Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2014, Article ID 835254, 8 pages
http://dx.doi.org/10.1155/2014/835254

Research Article

Research on Multiprincipals Selecting Effective Agency Mode in the Student Loan System

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Received 19 February 2014; Accepted 9 April 2014; Published 8 May 2014

Academic Editor: Ming Gao

An effective agency mode is the key to solve incentive problems in Chinese student loan system. Principal-agent frameworks are considered in which two principals share one common agent that is performing one single task but each prefers the different aspect of the task. Three models are built and decision mechanisms are given. The studies show that the three modes have different effects. Exclusive dealing mode is not good for long-term effect because sometimes it guides agent ignoring repayment. If effort proportionality coefficient and observability are both unchanged, principals all prefer common agency, but independent contracting mode may be more efficient in reality because not only the total outputs under that mode are larger than those under cooperation one, but also preferring independent contracting mode can stimulate the bank participating in the game.

1. Introduction

An effective management structure is a necessary condition for the student loans operation. Different countries have different structures of loan management system, such as bank, state agency, and other types of organization. In China, the student loans are operated by the most basic level agency called county-student financial assistance center which is regulated by government, and the funds are provided by policy bank. In this structure, one agent faces two or more principals; namely, various principals share one common agent. In these situations, conflicts typically arise among principals when the agent uses its time and effort to different principals; moreover, the agent's moral hazard and adverse selection problems can make the conflicts complicated. Usually, incentives must be provided to induce optimal performance when the agent's effort or his ability is unobservable, but the incentives provided by different principals could affect each other, which can decide how to use its time and effort alternatively by the agent.

Traditional principal-agent theory has offered many techniques dealing with optimal performance in principal-agent problems; some new techniques dealing with optimization problems in ambiguity environment are discussed by a study group [1–4], and backward stochastic differential equations are used in their important works in this field to deal with more complex problems [5–8]. In our study, multiprincipals sharing one agent which was called common agent and how to select an optimal agent mode are the core. Different principals sharing a common agent were first developed in the seminal paper of Bernheim and Whinston [9, 10]. In their studies, different principals simultaneously and independently influence a common agent. While complete and incomplete information were both contained in the studies, they show that implementation is always efficient and that noncooperative behavior induces an efficient action choice if and only if collusion among the principals would implement the first-best action at the first-best level of cost. They also investigate the existence of equilibria, the distribution of net rewards among principals, the characteristics of actions chosen in inefficient equilibria, and potential institutional remedies for welfare losses induced in noncooperative behavior. Subsequently, the studies about common agency are blooming so that more and more scholars focus on incentives in common agency, among which Martimort's series of work [11–17] forms a study framework of multiprincipals;

others also contribute to characteristics of common agency game equilibrium [18–22]. Some researchers are interested in designing incentive mechanism [14, 16, 23] and pay attention to agent facing multitasks [24, 25], and others pay more attention to the cooperation and competition among multiprincipals [24, 26, 27]. In the field of application, in addition to common sales agency problems, financial and insurance market, tax competition, and auction, researchers focus on multiprincipals problems of regulation or organizational design [12, 13, 28–31].

In Chinese current student loan system, the government is not only the regulator, but also a principal, who designs the management structure and selects the bank which takes part in the student loan system. So in this current paper, we consider a principal-agent framework, in which the model has multiple principals (basic level government and policy bank) and one single agent (staff of county-student financial assistance center) performing one single task, but the two principals have different preference in the same task's different aspects. From the government's point of view, the objective of the loan policy is to achieve the maximum of social welfare. The government hopes that students, as many as possible, from families with financial difficulties could be able to obtain loans to solve education problem. In the long run, the government's concern should not be the number of students who obtain loans but the students' repayment in order to facilitate the repeated game and obtain the long-term cooperation with banks. But in reality, the government often pays more attention to the short-term effect, which is manifested as its excessive emphasis on the quantity of students accepted by the agent, but does not pay enough attention to the effort of agent urging borrowers' repayment. In other words, the government prefers the agent paying more effort to handling more loan contracts. On the contrary, from the bank's point of view, more loan agreements often mean more benefits along with more risks; the bank pays more attention to the repayment of those students, so the agent's effort to urge the borrowers to repay on time is the key to the bank. In other words, the bank's preference is the effort to urge students to repay the loans. Resolving the conflicts of different preferences of principals is the key to guarantee the effective implementation of the student loans policy. In our hypothesis, the task's two aspects are regarded as two alternative tasks because the agent must reduce the effort and time in one aspect when he wants to take another aspect seriously. So incentive contracts offered by principals are the key to solve the conflict and meanwhile maximize their own profit.

