

Theory of radiative and rare B decays

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Abstract. We present a concise theoretical overview of radiative and rare B decays mediated by flavour-changing neutral-current transitions of the type $b \rightarrow s(d)\gamma$ and $b \rightarrow s(d)\bar{\ell}\ell$.

INTRODUCTION

Thanks to the efforts of B factories, the exploration of the mechanism of quark-flavour mixing is now entering a new interesting era. The precise measurements of mixing-induced CP violation and tree-level allowed semileptonic transition have provided an important consistency check of the SM, and a precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The next goal is to understand if there is still room for new physics (NP) or, more precisely, if there is still room for new sources of flavour symmetry breaking close to the electroweak scale. From this perspective, radiative and rare B decays mediated by flavour-changing neutral current (FCNC) amplitudes represent a fundamental tool (see e.g. Ref. [1]).

Beside the experimental sensitivity, the conditions which allow to perform significant NP searches in rare decays can be summarized as follows: i) decay amplitude dominated by electroweak dynamics, and thus enhanced sensitivity to non-standard contributions; ii) small theoretical error within the SM, or good control of both perturbative and non-perturbative corrections. In the rest of this talk we shall analyze at which level these conditions are satisfied in various decay modes.

INCLUSIVE FCNC B DECAYS

Inclusive rare B decays such as $B \rightarrow X_s\gamma$, $B \rightarrow X_s\ell^+\ell^-$ and $B \rightarrow X_s\nu\bar{\nu}$ are the natural framework for high-precision studies of FCNCs in the $\Delta B = 1$ sector [2]. Perturbative QCD and heavy-quark expansion form a solid theoretical framework to describe these processes: inclusive hadronic rates are related to those of free b quarks, calculable in perturbation theory, by means of a systematic expansion in inverse powers of the b -quark mass.

The starting point of the perturbative partonic calculation is the determination of a low-energy effective Hamil-

tonian, renormalized at a scale $\mu = \mathcal{O}(m_b)$, obtained by integrating out the heavy degrees of freedom of the theory. For $b \rightarrow s$ transitions –within the SM– this can be written as

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10, \nu} C_i(\mu) Q_i + \text{h.c.} \quad (1)$$

where $Q_{1\dots 6}$ are four-quark operators, Q_8 is the chromomagnetic operator and

$$\begin{aligned} Q_7 &= \frac{e}{4\pi^2} \bar{s}_L \sigma_{\mu\nu} m_b b_R F^{\mu\nu} \\ Q_9 &= \frac{e^2}{4\pi^2} \bar{s}_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \ell \\ Q_{10} &= \frac{e^2}{4\pi^2} \bar{s}_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \gamma_5 \ell \\ Q_\nu &= \frac{e^2}{4\pi^2 s_w^2} \bar{s}_L \gamma^\mu b_L \bar{\nu}_L \gamma_\mu \nu_L \end{aligned} \quad (2)$$

are the leading FCNC electroweak operators. Within the SM, the coefficients of all the operators in Eq. (2) receive a large non-decoupling contribution from top-quark loops at the electroweak scale. But the m_t dependence is not the same for the four operators, reflecting a different $SU(2)_L$ -breaking structure, which can be affected in a rather different way by new-physics contributions [3, 4].

The calculation of the rare decay rates then involves three distinct steps: i) the determination of the initial conditions of the Wilson coefficients at the electroweak scale; ii) the evolution by means of renormalization-group equations (RGEs) of the C_i down to $\mu = \mathcal{O}(m_b)$; iii) the evaluation of the hadronic matrix elements of the effective operators at $\mu = \mathcal{O}(m_b)$, including both perturbative and non-perturbative QCD corrections. Each of the three steps must be taken to matching orders of accuracy in powers of the strong coupling constants α_s and of the large logs generated by the RGE running. The interesting short-distance (electroweak) dynamics that we

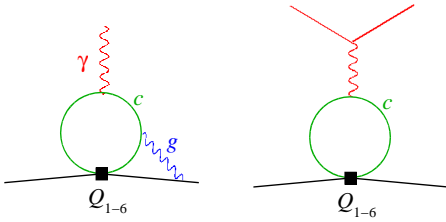


FIGURE 1. Representative diagrams for the mixing of four-quark operators into Q_7 (left) and Q_9 (right).

would like to test enters only in the first step; the following two steps are fundamental ingredients to reduce and control the theoretical error.

