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Study of the Stability of Fuzzy Controllers by an Estimation of the Attraction Regions: A Vector Norm Approach

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A fuzzy controller with singleton defuzzification can be considered as the association of a regionwise constant term and of a regionwise non linear term, the latter being bounded by a linear controller. Based on the regionwise structure of fuzzy controller, the state space is partitioned into a series of disjoint sets. The fuzzy controller parameters are tuned in order to ensure that the ith set is included into the domain of attraction of the preceding sets of the series. If the first set of the series is included into the region of attraction of the equilibrium point, the overall fuzzy controlled system is stable. The attractors are estimated with the help of the comparison principle, using Vector Norms, which ensures the robustness with respect to uncertainties and perturbations of the open loop system.

Key words: Fuzzy control, Vector norms, Nonlinear systems, Control design, Comparison systems

1 INTRODUCTION

Whereas the robustness properties of fuzzy controllers have been shown experimentally, [1, 2], a theoretical analysis of their stability and performances is a more difficult task ([3] and the references therein); in particular, it is difficult to find an appropriate domain for the fuzzy controllers parameters which guaranties that the controlled system is stable, owing to the locality and non-linearity of control.

The fuzzification algorithm partitions the controller input variables space into a set of regions, where the local controls designed therein are combined to make up the final global control. A fuzzy controller has been shown to behave as a non linear controller with a varying gain [4, 5], and a partition of the state space can be found where the controller has regionwise constant parameters [6].

A previous study has shown that a fuzzy controller with singleton defuzification and trapezoidal input membership functions can be split up into a non-linear state space feedback, with regionwise bounded parameters, and a regionwise valued constant term. The phase plane has been partitioned into two semi-planes by means of a switching line, a negative output has been generated above the switching surface and a positive below it. An admissible

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domain for the fuzzy controller parameters can be found for which the stability of the closed loop controlled system is ensured [7, 8]. These stability conditions are not very conservative, the controller behaviour is similar to a variable structure control with a boundary layer such as shown in Ref. [9, 10]. This approach is somewhat limited and does not explores all possibilities of fuzzy controllers design, since the fuzzy controller is forced into a nonlinear Variable Structure Controller.

Vector norms constitute a systematic mean of obtaining comparison systems, which help to overevaluate and analyse non-linear systems [11–13]. An adequate choice of the stable overevaluing system may prove the initial overvalued system stability. The method is robust with respect to bounded perturbations, and a good choice of the vector norms may allow to obtain little conservative stability conditions. The method can also provide an estimation of the attractor with respect to an initial domain for a nonlinear system [14].

The general idea of the paper is to cut the state space into a domain where the fuzzy controller will be tuned to ensure the closed-loop system stability, and a set of domains for which the former domain will be an attractor.

Firstly, some definitions on comparison systems and vector norms will be recalled. The general regionwise structure of fuzzy controllers will be presented, and in a third part, an algorithm will be provided which leads to stable fuzzy controllers design for nonlinear systems, using the previous stability theorems. An example will illustrate the method in the last section.

2 VECTOR NORMS AND OVERVALUING SYSTEMS

2.1 Vector Norms

DEFINITION 1 *Let $E = R^n$ and $E_1, E_2, \dots, E_k k$ subspaces of the vector space E, with $E = E_1 \cup E_2 \cup \cdots \cup E_k$ and $E_i \cap E_j \neq 0 \ \forall i \neq j$. Let x be an n vector defined on E and $x_i = P_i x$ the projection of x on E_i, where P_i is a projection operator from E into E_i, p_i is a scalar norm $(i = 1, \dots, k)$ defined on the subspace E_i and p is a vector norm (VN) of dimension k with ith component $p_i(x) = p_i(x_i)$, $p(x): \mathbb{R}^n \to \mathbb{R}^k_+$*

