An Examination of Linear and Nonlinear Causal Relationships Between Price Variability and Volume in Petroleum Futures Markets

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This article examines the relationship between returns and trading volume for three petroleum futures contracts. Using daily data on futures prices and trading volume, the study first tests for linear causality between returns and volume. The results of this linear causality test show that futures returns and volume have no predictive power for one another. However, because the distribution of the returns and volume series provides some evidence of nonlinear dependence, the study formally tests for and finds evidence of significant nonlinearities in the returns and volume for the three petroleum futures contracts. The returns and volume series are then filtered for linear dependence through the use of a VAR process. A nonparametric test statistic based on the correlation in-

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tegral reveals significant bidirectional nonlinear causal relationships between the filtered returns and volume series. Using a third-order moment test, this study finds that the nonlinear dependence in the futures returns and volume series arises from the variance, rather than the mean, of the process. Consequently, the filtered returns and volume series are adjusted for conditional heteroscedasticity. The study then examines the GARCH-filtered returns and volume series and finds that, even after adjusting for volatility effects, there is still strong evidence of bidirectional nonlinear Granger causality and concludes that the nonlinear process may influence both the mean and variance of futures returns and volume. The finding of strong nonlinear causal relationships between petroleum futures returns and trading volume implies that knowledge of current trading volume improves the ability to forecast futures prices. Thus, the results of this study should be useful to regulators, practitioners, and futures markets participants whose success hinges crucially on the ability to forecast futures price movements.

INTRODUCTION

This article extends and updates the existing empirical research on the relationship between asset price variability and volume by examining the daily returns and volume series for three petroleum futures contracts. This investigation is important for several reasons. First, causality tests can provide useful information on whether knowledge of past trading volume improves short-term forecasts of current and future movements in futures prices, and vice versa. Second, as noted by Cornell (1981), the volume-price variability relationship may have important implications for fashioning new contracts. For example, a positive volume-price variability relationship implies that a new futures contract will be successful only to the extent that there is enough price uncertainty associated with the underlying asset. Third, as noted by Karpoff (1987), price-volume relationships have important implications for futures markets-related research. For example, Cornell (1981) and Martell and Wolf (1985) show that price variability affects futures contracts trading volume. Fourth, a good understanding of futures price movements has significant implications for asset pricing models, regulators, hedgers, speculators, and other participants in futures markets.

With the exception of Hiemstra and Jones (1994), all previous investigations of the relationship between asset price variability and trading volume use tests that rely on the restrictive assumption of linearity. As shown by Baek and Brock (1992), however, these tests generally have low power against nonlinear relationships and may, therefore, fail to detect useful nonlinear relationships between asset price variability and trading volume. Consequently, this study also contributes to the literature on futures markets by explicitly testing for linear and nonlinear causal relationships between futures returns and trading volume.

Examining the possibility of nonlinear causal relationships is justified because the univariate time series of futures prices and trading volumes are likely to have been generated by nonlinear processes. As noted by Savit (1989), self-regulating systems are generally characterized by nonlinear processes. To the extent that financial and commodity markets are self-regulating systems with complex feedback and feedforward loops, the prices (and volumes) that these markets generate are expected to display significant nonlinearities. Hinich and Patterson (1985), Scheinkman and LeBaron (1989), Brock, Hsieh, and LeBaron (1991), and Hsieh (1991), among others, find evidence of significant nonlinear dependence in asset returns. Hiemstra and Jones (1992) also report evidence of nonlinearities in aggregate trading volume. In light of these findings, this study tests for and finds evidence of nonlinear dependence in the univariate time series of returns and trading volume for the three petroleum futures, and posits that the causal relationship between these futures returns and trading volume is largely of a nonlinear nature. Furthermore, the nonlinearities appear to influence both the mean and the variance of the time series.

The article proceeds as follows. The next section provides a review of the theoretical and empirical work on the price-volume relationship. Then the data are described, followed by a discussion of the methodology and estimation results of the linear and nonlinear tests in the fourth section. The final section closes the article by discussing the implications of the results and directions for future research.

PRIOR RESEARCH ON THE ASSET PRICE-VOLUME RELATIONSHIP

Theoretical Investigation

There are numerous studies that attempt to explain the possible existence of a causal linkage between asset prices and trading volume. Copeland (1976) proposes an approach, later extended by Jennings, Starks, and Felligham (1981), known as a *sequential arrival of information* model, in which a positive bidirectional causal relationship exists between absolute values of price changes and volume. This model hypothesizes that once new innovations reach the marketplace, they are not transmitted to all market participants at once. Instead, the model assumes that such innovations reach only one participant at a time, leading to a final information equilibrium only after a sequence of transitional equilibriums have occurred. According to this model, therefore, lagged absolute values of price changes may have the ability to predict current trading volume and vice versa.

A second explanation of the existence of a causal linkage between asset prices and trading volume is rooted in the *mixture of distributions* models of Clark (1973), Epps and Epps (1976), and Harris (1984). In this framework, asset prices and trading volume are positively correlated because the variance of the price change on a single transaction is conditional upon the volume of that transaction. Therefore, transaction price changes are mixtures of distributions, with volume as the mixing variable. This implies that trading volume and prices change synchronously in response to new information.

Another reason given for a causal relationship between asset returns and trading volume is the noise-trader models of DeLong, Shleifer, Summers, and Waldmann (1990). These models hold that because noise traders' activities are not based upon economic fundamentals, they tend to cause a temporary mispricing of stock prices in the short run. However, stock price changes revert to their means because of the disappearance of the transitory component in the long run. In these models, the positive causal relationship running from stock returns to trading volume is consistent with the positive-feedback trading strategies of noise traders who trade on the basis of past price changes. Also, a positive causal relationship from volume to price changes is consistent with the hypothesis made in these models that price changes are caused by the trading strategies of noise traders.

Empirical Investigation

Clark (1973) made one of the earliest examinations of the price-volume relationship in futures markets. Using cotton futures data, he finds a positive relationship between aggregated volume and the square of price movement. Cornell (1981) reports positive contemporaneous relationships between changes in price variability and changes in volume for 17 futures contracts. Tauchen and Pitts (1983) examine daily treasury-bill futures prices and report a positive relationship between trading volume and the variability of price changes. Grammatikos and Saunders (1986) study daily data for five foreign currency futures traded on the International Monetary Market (IMM) and find strong positive contemporaneous correlations between trading volume and price variability. Bessembinder and Seguin (1993) analyze a cross section of eight futures contracts and report a strong positive relationship between contemporaneous volume and price volatility.

The more recent work on the price-volume relationship focuses mainly on the stock market. Hiemstra and Jones (1994) analyze daily returns on the Dow Jones Industrial Average and percentage changes in New York Stock Exchange trading volume over the 1915–1946 and 1947–1990 periods and find no evidence of linear causality, but report highly significant bidirectional nonlinear causality. Recognizing that this could be because of volatility effects associated with the flow of information, Hiemstra and Jones (1994) estimate an ARCH model to control for the persistence of volatility in returns. They find that volume continues to have significant nonlinear explanatory power for stock returns even after accounting for volatility effects. They also argue that their evidence for bidirectional nonlinear causality between returns and volume cannot be explained entirely by a latent-variable effect attributed to the flow of information. Richardson and Smith (1994) and Lamoureux and Lastrapes (1994) further investigate this issue. Richardson and Smith apply the generalized method-of-moments procedure using daily price and volume data for 30 firms. They fail to find strong evidence to support the information-flow approach. Lamoureux and Lastrapes find that accounting for serial dependence in the flow of information does not eliminate the persistence in conditional volatility for returns.

DATA AND SUMMARY STATISTICS

The data set used in this study consists of daily futures price (PRI) and trading volume (VOL) series for crude oil, heating oil, and unleaded gasoline traded at the NYMEX from December 3, 1984 to September 30, 1993, a total of 2,217 observations. Because the maturity date changes over time, a single time series is constructed with the use of the nearby futures contract until the day prior to its last trading day, at which point the data are rolled over to the next deferred contract.[1] The price series are reexpressed in daily percentage changes as $R_t = 100 \times \ln(\mathrm{PRI}_{t,T}/\mathrm{PRI}_{t-1,T})$, and the volume series are expressed as $V_t = \mathrm{VOL}_{t,T}/1000$,

[1]Ma, Mercer, and Walker (1992) discuss the implications of linking futures price series and the possible biases that may arise when selecting a particular method to roll over futures contracts.

TABLE I

Summary Statistics for Futures Returns and Volume

	Crude Oil		*Heating Oil*		*Unleaded Gas*	
	R_t	V_t	R_t	V_t	R_t	V_t
Mean	0.0144	67.7683	0.0171	21.5036	0.0408	14.9741
SD[2]	0.0486	0.7579	0.0443	0.2340	0.0411	0.2345
t statistic	0.2970	89.4171[a]	0.3852	91.9023[a]	0.9909	63.8459[a]
Skewness	−2.2774[a]	0.3079[a]	−2.0670[a]	0.9877[a]	−1.5691[a]	0.4536[a]
Kurtosis	39.8308[a]	−0.2238[b]	37.8726[a]	1.2999[a]	27.3933[a]	−0.1716[a]
Minimum	−38.4071	4.4020	−35.0938	2.6100	−29.8099	0.0050
Maximum	12.3525	224.0120	12.8019	74.4570	10.6913	65.2310

The data are for the period, December 3, 1984 to September 30, 1993. $R_t = 100 \times \ln(PRI_{t,T}/PRI_{t-1,T})$, $V_t = VOL_{t,T}/1000$, where PRI = futures price and VOL = futures volume.
SD is the standard deviation of the mean. The *t* statistic is for the null hypothesis that the mean equals 0.
[a]Significant at the 1% level.
[b]Significant at the 5% level.

where $PRI_{t,T}$ and $VOL_{t,T}$ are, respectively, the daily futures price and volume at time t for futures contracts maturing at date T.[2]

Various descriptive statistics are provided in Table I. The distribution of the daily returns is negatively skewed and has excess kurtosis relative to the normal distribution. The volume data, on the other hand, are positively skewed and display significantly lower values for excess kurtosis. Some existing evidence [see, for example, Hall, Brorsen, and Irwin (1989)] suggests that the excess kurtosis is due to a possible time-varying variance. This hypothesis is examined in the following sections.

