Gauge Theories on Bound States of Fractional Branes [1]

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Abstract:
We study the $N = 2$ super Yang Mills theory living in the world volume of a bound state made of fractional D3/D7 branes at the orbifold $\mathbb{R}^{1,5} \times \mathbb{R}^4/\mathbf{Z_2}$, by using the probe technique. We also discuss the boundary action for the system.

1 Introduction

Since the formulation of the Maldacena conjecture which states the equivalence between N=4 super Yang-Mills theory in four dimensions and type IIB string theory compactified on $AdS_5 \times S_5$ [3] there have been many efforts to find new correspondences between string theories and non-conformal and less supersymmetric gauge theories. These attempts have led to explore in detail and in non trivial backgrounds the complementarity between the two-fold description of the low-energy properties of the D-branes, one based on the classical curved geometry generated by these non-perturbative extended objects and the other one given in terms of the gauge theory living in their world-volume. According to this gauge/gravity correspondence, one can exploit the classical solution generated by D-branes to study the gauge theory defined in their world-volume, or, vice-versa to use the quantum properties of the gauge theory defined on D-branes to study their dynamics. In particular in the last couple of years many efforts have been made to extend the Maldacena duality to non-conformal gauge theory with $N = 2$ [2, 4, 5, 6, 7, 8, 9] and $N = 1$ supersymmetries [10, 11].

One of these approaches that is also the topic of this paper is based on the study of the gauge/gravity correspondence for a bound state made of M D3 and N D7- fractional branes living at the singular point of the orbifold $\mathbb{R}^{1,5} \times \mathbb{R}^4/\mathbf{Z_2}^2$.

The gauge theory living in the world-volume of this bound state is a non-conformal $N = 2$ $SU(M)$ super Yang-Mills with N hypermultiplets in the fundamental representation of the gauge group and therefore such a bound state represents one of the simplest systems to consider in order to extend the Maldacena conjecture. Moreover as shown in Ref.s [1], the "gravity dual" of this system possesses a naked singularity of repulson

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[1]Talk given by the author at the RTN meeting "The Quantum Structure of Spacetime and the Geometric Nature of Fundamental Interactions", (Corfu', September 2001) and based on the papers in Ref.s [1, 2] done in collaboration with M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda and I. Pesando.

[2]For a review on fractional branes see Ref. [12]

type [13]. This seems a general feature of all classical solutions corresponding to non-conformal gauge theories and according to the cosmic censorship conjecture they must be rejected. However, in the case of the orbifold there exists a general mechanism to resolve the singularity, in fact by probing the background with a D3-fractional brane, one can see that the action of the probe becomes tensionless on a hypersurface called enhançon that contains the naked singularity. This aspect suggests that at the enhançon locus new light degrees of freedom come into play and therefore the supergravity approximation is not valid anymore; one also expects that string theory should resolve this problem. According to this phenomenon the region inside the enhançon must be excised obtaining a regular solution but in a region of the space that is far from the brane. Therefore for these kinds of solutions, it is not simple to take the decoupling limit and studying physics inside the enhançon is still an interesting and opened question (for a discussion on the physics inside the enhançon see Ref.s [14, 15].

2 D3/D7 bound state and its classical solution

In the first part of this section we will analyze in detail the D7 branes of type IIB at the orbifold $\mathbb{R}^{1,5} \times \mathbb{R}^4/\mathbf{Z_2}$. We closely follow the approach discussed in Ref. [1] and in particular we discuss the case of a D7 brane completely extended along the orbifold since this is the relevant configuration to yield four-dimensional N=2 gauge theories in the presence of fractional D3 branes. The peculiar aspect of the D7 branes extended along the orbifold is that they are sources not only of those "untwisted" fields typical of a D7 brane, but also of "twisted" scalar fields b and c defined as follows:

$$C_2 = c\ \omega_2 \qquad , \qquad B_2 = b\ \omega_2 \tag{1}$$

where B_2 and C_2 are respectively the Kalb-Ramond and the RR two form potentials, while ω_2 is the anti-self dual 2-form associated to the vanishing 2-cycle of the orbifold ALE space.