The first two steps (initial conditions and RGEs) are process independent and are common also to exclusive modes. Nonetheless, the organization of the leading-log (LL) series is not the same for the three underlying partonic processes, or the four operators in Eq. (2):

$b \rightarrow s\gamma$. Here only Q_7 has a non-vanishing matrix element at the tree level. The large logarithms generated by mixing of four-quark operators into Q_7 (see Fig. 1) play a very important role and enhance the partonic rate by a factor of almost three [5]. Since this mixing vanishes at the one-loop level, a full treatment of QCD corrections beyond lowest order is a rather non-trivial task. This has been achieved already a few years ago, thanks to the joint effort of many authors (see e.g. Ref. [2, 6] and references therein), and is nowadays a rather mature subject. All the ingredients of the partonic calculation have been cross-checked by more than one group. In particular, very recently an independent confirmation of the three-loop mixing of Q_7 and $Q_{1\dots 6}$ [7] – till few months ago the only piece of the calculation performed by one group only – has been presented [8].

$b \rightarrow s\ell^+\ell^-$. The three operators with non-vanishing tree-level matrix elements are Q_7 , Q_9 and Q_{10} . Similarly to Q_7 , QCD corrections are very important also for Q_9 . Since Q_9 mixes with four-quark operators already at the one-loop level (see Fig. 1), the organization of the LL series for $b \rightarrow s\ell^+\ell^-$ is different than in $b \rightarrow s\gamma$: the NLL level is much simpler (no three-loop mixing involved), but less precise. An accuracy below the 10% level on the decay rate (a precision similar to the NLL level in $b \rightarrow s\gamma$) is reached here only with a NNLL calculation. All the missing ingredients to reach this goal has finally become available. In particular, NNLL initial conditions and anomalous dimension matrix can be found in [9] and [8], respectively.

It is worth to stress that the impact of QCD corrections is very limited in the axial-current operator Q_{10} . This operator does not mix with four-quark operators and is completely dominated by short-distance contri-

butions. Together with Q_v , Q_{10} belongs to the theoretically clean $\mathcal{O}(G_F^2)$ hard-GIM-protected part of the effective Hamiltonian. Thus observables more sensitive to Q_{10} , such as the forward-backward (FB) lepton asymmetry in $b \rightarrow s\ell^+\ell^-$, have a reduced QCD uncertainty and a stronger sensitivity to possible non-standard phenomena.

$b \rightarrow s\nu\bar{\nu}$. In this case only Q_v is involved. Similarly to Q_{10} , QCD corrections play a very minor role since there is no mixing with four-quark operators. As a result, the only non-trivial step of the perturbative calculation for $b \rightarrow s\nu\bar{\nu}$ decays is the determination of the initial condition of C_v at the electroweak scale: this is known with a precision around 1% within the SM [10].

These two processes-independent steps of the calculation, namely the determination of the effective Hamiltonian renormalized at a low scale $\mu = \mathcal{O}(m_b)$, can easily be transferred from the $b \rightarrow s$ case to the $b \rightarrow d$ one. The only difference is the richer CKM structure of the $b \rightarrow d$ Hamiltonian, with two independent non-negligible terms ($V_{td}^*V_{tb}$ and $V_{ud}^*V_{ub}$).

The situation is very different for the last step of the calculation, namely the evaluation of the hadronic matrix elements. The latter strongly depend on the specific process and the specific observable we are interested in (e.g. fully inclusive rate or differential distribution). In the following we shall review the results of this step (and thus the final numerical predictions) for some of the most interesting $b \rightarrow s$ observables.

$B \rightarrow X_s\gamma$ [the most effective “NP killer”]

The inclusive $B \rightarrow X_s\gamma$ rate is the most precise and clean short-distance information that we have, at present, on $\Delta B = 1$ FCNCs. Combining the precise measurements by ALEPH, BaBar, Belle and CLEO, the world average reads [11]

$$\mathcal{B}(B \rightarrow X_s\gamma)^{\text{exp}} = (3.34 \pm 0.38) \times 10^{-4} \quad (3)$$

On the theory side, the NLL partonic calculation of the matrix elements, performed first in Ref. [12] for the leading terms, has recently been cross-checked and completed in Ref. [13]. Perturbative corrections due to higher-order electroweak effects have also been analyzed (see Ref. [14] and references therein).

