Hindawi Publishing Corporation
Journal of Control Science and Engineering
Volume 2016, Article ID 9735264, 8 pages
http://dx.doi.org/10.1155/2016/9735264

Research Article

An Approach of Tracking Control for Chaotic Systems

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Received 15 October 2015; Accepted 31 March 2016

Academic Editor: Jie Chen

Combining the ergodicity of chaos and the Jacobian matrix, we design a general tracking controller for continuous and discrete chaotic systems. The control scheme has the ability to track a bounded reference signal. We prove its globally asymptotic stability and extend it to generalized projective synchronization. Numerical simulations verify the effectiveness of the proposed scheme.

1. Introduction

Since Ott et al. firstly proposed the method of chaos control in 1990 [1], chaos control has attracted great attention in recent years and lots of successful methods have been reported [2–22], such as feedback control [7, 8], impulsive control [9, 10], backstepping method [11], adaptive control [12–15], adaptive fuzzy backstepping technique [16], adaptive sliding mode control [17, 18], neural network technique [19], and H_∞ synchronization [20–22].

However, due to the complexity of chaotic systems, most approaches are designed for a special chaotic system, or they only realize the control of unstable fixed point and synchronization for identical systems [7–10, 12–15]. The literatures on the tracking control of reference signal [23–32] are relatively less.

In 1998, Lin et al. applied the idea of Chen's method [23] to discrete chaotic systems [24], but the method needed calculating an error feedback matrix to assure that a certain matrix is negative semidefinite. When the dimension of these matrixes is large, the calculation becomes rather complex. Chen established the open-plus-closed-loop control law for discrete dynamical systems [25], but it required that the initial points were in the basin of entrainment. References [26–28] realized tracking control only for Henon chaotic system. References [29, 30] presented tracking control schemes for continuous chaotic systems and did not refer to the application to discrete systems. Zheng et al. gave a rapid synchronization algorithm [31], but it involved the calculation of high order derivative matrix. Rehan et al. discussed stabilization and tracking control using linear matrix inequalities for a class of continuous systems satisfying global Lipschitz condition [32].

In practical engineering, we need to eliminate the chaos or transform them into some useful signals. Therefore, tracking control, which transforms the chaos signal into desired bounded signal, is significant in practice. Moreover, generalized projective synchronization (GPS) can transform into the problem of tracking control.

Based on the above discussions, we present a general tracking control scheme based on the Jacobian matrix and ergodicity of chaos. It is simple and does not refer to high order derivative matrix or other requirements such as global Lipschitz condition. The rest of the paper is organized as follows. In Section 2, the tracking control schemes for continuous and discrete systems and their mathematical proofs are given, respectively. In Section 3, we have applied this method to three chaotic systems and make simulations. Finally, the conclusions are drawn in Section 4.

2. Theoretical Analysis

2.1. Tracking Control for Continuous Chaotic Systems. Consider the following n-dimensional continuous chaotic system:

$$\dot{x} = f(x, t) + u, \tag{1}$$

where the state variable vector $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, $f(x) = \{f_1(x), f_2(x), \ldots, f_n(x)\}$ is n-dimensional continuously differentiable nonlinear vector function, and u is the controller. It is chaotic with $u = 0$.

The control objective is to design a novel controller such that the vector x tracks the reference signal $r = (r_1, r_2, \ldots, r_n)^T \in R^n$ which satisfies

$$\dot{r} = g(r, t), \tag{2}$$

where $g(\cdot) = \{g_1(\cdot), g_2(\cdot), \ldots, g_n(\cdot)\}$ is n-dimensional continuously differentiable nonlinear vector function.

Set the range of chaotic system as F and the range of reference signal as G. As the range of chaotic system F is certain, we choose the reference signal r satisfying $F \cap G \neq \Phi$. It is easy to implement in practice.

Theorem 1. *For systems (1) and (2), if we set $e = x - r$ and add the controller*

$$u(x, r, t) = \lambda \left[\boldsymbol{\varepsilon} e + g(x, t) - f(r, t) - \left(\mathbf{Df}|_r + \mathbf{Dg}|_r \right) e \right], \tag{3}$$

where $\mathbf{Df}|_r$, $\mathbf{Dg}|_r$ are the Jacobian matrix of $f(x,t)$ and $g(x,t)$ at r, respectively, $\boldsymbol{\varepsilon} = \mathrm{diag}(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ is a constant feedback diagonal matrix, and λ is the switch-off controller, which is depicted as

$$\lambda = \begin{cases} 1 & \text{if } \|e\| < c \\ 0 & \text{else,} \end{cases} \tag{4}$$

where $\|\cdot\|$ denotes the Euclidean norm and c is a constant, then for the initial values $x(0)$ which make (1) chaotic,

$$\lim_{t \to +\infty} \|e(t)\| = 0. \tag{5}$$

That is, system (1) tracks system (2) asymptotically.