Let y be another vector in space E with $y_i = P_i y$, we have:

$$\begin{cases} p_i(x_i) \geq 0, & \forall x_i \in E_i, \ \forall i = 1, \dots, k \\ p_i(x_i) = 0 \Leftrightarrow x_i = 0, & \forall i = 1, \dots, k \\ p_i(x_i + y_i) \leq p_i(x_i) + p_i(y_i), & \forall x_i, y_i \in E_i, \ \forall i = 1, \dots, k \\ p_i(\lambda x_i) = |\lambda| p_i(x_i), & \forall \lambda \in R, \ \forall i = 1, \dots, k \end{cases}$$

If $k - 1$ of the subspaces E_i are insufficient to define the whole space E, the VN is surjective, and if in addition, the subspaces are in disjoint pairs, the VN is said to be regular.

2.2 Overvaluing Systems and Attractors

2.2.1 Overvaluing Systems

Let us consider the equation

$$\dot{x} = A(t, x, \omega)x + B(t, x, \omega), \tag{1}$$

Let p be a regular vector norm (NV) of size k, and S a compact set of $\mathbb{R}^n$ which includes the origin. ω represents the parameters of the perturbations on the state or the model, with $\omega \in P$.

DEFINITION 2 *The pair* $(M,N)M:\tau_0 \times S \to \mathbb{R}^{k\times k}$, $N:\tau_0 \times S \to \mathbb{R}^k_+$, *defines a non-homogeneous pseudo-overvaluing system (NHOS) of system* (1) *on the compact set* S, *relative to the VN* p, *if and only if*:

$$D^+p(x) \leq M(t,x)p(x) + N(t,x), \quad \forall(t,x) \in \tau_0 \times S. \tag{2}$$

$D^+p(x)$ *is the right-hand derivative taken along the motion of* (S) *into* E, *and* $\tau_0 = [t_0, +\infty[$

DEFINITION 3 *A matrix* A *is a M-matrix if all its off-diagonal elements are negative or zero, and in addition, if all its eigenvalues have a positive real part. The last condition is equivalent to all principal minor determinants of* A *are positive* (*Koteliansky criterion*) [13].

THEOREM 1 [14] *If it is possible to define in a domain* D *a time-invariant linear NHOS of* (1) *relative to a regular* p *VN*:

$$D^+p(x) \leq Mp(x) + N, \quad \forall(t,x) \in \tau_0 \times S,$$

for which M *is the opposite of a* (*constant*) *M-matrix, and* N *is a non negative* (*constant*) *vector, then, there is an asymptotically stable attractor* L_0 *and the set*:
$L = \{x \in \mathbb{R}^n;\ p(x) \leq -M^{-1}N\}$ *includes all the attractors of* (1), *if the outer frontier of* D *encloses the outer frontier of* L.

3 APPLICATION OF VECTOR NORMS TO FUZZY CONTROLLERS STABILITY

3.1 Fuzzy Controller Structure

Let us consider a fuzzy controller with one output u and n input variables $X_1, X_2, \ldots, X_n$ with the corresponding m_i predicates (membership functions) for every $X_i, i = 1, 2, \ldots, n$. The space vector is $x = X_1, X_2, \ldots, X_n$. The predicates will be noted A_{ij} with $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m_i$.

The class of fuzzy controllers to deal with meets the following assumptions:

- Input membership functions will be chosen as trapezoidal functions of the inputs.
- The AND operator will be chosen as the min(·) operator,
- Rule R_k reads if X_1 is A_{1,j_1} AND X_2 is A_{2,j_2} ... AND X_n is A_{n,j_n} THEN u is U_k, where $i = 1, 2, \ldots, n$, $j_i = 1, 2, \ldots, m_i$, and $U_k \in \mathbb{R}$.
- Singleton deffuzification will be undertaken.

