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Induced Charge Fluctuations in Semiconductor Detectors with a Cylindrical Geometry

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***Abstract*— Now, compound semiconductors are very appealing for hard X-ray room-temperature detectors for medical and astrophysical applications. Despite the attractive properties of compound semiconductors, such as high atomic number, high density, wide band gap, low chemical reactivity and long-term stability, poor hole and electron mobility-lifetime products degrade the energy resolution of these detectors. The main objective of the present study is in development of a mathematical model of the process of the charge induction in a cylindrical geometry with accounting for the charge carrier trapping. The formulae for the moments of the distribution function of the induced charge and the formulae for the mean amplitude and the variance of the signal at the output of the semiconductor detector with a cylindrical geometry were derived. It was shown that the power series expansions of the detector amplitude and the variance in terms of the inverse bias voltage allow determining the Fano factor, electron mobility lifetime product, and the nonuniformity level of the trap density of the semiconductor material.**

***Index Terms*— charge induction, charge carriers trapping, compound semiconductors, cylindrical geometry, energy resolution.**

I. Introduction

Now, compound semiconductors are very appealing for hard X-ray room-temperature detectors for medical and astrophysical applications. Despite the attractive properties of compound semiconductors, such as high atomic number, high density, wide band gap, low chemical reactivity and long-term stability, poor hole and electron mobility-lifetime products degrade the energy resolution of these detectors. The process of trapping in imperfections introduced during semiconductor crystal growth, such as impurity atoms, vacancies, and structural irregularities has a pronounced effect on the charge carrier transport and leads to the peak broadening. The main goal of the theory of radiation detectors is in putting forward mathematical models that adequately describe the processes occurring in the transformation of the energy of impinging radiation into the output detector signal. The energy resolution of a semiconductor detector depends on the fluctuations in the process of charge carriers generation, in the process of charge induction on the detector electrodes, caused by charge carriers trapping, on the fluctuations of the trapping centers concentration in the detector volume, on the fluctuations in the gain of amplifier and on the electronic noise. The main objective of the present study is the development of a mathematical model of the process of the charge induction in a cylindrical geometry with accounting for the charge carrier trapping. In this work, theoretical consideration of the stochastic process of the charge induction in cylindrical geometry with accounting for the charge carrier trapping was considered.

As in compound semiconductor detectors the electron mobility greatly exceeds the hole mobility, significant improvement in the energy resolution can be achieved by discarding the hole with the poorer transport properties. One way is in using special detector geometries to eliminate the contribution of holes. Two detector geometries that offer a marked increase in field strength near the collecting electrode are hemispherical and coaxial geometries. Theoretical consideration of the stochastic process of charge induction in a hemispherical geometry with accounting for electron trapping was considered in [1]. The main objective of the present study is in development of a mathematical model of the process of charge induction in a cylindrical geometry with accounting for the charge carrier trapping. The general formulae for the moments of the distribution function of the induced charge on the electrodes of a detector with a cylindrical geometry are applicable to all coaxial X-ray detectors. For example, they are applicable to the Reverse-Electrode Germanium detectors (REGe) with the ion-implanted boron p-type outer electrode, and the diffused lithium n-type inner electrode.

II. Moments of the Induced Charge Distribution Function in Coaxial Detectors

In coaxial X-ray detectors, the charge on the detector's electrodes is induced mainly in the very small high electric field region near the inner electrode. Thus, to eliminate the contribution of holes, the zero potential must be applied to the outer electrode and the potential V to the inner electrode, and X-rays must enter the detector from the outer electrode.

For low-energy X-rays the dominant interaction with the matter is a photoelectric absorption. If the length of the active region of the detector is much greater than the mean free path length of X-rays in semiconductor material, then we can assume that all X-rays interact with the detector close to the outer electrode. As the thickness of a detector is usually much greater than the size of the volume of the photoelectron energy conversion into the energy of electron-hole pairs, then we can

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consider that all electron-hole pairs are produced at the point of the X-quantum interaction, i.e. close to the outer electrode, and only electrons determine the detector signal.