METHODOLOGY AND ESTIMATION RESULTS

Tests for Linear Granger Causality

A process is said to exhibit Granger causality when past information on one variable improves the prediction of a second variable in a better fashion than when a prediction is based on past information on the second variable only. Consequently, Granger causality between two variables can

[2]A robust unit root test developed in Phillips (1987) and Phillips and Perron (1988) is used to further examine volume series in their levels and changes in futures prices. The Phillips–Perron method was chosen because it is a general test that can be used even in the presence of autocorrelated innovation sequences. It also handles departures from iid errors. The results of the unit root tests, not reported here but available from the authors upon request, support the null hypothesis of a unit root in each of the three futures price series, but reject the null of nonstationary volume time series. Consequently, this study differences the logarithm of prices to ensure stationarity. This transformation is necessary to avoid spurious results associated with the use of nonstationary variables.

run in either or both directions. Granger-causality tests are chosen from a number of alternative causality techniques, because Geweke, Meese, and Dent (1983) show that the one-sided Granger procedure conducted with the use of a Wald chi-square test statistic outperforms other causality tests in a series of Monte Carlo experiments.

Consider the bivariate covariance stationary stochastic process, $Z_t = \{R_t, V_t\}$, where R_t denotes the daily futures returns at time t and V_t is the daily trading volume at time t.[3] Assume that, at time $t - 1$, one attempts to forecast next-period returns, R_t. If R_t is better predicted by adding past values of trading volume to the past returns series than by using the past returns series alone, then volume is said to Granger-cause returns. Similarly, returns are said to Granger-cause volume if next-period volume, V_t, is better predicted by the bivariate time series than by the univariate volume series alone.

More formally, let R^*_{t-1}, V^*_{t-1}, and Z^*_{t-1} denote the set of past values of returns, volume, and the bivariate time series, respectively, so that $R^*_{t-1} = (R_{t-1}, R_{t-2}, \ldots)$, $V^*_{t-1} = (V_{t-1}, V_{t-2}, \ldots)$, and $Z^*_{t-1} = (R_{t-1}, R_{t-2}, \ldots, V_{t-1}, V_{t-2}, \ldots)$. Let $\sigma^2(R_t/Z^*_{t-1})$ denote the error in predicting period-t price change given that the predictions are based on the information set that includes both past returns and past trading volume time series.[4] By contrast, let $\sigma^2(R_t/Z^*_{t-1} - V^*_{t-1})$ be the error in predicting period-t returns given that the past volume time series is excluded for prediction purposes—in other words, period-t returns are predicted on the basis of past returns alone. If $\sigma^2(R_t/Z^*_{t-1}) < \sigma^2(R_t/Z^*_{t-1} - V^*_{t-1})$, then volume is said to Granger-cause returns. Similarly, if $\sigma^2(V_t/Z^*_{t-1}) < \sigma^2(V_t/Z^*_{t-1} - R^*_{t-1})$, then returns are said to Granger-cause volume. Feedback exists when both of the situations prevail, that is, returns and volume cause each other.

The empirical analysis begins with an examination of the relationship and the direction of *linear* causality between futures returns and trading volume. The multivariate model with *linear* causality is given by the following vector autoregressive (VAR) representation:

$$R_t = \sum_{i=1}^{p1} \phi_{1,i} R_{t-i} + \sum_{i=1}^{p2} \phi_{2,i} V_{t-i} + \zeta_{R,t} \tag{1}$$

$$V_t = \sum_{i=1}^{p3} \phi_{3,i} R_{t-i} + \sum_{i=1}^{p4} \phi_{4,i} V_{t-i} + \mu_{V,t} \tag{2}$$

[3]The random process (Y) is said to be covariance stationary if its statistical properties do not change over time.

[4]The variance symbol, σ^2, denotes the prediction error. Basically, $\sigma^2(R_t/Z^*_{t-1})$ is the minimum mean-square linear prediction error of R_t, given the information set, Z^*_{t-1}.

where R_t and V_t are the daily returns and volume, respectively, $\phi_{i,j}$'s are the parameters to be estimated, (ζ, μ) are the conventional zero-mean error terms with constant variance-covariance matrix, and the p_i's are the optimal lag lengths. In this study, the p_i parameters are obtained with the use of the final prediction error (FPE) criterion.[5]

The optimal equations using the FPE criterion are chosen such that

$$\mathrm{FPE}(\hat{p}) = \min(\mathrm{FPE}(s)|s = 1, \ldots, P)$$

where P is the prespecified upper bound on the maximum examined lag. In this study, maximum lags of 40 and 20 for P are considered for the univariate autoregressive process (dependent variable) and the cross-variable (independent variable), respectively.

Linear causal relationships are inferred from eqs. (1) and (2). If $\Sigma\phi_{2,i} = 0$ (i.e., $\phi_{2,i} = 0$ for all i), then eq. (1) implies that past volume has no influence on futures returns; volume does not cause returns. Similarly, returns do not cause volume if $\Sigma\phi_{3,i} = 0$ in eq. (2). On the other hand, if $\phi_{2,i} \neq 0$ for some values of i, then eq. (1) suggests that volume Granger-causes returns, implying that the information set used to predict next-period returns should also include current volume.

The test for causality used in this study is based on the following Wald test statistic:

$$\chi^2_{(p_i)} = (c - C\hat{\beta})' \, [C\Sigma_x C']^{-1} \, (c - C\hat{\beta}), \qquad i = 2,4 \tag{3}$$

where c is a $(p \times 1)$ vector of known constants, C is a $(p \times k)$ hypothesis design matrix of known constants, β is a $(k \times 1)$ vector of the regression coefficients, and Σ_x is the estimated covariance matrix of the regression coefficients. A statistically significant χ^2 implies that lagged values of the independent or pre-determined variable help to predict the dependent variable.

Tables II–IV report the estimated parameters of the *linear* model (1) and (2). Table II for the crude-oil results shows evidence of unidirectional linear causality from volume to returns. The χ^2 statistic that tests the exclusion of the volume series from the returns equation is significant at the 5% level, whereas the χ^2 statistic in the volume equation is insignificant. In the long run, however, crude-oil futures returns and volume appear to be unrelated because neither the summed impact of volume ($\Sigma\phi_{2,i} = 0.0015$) on returns nor the summed impact of returns ($\Sigma\phi_{3,i} = -0.0055$) on volume is statistically significant.

The results of the linear causality tests for heating oil, displayed in Table III, show no causal relationship between returns and volume. Neither of the χ^2 statistics is significant at any reasonable statistical level.

[5]The FPE criterion is based on the minimization of the prediction error.

TABLE II

Test Results of Linear Causality Between Futures Returns and Volume (Crude Oil)

Dependent Variable	*Coefficients* (**t** *statistics*) **R_t**		**V_t**	**Q** *Stat.*	χ^2 *Stat.*
R_t	$\phi_{1,1}$ = 0.021 (0.53)	$\phi_{1,16}$ = 0.016 (0.56)	$\phi_{2,1}$ = −0.004 (−1.87)[c]	1.996	20.35[b]
	$\phi_{1,2}$ = −0.018 (−0.49)	$\phi_{1,17}$ = 0.017 (0.54)	$\phi_{2,2}$ = 0.004 (1.69)[c]		
	$\phi_{1,3}$ = −0.086 (−1.73)[c]	$\phi_{1,18}$ = −0.011 (−0.41)	$\phi_{2,3}$ = −0.004 (−1.17)		
	$\phi_{1,4}$ = 0.046 (1.54)	$\phi_{1,19}$ = 0.004 (1.34)	$\phi_{2,4}$ = 0.000 (0.18)		
	$\phi_{1,5}$ = −0.083 (−2.53)[a]	$\phi_{1,20}$ = 0.010 (0.29)	$\phi_{2,5}$ = 0.006 (2.33)[b]		
	$\phi_{1,6}$ = 0.013 (0.43)	$\phi_{1,21}$ = 0.034 (0.93)	$\phi_{2,6}$ = −0.004 (−1.35)		
	$\phi_{1,7}$ = 0.030 (1.11)	$\phi_{1,22}$ = 0.039 (1.37)	$\phi_{2,7}$ = 0.006 (2.14)[b]		
	$\phi_{1,8}$ = −0.086 (−2.02)[b]	$\phi_{1,23}$ = −0.007 (−0.19)	$\phi_{2,8}$ = −0.008 (−2.70)[a]		
	$\phi_{1,9}$ = −0.012 (−0.34)	$\phi_{1,24}$ = 0.007 (0.18)	$\phi_{2,9}$ = 0.005 (1.44)		
	$\phi_{1,10}$ = 0.052 (1.49)	$\phi_{1,25}$ = −0.001 (−0.04)			
	$\phi_{1,11}$ = 0.051 (1.38)	$\phi_{1,26}$ = −0.020 (−0.81)			
	$\phi_{1,12}$ = 0.030 (0.95)	$\phi_{1,27}$ = 0.022 (0.81)			
	$\phi_{1,13}$ = −0.004 (−0.13)	$\phi_{1,28}$ = −0.009 (−0.18)			
	$\phi_{1,14}$ = 0.060 (2.07)[b]	$\phi_{1,29}$ = −0.006 (−0.20)			
	$\phi_{1,15}$ = −0.007 (−0.21)	$\phi_{1,30}$ = −0.029 (−1.07)			
		$\phi_{1,31}$ = 0.059 (1.08)			
	$\Sigma \phi_{1,i}$ = 0.1689 (0.8889)		$\Sigma \phi_{2,i}$ = 0.0015 (0.9003)		
V_t	$\phi_{3,1}$ = −0.0055 (−0.0301)	$\phi_{4,1}$ = 0.471 (16.81)[a]	$\phi_{4,21}$ = 0.024 (0.87)	2.102	0.001
		$\phi_{4,2}$ = 0.023 (0.82)	$\phi_{4,22}$ = 0.003 (0.12)		
		$\phi_{4,3}$ = 0.118 (4.10)[a]	$\phi_{4,23}$ = 0.066 (2.38)[b]		
		$\phi_{4,4}$ = 0.067 (2.32)[b]	$\phi_{4,24}$ = −0.022 (−0.81)		
		$\phi_{4,5}$ = 0.005 (0.19)	$\phi_{4,25}$ = −0.27 (−0.97)		
		$\phi_{4,6}$ = −0.019 (−0.62)	$\phi_{4,26}$ = −0.000 (−0.01)		
		$\phi_{4,7}$ = −0.045 (−1.47)	$\phi_{4,27}$ = −0.019 (−0.72)		
		$\phi_{4,8}$ = −0.002 (−0.09)	$\phi_{4,28}$ = 0.014 (0.53)		
		$\phi_{4,9}$ = 0.039 (1.52)	$\phi_{4,29}$ = 0.026 (0.89)		

