# Higher Order Curvature Theories of Gravity Matched with Observations: a Bridge Between Dark Energy and Dark Matter Problems

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Higher order curvature gravity has recently received a lot of attention due to the fact that it gives rise to cosmological models which seem capable of solving dark energy and quintessence issues without using "ad hoc" scalar fields. Such an approach is naturally related to fundamental theories of quantum gravity which predict higher order terms for loop expansions of quantum fields in curved spacetimes. In this framework, we obtain a class of cosmological solutions which are fitted against cosmological data. We reproduce reliable models able to fit high redshift supernovae and WMAP observations. The age of the universe and other cosmological parameters are recovered in this context. Furthermore, in the weak field limit, we obtain gravitational potentials which differ from the Newtonian one because of repulsive corrections increasing with distance. We evaluate the rotation curve of our Galaxy and compare it with the observed data in order to test the viability of these theories and to estimate the scale-length of the correction. It is remarkable that the Milky Way rotation curve is well fitted without the need of any dark matter halo and similar results hold also for other galaxies.

## I. INTRODUCTION

The Hubble diagram of type Ia supernovae (hereafter SNeIa) [1], the anisotropy spectrum of the cosmic microwave background radiation (hereafter CMBR) [2], the matter power spectrum determined by the large scale distribution of galaxies [3] and by the data on the Ly$\alpha$ clouds [4] are evidences in favor of a new picture of the universe, which is spatially flat and undergoing an accelerated expansion driven by a negative pressure fluid nearly homogeneously distributed and constituting up to $\sim 70\%$ of the energy content. This is called *dark energy*, while the model is usually referred to as the *concordance model*. Even if supported by the available astrophysical data, this new picture is not free of problems. Actually, while it is clear how dark energy works, its nature remains an unsolved problem. The simplest explanation claims for the cosmological constant $\Lambda$ thus leading to the so called $\Lambda$CDM model [5]. Although being the best fit to most of the available astrophysical data [2], the $\Lambda$CDM model is also plagued by many problems on different scales. If interpreted as vacuum energy, $\Lambda$ is up to 120 orders of magnitudes smaller than the predicted value. Furthermore, one should also solve the *coincidence problem*, i.e. the nearly equivalence, in magnitude orders, of matter and $\Lambda$ contributions to the total energy density. In order to address these issues, much interest has been devoted to models with dynamical vacuum energy, the so called *quintessence*. These models typically involve a scalar field rolling down its self interaction potential thus allowing the vacuum energy to become dominant at present epoch. Although quintessence by a scalar field is the most studied candidate for dark energy, it generally does not avoid *ad hoc* fine tuning to solve the coincidence problem. Moreover, it is not clear where this scalar field arises and how to choose the self interaction potential. Actually, there is a different way to face the problem of cosmic acceleration. It is possible that the observed acceleration is not the manifestation of another ingredient in the cosmic pie, but rather the first signal of a breakdown of our understanding of the laws of gravitation. From this point of view, it is thus tempting to modify the Friedmann equations to see whether it is possible to fit the astrophysical data with a model comprising only the standard matter. In this framework, there is the attractive possibility to consider the Einstein gravity as a particular case of a more general theory. This is the underlying philosophy of what are referred to as $f(R)$ theories [6–8]. In this case, the Friedmann equations have to be given away in favor of a modified set of cosmological equations that are obtained by varying a generalized gravity Lagrangian where the scalar curvature $R$ has been replaced by a generic function $f(R)$. The standard general relativity is recovered in the limit $f(R) = R$, while different results may be obtained for other choices of $f(R)$. With this paradigm in mind, the problems of dark energy and dark matter could be geometrically interpreted giving rise to a completely new picture of gravitational interaction. From a cosmological point of view, the key point of $f(R)$ theories is the presence of modified Friedmann equations, obtained by varying the generalized Lagrangian. However, here lies also the main problem of this approach

