pion scattering amplitudes are cut off by QCD resonances. Scaling from QCD, we expect that the masses of the various resonance will be of order a TeV. Unlike the situation in models with only fundamental scalars in the symmetry breaking sector, the scattering of longitudinal W and Z bosons can truly be strong. In section 3 we will discuss the resonances that can occur in these models.

The symmetry breaking sector must also couple to the ordinary fermions, allowing them to acquire mass. In models of a strong electroweak symmetry breaking sector there must either be additional flavor-dependent gauge interactions [16], the so-called "extended" technicolor (ETC) interactions, or Yukawa couplings to scalars [17] which communicate the breaking of the chiral symmetry of the technifermions to the ordinary fermions. As we are not discussing the physics of flavor, we refer the reader to ref. [18] for a recent review.

The technicolor theory may possess a global chiral symmetry group G larger than $SU(2)_L \times SU(2)_R$, which breaks to a subgroup H larger than $SU(2)$. For example, it is commonly assumed in ETC models that the ETC interactions commute with the ordinary strong and electroweak interactions. In order to explain the masses of all observed fermions these models must contain an entire family of technifermions with standard model gauge couplings. Such models are referred to as one-family models and possess an approximate $SU(8)_L \times SU(8)_R$ symmetry. In general, all that is necessary to break electroweak symmetry is that the electroweak $SU(2)_W \times U(1)_Y$ gauge group is embedded in G in such a way that the only unbroken subgroup of the electroweak interactions in H is electromagnetism.

One consequence of having a larger global symmetry is that the f-constant of the theory may be different from 246 GeV: if the theory contains N_D doublets, all of which contribute equally to the W and Z masses, the f-constant must be chosen to be $246/\sqrt{N_D}$ GeV. Furthermore, since there are generally more broken global symmetries than the three associated with the weak currents, chiral symmetry breaking produces additional (pseudo-)Nambu-Goldstone bosons. Since experiment tells us that these extra Nambu-Goldstone

bosons cannot be strictly massless, other interactions (generally electroweak, color, or ETC) must break the corresponding global symmetries.

Non-minimal models typically also possess a larger variety of resonances than the one-techni-doublet model. As in the simplest technicolor model, it is the exchange of resonances that cuts off the growth in the W_L and Z_L scattering amplitudes. In theories with many doublets (or, in general, with many flavors [19], see the third section), since the f-constant is generally smaller than 246 GeV, we expect that the masses of the resonances are smaller than in the one-doublet model. In addition, because of the existence of other pseudo-Nambu-Goldstone bosons, there may be sizable *inelastic* scattering amplitudes for W_L and Z_L scattering.

1.3. Other Theories of Dynamical Electroweak Symmetry Breaking

There are also theories in which the scale (M) of the dynamics responsible for electroweak symmetry breaking can, in principle, take any value of order a TeV or greater. We will describe two classes of such models.

The first class of models, inspired by the Nambu–Jona-Lasinio (NJL) model [20] of chiral symmetry breaking in QCD, involve a strong, but *spontaneously broken*, gauge interaction. Examples include top quark condensate (and related) models [21][22][23][24][25], as well as models with strong extended technicolor interactions [26]. When the strength of the effective four-fermion interaction describing the broken gauge interactions – *i.e.* the strength of the extended technicolor interactions in strong ETC models or the strength of other gauge interactions in top-condensate models – is adjusted close to the critical value for chiral symmetry breaking, the high-energy dynamics may play a role in electroweak symmetry breaking without driving the electroweak scale to a value of order M.

The second class are the Georgi-Kaplan Composite Higgs models [27]. In these, all *four* members of a Higgs doublet are Nambu-Goldstone bosons arising from chiral symmetry breaking due to a strong "hypercolor" interaction coupling to massless hyperfermions. In these theories $SU(2)_W \times U(1)_Y$ breaking is due to vacuum misalignment, typically because of the presence of an extra chiral gauge interaction. By adjusting the strength of the extra interaction responsible for the misalignment of the vacuum, it is possible to choose the scale of chiral-symmetry breaking of the hypercolor interactions to be larger, possibly much larger, than 1 TeV.

The high-energy dynamics must have the appropriate properties in order for it to play a role in electroweak symmetry breaking [28]: If the coupling constants of the high-energy theory are small, only low-energy dynamics (such as technicolor) can contribute to electroweak symmetry breaking. If the coupling constants of the high-energy theory are large and the interactions are attractive in the appropriate channels, chiral symmetry will be broken by the high-energy interactions and the scale of electroweak symmetry breaking will be of order M. If the transition between these two extremes is continuous, *i.e.* if the chiral symmetry breaking phase transition is *second order* in the high-energy couplings, then it is possible to adjust the high-energy parameters so that the dynamics at scale M can contribute to electroweak symmetry breaking. The adjustment of the high-energy couplings is a reflection of the fine-tuning required to create a hierarchy of scales.

What is crucial is that the transition be (at least approximately) second order in the high-energy couplings. If the transition is first order, then as one adjusts the high-energy couplings the scale of chiral symmetry breaking will jump discontinuously from approximately zero at weak coupling to approximately M at strong coupling. Therefore, if the transition is first order, it will generally not be possible to maintain any hierarchy between the scale of electroweak symmetry breaking and the scale of the high-energy dynamics.

If the transition is second order and if there is a large hierarchy of scales ($M \gg 1$ TeV), then close to the transition the theory may be described in terms of a low-energy effective Lagrangian with composite "Higgs" scalars – the Ginsburg-Landau theory of the chiral phase transition. However, if there is a large hierarchy, the arguments of triviality given in the first section apply to the effective low-energy Ginsburg-Landau theory describing the composite scalars: the effective low-energy theory would be one which describes a *weakly* coupled theory of (almost) fundamental scalars, despite the fact that the "fundamental" interactions are strongly self-coupled!

For this reason, only models in which M is of order 1 TeV can result in strong W_L and Z_L scattering amplitudes. In these models, while the extra "Higgs" scalars may be relatively heavy, they may still be light enough that they should be included in an effective-Lagrangian description of low-energy W_L and Z_L interactions. Furthermore, the interactions of these scalars can differ significantly from those of the standard-model Higgs boson [29]. The effective Lagrangian appropriate for describing the phenomenology of these models is discussed in section 3.

2. Effective Lagrangians and Electroweak Symmetry Breaking

The unknown high-energy physics responsible for electroweak symmetry breaking both provides the weak bosons with mass and influences their interactions with one another and with other particles. Hence, a meticulous investigation of the properties of the weak bosons can provide clues to the nature of the symmetry breaking sector. The most efficient way of proceeding is to identify a model-independent method of analyzing the relationship between the weak bosons' properties and the high-energy physics responsible for electroweak symmetry breaking. We discuss here the formalism of effective Lagrangians which will enable us to focus on the known symmetry properties of the broken theory and to classify interactions at energies below the symmetry-breaking scale in terms of their transformation properties under the symmetry remaining at low energies. This emphasis on symmetry will enable us to make quantitative statements about strongly-interacting dynamics for which direct calculation is problematic.

2.1. Effective Lagrangians

An "effective" Lagrangian is one that affords an approximate description of physics at energies below a designated cutoff scale Λ. The particle content and symmetry structure of the effective Lagrangian are dictated by what exists at scales below the cutoff. The presence of higher-energy physics and heavier particles is incorporated via the inclusion of appropriate non-renormalizable terms. The terms in an effective Lagrangian are arranged as an expansion in powers of momentum over the cutoff, Λ. Although there are an infinite number of terms in this expansion, at low energy the first few terms can give a good approximation. A familiar example of an effective Lagrangian is the $V-A$ description of the charged-current weak interactions at energies below M_W. The effective theory includes non-renormalizable four-fermion contact interactions that result from "integrating out" the propagating W boson that is present at higher energies.

The effective Lagrangian is in general non-renormalizable. That means that if calculated to an arbitrary number of loops, the renormalization of the theory would require an infinite number of counterterms. There must be some organizational principle by which some of the operators are included and others neglected in a particular calculation. Moreover this procedure has to be systematic, so that large contributions are not neglected at any order in the expansion.

In general, the requirement is that Λ be much larger than the momentum scale p at which the experiments are performed, and amplitudes are written as a power series in p/Λ.

When one computes a low-energy amplitude to a given accuracy, we compute the required numbers of terms in the momentum expansion. For example, in the $V-A$ theory, the cutoff scale is the mass of the W, and the momentum expansion is in terms of four-fermi operators that contain extra derivatives and are suppressed by additional powers of $1/M_W^2$. This expansion can be expected to work for momenta up to the cutoff.

In addition to powers of Λ suppressing the higher dimension operators, each operator has a dimensionless coefficient C. But if an operator in this expansion had a coefficient C very much greater than order 1, there would be some momentum scale $p \ll \Lambda$ at which it could compete with lower dimension operators. This would imply that the momentum expansion had broken down at p, well below Λ. Accordingly, every dimensionless coefficient in the expansion is expected to be smaller than or of order one, at least if the cutoff is really Λ.

