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Research Article

Improved Cross Entropy Algorithm for the Optimum of Charge Planning Problem

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To solve the charge planning problem involving charges and the orders in each charge, a traveling salesman problem based charge planning model and the improved cross entropy algorithm are proposed. Firstly, the charge planning problem with unknown charge number is modeled as a traveling salesman problem. The objective of the model is to minimize the dissimilarity costs between each order and its charge center order, the open order costs, and the unselected order costs. Secondly, the improved cross entropy algorithm is proposed with the improved initial state transition probability matrix which is constructed according to the differences of steel grades and order widths between orders. Finally, an actual numerical example shows the effectiveness of the model and the algorithm.

1. Introduction

Primary steel making is a main stage of steel production process [1], in which oxygen furnaces or electric furnaces convert molten iron into batches of molten steel. The batch, which is made to satisfy a group of orders requiring identical charge widths and steel grades, is called charge in steel production terms. Charge plan converts the primary order requirements into various charges subject to the process constraints of steel production. Reasonable design of charges can improve the productivity and cut down the resource and energy consumption. So, charge planning is a key element of the production operation management in the steel making industry and has been discussed a lot in the present literature. For example, Tang et al. [2] established a mixed integer programming model and a genetic algorithm for the problem. In another paper, Tang and Jiang [3] presented a novel mixed-integer programming model for the charge planning problem, and two kinds of Lagrangian relaxation methods are proposed to solve the problem by using different relaxation methods. The number of charges in these articles is known previously, while it is better to be determined by the planner in the steel production process in order to raise productivity. Dong et al. [4] transformed the objective of minimizing the number of charges into a constraint of minimum and maximum number of charges. However, when we derive a charge plan, the number of charges also needs to be settled according to the constraint firstly. Xue et al. [5] modeled the charge planning problem with unknown charge number and transformed the model into pseudo-traveling salesman model. In this model, the dissimilarity costs in a charge are calculated as total differences among orders in the same charge, while it is better to be calculated as differences between orders in each charge and the charge center order according to the practical steel production process. Wang et al. [6] developed a new charge plan model and a modified parthenogenetic algorithm for billet continuous cast process, but the penalties for the contract products' widths and steel grades were removed from the model. Based on the analysis of the papers mentioned above, a precise charge planning model, especially with the optimum charge number and minimum of the dissimilarity costs between each order and its charge center order, needs further research.

Charge planning problem is a NP-hard combinatorial optimization problem, so how to improve the speed and accuracy of the algorithm for this problem is a key point. As mentioned above, the charge plan is made under some production constraints, which can be used to improve the speed and accuracy of the charge planning algorithm. The existing algorithms only consider these constraints as variables in the charge planning models while the proposed improved cross entropy (ICE) algorithm in this study could use the constraints efficiently. Cross entropy (CE) method [7] is a simple, efficient, and general method for solving NP-hard combinatorial optimization problems. It has been used to solve the traveling salesman problem [8], the vehicle routing problem [9], the buffer allocation problem [10], and the max-cut problem [11]. These applications demonstrate the power of the CE method as a generic and practical tool for solving NP-hard problems.

In this research, the improved cross entropy algorithm is proposed to solve the charge planning problem with unknown charge numbers and its objective is to minimize the dissimilarity costs between each order and its charge center order. The rest of the paper is organized as follows. Section 2 states the charge planning model based on the traveling salesman problem. Section 3 introduces the improved cross entropy algorithm in detail. The proposed approach is applied to an example, and the result is compared with cross entropy method in Section 4. This study is summarized and concluded in Section 5.

2. Charge Planning Model Based on Traveling Salesman Problem

2.1. Problem Description. Charge is the basic unit for the steel making process, and the problem in this study is to establish the optimum charge plan including the charge center orders and the orders merged into them. Each order has its own requirement on steel grade, specification, and due date. The requirement differences may cause some costs when an order merged into a charge center order. Therefore, minimize the requirement differences between charge center orders and the orders merged into them are one of three objectives of charge planning problem. The other two objectives are to minimize the costs of unselected orders and the amount of open orders which are used to fill in the furnace but do not belong to any current orders. The requirements for establishing a charge plan are listed as follows.

(1) The steel grades of the orders in a charge should be in the same steel grade class.

(2) The widths differences of the orders in a charge should not be larger than the allowed maximum adjustment width E.