In Mezzetti's model [27], the single agent performs related tasks for different principals who are horizontally differentiated and each principal requires that a task should be performed. The equilibrium under cooperation between two principals, exclusive dealing, and independent contracting are discussed in Mezzetti's article. Firstly, the principals offer the common agent an incentive contract that maximizes their joint payoff under cooperation. Secondly, each principal chooses an incentive contract noncooperatively and cannot contract on the agent's output for the other principal under independent contracting. Thirdly, each principal makes contract with a different, but ex ante identical, agent under exclusive dealing. In our paper, ideas are borrowed from Mezzetti [27] to discuss the incentive contracts offered by government and bank (principals) to county-student financial assistance center staff (agent) and help the principals having different preferences select effective agency mode. In any kind of agency mode, the agent will select the optimal effort level to maximize his expected utility when his effort or ability is unobservable.

2. Major Assumptions and Variable Declaration

(1) Two principals i ($i = 1, 2$; 1 is the government; 2 is the bank) contract with a common agent (county-student financial assistance center staff) to perform the student-loan-management task. Government prefers the agent paying more effort to handling more loan contracts; the bank prefers more the agent's efforts to urge students' repayment. The principals are all risk neutral whose expected utility is equal to their expected return. The agent is risk averse: his utility function has the characteristics of constantly absolute risk aversion and $\rho = -u''/u' > 0$ is the parameter of risk aversion degree.

(2) Principal's utility function is v_i; the agent's corresponding utility function is u_1, u_2; reservation wage $\overline{\omega}_i > 0, \overline{\omega}_2 > \overline{\omega}_1 > 0$, means the agent's opportunity income obtaining from the bank is higher than that from the government.

(3) The effort level a_1, a_2, agent working for different principals' preference, is unobservable. Let $k_i > 0$ be the proportionality coefficient between agent's effort and his output for two principals. θ_i is private information of the agent and as a random variable, normally distributed in $[0, \sigma_i^2]$: variance $\sigma_1^2 < \sigma_2^2$ means the bank's preference is more difficult than government's preference to be completed. Thus, the agent's output on principals' task is $\pi_i = k_i a_i + \theta_i$.

(4) Let α_i and β_i be the flat fee and the incentive coefficient, respectively, that each principal pays to the agent. The principals offer incentives contracts to the agent, and the agent's payoff is

$$s(\pi_i) = \alpha_i + \beta_i \pi_i, \quad (0 \leq \beta_i \leq 1). \tag{1}$$

(5) The common agent's effort cost in different tasks is alternative; let t be the alternative coefficient; $t = 1$ means the maximum alternative. The cost function is $C(a_1, a_2) = a_1^2/2 + a_2^2/2 - t a_1 a_2, 0 \leq t \leq 1$. The cost function in exclusive dealing mode is $C(a_i) = a_i^2/2$.

3. Exclusive Dealing Mode

Under exclusive dealing mode, the optimal incentive contracts offered by two principals exclusively are similar to different principals selecting different agents and offering his agent exclusive contract, which is a model containing the single principal and single agent. Thus, the agent's real income is

$$\omega_i = s(\pi_i) - c(a_i) = \alpha_i + \beta_i k_i (a_i + \theta) - \frac{a_i^2}{2}, \tag{2}$$

and the agent's certainty equivalence wealth (CEW) is

$$\overline{\omega}_i = E\omega_i - \frac{\rho\beta_i^2\sigma_i^2}{2} = \alpha_i + \beta_i k_i a_i - \frac{a_i^2}{2} - \frac{\rho\beta_i^2\sigma_i^2}{2}. \tag{3}$$

Under exclusive dealing incentive contract, each agent, using his reservation wage as a benchmark, performs his task maximizing his own certainty equivalence wealth.

The agent's incentive compatibility constraint (IC) is

$$\max_{a_i}\left(\alpha_i + \beta_i k_i a_i - \frac{a_i^2}{2} - \frac{\rho\beta_i^2\sigma_i^2}{2}\right), \tag{4}$$

and the agent's individual rationality constraint (IR) is

$$\alpha_i + \beta_i k_i a_i - \frac{a_i^2}{2} - \frac{\rho\beta_i^2\sigma_i^2}{2} \geq \overline{\omega}_i. \tag{5}$$

Each risk-neutral principal's expected utility, equal to his expected return, is

$$\begin{aligned} Ev_i[\pi_i - s(\pi_i)] &= v_i\{E[\pi_i - s(\pi_i)]\} \\ &= v_i[-\alpha_i + (1-\beta_i)k_i a_i] \\ &= -\alpha_i + (1-\beta_i)k_i a_i, \end{aligned} \tag{6}$$

and each principal will select the optimal incentive scheme (α_i, β_i), to maximize his own expected income.