Non-perturbative $1/m_b$ corrections are well under control in the total rate. In particular, $\mathcal{O}(1/m_b)$ corrections vanish in the ratio $\Gamma(B \rightarrow X_s\gamma)/\Gamma(B \rightarrow X_c\ell\nu)$, and the $\mathcal{O}(1/m_b^2)$ ones are known and amount to few per cent [15]. Also non-perturbative effects associated to charm-quark loops have been estimated and found to be very small [16, 17, 18]. The most serious problem of non-perturbative origin is related to the (unavoidable) experimental cut in the photon energy spectrum that prevents

the measurement from being fully inclusive [17, 19]. With the present cut by CLEO $E_\gamma > 2.0$ GeV [20], this uncertainty is smaller but non-negligible with respect to the error of the perturbative calculation. The latter is around 10% and its main source is the uncertainty in the ratio m_c/m_b that enters through charm-quark loops [21].

According to the detailed analysis of theoretical errors presented in Ref. [21], the SM expectation is

$$\mathcal{B}(B \rightarrow X_s \gamma)^{\text{SM}} = (3.73 \pm 0.30) \times 10^{-4}, \quad (4)$$

in good agreement with Eq. (3). It must be stressed that the overall scale dependence is very small: for $\mu \in [m_b/2, 2m_b]$ the central value moves by about 1%. The error in Eq. (4) is an educated guess about the size of possible NNLL terms. In particular, the largest source of uncertainty is obtained by the variation of $\bar{m}_c(\mu)/m_b^{\text{pole}}$ for $\mu \in [m_c, m_b]$. A critical discussion about the error in Eq. (4), with alternative more conservative estimates, can be found in Ref. [22].

The comparison between theory and experiments in $\mathcal{B}(B \rightarrow X_s \gamma)$ is a great success of the SM and has led us to derive many significant bounds on possible new-physics scenarios. For instance, the $\mathcal{B}(B \rightarrow X_s \gamma)$ constraint is one of the main obstacles to build consistent models that predict a sizable difference between $\mathcal{A}_{\text{CP}}(B \rightarrow \phi K_S)$ and $\mathcal{A}_{\text{CP}}(B \rightarrow \psi K_S)$. This constraint can be avoided (see e.g. Ref. [23] and references there in), but the resulting models require a considerable amount of fine tuning. By far more natural are the so-called MFV models [4], where $\mathcal{A}_{\text{CP}}(B \rightarrow \phi K_S) \approx \mathcal{A}_{\text{CP}}(B \rightarrow \psi K_S)$ and deviations from the SM in $\mathcal{B}(B \rightarrow X_s \gamma)$ do not exceed the 10%–30% level [4]. Improved measurements of $\mathcal{B}(B \rightarrow X_s \gamma)$ are certainly useful to further constrain this possibility. However, since the experimental error has reached the level of the theoretical one, it will be very difficult to clearly identify possible deviations from the SM, if any, in this observable.

Hopes to detect new-physics signals are still open through the CP-violating asymmetry

$$\Delta\Gamma_{\text{CP}}(B \rightarrow X_s \gamma) = \frac{\Gamma(B \rightarrow X_s \gamma) - \Gamma(\bar{B} \rightarrow X_s \gamma)}{\Gamma(B \rightarrow X_s \gamma) + \Gamma(\bar{B} \rightarrow X_s \gamma)}. \quad (5)$$

This is expected to be below 1% within the SM [22, 24], but could easily reach $\mathcal{O}(10\%)$ values beyond the SM, even in the absence of large effects in the total $B \rightarrow X_s \gamma$ rate. This is indeed one of the main expectations in models with sizable NP effects in $\mathcal{A}_{\text{CP}}(B \rightarrow \phi K_S)$. The present measurement of $\Delta\Gamma_{\text{CP}}(B \rightarrow X_s \gamma)$ is consistent with zero [11], but the sensitivity is still one order of magnitude above the SM level.