*Proceedings of XVI SIGRAV Conference, 13-16 September 2004, Vietri (Italy)

since it is not clear how the variation has to be performed. Actually, once the Friedmann - Robertson - Walker (FRW) metric has been assumed, the equations governing the dynamics of the universe are different depending on whether one varies with respect to the metric only or with respect to the metric components and the connections. It is usual to refer to these two possibilities as the *metric approach* and the *Palatini approach* respectively. The two methods give the same results only in the case $f(R) = R$, while they lead to significantly different dynamical equations for every other choice of $f(R)$ (see [8] and references therein). The debate on what is the true physical approach is still open [9], nevertheless several positive results have been achieved in both of them. In [6] and then in [7,8], it has been showed that it is possible to obtain the observed accelerating dynamics of the universe expansion by taking into account higher order curvature terms into the gravitational Lagrangian. Furthermore, in [6], a successful test with SNeIa data has been performed. Having tested such a scheme on cosmological scales, it is straightforward to try to complement the approach by analyzing the low energy limit of these theories in order to see whether this approach is consistent with the *local* physics, i.e. on galactic scale. In [10], it has been found that, in the weak field limit, the Newtonian potential is modified by an additive term which scales with the distance $r$ as a power law. Having obtained the corrected gravitational potential, the theoretical rotation curve of our Galaxy has been evaluated and compared with the observational data. This test shows that the correction term allows to well fit the Milky Way rotation curve without the need of dark matter. These results suggest that considering $f(R)$ theories of gravity can provide both an explanation to dark energy and dark matter issues. In this lecture, we outline the basic features of the $f(R)$-theories in the metric approach regarding the dark energy and the dark matter problems, stressing, in particular, the matching with astrophysical and cosmological data. Far from being exhaustive on the whole argument, we want to point out that these families of extended theories of gravity have to be seriously taken into account since they give rise to viable and reliable pictures of the observed universe.

## II. CURVATURE QUINTESSENCE

A generic fourth–order theory of gravity, in four dimensions, is given by the action [6],

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[f(R) + \mathcal{L}_{(matter)}\right] \, , \tag{1}$$

where $f(R)$ is a function of Ricci scalar $R$ and $\mathcal{L}_{(matter)}$ is the standard matter Lagrangian density. We are using physical units $8\pi G_N = c = \hbar = 1$. The field equations are

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = T^{(curv)}_{\alpha\beta} + T^{(matter)}_{\alpha\beta} \, , \tag{2}$$

where the stress-energy tensor has been defined for the curvature contributes

$$T^{(curv)}_{\alpha\beta} = \frac{1}{f'(R)}\left\{\frac{1}{2}g_{\alpha\beta}\left[f(R) - Rf'(R)\right] + f'(R)^{;\mu\nu}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu})\right\} \tag{3}$$

and the matter contributes

$$T^{(matter)}_{\alpha\beta} = \frac{1}{f'(R)}\tilde{T}^{(matter)}_{\alpha\beta} \, . \tag{4}$$

We have taken into account the nontrivial coupling to geometry; prime means the derivative with respect to $R$. If $f(R) = R + 2\Lambda$, we recover the standard second–order Einstein gravity (plus a cosmological constant term). In a FRW metric, the action (1) reduces to the point-like one:

$$\mathcal{A}_{(curv)} = \int dt\left[\mathcal{L}(a,\dot{a};R,\dot{R}) \, + \mathcal{L}_{(matter)}\right] \tag{5}$$

where the dot means the derivative with respect to the cosmic time. In this case the scale factor $a$ and the Ricci scalar $R$ are the canonical variables. It has to be stressed that the definition of $R$ in terms of $a, \dot{a}, \ddot{a}$ introduces a constraint in the action (5) [6], by which we obtain

$$\mathcal{L} = a^3\left[f(R) - Rf'(R)\right] + 6a\dot{a}^2 f'(R) + 6a^2\dot{a}\dot{R}f''(R) - 6kaf'(R) + a^3 p_{(matter)} \, , \tag{6}$$