TABLE II (Continued)

Test Results of Linear Causality Between Futures Returns and Volume (Crude Oil)

Dependent Variable	Coefficients (t statistics): R_t	Coefficients (t statistics): V_t	Q *Stat.*	χ^2 *Stat.*
	$\phi_{4,10}$ = 0.027 (0.99)	$\phi_{4,30}$ = −0.019 (−0.69)		
	$\phi_{4,11}$ = 0.006 (0.21)	$\phi_{4,31}$ = 0.017 (0.61)		
	$\phi_{4,12}$ = −0.020 (−0.71)	$\phi_{4,32}$ = −0.039 (−1.49)		
	$\phi_{4,13}$ = −0.012 (−0.42)	$\phi_{4,33}$ = 0.013 (0.48)		
	$\phi_{4,14}$ = 0.034 (1.12)	$\phi_{4,34}$ = 0.030 (1.12)		
	$\phi_{4,15}$ = 0.004 (0.15)	$\phi_{4,35}$ = 0.004 (0.14)		
	$\phi_{4,16}$ = 0.008 (0.27)	$\phi_{4,36}$ = −0.052 (−0.86)		
	$\phi_{4,17}$ = −0.017 (−0.62)	$\phi_{4,37}$ = −0.008 (−0.27)		
	$\phi_{4,18}$ = 0.040 (1.43)	$\phi_{4,38}$ = 0.036 (1.25)		
	$\phi_{4,19}$ = 0.052 (1.89)[c]	$\phi_{4,39}$ = 0.048 (1.62)[c]		
	$\phi_{4,20}$ = 0.028 (1.05)	$\phi_{4,40}$ = 0.070 (2.60)[a]		
	$\Sigma\,\phi_{3,i}$ = −0.0055 (−0.0301)	$\Sigma\,\phi_{4,i}$ = 0.9741 (79.2971)[a]		

The sample period is from December 3, 1984 to September 30, 1993. $R_t = 100 \cdot \ln(PRI_{t,T}/PRI_{t-1,T})$, where PRI = futures price and $V_t = VOL_{t,T}/1000$, where VOL = futures volume. The t statistics are reported in parentheses next to the estimated coefficients. The coefficients show the impact of a specific lag of a right-hand variable on the left-hand-side variable. For example, $\phi_{2,1}$ represents the impact of volume (variable 2) on returns (variable 1) for a given lag of 1.

Q Stat. is the Q statistic for serial independence. This statistic is based on the revised Box–Ljung Q test for serial correlation among the regression residuals.

χ stat. is the chi-square statistic for testing for the joint significance of the lags on the right-hand-side variables.

For a given sum coefficient, the t statistic, t, is calculated as $t = s/\sigma$, where $s = \Sigma\, a_i$ and n = number of lags on the independent variable whose impact is being investigated. For example, if $n = 3$, then $s = \Sigma\, a_i = a_1 + a_2 + a_3$, and

$$\sigma_s = \sigma_{(a1+a2+a3)} = \sqrt{\sigma^2_{a_1} + \sigma^2_{a_2} + \sigma^2_{a_3} + 2\sigma_{a_1a_2} + 2\sigma_{a_2a_3} + 2\sigma_{a}1a_3},$$

[a]Significant at the 1% level.
[b]Significant at the 5% level.
[c]Significant at the 10% level.

TABLE III

Test Results of Linear Causality Between Futures Returns and Volume (Heating Oil)

Dependent Variable	*Coefficients* (**t** *statistics*) $\mathbf{R}_t$	$\mathbf{V}_t$		*Q Stat.*	χ^2 *Stat.*
R_t	$\phi_{1,1}$ = 0.040 (1.21)	$\phi_{2,1}$ = −0.001 (−0.25)		1.899	0.06
	$\phi_{1,2}$ = −0.007 (0.19)				
	$\phi_{1,3}$ = −0.010 (−1.96)[b]				
	$\phi_{1,4}$ = 0.006 (0.21)				
	$\phi_{1,5}$ = −0.065 (−2.18)[b]				
	$\phi_{1,6}$ = −0.030 (−1.00)				
	$\phi_{1,7}$ = 0.031 (1.11)				
	$\phi_{1,8}$ = −0.078 (1.81)[c]				
	$\phi_{1,9}$ = −0.021 (−0.81)				
	$\phi_{1,10}$ = 0.006 (1.88)[c]				
	$\phi_{1,11}$ = 0.090 (2.12)[b]				
	$\phi_{1,12}$ = 0.035 (0.13)				
	$\phi_{1,13}$ = −0.004(1.42)				
	$\phi_{1,14}$ = 0.033 (1.12)				
	$\Sigma\ \phi_{1,i}$ = 0.0590 (0.5148)	$\Sigma\ \phi_{2,i}$ = 0.0010 (−0.2461)			
V_t	$\phi_{3,1}$ = −0.036 (0.63)	$\phi_{4,1}$ = 0.459(16.90)[a]	$\phi_{4,19}$ = 0.049 (1.65)[c]	1.988	8.30
	$\phi_{3,2}$ = 0.038 (0.66)	$\phi_{4,2}$ = 0.000 (0.02)	$\phi_{4,20}$ = 0.059 (1.72)[c]		
	$\phi_{3,3}$ = −0.072 (−1.23)	$\phi_{4,3}$ = 0.051 (1.89)[c]	$\phi_{4,21}$ = 0.001 (0.02)[c]		
	$\phi_{3,4}$ = 0.089 (1.57)	$\phi_{4,4}$ = 0.106 (3.80)[a]	$\phi_{4,22}$ = 0.053 (1.71)		
	$\phi_{3,5}$ = 0.045 (0.84)	$\phi_{4,5}$ = 0.054 (2.13)[b]	$\phi_{4,23}$ = 0.015 (0.48)		
	$\phi_{3,6}$ = −0.105 (−1.80)[c]	$\phi_{4,6}$ = −0.041 (−1.55)	$\phi_{4,24}$ = −0.043 (−1.44)		
		$\phi_{4,7}$ = 0.049 (1.70)[c]	$\phi_{4,25}$ = 0.032 (1.09)		
		$\phi_{4,8}$ = −0.072 (−2.64)[a]	$\phi_{4,26}$ = −0.013 (−0.43)		
		$\phi_{4,9}$ = 0.028 (1.03)	$\phi_{4,27}$ = 0.042 (−1.36)		
		$\phi_{4,10}$ = 0.079 (2.71)[a]	$\phi_{4,28}$ = 0.067 (2.36)[b]		
		$\phi_{4,11}$ = 0.012 (0.48)	$\phi_{4,29}$ = 0.002 (0.07)		
		$\phi_{4,12}$ = −0.005 (−0.21)	$\phi_{4,30}$ = 0.037 (1.32)		
		$\phi_{4,13}$ = 0.024 (0.81)	$\phi_{4,31}$ = −0.029 (−1.11)		

TABLE III (Continued)

Test Results of Linear Causality Between Futures Returns and Volume (Heating Oil)

Dependent Variable	Coefficients (**t** statistics) R_t	V_t		*Q Stat.*	χ^2 *Stat.*
		$\phi_{4,14}$ = 0.075 (2.58)[a]	$\phi_{4,32}$ = 0.030 (1.07)		
		$\phi_{4,15}$ = −0.032 (−1.09)	$\phi_{4,33}$ = −0.061 (−2.23)[b]		
		$\phi_{4,16}$ = 0.047 (1.66)[c]	$\phi_{4,34}$ = 0.082 (3.06)[a]		
		$\phi_{4,17}$ = −0.004 (−0.13)	$\phi_{4,35}$ = −0.037 (−1.33)		
		$\phi_{4,18}$ = −0.029 (−1.06)	$\phi_{4,36}$ = −0.039 (−1.52)		
	$\Sigma\ \phi_{3,i}$ = 0.0324 (0.2057)		$\Sigma\ \phi_{4,i}$ = 0.9687 (54.7021)[a]		

The sample period is from December 3, 1984 to September 30, 1993. $R_t = 100 \cdot \ln(PRI_{t,T}/PRI_{t-1,T})$, where PRI = futures price and $V_t = VOL_{t,T}/1000$, where VOL = futures volume. The *t* statistics are reported in parentheses next to the estimated coefficients. The coefficients show the impact of a specific lag of a given right-hand variable on the left-hand-side variable. For example, $\phi_{2,1}$ represents the impact of volume (variable 2) on returns (variable 1) for a given lag of 1.