It has been previously demonstrated [7, 8] that there exists a number of regions $\Re_h$ with $h = 1, 2, \ldots, M$, for which any active membership function attached to any variable X_i is an affine function of the variable X_i. As the AND operator is chosen as the min(·) operator, the membership function to rule R_k, which we will call μ_k, remains an affine function of the variables $X_1, X_2, \ldots, X_n$ in the hypervolumes $\Re_h$ $(h = 1, \ldots, M)$.

In every region $\Re_h$, the control output u is the sum of a non linear state space feedback and of a constant term V_h $(h = 1, \ldots, M)$ that is a combination of the fuzzy output membership functions U_k $(k = 1, \ldots, N)$ defined in (1) (see [7, 8]):

$$u(X_1, X_2, \ldots, X_n) = \frac{\sum_{k=1}^{N} \mu_k U_k}{\sum_{k=1}^{N} \mu_k} = V_h + \frac{\alpha_h x}{N_h(x)}, \tag{3}$$

where both V_h and $\alpha_h = (\alpha_{h,1}, \ldots, \alpha_{h,n})$ are constant parameters in the region $\Re_h$ and depend on the input membership functions and on the parameters U_k; x is the state vector, $x = (X_1, \ldots, X_n)$. $N_h(x)$ is a linear bounded function of the state space x (there exist $\underline{N_h}$ and $\overline{N_h}$ such that for every x, $\underline{N_h} \leq N_h(x) \leq \overline{N_h}$.

Other regionwise structures can be found for different schemes of fuzzy controllers [4–6].

3.2 Design of Stable Fuzzy Controlled Systems

3.2.1 General Algorithm

Let us consider the continuous state-space system governed by a fuzzy controller:

$$\dot{x} = A(t, x, \omega)x + B(t, x, \omega)u, \tag{4}$$

with $u = f_h(x)$, where the functions f_h are nonlinear functions of x with constant parameters in some disjoints regions $\Re_h$ (in our application case, we have $u(X_1, X_2, \ldots, X_n) = V_h + \alpha_h x / N_h$ in every region $\Re_h$ defined above, where the control is the sum of a non-linear term and of a regionwise constant term).

The stability algorithm will be threefold:

1. Partition the state space into several subsets $D_i (i = 1, \ldots, d)$, every subset being a merging of some of the regions $\Re_h$,
2. Among these domains, find a domain D_1 containing the origin, where the closed-loop system (4) is asymptotically stable,
3. For every $D_i (i = 2, \ldots, d)$, find a system of comparison (M_i, N_i) for the closed-loop system (4), for which M_i is the opposite of a (constant) M-matrix, N_i is a non negative (constant) vector, and every set L, defined in Theorem 1, containing the attractor of D_i $(i = 2, \ldots, d)$ is included into D_{i-1}.

From Theorem 1, it is straightforward to check that the fuzzy controlled system is stable; every trajectory starting in D_i arrives in L_i, which is included into D_{i-1}, and so forth, until trajectories end into D_1, and converge to the equilibrium (implicitly, D_1 is a neighbourhood of the equilibrium).

3.2.2 Design Considerations and Discussion

It is worthy to note that the last statement of the algorithm only means that $\{0\} \subset L_2 \subset D_1 \subset L_3 \subset D_2 \cdots L_{i-1} \subset D_i$. It suffices to prove, then, that the controller is stable for every trajectory starting in D_1.

Unlike some other design methods for the obtaining of stable fuzzy controlled systems, such as turning these into a particular case of Variable Structure Fuzzy Controllers, the algorithm respects the philosophy of fuzzy control, where local sets in the state space are defined, and where trajectories go from set to set until reaching the origin.

As an example of the method, we can see in Figure 1 that domain D_3 is a ring (domain with horizontal hatchings), its attractor L_3 (the interior of the dotted line) is included into domain D_2 (the interior of the thick line). The attractor L_2 of domain D_2 is included into domain D_1 where the system is stable (and thus the trajectory converges to the equilibrium).