It is possible to judge whether a given experiment can place useful limits on the coefficients of terms in the effective Lagrangian. Say, for example, that given the cutoff Λ we expect a particular coefficient C to be of order 1. If a proposed experiment can only place an upper bound of 100 on that coefficient, the measurement is not likely to be informative. On the other hand, if an experiment appeared to measure a definite value of 50 for that coefficient, it would indicate that new physics enters at a scale lower than expected – an informative outcome indeed!

As mentioned above, the longitudinal modes of the W and Z are the Nambu-Goldstone bosons of a spontaneously broken $SU(2)\times U(1)$ symmetry. As we will see, their lowest-order interactions are completely determined by the symmetry structure. Therefore, distinguishing among different models of symmetry breaking will require more precise measurements than might seem necessary at first glance, because any dynamics that *is* sensitive to the precise nature of the symmetry-breaking sector is suppressed by powers of $1/\Lambda^2$. As in the $V-A$ example, the cutoff is at the mass of the physics that was integrated out – a characteristic scale of the symmetry breaking. For example, in the standard model with $M_W \ll m_H \lesssim 1\,\mathrm{TeV}$, $\Lambda = m_H$, while in a technicolor model, Λ might be of order the mass of the lightest techni-resonance.

Interestingly, there are additional constraints on the effective Lagrangian: the cutoff scale Λ may not get arbitrarily large, and the C's of the operators cannot get too small. This is once again due to the non-renormalizable nature of the theory. The operators that appear at any given order in the momentum expansion are needed as counterterms for loop diagrams involving lower-order operators. If the cutoff Λ were very large or a particular

C were very small, it would imply that the corresponding higher-dimension operator was unimportant. On the other hand, the operator is a counterterm for loop diagrams involving lower-dimension operators, and so it is unnatural to assume that the small renormalized value of the coefficient is the result of a cancellation of a large bare coupling with a large loop diagram. It is more natural to assume that the coefficients in the effective Lagrangian are not too small, and Λ is not too big. This argument, known as Naive Dimensional Analysis (NDA), implies that, for electroweak symmetry breaking, $\Lambda \lesssim 4\pi v$. If this limit is saturated, the coefficients C are of order one[30].

Our discussion in this section of the effective Lagrangian for electroweak symmetry breaking is subject to the following constraints. We will assume that the longitudinal W and Z are the only quanta in the strongly-interacting symmetry-breaking sector that are light compared to the symmetry-breaking scale. This necessarily constrains the global symmetry-breaking pattern to be $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ or $SU(2)_L \times U(1)_R/U(1)_{L+R}$ [31]. We remain mindful that the formalism is only valid in the energy regime in which the momentum expansion is valid.

In discussing high-energy tests of the strongly-interacting symmetry-breaking sector, we shall also rely on the "equivalence theorem" [2][32][33]. This states that in calculating scattering amplitudes at center-of-mass energy E, one may replace external longitudinal W and Z bosons by the corresponding Nambu-Goldstone bosons, up to corrections of order M_W/E. The resulting simplification is of particular use in discussing the two-body scattering of longitudinal weak bosons.

2.2. The effective Lagrangian at order p^2

Our next task is to construct an effective Lagrangian that will enable us to study the interactions of the W and Z bosons. We consider the most general Lagrangian consistent with the observed symmetry breaking pattern. We begin by considering a Lagrangian for global symmetry breaking, in terms of the "eaten" Nambu–Goldstone bosons π^a. These fields are most conveniently written in the non-linear representation

$$\Sigma = \exp(2i\pi^a T^a/f) \quad . \tag{2.1}$$

Here the T^a are $SU(2)$ generators normalized to $\mathrm{Tr}\left[T^a T^b\right] = \delta^{ab}/2$, and f is the analogue of the pion decay constant. Under a global chiral transformation, the field Σ transforms as $\Sigma \to L\Sigma R^\dagger$, with $L \in SU(2)_L$ and $R \in SU(2)_R$ or $U(1)_R$.

If the only low-energy degrees of freedom of interest are the Nambu-Goldstone boson fields themselves, the most general chirally invariant Lagrangian can be written as an expansion in powers of derivatives of Σ [34][35]. There are no nontrivial chirally invariant terms involving no derivatives. And there are only two terms with two derivatives[31]:

$$\mathcal{L}_2^u = \frac{f^2}{4}\mathrm{Tr}\ \partial^\mu\Sigma^\dagger\partial_\mu\Sigma + \frac{f^2}{2}(\rho-1)\left[\mathrm{Tr}T_3\Sigma^\dagger\partial^\mu\Sigma\right]^2 \quad . \tag{2.2}$$

Here ρ is an arbitrary coefficient; we will see below that it corresponds precisely to the ρ parameter defined in eqn. (1.14) above. Note that the second term is only invariant under $U(1)_R$ and not the full $SU(2)_R$ symmetry group. Terms with more derivatives are suppressed by inverse powers of the momentum cutoff corresponding to the scale, Λ, at which additional physics enters; we will discuss these operators shortly.

Up to now the discussion has been about global symmetries only, but to study the interactions of the weak bosons, one gauges the chiral symmetries, identifying $SU(2)_L$ with $SU(2)_W$ and the diagonal generator of $SU(2)_R$ (or the generator of $U(1)_R$) with $U(1)_Y$, and employs the corresponding gauged Lagrangian. To lowest order this amounts to gauging (2.2),

$$\mathcal{L}_2 = \frac{f^2}{4}\mathrm{Tr}\left[D^\mu\Sigma^\dagger D_\mu\Sigma\right] + \frac{f^2}{2}(\rho-1)\left[\mathrm{Tr}T_3\Sigma^\dagger D^\mu\Sigma\right]^2 \quad , \tag{2.3}$$

where the covariant derivative is $D_\mu\Sigma = \partial_\mu\Sigma + \mathrm{i}g\mathbf{W}_\mu\Sigma - \mathrm{i}\Sigma g'\mathbf{B}_\mu$. and the gauge boson fields are $\mathbf{W}_\mu = W_\mu^a T^a$ and $\mathbf{B}_\mu = B_\mu T^3$. The full effective Lagrangian for the theory of gauge and Nambu-Goldstone bosons is the sum of the lowest-order Lagrangian (2.3), the usual gauge-boson kinetic energy terms

$$\begin{aligned}\mathcal{L}_{gauge} = &-\frac{1}{2}\mathrm{Tr}\left[\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}\right] - \frac{1}{2}\mathrm{Tr}\left[\mathbf{B}^{\mu\nu}\mathbf{B}_{\mu\nu}\right], \\ \mathbf{B}_{\mu\nu} &= \left(\partial_\mu\mathbf{B}_\nu - \partial_\nu\mathbf{B}_\mu\right)T^3 \\ \mathbf{W}_{\mu\nu} &= \left(\partial_\mu\mathbf{W}_\nu - \partial_\nu\mathbf{W}_\mu - \frac{i}{2}\,g[\mathbf{W}_\mu,\mathbf{W}_\nu]\right) \quad ,\end{aligned} \tag{2.4}$$

and gauge-fixing and Fadeev-Popov ghost terms.

To find expressions for the gauge boson masses in this effective theory, we rewrite the order p^2 Lagrangian in unitary gauge (where $\Sigma = 1$) and diagonalize the $W_3 - B$ mixing matrix. The result is

$$\frac{g^2 f^2}{4}W_-^\mu W_{+\mu} + \frac{g^2 f^2}{8\rho\cos^2\theta}Z^\mu Z_\mu \quad . \tag{2.5}$$

The photon $A_\mu = \sin\theta W^3_\mu + \cos\theta B_\mu$ is massless. Since the mass of the W boson is $M_W = gf/2$, the Nambu-Goldstone boson decay constant f is equal to $v \equiv 246$ GeV. As noted earlier, the parameter ρ equals 1 for a theory in which a custodial symmetry $SU(2)_{L+R}$ remains after chiral symmetry breaking; otherwise the deviation of ρ from 1 measures the degree of custodial symmetry violation in the theory.

We can also obtain definitive expressions for two-body scattering of the Nambu-Goldstone bosons (W_L and Z_L) that, like the W and Z masses, depend on v and ρ. In the energy range where the effective Lagrangian and the equivalence theorem are both valid, $M_W^2 << s << \Lambda^2$, this can be done by expanding the Lagrangian (2.2) to determine the 3-π and 4-π vertices, and then forming the amplitudes. The result is

$$\begin{aligned} \mathcal{M}[W_L^+W_L^- \to W_L^+W_L^-] =& \frac{iu}{v^2\rho} \\ \mathcal{M}[W_L^+W_L^- \to Z_LZ_L] =& \frac{is}{v^2}\left(4 - \frac{3}{\rho}\right) \\ \mathcal{M}[Z_LZ_L \to Z_LZ_L] =& 0 \quad . \end{aligned} \tag{2.6}$$

and the expressions for the $W_L^\pm Z_L$ and $W_L^\pm W_L^\pm$ channels follow by crossing symmetry. What is striking is that these tree-level expressions for longitudinal gauge boson scattering at energies below the symmetry-breaking scale will be identical for *any* theory with an $SU(2)_L \times SU(2)_R$ global symmetry structure at high energies. Hence, the expressions (2.6) are known as the "low-energy theorems" for a strongly interacting symmetry-breaking sector.