Proof. The Taylor series of $f(x,t)$ at r is

$$\begin{aligned} f(x,t) &= f(r+e,t) = f(r,t) + \mathbf{Df}|_r e \\ &+ \sum_{q=2}^{n} \frac{1}{q!} \left[e_1 \frac{\partial}{\partial x_1} + e_2 \frac{\partial}{\partial x_2} + \cdots + e_n \frac{\partial}{\partial x_n} \right]^q f(r,t) \\ &+ \frac{1}{(n+1)!} \left[e_1 \frac{\partial}{\partial x_1} + e_2 \frac{\partial}{\partial x_2} + \cdots + e_n \frac{\partial}{\partial x_n} \right]^{n+1} \\ &\cdot f(r + \theta e, t) \overset{\text{def}}{=} f(r,t) + \mathbf{Df}|_r e + p(e,t), \\ &\qquad (0 < \theta < 1), \end{aligned} \tag{6}$$

where $p(e,t)$ is a polynomial vector which contains quadratic term and finite higher order terms of e and $p(0,t) = 0$.

Similarly,

$$g(x,t) = g(r,t) + \mathbf{Dg}|_r e + q(e,t), \tag{7}$$

where $q(e,t)$ is a polynomial vector like $p(e,t)$ and $q(0,t) = 0$.

When $\|e\| < c$, according to (1)–(4), we have

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{r} \\ &= f(x,t) + \boldsymbol{\varepsilon} e + g(x,t) - f(r,t) - \left(\mathbf{Df}|_r + \mathbf{Dg}|_r \right) e \\ &\quad - g(r,t) = p(e,t) + q(e,t) + \boldsymbol{\varepsilon} e. \end{aligned} \tag{8}$$

We introduce the following nonnegative Lyapunov function:

$$V(e,t) = 0.5 e^T e. \tag{9}$$

Then

$$\dot{V}(e,t) = e^T \dot{e} = e^T \left(p(e,t) + q(e,t) \right) + e^T \boldsymbol{\varepsilon} e. \tag{10}$$

Since $\|e\| < c$ and $p(e,t)$, $q(e,t)$ are polynomial vector composed of quadratic term and finite higher order terms of e, there always exists $\varepsilon_i < 0$ $(i = 1, 2, \ldots, n)$ satisfying $\dot{V}(e,t) < 0$.

When $\|e\| \geq c$, the controller does not work. As there exist x and r satisfying $F \cap G \neq \Phi$, we let $K = F \cap G$. According to the ergodicity of chaos, there always exists a certain time t_0 satisfying $e = g(x, t_0) - f(r, t_0) = 0$ in the set K, and system (1) always comes into the domain of $\|e\| < c$ in a limited time; thus, it asymptotically tracks the reference signal with controller (3). The proof is completed. □

Remark 2. The value of c is related to ε_i $(i = 1, 2, \ldots, n)$ according to (10). The larger the ε_i is, the quicker the convergence is. However, it is difficult to calculate specific value of c. In practice, the value of c can be increased from zero for an appropriate convergence speed.

2.2. Tracking Control for Discrete Chaotic Systems. Consider n-dimensional discrete chaotic system or data-sampling system:

$$x_{k+1} = f(k, x_k) + u, \tag{11}$$

where the state variable $x_k = (x_k(1), x_k(2), \ldots, x_k(n))^T \in R^n$, $f(\cdot) = \{f_1, f_2, \ldots, f_n\}$ is n-dimensional continuously differentiable nonlinear vector function, and u is the controller. It is chaotic with $u = 0$.

The drive system or the reference signal is denoted as

$$r_{k+1} = g(k, r_k), \tag{12}$$

where $g(\cdot) = \{g_1(\cdot), g_2(\cdot), \ldots, g_n(\cdot)\}$ is n-dimensional continuously differentiable nonlinear vector function. Like continuous systems, we assume there exist x_k and r_k satisfying $F \cap G \neq \Phi$.

Lemma 3 (see [33]). *Assume that there exists a function V such that*

(i) $V : N_{n_0}^{+} \times B_H \to R^{+} = [0, +\infty)$, $B_H = \{x \mid \|x\| \leq H, x \in R^s\}$; $V(n, 0) = 0$; *V is positive definite and continuous with respect to the second argument;*

(ii) *the variation of the function* V *along the system* $y_{n+1} = h(n, y_n)$,

$$\Delta V(n, y_n) = V(n+1, y_{n+1}) - V(n, y_n), \tag{13}$$

is negative definite. That is, there exists function Ψ *satisfying* $\Psi(x) > 0$ $(x > 0)$, $\Psi(0) = 0$, $(\Psi(x_1) - \Psi(x_2))(x_1 - x_2) > 0$, *such that* $\Delta V(n, y_n) \leq -\Psi(\|y_n\|)$, *where* $h : N_{n_0}^{+} \times R^s \to R^s$, $h(n, 0) = 0$.

