

Research Article

Intuitionistic Trapezoidal Fuzzy Multiple Criteria Group Decision Making Method Based on Binary Relation

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The aim of this paper is to develop a methodology for intuitionistic trapezoidal fuzzy multiple criteria group decision making problems based on binary relation. Firstly, the similarity measure between two vectors based on binary relation is defined, which can be utilized to aggregate preference information. Some desirable properties of the similarity measure based on fuzzy binary relation are also studied. Then, a methodology for fuzzy multiple criteria group decision making is proposed, in which the criteria values are in the terms of intuitionistic trapezoidal fuzzy numbers (ITFNs). Simple and exact formulas are also proposed to determine the vector of the aggregation and group set. According to the weighted expected values of group set, it is easy to rank the alternatives and select the best one. Finally, we apply the proposed method and the Cosine similarity measure method to a numerical example; the numerical results show that our method is effective and practical.

1. Introduction

With the development of internet technology, multiple criteria group decision making (MCGDM) becomes an important part of modern decision science. It has been extensively applied to various areas, such as society, economics, management, military, and engineering technology. Since Bellman and Zadeh [1] initially proposed the fuzzy decision making model based on the fuzzy mathematical theory, many achievements have been made in literature on fuzzy multiple criteria group decision making (FMCDM) problems [2]. Some related fuzzy decision making methods were proposed in the references, for example, weighted aggregation operators [3], ordered weighted aggregation operators [4], weighted geometric aggregation operators [5], TOPSIS method [6], analytic hierarchy process method [7], grey relational analysis method [8], similarity measures [9], and so on. Similarity measure is an important tool to determine the degree of similarity between two fuzzy uncertain objects, which has successfully applied in FMCDM problem. Many different similarity measures for FMCDM problems have been investigated in the literature [10–14]. Based on the

extension of the Hamming distance on fuzzy sets, Szmidt and Kacprzyk [15] introduced the Hamming distance between two intuitionistic fuzzy sets (IFSs) to define the similarity measure for IFSs and applied the method to group decision making. On the other hand, Xu [16] developed some similarity measures of IFSs based on the geometric distance and defined the notions of positive ideal IFS and negative ideal IFS, which can be applied to multiple attribute decision making under intuitionistic fuzzy environment. But as an effective method to solve FMCDM problem, it is rarely applied in intuitionistic trapezoidal fuzzy group decision making.

Nehi and Maleki [17] firstly introduced the ITFN and the operation law of ITFNs. Because the ITFN is intuitive and easy to use, many researchers used it to express the preference information of the experts and the criteria values. Ye [18] used the ITFN to express the decision information. Based on the expected values for ITFNs, he presented a handling method for intuitionistic trapezoidal fuzzy multiple attribute decision making. In [19], Ye used the vector similarity measures between two ITFNs to rank the alternative, through the weighted similarity measures between each alternative and ideal alternative, and it was easy to select the best one.

However, the above methods are without considering the binary relation between decision criteria. In the complex real-life case, it has complex binary relation between each criterion. For example, in emergency resource allocation decision making problem, food and water (i.e., criteria) have “proportion” relationship (i.e., binary relation). In this case, some traditional FMCGDM methods are unsuitable to solve this problem. So in this paper, we introduce a new method for FMCGDM based on binary relation of criteria, with intuitionistic trapezoidal fuzzy information. The rest of this paper is arranged as follows: in the next section, we firstly introduce some basic notions and preliminary definitions of fuzzy numbers and intuitionistic trapezoidal fuzzy numbers; Section 3 contains a new degree of similarity between two vectors based on binary relation; in Section 4, the clustering algorithm based on binary relation similarity measure is given, and we use an example to compare with the other two clustering algorithms; we propose a new FMCGDM method based on binary relation in Section 5; then, we give an illustrative example to authenticate the method and discuss the result of numerical example in Section 6; the results of the Cosine similarity measure are also given for comparison; the paper ends with conclusion in Section 7.

2. Preliminaries

We consider the following well-known description of a fuzzy number α .