Owing to the number of existing vector norms and of domains that can be chosen for the D_i, it is difficult to obtain the "best" system of comparison for system (1). An adequate change of vector base can be interesting to find a convenient comparison system [12]. The choice of the vector norm remains intuitive; the VN, as will be demonstrated in the

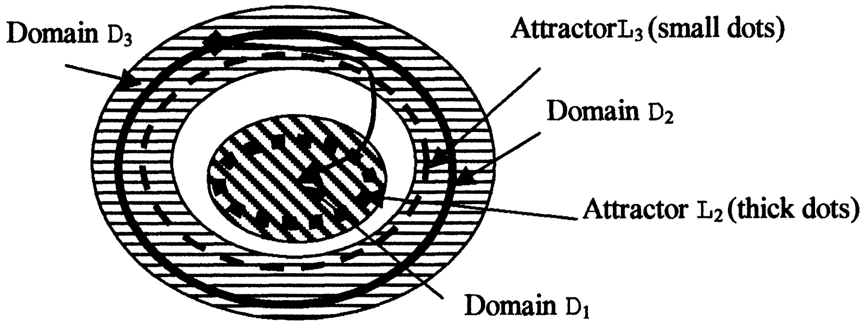

FIGURE 1 Domains and their Attractors.

example section, can be taken to obtain a shape of the domains L_i which is as close as possible (homothetic at best) from the ultimate domain D_1. For example, if D_1 is an hypercube, it is convenient to take the VN $|x|$ for which the domain $|x| \leq V$ (where V is a vector) is also an hypercube. The estimate L_i of the attractor of the domain D_i, obtained with Theorem 1, will be an hypercube.

For regions where $V_h = 0$, the closed loop system is stable if a positive matrix P can be found such that $(A + B\alpha_h^{\mathrm{T}}/N_h)^{\mathrm{T}} P + P(A + B\alpha_h^{\mathrm{T}}/N_h)$ is negative (proof is immediate using the Lyapunov function $V = x^{\mathrm{T}}Px$. A practical choice for the domain D_1 might thus be the set of regions where $V_h = 0$.

The robustness issue of fuzzy controllers will be enhanced into the example section. As shown in the Vector Norm method [13], the perturbation or uncertainties ω need not to be exactly known, but an upper bound has to be given. It is only needed, then, that a system of comparison still exists for system (4) which verifies the assumptions of Theorem 1. Parameters can be tuned so as to cope with these uncertainties. Stability conditions might not be strongly conservative, for the system needs to be asymptotically stable only into D_1. A remark is that the number of regions $\Re_h$, and the number of possible choices for the domains D_i increases with the number of fuzzy predicates of the inputs X_i.

The method will be applied to the particular fuzzy controller structure of Section 3.1. However, it can be extended to other schemes of fuzzy controllers, particularly to those with a nonlinear structure with regionwise parameters.

4 EXAMPLE

4.1 Controller Structure

Let us take the non linear system:
$\dot{x} = Ax + Bu$ with $x = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$. We take $A = \begin{pmatrix} -3.5 + 0.5\cos(t) & 0.5 + 0.5\cos(t) \\ 1 & -5 + 2\omega \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$ and $0 < \omega < 0.5$. The system output is X_2.

The membership functions are defined in Figure 2 and the rules in Table I (the Negative, Zero and Positive predicates correspond to the A_{ij} defined in Section 3.)

For example, rule R_1 reads "if X_1 is Negative AND X_2 is Positive then u is U_1". We introduce $x_1 = X_1/a$, $x_2 = X_2/b$.

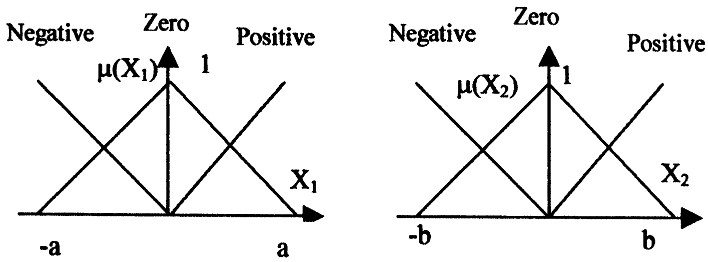

FIGURE 2 Input membership functions.