A wealth of data from LEP, SLAC and Fermilab now tell us that ρ equals 1 to a few parts in a thousand [1][36] :

$$\rho - 1 = \pm .004 \quad . \tag{2.7}$$

Therefore, for the rest of this article we shall assume that the pattern of symmetry breaking is $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$; the custodial symmetry that enforces $\rho = 1$ is present. The only source of custodial symmetry breaking in our effective Lagrangian will be the non-zero hypercharge coupling, g'.

2.3. The effective Lagrangian at order p^4

So far, we have constructed an effective Lagrangian whose predictions depend only on the symmetries of the electroweak symmetry breaking sector. In order to probe other

properties and differentiate among competing models, it will be necessary to include terms in the Lagrangian that arise at higher order in the momentum expansion.

The next-to-leading order effective Lagrangian for the Nambu-Goldstone fields includes several terms containing four derivatives [34][37]:

$$\mathcal{L}_4^u = \frac{L_1}{16\pi^2}\{\mathrm{Tr}(\partial^\mu\Sigma^\dagger\partial_\mu\Sigma)\}^2 + \frac{L_2}{16\pi^2}\{\mathrm{Tr}(\partial^\mu\Sigma^\dagger\partial^\nu\Sigma)\}^2 \quad . \tag{2.8}$$

All other possible four-derivative terms are linear combinations of these two or vanish by the equations of motion.

The coefficients L_1 and L_2 are new parameters of the effective Lagrangian which are not determined by the low-energy terms. The coefficients $L_i/16\pi^2$ of the operators in (2.8) are of order v^2/Λ^2. Therefore, the L_i are of order one in a theory in which $\Lambda \approx 4\pi v$. NDA implies that Λ cannot be larger than this value. Different underlying theories of the high-energy physics responsible for electroweak symmetry breaking will predict different values for the L_i. It is by measuring the physical observables related to these coefficients that experiments will be able to constrain such models. If the L_i are found to be significantly larger than one, the scale Λ is less than $4\pi v$.

Again, if we are interested in studying the loop-level properties of scattering amplitudes involving the weak bosons, we employ a gauged effective Lagrangian. This looks like:

$$\begin{aligned}\mathcal{L}_4 \;=\; & \frac{L_1}{16\pi^2}\left[\mathrm{Tr}D^\mu\Sigma^\dagger D_\mu\Sigma\right]^2 \;+\; \frac{L_2}{16\pi^2}\,\mathrm{Tr}D_\mu\Sigma^\dagger D_\nu\Sigma\mathrm{Tr}D^\mu\Sigma^\dagger D^\nu\Sigma \\ & - ig\frac{L_{9L}}{16\pi^2}\,\mathrm{Tr}\mathbf{W}^{\mu\nu}D_\mu\Sigma D_\nu\Sigma^\dagger \;-\; ig'\frac{L_{9R}}{16\pi^2}\,\mathrm{Tr}\mathbf{B}^{\mu\nu}D_\mu\Sigma^\dagger D_\nu\Sigma \\ & +\; gg'\frac{L_{10}}{16\pi^2}\,\mathrm{Tr}\Sigma\mathbf{B}^{\mu\nu}\Sigma^\dagger\mathbf{W}_{\mu\nu} \quad .\end{aligned} \tag{2.9}$$

Unlike [38], we are not restricting ourselves to vectorial models with $L_9^L = L_9^R$.

We now relate these various coefficients to physical quantities that colliders are currently measuring or hope to bound in the future. We shall address sequentially the information provided by 2-point, 3-point and 4-point vertices involving gauge and Nambu-Goldstone bosons.

• `2-point vertices`

Radiative corrections from non-standard physics that alter the vacuum polarization of the electroweak gauge bosons are known as "oblique" corrections [39]. Due to their effects on many well-measured quantities, the oblique corrections provide some of the most important limits on the electroweak symmetry breaking sector [35][40][41].

It is conventional to describe the oblique corrections in terms of three ultraviolet-finite combinations of vacuum polarizations [41]:

$$\begin{aligned}\alpha S \equiv & 4e^2 \left[\Pi'_{33}(0) - \Pi'_{3Q}(0)\right] \\ \alpha T \equiv & \frac{g^2}{\cos^2\theta m_Z^2}\left[\Pi_{11}(0) - \Pi_{33}(0)\right] \\ \alpha U \equiv & 4e^2\left[\Pi'_{11}(0) - \Pi'_{33}(0)\right] \quad .\end{aligned} \tag{2.10}$$

where $\Pi'(q^2) \equiv d\Pi(q^2)/dq^2$. After calculating radiative corrections to an observable x, one can write

$$x = x_{sm}(m_t, m_H) + \lambda_1^x S + \lambda_2^x T + \lambda_3^x U \quad , \tag{2.11}$$

where x_{sm} includes all standard model contributions to x for given masses of the top quark and Higgs boson, and the λ_i^x are coefficients independent of m_t and m_H. When the observables α, G_F and M_Z are used to define the parameters g, g' and v in the electroweak theory, α_3^x is zero for all neutral-current and low-energy observables. The only measured quantity depending on U is the ratio of the W and Z masses [41]; furthermore, since we are assuming an approximate custodial symmetry holds, $U/T \sim m_Z^2/\Lambda^2 << 1$. If one takes $U \approx 0$, the S parameter measures weak-isospin-conserving oblique corrections from new physics and T measures weak-isospin-violating contributions.

Examining the effective Lagrangian (2.9) in unitary gauge, we find that the only term that includes a 2-point vertex is the operator with coefficient L_{10}. This, then, is the only operator that contributes to the oblique corrections at order p^4. Since the L_{10} term contributes an amount $-q^2 L_{10}(M_Z)/16\pi^2$ to the vacuum polarization $\Pi_{33} - \Pi_{3Q}$, one has [42]

$$L_{10} = -\frac{1}{\pi} S \quad . \tag{2.12}$$

We will find that this correspondence between L_{10} and S means that L_{10} is better constrained at present than any of the other L_i.

The T parameter as defined above is related to the isospin-violating parameter ρ encountered in the discussion of weak gauge boson masses

$$\alpha T = \rho - 1 \quad . \tag{2.13}$$

We have already limited our discussion to theories in which the presence of an approximate custodial $SU(2)_{L+R}$ symmetry enforces $\rho \approx 1$. A non-zero value for the hypercharge coupling does break the custodial symmetry, so that loop diagrams involving exchange

of hypercharge bosons do contribute to non-zero T. If one is studying the energy range $M_Z < E < m_t$, where the top quark is not present in the effective theory, then the absence of a partner for the bottom quark introduces additional contributions to T.

Current limits on S and T derived from a global fit to data [1] are

$$S = -0.15 \pm 0.25^{-0.08}_{+0.17}, \qquad T = -0.08 \pm 0.32^{+0.18}_{-0.11} \quad . \tag{2.14}$$

This implies the constraint

$$-.09 < L_{10} < 0.15 \tag{2.15}$$

on the effective Lagrangian at order p^4.

•3-point vertices

A popular topic in recent years [4] has been the study of the ability of collider experiments to test the form and strength of the three-weak-gauge-boson vertices. While much effort has been devoted to studying the potential of FNAL, LEP, LEP II, LHC, HERA and various NLCs for measuring small deviations from the standard model predictions. It seems clear that the prospects are dim [43]. Simply put, the only values of the L_i that would be accessible to *any* current experiment are so large that for any reasonable Λ they contradict the rules discussed in section 2.1, which are an intrinsic part of the effective Lagrangian. A similar statement can be made for any but the highest energy experiments being planned. If any experiment at FNAL, LEP, LEP II, or HERA were to measure a deviation from the standard model predictions, it would imply an L_i so large that the scale of new physics would have to be nearly as small as M_W, invalidating the entire effective Lagrangian approach.