The coaxial geometry on the one hand eliminates the contribution of holes, but on the other hand increases the fluctuations of the induced charge on the detector electrodes. These fluctuations are determined by the process of electron trapping in imperfections, impurity atoms, vacancies, and structural irregularities. As an electron spends most of the time in the low electric field region, the process of electron trapping has a pronounced effect on the process of charge induction on the detector electrodes. For the best performance of the coaxial detector the correct ratio of the outer to inner radii and the operating voltage must be chosen according to the electron mobility-lifetime product.

Let us consider a coaxial detector with the zero electric potential applied to the outer electrode of the radius R_2, and the electric potential V applied to the inner electrode of the radius R_1. In [2], the general formulae for the moments of the distribution function of the relative charge induced on the detector electrodes were derived

$$\langle q^n \rangle = n\int_0^1 dq \cdot q^{n-1} \exp(-F(q)) \quad (1)$$

The most important are the first two ones

$$\langle q \rangle = \int_0^1 dq \exp(-F(q)), \quad (2)$$

$$\langle q^2 \rangle = 2\int_0^1 dq \cdot q \exp(-F(q)), \quad (3)$$

that determine the mean value and the variance

$$\sigma_q^2 = \langle q^2 \rangle - \langle q \rangle^2 = 2\int_0^1 dq \cdot q \exp(-F(q)) - \left(\int_0^1 dq \exp(-F(q))\right)^2 \quad (4)$$

of the relative induced charge on the detector electrodes.

In the above formulae, according to the Ramo's theorem [3], the relative induced charge on the detector electrodes is

$$q(r) = \frac{\varphi(r) - \varphi(R_2)}{V} = \frac{\varphi(r)}{V}, \quad (5)$$

where $\varphi(r) - \varphi(R_2)$ is the electric potential difference between the electron absorption point at the radius r from the center and the creation point, which for low energy X-rays coincides with the radius of the outer electrode R_2.

The dimensionless function $F(q)$ is the analogue of the optical path length, and is equal to the ratio of the time to induce the relative induced charge on the detector electrodes q to the mean electron trapping time, usually referred to as the electron lifetime τ_e.

The function $F(q)$ has the form of the line integral along the electric field line

$$F(q) = \int_0^{s(q)} ds / \mu_e(s)\tau_e(s)E(s) \quad (6)$$

where $E(s)$ is the magnitude of the electric field strength, $\mu_e(s)$ is the electron mobility, and $\tau_e(s)$ the electron lifetime at the distance s from the electron creation point; $s(q)$ is the inverse function of the relative induced charge on the detector electrodes (5).

The electric potential and the magnitude of the electric field strength in a coaxial detector at the radius r from the center are

$$\varphi(r) = V\frac{\ln(R_2/r)}{\ln(R_2/R_1)}, \quad (7)$$

$$E(r) = E_r(r) = \frac{V}{r\ln(R_2/R_1)}. \quad (8)$$

For our case, the relative induced charge on the detector electrodes (5) is determined by the equation

$$q(r) = \frac{\ln(R_2/r)}{\ln(R_2/R_1)}, \quad (9)$$

and the inverse function of $q(r)$ is

$$r(q) = R_2 \exp(-pq). \quad (10)$$

In the formula (10), the dimensionless parameter

$$p = \ln(R_2/R_1) \quad (11)$$

is the first main parameter that characterizes the coaxial geometry of a given detector.

After integration along the electric field line, the function $F(q)$ has the form

$$F(q) = \frac{1}{2}p[1 - \exp(-2pq)]\xi, \quad (12)$$

where

$$\xi = \frac{R_2^2}{\mu_e\tau_e V} \quad (13)$$

is the second main dimensionless parameter that includes the physical characteristics of the coaxial detector and varies with the applied bias voltage V.

The average value of the relative induced charge on the electrodes of a coaxial detector can be received by expanding integrand in (2) in power series of the parameter ξ

$$\langle q \rangle = \int_0^1 dq \exp(-F(q)) = \sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{p\xi}{2}\right)^n \int_0^1 dq[1 - \exp(-2pq)]^n. \quad (14)$$

After term-by-term integration of the binomial expansion for power n and rearranging terms, the average value of the relative induced charge on the electrodes of a coaxial detector in power series of the parameter ξ has the form

$$\langle q\rangle = \sum_{n=0}^{\infty} a_n \xi^n , \quad (15)$$

where the expansion coefficients are determined by the formula

$$a_n = \frac{1}{n!}\left(-\frac{p\xi}{2}\right)^n \left\{1 - \frac{1}{2p}\begin{bmatrix}\Psi(n+1)+\gamma \\ -e^{-2p}{}_3F_2\left(1,1,1-n;2,2;e^{-2p}\right)\end{bmatrix}\right\} . \quad (16)$$

In the formula (16) ${}_rF_s\left(a_1,...a_r;b_1,...b_s;x\right)$ is the generalized hypergeometric series [4], $\Psi(x)$ is the digamma function, and $\gamma = 0.57721\ 56649$ is the Euler's constant [5].