Definition 1 (Dubois and Prade [20]). Let α be a fuzzy number in the set of real number R ; its membership function is defined as follows:

$$\mu_{\alpha}(x) = \begin{cases} 0, & x < a_1, \\ f_{\alpha}(x), & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ g_{\alpha}(x), & a_3 \leq x \leq a_4, \\ 0, & a_4 < x, \end{cases} \quad (1)$$

where $a_1, a_2, a_3, a_4 \in R$, $f_{\alpha}: [a_1, a_2] \rightarrow [0, 1]$ is a nondecreasing continuous function, $f_{\alpha}(a_1) = 0$, $f_{\alpha}(a_2) = 1$, called the left side of fuzzy number α and $g_{\alpha}: [a_3, a_4] \rightarrow [0, 1]$ is a nonincreasing continuous function, $g_{\alpha}(a_3) = 1$, $g_{\alpha}(a_4) = 0$, called the right side of fuzzy number α .

The intuitionistic trapezoidal fuzzy number, which is expressed by a membership function and nonmembership function, is shown as follows.

Definition 2 (Ye [19]). An ITFN α with parameters $b_1 \leq a_1 \leq b_2 \leq a_2 \leq b_3 \leq a_3 \leq b_4 \leq a_4$ is denoted as $\alpha = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4) \rangle$ in the set of real number R .

In this case, its membership function and nonmembership function can be given as follows:

$$\mu_{\alpha}(x) = \begin{cases} 0, & x < a_1, \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \frac{x - a_4}{a_3 - a_4}, & a_3 \leq x \leq a_4, \\ 0, & a_4 < x, \end{cases} \quad (2)$$

$$\nu_{\alpha}(x) = \begin{cases} 0, & x < b_1, \\ \frac{x - b_1}{b_1 - b_2}, & b_1 \leq x \leq b_2, \\ 1, & b_2 \leq x \leq b_3, \\ \frac{x - b_3}{b_4 - b_3}, & b_3 \leq x \leq b_4, \\ 0, & b_4 < x. \end{cases}$$

The following properties for ITFN can be seen in [17]. Let $\alpha_1 = \langle (a_{11}, a_{12}, a_{13}, a_{14}), (b_{11}, b_{12}, b_{13}, b_{14}) \rangle$ and $\alpha_2 = \langle (a_{21}, a_{22}, a_{23}, a_{24}), (b_{21}, b_{22}, b_{23}, b_{24}) \rangle$ be two ITFNs and let γ be a positive scalar number. Then,

$$\begin{aligned} \alpha_1 + \alpha_2 &= \langle (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24}), \\ &\quad (b_{11} + b_{21}, b_{12} + b_{22}, b_{13} + b_{23}, b_{14} + b_{24}) \rangle, \\ \gamma\alpha_1 &= \langle (\gamma a_{11}, \gamma a_{12}, \gamma a_{13}, \gamma a_{14}), (\gamma b_{11}, \gamma b_{12}, \gamma b_{13}, \gamma b_{14}) \rangle. \end{aligned} \quad (3)$$

Definition 3 (Grzegorzewski [21]). Let $\alpha = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4) \rangle$ be an ITFN in the set of real number R ; then the expected value of α is defined by

$$EV(\alpha) = \frac{E_*(\alpha) + E^*(\alpha)}{2}, \quad (4)$$

where

$$E_*(\alpha) = \frac{b_1 + a_2}{2} + \frac{1}{2} \int_{b_1}^{b_2} h_{\alpha}(x) dx - \frac{1}{2} \int_{a_1}^{a_2} f_{\alpha}(x) dx, \quad (5)$$

$$E^*(\alpha) = \frac{b_4 + a_3}{2} + \frac{1}{2} \int_{a_3}^{a_4} g_{\alpha}(x) dx - \frac{1}{2} \int_{b_3}^{b_4} k_{\alpha}(x) dx.$$

Therefore, the expected value of ITFNs based on the expected value of IFN can be obtained as the following theorem.

Theorem 4 (Ye [19]). Let $\alpha = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4) \rangle$ be an ITFN in the set of real number R . Thus, when $f_{\alpha}(x) = (x - a_1)/(a_2 - a_1)$, $g_{\alpha}(x) = (x - a_4)/(a_3 - a_4)$, $h_{\alpha}(x) = (x - b_1)/(b_2 - b_1)$, $k_{\alpha}(x) = (x - b_3)/(b_4 - b_3)$, and $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in R$, its expected value is obtained by

$$EV(\alpha) = \frac{1}{8} (a_1 + a_2 + a_3 + a_4 + b_1 + b_2 + b_3 + b_4). \quad (6)$$

3. The New Similarity Based on Binary Relation on Finite Set

Firstly, we introduce the definition of vector similarity measure.