Let us consider the quadrant called Q_1 where $-1 < x_1 < 0$ and $0 < x_2 < 1$; (the others are called, turning clockwise, Q_2, Q_3, Q_4) where only rules R_1, R_2, R_0 and R_4 are active.

The membership functions for these rules are respectively, using the operator min as AND:

$$\mu_{R_1} = \min(-x_1, x_2),\ \ \mu_{R_2} = \min(1 + x_1, x_2),$$
$$\mu_{R_0} = \min(1 + x_1, 1 - x_2),\ \ \mu_{R_4} = \min(-x_1, 1 - x_2).$$

We thus have 4 regions $\Re_1, \Re_2, \Re_3, \Re_4$, for which the expression of the membership functions are all linear with respect to x_1 and x_2 and have constant parameters (see Fig. 3). For example, if $-x_1 > x_2$, $1 + x_1 < 1 - x_2$ so that in region $\Re_1$ where $x_1 + x_2 < 0$, we obtain $\mu_{R_1} = \min(-x_1, x_2) = x_2$, and $\mu_{R_0} = \min(1 + x_1, 1 - x_2) = 1 + x_1$.

$$\Re_1\colon -x_1 > x_2 \quad \text{and} \quad 1 + x_1 < x_2, \qquad \Re_2\colon -x_1 < x_2 \quad \text{and} \quad 1 + x_1 < x_2,$$
$$\Re_3\colon -x_1 < x_2 \quad \text{and} \quad 1 + x_1 > x_2, \qquad \Re_4\colon -x_1 > x_2 \quad \text{and} \quad 1 + x_1 > x_2.$$

For example, for region $\Re_1$, we have: $\mu_{R_1} = x_2$, $\mu_{R_2} = 1 + x_1$, $\mu_{R_0} = 1 + x_1$, $\mu_{R_4} = 1 - x_2$, which finally gives: The set of regions in light grey (Fig. 3) is called D_2. The set of regions in white is called D_1.

Using Eq. (3), we have:

$$\Re_1\colon u = \frac{U_1 x_2 + (1 + x_1)U_2 + U_0(1 + x_1) + U_4(1 - x_2)}{3 + 2x_1},$$

TABLE I Lookup Table and Rule Number (in Parenthesis) for the Fuzzy Controller Output U.

	x_1		
x_2	*Negative*	*Zero*	*Positive*
Positive	$U_1(R_1)$	$U_2(R_2)$	$U_3(R_3)$
Zero	$U_4(R_4)$	$U_0(R_0)$	$U_5(R_5)$
Negative	$U_6(R_6)$	$U_7(R_7)$	$U_8(R_8)$

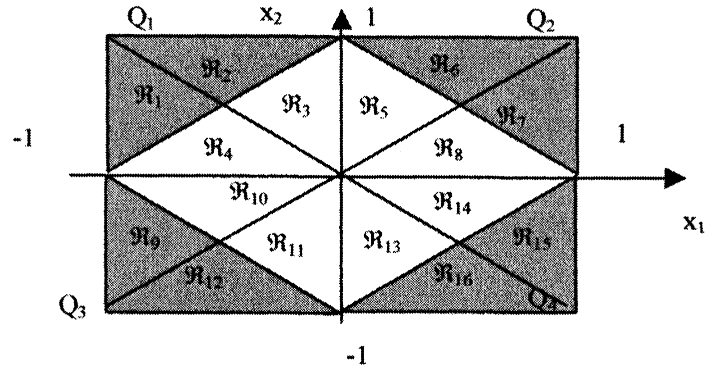

FIGURE 3 Regions with constant parameters membership functions.