The first several coefficients of expansion are

$$a_0 = 1, \quad (17)$$

$$a_1 = -\frac{1}{4}(2p - 1 + e^{-2p}), \quad (18)$$

$$a_2 = \frac{1}{32}p(4p - 3 + 4e^{-2p} - e^{-4p}), \quad (19)$$

$$a_3 = -\frac{1}{576}p^2(12p - 11 + 18e^{-2p} - 9e^{-4p} + 2e^{-6p}) . \quad (20)$$

In a similar way, the second moment of the distribution function of the relative induced charge on the electrodes of a coaxial detector has the form

$$\left\langle q^2(\tau_e^{-1})\right\rangle = 2\int_0^1 dq \cdot q \exp(-F(q)) = \sum_0^{\infty} b_n \xi^n , \quad (21)$$

where the expansion coefficients are determined by the formula

$$b_n = \frac{1}{n!}\left(-\frac{p\xi}{2}\right)^n \left\{\begin{matrix} 1 - \frac{n}{2p^2}\cdot{}_4F_3\left(1,1,1,1-n;2,2,2;1\right) + \frac{n}{2p^2} \\ \times e^{-2p}\begin{bmatrix} 2p\cdot{}_3F_2\left(1,1,1-n;2,2;e^{-2p}\right) \\ +{}_4F_3\left(1,1,1,1-n;2,2,2;e^{-2p}\right)\end{bmatrix}\end{matrix}\right\} , \quad (22)$$

where ${}_rF_s\left(a_1,...a_r;b_1,...b_s;x\right)$ is the generalized hypergeometric series [4].

The first several coefficients of expansion are

$$b_0 = 1, \quad (23)$$

$$b_1 = -\frac{1}{4p}(2p^2 - 1 + e^{-2p}(2p+1)), \quad (24)$$

$$b_2 = \frac{1}{64}(8p^2 - 7 + e^{-2p}8(2p+1) - e^{-4p}(4p+1)), \quad (25)$$

$$b_3 = -\frac{p}{3456}(72p^2 - 85 + e^{-2p}108(2p+1) - e^{-4p}27(4p+1) + e^{-6p}4(6p+1)) . \quad (26)$$

In the limit $R_2 \to \infty$ and $R_1 \to \infty$, providing that $R_2 - R_1 = D$, the formulae (15) and (21) reduce to the formulae for the first two moments of the distribution function of the induced charge in a planar detector with uniform electric field [6].

III. The bias dependence of the variance of coaxial detectors

In [7], the general formulae for the mean value and the variance of the semiconductor detector signal were derived. In the case, when all electron-hole pairs are produced at the point of the soft X-quantum interaction close to the outer electrode, and the detector signal is determined only by electrons, the dependence of the mean value and the variance of the semiconductor detector signal on the bias voltage V have forms

$$\langle Q(V)\rangle = \frac{E_0}{\varepsilon}\langle q(V)\rangle\langle g\rangle , \quad (27)$$

$$\sigma_Q^2(V) = \frac{E_0}{\varepsilon}\begin{pmatrix} F\langle q(V)\rangle^2 + \langle q^2(V)\rangle \\ -\langle q(V)\rangle^2 + \langle q(V)\rangle\eta_g^2\end{pmatrix}\langle g\rangle^2 + \sigma_{\text{noise}}^2, \quad (28)$$

where E_0 is the energy of incident monoenergetic particles, ε is the effective energy to produce one electron-hole pair, F is the Fano factor, $\langle g\rangle$ and $\eta_g^2 = \sigma_g^2/\langle g\rangle^2$ are the mean value and the relative variance of the electronic amplifier gain, $\sigma_{\text{noise}}^{\mathbf{2}}$ is the electronic noise at the output of spectrometer. In the formulae (27) and (28)

$$\langle q(V)\rangle = \int d\tau_e^{-1}\rho(\tau_e^{-1})\left\langle q(\tau_e^{-1},V)\right\rangle , \quad (29)$$

$$\left\langle q^2(V)\right\rangle = \int d\tau_e^{-1}\rho(\tau_e^{-1})\left\langle q^2(\tau_e^{-1},V)\right\rangle , \quad (30)$$

are the first and the second moments of the distribution function of the relative charge induced by one electron on the detector electrodes averaged over the distribution of the inverse electron drift time τ_e^{-1} [8].