Q Stat. is the *Q* statistic for serial independence. This statistic is based on the revised Box–Ljung *Q* test for serial correlation among the regression residuals.

χ stat. is the chi-square statistic for testing for the joint significance of the lags on the right-hand-side variables.

For a given sum coefficient, the *t* statistic, *t*, is calculated as $t = s/\sigma_s$ where $s = \Sigma^a\ a_i$ and n = number of lags on the independent variable whose impact is being investigated. For example, if $n = 3$, then, $s = \Sigma\ a_i = a_1 + a_2 + a_3$, and

$$\sigma_s = \sigma_{(a1+a2+a3)} = \sqrt{\sigma^2_{a_1} + \sigma^2_{a_2} + \sigma^2_{a_3} - 2\sigma_{a_1a_2} + 2\sigma_{a_2a_3} + 2\sigma_a 1 a_3}$$

[a]Significant at the 1% level.
[b]Significant at the 5% level.
[c]Significant at the 10% level.

TABLE IV

Test Results of Linear Causality Between Futures Returns and Volume (Unleaded Gas)

Dependent Variable	Coefficients (**t** statistics) **R_t**		**V_t**		**Q** *Stat.*	χ^2 *Stat.*
R_t	$\phi_{1,1} = 0.061\ (1.65)$[c]	$\phi_{1,18} = -0.036\ (-1.57)$	$\phi_{2,1} = 0.000\ (0.01)$		1.995	4.67
	$\phi_{1,2} = 0.048\ (1.21)$	$\phi_{1,19} = 0.036\ (1.49)$	$\phi_{2,2} = 0.015\ (1.64)$			
	$\phi_{1,3} = -0.005\ (-1.19)$	$\phi_{1,20} = 0.028\ (1.11)$	$\phi_{2,3} = -0.015\ (-2.10)$[b]			
	$\phi_{1,4} = -0.005\ (-0.19)$	$\phi_{1,21} = 0.031\ (1.10)$				
	$\phi_{1,5} = -0.013\ (-0.44)$	$\phi_{1,22} = 0.023\ (0.87)$				
	$\phi_{1,6} = -0.010\ (-0.32)$	$\phi_{1,23} = 0.038\ (1.33)$				
	$\phi_{1,7} = 0.011\ (0.40)$	$\phi_{1,24} = 0.005\ (0.21)$				
	$\phi_{1,8} = -0.092\ (-2.22)$[b]	$\phi_{1,25} = -0.036\ (-1.40)$				
	$\phi_{1,9} = 0.000\ (0.02)$	$\phi_{1,26} = -0.005\ (-0.21)$				
	$\phi_{1,10} = 0.065\ (2.21)$[b]	$\phi_{1,27} = 0.001\ (0.02)$				
	$\phi_{1,11} = 0.027\ (1.01)$	$\phi_{1,28} = -0.006\ (-0.17)$				
	$\phi_{1,12} = 0.010\ (0.40)$	$\phi_{1,29} = -0.008\ (-0.28)$				
	$\phi_{1,13} = 0.005\ (0.19)$	$\phi_{1,30} = -0.050\ (-1.51)$				
	$\phi_{1,14} = 0.009(0.35)$	$\phi_{1,31} = 0.040\ (1.15)$				
	$\phi_{1,15} = 0.009\ (0.34)$	$\phi_{1,32} = 0.015\ (0.58)$				
	$\phi_{1,16} = 0.009\ (0.37)$	$\phi_{1,33} = -0.038\ (-1.57)$				
	$\phi_{1,17} = -0.003\ (-0.11)$	$\phi_{1,34} = -0.031\ (-1.28)$				
		$\Sigma\ \phi_{1,i} = 0.0836$ (0.5094	$\Sigma\ \phi_{2,i} = -0.0002\ (-0.0618)$			
V_t	$\phi_{3,1} = 0.102\ (2.38)$[b]		$\phi_{4,1} = 0.442(14.73)$[a]	$\phi_{4,21} = 0.017\ (0.51)$	2.011	23.59[b]
	$\phi_{3,2} = 0.027\ (0.66)$		$\phi_{4,2} = 0.035\ (0.97)$	$\phi_{4,22} = 0.768\ (2.18)$[b]		
	$\phi_{3,3} = 0.073\ (1.79)$[c]		$\phi_{4,3} = 0.030\ (0.97)$	$\phi_{4,23} = 0.006\ (0.17)$		
	$\phi_{3,4} = 0.045\ (1.11)$		$\phi_{4,4} = 0.151\ (5.12)$[a]	$\phi_{4,24} = -0.030\ (-0.95)$		
	$\phi_{3,5} = 0.039\ (0.94)$		$\phi_{4,5} = 0.018\ (0.58)$	$\phi_{4,25} = 0.002\ (0.05)$		
	$\phi_{3,6} = 0.048\ (1.09)$		$\phi_{4,6} = -0.013\ (-0.41)$	$\phi_{4,26} = -0.019\ (-0.64)$		
	$\phi_{3,7} = -0.052\ (-1.33)$		$\phi_{4,7} = -0.026\ (-0.73)$	$\phi_{4,27} = -0.011\ (-0.36)$		
	$\phi_{3,8} = 0.023\ (0.58)$		$\phi_{4,8} = 0.020\ (0.64)$	$\phi_{4,28} = 0.049\ (1.44)$		
	$\phi_{3,9} = 0.071\ (1.67)$		$\phi_{4,9} = 0.061\ (2.07)$[b]	$\phi_{4,29} = 0.040\ (1.27)$		
	$\phi_{3,10} = -0.061\ (-1.56)$		$\phi_{4,10} = 0.064\ (2.11)$[b]	$\phi_{4,30} = -0.021\ (-0.70)$		

TABLE IV (Continued)

Test Results of Linear Causality Between Futures Returns and Volume (Unleaded Gas)

Dependent Variable	*Coefficients* (**t** *statistics*)			**Q** *Stat.*	χ^2 *Stat.*
		$\mathbf{R}_t$	$\mathbf{V}_t$		
	$\phi_{3,11}$ − 0.026 (0.66)	$\phi_{4,11}$ = −0.022 (−0.79)	$\phi_{4,31}$ = 0.014 (0.45)		
	$\phi_{3,12}$ = 0.077 (1.89)[c]	$\phi_{4,12}$ = −0.67 (−2.31)[b]	$\phi_{4,32}$ = −0.064 (−2.13)[b]		
		$\phi_{4,13}$ = 0.006 (1.82)[c]	$\phi_{4,33}$ = −0.024 (−0.77)		
		$\phi_{4,14}$ = 0.020 (0.67)	$\phi_{4,34}$ = 0.065 (2.24)[b]		
		$\phi_{4,15}$ = 0.024 (0.70)	$\phi_{4,35}$ = −0.056 (−1.85)[c]		
		$\phi_{4,16}$ = 0.018 (0.51)	$\phi_{4,36}$ = −0.002 (−0.08)		
		$\phi_{4,17}$ = −0.033 (−0.95)	$\phi_{4,37}$ = −0.013 (−0.46)		
		$\phi_{4,18}$ = −0.026 (−0.82)	$\phi_{4,38}$ = 0.009 (0.30)		
		$\phi_{4,19}$ = 0.036 (1.11)	$\phi_{4,39}$ = 0.099 (3.34)[a]		
		$_{4,20}$ = 0.060 (1.66)			
	$\Sigma\, \phi_{3,i}$ = 0.4174 (3.044)[a]	$\Sigma\, \phi_{4,i}$ = 0.9883 (95.2687)[a]			

The sample period is from December 3, 1984 to September 30, 1993. $R_t = 100 \cdot \ln(\mathrm{PRI}_{t,T}/\mathrm{PRI}_{t-1,T})$, where PRI = futures price and $V_t = \mathrm{VOL}_{t,T}/1000$, where VOL = futures volume. The t statistics are reported in parentheses next to the estimated coefficients. The coefficients show the impact of a specific lag of a right-hand variable on the left-hand-side variable. For example, $\phi_{2,1}$ represents the impact of volume (variable 2) on returns (variable 1) for a given lag of 1.

Q Stat. is the Q statistic for serial independence. This statistic is based on the revised Box–Ljung Q test for serial correlation among the regression residuals.

χ stat. is the chi-square statistic for testing for the joint significance of the lags on the right-hand-side variables.

For a given sum coefficient, the t statistic, t, is calculated as $t = s/\sigma_s$, where $s = \Sigma\, a_i$ and n = number of lags on the independent variable whose impact is being investigated. For example, if $n = 3$, then $s = \Sigma\, a_i = a_1 + a_2 + a_3$, and

$$\sigma_s = \sigma_{(a1+a2+a3)} = \sqrt{\sigma^2_{a_1} + \sigma^2_{a_2} + \sigma^2_{a_3} + 2\sigma_{a_1a_2} + 2\sigma_{a_2a_3} + 2\sigma_a 1 a_3,}$$

[a]Significant at the 1% level.
[b]Significant at the 5% level.
[c]Significant at the 10% level.