(the standard fluid matter contribution acts essentially as a pressure term). The Euler-Lagrange equations coming from (6) give the system:

$$2\left(\frac{\ddot{a}}{a}\right)+\left(\frac{\dot{a}}{a}\right)^2+\frac{k}{a^2}=-p_{(tot)}\,, \tag{7}$$

and

$$f''(R)\left\{R+6\left[\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2+\frac{k}{a^2}\right]\right\}=0\,, \tag{8}$$

constrained by the energy condition

$$\left(\frac{\dot{a}}{a}\right)^2+\frac{k}{a^2}=\frac{1}{3}\rho_{(tot)}\,. \tag{9}$$

Using Eq.(9), it is possible to write down Eq.(7) as

$$\left(\frac{\ddot{a}}{a}\right)=-\frac{1}{6}\left[\rho_{(tot)}+3p_{(tot)}\right]\,. \tag{10}$$

The accelerated behavior of the scale factor is achieved for

$$\rho_{(tot)}+3p_{(tot)}<0\,. \tag{11}$$

To understand the actual effect of these terms, we can distinguish between the matter and the geometrical contributions

$$p_{(tot)}=p_{(curv)}+p_{(matter)}\qquad \rho_{(tot)}=\rho_{(curv)}+\rho_{(matter)}\,. \tag{12}$$

Assuming that all matter components have non-negative pressure, Eq.(11) becomes:

$$\rho_{(curv)}>\frac{1}{3}\rho_{(tot)}\,. \tag{13}$$

The curvature contributions come from the stress-energy tensor (3) and then the *curvature pressure* is

$$p_{(curv)}=\frac{1}{f'(R)}\left\{2\left(\frac{\dot{a}}{a}\right)\dot{R}f''(R)+\ddot{R}f''(R)+\dot{R}^2f'''(R)-\frac{1}{2}\left[f(R)-Rf'(R)\right]\right\}\,, \tag{14}$$

and the *curvature energy-density* is

$$\rho_{(curv)}=\frac{1}{f'(R)}\left\{\frac{1}{2}\left[f(R)-Rf'(R)\right]-3\left(\frac{\dot{a}}{a}\right)\dot{R}f''(R)\right\}\,, \tag{15}$$

which account for the geometrical contributions into the thermodynamical variables. It is clear that the form of $f(R)$ plays an essential role for this model. For the sake of simplicity, we choose the $f(R)$ function as a generic power law of the scalar curvature and we ask also for power law solutions of the scale factor, that is

$$f(R)=f_0R^n\,,\qquad a(t)=a_0\left(\frac{t}{t_0}\right)^{\alpha}\,. \tag{16}$$

The interesting cases are for $\alpha\geq 1$ which give rise to accelerated expansion. For $\rho_{(matter)}=0$ and for spatially flat space-time ($k=0$), we get the algebraic relations $n$ and $\alpha$

$$\alpha[\alpha(n-2)+2n^2-3n+1]=0\,,\quad \alpha[n^2+\alpha(n-2-n-1)]=n(n-1)(2n-1) \tag{17}$$

from which the allowed solutions are

$$\alpha=0\;\rightarrow\;n=0,\;1/2,\;1\,,\quad \alpha=\frac{2n^2-3n+1}{2-n}\,,\;\forall n\;\;\text{but}\;\;n\neq 2\,. \tag{18}$$

The solutions for $\alpha=0$ are not interesting since they provide static cosmologies with a non evolving scale factor. On the other hand, the cases with generic $\alpha$ and $n$ furnish an entire family of significative cosmological models. We see that such a family of solutions admits negative and positive values of $\alpha$ which give rise to accelerated behaviors (see also [11] for a detailed discussion). The curvature-state equation is given by

$$w_{(curv)} = -\left(\frac{6n^2 - 7n - 1}{6n^2 - 9n + 3}\right) , \tag{19}$$

which clearly is $w_{(curv)} \to -1$ for $n \to \infty$. This fact shows that the approach is compatible with the recovering of a cosmological constant. The accelerated behavior is allowed only for $w_{(curv)} < 0$ as requested for a cosmological fluid with negative pressure. From these straightforward considerations, the accelerated phase of the universe expansion can be described as an effect of higher order curvature terms which provide an effective negative pressure contribution. In order to see if such behavior is possible for today epoch, we have to match the model with observational data. The presence of standard fluid matter ($\rho_{(matter)} \neq 0$) does not affect greatly this overall behavior as widely discussed in [11].