It was shown in [7], that for quasi-uniform semiconductor material with spatial fluctuations in the trap density, the inverse charge carrier drift time is the normal distribution $\rho(\tau_e^{-1})$ with the mean value $\left\langle\tau_e^{-1}\right\rangle$, the variance $\sigma_{\tau_e^{-1}}^2$, and the relative variance $\eta_{\tau_e^{-1}}^2 = \sigma_{\tau_e^{-1}}^2/\left\langle\tau_e^{-1}\right\rangle^2$ equals to the relative variance of the trap density $\eta_{N_t}^2 = \sigma_{N_t}^2/\langle N_t\rangle^2$.

As $\tau_e^{-1} = \left\langle\tau_e^{-1}\right\rangle + \Delta\tau_e^{-1}$, after introducing the new parameter

$$\chi = \xi\frac{V}{\tau_e^{-1}} = \frac{R_2^2}{\mu_e} , \quad (31)$$

averaged over the Gaussian distribution $\rho(\tau_e^{-1})$ the first and the second moments of the distribution function have forms

$$\left\langle q(V)\right\rangle = \int d\tau_e^{-1}\rho(\tau_e^{-1})\left\langle q(\tau_e^{-1},V)\right\rangle = \int d\tau_e^{-1}\rho(\tau_e^{-1})\sum_0^\infty a_n\xi^n$$

$$= \sum_0^\infty a_n\left(\frac{\chi}{V}\left\langle\tau_e^{-1}\right\rangle\right)^n \cdot \int d\Delta(\tau_e^{-1})\rho(\Delta(\tau_e^{-1}))\left(1+\frac{\Delta(\tau_e^{-1})}{\left\langle\tau_e^{-1}\right\rangle}\right)^n$$

$$= \sum_0^\infty a_n\left(\frac{\chi}{V}\left\langle\tau_e^{-1}\right\rangle\right)^n \int d\Delta(\tau_e^{-1})\rho(\Delta(\tau_e^{-1}))\cdot\left(1+n\frac{\Delta(\tau_e^{-1})}{\left\langle\tau_e^{-1}\right\rangle}\right)$$

$$= \sum_0^\infty a_n\left(\frac{\chi}{V}\left\langle\tau_e^{-1}\right\rangle\right)^n$$

, (32)

$$\left\langle q^2(V)\right\rangle = \int d\tau_e^{-1}\rho(\tau_e^{-1})\sum_0^\infty b_n\xi^n$$

$$= \sum_0^\infty b_n\left(\frac{\chi}{V}\left\langle\tau_e^{-1}\right\rangle\right)^n \cdot \int d\Delta(\tau_e^{-1})\rho(\Delta(\tau_e^{-1}))\left(1+\frac{\Delta(\tau_e^{-1})}{\left\langle\tau_e^{-1}\right\rangle}\right)^n$$

$$= \sum_0^\infty b_n\left(\frac{\chi}{V}\left\langle\tau_e^{-1}\right\rangle\right)^n \int d\Delta(\tau_e^{-1})\rho(\Delta(\tau_e^{-1})) ,$$

$$\times\left(1+n\frac{\Delta(\tau_e^{-1})}{\left\langle\tau_e^{-1}\right\rangle}+\frac{n(n-1)}{2}\frac{\left(\Delta(\tau_e^{-1})\right)^2}{\left\langle\tau_e^{-1}\right\rangle^2}\right)$$

$$= \sum_0^\infty b_n\left(\frac{\chi}{V}\left\langle\tau_e^{-1}\right\rangle\right)^n\left(1+\frac{n(n-1)}{2}\eta^2_{\tau_e^{-1}}\right)$$