In order to study non-standard contributions to the three-gauge-boson vertices, we expand the effective Lagrangian (2.9) in unitary gauge and extract the terms with three-point vertices. To make contact with the literature on this topic, it is convenient to organize the three-point terms as follows:

$$\begin{aligned} \mathcal{L}_4^{3-point} = & - \mathrm{i}e_* \frac{\cos\theta}{\sin\theta} g_Z \left(W^\dagger_{\mu\nu} W^\mu - W_{\mu\nu} W^{\mu\dagger} \right) Z^\nu \\ & - \mathrm{i}e_* \left(W^\dagger_{\mu\nu} W^\mu - W_{\mu\nu} W^{\mu\dagger} \right) A^\nu \\ & - \mathrm{i}e_* \frac{\cos\theta}{\sin\theta} k_Z W^\dagger_\mu W_\nu Z^{\mu\nu} - \mathrm{i}e_* k_\gamma W^\dagger_\mu W_\nu A^{\mu\nu} \quad , \end{aligned} \tag{2.16}$$

[4] see e.g. Proceedings, International Symposium on Vector Boson Self Interactions, ed. by U. Baur, S. Errede, T. Muller, UCLA, Feb. 1-3, 1995.

where

$$
\begin{aligned}
g_Z &= \frac{e_*^2 L_9^L}{32\pi^2 \sin^2\theta\cos^2\theta} + \frac{e_*^2 L_{10}}{16\pi^2\cos^2\theta(\cos^2\theta - \sin^2\theta)}, \\
k_Z &= \frac{e_*^2(\cos^2\theta L_9^L - \sin^2\theta L_9^R)}{16\pi^2\cos^2\theta\sin^2\theta} + \frac{2e_*^2 L_{10}}{16\pi^2\cos^2\theta\sin^2\theta}, \\
k_\gamma &= -\frac{e_*^2\left(L_9^L + L_9^R + 2L_{10}\right)}{32\pi^2\sin^2\theta} \quad .
\end{aligned}
\tag{2.17}
$$

The coupling e_* and mixing angle are defined by

$$
\begin{aligned}
e_*^2/4\pi =& \alpha_*(M_Z) \\
\sin^2\theta\cos^2\theta \equiv& \frac{\pi\alpha_*}{\sqrt{2}G_F m_Z^2} \quad .
\end{aligned}
\tag{2.18}
$$

Before discussing possible experimental limits, we should demonstrate the relationship between our effective Lagrangian and a related formalism often used for discussion of weak boson three-point vertices. The notation introduced in [44] for describing non-standard C and P conserving contributions to weak-gauge-boson self-interactions is

$$
\begin{aligned}
\frac{i}{e\cot\theta}\mathcal{L}_{WWZ} =& g_1(W^\dagger_{\mu\nu}W^\mu Z^\nu - W^\dagger_\mu Z_\nu W^{\mu\nu}) + \kappa_Z W^\dagger_\mu W_\nu Z^\mu\nu + \frac{\lambda_Z}{M_W^2}W^\dagger_{\lambda\mu}W^\mu_\nu Z^{\nu\lambda} \\
\frac{i}{e}\mathcal{L}_{\mathcal{WW}\gamma} =& (W^\dagger_{\mu\nu}W^\mu A^\nu - W^\dagger_\mu A_\nu W^{\mu\nu}) + \kappa_\gamma W^\dagger_\mu W_\nu F^\mu\nu + \frac{\lambda_\gamma}{M_W^2}W^\dagger_{\lambda\mu}W^\mu_\nu F^{\nu\lambda} \quad .
\end{aligned}
\tag{2.19}
$$

In the standard model, one has $g_1 = \kappa_Z = \kappa_\gamma = 1$ and $\lambda_Z = \lambda_\gamma = 0$; deviations from these values are intended to parametrize the contributions of new physics. By comparing (2.19) with (2.9) and (2.17) above, we find that g_1, κ_Z and κ_γ are related to the L_i by

$$
\left.\begin{array}{c} g_1 - 1 \\ \kappa_Z - 1 \\ \kappa_\gamma - 1 \end{array}\right\} \approx \frac{\alpha_* L_i}{4\pi\sin^2\theta} \quad .
\tag{2.20}
$$

If the L_i are of order 1, the parameters κ_γ and κ_Z differ from unity by an amount that is of order 10^{-3}. It will be *crucial* to bear this in mind when evaluating experimental measurements of deviations of three-point vertices from the standard model predictions.

The coefficients λ_γ and λ_Z in (2.19) accompany terms that are of higher order, p^6, in the momentum expansion. Therefore, they are related not to the L_i discussed above, but to coefficients of higher-order operators. Because we are constructing our effective Lagrangian (2.9) as a systematic expansion in powers of p^2/Λ^2, if we were to include terms of order p^6, they would naturally be suppressed by a factor of $1/\Lambda^2$. For example, our order-p^6 Lagrangian would include a term like:

$$\frac{Cv^2}{\Lambda^4}\,\mathrm{Tr}\left[D_\mu\Sigma^\dagger D^\mu\Sigma\right]^3 \quad , \tag{2.21}$$

where C is order 1. Expressing the order p^6 terms in (2.19) as part of such an effective Lagrangian, we have

$$\frac{\lambda_{Z,\gamma}}{M_W^2} = \frac{Cv^2}{\Lambda^4} \quad , \tag{2.22}$$

This is consistent with the fact that we expect the effects of a strongly-interacting symmetry breaking sector to vanish in both the limit of vanishing W mass (since no symmetry-breaking will have been effected) and in the limit of a large symmetry-breaking scale. For $\Lambda \sim 1$ TeV, we expect

$$\lambda_{Z,\gamma} = C\frac{M_W^2}{\Lambda^4} \approx 10^{-4} \quad . \tag{2.23}$$

Again, the small value expected for $\lambda_{Z,\gamma}$ will strongly influence our assessment of the utility of planned experimental measurements.

Much has been written about how to use present or anticipated data to constrain 3-gauge-boson vertices; a compendium of results from energies high and low appears in [45] . We shall summarize the salient points and indicate where the interested reader may look for further details. We have chosen this route in large part because most present and anticipated limits on the three-point L_i (or equivalently on the λ_i and κ_i) are woefully loose.

A straightforward calculation starting from the effective Lagrangian (2.16) reveals the contribution that higher-dimension operators make to scattering processes involving three-vector-boson vertices. It has been demonstrated that the various operators make complementary contributions to different processes. Production (at an e^+e^- or hadron collider) of pairs of Z bosons or of a $W^\pm W^\pm$ final state does not involve a three-gauge-boson vertex, and so is independent of the L_i considered here. The process $f\bar{f} \to W^\pm Z$ involves L_{9L}; the channel $f\bar{f} \to W^\pm W^\mp$ involves L_{9L} and L_{9R}; the channel $f\bar{f} \to W^\pm\gamma$ involves L_{9L}, L_{9R} and L_{10} [46][47][48][49][50][51]. Precision measurements of Z decays at LEP are indirectly sensitive to L_{9L}, L_{9R} and L_{10} [52][53]; measurements of $ep \to \nu\gamma X$ at HERA could also potentially access those three L_i [54]. At an $e\gamma$ collider, the process $e\gamma \to \nu WZ$ is affected by the L_{9L} and L_{9R} couplings [55].

When the cross-sections are compared with existing or projected data, the following pattern emerges. The integrated luminosity accumulated at the Tevatron should restrict the $|\kappa_i - 1|$ and $|\lambda_i|$ to be smaller than about 1 [46][56]. The limits from HERA are,

perhaps, a little looser at present [54]. In other words, the constraints derivable from existing data greatly exceed the natural values of the coefficients[5]

This situation will gradually improve at future colliders [43]. Experiments at LEPII may improve the bounds on $|\kappa_i - 1|$ and $|\lambda_i|$ to something of order 0.1 [47], which would imply new strongly coupled physics at $\Lambda \approx 300$ GeV. Either the LHC [49][50][51][48][56] or an NLC [47] with a center-of-mass energy of half a TeV could push this to roughly 0.01, which would imply strongly interacting new physics at $\Lambda \approx 1$ TeV. It would take an NLC with $\sqrt{s} \geq 1$ TeV to probe $|\kappa_i - 1|$ or $|\lambda_i|$ to anything near their minimum size of a few times 10^{-3} [47]. In other words, only the highest-energy electron-positron colliders being discussed today would have the resolution required to probe $\Lambda \approx 4\pi v$.

• `4-point vertices`

Direct tests of the four-point vertices must await the advent of high-energy colliders capable of producing large numbers of high-momentum weak boson pairs. Two-body scattering of weak bosons occurs at high-energy colliders like the LHC or NLC when gauge bosons are radiated from the incoming fermions and then rescatter via a four-point vertex. The four-point vertices of greatest interest for experimentally probing the nature of the electroweak symmetry-breaking sector are those involving only longitudinal gauge bosons. The $V_L V_L \to V_L V_L$ processes that they mediate are precisely those which the dynamics associated with electroweak symmetry breaking must unitarize at an energy of 2.5 TeV or less. As can be seen by inspecting the effective Lagrangian (2.9), four-point vertices involving transverse, as well as longitudinal, gauge bosons will be affected by the higher-order terms. However, scattering processes involving transverse gauge bosons suffer from much larger backgrounds which would obscure the effects of the symmetry-breaking sector.

Many terms in our effective Lagrangian include pieces that correspond to four-point vertices, but only two are relevant here. Since we care only about the four-point scattering of longitudinal weak bosons, we can work in terms of the ungauged Lagrangian (2.2) and (2.8). This eliminates the L_9 and L_{10} terms, for example. Furthermore, the contributions of the leading-order Lagrangian (2.2) will, by the low-energy theorems, be identical in any symmetry-breaking sector with a given symmetry structure. This leaves us with the order

[5] Current LEP data have been shown to place indirect bounds of order 20 on $|L_{9L}|$ and of order 80 on $|L_{9R}|$ [52][53]. These bounds are based on loop-level calculations assuming that no large tree-level contribution causes a significant cancellation of the effect.

p^4 Lagrangian (2.8); as we assuming an $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ global symmetry-breaking pattern, only the terms proportional to L_1 and L_2 are present.