## III. MATCHING WITH DARK ENERGY OBSERVATIONS

To verify if the curvature quintessence approach is an interesting perspective, we have to match the model with the observational data. In this way, we can constrain the parameters of the theory to significant values. First we compare our theoretical setting with the SNeIa results. As a further analysis, we check also the capability of the model with the universe age predictions. It is worth noticing that the SNeIa observations have represented a cornerstone in the recent cosmology, pointing out that we live in an expanding accelerating universe. This result has been possible in relation to the main feature of supernovae which can be considered reliable standard candles via thanks to the *Phillips amplitude-luminosity relation*. To test our cosmological model, we have taken into account the supernovae observations reported in [1] and compiled a combined sample of these data. Starting from these data, it is possible to perform a comparison between the theoretical expression of the distance modulus and its experimental value for SNeIa. The best fit is performed minimizing the $\chi^2$ calculated between the theoretical and the observational value of distance modulus. In our case, the luminosity distance is

$$d_L(z, H_0, n) = \frac{c}{H_0}\left(\frac{\alpha}{\alpha - 1}\right)(1+z)\left[(1+z)^{\frac{\alpha}{\alpha-1}} - 1\right] , \tag{20}$$

where $c$ is the light speed and $z$ is the red-shift. The range of $n$ can be divided into intervals taking into account the existence of singularities in (20). In order to define a limit for $H_0$, we have to note that the Hubble parameter, being a function of $n$, has the same trend of $\alpha$. We find that for $n$ lower than $-100$, the trend is strictly increasing while for $n$ positive, greater than 100, it is strictly decreasing. The results of the fit are showed in Table 1.

| Range | $H_0^{best}(km\,s^{-1}Mpc^{-1})$ | $n^{best}$ | $\chi^2$ |
|---|---|---|---|
| $-100 < n < 1/2(1-\sqrt{3})$ | 65 | $-0.73$ | 1.003 |
| $1/2(1-\sqrt{3}) < n < 1/2$ | 63 | $-0.36$ | 1.160 |
| $1/2 < n < 1$ | 100 | 0.78 | 348.97 |
| $1 < n < 1/2(1+\sqrt{3})$ | 62 | 1.36 | 1.182 |
| $1/2(1+\sqrt{3}) < n < 3$ | 65 | 1.45 | 1.003 |
| $3 < n < 100$ | 70 | 100 | 1.418 |

TABLE I. Results obtained by fitting the curvature quintessence models against SNeIa data. First column indicates the range of $n$, column two gives the relative best fit value of $H_0$, column three $n^{best}$, column four the $\chi^2$ index.

The age of the universe can be obtained, from a theoretical point of view, if one knows the today value of the Hubble parameter. In our case, it is

$$t=\left(\frac{2n^2-3n+1}{2-n}\right)H_0^{-1}. \tag{21}$$

We evaluate the age taking into account the intervals of $n$ and the $3\sigma$-range of variability of the Hubble parameter deduced from the SNeIa fit. We have considered, as good predictions, age estimates included between $10Gyr$ and $18Gyr$. By this test, we are able of refine the allowed values of $n$. The results are shown in Table 2. First of all, we discard the intervals of $n$ which give negative values of $t$. Conversely, the other ranges, tested by SNIa fit (Tab.1), become narrower, strongly constraining $n$.

| $Range$ | $\Delta H(km\,s^{-1}Mpc^{-1})$ | $\Delta n$ | $t(n^{best})(Gyr)$ |
|---|---|---|---|
| $-100<n<1/2(1-\sqrt{3})$ | $50-80$ | $-0.67\leq n<-0.37$ | 23.4 |
| $1/2(1-\sqrt{3})<n<1/2$ | $57-69$ | $-0.37<n\leq -0.07$ | 15.6 |
| $1<n<1/2(1+\sqrt{3})$ | $56-70$ | $1.28\leq n<1.36$ | 15.3 |
| $1/2(1+\sqrt{3})<n<2$ | $54-78$ | $1.37<n\leq 1.43$ | 24.6 |

TABLE II. The results of the age test. In the first column is presented the tested range. Second column shows the $3\sigma$-range for $H_0$ obtained by SNeIa test, while in the third we give the $n$ intervals, i.e. the values of $n$ which allow to obtain ages of the universe ranging between $10Gyr$ and 18 $Gyr$. In the last column, the best fit age values of each interval are reported.