It is interesting to observe that these twisted fields are the same of those emitted by the fractional D3 branes studied for examples in Ref.s [2, 5].

These features can be clearly seen by computing the vacuum energy Z of the open string stretched between two such D7 branes that is given by:

$$Z = \int_0^\infty \frac{ds}{s}\ \mathrm{Tr_{NS-R}}\left[P_{\mathrm{GSO}}\left(\frac{\mathbb{1}+g}{2}\right)\mathrm{e}^{-2\pi s(L_0-a)}\right] \tag{2}$$

where P_{GSO} is the GSO projection, g is the orbifold $\mathbf{Z}_2$ parity, and $a = 1/2$ in the NS sector and $a = 0$ in the R sector. When one takes the $\mathbb{1}$ inside the bracket, one gets half of the usual contribution of the open strings stretched between two D7 branes in flat space, whereas as one takes the g inside the bracket one obtains $1/16$ times the contribution of the twisted sectors of the fractional D3 branes of Ref.s [2, 5] After performing the modular transformation $s \to 1/s$ and factorizing the resulting expression in the closed string channel, one can derive the boundary state associated to the D7 brane along the orbifold. The final result is the sum of two terms:

$$|D7\rangle = |D7\rangle^{\mathrm{U}} + |D7\rangle^{\mathrm{T}} \tag{3}$$

The untwisted part $|D7\rangle^{\mathrm{U}}$ is the same as that of the D7 branes in flat space but with a normalization differing by a factor of $1/\sqrt{2}$; the twisted part $|D7\rangle^{\mathrm{T}}$ is, instead, similar

to that of the fractional D3 branes but with a normalization differing by a factor of $1/4$. By saturating the boundary state (3) with the massless closed string states of the various sectors, one can determine which are the fields that couple to the fractional D7 brane.

From the explicit couplings it is possible to infer the form of the terms of the world-volume action that are linear in the bulk fields obtaining the following expression:

$$S_{\text{b}}^{\text{D7}} \simeq -\tau_7 \int d^8x \; e^\phi \sqrt{-\det g_{ab}} + \tau_7 \int C_8$$
$$+ \frac{\tau_3}{2(2\pi\sqrt{\alpha'})^2} \int d^4x \; \sqrt{-\det g_{\alpha\beta}} \; \tilde{b} - \frac{\tau_3}{2(2\pi\sqrt{\alpha'})^2} \int A_4 + \dots \tag{4}$$

where $\tau_p \equiv T_p/(\sqrt{2}\,\kappa_{\text{orb}})$ and g_{ab} is the induced metric. The previous expression represents the whole information that can be extracted from the boundary state but it is enough to compute the classical solution generated by a bound state made of D3/D7 fractional branes.

In order to determine the complete boundary action for our D7 brane completely extended along the orbifold we have first to compute the classical solution and after, by imposing the zero force condition for the system, we will be able to fix the higher order couplings appearing in the boundary action. However this is exactly the argument of the forthcoming discussion.

In the last part of this section we compute the classical solution of the type IIB supergravity equation of motion, for a bound state made of M D3-fractional branes and N D7-branes on the orbifold $\mathbb{R}^{1,5} \times \mathbb{R}^4/\mathbf{Z_2}$. In particular we will consider configurations in which the D7 branes are entirely extended along the orbifold; i.e. in the directions $x^0 \dots x^3$ and in the orbifolded directions $x^6 \dots x^9$, while the D3-branes are transverse to the orbifold and therefore extended along the directions $x^0 \dots x^3$.