Definition 5. Let $X = (x_1, x_2, \dots, x_n)$ and $Y = (y_1, y_2, \dots, y_n)$ be two vectors of length n , where all the coordinates are positive; if $S(X, Y)$ satisfies the following properties, $S(X, Y)$ is called a similarity measure between X and Y :

- (P1) $0 \leq S(X, Y) \leq 1$;
- (P2) $S(X, Y) = S(Y, X)$;
- (P3) if $X = Y$, that is, $x_i = y_i$, for $i = 1, 2, \dots, n$,

$$S(X, Y) = 1. \quad (7)$$

Let Q be a binary relation on a finite vector X . This relation is defined by its membership function $Q(x_1, x_2)$, which maps a direct product $X \times X$ into the unit interval $[0, 1]$. For $1 \leq i, j \leq n$, $M(Q) = (m_{ij})_{n \times n}$ is the relation matrix; if and only if $(x_i, x_j) \in Q$, $m_{ij} = 1$, or else $m_{ij} = 0$. Obviously, $M(Q)$ is a 0-1 matrix. For example, in emergency resource allocation decision making problem, food and water (i.e., criteria) have "proportion" relationship (i.e., binary relation); the preference vector of the alternative is $V = (v_1, v_2)$; Q is defined by its membership function $Q(v_1, v_2)$.

Let $M^1(Q)$ and $M^2(Q)$ be two 0-1 matrixes; the following equation holds true:

$$M^1(Q) + M^2(Q) = (\text{Max}(m_{ij}^1, m_{ij}^2))_{n \times n}. \quad (8)$$

Definition 6. For the binary relation Q , the relation matrixes of the two vectors X and Y are M and N , respectively. By using the norm of the matrix, the similarity measure between two vectors is defined as follows:

$$r(X, Y) = \frac{1}{\sqrt{n}} \frac{\|M + N\|_2}{\|M\|_\infty + \|N\|_\infty}, \quad (9)$$

where $\|M\|_2 = (\rho(M^T M))^{1/2}$, $\|M\|_\infty = \max_{1 \leq i \leq n} \sum_{j=1}^n |m_{ij}|$, and $\rho(M^T M)$ is the spectral radius of $M^T M$, which is the maxis of characteristic value of the matrix $M^T M$.

Theorem 7. *The similarity measure in Definition 6 satisfies the following properties:*

- (P1) $0 \leq r_{ij}(X_i, X_j) \leq 1$;
 - (P2) $r_{ii}(X_i, X_i) = 1$.
- (10)

Proof. (P1) As $\|M_i + M_j\|_2 \geq 0$, $\|M_i\|_\infty \geq 0$, and $\|M_j\|_\infty \geq 0$, it is easy to see that $r_{ij}(X_i, X_j) \geq 0$. Thus, we only need to prove $r_{ij}(X_i, X_j) \leq 1$. By using the property of the norm of the 0-1 matrix,

$$\|M\|_2 = \sqrt{n}\|M\|_\infty, \quad \|M_i + M_j\|_2 \leq \|M_i\|_2 + \|M_j\|_2. \quad (11)$$

Taking (11) into (9), we get

$$\begin{aligned} r_{ij}(X_i, X_j) &= \frac{1}{\sqrt{n}} \frac{\|M_i + M_j\|_2}{\|M_i\|_\infty + \|M_j\|_\infty} \\ &\leq \frac{1}{\sqrt{n}} \frac{\|M_i\|_2 + \|M_j\|_2}{\|M_i\|_\infty + \|M_j\|_\infty} \\ &\leq \frac{\sqrt{n} \|M_i\|_\infty + \sqrt{n} \|M_j\|_\infty}{\sqrt{n} \|M_i\|_\infty + \sqrt{n} \|M_j\|_\infty} \\ &\leq 1. \end{aligned} \quad (12)$$