The model is

$$\begin{aligned} &\max_{\alpha_i,\beta_i} \quad [-\alpha_i + (1-\beta_i)k_i a_i] \\ &\text{s.t.} \quad (\text{IR})\ \alpha_i + \beta_i k_i a_i - \frac{a_i^2}{2} - \frac{\rho\beta_i^2\sigma_i^2}{2} \geq \overline{\omega}_i \\ &\qquad (\text{IC})\ \max_{a_i} CE_A = \overline{\omega}_i = \left(\alpha_i + \beta_i k_i a_i - \frac{a_i^2}{2} - \frac{\rho\beta_i^2\sigma_i^2}{2}\right). \end{aligned} \tag{7}$$

Under each optimal incentive scheme (α_i, β_i), the agent's IC should ensure maximizing his CEW, $\overline{\omega}_i$, and the first-order condition is

$$\frac{\partial\overline{\omega}_i}{\partial a_i} = \beta_i k_i - a_i = 0, \quad \text{thus, } a_i = \beta_i k_i. \tag{8}$$

We denote by $\beta_E^*, a_{Ei}^*, \alpha_E^*$ (subscript E on behalf of the exclusive dealing situation) the second-best solution when feeding IC, IR, and formula (8) to objective function. The second-best solution is

$$\begin{aligned} &\beta_E^* = \frac{k_i^2}{k_i^2 + \rho\sigma_i^2}, \qquad a_{Ei}^* = \frac{k_i^3}{k_i^2 + \rho\sigma_i^2}, \\ &\alpha_E^* = \overline{\omega}_i + \frac{\rho\sigma_i^2 k_i^4 + k^6 - 2k^9}{2(k_i^2 + \rho\sigma_i^2)^2}. \end{aligned} \tag{9}$$

Proposition 1. *Under exclusive dealing mode, the decision mechanism of principal is to determine the second-optimal incentive coefficient which satisfies the following:*

$$\beta_{Ei}^* = \frac{k_i^2}{k_i^2 + \rho\sigma_i^2}. \tag{10}$$

In order to obtain the agent's optimal response

$$a_{Ei}^* = \frac{k_i^3}{k_i^2 + \rho\sigma_i^2}. \tag{11}$$

The incentive coefficient was determined by the agent's risk aversion degree, variances, and proportionality coefficient.

4. Independent Contracting Mode

Under independent contracting mode, each principal designs incentive contract to common agent noncooperatively meanwhile maximizing his own profit:

$$s(\pi_i) = \alpha_i + \beta_i \pi_i, \quad (0 \leq \beta_i \leq 1), \tag{12}$$

and the agent's effort costs in two principals' preference are correlative. In two principals' separate incentive mechanism, the agent's response selects the optimal effort level to adapt to the incentive contracts; meanwhile its IC should ensure that its separate real income is not less than the separate $\overline{\omega}_i$, and the IR should ensure maximizing agent's own total CEW:

$$CE_A = \alpha_1 + \alpha_2 + \beta_1 k_1 a_1 + \beta_2 k_2 a_2 - \frac{\rho\beta^T\Sigma\beta}{2} - C(a_1, a_2). \tag{13}$$

Principals will determine their separate optimal incentive scheme (α_i, β_i), and their maximization problems can be written as follows:

$$\begin{aligned} &\max_{\alpha_i,\beta_i} \quad [-\alpha_i + (1-\beta_i)k_i a_i] \\ &\text{s.t.} \quad (\text{IR})\ \alpha_i + \beta_i k_i a_i - \frac{\rho\beta_i^2\sigma_i^2}{2} - C(a_1, a_2) \geq \overline{\omega}_i \\ &\qquad (\text{IC})\ \max_{a_1,a_2} CE_A = \alpha_1 + \alpha_2 + \beta_1 k_1 a_1 + \beta_2 k_2 a_2 \\ &\qquad\qquad - \frac{\rho\beta^T\Sigma\beta}{2} - C(a_1, a_2) \end{aligned} \tag{14}$$

and the results of calculating the partial derivative of CEW about a_1, a_2 are

$$\begin{aligned} \frac{\partial CE_A}{\partial a_1} &= \beta_1 k_1 - a_1 + t a_2 \\ \frac{\partial CE_A}{\partial a_2} &= \beta_2 k_2 - a_2 + t a_1, \\ a_1 &= \frac{\beta_1 k_1 + t\beta_2 k_2}{1 - t^2} \\ a_2 &= \frac{\beta_2 k_2 + t\beta_1 k_1}{1 - t^2}. \end{aligned} \tag{15}$$