Another check for the allowed values of $n$ is to verify if the interesting ranges of $n$ provide also accelerated expansion rates. This test can be easily performed considering the definition of the deceleration parameter $q_0 = -(\ddot{a}a)/(\dot{a}^2)_0$, using the relation (16) and the definition of $\alpha$ in term of $n$. To obtain an accelerated expanding behaviour, the scale factor $a(t) = a_0 t^\alpha$ has to get negative (pole-like) or positive values of $\alpha$ greater than one. We obtain that only the intervals $-0.67 \leq n \leq 0.37$ and $1.37 \leq n \leq 1.43$ provide a negative deceleration parameter with $\alpha > 1$. Conversely the other two intervals of Tab.2 do not give interesting cosmological dynamics, being $q_0 > 0$ and $0 < \alpha < 1$ (standard Friedmann behaviour).

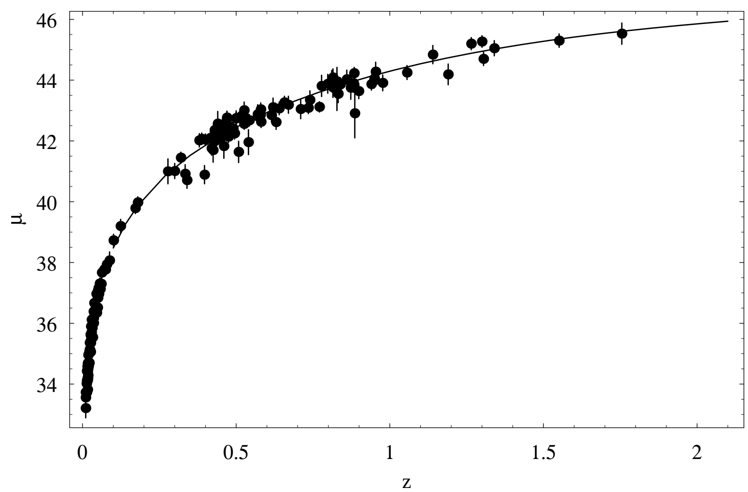


FIG. 1. Best fit curve to the SNeIa Hubble diagram for the power law Lagrangian model.

A further test of the model can be performed by the age estimate obtained by the WMAP campaign [2]. Using these data, we can improve the constraints on $n$ in relation to the very low error (1%) of WMAP age estimator which range between $13.5Gyr$ and $13.9Gyr$ [6].

## IV. MATCHING WITH DARK MATTER: THE MILKY WAY ROTATION CURVE

Beside cosmology, the consistency of $f(R)$ gravity may be verified also at shorter astrophysical scales, e.g. at galactic scales, in order to check the full viability of the theory. In the low energy limit, assuming as above $f(R) = f_0 R^n$, we obtain the gravitational potential [10]

$$\Psi(r) = -\frac{c^2}{2}\left[\left(\frac{r}{\xi_1}\right)^{-1} - \left(\frac{r}{\xi_2}\right)^{\beta(n)}\right] . \tag{22}$$

where $c$ is the light speed,

$$\beta(n) = \sqrt{\frac{4n-1}{2(n-1)}} \times [\mathcal{P}(n) + \mathcal{Q}(n)] , \tag{23}$$

and $\xi_{1,2}$ are scale-lengths. A first estimate of $\xi_1$ may be obtained observing that, for $r << \xi_2$, Eq.(22) reduces to

$$\Psi(r) \sim -\frac{c^2}{2}\left(\frac{r}{\xi_1}\right)^{-1} .$$

Since we have to recover the Newtonian potential at these scales, we have to fix :

$$\xi_1 = \frac{2GM}{c^2} \simeq 9.6 \times \frac{M}{M_\odot} \times 10^{-17} \text{ kpc} ,$$