(3) The thicknesses of orders in a charge should be the same. In this study, the thicknesses of orders are supposed all same according to the actual production process.

(4) The total weight of the contract products in a charge should not surpass the maximum furnace capacity.

(5) The due dates of the contract products in a charge should be similar.

2.2. Charge Planning Model. Given n orders, assume that each single order is less than furnace capacity and cannot be split. The charge planning problem is modeled as follows.

Minimize

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{i,j} X_{ij} X_{jj} + \sum_{j=1}^{n} p_j X_{jj}\left(T - \sum_{i=1}^{n} g_i X_{ij}\right) + \sum_{i=1}^{n}\left(1 - \sum_{j=1}^{n} X_{ij} X_{jj}\right) h_i \tag{1}$$

subject to

$$\sum_{j=1}^{n} X_{ij} \leqslant 1, \quad i = 1,\ldots,n,$$
$$\sum_{i=1}^{n} g_i X_{ij} \leqslant T, \quad j = 1,\ldots,n,$$
$$X_{ij} \in \{0,1\}, \quad X_{jj} \in \{0,1\}, \quad i = 1,\ldots,n, \quad j = 1,\ldots,n, \tag{2}$$

where p_j is a penalty coefficient for open order in the charge of center order j, g_i is the weight of charge i, h_i is the unselected penalty of charge i, $c_{i,j}$, which can be calculated via $c_{i,j} = c_{ij}^1 + c_{ij}^2 + c_{ij}^3$, is the dissimilarity cost for the differences between order i and the charge center order j that i merged in, c_{ij}^1, c_{ij}^2, and c_{ij}^3 are the costs caused by the differences of steel grades, widths, and due dates, respectively, between i and j, which can be calculated as follows:

$$c_{ij}^1 = \begin{cases} F_1\left(ST_i - ST_j\right), & \text{if } 0 \leqslant ST_i - ST_j \leqslant TH_1, \\ +\infty, & \text{otherwise}, \end{cases}$$
$$c_{ij}^2 = \begin{cases} F_2\left|W_i - W_j\right|, & \text{if } \left|W_i - W_j\right| \leqslant E, \\ +\infty, & \text{otherwise}, \end{cases} \tag{3}$$
$$c_{ij}^3 = \begin{cases} F_3\left(d_i - d_j\right), & \text{if } 0 \leqslant d_i - d_j \leqslant D, \\ F_4\left(d_i - d_j\right), & \text{if } -D \leqslant d_i - d_j \leqslant 0. \end{cases}$$

ST_i, W_i, and d_i are the steel grade value, width, and due date of order i, respectively. F_1 is the unit cost coefficient with the difference of steel grade. F_2 is the unit width dissimilarity penalty cost coefficient. F_3 is the earliness penalty coefficient and F_4 is the tardiness penalty coefficient. D is the due date piecewise parameter.

X_{ij} and X_{jj} are decision variables, where

$$X_{ij} = \begin{cases} 1, & \text{if order } i \text{ is merged into order } j, \\ 0, & \text{otherwise}. \end{cases}$$
$$X_{jj} = \begin{cases} 1, & \text{if order } j \text{ is charge center}, \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

In the charge planning model mentioned above, objective function (1) minimizes the sum of dissimilarity costs caused by each order and its charge center order, and the costs resulted from open order and the penalty costs for unselected orders. The first part of constraint formula (2) represents that each order can be assigned to only one charge center at most, the second part of constraint formula (2) represents that total charge weight is less than furnace capacity T, and the third part of constraint formula (2) expresses that variables X_{ij} and X_{jj} have two possible values: 0 or 1.

2.3. Traveling Salesman Problem Model for Charge Planning. Regarding the orders as cities, the sum of dissimilarity costs caused by orders merged into charge centers, the costs resulted from open order and the penalties for unselected orders as distances, furnace capacity as the maximum distance a salesman travels, the charge planning problem can be regarded as a traveling salesman problem. The objective of the problem is to find an optimum order sequence minimum the penalty costs that calculated by formula

$$\min_{r \in \chi} S(r) = \min_{r \in \chi} \sum_k C_k^r, \tag{5}$$

where C_k^r is the penalty cost of charge k, which can be calculated as follows:

$$C_k^r = \begin{cases} \sum_{i=1}^{n} I_{r_i}^k h_{r_i}, & \text{if } \sum_{i=1}^{n} \left[I_{r_i}^k c_{r_i, r_{j_k^r}} + p_{j_k^r} \left(T - I_{r_i}^k g_{r_i} \right) \right] > \sum_{i=1}^{n} I_{r_i}^k h_{r_i}, \\ \sum_{i=1}^{n} \left[I_{r_i}^k c_{r_i, r_{j_k^r}} + p_{j_k^r} \left(T - I_{r_i}^k g_{r_i} \right) \right], & \text{otherwise.} \end{cases} \tag{6}$$