For the sake of simplicity, we suppose that $U_0 = 0$, and we obtain:

$$\Re_1 : u = \frac{1}{3}(U_2 + U_4) + \frac{(U_2 - (2/3)U_4)x_1 + (U_1 - U_4)x_2}{3 + 2x_1},$$
$$\text{with } N_1 = 3 + 2x_1 \text{ and } 1 < N_1 < 2,$$

so as to obtain

$$V_1 = \frac{1}{3}(U_2 + U_4), \qquad \alpha_1 = \left(U_2 - \frac{2}{3}U_4, (U_1 - U_4)\right).$$

The other controls are summarised Table II.

Remark This structure will remain the same for a 2-dimensional controller with the above membership functions and a singleton deffuzification. It can be noticed that the constant terms V_h are nonzero for domain D_2, and depend only on parameters U_4, U_5, U_7 and U_2, and are zero in domain D_1.

The same method is extendable to other kinds of piecewise affine membership functions or to higher dimensions (but the number of regions grows quickly). Finally, it can be checked easily that, for each region, $1 \leq N_h \leq 2$.

4.2 Controller Stability

As domain D_1 is a rhombus with equal sides, it is convenient to use the following change of variable ξ:

Let us use the change of variable $\begin{cases} \xi_1 = x_1 + x_2 \\ \xi_2 = x_1 - x_2 \end{cases}$.

We obtain $\dot{\xi} = \tilde{A}\xi + \tilde{B}u$ with $\tilde{A} = \begin{pmatrix} -3.5 + 0.5\cos(t) + \omega & 1 - \omega \\ 0.5 + 0.5\cos(t) - \omega & -5 + \omega \end{pmatrix}$ and $\tilde{B} = \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}$.

TABLE II Controller Structure.

$\Re_1: u = \frac{1}{3}(U_2 + U_4) + \left(\frac{U_2 - 2U_4}{3}x_1 + (U_1 - U_4)x_2\right)/(3 + 2x_1),$	$\Re_9: u = \frac{1}{3}(U_7 + U_4) + \left(\frac{U_7 - 2U_4}{3}x_1 + (U_4 - U_6)x_2\right)/(3 + 2x_1)$
$\Re_2: u = \frac{1}{3}(U_2 + U_4) + \left((U_2 - U_1)x_1 + \frac{2U_2 - U_4}{3}x_2\right)/(3 - 2x_2),$	$\Re_{10}: u = (-U_4x_1 - (U_6 + U_7)x_2)/(1 - 2x_2)$
$\Re_3: u = ((-U_1 - U_4)x_1 + U_2x_2)/(1 - 2x_1),$	$\Re_{11}: u = ((-U_6 - U_4)x_1 - U_7x_2)/(1 - 2x_1)$
$\Re_4: u = (-U_4x_1 + (U_1 + U_2)x_2)/(1 + 2x_1),$	$\Re_{12}: u = \frac{1}{3}(U_7 + U_4) + \left((U_7 - U_6)x_1 + \frac{U_4 - 2U_7}{3}x_2\right)/(3 + 2x_2)$
$\Re_5: u = ((U_5 + U_3)x_1 + U_2x_2)/(1 + 2x_1),$	$\Re_{13}: u = ((U_8 + U_5)x_1 - U_7x_2)/(1 + 2x_1)$
$\Re_6: u = \frac{1}{3}(U_2 + U_5) + \left((U_3 - U_2)x_1 + \frac{2U_2 - U_5}{3}x_2\right)/(3 - 2x_2),$	$\Re_{14}: u = (U_5x_1 - (U_8 + U_7)x_2)/(1 - 2x_2)$
$\Re_7: u = \frac{1}{3}(U_2 + U_5) + \left(\frac{2U_2 - U_2}{3}x_1 + (U_3 - U_5)x_2\right)/(3 - 2x_1),$	$\Re_{15}: u = \frac{1}{3}(U_7 + U_5) + \left(\left(\frac{2U_5 - U_7}{3}\right)x_1 + (U_5 - U_8)x_2\right)/(3 - 2x_1)$
$\Re_8: u = (U_5x_1 + (U_3 + U_2)x_2)/(1 + 2x_2),$	$\Re_{16}: u = \frac{1}{3}(U_7 + U_5) + \left((U_8 - U_7)x_1 + \frac{U_5 - 2U_7}{3}x_2\right)/(3 + 2x_2)$