The starting point are the equations of motion of type IIB supergravity in ten dimensions that, for the fields that we are interested in, can be combined in two complex and compact expressions given by:

$$d^*d\tau + ie^\phi d\tau \wedge {}^*d\tau + \frac{i}{2}G_3 \wedge {}^*G_3 = 2i\kappa^2 e^{-\phi} \left[\frac{\delta\mathcal{L}_b}{\delta\phi} + ie^{-\phi}\frac{\delta\mathcal{L}_b}{\delta C_0} \right] \tag{5}$$

and

$$d^*G_3 + d\tau \wedge \left[ie^{\phi\,*}G_3 + {}^*H_3 \right] - i\tilde{F}_5 \wedge G_3 = -2i\kappa^2 \left[\frac{\delta\mathcal{L}_b}{\delta B_2} - \tau\frac{\delta\mathcal{L}_b}{\delta C_2} \right] \tag{6}$$

where $G_3 = dC_2 + \tau dB_2$, $\tau = C_0 + ie^{-\phi}$ is the standard combination of the axion and dilaton fields and $\mathcal{L}_b$ is the boundary lagrangian for our bound system. The equation of motion for the other fields that couple to our system are not relevant for the aim of this paper and therefore they will not be taken into account in the following discussion. The solution of the previous system of equations of motion compatible with the symmetries of the system and preserving eight real supersymmetries is:

$$\tau = i\left(1 - \frac{Ng_s}{2\pi} \log\frac{z}{\epsilon} \right) \quad, \tag{7}$$

and

$$\gamma = i\,2\pi\alpha' g_s \left[\frac{\pi}{g_s} + (2M - N)\log\frac{z}{\epsilon} \right] \tag{8}$$

where $z = x^4 + i\,x^5$ and we have chosen the integration constants to enforce the appropriate background values. It is interesting to observe that, by rewriting the solution in terms of the real component fields, and expanding them in powers of $\lambda = N\,g_s$ we realize that C_0 does not receive corrections to higher orders in λ while the twisted fields b and c acquire an infinite tail of logarithmic terms. This is to be contrasted with the solution of the pure fractional D3 branes [2, 5] where the twisted scalars had, instead, only terms at first order. Thus, if one wants to determine the classical profile of the twisted scalars using the boundary state formalism in the presence of fractional D7 branes, it is not sufficient to consider contributions with just one boundary, but it is necessary to sum over all contributions with an arbitrary number of boundaries as explained in Ref. [16], which, due to the open/closed string duality, is equivalent to sum over an arbitrary number of open-string loops.

Having the complete solution, we can verify the no-force condition and check the structure of the world-volume action of the bound state. If we substitute our solution in the boundary action of the D7-brane given in the eq.(4), we see the missing of zero force condition. This problem is overcome by remembering that the boundary action (4) only contains the linear coupling emerging from the boundary states. By imposing the zero force condition and remembering that the D7 branes being extended along the orbifolded direction couple also with the twisted fields, it is possible to deduce all the higher order couplings, obtaining:

$$
\begin{aligned}
S_{\mathrm{b}}^{\mathrm{D7}} &= -\tau_7 \int d^8x \; e^\phi \sqrt{-\det g_{ab}} \; + \; \tau_7 \int C_8 \\
&+ \; \frac{\tau_3}{2(2\pi\sqrt{\alpha'})^2} \int d^4x \; \sqrt{-\det g_{\alpha\beta}} \; \widetilde{b}\left(1 + \frac{\widetilde{b}}{4\pi^2\alpha'}\right) - \frac{\tau_3}{2(2\pi\sqrt{\alpha'})^2} \int A_4 \left(1 + \frac{\widetilde{b}}{4\pi^2\alpha'}\right) \\
&- \; \frac{\tau_3}{2(2\pi\sqrt{\alpha'})^2} \int C_4 \; \widetilde{b}\left(1 + \frac{\widetilde{b}}{4\pi^2\alpha'}\right) \; .
\end{aligned}
\tag{9}
$$

It would be also interesting to confirm the boundary action by string calculations or by geometrical considerations.

3   The probe action and the $\mathcal{N} = 2$ gauge theory

The supergravity solution found in the previous section can provide, through the probe technique, non-trivial information on its dual four-dimensional gauge theory.

According to the probe technique, we consider a fractional D3 brane carrying a gauge field $F_{\mu\nu}$ slowly moving in a given supergravity background. By studying the boundary action of this probe we can get non trivial information on the Coulomb phase of the $SU(M+1)$ gauge theory broken to $SU(M) \times U(1)$ that corresponds to taking the probe at a distance $\rho = |z|$ from the source.