Then the origin of the system $y_{n+1} = h(n, y_n)$ is asymptotically stable.

Similar to continuous system, we have the following.

Theorem 4. *For systems (11) and (12), if we set* $e_k = x_k - r_k$ $(k = 1, 2, \ldots, n)$ *and add the controller*

$$\begin{aligned} u(k, x_k, e_k) = \mu\big[&\delta_k e_k + g(k, x_k) - f(k, r_k) \\ &- \left(\mathbf{Df}|_{r_k} + \mathbf{Dg}|_{r_k}\right) e_k\big], \end{aligned} \tag{14}$$

where $\mathbf{Df}|_{r_k}, \mathbf{Dg}|_{r_k}$ *are the Jacobian matrix of* $f(k, x_k)$ *and* $g(k, x_k)$ *at* r_k*, respectively,* $\boldsymbol{\delta} = \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_n)$ *is a constant feedback diagonal matrix, and the switch-off controller* μ *is depicted as*

$$\mu = \begin{cases} 1 & \text{if } \|e_k\| < d \\ 0 & \text{else,} \end{cases} \tag{15}$$

where d *is a constant less than 1, then for the initial values* x_1 *which make (11) chaotic we have*

$$\lim_{k \to +\infty} \|e_k\| = 0. \tag{16}$$

That is, system (11) tracks system (12) asymptotically.

Proof. The Taylor series of $f(k, x_k)$ at r_k is

$$\begin{aligned} f(k, x_k) &= f(k, r_k + e_k) = f(k, r_k) + \mathbf{Df}|_{r_k} e_k \\ &+ \sum_{q=2}^{n} \frac{1}{q!}\left[e_k(1)\frac{\partial}{\partial x_k(1)} + e_k(2)\frac{\partial}{\partial x_k(2)} + \cdots \right. \\ &\left. + e_k(n)\frac{\partial}{\partial x_k(n)}\right]^q f(k, r_k) \\ &+ \frac{1}{(n+1)!}\left[e_k(1)\frac{\partial}{\partial x_k(1)} + e_k(2)\frac{\partial}{\partial x_k(2)} + \cdots \right. \\ &\left. + e_k(n)\frac{\partial}{\partial x_k(n)}\right]^{n+1} f(k, r_k + \theta e_k) \overset{\text{def}}{=} f(k, r_k) \\ &+ \mathbf{Df}|_{r_k} e_k + w(k, e_k), \quad (0 < \theta < 1), \end{aligned} \tag{17}$$

where $w(k, e_k)$ is a polynomial vector which contains quadratic term and finite higher order terms of e_k and $w(k, 0) = 0$.

Similarly,

$$g(k, x_k) = g(k, r_k) + \mathbf{Dg}|_{r_k} e_k + v(k, e_k), \tag{18}$$

where $v(k, e_k)$ is a polynomial vector like $w(k, e_k)$ and $v(k, 0) = 0$.

If $\|e_k\| < d$, according to (11)–(14), we have

$$\begin{aligned} e_{k+1} &= x_{k+1} - r_{k+1} \\ &= f(k, x_k) + \boldsymbol{\delta} e_k + g(k, x_k) - f(k, r_k) \\ &\quad - \left(\mathbf{Df}|_{r_k} + \mathbf{Dg}|_{r_k}\right) e_k - g(k, r_k) \\ &= w(k, e_k) + v(k, e_k) + \boldsymbol{\delta} e_k. \end{aligned} \tag{19}$$

We introduce the following nonnegative Lyapunov function:

$$V(k, e_k) = e_k^T e_k. \tag{20}$$

Then

$$\begin{aligned} \Delta V(k, e_k) &= e_{k+1}^T e_{k+1} - e_k^T e_k = \big(w^T(k, e_k) w(k, e_k) \\ &+ v^T(k, e_k) v(k, e_k) + e_k^T \boldsymbol{\delta}^T \boldsymbol{\delta} e_k \\ &+ 2w^T(k, e_k) v(k, e_k) + 2w^T(k, e_k) \boldsymbol{\delta} e_k \\ &+ 2v^T(k, e_k) \boldsymbol{\delta} e_k\big) - e_k^T e_k. \end{aligned} \tag{21}$$