The resulting controlled system is now:

$$\dot{\xi} = A\xi + B\left(V_h + \frac{\alpha_h}{N_h}x\right) = \begin{pmatrix} -3.5 + 0.5\cos(t) + \omega & 1-\omega \\ 0.5 + 0.5\cos(t) - \omega & -5+\omega \end{pmatrix}\xi + \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}\left(V_h + \frac{1}{N_h}\left(\frac{\alpha_{1,h}+\alpha_{2,h}}{2} \;\; \frac{\alpha_{1,h}-\alpha_{2,h}}{2}\right)\right)$$

$$\dot{\xi} = \begin{pmatrix} -3.5 + 0.5\cos(t) + \omega + \dfrac{0.75(\alpha_{1,h}+\alpha_{2,h})}{N_h} & 1 - \omega + \dfrac{0.75(\alpha_{1,h}-\alpha_{2,h})}{N_h} \\ 0.5 + 0.5\cos(t) - \omega + \dfrac{0.25(\alpha_{1,h}+\alpha_{2,h})}{N_h} & -5 + \omega + \dfrac{0.25(\alpha_{1,h}-\alpha_{2,h})}{N_h} \end{pmatrix}\xi + \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix} V_h.$$

Domain D_2 defined in Figure 3 is the union of the regions $\Re_h$ with $(h = 1, 2, 6, 7, 9, 12, 15, 16)$.

As $1 \le N_h \le 2$, for every region $\Re_h$ included in D_2, the system admits the following overvaluing system relative to the vector norm $p(x) = (|x_1|, |x_2|)$,

$$\dot{z} = M_2 z + N_2 = \begin{pmatrix} -2.5 + 0.75 \sup_h |\alpha_{1,h}+\alpha_{2,h}| & 0.5 + 0.75 \sup_h |\alpha_{1,h}-\alpha_{2,h}| \\ 0.5 + 0.25 \sup_h |\alpha_{1,h}+\alpha_{2,h}| & -4.5 + 0.25 \sup_h |\alpha_{1,h}-\alpha_{2,h}| \end{pmatrix} \times z + \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix} \sup_h |V_h|,$$

$(h = 1, 26, 7, 9, 12, 15, 16)$, where we have $D^+p(x) \le Mp(x) + N$,

Calling $m_1 = \sup_h |\alpha_{1,h}+\alpha_{2,h}|$, $m_2 = \sup_h |\alpha_{1,h}-\alpha_{2,h}|$, $(h = 1, 26, 7, 9, 12, 15, 16)$, and applying Kotelianski Criterion, M_2 is a the opposite of a M-Matrix iff:

$$-2.5 + 0.75m_1 < 0 \quad \text{and} \quad -(11 - m_1 - 3.5m_2) > 0. \tag{5}$$

Condition (5) gives $|m_1| < 3.33$, $|m_2| > (22 - 2|m_1|)/7$.

These conditions hold only in domain D_2.