There are several different physical scattering processes encompassed in the expression $V_L V_L \to V_L V_L$. Since we are assuming that our theory possesses an unbroken custodial $SU(2)_{L+R}$ symmetry, the scattering amplitudes for the different proceses are related to one another by crossing and $SU(2)_{L+R}$ symmetries. More precisely, if the amplitude for the process $W_L^+ W_L^- \to Z_L Z_L$ is given by

$$\mathcal{M}(W_L^+ W_L^- \to Z_L Z_L) \equiv A(s,t,u) \quad , \tag{2.24}$$

where s,t,u are the usual Mandelstaam kinematic variables, then the amplitudes for the other $V_L V_L \to V_L V_L$ processes are

$$\begin{aligned} \mathcal{M}(W_L^+ W_L^- \to W_L^+ W_L^-) &= A(s,t,u) + A(t,s,u) \\ \mathcal{M}(Z_L Z_L \to Z_L Z_L) &= A(s,t,u) + A(t,s,u) + A(t,s,u) \\ \mathcal{M}(W_L^\pm Z_L \to W_L^\pm Z_L) &= A(t,s,u) \\ \mathcal{M}(W_L^\pm W_L^\pm \to W_L^\pm W_L^\pm) &= A(t,s,u) + A(u,t,s) \quad . \end{aligned} \tag{2.25}$$

The tree-level contribution to $A(s,t,u)$ from the low-energy theorem is

$$A(s,t,u)_{L.E.T.} = \frac{s}{f^2} \quad . \tag{2.26}$$

The tree-level contribution at order p^4 from (2.8) is [50]

$$A(s,t,u)_4 = \frac{4}{f^4}\left(2L_1 s^2 + L_2(t^2+u^2)\right) \quad . \tag{2.27}$$

The form and symmetries of the amplitudes for the 2-body scattering of Nambu-Goldstone bosons arising from the effective Lagrangian (2.8) have been discussed at length in [19][37].

The amplitudes we have written down for $V_L V_L \to V_L V_L$ scattering make it clear that production of all of the different $V_L V_L$ final states will be affected by the order p^4 effective Lagrangian coefficients L_1 and L_2. However, some final states lend themselves more readily to the study of four-point vertices than others do. The $W_L^+ W_L^-$ and $W_L^\pm Z_L$ final states are produced mostly through $f\bar{f}$ annihilation rather than weak boson re-scattering; therefore production of these states is more sensitive to alteration of the 3-gauge-boson vertex by the L_9 terms of (2.9) than to alteration of the 4-point vertex by L_1 or L_2 [51][49]. The $Z_L Z_L$ final state cannot be produced through a 3-weak-boson vertex and therefore lacks

the large $f\bar{f}$ annihilation background; however there are backgrounds from continuum ZZ production and (at hadron colliders) from gluon fusion through a top quark loop [57]. The $W_L^\pm W_L^\pm$ final state has the distinct advantage of being free from order α^2 continuum backgrounds; the largest backgrounds are from $t\bar{t}$ production and decay and from a mixed electroweak-strong process in which a gluon is exchanged between two initial-state quarks which then each radiate a weak boson. The lower background rates in this channel should allow the observation of signal events at relatively low invariant mass; since the WW distribution functions fall with increasing invariant mass, this increases the signal-to-background ratio. Indeed, the $W_L^\pm W_L^\pm$ channel appears to be most sensitive to L_1 and L_2 once backgrounds, branching fractions and cuts are taken into account [58][49][57][59].

A good deal of effort has been directed at estimating the ability of proposed high-energy colliders to constrain L_1 and L_2[6]. It has been found that an NLC can probe the coefficients L_1 and L_2 down to the level of 1-5 [60]. The LHC is projected to do even better – measuring them to within their natural size of order 1 [57][49][45].

3. Beyond the Effective Lagrangian

The effective Lagrangians discussed in previous sections can never provide anything more than a low energy description of symmetry breaking physics. Since they are non-renormalizable, effective Lagrangians cannot be extended to arbitrarily high energies. Ultimately one wants to know the true structure of the strongly interacting theory.

Consider the interactions of the ordinary hadrons. The effective Lagrangian for the low-energy states in QCD describes only the scattering of pions near zero momentum. At energies above a few GeV one may use perturbative QCD to describe features of the physics such as the rate of multijet events. In a sense, it is most difficult to describe the range of energies between approximately 1 and 10 GeV. This is the region that contains bound state resonances such as the $\rho(770)$ and the baryons. The techniques for describing this region are nowhere near as simple and beautiful as those that work for either low or high energies.

In this section we wade into the bog of intermediate energy. We will go beyond the dynamics of the longitudinal gauge bosons, to describe what happens when other

[6] Current LEP data have been shown to place indirect bounds of order 20 on L_1 and L_2[52]. These bounds are based on loop-level calculations assuming that no large tree-level contribution causes a significant cancellation of the effect.

resonances appear. This discussion is speculative, because even in QCD the understanding of this physics is difficult. No one really knows how it will look in theories that are significantly different.

3.1. QCD-like Theories

In a technicolor theory with one doublet of techniquarks and three technicolors, the spectrum should be the same as that of QCD, but scaled up by a factor of v/f_π [61]. Furthermore, all of the interactions of these techniparticles would be expected to mirror those of the corresponding particles in QCD. For example, the ratio of the mass to the width of the technirho should be the same as that of the $\rho(770)$. This scenario is the simplest technicolor model (though it is probably disallowed by experiment).

Gauging the lowest-order (two-derivative) effective Lagrangian for this model is not very interesting. As we saw in the last section, in unitary gauge the symmetry breaking sector is nothing more than a mass term for the gauge bosons, with no hint of the structure of the higher energy theory. It is therefore imperative to find some way of going beyond the lowest-order Lagrangian.

Section 2 discussed the most straightforward approach: include the four-derivative terms of the effective Lagrangian. As shown there, this yields a description of the three- and four-gauge-boson vertices. However, this method is not really adequate for the energy regime in which a strongly interacting theory can be expected to become distinctive: only in the region above 1 TeV will one expect that the π^a scattering amplitudes become strong, leading to the formation of resonances.

Consider the analogous situation in the ordinary strong interactions. At present, there is no ideal way of parametrizing the energy region in which the bound-state resonances occur. The techniques currently used to describe the plethora of resonances, such as the non-relativistic quark model, are *ad hoc* and not based on either chiral symmetry or QCD. In a QCD-like technicolor theory it is essential to preserve the chiral symmetry, because that symmetry is gauged. What is frequently done is to include the lightest resonances into the effective Lagrangian. So long as the dynamics are QCD-like, the lightest particles are expected to be the vector resonances, the technirho and techniomega, which are the analogues of the $\rho(770)$ and $\omega(783)$. One hopes that in this way at least some of the intermediate-energy region can be described.

We now discuss two equivalent methods for including the vector resonances into the effective Lagrangian [62]. The simplest way to include the $\rho(770)$ into the effective Lagrangian for the strong interactions is as a "matter" field, an object that transforms homogeneously under chiral rotations. In order to include matter fields, one proceeds in two steps. First, a field ξ is defined by

$$\xi^2 = \Sigma \quad . \tag{3.1}$$

This field does not transform linearly under $SU(2)_L \times SU(2)_R$ rotations. Instead one must define the matrix U by

$$\xi \to L\xi U^\dagger = U\xi R^\dagger \quad . \tag{3.2}$$

Note that U will depend on ξ and hence on spacetime. The ρ field can now be included. It written as a matrix

$$\rho^\mu = \frac{1}{2}\rho_i^\mu \sigma_i \quad , \tag{3.3}$$

where σ_i are the Pauli matrices and ρ_i^μ are three real fields. This field is taken to transform as

$$\rho^\mu \to U\rho^\mu U^\dagger \quad . \tag{3.4}$$

The kinetic energy term for ρ is then

$$\mathcal{L}_{KE} = -\frac{1}{2}\mathrm{Tr}(d^\mu\rho^\nu - d^\nu\rho^\mu)^2 \quad , \tag{3.5}$$

where d^μ is a chirally covariant derivative, defined by

$$d^\mu\rho^\nu \equiv \partial^\mu\rho^\nu + iV^\mu\rho^\nu - i\rho^\nu V^\mu \quad , \tag{3.6}$$

and

$$V^\mu = -\frac{i}{2}(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger) \quad . \tag{3.7}$$

This is a chirally covariant derivative, in the sense that

$$d^\mu\rho^\nu \to d^\mu(U\rho^\nu U^\dagger) = U(d^\mu\rho^\nu)U^\dagger \quad . \tag{3.8}$$

The mass term for the ρ is

$$\mathcal{L}_m = m_\rho^2 \mathrm{Tr}\rho^\mu\rho_\mu \quad . \tag{3.9}$$

The chirally covariant derivative can act on the fields of other particles that are included in the effective Lagrangian. For example, if the nucleon doublet N is included, it transforms as $N \to UN$. Its kinetic energy and mass terms are

$$\mathcal{L}_B = \bar{N}(i\not{d} + m_N)N \quad . \tag{3.10}$$

One then proceeds to add all possible chirally invariant terms to the Lagrangian. For example, the term that couples the ρ to the nucleon is just

$$\mathcal{L}_{\rho NN} = g_\rho \bar{N} \not{\rho} N \quad . \tag{3.11}$$

In this formulation, it is a mystery why the ρ appears to dominate the vector current form factor of the nucleon, and why its couplings appear to be universal.