(P2) As the relation matrix is 0-1 matrix, we get $\|M\|_2 = \sqrt{n}\|M\|_\infty$. Then

$$r_{ii}(X_i, X_i) = \frac{1}{\sqrt{n}} \frac{\|M_i + M_i\|_2}{\|M_i\|_\infty + \|M_i\|_\infty} = \frac{1}{\sqrt{n}} \frac{2\sqrt{n}\|M_i\|_\infty}{2\|M_i\|_\infty} = 1. \quad (13)$$

□

4. The Clustering Algorithm Based on Binary Relation Similarity Measure

Let $\Phi = \{X_1, X_2, X_3, \dots, X_p\}$ be a set of vector, let Q be a binary relation on vector X_i , and let δ be a threshold value, which is used to determine the similarity between two vectors. If $r_{ij}(X_i, X_j) \geq \delta$, that means X_i and X_j are similar. The clustering algorithm based on binary relation similarity measure can be summarized as follows.

Step 1. We form the relation matrix M_i ($i = 1, 2, \dots, p$) based on the binary relation Q . Let $U = \{M_1, M_2, \dots, M_p\}$ be a set of relation matrix and let T be a temporary set.

Step 2. Let k be number of aggregation, initially, $k = 1, i = 1$; the threshold value δ is a real number, such as 0.2, 0.3, and 0.6.

Step 3. Select M_i by orders from the finite set U , $M_i \in U$, and then put it into the aggregation C_k ; at the same time, remove M_i from U ; the number of matrixes in C_k is $n_k = 1$.

Step 4. The linear combination of the relation matrix in C_k is obtained by

$$Z_k = \sum_{i=1}^{n_k} M_i. \quad (14)$$

Step 5. If U is nonempty, select the next relation matrix M_i ($i = i + 1$) by order from U ; if U is empty, go to Step 7.

Step 6. Using (9), the similarity measure between M_i and Z can be got. If $r_i(Z, M_i) \geq \delta$, assign M_i to the aggregation C_k ; at the same time, remove M_i from U , $n_k = n_k + 1$; if $r_i(Z, M_i) < \delta$, put M_i into the temporary set T ; at the same time, remove M_i from U . Then, go to Step 4.

Step 7. If T is nonempty, let $U = T$, $k = k + 1$; go to Step 3, or else, T is empty; go to Step 8.

Step 8. Record the clustering results.

Example 8. Let $\Phi = \{X_1, X_2, X_3, X_4, X_5, X_6\}$ be a set of vector,

$$\begin{aligned} X_1 &= (0.74, 0.71, 0.64, 0.69, 0.93); \\ X_2 &= (0.86, 0.03, 0.81, 0.14, 0.28); \\ X_3 &= (0.03, 1.00, 0.28, 0.26, 0.44); \\ X_4 &= (0.65, 0.01, 0.88, 0.61, 0.14); \\ X_5 &= (0.12, 0.61, 0.97, 0.68, 0.86); \\ X_6 &= (0.03, 0.72, 0.67, 0.79, 0.94). \end{aligned} \quad (15)$$

In the real life, a binary relation can be got through by the analysis of the past data. In an illustrated example, a binary can be given by the decision maker. So in this case, we give the binary relation between the five elements for each vector is “>”; “>” means better or more important, which can be described in mathematic language as $x_i - x_j \geq 0.4$ ($i = 1, 2, \dots, 5$; $j = 1, 2, \dots, 5$). Using the above clustering algorithm, we get the clustering results as in Table 1; for comparison, we also use the other clustering algorithm in [22, 23]; the results are shown in Table 1.

In [22], Xu and Chen defined the consistency index ρ to show the effectiveness of the clustering algorithm; the larger ρ is, the more effective the clustering algorithm is. From Table 1, we can see that the consistency index of our clustering algorithm is larger than the other two clustering algorithms; so our clustering algorithm is more effective.