From Theorem 1, the attractor relative to domain D_2 is then contained in domain L_2: $|\xi| \le -\mathrm{M}^{-1}\mathrm{N}$ *i.e.*,

$$|\xi| \le \begin{pmatrix} 7\dfrac{\sup_h V_h}{(-11 + |m_1| + 3.5|m_2|)} \\ 2\dfrac{\sup_h V_h}{(-11 + |m_1| + 3.5|m_2|)} \end{pmatrix} \text{ with } \xi = \begin{pmatrix} |x_1 + x_2| \\ |x_1 - x_2| \end{pmatrix}.$$

All trajectories starting from domain D_2 converge into domain L_2 which should be included into D_1. The domain L_2 is a rhombus in the plane (x_1, x_2) which is of interest since D_1 is also a rhombus, which justifies the change of base. If L_2 is included into D_1, we must have $|\xi| < \binom{1}{1}$ which gives:

$$7 \sup_h V_h \le (-11 + |m_1| + 3.5|m_2|) \tag{6}$$

Choosing for example $P = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$, if $P(A + B\alpha_h{}^{\mathrm{T}}) + (A + B\alpha_h{}^{\mathrm{T}})^{\mathrm{T}} P < 0$ for each region $\Re_h$ included into domain D_1, then the closed loop system is asymptotically stable in D_1, (which is the union of regions $\Re_h$ with $h = (3, 4, 6, 8, 10, 11, 13, 14)$) which gives:

$$\alpha_{1,h} < 1.55, \quad \alpha_{2,h} < 1.55 \quad \text{with } h = (3, 4, 6, 8, 10, 11, 13, 14), \tag{7}$$

Choosing $a = b = 1$, these conditions can be summarised in

(a) $U4 - U_1 < 1.55$, $-U4 < 1.55$, $U_3 + U_5 < 1.55$, $U_5 < 1.55$, $-U_4 - U_6 < 1.55$, $U_8 + U_5 < 1.55$, $U_1 + U_2 < 1.55$, $U_2 < 1.55$, $U_3 + U_2 < 1.55$, $U_6 + U_7 < 1.55$, $-U_7 < 1.55$, $-U_8 - U_7 < 1.55$.

(*b*) $$|m_1| = \sup\left(\left|\frac{U_2 - 2U_4}{3}\right|, |U_2 - U_1|, |U_3 + U_5|, |U_3 - U_2|, \left|\frac{U_7 - 2U_4}{3}\right|, |U_7 - U_6|, \left|\frac{2U_5 - U_7}{3}\right|, |U_8 - U_7|\right) < 3.33$$

$$|m_2| = \sup\left(\left|\frac{2U_2 - U_4}{3}\right|, |U_1 - U_4|, \left|\frac{2U_2 - U_5}{3}\right|, |U_3 - U_5|, |U_4 - U_6|, \left|\frac{U_4 - 2U_7}{3}\right|, |U_5 - U_8|, \left|\frac{U_5 - 2U_7}{3}\right|\right) > \frac{22}{7}$$

(c) $7\sup(|U_2 + U_4|, |U_2 + U_5|, |U_6 + U_4|, |U_8 + U_5|) \leq (-11 + |m_1| + 3.5|m_2|)$.

Conditions (b) and (c) resulting from Theorem 1 are not very conservative and let a number of choices for the fuzzy controller parameters. A stronger values of perturbations for the initial system results in more conservative results since the overvaluing system will be less stable.

An example of a set of parameters satisfying the above conditions (a), (b), (c) is:

$$U_1 = 2, U_2 = -5, U_3 = 2, U_4 = 2, U_5 = -2, U_6 = -2, U_7 = 1, U_8 = -1, U_0 = 0.$$

5 CONCLUSION

Comparison systems and vector norms can be used to estimate the attractor of a set for a nonlinear system. To ensure the stability of a fuzzy controlled system, a series of domains has been built whose attractors lies within a domain where the closed-loop system is asymptotically stable. Some constraints on the fuzzy controller parameter values come from the design of the domains and the stability conditions. An admissible domain for controller parameter design is thus provided for which the closed-loop fuzzy controlled system is stable. An advantage of the method is to respect fuzzy control philosophy, consisting in driving trajectories from a delimited domain of the state space into another and so forth until the origin is reached. Moreover, this method allows to handle bounded parameters uncertainties proving the robustness of fuzzy control. Future work should focus into the obtaining of a synthesis method which will allow to tune more readily systems parameters.

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