Also, neither the summed impact of volume ($\Sigma\phi_{2,i} = -0.0010$) on returns nor the summed impact of returns ($\Sigma\phi_{3,i} = 0.0324$) on volume is statistically significant, indicating the lack of a long-run linear causal relationship between heating-oil futures returns and volume.

Finally, the linear causality test results for unleaded gasoline in Table IV reveal a unidirectional causal relationship from returns to volume. The χ^2 statistic, which tests the exclusion of the returns time series from the volume equation, is significant at the 5% level, whereas the χ^2 statistic in the returns equation is insignificant. The unidirectional causal relationship from returns to volume is confirmed in the long run because the summed impact of returns ($\Sigma\phi_{3,i} = 0.4174$) on volume is statistically significant at the 1% level, whereas the summed impact of volume ($\Sigma\phi_{2,i} = -0.0002$) on returns is not.

Overall, the findings in Tables II–IV imply that petroleum futures returns series and volume series have no strong linear predictive power for one another. These results seem to contradict the findings of Clark (1973), Cornell (1981), Tauchen and Pitts (1983), Grammatikos and Saunders (1986), and Bessembinder and Seguin (1993), who all report a positive linear relationship between futures volume and price variability. This study argues, however, that the results reported in Tables II–IV, as well as those of earlier studies, may be spurious, because they rely on the assumption of linear relationships between returns and volume and a constant-variance error term.

Tests for Nonlinear Granger Causality

The estimates and tests in Tables II–IV are based on the model (1) and (2) that presumes that any causal relationship between futures returns and volume is *linear*. Therefore, this method is not able to detect certain types of nonlinear causal relationships.[6] This study conducts a preliminary nonlinearity test with the use of the method described in McLeod and Li (1983). This test is based on the Box–Pierce Q statistic for the squared data, and has been used in the literature to gauge the existence of conditional heteroscedasticity.[7] The residuals from the model (1) and (2) are analyzed, and it is found that the Q statistics, given in Table V, are highly significant, indicating that the daily returns and volume resid-

[6]See Brock (1991) for an illustration of how linear causality tests, such as the Granger test, may fail to uncover nonlinear predictive power.
[7]See, for example, Hsieh (1989a, 1989b). Hsieh (1989a) points out that the McLeod and Li Q statistic is related to Engle's (1982) test for heteroscedasticity, because the former uses the autocorrelation coefficients of the squared data, and the latter relies on the partial autocorrelation coefficients.

TABLE V

Summary Statistics for Futures Returns and Volume Residuals

	Crude Oil		Heating Oil		Unleaded Gas	
	$\zeta_{R,t}$	$\mu_{V,t}$	$\zeta_{R,t}$	$\mu_{V,t}$	$\zeta_{R,t}$	$\mu_{V,t}$
Skewness	−1.7755[a]	0.8022[a]	−1.6928[a]	0.9163[a]	−1.3148[a]	0.5704[a]
Kurtosis	30.4719[a]	2.8119[a]	29.5742[a]	4.1760[a]	22.2361[a]	4.0162[a]
Minimum	−34.9192	−80.3958	−32.3494	−28.6365	−27.6850	−23.0494
Maximum	12.5521	123.9531	13.3602	42.4934	10.5219	30.2370
$Q^2(20)$	136.1671	265.1627	126.6425	235.2200	177.9085	876.5951
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q^2(40)$	264.1217	389.6911	153.4466	432.7387	250.7508	1418.4152
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q^2(60)$	314.2478	522.5071	171.6863	592.5119	289.5217	2061.9377
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

The data are for the period, December 3, 1984 to September 30, 1993, $\zeta_{R,t}$ and $\mu_{V,t}$ represent returns and volume residuals obtained from eqs. (1) and (2), respectively. $Q^2(p)$ is the Box–Pierce Q statistic for conditional heteroscedasticity for the squared data for lag p. Marginal significance levels are given in brackets.
[a]Significant at the 1% level.

ual series exhibit conditional heteroscedasticity. Indeed, the results of formal nonlinear dependence tests reported in Tables VIII and IX of the Appendix reveal significant nonlinearities in the univariate futures returns and volume residual time series. Given this preponderance of evidence for nonlinearities in all the univariate time series examined, it seems reasonable to conduct tests for nonlinear causality between futures returns and trading volume.[8]

Baek and Brock (1992) suggest a nonparametric statistical technique for uncovering nonlinear causal relationships that cannot be discovered by standard linear causality tests. Consider the two time series of futures returns ($\{R_t\}$) and volume ($\{V_t\}$). Let the m-length lead vector of R_t be denoted by R_t^m, and the Lr- length and Lv-length lag vectors of R_t and V_t be denoted, respectively, by R_{t-Lr}^{Lr} and V_{t-Lv}^{Lv}. For given values of m, Lr, and $Lv \geq 1$, and for $d > 0$, V does not strictly Granger-cause R if

$$\begin{aligned}&\text{Prob}(\|R_t^m - R_s^m\| \langle d| \, \|R_{t-Lr}^{Lr} - R_{s-Lr}^{Lr}\| \langle d, \|V_{t-Lv}^{Lv} - V_{s-Lv}^{Lv}\| \langle d) \\ &\quad = \text{Prob}(\|R_t^m - R_s^m\| \langle d| \, \|R_{t-Lr}^{Lr} - R_{s-Lr}^{Lr}\| \langle d) \qquad (4)\end{aligned}$$

where Prob( ) designates probability and $\| \, \|$ denotes the maximum norm.

[8]Similar to Hsieh (1991), this study argues that nonstationarity is not likely to be an important issue given the use of daily data. To the extent that nonstationarity can be associated largely with structural changes, it is reasonable to assume that such changes occur infrequently. Thus, the use of high-frequency data should minimize the effects of such nonstationary behavior such as that which occurred during the market crash of 1985–1986 and the Persian Gulf crisis in 1990–1991.

The left-hand side of eq. (4) is the conditional probability that two arbitrary m-length lead vectors of $\{R_t\}$ are within a distance, d, of each other, given that the corresponding Lr-length lag vectors of $\{R_t\}$ and Lv-length lag vectors of $\{V_t\}$ are within a distance, d, of each other. The right-hand side of eq. (4) is the conditional probability that two arbitrary m-length lead vectors of $\{R_t\}$ are within a distance, d, of each other, assuming that their corresponding Lr-length lag vectors are within a distance, d, of each other.

The strict Granger-noncausality condition in eq. (4) is reexpressed as

$$\frac{CI_1(m + Lr, Lv, d)}{CI_2(Lr, Lv, d)} = \frac{CI_3(m + Lr, d)}{CI_4(Lr, d)} \tag{5}$$

where the $\mathrm{CI}_i(\quad)$'s are the correlation-integral estimators of the joint probabilities.[9] Assuming that $\{R_t\}$ and $\{V_t\}$ are strictly stationary, weakly dependent, and satisfy the mixing conditions of Denker and Keller (1983), if $\{V_t\}$ does not strictly Granger-cause $\{R_t\}$, then,

$$\sqrt{n}\left[\frac{CI_1(m + Lr, Lv, d, n)}{CI_2(Lr, Lv, d, n)} - \frac{CI_3(m + Lr, d, n)}{CI_4(Lr, d, n)}\right] \sim N(0, \sigma^2(m, Lr, Lv, d)) \tag{6}$$

Hiemstra and Jones (1994) show that a consistent estimator of the variance, $\sigma^2(m, Lr, Lv, d)$, in eq. (6) is $\hat{\sigma}^2(m, Lr, Lv, d, n) = \hat{\delta}(n)\,\hat{\Sigma}(n)\,\hat{\delta}(n)'$.

The two test statistics described in eqs. (5) and (6) are applied to the two estimated residual series from the VAR model in eqs. (1) and (2), $\{\hat{\zeta}_{R,t}\}$ and $\{\hat{\mu}_{V,t}\}$. The null hypothesis is that $\{V_t\}$ does not nonlinearly strictly Granger-cause $\{R_t\}$, and eq. (6) holds for all m, Lr, and $Lv \geq 1$ and for all $d > 0$. Although the evidence for linear causality is not overwhelming, this study removes any linear predictive power with a VAR model. Thus, any remaining incremental predictive power of one residual series for another can be considered nonlinear predictive power [see Baek and Brock (1992)].

Values for the lead length, m, the lag lengths, Lr and Lv, and the scale parameter, d, must be selected to implement the Baek and Brock test. However, in contrast to linear causality testing, no methods have been developed for choosing optimal values for lag lengths and the scale

[9]See Hiemstra and Jones (1994) for the derivation of the joint probabilities and their corresponding correlation-integral estimators.

parameter. Therefore, this study relies on the Monte Carlo results in Hiemstra and Jones (1993) by setting the lead length at $m = 1$ and $Lr = Lv$ for all cases. This study also uses common lag lengths of 1–6 lags and a common scale parameter of $e = 1.0\sigma$, where $\sigma = 1$ denotes the standard deviation of the standardized time series.

The empirical results of the nonlinear Granger-causality tests reported in Table VI show that the minimum value attained by the standardized test statistic (NORM) is 1.900 and is significant at the 5% level. This indicates strong evidence of nonlinear Granger-causality between futures returns and trading volume in both directions. These results are clearly at odds with those in Tables II–IV, where no evidence of linear causality is reported.