5. Multiple Criteria Group Decision Making Method Based on Binary Relation

In this section, we present a handling method for ITFNs multiple criteria decision making problems. Let $A = \{A_1, A_2, \dots, A_n\}$ be a set of alternatives, let $C = \{C_1, C_2, \dots, C_m\}$ be a set of criteria, and let $B = \{B_1, B_2, \dots, B_l\}$ be a set of group members. Assume Q is a binary relation on a finite criteria set C . The characteristic of alternative A_i for group member B_j is represented by the following ITFN:

$$\begin{aligned} v_i^j &= \left\{ \left\langle C_1, (a_{11i}^j, a_{12i}^j, a_{13i}^j, a_{14i}^j), (b_{11i}^j, b_{12i}^j, b_{13i}^j, b_{14i}^j) \right\rangle, \right. \\ &\quad \left\langle C_2, (a_{21i}^j, a_{22i}^j, a_{23i}^j, a_{24i}^j), (b_{21i}^j, b_{22i}^j, b_{23i}^j, b_{24i}^j) \right\rangle, \dots, \\ &\quad \left. \left\langle C_m, (a_{m1i}^j, a_{m2i}^j, a_{m3i}^j, a_{m4i}^j), (b_{m1i}^j, b_{m2i}^j, b_{m3i}^j, b_{m4i}^j) \right\rangle \right\}, \end{aligned} \quad (16)$$

for $i = 1, 2, \dots, n$, $j = 1, 2, \dots, l$. The decision procedure for the proposed method can be summarized as follows.

TABLE 1: The clustering results for different clustering algorithm.

Different clustering algorithm	n_k	Group member	The consistency index ρ [22]
Binary relation clustering algorithm (BRCA)	5	X_1, X_2, X_4, X_5, X_6	0.6450 (BRCA)
	1	X_3	
Vector clustering algorithm (VCA)	2	X_1, X_2	0.3029 (VCA)
	1	X_3	
	2	X_4, X_6	
Ants-clustering algorithm (ACA)	1	X_5	0.6442 (ACA)
	1	X_1	
	2	X_2, X_3	
	1	X_4	
	2	X_5, X_6	

Step 1. Using (6), we get the expected vector of all group members for each alternative A_i ($i = 1, 2, \dots, n$). Let $\Omega_i = \{E_i^1, E_i^2, \dots, E_i^l\}$ be a set of group member's expected vectors for the alternative A_i .

Step 2. Using the similarity model (9) and the clustering algorithm in Section 4, for the alternative A_i , we get K ($1 \leq K \leq m$) aggregations; if n_k is the number of expected vector of k th aggregation, then $\sum_{k=1}^m n_k = m$; the group members in the same aggregation have the relative closeness preference.

Step 3. The expected vector of the aggregation C_i^k of alternative A_i can be obtained by

$$H_i^k = \frac{h_i^k}{|h_i^k|}, \quad (17)$$

where $h_i^k = \sum_{E_i^j \in C_i^k} E_i^j$ and $|h_i^k|$ is the Euclidean norm of the expected vector of aggregation C_i^k . Then, we calculate the expected vector of the whole group for the alternative A_i :

$$G_i = \frac{g_i}{|g_i|}, \quad (18)$$

where $g_i = \sum_{k=1}^m (n_k/m) H_i^k$ and $|g_i|$ is the length of the expected vector of g_i .

Step 4. For each alternative A_i in $A = \{A_1, A_2, \dots, A_n\}$ (where $i = 1, 2, \dots, n$), using Steps 1–3, we can get the group expected matrix; that is,

$$G = (G_1, G_2, \dots, G_n)^T = \begin{pmatrix} g_1^1 & g_1^2 & \dots & g_1^m \\ g_2^1 & g_2^2 & \dots & g_2^m \\ \vdots & \vdots & \ddots & \vdots \\ g_n^1 & g_n^2 & \dots & g_n^m \end{pmatrix}. \quad (19)$$

Step 5. For the weight of the criteria C_j ($j = 1, 2, \dots, m$), it is a trapezoidal intuitionistic fuzzy weight $w_j = \langle (c_1, c_2, c_3, c_4), (d_1, d_2, d_3, d_4) \rangle$ ($j = 1, 2, \dots, n$). The expected weight

value w_j ($j = 1, 2, \dots, n$) for an intuitionistic trapezoidal fuzzy weight is obtained by (6). Then we normalize the expected weight value w_j ($j = 1, 2, \dots, n$) by using the following formula:

$$w_j = \frac{EV(w_j)}{\sum_{j=1}^n EV(w_j)}. \quad (20)$$

Step 6. The ranking order of all alternatives $A = \{A_1, A_2, \dots, A_n\}$ can be obtained as follows:

$$F = W^T G = (w_1, w_2, \dots, w_n) \begin{pmatrix} g_1^1 & g_1^2 & \dots & g_1^m \\ g_2^1 & g_2^2 & \dots & g_2^m \\ \cdot & \cdot & \cdot & \cdot \\ g_n^1 & g_n^2 & \dots & g_n^m \end{pmatrix} \quad (21)$$

$$= (F_1, F_2, \dots, F_n).$$

According to the maximum principle, we can select the best one by

$$\text{Best alternative} = \max_{1 \leq i \leq n} \{F_i\}. \quad (22)$$

Then by using (22), we can easily select the best one in the alternative set.

6. Application and Comparison

In this section, we use the illustrative example in [19] for a multiple criteria decision making problem, which is a demonstration of the application of the proposed FMCGDM in Section 5.

There is a panel with four possible alternatives to invest the money (adapted from [19]): (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is a television company. The investment company must take a decision according to the following three criteria: (1) C_1 is the social benefit; (2) C_2 is the economical benefit; (3) C_3 is the environmental impact, where C_1 and C_2 are benefit criteria and C_3 is cost criterion. The four possible alternatives are to be evaluated under the above three criteria by corresponding to the linguistic values of ITFNs for the linguistic terms, as shown in Table 2.

Suppose we invite 3 experts to make judgment on four alternatives. They are expected to give linguistic terms. Then, from Table 2, we convert the linguistic terms to ITFNs, as listed in Table 3.

The decision procedure for the proposed method can be summarized as follows.

Step 1. Using formulation (6), we get the expected value of 3 experts for each alternative, as listed in Table 4.

Step 2. The decision maker gives the binary relation as “>” which has been shown in detail in Section 4. The binary relation between criteria is $C_2 > C_3 > C_1$; “>” can be described in mathematic as $x_i - x_j \geq 0.4$. For each alternative

TABLE 2: Nine-member linguistic values of ITFNs.

Linguistic term	Linguistic values of ITFN
Absolutely low	$\langle(0.0001, 0.001, 0.001, 0.001), (0.001, 0.001, 0.001, 0.001)\rangle$
Low	$\langle(0.0, 0.1, 0.2, 0.3), (0.0, 0.1, 0.2, 0.3)\rangle$
Fairly low	$\langle(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)\rangle$
Medium	$\langle(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)\rangle$
Fairly high	$\langle(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)\rangle$
High	$\langle(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)\rangle$
Absolutely high	$\langle(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)\rangle$

A_i ($i = 1, 2, 3, 4$), let the threshold value $\delta = 0.6$; using similarity model (9) and the clustering method in Section 4, we can get the aggregation C_i^k and the group members in C_i^k for each alternative A_i , as listed in Table 5.

Step 3. Using (17) and (18), we can get the expected vectors of aggregation and group, as listed in Table 5.

Step 4. The weight value of criteria C_i is shown in Table 3.

The expected weight values can be obtained by (6). Then we normalize the expected weight value by (20) and get the weight vector $W^T = (0.3490, 0.3020, 0.3490)$.

Step 5. Using the dates in the last row of Table 5, we get the group expected matrix:

$$G = \begin{pmatrix} 0.5537 & 0.5632 & 0.5649 & 0.7353 \\ 0.7665 & 0.6940 & 0.7107 & 0.6199 \\ 0.3145 & 0.4407 & 0.4192 & 0.2739 \end{pmatrix}. \quad (23)$$

Using (21),

$$F = W^T G = (0.3490, 0.3020, 0.3490)$$

$$\times \begin{pmatrix} 0.5537 & 0.5632 & 0.5649 & 0.7353 \\ 0.7665 & 0.6940 & 0.7107 & 0.6199 \\ 0.3145 & 0.4407 & 0.4192 & 0.2739 \end{pmatrix} \quad (24)$$

$$= (0.5345, 0.5599, 0.5581, 0.5394).$$

Step 6. Rank the alternatives as follows: $A_2 > A_4 > A_3 > A_1$.

Thus, the most desirable alternative is A_2 in accordance with the above result, which is the same as the C-similarity measure in [19].