Feed a_1, a_2 into IC, and then get results as follows:

$$\alpha_1 = \overline{\omega}_1 - \beta_1 k_1 a_1 + \frac{\rho\beta_1^2\sigma_1^2}{2} + \frac{a_1^2}{2} + \frac{a_2^2}{2} - t a_1 a_2,$$
$$\alpha_2 = \overline{\omega}_2 - \beta_2 k_2 a_2 + \frac{\rho\beta_2^2\sigma_2^2}{2} + \frac{a_1^2}{2} + \frac{a_2^2}{2} - t a_1 a_2. \quad (16)$$

Feed α_i into two principals' separate objective function (subscript I on behalf of the independent contracting situation):

$$\max v_{I1} = -\overline{\omega}_1 - \frac{\rho\beta_1^2\sigma_1^2}{2} - \frac{a_1^2}{2} - \frac{a_2^2}{2} + t a_1 a_2 + k_1 a_1$$
$$= -\overline{\omega}_1 - \frac{\rho\beta_1^2\sigma_1^2}{2} - \left(\left(\beta_1^2 k_1 + \beta_2^2 k_2^2\right) + 2t\left(\beta_1 k_1 + \beta_2 k_2\right) - 2k_1\left(\beta_1 k_1 + t\beta_2 k_2\right)\right)\left(2\left(1-t^2\right)\right)^{-1},$$
$$\max v_{I2} = -\overline{\omega}_2 - \frac{\rho\beta_2^2\sigma_2^2}{2} - \frac{a_1^2}{2} - \frac{a_2^2}{2} + t a_1 a_2 + k_2 a_2$$
$$= -\overline{\omega}_2 - \frac{\rho\beta_2^2\sigma_2^2}{2} - \left(\left(\beta_1^2 k_1 + \beta_2^2 k_2^2\right) + 2t\left(\beta_1 k_1 + \beta_2 k_2\right) - 2k_2\left(\beta_2 k_2 + t\beta_1 k_1\right)\right)\left(2\left(1-t^2\right)\right)^{-1}. \quad (17)$$

Calculate the partial derivative of the previous two formulas about β_1, β_2. We have

$$\beta_1 = \frac{k_1^2 - t k_1 k_2 \beta_2}{\rho\sigma_1^2\left(1-t^2\right) + k_1^2},$$
$$\beta_2 = \frac{k_2^2 - t k_1 k_2 \beta_1}{\rho\sigma_2^2\left(1-t^2\right) + k_2^2}. \quad (18)$$

We denote by $\beta_{I1}^*, \beta_{I2}^*$ the second-best solutions under independent contracting mode of simultaneous equations (15) and (18). Thus,

$$\beta_{I1}^* = \frac{\rho\sigma_2^2\left(1-t^2\right)k_1^2 + k_1^2 k_2^2 - t k_1 k_2^3}{\left[\rho\sigma_1^2\left(1-t^2\right) + k_1^2\right]\left[\rho\sigma_2^2\left(1-t^2\right) + k_2^2\right] - t^2 k_1^2 k_2^2},$$
$$\beta_{I2}^* = \frac{\rho\sigma_1^2\left(1-t^2\right)k_2^2 + k_1^2 k_2^2 - t k_2 k_1^3}{\left[\rho\sigma_1^2\left(1-t^2\right) + k_1^2\right]\left[\rho\sigma_2^2\left(1-t^2\right) + k_2^2\right] - t^2 k_1^2 k_2^2},$$
$$a_{I1}^* = \frac{\rho\left[\sigma_2^2 k_1^3 + t\sigma_1^2 k_2^3\right] + k_1^3 k_2^2}{\left[\rho\sigma_1^2\left(1-t^2\right) + k_1^2\right]\left[\rho\sigma_2^2\left(1-t^2\right) + k_2^2\right] - t^2 k_1^2 k_2^2},$$
$$a_{I2}^* = \frac{\rho\left[\sigma_1^2 k_2^3 + t\sigma_2^2 k_1^3\right] + k_2^3 k_1^2}{\left[\rho\sigma_1^2\left(1-t^2\right) + k_1^2\right]\left[\rho\sigma_2^2\left(1-t^2\right) + k_2^2\right] - t^2 k_1^2 k_2^2}. \quad (19)$$