Tests for Nonlinear Granger-Causality with the Use of GARCH-Filtered Data

Although the nonlinear Granger causality is a nonparametric procedure, additional information exists to characterize the nonlinear structure between returns and volume. In particular, Hsieh's third-order moment test discussed in the Appendix, suggests that the underlying nonlinear relationship is driven by the variance of the stochastic processes.[10] Moreover, the Box–Pierce Q statistics of the squared data suggest that the behavior of the variance may be characterized by an ARCH process. Based on the preceding evidence, this study considers whether the nonlinear Granger causality between returns and trading volume can be attributed to volatility effects associated with the flow of information. Hiemstra and Jones (1994) point out that the test for nonlinear Granger causality may detect spurious causality between lagged volume and current returns variance if lagged volume captures the temporal dependence in the rate of information flow.

This study accounts for the excess kurtosis in the returns and volume data by estimating the generalized ARCH (GARCH) model of Bollerslev (1986). In particular, this study employs a GARCH(1,1) model that has been shown by various researchers [see, for example, McCurdy and Morgan (1987) and Baillie and Bollerslev (1989)] to provide a parsimonious description of the data. With the use of eq. (1), the GARCH(1,1) model for returns can be described as[11]

[10]However, the third-order moment test has low power against the GARCH-in-mean model.
[11]A similar specification is used for volume, with the conditional volatility depending only on the lagged squared residuals and lagged conditional variance.

TABLE VI

Results of Nonlinear Granger Causality Test Between Futures Returns and Volume

	H_0: *Futures Returns Do Not Cause Volume*						H_0: *Volume Does Not Cause Futures Returns*					
Lr = Lv	*1*	*2*	*3*	*4*	*5*	*6*	*1*	*2*	*3*	*4*	*5*	*6*
						Panel A: Crude Oil						
DIFF	0.0073	0.0020	0.0109	0.0401	0.0493	0.0790	0.0079	0.0289	0.0503	0.0701	0.0666	0.0801
NORM	2.402[b]	2.059[b]	2.955[a]	1.988[b]	2.097[b]	3.277[a]	2.211[b]	4.003[a]	4.034[a]	3.989[a]	3.112[a]	2.658[a]
						Panel B: Heating Oil						
DIFF	0.0040	0.0037	0.0176	0.0071	0.0294	0.0041	0.0020	0.0062	0.0175	0.0292	0.0266	0.0060
NORM	1.900[b]	2.204[b]	2.558[a]	3.033[a]	4.156[a]	3.966[a]	2.874[a]	2.731[a]	2.304[b]	2.880[a]	3.449[a]	2.202[b]
						Panel C: Unleaded Gas						
DIFF	0.0070	0.0122	0.0394	0.0442	0.0452	0.0777	0.0089	0.0203	0.0121	0.0279	0.0296	0.0465
NORM	4.042[a]	3.323[a]	2.395[b]	2.906[a]	1.990[b]	4.003[a]	3.566[a]	2.155[b]	2.363[b]	2.239[b]	3.128[a]	3.175[a]

The data are for the period, December 3, 1984 to September 30, 1993. The results are based on the residual series, $\zeta_{R,t}$ and $\mu_{V,t}$, from eq. (1) and (2). $Lr = Lv$ designates the number of lags on the residuals series, $\zeta_{R,t}$ and $\mu_{V,t}$. DIFF and NORM, respectively, denote the difference between the two conditional probabilities in eq. (5) and the standardized test statistic in eq. (6). Under the null hypothesis of nonlinear Granger noncausality, the test statistic is asymptotically distributed $N(0,1)$. The tests employ the unconditionally standardized series with the lead length, m, set to unity, and the length scale, d, set to 1.0.

[a]Significant at the 1% level.

[b]Significant at the 5% level.

$$R_t = \sum_{i=1}^{p_1} \phi_{1,i} R_{t-i} + \sum_{j=1}^{p_2} \phi_{2,j} V_{t-j} + \zeta_{R,t}$$

$$\zeta_{R,t} | \Omega_{t-1} \sim t.d.\ (0, h_t, v)$$

$$h_{R,t} = \alpha_0 + \alpha_1 \zeta_{R,t-1}^2 + \alpha_2 h_{R,t-1} + V_t$$

The error term, $\zeta_{R,t}$, is assumed to follow a conditional student t density $t.d.$, with v degrees of freedom, and a conditional variance, $h_{R,t}$. Ω_{t-1} is the set of all relevant and available information at time $t - 1$. The sum, $\alpha_1 + \alpha_2$, is a measure of the persistence of a shock to the variance. As noted by Engle and Bollerslev (1986), if this sum equals one, an integrated GARCH (IGARCH) process is said to be exhibited, implying that shocks to the conditional variance will persist over future horizons.

The conditional variance for the returns series is estimated with current volume as an explanatory variable, following Lamoureux and Lastrapes (1990), who demonstrated that contemporaneous volume is able to explain the persistence in volatility of common stock returns. This study finds that current volume is statistically significant, but, in contrast to Lamoureux and Lastrapes, is unable to account for the persistence in the conditional variance. The parameter estimates for the conditional variance specification indicate that both returns and volume exhibit a high degree of persistence, characteristic of the IGARCH process of Engle and Bollerslev (1986).

This study then tests for nonlinear Granger causality between returns and volume with the use of the estimated VAR residuals for conditionally standardized returns and volume series, namely, $\zeta_{R,t}/\sqrt{\hbar_{R,t}}$ and $\mu_{V,t}/\sqrt{\hbar_{V,t}}$.

The results of the nonlinear Granger-causality test applied to the GARCH-filtered returns and volume series are reported in Table VII. There is still strong evidence of bidirectional nonlinear Granger causality between returns and volume, even after the data have been adjusted for heteroscedasticity. The lone exception occurs in the case of crude oil, where the null hypothesis of strict nonlinear Granger causality from trading volume to returns is not rejected at lag 5. Overall, these findings are consistent with those reported in Table VI and point to the fact that trading volume and returns have significant nonlinear predictive power for one another over and above volatility effects. Thus, the initial conjecture of this study that the underlying relationship is directed by the volatility of the stochastic process appears to be rejected.

TABLE VII

Results of GARCH-Filtered Nonlinear Granger Causality Test Between Futures Returns and Volume

	H_0: *Futures Returns Do Not Cause Volume*						H_0: *Volume Does Not Cause Futures Returns*					
Lr = *Lv*	*1*	*2*	*3*	*4*	*5*	*6*	*1*	*2*	*3*	*4*	*5*	*6*
						Panel A: Crude Oil						
DIFF	0.0069	0.0155	0.0188	0.0241	0.0299	0.0248	0.0075	0.0122	0.0160	0.0311	0.0162	0.0123
NORM	4.011[a]	4.966[a]	3.977[a]	3.548[a]	1.955[b]	2.026[b]	3.228[a]	3.533[a]	2.365[b]	2.789[a]	1.485	3.096[a]
						Panel B: Heating Oil						
DIFF	0.0035	0.0050	0.0079	0.0021	0.00159	0.0021	0.0013	0.0037	0.0088	0.0076	0.0028	0.0045
NORM	1.902[b]	3.445[a]	4.333[a]	3.689[a]	3.699[a]	3.298[a]	2.877[a]	3.166[a]	2.864[a]	3.005[b]	2.383[b]	1.599[c]
						Panel C: Unleaded Gas						
DIFF	0.0020	0.0012	0.0042	0.0065	0.0010	0.0043	0.0031	0.0088	0.0059	0.0032	0.0036	0.0050
NORM	4.119[a]	1.909[b]	3.533[a]	2.240[b]	3.662[a]	3.555[a]	3.007[a]	2.893[a]	2.188[b]	3.422[a]	2.305[b]	3.698[a]

The data are for the period, December 3, 1984 to September 30, 1993. The results are based on the residual series, $\zeta_{R,t}$ and $\mu_{V,t}$, from eq. (1) and (2). $Lr = Lv$ designates the number of lags on the residuals series, $\zeta_{R,t}$ and $\mu_{V,t}$. DIFF and NORM, respectively, denote the difference between the two conditional probabilities in eq. (5) and the standardized test statistic in eq. (6). Under the null hypothesis of nonlinear Granger noncausaility, the test statistic is asymptotically distributed $N(0,1)$. The tests employ the unconditionally standardized series with the lead length, *m*, set to unity, and the length scale, *d*, set to 1.0.

[a]Significant at the 1% level.

[b]Significant at the 5% level.

[c]Significant at the 10% level.

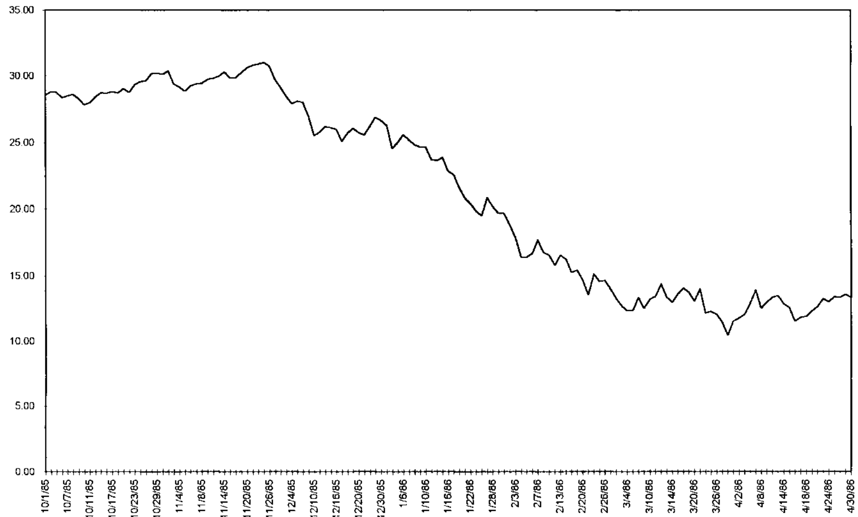

FIGURE 1
The behavior of crude oil closing prices during the OPEC crisis (10/1/85–4/30/86).