Frequently, a different approach is used: the $\rho(770)$ is included as a gauge particle of a broken "hidden local symmetry"[7]. The goal is to give some explanation of the universality of the ρ couplings to other particles. One can show the equivalence of the two approaches by defining

$$\rho_{new} = \rho + g_\rho V^\mu \quad , \tag{3.12}$$

where here g_ρ acts as a gauge coupling constant. The chirally covariant derivative that acts on other fields in the Lagrangian is

$$d^\mu_{new} = \partial^\mu + i g_\rho \rho^\mu_{new} \quad . \tag{3.13}$$

One may as usual define a "field strength" tensor

$$F^{\mu\nu} = \frac{1}{i}[d^\mu_{new}, d^\nu_{new}] \quad , \tag{3.14}$$

and then the kinetic energy term of the ρ meson is

$$\mathcal{L}_{KE} = -\frac{1}{2}\mathrm{Tr} F^{\mu\nu} F_{\mu\nu} \quad , \tag{3.15}$$

and the mass term of the ρ field is

$$\mathcal{L}_m = m_\rho \mathrm{Tr}(\rho^\mu_{new} - g_\rho V^\mu)^2 \quad . \tag{3.16}$$

[7] When this method of including the vector mesons is used in the electroweak theory, it is known as the BESS model[63].

When an effective Lagrangian is built out of d_{new}, the ρ has universal couplings to the nucleons and other particles. However, any chirally invariant term that can be built in the "matter" formulation of the previous paragraph will continue to appear in this approach. In other words, it is still possible to add additional ρ–nucleon couplings by adding additional operators, such as

$$\mathcal{L}_{\rho NN} = h\bar{N}(\not{\rho}_{new} - g_\rho \not{V})N \quad . \tag{3.17}$$

The mystery of the universality of the ρ couplings persists in this formulation of the theory, but now it takes a different form. Now we need to know why the new coupling h is so much smaller than one expects.

While it is true that every vertex in the "matter field ρ" Lagrangian can be written in the "gauge field ρ" Lagrangian, the power counting of the operators is somewhat different in the two approaches. In the latter approach, m_ρ gets renormalized only by terms proportional to powers of g_ρ. Only in the "gauge field ρ" method can one understand a light, weakly coupled vector.

On the other hand, it is not really strictly valid to include the $\rho(770)$ in the effective Lagrangian description of QCD. The effective Lagrangian breaks down at a scale near Λ. If Λ were much greater than 770 MeV, then one would expect to see a big hierarchy between the $\rho(770)$ and other physics. This does not appear to be the case; once 770 MeV is reached the resonances come thick and fast. Another way of saying this is that when the ρ is included as a gauge field, the coupling g_ρ is of order 4π. By analogy, it is likely to be invalid in a strict sense to include the technirho in the effective Lagrangian for a technicolor sector, because neither formulation has a valid procedure for the inclusion of technirho loops or higher-derivative multipion operators.

However, for the simpler purposes of calculating event rates, inclusion of the technirho into the effective Lagrangian and working at tree level is actually quite useful. It does yield a qualitatively reasonable, gauge invariant amplitude for gauge boson scattering. There have been numerous papers that have looked at the possibility of seeing the low-lying hadronic resonances at colliders[4][33][64][65], but many of these papers discuss observability at 17 TeV or 40 TeV machines. At present, more work is needed to determine exactly where the window of discovery is at the 14 TeV LHC.

3.2. Limits of the Effective Lagrangian

In this paper we have mentioned examples of technicolor models with various numbers of flavors. In such models the symmetry breaking pattern is $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_{L+R}$. For example, $N_f = 2$ in the simplest technicolor model and $N_f = 8$ in the one-family model. We begin by deriving some formulas implementing the extension to $SU(N_f)$ of the custodial symmetry. We will use these results to explore the scale Λ at which the effective Lagrangian breaks down, and to argue that the number of flavors directly influences the mass of resonances, such as the technirho [19].

As stated in the introduction, the chiral symmetries of technicolor are generally only approximate symmetries. In addition to the three "eaten," exact Nambu-Goldstone bosons, there are often additional pseudo-Goldstone bosons. If the mass of a typical pseudo-Goldstone boson is m, the effective Lagrangian is also an expansion in m^2. For simplicity, we consider a chiral symmetry breaking interaction that does not break the conserved $SU(N_f)_{L+R}$ vector symmetry. Such a chiral symmetry breaking term gives the same mass to all Nambu–Goldstone bosons.

Consider the scattering process $\pi^a\pi^b \to \pi^c\pi^d$. The amplitude for such a process may be decomposed into irreducible representations of the unbroken $SU(N_f)_{L+R}$. Since the π's are in the adjoint representation of this symmetry, one needs to know the representation content of adjoint $\otimes$ adjoint. For $N_f = 2$ there are three representations, corresponding to the isospin 0, 1, and 2 channels. For $N_f = 3$, the representations are the familiar $\mathbf{1}, \mathbf{8_a}, \mathbf{8_s}, \mathbf{27}, \mathbf{10}$, and $\overline{\mathbf{10}}$ in $\mathbf{8} \otimes \mathbf{8}$. For $N_f > 3$, there are always seven representations. Of greatest interest for our purposes will be the singlet representation, in which the incoming π's have the same flavor: $a = b$.

We can construct the most general amplitude for $\pi\pi$ scattering consistent with Bose symmetry, crossing invariance, and $SU(N_f)_{L+R}$ conservation. If we define d^{abc} and f^{abc} by

$$f^{abc} = -2i\mathrm{Tr}[T^a, T^b]T^c \quad \text{and} \quad d^{abc} = 2\mathrm{Tr}\{T^a, T^b\}T^c \ , \tag{3.18}$$

then the most general amplitude is

$$\begin{aligned} a(s,t,u)^{a,b;c,d} =& \delta^{ab}\delta^{cd}A(s,t,u) + \delta^{ac}\delta^{bd}A(t,s,u) + \delta^{ad}\delta^{bc}A(u,t,s) \\ &+ d^{abe}d^{cde}B(s,t,u) + d^{ace}d^{bde}B(t,s,u) + d^{ade}d^{bce}B(u,t,s) \quad , \end{aligned} \tag{3.19}$$

where s, t, and u are the Mandelstam variables and A and B are unknown functions. Bose symmetry implies that the functions A and B must be symmetric under the exchange

of their second and third arguments. From this amplitude we may derive the scattering amplitude of, for example, the singlet representation:

$$\begin{aligned} a_0(s,t,u) =&(N_f^2-1)A(s,t,u) + A(t,s,u) + A(u,t,s) \\ &+ \frac{(N_f^2-4)}{N_f}(B(t,s,u)+B(u,t,s)) \quad . \end{aligned} \tag{3.20}$$

We may, as usual, project this amplitude onto its various (even) orbital angular momentum components. The functions A and B will be such that all these partial wave amplitudes obey the usual unitarity relations.

One may use the two-derivative effective Lagrangian to compute the invariant functions A and B: $A(s,t,u) = (2/N_f)(s-m^2)/f^2$, and $B(s,t,u) = (s-m^2)/f^2$. The isosinglet spin-zero scattering amplitude is therefore [66]:

$$a_{00} = \frac{N_f s}{32\pi f^2} - \frac{m^2}{16\pi N_f f^2} \quad . \tag{3.21}$$

It will be important below to note that this scattering amplitude is enhanced by a factor of N_f.

We have seen in previous sections that the most general chirally invariant Lagrangian can be written as an expansion in powers of derivatives. Additional terms with more derivatives are suppressed by powers of the momentum scale that we have denoted Λ. The effective Lagrangian is an expansion in p^2/Λ^2 (and m^2/Λ^2).

At energies near or above Λ, all terms in the expansion contribute and the effective Lagrangian becomes useless. The amplitude a_{00} calculated at tree level is real, and (for small m^2) exceeds 1 when $\sqrt{s} > 4\pi f/\sqrt{N_f}$. A physical scattering amplitude must lie on or inside the Argand circle, but the point $a_{00} = 1$ is far outside. At these energies, therefore, loop corrections and higher order terms in the effective Lagrangian must make as large a contribution as the two-derivative term, and the calculation using just the lowest order effective Lagrangian ceases to be useful. This suggests that Λ is less than or of order $4\pi f/\sqrt{N_f}$, as was emphasized in [67][8].

An alternative approach which puts the same limit on Λ is based on an estimate of the size of loop corrections [30]. Since the theory is not renormalizable, the terms of order p^4 are required as counterterms to loops involving the lowest order interactions. In calculating the scattering amplitude to order p^4, one must consider tree-level diagrams with interactions

[8] Note that in section 2, $N_f = 2$ and the factor of $1/\sqrt{N_f}$ was neglected.

coming from operators of fourth order in momenta, and one-loop diagrams using the two-derivative terms in the effective Lagrangian. Similarly, the two-loop calculation using the lowest order effective Lagrangian will require counterterms of order p^6, etc.