Because $\|e_k\| < d \leq 1$, $\|w(k, e_k)\| = O(\|e_k\|^2)$ and $\|v(k, e_k)\| = O(\|e_k\|^2)$. Assume $\boldsymbol{\delta} = 0$; there always exists $M > 0$ satisfying

$$\begin{aligned} &w^T(k, e_k) w(k, e_k) + v^T(k, e_k) v(k, e_k) \\ &\quad + 2w^T(k, e_k) v(k, e_k) < M\|e_k\|^2. \end{aligned} \tag{22}$$

Therefore,

$$\begin{aligned} \Delta V(k, e_k) &< M\|e_k\|^2 - 0.99 e_k^T e_k - 0.01 e_k^T e_k \\ &< -0.01 e_k, \end{aligned} \tag{23}$$

under the condition that

$$M\|e_k\|^2 - 0.99 \leq 0. \tag{24}$$

It means $d \leq \sqrt{0.99/M} \approx \sqrt{1/M}$.

If $\|e_k\| \geq d$, the controller does not work. As there exist x_k and r_k satisfying $F \cap G \neq \Phi$; according to the ergodicity of chaos, system (11) always comes into the range of $\|e_k\| < d$ in a limited time; thus, it asymptotically tracks (12). The proof is completed. □

Remark 5. From (21), we know $\boldsymbol{\delta} = 0$ is the simplest situation for discrete systems. It can be extended to $|\delta_k| < a$ $(k = 1, 2, \ldots, n)$ for specific system, where a is a positive constant, which will be illustrated in the following simulations.

Remark 6. The value of d is related to δ_k $(k = 1, 2, \ldots, n)$ according to (21)–(24).

Remark 7. Compared with the controllers in [23–32], the proposed scheme is simpler and more general. It is suitable for continuous and discrete chaotic systems.

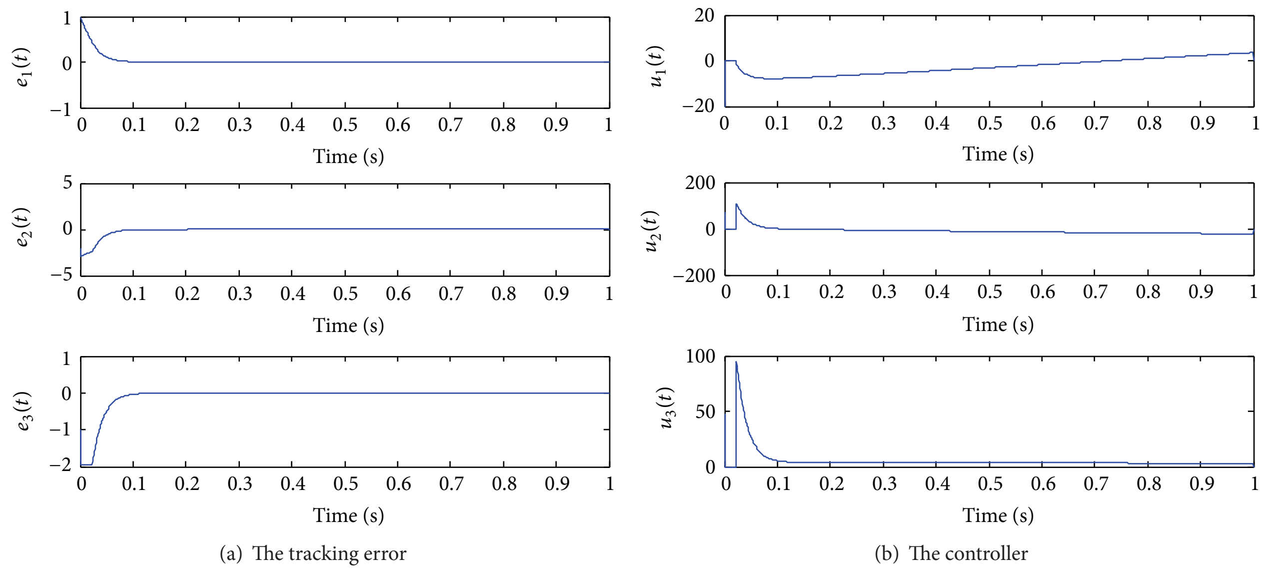


Figure 1: The Lorenz system tracks $(\sin t, \cos t, \sin t + \cos t)$ starting from $(1, -2, -1)$ with $c = 10$ and $\varepsilon_i = -50$ $(i = 1, 2, 3)$.

Remark 8. The proposed scheme has the ability to track a bounded signal satisfying $F \cap G \neq \Phi$.

Remark 9. The method can be extended to GPS.

For response system (1) (or (11) for discrete system) and a drive system (2) (or (12)), if $\lim_{t\to+\infty}\|\rho x - r\| \to 0$ (or $\lim_{k\to+\infty}\|\rho x_k - r_k\| \to 0$), where ρ is the reversible scaling factor diagonal matrix, it is said that systems (1) and (2) (or (11) and (12)) realize GPS. Set $r' = \rho^{-1} r$/(or $r'_k = \rho^{-1} r_k$); therefore, GPS becomes complete synchronization between x and r' (or x_k and r'_k).