Further Investigation

The time period covered by this study includes two major events for petroleum markets. The first is the market crash of 1986. Prior to 1986, competition among non-OPEC and OPEC members escalated. Meanwhile, Saudi Arabia had been playing the swing leader role by adjusting its oil output to changes in the level of the demand for OPEC oil. It sought to absorb overproduction from other OPEC member countries in an attempt to stabilize total OPEC production levels and maintain the high official price. However, because of declining market share and falling oil revenues, Saudi Arabia increased its daily production substantially, resulting in an oil glut in the world. The predictable outcome of this action was a sharp decline of spot crude-oil prices from $31.01 per barrel on November 25, 1985 to less than $10.50 per barrel on March 31, 1986 (see Figure 1). The second important event is the 1990–1991 Gulf War. On August 2, 1990, Iraq invaded Kuwait and a war broke out on January 16, 1991 between Iraq and Kuwait and its allies. The war ended on February 27, 1991 with the liberation of Kuwait. If Iraq had won the war, it would have controlled a significant proportion of the worldwide oil reserves. As shown in Figure 2, crude-oil prices fluctuated dramatically during the war, reaching a high of $39.54 per barrel on September 27, 1990 and a low of $17.91 per barrel on February 22, 1991. Between January 16, 1991 and January 17, 1991, crude-oil prices tumbled almost 32%, from $30.29 to $20.63 per barrel.

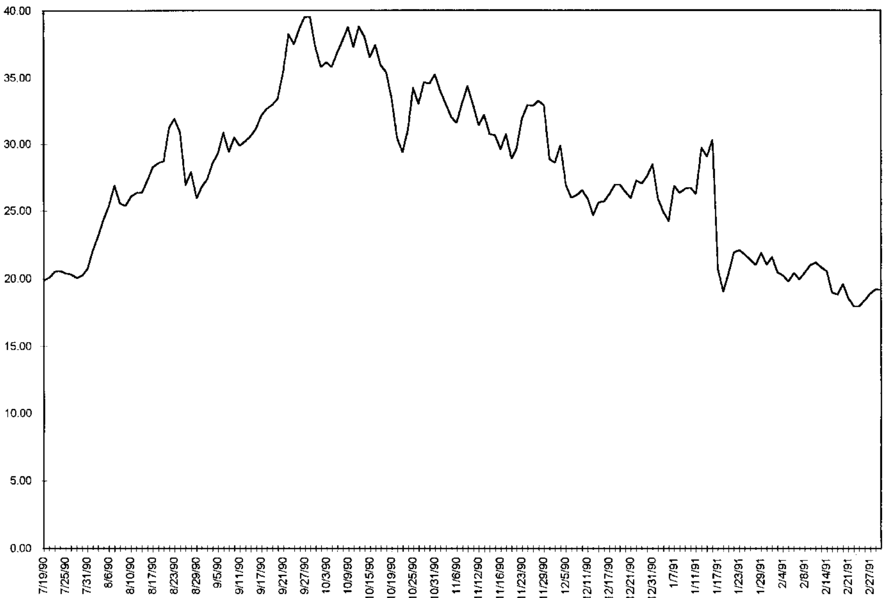

FIGURE 2
The behavior of crude oil closing prices during the Persian Gulf War (7/19/90–2/27/91).

To examine whether the empirical results of this study are contaminated by these two significant events, all the empirical tests are repeated based on three subsamples: December 3, 1984 to March 31, 1986; April 1, 1986 to January 16 1991; and January 17, 1991 to September 30, 1993. The findings for the subperiods (available upon request from the authors) are qualitatively similar to those reported for the full 1984–1993 sample period. That is, little evidence of significant *linear* causality between futures returns and volume is found. By contrast, significant *nonlinear* causal relationships are uncovered with the use of a nonparametric test. Finally, the results of this study do not change significantly when the periods immediately surrounding the market crash of 1986 and the Persian Gulf War of 1990–1991 are omitted from the empirical analysis.

CONCLUSION

This article examines the relationship between returns and trading volume for three petroleum futures contracts. Using daily data on futures prices and trading volume from the NYMEX for the December 3, 1984 to September 30, 1993 period, the study first tests for linear causality between returns and volume. The results of this linear causality test show that futures returns and volume have no predictive power for one another.

However, because the distribution of the returns and volume series provides some evidence of nonlinear dependence, the study formally tests for and finds evidence of significant nonlinearities in the returns and volume for the three petroleum futures contracts. The returns and volume series are then filtered for linear dependence through the use of a VAR process. A nonparametric test statistic based on the correlation integral reveals significant bidirectional nonlinear causal relationships between the filtered returns and volume series.

Using a third-order moment test, this study finds that the nonlinear dependence in the futures returns and volume series arises from the variance, rather than the mean, of the process. Consequently, the filtered returns and volume series are adjusted for conditional heteroscedasticity. The study then examines the GARCH-filtered returns and volume series and finds that, even after adjusting for volatility effects, there is still strong evidence of bidirectional nonlinear Granger causality and concludes that the nonlinear process may influence both the mean and variance of futures returns and volume.

Overall, the results of this study are very similar to those of Hiemstra and Jones (1994) for Dow Jones stock returns and percentage changes in New York Stock Exchange trading volume. They find no evidence of linear causality but report strong significant bidirectional nonlinear causality between the two variables over the 1915–1946 and 1947–1990 periods based on a modified version of the Baek and Brock test. The results of this study are also similar to those of Gallant, Rossi, and Tauchen (1993) with respect to the nonlinear causality flow from returns to trading volume. With the use of nonlinear impulse response functions to examine the joint dynamics between daily Standard and Poor's 500 Index stock returns and NYSE trading volume over the 1928 to 1987 period, they detect evidence of strong nonlinear causality from lagged stock returns to current and future trading volume. However, in contrast to the results of this study and those of Hiemstra and Jones (1994), only weak evidence of a nonlinear impact from lagged volume to current and future returns is uncovered with the use of their technique. Gallant, Rossi, and Tauchen (1993) contend that their results suggest that stock returns are nearly Granger causally prior to trading volume. However, such an interpretation is not supported by the findings of strong bidirectional nonlinear Granger causality between futures returns and trading volume reported in this study.

It is worth noting that the results of nonlinear causality tests reported in this study are consistent with the predictions of more than one of the competing explanations for the presence of a causal relationship between asset price variability and trading volume previously discussed. For example, causality from futures trading volume to price variability is consistent with the sequential information arrival models of Copeland (1976) and Jennings, Starks, and Fellingham (1981) and the mixture of distributions model of Clark (1973) and Epps and Epps (1976). Also, a significant causal relationship from futures price variability to trading volume is implied by the noise trading models of DeLong, Shleifer, Summers, and Waldmann (1990).

The finding of a significant *nonlinear* causal relationship between price variability and trading volume can be of interest to market regulators as they decide on the effectiveness or the appropriateness of market restrictions such as daily price movement limits and position limits. For example, increased volume in futures may lead to increased price fluctuations, which in turn may prompt more regulation of futures markets. However, the appropriateness of such regulation may hinge on the cause of price variability. Greater regulatory restrictions may be warranted if increased price fluctuations are caused by increased trading volume. On the other hand, further regulation may be detrimental to the price responsiveness in futures markets if increased price variability and volume are attributed to liquid and efficient markets.

The empirical findings of this study also have practical implications for traders and other futures markets participants. For example, it is now well known that successful hedging and speculative activities in futures markets depend crucially on the ability to forecast futures price movements. The finding of strong nonlinear causal relationships between petroleum futures price variability and trading volume reported in this study implies that knowledge of current trading volume improves the ability to forecast futures prices. This improvement of short-term price predictability should lead to the construction of more accurate hedge ratios and improvements in investment strategies.

Finally, the empirical findings of this study may be analyzed in terms of their implications for the efficiency of petroleum futures markets. The fact that lagged volume contains information useful for the prediction of current price variability may imply a degree of inefficiency in petroleum futures prices. Such inefficiency may be caused by a form of mimetic contagion, where traders set their prices with reference to the trading patterns of other traders. Another possible explanation for this apparent inefficiency may be that futures traders condition their prices on previous day's trading volume as a measure of the market consensus. Further research should be directed toward detecting the possible sources or causes of this inefficiency.

APPENDIX

This Appendix conducts nonlinear dependence tests beginning with Brock, Dechert, and Scheinkman (1987), who propose a test (BDS) for deviations from iid behavior. For a sequence of observations $\{x : t = 1, \ldots, T\}$ that are iid, an m-dimensional vector, $X_t^m = (x_t, x_{t+1}, \ldots, x_{t+m-1})$, can be formed. The test computes a statistic based on the correlation integral defined by[12]

$$C_m(\varepsilon, T) = \frac{2}{n(n-1)} \sum_{t<s} I_\varepsilon(X_t^m, X_s^m) \tag{A1}$$

where $n = T - m + 1$ and $I_\varepsilon(X_t^m, X_s^m)$ is an indicator function defined as

$$\begin{aligned} I_\varepsilon(X_t^m, X_s^m) &= 1, \qquad \text{if } \|X_t^m - X_s^m\| < \varepsilon \\ &= 0, \qquad \text{otherwise} \end{aligned} \tag{A2}$$

and $\|\ \|$ denotes the maximum norm.