The piecewise function (6) means if the sum of dissimilarity costs caused by the orders merged into the kth charge and the open order costs in this charge is larger than the total unselected penalties, the orders in this charge are canceled, and then the total penalty of the kth charge is set to the total unselected penalties. j_k^r means the kth charge center order:

$$j_k^r = \begin{cases} 0, & \text{if } 0 \leqslant ST_{r_i} - ST_{j_k^r} \leqslant TH_1 \text{ and } \sum_{i=j_{k-1}^r}^{r_i} g_i \leqslant T, \\ r_i, & \text{otherwise.} \end{cases} \tag{7}$$

$I_{r_i}^k$ is the indicator function that indicates whether r_i is merged into charge k:

$$I_{r_i}^k = \begin{cases} 1, & \text{if } j_k^r \leqslant r_i \leqslant j_{k+1}^r, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

3. Improved Cross Entropy Algorithm

In the last section, the traveling salesman problem based model of charge planning problem is constructed. We can use cross entropy method, which has been proven as an efficient method for solving the traveling salesman problem, to solve the charge planning problem as the method. The result did not meet performance expectation when we use the method directly, so we need to improve it. The key point in using cross entropy method to solve a combinatorial optimization problem is the state transition probability matrix, which can be improved according to the characteristics of the problem. In the charge planning problem, two orders with different steel grades or widths have different dissimilarity costs when merged into a same charge center, so the matrix elements can be settled according to the dissimilarity costs as follows.

(1) Assign values to matrix elements according to the process constraints of the problem:

$$p'_{ij} = \begin{cases} 0, & \text{if } i = j, \\ \frac{1}{n}, & \text{if } i = 1, j \neq 1, \\ \frac{1}{N}, & \text{if } ST_i - ST_j < 0 \text{ or } ST_i - ST_j > TH_1 \text{ or } \left| W_i - W_j \right| > E, \\ \frac{1}{1 - \sum_{i=1}^{n+1} p_{ij}}, & \text{if } j = 1, j \neq i, \\ \frac{1}{2}, & \text{if } ST_i = ST_j \text{ and } W_i = W_j, \\ \frac{1}{4}, & \text{if } ST_i = ST_j, \left| W_i - W_j \right| = 50 \text{ or } W_i = W_j, 1 \leqslant ST_i - ST_j \leqslant 2, \\ \frac{1}{8}, & \text{otherwise.} \end{cases} \tag{9}$$

(2) Normalize P:

$$p_{ij} = \frac{p'_{ij}}{\sum_{j=1}^{n+1} p'_{ij}}, \quad i = 1, \ldots, n+1. \tag{10}$$

Based on the state transition probability matrix above, we proceed the improved cross entropy algorithm as follows.

Step 1. Generate N order paths $\{(r^0, r^1, \ldots, r^n), r\}$, $r = 1, \ldots, N$ via a Markov process with the initial state transition probability matrix P described by (10), where r^0 is the virtual starting order.

Step 2. Calculate $S(r)$ for every order path using formula (5).

Step 3. Order $S(r)$ from the smallest to the biggest: $\widetilde{S}(1) \leqslant \widetilde{S}(2) \leqslant \cdots \leqslant \widetilde{S}(N)$, and evaluate the ρ sample quantile as $\gamma_t = \widetilde{S}_{\lceil \rho \cdot N \rceil}$, where $\widetilde{S}(i)$, $i = 1, \ldots, N$ is called the ith order

Table 1: Comparison of results by ICE and CE.

	ICE	CE
Optimum value	2352	2354
Mean running time	26.06	28.98
Mean deviation	8.6	55.3

statistic of the sequence $\widetilde{S}(1) \leqslant \widetilde{S}(2) \leqslant \cdots \leqslant \widetilde{S}(N)$ and $\lceil\rceil$ means round up to the nearest integer.