3. Examples

Example 1 (Lorenz system). The Lorenz system is depicted as

$$\begin{aligned} \dot{x}_1 &= \alpha\left(-x_1 + x_2\right) + u_1, \\ \dot{x}_2 &= \beta x_1 - x_2 - x_1 x_3 + u_2, \\ \dot{x}_3 &= x_1 x_2 - \gamma x_3 + u_3, \end{aligned} \tag{25}$$

where $\alpha, \beta, \gamma, x_1, x_2, x_3, u_1, u_2, u_3 \in R$. The system is chaotic with $u_1 = u_2 = u_3 = 0$, $\alpha = 10$, $\beta = 28$, and $\gamma = 8/3$. The fourth-order Runge-Kutta scheme is utilized to solve the differential equations with $\Delta t = 10^{-3}$. The initial point of the system is $(1, -2, -1)$.

The reference signal is $(\sin t, \cos t, \sin t + \cos t)$, and it also can be depicted as

$$\begin{aligned} \dot{r}_1 &= r_2, \\ \dot{r}_2 &= -r_1, \\ \dot{r}_3 &= r_2 - r_1. \end{aligned} \tag{26}$$

According to (3), we have

$$\begin{aligned} u_1 &= \varepsilon_1 e_1 + x_2 - \alpha\left(-r_1 + r_2\right) + \alpha e_1 - \left(\alpha + 1\right) e_2, \\ u_2 &= \varepsilon_2 e_2 - x_1 - \left(\beta r_1 - r_2 - r_1 r_3\right) - \left(\beta - 1 - r_3\right) e_1 \\ &\quad + e_2, \\ u_3 &= \varepsilon_3 e_3 + x_2 - x_1 - \left(r_1 r_2 - \gamma r_3\right) - \left(r_2 - 1\right) e_1 \\ &\quad - \left(r_1 + 1\right) e_2 + \gamma e_3. \end{aligned} \tag{27}$$

The tracking error and the controller are shown in Figure 1. From the track error in Figure 1, we conclude that the Lorenz system quickly tracks the reference signal.

To display the robustness of the proposed method, we add a uniformly distributed random noise to $x(t)$. Figure 2 indicates that $x(t)$ eventually tracks the reference signal and ultimately slightly fluctuates around it.

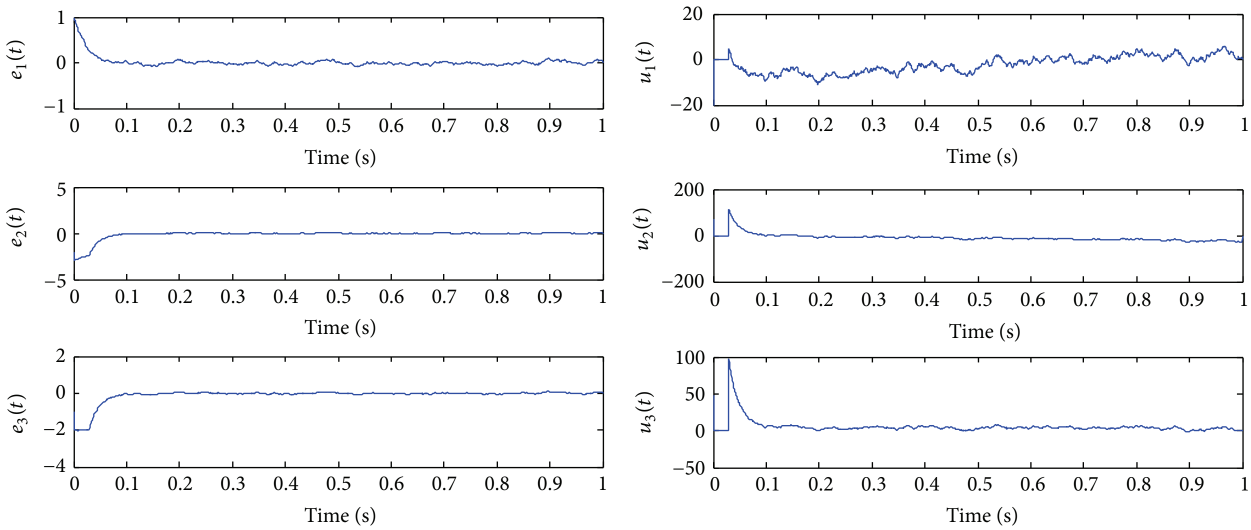
Example 2 (Duffing system). Consider the following Duffing system:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\theta_1 x_1 - x_1^3 - \theta_2 x_2 + \theta_3 \cos\omega t + u, \end{aligned} \tag{28}$$

where $\theta_1 = -1.1$, $\theta_2 = 0.4$, $\theta_3 = 1.8$, and $\omega = 1.8$. The Duffing equation with $u = 0$ has a chaotic solution shown as in Figure 3. We utilize the fourth-order Runge-Kutta scheme to solve the differential equations with $\Delta t = 10^{-3}$. The initial point is $(1, 2)$, and the reference signal is $(10\sin t, 10\cos t)$.