The test statistic is given by

$$BDS_m(\varepsilon, T) = \frac{\sqrt{T}[C_m(\varepsilon, T) - C_1(\varepsilon, T)^m]}{\sigma_m(\varepsilon, T)} \tag{A3}$$

Under the null hypothesis that $\{x_t\}$ is iid, the term, $\sqrt{T}[C_m(\varepsilon, T) - C_1(\varepsilon, T)^m]$, has a normal limiting distribution with mean zero and standard deviation, $\sigma_m(\varepsilon, T)$.[13] The null hypothesis of a random iid process is rejected if the probability of any two m histories being close together exceeds the mth power of the probability of any two points being close together.

In general, a rejection of the null hypothesis is consistent with some type of dependence in the returns and volume series that could be due to a linear stochastic process, nonstationarity, a nonlinear stochastic process, or a nonlinear deterministic system. However, following Hsieh (1991), linear dependence can be ruled out by filtering the returns and volume series with the use of the VAR process estimated in (1) and (2) in the text. In addition, nonstationarity is not likely to be an important issue given the use of daily data. To the extent that the nonstationarity

[12]The correlation integral measures the fraction of the pairs of points of $\{x_t\}$ that are within a distance of ε from each other. The value for ε is chosen relative to the standard deviation divided by the spread of the raw data.

[13]Brock, Hsieh, and LeBaron (1991) note that $C_m(\varepsilon, T) = C_1(\varepsilon, T)^m$ does not imply an iid series.

can be associated largely with structural changes, this study posits that it is reasonable to assume that such changes occur infrequently. Thus, the use of high-frequency data should minimize the effects of such nonstationary behavior. As a result, because the test analyzes the residual series from an autoregressive process of the raw data, the rejection of the null hypothesis can be associated with evidence of nonlinearity.

Table VIII provides the results of the BDS test applied to the residual series from the linear filter.[14] The values of the test statistics are all positive and significantly greater than zero, leading to rejection of the null of iid behavior for the residual series. Although the BDS test reveals the presence of nonlinearities, it does not indicate whether the stochastic process affects the mean or variance (possibly both) of the series. Following Hsieh (1989a), two types of nonlinearities in a series, v_t, which represents the residual from the filtered raw data series, x_t (R_t or V_t), may be expressed as follows:

1. *Additive dependence*

$$v_t = f[v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}] + w_t \tag{A4}$$

2. *Multiplicative dependence*

$$v_t = g[v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}]w_t \tag{A5}$$

where w_t is an iid random variable with mean zero and independent of past v_t's and x_t's.

The functions $f(\ )$ and $g(\ )$ are some nonlinear functions of the v_t's and x_t's or finite p. Given the above formulations, additive dependence indicates that the nonlinearity enters through the mean of the stochastic process. Multiplicative dependence implies that the presence of the nonlinearity is transmitted through the variance, as suggested by the autoregressive conditional heteroscedasticity (ARCH) process of Engle (1982).[15] Both additive and multiplicative dependence can influence the random variable, v_t, as in the case of the ARCH-in-mean (ARCH-M) model given by:

$$v_t = f[v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}] + g[v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}]w_t. \tag{A6}$$

[14]The tests are computed using an algorithm developed by Dechert and contained in Brock et al. (1991).

[15]The dependence in commodity price changes due to serially correlated variances is suggested by Hall, Brorsen, and Irwin (1989).

TABLE VIII

BDS Test Statistics for Futures Returns and Volume Residuals

		Crude Oil		Heating Oil		Unleaded Gas	
m	**ε/σ**	$\zeta_{R,t}$	$\mu_{V,t}$	$\zeta_{R,t}$	$\mu_{V,t}$	$\zeta_{R,t}$	$\mu_{V,t}$
2	0.5	11.616	14.073	9.484	9.246	11.264	20.238
3	0.5	15.493	23.147	12.337	12.460	15.117	35.480
4	0.5	19.855	37.122	15.048	16.433	19.098	59.858
5	0.5	24.229	61.119	17.703	22.122	22.583	103.460
6	0.5	29.567	105.770	20.779	29.034	27.160	191.240
2	1.0	13.364	10.335	11.602	8.312	12.625	12.986
3	1.0	17.044	14.281	14.386	10.810	15.911	19.973
4	1.0	20.398	18.247	16.904	12.865	18.728	25.703
5	1.0	23.416	22.777	19.227	15.236	20.875	32.038
6	1.0	26.770	28.279	21.537	17.301	23.295	40.208
2	1.5	13.912	8.411	13.249	7.209	13.672	9.700
3	1.5	17.107	10.738	15.947	9.430	16.431	14.612
4	1.5	19.685	12.573	18.000	10.706	18.410	17.494
5	1.5	21.518	14.161	19.533	12.047	19.574	20.088
6	1.5	23.329	15.759	20.954	13.023	20.934	22.554
2	2.0	12.682	7.457	13.774	5.540	14.561	7.619
3	2.0	15.035	8.947	16.439	7.700	16.644	11.693
4	2.0	17.309	10.222	18.248	8.817	17.752	13.527
5	2.0	18.614	11.091	19.284	9.867	18.255	14.926
6	2.0	19.848	11.953	20.173	10.413	18.948	16.047

$\zeta_{R,t}$ and $\mu_{V,t}$ represent returns and volume residuals obtained from eqs. (1) and (2), respectively. The data are for the period, December 3, 1984 to September 30, 1993. The numbers in the table are the BDS test statistics, and are calculated as

$$BDS_m(\varepsilon, T) = \frac{\sqrt{T}[C_m(\varepsilon, T) - C_1(\varepsilon, T)^m]}{\sigma_m(\varepsilon, T)}$$

The BDS statistic has a standard normal limiting distribution. The null hypothesis of a random iid process is rejected if the probability of any two *m* histories being close together exceeds the *m*th power of the probability of any two points being close together, where *m* is the vector dimension.
The 10, 5, and 1% critical levels are 1.645, 1.960, and 2.575, respectively.

Hsieh (1989a, 1991) points out that $x(t)^2$ will be correlated with its own lags in both the additive and multiplicative cases, which is what the McLeod and Li (1983) test detects. The distinguishing feature, however, is that under additive dependence:

$$E[v_t | v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}] \neq 0 \qquad \text{(A7)}$$

whereas multiplicative dependence implies

$$E[v_t | v_{t-1}, \ldots, v_{t-p}, x_{t-1}, \ldots, x_{t-p}] = 0 \qquad \text{(A8)}$$

Hsieh provides a third-order moment test to detect the presence of ad-

TABLE IX

Third-Order Moment Test Statistics for Futures Returns and Volume Residuals

Lag		*Crude Oil*		Heating Oil		Unleaded Gas	
i	j	$\zeta_{R,t}$	$\mu_{V,t}$	$\zeta_{R,t}$	$\mu_{V,t}$	$\zeta_{R,t}$	$\mu_{V,t}$
1	1	−0.0193	−0.0009	−0.0269	0.0000	−0.0170	0.0055
2	1	0.0344	−0.0437	0.0022	−0.0038	0.0225	−0.0270
2	2	0.0185	−0.0265	0.0200	−0.0469	0.0206	−0.0028
3	1	−0.0264	−0.0103	−0.0212	0.0211	−0.0295	0.0019
3	2	0.0138	0.0126	0.0069	0.0084	0.0167	−0.0292
3	3	0.0019	−0.0001	−0.0102	−0.0397	0.0100	−0.0351
4	1	0.0220	0.0126	0.0514	0.0163	0.0408	0.0374
4	2	0.0017	0.0104	0.0009	−0.0138	−0.0002	0.0144
4	3	0.0346	−0.0149	0.0250	0.0034	0.0001	−0.0042
4	4	−0.0024	−0.0165	−0.0107	−0.0243	−0.0163	−0.0362
5	1	−0.0139	0.0401	−0.0063	−0.0043	−0.0042	−0.0024
5	2	−0.0209	0.0025	−0.0265	0.0214	−0.0120	−0.0229
5	3	0.0172	−0.0082	0.0435	−0.0072	0.0266	0.0107
5	4	0.0181	0.0096	−0.0255	−0.0212	0.0074	−0.0219
5	5	−0.0191	−0.0298	0.0023	−0.0587	0.0144	−0.0049

$\zeta_{R,t}$ and $\mu_{V,t}$ represent returns and volume residuals obtained from eqs. (1) and (2), respectively. The data are for the period, December 3, 1984 to September 30, 1993.
The 10, 5, and 1% critical levels are 1.645, 1.960, and 2.575, respectively.

ditive dependence, given the null hypothesis of multiplicative dependence. The test statistic exploits the fact that, under multiplicative dependence, v_t is not correlated with terms such as $v_{t-i}v_{t-j}$, whereas additive dependence implies that the variable, v_t, is correlated with terms like $v_{t-i}v_{t-j}$. Thus, the null hypothesis of multiplicative dependence makes use of the fact that $E[v_t v_{t-i} v_{t-j}] = 0$, whereas the alternative hypothesis is $E[v_t v_{t-i} v_{t-j}] \neq 0$, implying additive dependence. The test is designed to reject only in the presence of additive nonlinearity, not multiplicative nonlinearity. Thus, the third-moment procedure is able to detect a specification such as the ARCH-M.

To uncover the source of the nonlinear behavior, this study reports the results of Hsieh's (1989a, 1991) third-order moment test statistics in Table IX for $i,j = 1,2,3,4,5$.[16] None of these third-order moment test statistics is significantly different from zero, indicating a failure to reject the null hypothesis of multiplicative dependence. These results suggest that nonlinearity arises solely from the variance of the process.

[16]Results for $i,j = 6, 7, 8, 9, 10$, not reported here but available on request, also fail to reject the null hypothesis.

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