Proposition 2. *Under independent contracting mode, the different incentive coefficients given by different principal are as follows:*

$$\beta_{I1}^* = \frac{\rho\sigma_2^2\left(1-t^2\right)k_1^2 + k_1^2 k_2^2 - t k_1 k_2^3}{\left[\rho\sigma_1^2\left(1-t^2\right) + k_1^2\right]\left[\rho\sigma_2^2\left(1-t^2\right) + k_2^2\right] - t^2 k_1^2 k_2^2},$$
$$\beta_{I2}^* = \frac{\rho\sigma_1^2\left(1-t^2\right)k_2^2 + k_1^2 k_2^2 - t k_2 k_1^3}{\left[\rho\sigma_1^2\left(1-t^2\right) + k_1^2\right]\left[\rho\sigma_2^2\left(1-t^2\right) + k_2^2\right] - t^2 k_1^2 k_2^2}, \quad (20)$$

which are determined jointly by the agent's risk aversion degree, variances, alternative coefficient, and proportionality coefficient. The best corresponding responses of agent are

$$a_{I1}^* = \frac{\rho\left[\sigma_2^2 k_1^3 + t\sigma_1^2 k_2^3\right] + k_1^3 k_2^2}{\left[\rho\sigma_1^2\left(1-t^2\right) + k_1^2\right]\left[\rho\sigma_2^2\left(1-t^2\right) + k_2^2\right] - t^2 k_1^2 k_2^2},$$
$$a_{I2}^* = \frac{\rho\left[\sigma_1^2 k_2^3 + t\sigma_2^2 k_1^3\right] + k_2^3 k_1^2}{\left[\rho\sigma_1^2\left(1-t^2\right) + k_1^2\right]\left[\rho\sigma_2^2\left(1-t^2\right) + k_2^2\right] - t^2 k_1^2 k_2^2}. \quad (21)$$

5. Cooperation between Principals Mode

Under cooperation mode, two principals offer common incentive contract (α, β) to common agent in order to maximize their joint profit:

$$s\left(\pi_1, \pi_2\right) = \alpha + \beta\left(\pi_1 + \pi_2\right), \quad (0 \le \beta \le 1), \quad (22)$$

and the total expected return of two principals is

$$E\left(v_1 + v_2\right) = Ev\left[\pi_1 + \pi_2 - s\left(\pi_1 + \pi_2\right)\right]$$
$$= v\left\{E\left[\pi_1 + \pi_2 - s\left(\pi_1 + \pi_2\right)\right]\right\}$$
$$= v\left[-\alpha + (1-\beta)\left(k_1 a_1 + k_2 a_2\right)\right]$$
$$= -\alpha + (1-\beta)\left(k_1 a_1 + k_2 a_2\right). \quad (23)$$

Under cooperation, we consider that (α, β) must satisfy IC with the sum of reservation wages of two principals' separate contract in order for incentive agent to perform the tasks, and IR is to maximize agent's CEW:

$$CE_A = \alpha + \beta\left(k_1 a_1 + k_2 a_2\right) - \frac{\rho\beta^2\left(\sigma_1^2 + \sigma_2^2\right)}{2} - C\left(a_1, a_2\right). \quad (24)$$

We can write principals' maximization problem as follows:

$$\max_{\alpha,\beta} \quad \left[-\alpha + (1-\beta)\left(k_1 a_1 + k_2 a_2\right)\right]$$
$$\text{s.t.} \quad \text{(IR)}\ \alpha + \beta k_1 a_1 + \beta k_2 a_2 - \frac{\rho\beta^2\left(\sigma_1^2 + \sigma_2^2\right)}{2} - C\left(a_1, a_2\right) \ge \overline{\omega}_1 + \overline{\omega}_2$$
$$\text{(IC)}\ \max_{a_1, a_2} CE_A = \alpha + \beta k_1 a_1 + \beta k_2 a_2 - \frac{\rho\beta^2\left(\sigma_1^2 + \sigma_2^2\right)}{2} - C\left(a_1, a_2\right). \quad (25)$$