In doing the one-loop calculation, it is unnatural to assume that the contribution from the loop diagrams is much larger than that from the tree-level four-derivative operators, since such a statement could only be true for a particular choice of renormalization scale[9]. Therefore it is inconsistent to assume that the corrections to $\pi\pi$ scattering of order p^4 are less than or of order $(\sqrt{N_f}p/4\pi f)^2$ where p is a typical momentum in the process – meaning that it is unrealistic to assume that the coefficients of the higher order four-derivative terms in the effective Lagrangian are smaller than about $N_f/16\pi^2 f^2$.

It is possible to show that this pattern persists to all orders: with each additional loop, the corrections are a factor of order $(\sqrt{N_f}p/4\pi f)^2$ times the previous correction. Again, this implies that at any order in the momentum expansion, the mass scale Λ suppressing the higher derivative terms cannot be much larger than $4\pi f/\sqrt{N_f}$.

3.3. Implications for New Physics

Some interesting questions arise at this point. We have argued that the momentum expansion breaks down at or before Λ, but what actually happens to the amplitudes as s increases beyond this value? What is the significance of Λ? The amplitudes for the partial waves other than a_{00} are all below their unitarity limits when $\sqrt{s} = 4\pi f/\sqrt{N_f}$. Is it possible that, like Λ_{QCD}, Λ is a purely calculational artifact corresponding to no particular physical structure?[10]

In the effective Lagrangian the multiderivative terms contribute an arbitrary polynomial in s, t, and u to the scattering amplitude:

$$\sum_k a_k \frac{p^2}{f^2}\left(\frac{p^2}{\Lambda^2}\right)^{k-1} \tag{3.22}$$

[9] The explicit calculation [19] of the one loop corrections to the tree-level functions A and B shows that the results are factors of order $N_f s/(16\pi^2 f^2)$ or $N_f m^2/(16\pi^2 f^2)$.

[10] Ref [68] argues for this interpretation. In the absence of any well-motivated way to calculate these field theories, any such argument, including the one presented in this paper, is fairly speculative. However, in certain toy models the argument presented here can be made rigorous [69].

where all the a_i are numbers of order 1. When does such a series fail to converge? Since the a_k are of order one, the radius of convergence is Λ. Because the series diverges at energies higher than Λ, the momentum expansion (to any, arbitrarily high, finite order) cannot give a good approximation to the scattering amplitude at energies beyond Λ.

It is plausible that the effective Lagrangian can accurately match the scattering amplitude out to the first non-analytic structure representing new physics. This is because the S matrix is an analytic function of momenta, except at isolated points where intermediate states go on shell. All the non-analytic structures in the scattering amplitudes corresponding to multipion states are correctly included at some order in the momentum expansion by the pion loop calculations - what is not properly included are effects of other states. For example, in QCD there is a pole in the S matrix at the $\rho(770)$ mass. Above m_ρ, the effects of a term like $1/(p^2 - m_\rho^2)$ in the S-matrix this can never be reproduced as a finite power series in positive powers of p. The series has to be resummed in some way.

If this argument is correct, then it follows that Λ is precisely the mass of the lightest non-analytic structure in the S-matrix. The conclusion is that new physics is lighter than a scale of order $4\pi f/\sqrt{N_f}$.

The implications for technicolor and strongly interacting field theories in general may be substantial. In the case of technicolor, it may be that the new physics that comes in at this low scale is the technirho. If this is true then the technirho mass suggested by the simple scaling argument may be a significant overestimate. In this case, if the vector dominance relations continue to hold, then the simplest estimates of oblique radiative corrections in technicolor models, such as those in [35] and [41], may be rather unreliable [70]. If technicolor somehow manages to evade the problems from radiative corrections, the lightness of the vector bosons may make them interesting for future colliders [71].

Even if non-perturbative physics has nothing to do with electroweak symmetry breaking, the arguments of this section may be of interest for ordinary QCD. The idea that the masses of the resonances depend on the number of light flavors may seem counterintuitive. If the charm, strange, top, and bottom quarks were as light as the up and down, the ratio m_ρ/f_π would be substantially altered. However, in a non-relativistic quark model neither the ρ nor π contains anything other than the first generation quarks. Understanding the effects discussed in this section may have something to do with understanding the difficulties [72] with quenched chiral perturbation theory [73].

3.4. A "Hidden" Symmetry Breaking Sector

As we have seen, the most direct probe of the symmetry breaking sector is the scattering of longitudinal W and Z bosons. That is because at energies large compared to their mass, the longitudinal components of these particles are (essentially) the eaten Nambu–Goldstone Bosons of $SU(2)_W \times U(1)_Y$ symmetry breaking [33]. Section 1 discussed various possibilities for the new physics that enters at the scale Λ: in the weakly coupled one-doublet Higgs model, it was the light and narrow Higgs boson; in minimal technicolor, the exchange of the technirho and other particles unitarizes gauge boson scattering.

It is frequently assumed that these two types of behavior for elastic W and Z scattering are generic (see, for example [4],[59]). If the symmetry breaking sector is weakly coupled, the growth of the $W_L W_L$ scattering amplitudes is cut off by narrow resonances (like a light Higgs boson) at a mass scale well below a TeV. For strongly coupled theories, it is assumed that the amplitudes saturate unitarity and that there are broad resonances in the TeV region where the strong interaction sets in.

There is another possibility: if the electroweak symmetry breaking sector has a large number of particles, the *elastic* W and Z scattering amplitudes can be small and structureless, i.e. lacking any discernible resonances. Nonetheless, the theory can be strongly interacting and the *total* W and Z cross sections large: most of the cross section is for the production of particles other than the W or Z. In such a model, termed a "Hidden Symmetry Breaking Sector" [74], discovering the electroweak symmetry breaking physics depends on the observation the other particles and the ability to associate them with symmetry breaking. Physicists should keep an open mind about the experimental signatures of the electroweak symmetry breaking sector because discovery of electroweak symmetry breaking may not rely solely on two-gauge-boson final states.

This scenario may be illustrated by considering a toy model of the electroweak symmetry breaking sector based on an $O(N)$ linear sigma model. This model is particularly interesting since it can be solved (even for strong coupling) in the limit of large N [75]. One constructs a model with both exact Nambu–Goldstone bosons (which will represent the longitudinal components of the W and Z) and pseudo-Goldstone bosons. To this end let $N = j + n$ and consider a model with j- and n-component real scalar fields. One can construct a theory that has an approximate $O(j + n)$ symmetry which is broken softly but explicitly to $O(j) \times O(n)$. A vacuum expectation value, breaks the $O(j)$ symmetry to $O(j - 1)$, and the theory has $j - 1$ massless Nambu–Goldstone bosons and one massive Higgs boson. The $O(n)$ symmetry is unbroken, and there are n degenerate massive

pseudo-Goldstone bosons, which we refer to as ψs. It is possible to solve this model in the limit that $j, n \to \infty$ with j/n held fixed[11].

The scalar sector of the standard one-doublet Higgs model has a global $O(4) \approx SU(2) \times SU(2)$ symmetry, where the 4 of $O(4)$ transforms as one complex scalar doublet of the $SU(2)_W \times U(1)_Y$ electroweak gauge interactions. This symmetry is enlarged in the $O(N)$ model: the spin-0 weak isosinglet scattering amplitude of longitudinal gauge bosons is modeled by the spin-0 $O(j)$ singlet scattering of the Nambu–Goldstone bosons in the $O(j+n)$ model solved in the large j and n limit. Of course, $j = 4$ is not particularly large. Nonetheless, the resulting model will have all of the qualitative features needed, and the Nambu–Goldstone boson scattering amplitudes will be unitary (to the appropriate order in $1/j$ and $1/n$). Thus this theory can be used to investigate the scattering of Nambu–Goldstone bosons at moderate to strong coupling [77]. Since they are mostly produced via their strong interactions, the electroweak quantum numbers of the pseudo-Goldstone bosons can be anything; here we assume that the pseudo-Goldstone bosons are $SU(3)$ color singlets[12].

Nambu–Goldstone boson scattering in an $O(N) \to O(N-1)$ model is in some ways similar to that in the $SU(N_f) \times SU(N_f)$ model considered above. For example, the amplitude $a^{ij;kl}(s,t,u)$ for the process $\pi^i\pi^j \to \pi^k\pi^l$ is

$$a^{ij;kl}(s,t,u) = A(s;M)\delta^{ij}\delta^{kl} + A(t;M)\delta^{ik}\delta^{jl} + A(u;M)\delta^{il}\delta^{jk} \quad , \tag{3.23}$$

where $A(s;M)$ is some function. This $O(N)$ theory is soluble to leading order in $1/N$, so A may in fact be computed to this order without any assumptions. In the equation above M is a parmeter with dimensions of mass that specifies the strength of the self-coupling of the symmetry breaking sector. It is essentially a cutoff, and so the smaller M is, the stronger the self-coupling. The isospin-zero amplitude spin-zero is therefore calculable to order $1/N$ too; it is

$$a_{00}(s) = \frac{jA(s;M)}{32\pi} \quad . \tag{3.24}$$

Plotted in fig. 1 is the absolute value of a_{00} *vs.* the center-of-mass energy for different values of M. We have set $j = 4$, as always. We have also set the number of pseudo-Goldstone bosons, n, to 32. The pseudogoldstone bosons have a mass $m_\psi = 125$ GeV.