(33)

With these results, and after rearranging in powers of

$$\gamma = \chi\left\langle\tau_e^{-1}\right\rangle \quad (34)$$

for the mean value of the signal of coaxial detector we have

$$\left\langle Q(V)\right\rangle = \frac{E_0}{\varepsilon}\sum_0^\infty a_n\left(\frac{\gamma}{V}\right)^n\left\langle g\right\rangle . \quad (35)$$

$$\sigma_Q^2(V) = \frac{E_0}{\varepsilon}\sum_0^\infty c_n\left(\frac{\gamma}{V}\right)^n\left\langle g\right\rangle^2+\sigma^2_{\text{noise}} . \quad (36)$$

where

$$c_n = b_n\left(1+\frac{n(n-1)}{2}\eta^2_{\tau_e^{-1}}\right)+(F-1)\cdot\sum_{k=0}^n a_k a_{n-k}+a_n\eta_g^2 ,$$

(37)

and coefficients a_n and b_n are given by (16) and (22).

The first several coefficients of c_n are

$$c_0 = F+\eta_g^2 , \quad (38)$$

$$c_k = \alpha_k\left[d_k+d_k^F F+d_k^{\tau_e^{-1}}\eta^2_{\tau_e^{-1}}+d_k^g\eta_g^2\right], \quad k\geq 1 . \quad (39)$$

$$k=1,\ \alpha_1=\frac{1}{4},\ d_1 = 2(p-1)+\frac{1}{p}\left(1-e^{-2p}\right),$$

$$d_1^F = -2\left(2p-1+e^{-2p}\right),\ d_1^{\tau_e^{-1}}=0,\ d_1^g = \left(2p-1+e^{-2p}\right).$$

(40)

$$k=2,\ \alpha_2=-\frac{1}{64},$$

$$d_2 = 24p^2-28p+11+e^{-2p}16(p-1)+e^{-4p}5,$$

$$d_2^F = -\left(32p^2-28p+4+e^{-2p}8(4p-1)-e^{-4p}4(p-1)\right),$$

$$d_2^{\tau_e^{-1}} = -\left(8p^2-7+e^{-2p}8(2p+1)-e^{-4p}(4p+1)\right),$$

$$d_2^g = -2p\left(4p-3+e^{-2p}4-e^{-4p}\right). \quad (41)$$

$$k=3,\ \alpha_3=\frac{1}{3456},$$

$$d_3 = \alpha\begin{pmatrix}504p^2-672p+247+e^{-2p}2(324p-243)\\ -e^{-4p}27(4p-11)\end{pmatrix},$$

$$d_3^F = -6p\begin{pmatrix}96p^2-112p+27+e^{-2p}(144p-63)\\ -e^{-4p}9(4p-5)+e^{-6p}(4p-9)\end{pmatrix},$$

$$d_3^{\tau_e^{-1}} = -3p\begin{pmatrix}72p^2-85+e^{-2p}108(2p+1)\\ -e^{-4p}27(4p+1)+e^{-6p}4(6p+1)\end{pmatrix},$$

$$d_3^g = -6p^2\left(12p-11+e^{-2p}18-e^{-4p}9+e^{-6p}2\right). \quad (42)$$

In all above formulae only the first order of small parameters $F<<1$, $\eta^2_{\tau_e^{-1}} = \sigma^2_{\tau_e^{-1}}/\left\langle\tau_e^{-1}\right\rangle^2<<1$ and $\eta_g^2 = \sigma^2_{g_e}/\left\langle g\right\rangle^2<<1$ are retained.

The formulae (38) - (45) in the limit $R_2\to\infty$ and $R_1\to\infty$, providing that $R_2-R_1=D$, reduce to the formulae for the planar detector with uniform electric field [8].

The coefficients of expansions (35) and (36) allow determining characteristics of a coaxial detector from experimental data.

IV. Conclusion

The exact mathematical description of the processes in a semiconductor coaxial detector at registration of low energy X-rays gives the correct formulae for the mean and the variance of the output signal of a detector. It was shown that the power series expansions of the detector amplitude and the variance in terms of the inverse bias voltage allow determining the characteristics of a coaxial detector. These formulae are useful for analysis of the influence of different factors on the energy resolution of semiconductor coaxial detectors.

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