The calculation process and results are as follows:

$$\frac{\partial CE_A}{\partial a_1} = k_1\beta - a_1 + ta_2$$
$$\frac{\partial CE_A}{\partial a_2} = k_2\beta - a_2 + ta_1, \quad (26)$$
$$a_1 = \frac{(k_1 + tk_2)\beta}{1-t^2} \qquad a_2 = \frac{(k_2 + tk_1)\beta}{1-t^2}.$$

Feed a_1, a_2 into IC separately. Then,

$$\alpha = \overline{\omega}_1 + \overline{\omega}_2 - \left(\frac{2k_1^2 + 3tk_1k_2 + 2k_2^2}{2(1-t^2)} - \frac{\rho(\sigma_1^2+\sigma_2^2)}{2}\right)\beta^2. \quad (27)$$

Feed α into principals' joint objective function (subscript C on behalf of the cooperation situation):

$$\max_{\alpha,\beta} v_C = -(\overline{\omega}_1 + \overline{\omega}_2) + \left(\frac{(k_1^2 + tk_1k_2 + k_2^2)}{2(1-t^2)} - \frac{\rho(\sigma_1^2+\sigma_2^2)}{2}\right)\beta^2 + \frac{(k_1^2 + k_2^2 + 2tk_1k_2)}{1-t^2}\beta, \quad (28)$$

and the first-order condition is

$$\left(\frac{k_1^2 + tk_1k_2 + k_2^2}{1-t^2} - \rho(\sigma_1^2+\sigma_2^2)\right)\beta + \frac{k_1^2 + k_2^2 + 2tk_1k_2}{1-t^2} = 0. \quad (29)$$

We denote by β_C^* the second-best solutions under cooperation; the results are

$$\beta_C^* = \frac{k_1^2 + k_2^2 + 2tk_1k_2}{(k_1^2 + tk_1k_2 + k_2^2) - \rho(\sigma_1^2+\sigma_2^2)(1-t^2)},$$
$$a_{C1}^* = \frac{(k_1 + tk_2)(k_1^2 + k_2^2 + 2tk_1k_2)}{(1-t^2)[\rho(\sigma_1^2+\sigma_2^2)(1-t^2) - (k_1^2 + tk_1k_2 + k_2^2)]}$$
$$a_{C2}^* = \frac{(k_2 + tk_1)(k_1^2 + k_2^2 + 2tk_1k_2)}{(1-t^2)[\rho(\sigma_1^2+\sigma_2^2)(1-t^2) - (k_1^2 + tk_1k_2 + k_2^2)]}. \quad (30)$$

Proposition 3. *Under cooperation contracting mode, the joint decision mechanism of two principals is*

$$\beta_C^* = \frac{k_1^2 + k_2^2 + 2tk_1k_2}{(k_1^2 + tk_1k_2 + k_2^2) - \rho(\sigma_1^2+\sigma_2^2)(1-t^2)}. \quad (31)$$

The best effort responses of common agent to different tasks are

$$a_{C1}^* = \frac{(k_1 + tk_2)(k_1^2 + k_2^2 + 2tk_1k_2)}{(1-t^2)[\rho(\sigma_1^2+\sigma_2^2)(1-t^2) - (k_1^2 + tk_1k_2 + k_2^2)]},$$
$$a_{C2}^* = \frac{(k_2 + tk_1)(k_1^2 + k_2^2 + 2tk_1k_2)}{(1-t^2)[\rho(\sigma_1^2+\sigma_2^2)(1-t^2) - (k_1^2 + tk_1k_2 + k_2^2)]}. \quad (32)$$

6. Numerical Analysis and Discussions

Numerical analysis is discussed in this section in order to illustrate the decision mechanism of both sides and compare the incentive efficient further in different modes.

Firstly, parameters are set according to their ranges in the models' assumption as follows:

$$\sigma_1^2 = 0.1, \qquad \sigma_2^2 = 1, \qquad \rho = 0.005,$$
$$k_1 = 1, \qquad k_2 3, \qquad \overline{\omega}_1 = 1, \qquad \overline{\omega}_2 = 2. \quad (33)$$

The results of β, a, π in three modes are compared when $t = 0.1, 0.3, 0.5, 0.8$; the influence of alternative coefficient on principals and the agent's decision mechanism are illustrated in Table 1.

Under the condition of unchangeable alternative coefficient the following can be drawn from Table 1.