According to (3), we have

$$\begin{aligned} u &= \varepsilon_2 e_2 + x_1 - \left(-\theta_1 r_1 - r_1^3 - \theta_2 r_2 + \theta_3 \cos\omega t\right) \\ &\quad + \left(\theta_1 + 3r_1^2 + 1\right) e_1 + \theta_2 e_2. \end{aligned} \tag{29}$$



(a) The tracking error (b) The controller

FIGURE 2: The Lorenz system tracks $(\sin t, \cos t, \sin t + \cos t)$ starting from $(1, -2, -1)$ with $c = 10$ and $\varepsilon_i = -50$ $(i = 1, 2, 3)$ under random noise jamming.

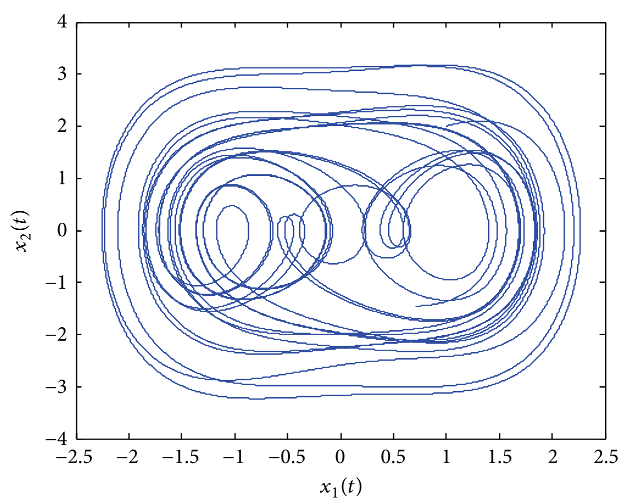


FIGURE 3: The chaotic behavior of Duffing system.

The tracking trajectory and the controller are shown in Figure 4, where the dotted line denotes the reference signals. The controlled Duffing system quickly tracks the reference signal whose range is larger than that of Duffing system.

Example 3 (Henon system). Consider the well-known Henon mapping:

$$\begin{aligned} x_{k+1,1} &= 1 - a x_{k,1}^2 + x_{k,2} + u_{k,1}, \\ x_{k+1,2} &= b x_{k,1}, \end{aligned} \tag{30}$$

where $a, b, u_{k,1} \in R$. When $a = 1.4$, $b = 0.3$, the Henon mapping exhibits a chaotic behavior with $u_{k,1} = 0$. The initial point is $(0, 0)$.

Given the goal orbit [25]

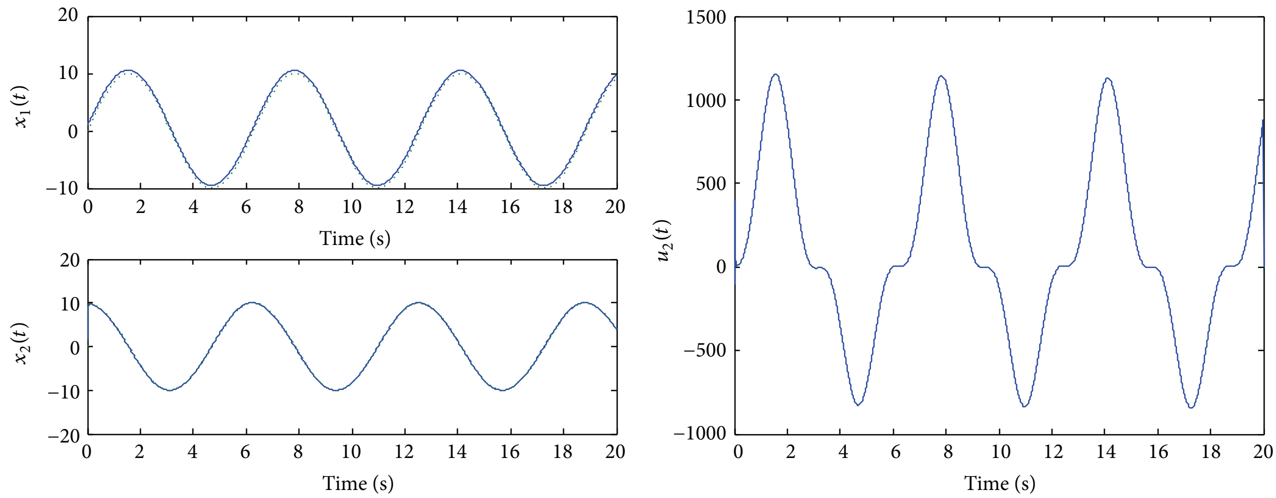
$$\begin{aligned} r_{k+1,1} &= 3.5 r_{k,1} \left(1 - r_{k,1}\right), \\ r_{k+1,2} &= b r_{k,1}, \end{aligned} \tag{31}$$

which is a period-4 orbit.