$B \rightarrow X_s \ell^+ \ell^-$ [*the present frontier*]

Both Belle [25] and BaBar [26] have recently announced a clear evidence ($\approx 5\sigma$) of the $B \rightarrow X_s \ell^+ \ell^-$ decay. The two results are compatible and are both based on a semi-inclusive analysis (the hadronic system is reconstructed from a kaon plus 0 to 4 pions, with at most one π^0). Their combination [11]

$$\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)^{\text{exp}} = (6.2 \pm 1.1_{-1.3}^{+1.6}) \times 10^{-6}. \quad (6)$$

represents a very useful new piece of information about $\Delta B = 1$ FCNCs, with considerable margin of improvement in the near future.

In principle, these decays offer a phenomenology richer than $B \rightarrow X_s \gamma$, with more than one interesting observable. The joint effort of several groups has recently allowed to evaluate at the NNLL level all the matrix element necessary for the two main kinematical distributions: the dilepton spectrum [27, 28, 29] and the lepton FB asymmetry [30, 31].

In addition to the non-perturbative corrections due to the finite b quark mass, $B \rightarrow X_s \ell^+ \ell^-$ transitions suffer from specific non-perturbative effects due to long-lived $c\bar{c}$ intermediate states ($B \rightarrow X_s c\bar{c} \rightarrow X_s \ell^+ \ell^-$). The heavy-quark expansion, which allow to evaluate the Λ_{QCD}/m_b terms, is rapidly convergent and leads to small corrections for *sufficiently inclusive* observables [32, 33]. A consistent treatment of the second type of effects requires *kinematical cuts* in order to avoid the large non-perturbative background of the narrow $c\bar{c}$ resonances (see Fig. 2). These two requirements are somehow in conflict [28, 33]; nonetheless, we can identify two perturbative windows, defined by:

$$\begin{aligned} q^2 &\equiv M_{\ell^+ \ell^-}^2 \in [1 \text{ GeV}^2, 6 \text{ GeV}^2] \quad (\text{low}), \\ q^2 &> 14.4 \text{ GeV}^2 \quad (\text{high}), \end{aligned}$$

where reliable predictions can be performed [28]. It is worth to emphasize that the two regions have complementary virtues and disadvantages:

- *Virtues of the low- q^2 region:* reliable q^2 spectrum; small $1/m_b$ corrections; sensitivity to the interference of C_7 and C_9 ; high rate.
- *Disadvantages of the low- q^2 region:* difficult to perform a fully inclusive measurement (severe cuts on the dilepton energy and/or the hadronic invariant mass); long-distance effects due to processes of the type $B \rightarrow \Psi X_s \rightarrow X_s + X' \ell^+ \ell^-$ not fully under control; non-negligible scale and m_c dependence.
- *Virtues of the high- q^2 region:* negligible scale and m_c dependence due to the strong sensitivity to $|C_{10}|^2$; easier to perform a fully inclusive measurement (small hadronic invariant mass); negligible long-distance effects of the type $B \rightarrow \Psi X_s \rightarrow X_s + X' \ell^+ \ell^-$.

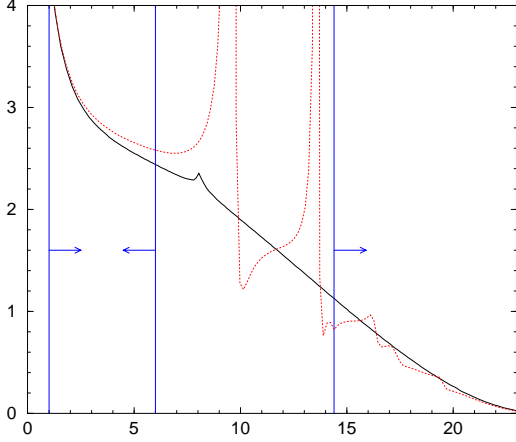


FIGURE 2. Dilepton spectrum of the inclusive $B \rightarrow X_s e^+ e^-$ decay within the SM. Vertical axis: $d\mathcal{B}(B \rightarrow X_s e^+ e^-)/dq^2$ in units of $10^{-7} \times \text{GeV}^{-2}$; horizontal axis: q^2 in GeV^2 . The dotted line denotes the NNLL pure perturbative result, the full line includes an estimates of the non-perturbative $c\bar{c}$ effects [28].

- *Disadvantages of the high- q^2 region:* q^2 spectrum not reliable (only the integrated rate can be predicted); sizeable $1/m_b$ corrections (effective expansion in $1/(m_b - \sqrt{q_{\min}})$ [28]); low rate.