(1) Under exclusive dealing mode, $\beta_{E1}^* > \beta_{E2}^*$; namely, the principals offer greater incentive on the easy supervision task. Under independent contracting mode, $\beta_{I2}^* > \beta_{I1}^*$; namely, the principal offers greater incentive on the difficult supervision task.

(2) Both principals prefer to select common agency who only considers the influence of alternative coefficient and $\beta_C^* > \beta_{I1}^*$ and $\beta_C^* > \beta_{E1}^*$ mean principal with easy supervision task prefers to select cooperation mode, but principal with difficult supervision task will select cooperation mode when alternative coefficient (t) is small; otherwise independent mode will be selected when t gradually becomes larger and $\pi_{I(\text{sum})}^* > \pi_{C(\text{sum})}^*$ and $\pi_{I(\text{sum})}^* > \pi_{E(\text{sum})}^*$ mean that total outputs under independent mode are always larger than those under the other two modes. The changing of alternative coefficient (t) will not influence the incentive under exclusive dealing mode, but it can influence that in common agency. That is to say, under cooperation mode, the incentive will change in the same direction with alternative coefficient. And under independent contracting mode, it will change still in the same direction on the difficult supervision task but change inversely on the easy one.

(3) Consider that $a_{E2}^* > a_{E1}^*$, $a_{I2}^* > a_{I1}^*$, and $a_{C2}^* > a_{C1}^*$ mean that agent makes more efforts on the difficult supervision task under any agency mode because of the principal's different incentives in different mode. When other conditions remain unchanged, the effort becomes greater, while the alternative coefficient gets larger. When other conditions remain unchanged, the efforts on two tasks both become greater gradually with the difficult supervision task's variance getting larger under cooperation mode. On the contrary, the effort becomes smaller under independent mode.

Secondly, parameters are set according to its range in models assumption as follows:

$$\overline{\omega}_1 = 1, \qquad \overline{\omega}_2 = 2, \qquad \rho = 0.005,$$
$$t = 0.3, \qquad k_1 = 1, \qquad k_2 = 3. \quad (34)$$

Table 1: Different outputs under three modes when alternative coefficient (t) changes.

Common agency										Exclusive dealing	
Independent contracting mode					Cooperation mode						
t	0.1	0.3	0.5	0.8	t	0.1	0.3	0.5	0.8		
β^*_{I1}	0.7072	0.1103	−0.6658	−3.8863	β^*_{C}	1.0296	1.0830	1.1308	1.1937	β^*_{E1}	1.0000
β^*_{I2}	0.9759	0.9885	1.1105	2.0358						β^*_{E2}	0.9994
a^*_{I1}	1.2345	2.0407	3.9995	24.9909	a^*_{C1}	1.3520	2.2613	3.7694	11.2741	a^*_{E1}	1.0000
a^*_{I2}	3.7017	6.1193	11.9940	74.9600	a^*_{C2}	3.2241	3.9275	5.2771	12.6004	a^*_{E2}	2.9983
π^*_{I1}	1.2345	2.0407	3.9995	24.9909	π^*_{C}	4.5761	6.1887	9.0465	23.8745	π^*_{E1}	1.0000
π^*_{I2}	11.1051	18.3580	35.9821	224.8799						π^*_{E2}	8.9950
$\pi^*_{I\,(\text{sum})}$	12.3396	20.3987	39.9821	249.8708	$\pi^*_{C\,(\text{sum})}$	4.5761	6.1887	9.0465	23.8745	$\pi^*_{E\,(\text{sum})}$	9.9950

Table 2: Different outputs under three modes when one variance (σ_2^2) changes.

Common agency								Exclusive dealing			
Independent contracting mode				Cooperation mode							
σ_2^2	100	36	9	σ_2^2	100	36	9	σ_2^2	100	36	9
β^*_{I1}	0.1490	0.1231	0.1116	β^*_{C}	1.1317	1.1010	1.0885	β^*_{E1}	0.9804	0.9804	0.9804
β^*_{I2}	0.9356	0.9681	0.9826					β^*_{E2}	0.9474	0.9804	0.9950
a^*_{I1}	2.0226	2.0295	2.0325	a^*_{C1}	2.3629	2.2987	2.2726	a^*_{E1}	0.9804	0.9804	0.9804
a^*_{I2}	5.8193	6.0027	6.0840	a^*_{C2}	4.1040	3.9925	3.9472	a^*_{E2}	2.8421	2.9412	2.9851
π^*_{I1}	2.0226	2.0295	2.0325	π^*_{C}	6.4669	6.2912	6.2199	π^*_{E1}	0.9804	0.9804	0.9804
π^*_{I2}	17.4578	18.0082	18.2520					π^*_{E2}	8.5263	8.8235	8.9552
$\pi^*_{I\,(\text{sum})}$	19.4804	20.0377	20.2845	$\pi^*_{C\,(\text{sum})}$	6.4669	6.2912	6.2199	$\pi^*_{E\,(\text{sum})}$	9.5067	9.8039	9.9366