Applying control law (14), we have

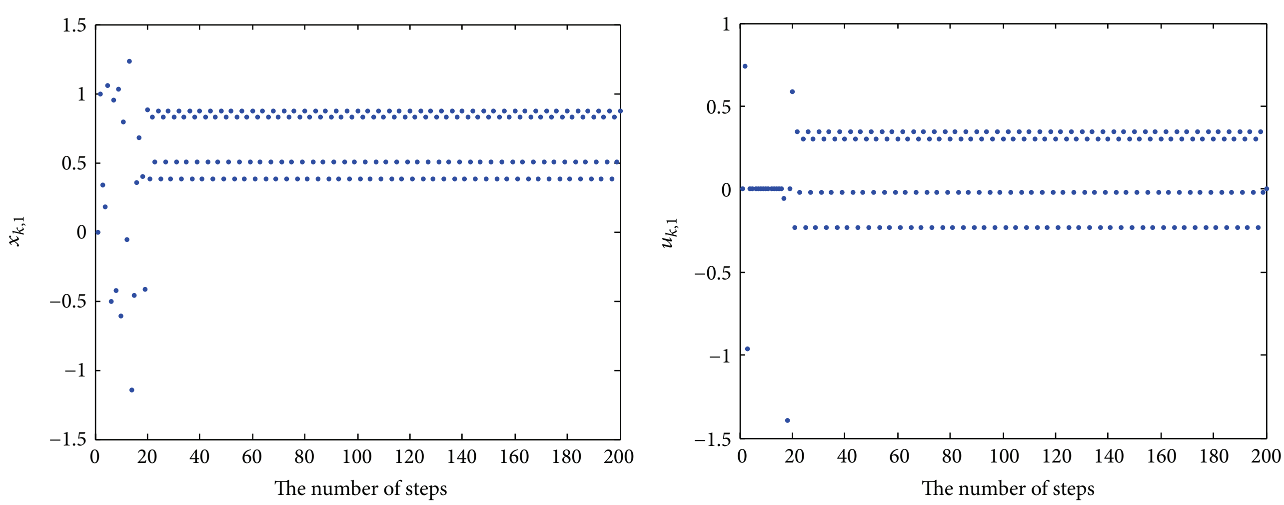
$$\begin{aligned} u_{k,1} &= \delta_1 e_1 + 3.5 x_{k,1} \left(1 - x_{k,1}\right) - \left(1 - a y_{k,1}^2 + y_{k,2}\right) \\ &\quad - \left(3.5 - 7 r_1 - 2 a r_1\right) e_1 - e_2. \end{aligned} \tag{32}$$

The results with different δ_1 are shown in Figures 5–7. It can be seen that the system arrives at the desired goal in



(a) The system trajectory (b) The controller

FIGURE 4: The Duffing system tracks $(10\sin t, 10\cos t)$ starting from (1, 2) with $c = 80$ and $\varepsilon_2 = -50$.



(a) The system trajectory (b) The controller

FIGURE 5: The Henon system tracks a period-4 orbit starting from (0, 0) with $d = 0.2$ and $\delta_1 = 0$.

a short time. It is quicker than [25] and does not have any requirement of initial point.

However, the speed of convergence is related to the value of d and δ_1. We should choose appropriate δ_1 (choosing 0 as initial value) to get a desired speed of convergence, not the larger the better.