Given this situation, we believe that future experiments should try to measure the branching ratios in both regions and report separately the two results. These two measurements are indeed affected by different systematic uncertainties (of theoretical nature) and provide a different short-distance information. The NNLL SM predictions for the two clean windows [28],

$$\begin{aligned} \mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{\text{low}}^{\text{SM}} &= (1.63 \pm 0.20) \times 10^{-6}, \\ \mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{\text{high}}^{\text{SM}} &= (4.04 \pm 0.78) \times 10^{-7}, \end{aligned} \quad (7)$$

are still affected by a considerable error; however, in both cases the uncertainty is mainly of parametric nature and could be substantially improved in the future. In particular, the large error in the high- q^2 region is mainly due to the uncertainty in the relation between the physical q^2 interval and the corresponding interval for the partonic calculation (i.e. the uncertainty in the relation between m_b and the physical hadron mass), which can be improved with better data on charged-current semileptonic modes. According to the recent analysis of Ref. [29], both results in (7) should be decreased by $\approx 4\%$ to take into account the leading electroweak corrections.

The two results in (7) cannot be directly confronted with (6), which includes an extrapolation to the full q^2 spectrum. The updated SM expectation for this extrapolated branching ratio is $(4.6 \pm 0.8) \times 10^{-6}$ [28] (in agreement with the previous estimate of Ref. [34]), and it is consistent with the present experimental world average.

We stress that this prediction is already saturated by irreducible theoretical errors and, contrary to the results in (7), is very difficult to improve it further.

As anticipated, some of the most interesting short-distance tests in $\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)$ decays can be performed by means of the FB asymmetry of the dilepton distribution:

$$\mathcal{A}_{\text{FB}}(q^2) = \frac{1}{d\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)/dq^2} \int_{-1}^1 d \cos \theta_\ell \frac{d^2 \mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)}{dq^2 d \cos \theta_\ell} \text{sgn}(\cos \theta_\ell), \quad (8)$$

where θ_ℓ is the angle between ℓ^+ and B momenta in the dilepton centre-of-mass frame. Here the SM predict a zero for $s_0 = q_0^2/m_b^2 = 0.162 \pm 0.008$ [30, 31]: a very precise prediction which could easily be modified beyond the SM, even in absence of significant non-standard effects on the total rate.

EXCLUSIVE MODES

On general grounds, theoretical predictions for exclusive FCNC decays are more difficult. The simplest cases are processes with at most one hadron in the final state. Here there has been a substantial progress in the last few years, both by means of analytic approaches [35] and by means of Lattice QCD [36], but still the overall theoretical uncertainty is around 20% at the amplitude level. The largest source of uncertainty is typically the normalization of the hadronic form factors, an error that can be substantially reduced in appropriate ratios or differential distributions. These type of observables become particularly interesting in channels where, because of irreducible experimental problems, the short-distance amplitude cannot be extracted from corresponding inclusive modes. Two of such examples are the ratio

$$R_\gamma(\rho/K^*) = \frac{\mathcal{B}(B \rightarrow \rho \gamma)}{\mathcal{B}(B \rightarrow K^* \gamma)}, \quad (9)$$

and the normalized FB asymmetry in $B \rightarrow K^* \ell^+ \ell^-$.

The ratio $R_\gamma(\rho/K^*)$ is one of the most promising tool to extract short-distance properties about the $b \rightarrow s \gamma$ amplitude. On the experimental side, the combination of the bounds on charged and neutral channels, in the isospin limit, leads to $R_\gamma(\rho/K^*) < 0.047$ at 90% C.L. [37]. On the theory side, the $B \rightarrow V \gamma$ amplitudes which determine this ratio have been analyzed beyond naive factorization by several authors [38, 39, 40]. Within the SM one can write

$$R_\gamma(\rho/K^*) = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{(m_B^2 - m_{rho}^2)^3}{(m_B^2 - m_{K^*}^2)^3} \zeta^2 (1 - \Delta R), \quad (10)$$