Applying this technique to the background produced by our bound state made of M D3 and N D7 fractional branes, we find that in the probe action all the dependence on the untwisted fields drops out only leaving the following expression:

$$
S_{gauge} = -\frac{1}{g_{\mathrm{YM}}^2(\mu)} \int d^4x \left\{ \frac{1}{2}\partial_\alpha \Phi^i \partial^\alpha \Phi^i + \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} \right\} + \frac{\theta_{\mathrm{YM}}}{32\pi^2} \int d^4x F_{\alpha\beta}\widetilde{F}^{\alpha\beta}
\tag{10}
$$

where

$$\frac{1}{g_{\mathrm{YM}}^2(\mu)} \;=\; \frac{1}{g_{\mathrm{YM}}^2} + \frac{2M-N}{8\pi^2}\log\mu \quad ; \quad g_{\mathrm{YM}}^2 = 8\pi g_s \tag{11}$$

$$\theta_{\mathrm{YM}} \;=\; (2M-N)\,\theta \tag{12}$$

are the effective Yang-Mills gauge coupling and θ-angle, respectively, while $\Phi^i = (2\pi\alpha')^{-1}x^i$ being x^i the two coordinates transverse both to the probe and to the orbifold.

It is interesting to observe that the coefficients in front the two kinetic terms in the expression (10) are the same, this is in agreement with the fact that the gauge theory living on the brane has $\mathcal{N}=2$ supersymmetry.

Eq.(11) clearly shows that $g_{\mathrm{YM}}(\mu)$ is the running coupling constant of an $\mathcal{N}=2$ supersymmetric gauge theory with gauge group $SU(M)$ and N hypermultiplets in the fundamental representation, while the renormalization group scale is related to the supergravity variables by: $\mu \equiv |z|/\epsilon$. Indeed this is the field theory living on the system of M D3-branes and N D7-branes.

Furthermore by studying the boundary action of a fractional D3-probe in the background defined by the eq.s (7) and (8) one sees that on the geometric locus:

$$|z_e| \;=\; \epsilon\,\mathrm{e}^{-\pi/(2M-N)\,g_s} \quad, \tag{13}$$

the probe becomes tensionless, thus indicating the presence of an enhançon. At distances smaller than $|z_e|$ the probe has negative tension, while at the enhançon extra light degrees of freedom come into play [5]. This means that the supergravity approximation leading to the solution described in section 2 is not valid anymore, and that the region of space-time $\rho < |z_e|$ is excised.

A distinctive feature of the D3/D7 system with respect to that of pure fractional D3-branes of Ref. [2] is that the twisted scalars given in the eq.s (7) and (8) are expressed as an infinite series in the open string coupling. However, the scalar field combinations which have a meaning at the gauge theory level are again exact at one-loop, as expected for a $\mathcal{N}=2$ super Yang-Mills theory. This non-trivial cancellation is a (higher loop) check of the validity of the gauge/gravity correspondence.

In conclusion we have seen that by using the classical solution generated by bound states of fractional branes it is possible to have information on the perturbative region of the moduli space of $N=2$ supersymmetric gauge theory.

These solutions, also for the presence in the infrared of a naked singularity, prevent the possibility to perform the decoupling limit, and therefore they seem in contrast with a duality interpretation á la Maldacena where the supergravity solution gives a good description of the gauge theory at large 't Hoof coupling. Our results instead can be easily understood if we regard the classical supergravity solution as a large distance expansion around the asymptotic flat background. The curvature is clearly small and the supergravity is reliable, while the expansion, as explained in Ref. [16], can be seen as a sum over the tree level close string diagrams, or because the open/closed string duality, as a sum over open string loops. From this point of view, the supergravity solutions are considered as an effective way of summing over open string loops and therefore they can only encode the perturbative properties of the $\mathcal{N}=2$ gauge theory living on the world-volume of a fractional D3-brane.

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