[11] For the complete details of the construction and solution of this model, see [74] and [76].

[12] Gauge boson pair production in models with colored pseudo-Goldstone bosons is discussed in detail in [78].

The curves plotted correspond to approximately $8M/m_\psi = 10000$, 600, 200, 100, and 60. For the weakly coupled theory, for example the 10000 curve, there is a light Higgs boson which decays to π's. When the Higgs boson is light, its width is more or less unaffected by the heavy ψ's, and thus its properties are identical to those of the Higgs boson of similar mass in the $O(j)$ model [77]. As the Higgs resonance gets closer to the two ψ threshold, it gets relatively narrower than it would have been were the ψ's absent. As the theory becomes more strongly coupled still, the resonance gets heavier and broader. Eventually, for small enough M, the imaginary part of the location of the pole is so great that there is no discernible resonance in a_{00}.

When the Higgs resonance is heavier than twice m_ψ, it no longer decays exclusively to π's, and thus the absolute value of the amplitude for elastic $\pi\pi$ scattering never gets anywhere near 1. Probability is leaking out of this channel into that for the production of pairs of ψ's. For comparison, the dashed line shows the scattering amplitude in the limit $m_\psi \to \infty$ with M adjusted to produce a Higgs resonance at approximately 500 GeV.

In the gauged model, the π's are eaten by the gauge bosons, and become their longitudinal components. Therefore, $\pi\pi$ final states correspond to two-gauge-boson events. In this toy model the Higgs resonance may be light but so broad that at no energy is the number of WW or ZZ events large; discovering the Higgs boson depends on its observation in the two $\psi\psi$ channel. Depending on how the ψ's decay, this may be easy or hard. Nonetheless, it is clear that an experiment looking for electroweak symmetry breaking may not be able to rely exclusively on the two-gauge-boson events. Parton level computations have indicated [79] that it is probably not possible to detect this symmetry breaking sector at the proposed LHC by examining the gauge-boson-pair modes exclusively[13].

This section has shown that it is possible for the W and Z scattering amplitudes to be small and structureless: if the symmetry breaking sector contains a large number of particles in addition to the longitudinal gauge bosons, there may be light but very broad

[13] This claim is disputed in [80]. There it was shown that the numbers of final state gauge boson pairs from gauge boson scattering is roughly independent of N if $\sqrt{N}M$ is held fixed. This is because as N increases for fixed $\sqrt{N}M$, M and the mass and width of the Higgs boson decrease like $1/\sqrt{N}$. The increased production of Higgs bosons due to their smaller mass is approximately cancelled by the Higgs boson's smaller branching ratio into Ws and Zs. The number of signal events, therefore, is approximately independent of N and is the same as the number which would be present in the standard model. However, the *background* rate is much larger, because the Higgs boson is much lighter, and this renders the signal unobservable.

"resonances". The ability to discover the electroweak symmetry breaking sector depends on the observability of technipions other than the longitudinal gauge bosons [64]. Associating the technipions of this model with electroweak symmetry breaking will be crucial.

4. Conclusions

In this review we have discussed theories in which the W and Z interactions become strong at an energy scale of order a TeV or less. We began with a survey of the range of theories which have been constructed to explain electroweak symmetry breaking. We argued that the putative triviality of theories with fundamental scalar particles implies that *any* theory with a large hierarchy between the weak scale and the scale of the dynamics responsible for producing electroweak symmetry breaking must be *weakly interacting*. This implies that if the W and Z interactions are strong at energies of order a TeV or less, the physics responsible for electroweak symmetry must become apparent at *the same energy scale*.

We then reviewed the use of the effective Lagrangian to describe the physics of any strongly-interacting symmetry breaking sector at energies lower than the mass of the lightest resonance. Limits on the values of low-energy parameters (*e.g.* S or L_{10}, and T) provide the most significant constraints on the strongly-interacting symmetry breaking sector.

In order to discover the physics of the symmetry breaking sector it will be necessary to probe physics at energy scales of order a TeV. In a strongly-interacting symmetry breaking sector we expect that a plethora of new resonances will appear at these energies to cut off the growth of the W_L and Z_L scattering amplitudes. As we discuss in the last section, the effective Lagrangian ceases to be a useful description at an energy scale of order the mass of these resonances. Further, we argued that, in order for the effective chiral Lagrangian to be self-consistent, the mass scale of the resonances must be lighter than or of order $4\pi f/\sqrt{N_f}$.

Finally, it is often assumed that if the W and Z interactions are strong, there will always be large W and Z scattering cross section at high energies. We concluded with a description of the "hidden" symmetry breaking sector in which, although the W_L and Z_L interactions are strong, the *elastic* scattering amplitudes are always small and structureless.

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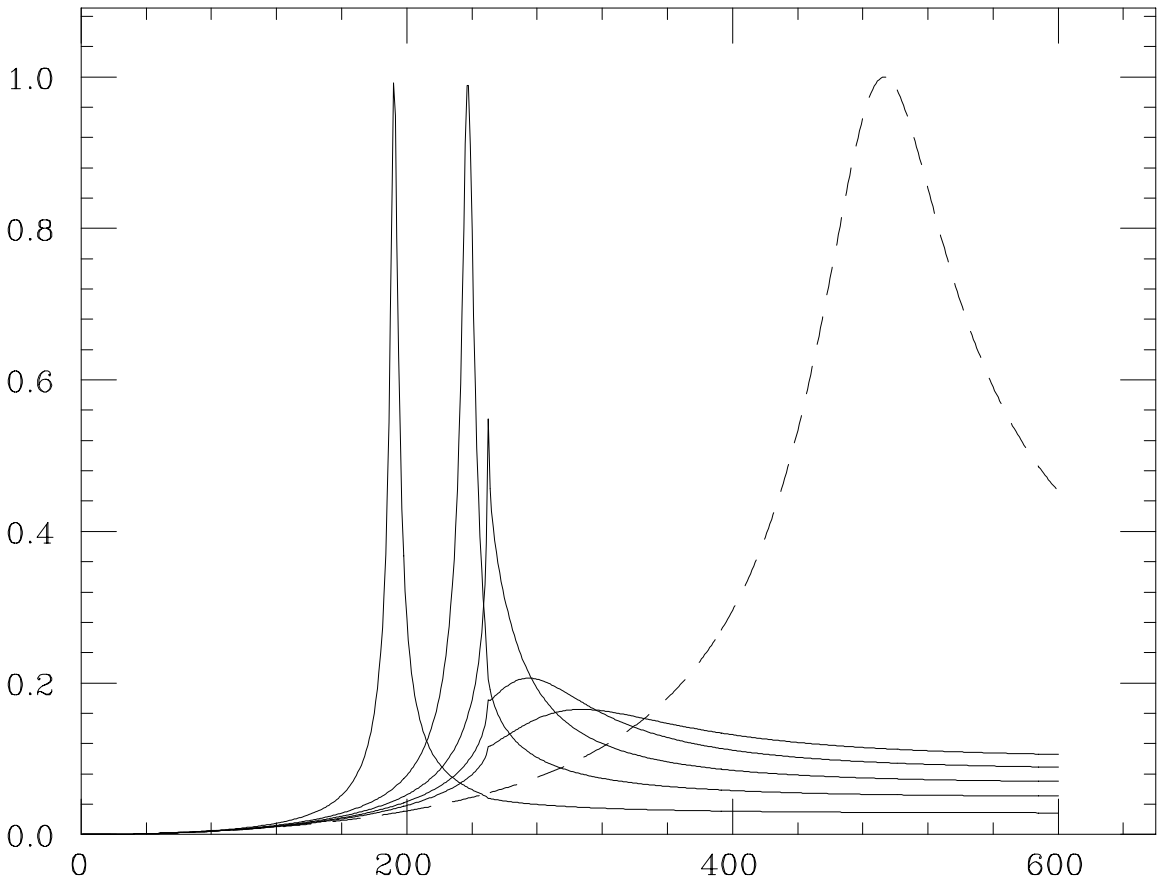
1.0
0.8
0.6
0.4
0.2
0.0
0
200
400
600

Figure Captions

Fig. 1. The absolute value of the $\pi\pi \to \pi\pi$ scattering amplitude vs. CM energy for different values of M. Here $j = 4$, $n = 32$, $m_\psi = 125$ GeV, and $f = 250$ GeV. The curves correspond to roughly $8M/m_\psi \sim 10000$, 600, 200, 100, and 60. The curve with the leftmost bump is 10000, and the low nearly structureless curve is $8M/m_\psi \sim 60$. For comparison, the dashed line shows the scattering amplitude in the limit $m_\psi \to \infty$ with M adjusted to produce a Higgs resonance at approximately 500 GeV.