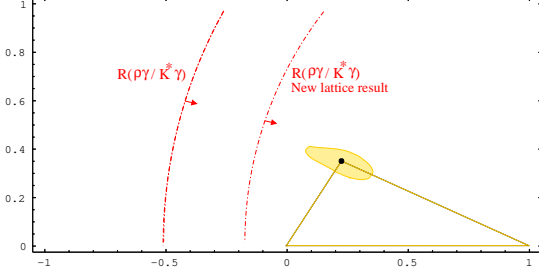


FIGURE 3. Bounds on the ρ - η plane from the BaBar bound on $R_\gamma(\rho/K^*)$: the two curves correspond to different values of ζ [39].

where ζ denotes the ratio of the form factors at $q^2 = 0$ in the $m_b \rightarrow \infty$ limit, and ΔR the additional $SU(3)$ (and isospin) breaking due to $1/m_b$ and $\mathcal{O}(\alpha_s)$ corrections. The largest source of uncertainty is ζ : according to the light-cone sum rule estimate $\zeta = 0.76 \pm 0.10$ [39], the present experimental bound on $R_\gamma(\rho/K^*)$ is about twice the SM expectation. However, preliminary Lattice results indicate a larger value, $\zeta = 0.91 \pm 0.08$ [41], which would imply more stringent bounds on $|V_{td}/V_{ts}|$. The two curves in Fig. 3 summarizes the present status.

While the inclusive $B \rightarrow X_s \ell^+ \ell^-$ rate is already accessible at the B factories, a differential study of the inclusive FB asymmetry is beyond their present and near-future reach. More accessible from the experimental point of view is the FB asymmetry in $B \rightarrow K^* \ell^+ \ell^-$ (defined as in Eq. (8) with $X_s \rightarrow K^*$). Assuming that the leptonic current has only a vector (V) or axial-vector (A) structure (as in the SM), the FB asymmetry provides a direct measure of the A - V interference. Indeed, at the lowest-order one can write

$$\mathcal{A}_{\text{FB}}(q^2) \propto \text{Re} \left\{ C_{10}^* \left[\frac{q^2}{m_b^2} C_9^{\text{eff}} + r(q^2) \frac{m_b C_7}{m_B} \right] \right\},$$

where $r(q^2)$ is an appropriate ratio of $B \rightarrow K^*$ vector and tensor form factors [42]. There are three main features of this observable that provide a clear and independent short-distance information: 1) The position of the zero of $\mathcal{A}_{\text{FB}}(q^2)$ in the low- q^2 region (see Fig. 4) [42]. As shown by means of a full NLO calculation [40], the experimental measurement of q_0^2 could allow a determination of C_7/C_9 at the 10% level. 2) The sign of $\mathcal{A}_{\text{FB}}(q^2)$ around the zero. This is fixed unambiguously in terms of the relative sign of C_{10} and C_9 [43]: within the SM one expects $\mathcal{A}_{\text{FB}}(q^2 > q_0^2) > 0$ for $|\bar{B}\rangle \equiv |b\bar{d}\rangle$ mesons. 3) The relation $\mathcal{A}[\bar{B}]_{\text{FB}}(q^2) = -\mathcal{A}[B]_{\text{FB}}(q^2)$. This follows from the CP-odd structure of \mathcal{A}_{FB} and holds at the 10^{-3} level within the SM [43], where C_{10} has a negligible CP-violating phase.

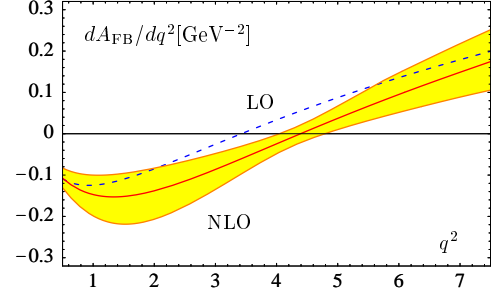


FIGURE 4. Zero of the forward-backward asymmetry in $B^- \rightarrow K^{*-} \ell^+ \ell^-$ at LO and NLO. The band reflects all theoretical uncertainties from parameters and scale dependence combined [40].