Step 4. Update P by taking only those paths that have a total length less than or equal to γ_t into account, and the updated value for p_{ij} can be estimated as $p_{ij} = \sum_{r=1}^{N} I_{\{S(r)\leqslant\gamma_t\}} I_{\{r\in\chi_{ij}\}} / \sum_{r=1}^{N} I_{\{S(r)\leqslant\gamma_t\}}$, where χ_{ij} is the set of tours in which the transition from v_i to v_j is made and I is indicator function.

Step 5. If $\widehat{S}_{t,(1)} = \widehat{S}_{t+1,(1)} = \cdots = \widehat{S}_{t+5,(1)}$ for every $t \geq 1$, calculate $S(r)$ and then stop; otherwise, reiterate from Step 1, $\widehat{S}_{t,(1)}$ denotes the value of smallest cost in iteration t.

To avoid local optimum, instead of updating the transition matrix P directly via Step 4, we use a smoothed updating procedure in which $P_{ij}^{t} = \alpha p_{ij} + (1-\alpha) p_{ij}^{t-1}$, where α is smoothing parameter.

4. Computational Results and Discussion

We test our algorithm on the practical production data shown in [5] with 30 orders to be arranged. The basic model parameters are $TH_1 = 5$, $E = 100$, $T = 100$, $D = 30$, $F_1 = 5$, $F_2 = 0.1$, and $F_3 = F_4 = 2$.

To verify the efficiency of the proposed algorithm, we compare the results obtained by ICE and CE. The algorithms are run on Matlab 7.0 and a personal computer of Pentium R, 2 GB RAM. There are three parameters in the proposed improved cross entropy algorithm and cross entropy method: sample size N, rarity parameter ρ, and smoothing parameter α. Different values of the parameters may result in different solution results. We test $\{100, 300, 500, 700, 1000\}$ for N, $\{0.01, 0.02, 0.05, 0.1, 0.2\}$ for ρ, and $\{0.5, 0.6, 0.7, 0.8, 0.9\}$ for α. The parameters maintain the same when they are not the tested parameter. Ten computational tests have been conducted for each of different values of each parameter. Mean deviations between the results of the ten tests and the best result as well as the mean running times are regarded as performance criteria of the algorithms. According to the results of the tests, we choose $N = 500$, $\rho = 0.02$ and $\alpha = 0.8$ in this study. The results of the charge planning problem mentioned above are shown in Table 1.

The optimum values are the best values obtained by the two algorithms in 10 times, the mean times are the average times that the two algorithms run 10 times, and the mean deviations are the average distances between the 10 values and the optimum values of the two algorithms in 10 times. The details of the optimum charge plan obtained by ICE and CE are shown in Table 2.

Table 2: Optimum charge plans obtained by ICE and CE.

Charges	ICE		CE	
	Slabs	Weights	Slabs	Weights
1	12, 13, 17, 15	98	4, 8, 15, 21	99
2	14, 21, 20, 19	95	2, 1, 5, 3	100
3	26, 23, 25, 24	96	26, 24, 23, 25	96
4	4, 9, 16, 18	99	14, 18, 12, 20	98
5	5, 3, 1, 2	100	13, 16, 19, 17	95
6	6, 8, 11, 10, 7	100	11, 10, 7, 9, 6	100
7	29, 30	20	29, 30	20
Unselected	22, 27, 28	63	22, 27, 28	63

From Tables 1 and 2, we can observe that ICE makes several improvements compared to the CE method: (1) the optimum value is decreased; (2) the average run time is decreased by about 11.2%; (3) the mean deviations are decreased significantly. These observations prove that the proposed ICE algorithm is better than CE method when solving the charge planning problems, which means better productivity and less resource consumption in the actual production process.

5. Conclusion

In this paper, we described the charge planning problem that focuses on the unknown charge number and the dissimilarity costs between orders and those charge center orders in iron and steel production process. The charge planning model, which is based on traveling salesman problem, is formulated and the improved cross entropy algorithm is proposed. Through the improvement of state transition probability matrix, the proposed ICE algorithm could take advantage of the problem's characteristics. As a result, the speed and accuracy of the algorithm are improved, which are proved by an actual production example. Future work will focus on improving the stability and accuracy of ICE and extending the model and algorithm to the actual charge planning systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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