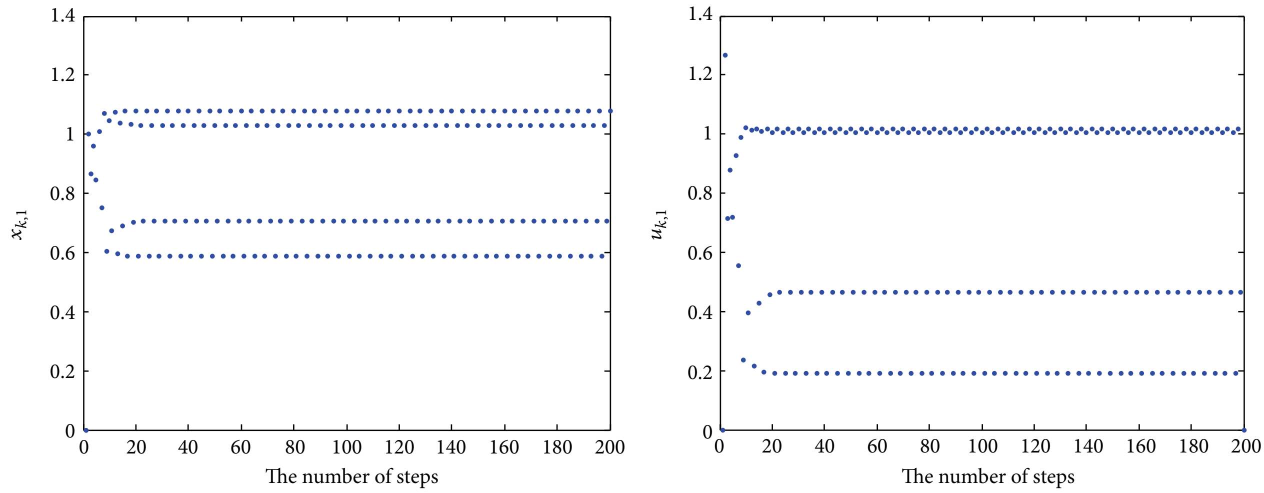
4. Conclusions

We design a general tracking controller for chaotic systems combining the ergodicity of chaos and Jacobian matrix. It is suitable for continuous and discrete chaotic systems. For continuous systems, the element of feedback matrix $\varepsilon_i < 0$ $(i = 1, 2, \ldots, n)$ and the norm of error $c > 0$. For discrete system, the element of feedback matrix $|\delta_i| < a$ $(i = 1, 2, \ldots, n)$ and the norm of error $0 < d < 1$.

The control scheme has the ability to track a bounded reference signal satisfying $F \cap G \neq \Phi$. Moreover, it can be generalized into GPS. The simulations demonstrate its good performance in terms of simplicity, feasibility, and robustness, which indicate it has better practical significance for real world applications.

Competing Interests

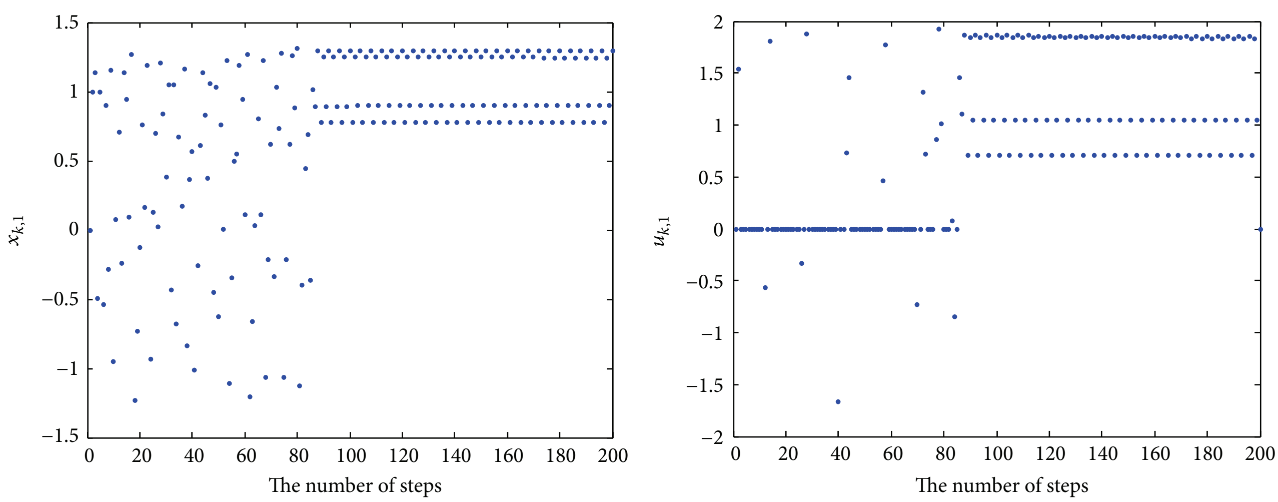
The authors declare that they have no competing interests.



(a) The system trajectory

(b) The controller

FIGURE 6: The Henon system tracks a period-4 orbit starting from (0, 0) with $d = 0.2$ and $\delta_1 = 2$.



(a) The system trajectory

(b) The controller

FIGURE 7: The Henon system tracks a period-4 orbit starting from (0, 0) with $d = 0.2$ and $\delta_1 = 3$.

Acknowledgments

This work is partially supported by Science and Technology Project of Major Production Safety Accident Prevention and Control of Key Technologies, Science and Technology Development Plan Project in Safe Production of Shandong Province (LAJK2013-15).

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