$B_{s,d} \rightarrow \ell^+ \ell^-$ [the future frontier]

The purely leptonic decays constitute a special case among exclusive transitions. Within the SM only the axial-current operator, Q_{10} , induces a non-vanishing contribution to these decays. As a result, the short-distance contribution is not *diluted* by the mixing with four-quark operators. Moreover, the hadronic matrix element involved is the simplest we can consider, namely the B -meson decay constant

$$\langle 0 | \bar{q} \gamma_\mu \gamma_5 b | \bar{B}_q(p) \rangle = i p_\mu f_{B_q} \quad (11)$$

Reliable estimates of f_{B_d} and f_{B_s} are obtained at present from lattice calculations and in the future it will be possible to cross-check these results by means of the $B^+ \rightarrow \ell^+ \nu$ rate. Modulo the determination of f_{B_q} , the theoretical cleanliness of $B_{s,d} \rightarrow \ell^+ \ell^-$ decays is comparable to that of the *golden modes* $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $B \rightarrow X_{s,d} \nu \bar{\nu}$.

The price to pay for this theoretically-clean amplitude is a strong helicity suppression for $\ell = \mu$ (and $\ell = e$), or the channels with the best experimental signature. Employing the full NLO expression of C_{10} [10], we can write

$$\begin{aligned} \mathcal{B}(B_s \rightarrow \mu^+ \mu^-)^{\text{SM}} &= 3.1 \times 10^{-9} \left(\frac{|V_{ts}|}{0.04} \right)^2 \\ &\times \left(\frac{f_{B_s}}{0.21 \text{ GeV}} \right)^2 \left(\frac{\tau_{B_s}}{1.6 \text{ ps}} \right) \left(\frac{m_t(m_t)}{166 \text{ GeV}} \right)^{3.12} \\ \frac{\mathcal{B}(B_s \rightarrow \tau^+ \tau^-)^{\text{SM}}}{\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)^{\text{SM}}} &= 215. \end{aligned}$$

The corresponding B_d modes are both suppressed by an additional factor $|V_{td}/V_{ts}|^2 = (4.0 \pm 0.8) \times 10^{-2}$. The present experimental bound closest to SM expectations is the one obtained by CDF on $B_s \rightarrow \mu^+ \mu^-$ [11, 44]:

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) < 9.5 \times 10^{-7} \quad (95\% \text{ CL}),$$

which is still very far from the SM level. The latter will certainly not be reached before the LHC era.

As emphasized in the recent literature [45, 46, 47], the purely leptonic decays of B_s and B_d mesons are excellent probes of several new-physics models and, particularly, of scalar FCNCs. Scalar FCNC operators, such as $\bar{b}_{RS}L\bar{\mu}_R\mu_L$, are present within the SM but are absolutely negligible because of the smallness of down-type Yukawa couplings. On the other hand, these amplitudes could be non-negligible in models with an extended Higgs sector. In particular, within the MSSM, where two Higgs doublets are coupled separately to up- and down-type quarks, a strong enhancement of scalar FCNCs can occur at large $\tan\beta = v_u/v_d$ [45]. This effect is very small in non-helicity-suppressed B decays and in K decays (because of the small Yukawa couplings), but could enhance $B \rightarrow \ell^+\ell^-$ rates by orders of magnitude. As pointed out in Ref. [48], $\mathcal{O}(100)$ enhancements in $\mathcal{B}(B \rightarrow \ell^+\ell^-)$ correspond to $\mathcal{O}(10\%)$ breaking of universality in $\mathcal{B}(B \rightarrow K\mu^+\mu^-)$ vs. $\mathcal{B}(B \rightarrow Ke^+e^-)$. Therefore, the present search for $B \rightarrow \ell^+\ell^-$ at CDF is already quite interesting, even if the sensitivity is well above the SM level. In a long-term perspective, the discovery of such processes is definitely one of the most interesting items in the B -physics program of hadron colliders.

CONCLUSIONS

Rare FCNC decays of B mesons provide a unique opportunity to perform high-precision studies of quark-flavour mixing. The $B \rightarrow X_s\gamma$ rate, where both experimental and theoretical errors have reached a comparable level around 10%, represents the highest peak in our present knowledge of FCNCs. The lack of deviations from SM expectations in $\Gamma(B \rightarrow X_s\gamma)$ should not discourage the measurement of other clean and independent FCNC observables, such as the forward-backward asymmetry in $B \rightarrow X_s\ell^+\ell^-$ or the $B \rightarrow \ell^+\ell^-$ rates. Even if new physics will first be discovered elsewhere, the experimental study of these theoretically-clean observables would still be very useful to investigate the flavour structure of any new-physics scenario.

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