Our method’s decision results are the same as the Cosine similarity measure’s decision results, but our method gets the same results more rapidly than the Cosine similarity measure, especially when the number of the experts is large (for example 100 or 1000). Because an ITFN has 8 elements, each preference vector has 32 elements or more; when the group members are many, the Cosine similarity measure has more complex computations. The decision results of methods in [19] were got by the weighted similarity measure between

TABLE 3: Preference values of alternative and weights given from 3 experts by ITFN.

A	M	C ₁ C ₃	C ₂
A ₁	1	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩ ⟨(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)⟩	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩
	2	⟨(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)⟩ ⟨(0.0, 0.1, 0.2, 0.3), (0.0, 0.1, 0.2, 0.3)⟩	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩
	3	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩ ⟨(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)⟩	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩
A ₂	1	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩ ⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩
	2	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩ ⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩	⟨(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)⟩
	3	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩ ⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩
A ₃	1	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩ ⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩
	2	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩ ⟨(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)⟩	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩
	3	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩ ⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩
A ₄	1	⟨(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)⟩ ⟨(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)⟩	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩
	2	⟨(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)⟩ ⟨(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)⟩	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩
	3	⟨(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)⟩ ⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩	⟨(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)⟩
Weights	1	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩ ⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩
	2	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩ ⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩
	3	⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩ ⟨(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)⟩	⟨(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)⟩

TABLE 4: The expected values of ITFNs.

Alternatives	Group member	C ₁	C ₂	C ₃	Alternatives	Group member	C ₁	C ₂	C ₃
A ₁	1	0.450	0.450	0.250	A ₂	1	0.650	0.650	0.450
A ₁	2	0.250	0.450	0.150	A ₂	2	0.650	0.850	0.450
A ₁	3	0.450	0.650	0.250	A ₂	3	0.450	0.650	0.450
A ₃	1	0.450	0.650	0.450	A ₄	1	0.850	0.650	0.250
A ₃	2	0.450	0.650	0.250	A ₄	2	0.850	0.650	0.250
A ₃	3	0.650	0.650	0.450	A ₄	3	0.850	0.850	0.450

TABLE 5: The aggregation of group member based on binary relation, when $\delta = 0.6$.

Alternatives	n _k	Group member	EV of aggregation	EV of group set
A ₁	2	1, 3	(0.5974, 0.7301, 0.3319)	
A ₁	1	2	(0.4663, 0.8393, 0.2798)	(0.5537, 0.7665, 0.3145)
A ₂	1	1	(0.6351, 0.6351, 0.4397)	
A ₂	1	2	(0.5599, 0.7322, 0.3877)	
A ₂	1	3	(0.4947, 0.7145, 0.4947)	(0.5632, 0.6940, 0.4407)
A ₃	3	1, 2, 3	(0.5649, 0.7107, 0.4192)	(0.5649, 0.7107, 0.4192)
A ₄	3	1, 2, 3	(0.7353, 0.6199, 0.2739)	(0.7353, 0.6199, 0.2739)

each alternative and the ideal alternative. In real-world emergence decision making problem, it is difficult to get the ideal alternative; so our method is more applicable in fuzzy multiple criteria large group decision making problem.

7. Conclusion

In the complex real-life case, it has complex binary relation between each criterion. Thus, in this paper, we propose a new method for intuitionistic trapezoidal fuzzy multiple criteria group decision problem based on binary relation, in which the preference values for an alternative and the weight values of criteria take the form of ITFNs. The similarity measure between two expected vectors of ITFNs in the vector space based on the binary relation is suggested, and some desirable properties are also proved. Using the similarity measure model, we establish the clustering algorithm for FMCGDM. We also compare our clustering algorithm with the other two clustering algorithms in the reference. Finally, a practical example is given; from the weighted expected vector of group, the ranking order of all alternatives can be determined and the best one(s) can be easily identified as well. The illustrative example shows that the proposed method in this paper is applicable. The comparison between our method and the Cosine similarity measure is given to show our method's effectiveness; the two methods have the same decision results, but our method has simpler computation and gets the same results more rapidly than the Cosine similarity measure method. In the future, we will implement the proposed method into a business decision support system and solve the some real-world problem, such as facility location selection problem, emergence decision making problem, and environment issues.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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