with $M_\odot$ the mass of the Sun. The value of $\xi_2$ is a free parameter of the theory. Up to now, we can only say that $\xi_2$ should be much larger than the Solar System scale in order not to violate the constraints coming from local gravity experiments. Eq.(22) gives the gravitational potential of a pointlike source. Since real galaxies are not pointlike, we have to generalize Eq.(22) to an extended source. To this aim, we may suppose to divide the Milky Way in infinitesimal mass elements, to evaluate the contribution to the potential of each mass element and then to sum up these terms to get the final potential. In order to test whether the theory is in agreement with observations and to determine the parameter $\xi_2$, we have computed the Milky Way rotation curve modelling our Galaxy as a two components system, a spheroidal bulge and a thin disk. In particular, we assume:

$$\rho_{bulge} = \rho_0 \left(\frac{m}{r_0}\right)^{-1.8} \exp\left(-\frac{m^2}{r_t^2}\right) , \quad \rho_{disk} = \frac{\Sigma_0}{2 z_d} \exp\left(-\frac{R}{R_d} - \left|\frac{z}{z_d}\right|\right) \tag{24}$$

where $m^2 = R^2 + z^2/q^2$, $R$ is the radial coordinate and $z$ is the height coordinate. The central densities $\rho_0$ and $\Sigma_0$ are conveniently related to the bulge total mass $M_{bulge}$ and the local surface density $\Sigma_\odot$ by the following two relations:

$$\rho_0 = \frac{M_{bulge}}{4\pi q \times 1.60851} , \ \Sigma_0 = \Sigma_\odot \exp\left(\frac{R_0}{R_d}\right) ,$$

being $R_0 = 8.5$ kpc the distance of the Sun to the Galactic Centre. We fix the Galactic parameters as follows:

$$M_{bulge} = 1.3 \times 10^{10} \ \mathrm{M}_\odot \ , \ r_0 = 1.0 \ \mathrm{kpc} \ , \ r_t = 1.9 \ \mathrm{kpc} \ ,$$

$$\Sigma_\odot = 48 \ \mathrm{M}_\odot \ \mathrm{pc}^{-2} \ , R_d = 0.3 R_0 \ , \ z_d = 0.18 \ \mathrm{kpc} \ .$$

The Milky Way rotation curve $v_c(R)$ can be reconstructed starting from the data on the observed radial velocities $v_r$ of test particles. We have used the data coming from the H II regions, molecular clouds and those coming from classical Cepheids in the outer disc obtained by Pont et al. [13].

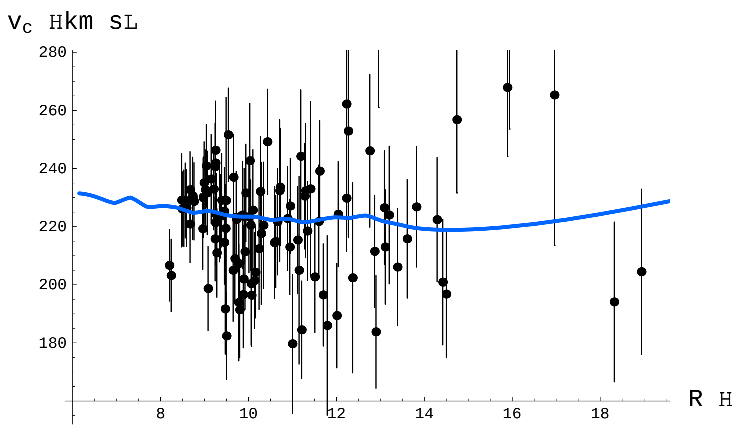


FIG. 2. Observed data and theoretical Milky Way rotation curve computed using the modified gravitational potential with $n = 0.35$ and $\xi_2 = 14.88$ kpc. Note that the points with $R$ between 15.5 and 17.5 kpc are likely affected by systematic errors.