The results of β, a, π in three modes are compared when

$$\begin{aligned} &\sigma_1^2 = 4, \quad \sigma_2^2 = 100, \quad k_1^2\sigma_2^2 > k_2^2\sigma_1^2, \\ &\sigma_1^2 = 4, \quad \sigma_2^2 = 36, \quad k_1^2\sigma_2^2 = k_2^2\sigma_1^2, \\ &\sigma_1^2 = 4, \quad \sigma_2^2 = 9, \quad k_1^2\sigma_2^2 < k_2^2\sigma_1^2. \end{aligned} \tag{35}$$

The influence of variance on principals and the agent's decision mechanism are illustrated in Table 2.

According to Table 2, if the influence of task's variance was considered merely it can be obtained as follows.

$\beta^*_C > \beta^*_{Ii}$ and $\beta^*_C > \beta^*_{Ei}$ mean that principals always prefer cooperation mode. When other conditions remain unchanged, with the difficult supervision task's variance getting larger, the incentive offered by the principal whose task is difficult to be supervised becomes smaller gradually under exclusive dealing mode. The incentive becomes greater on the easy supervision task, but it becomes smaller on the difficult supervision task under independent contracting mode, while the incentive becomes greater gradually under cooperation mode.

Through the above analysis, implications and suggestions on how to select the effective agency mode can be got as follows.

(1) Because the government's ultimate goal is to realize the maximum social welfare, it should think highly of urging borrowers' repayment rather than merely consider the quantity of loan contracts just like what they do in reality. Because the principal whose task is easy to be supervised prefers to offer more incentives under exclusive dealing mode, selecting exclusive dealing mode will lead the staff to pay more attention to sign more loan contracts but ignore to urge repayment, which is not good for the long-term effect of national student financial aid policy.

(2) If the effort proportionality coefficient and variance are both unchanged, both principals prefer to select common agency, but each principal's preference degree of selecting cooperation or independent mode is different according to the difficulty degree of the task. We consider that the government prefers cooperation mode, although under it the total output is less than that under independent mode. In order to stimulate the bank participating in the policy, the government should select the mode that the bank prefers.

(3) The study shows that although principals offer different incentives in different modes, the agent always offers more effort to the difficult supervision task under any mode, which not only gives enlightenment that the student loans repayment is the key in financial aid policy, but also warns that incentive mechanism designing absolutely according to the study results may lead us to ignore the quantity of student loans which is the base to realize national policy objective. So in the practical mechanism designing, the government that is not just a principal but more importantly a regulator should comprehensively consider more

affecting factors such as total output, bank and staff's enthusiasm, and the continuity of policy.

7. Conclusions

The research on multiprincipals and how to select effective agency mode in the student loan system has been carried out. Three models of cooperation between principals, exclusive dealing, and independent contracting have been investigated and discussed. Decision mechanisms are given and efficiencies among three modes are contrasted by numerical analysis. Under the condition of unchangeable alternative coefficient three main conclusions were obtained and discussed. Under exclusive dealing mode and independent contracting mode the principals offer greater incentive on the easy supervision task and difficult supervision task, respectively. And both principals prefer to select common agency who only considers the influence of alternative coefficient. Considering the influence of task's variance principals always prefer cooperation mode. The studies show that exclusive dealing mode is not good for student financial aid policy's long-term effect because it sometimes guides agent ignoring repayment; if effort proportionality coefficient and observability are both unchanged, both principals prefer common agency, but independent contracting mode may be more efficient in reality because not only the total outputs under it are larger than those under cooperation mode, but also preferring independent contracting mode could stimulate the bank participating in the game; the conclusion that agent always offers more efforts to the difficult supervision task under any mode warns that incentive mechanisms designing absolutely according to the study results may lead us to ignore loans quantity, so the government, which is not just a principal but more importantly a regulator, should consider comprehensively more affecting factors in practice.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

The research was supported by Scientific Research Projects of Education Department of Shandong Province (no. J13WH119).

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