For a given $n$, we perform a $\chi^2$ test to see whether the modified gravitational potential is able to fit the observed rotation curve and to constrain the value of $\xi_2$. Since *a priori* we do not know what is the range for $\xi_2$, we get a first estimate of $\xi_2$ by a simple approach. For a given $R$, we compute $\xi_2$ imposing that the theoretical rotation curve is equal to the observed one. Then, we study the distribution of the $\xi_2$ values thus obtained and evaluate both the median $\xi_2^{med}$ and the median deviation $\delta\xi_2$. The usual $\chi^2$ test is then performed with the prior that $\xi_2$ lies in the range $(\xi_2^{med} - 5\ \delta\xi_2, \xi_2^{med} + 5\ \delta\xi_2)$. As a first test, we arbitrarily fix $n = 0.35$. We get $\xi_2 = 14.88$ kpc , $\chi^2 = 0.96$. In Fig. 2, we show both the theoretical rotation curve for $(n, \xi_2) = (0.35, 14.88)$ and the observed data. The agreement is quite good even if we have not added any dark matter component to the Milky Way model. This result seems to suggest that our modified theory of gravitation is able to fit galaxy rotation curves without the need of dark matter. As a final remark, we note that $\xi_2^{med} = 14.37$ kpc that is quite similar to the best fit value. Actually, a quite good estimate is also obtained considering the value of $\xi_2$ evaluated using the observed rotational velocity at $R_0$. This suggest that a quick estimate of $\xi_2$ for other values of $n$ may be directly obtained imposing $v_{c,theor}(R_0; n, \xi_2) = v_{c,obs}(R_0)$.

## V. CONCLUSIONS

In this lecture, we have considered $f(R)$ theories of gravity to address the problems of dark energy and dark matter. Such an approach has a natural background in several attempts to quantize gravity, because higher-order curvature invariants come out in the renormalization process of quantum field theories on curved space times. We have obtained a family of cosmological solutions [6] which we have fitted against several classes of observational data. A straightforward test is a comparison with SNIa observations [1]. The model fits these data and provides a constrain on the family of possible cosmological solutions. To improve this result, we have performed a test with the age of the universe giving encouraging results in the range between $10Gyr$ and $18Gyr$. In order to better refine these ranges, we have then considered a test based on WMAP age evaluation. In this case, the age ranges between $13.5Gyr$ and $13.9Gyr$. In conclusion, we can say that a fourth order theory of gravity of the form $f(R) = f_0 R^{1+\varepsilon}$ with $\varepsilon \simeq -0.6$ or $\varepsilon \simeq 0.4$ can give rise to reliable cosmological models which well fit SNeIa and WMAP data. In this sense, we need only "small" corrections to the Einstein gravity in order to achieve quintessence issues. Indications in this sense can be found also in a detailed analysis of $f(R)$ cosmological models performed against CMBR constraints, as shown in [12].

Furthermore, it has been analyzed the low energy limit of $f(R) = f_0 R^n$ theories of gravity considering stationary solutions. An exact solution of the field equations has been obtained. The resulting gravitational potential for a point-like source is the sum of a Newtonian term and a contribution whose rate depends on a function of the exponent $n$ of Ricci scalar. The potential agrees with experimental data if $n$ ranges into the interval $(0.25, 1)$, so that the correction term scales as $r^\beta$ with $\beta > 0$. The following step is the generalization of this result to an extended source as a galaxy. To this aim the experimental data and the theoretical prediction for the rotation curve of Milky Way have been compared. The final result has been that the modified potential is able to provide a rotation curve which fits data *without adding any dark matter component*. This result has to be tested further before drawing a definitive conclusion against the need for galactic dark matter. To this aim, one has to show that a potential like that predicted by our model is able to fit rotation curves of a homogeneous sample of external galaxies with both well measured rotation curves and detailed surface photometry. In particular, the exponent $n$ coming out from the fit must be the same for all the galaxies, while $\xi_2$ could be different being related to the scale where deviations from the Newtonian potential sets in.

In conclusion, we have given indications that it is possible to reduce the dark energy and dark matter issues under the same standard of $f(R)$ theories of gravity which could give rise to realistic models working at very large scales (cosmology) and astrophysical scales (galaxies).

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