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Research Article

Symmetric SOR Method for Absolute Complementarity Problems

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We study symmetric successive overrelaxation (SSOR) method for absolute complementarity problems. Solving this problem is equivalent to solving the absolute value equations. Some examples are given to show the implementation and efficiency of the method.

1. Introduction

Absolute complementarity problem seeks real vectors $x \geq 0$ and $Ax - |x| - b \geq 0$, such that

$$\langle x, Ax - |x| - b \rangle = 0, \quad (1)$$

where $A \in R^{n\times n}$ and $b \in R^n$. The complementarity theory was introduced and studied by Lemke [1] and Cottle and Dantzig [2]. The complementarity problems have been generalized and extended to study a wide class of problems, which arise in pure and applied sciences; see [1–9] and the references therein. Equally important is the variational inequality problem, which was introduced and studied in the early sixties.

In this paper, we suggest and analyze SSOR [5] method for absolute complementarity problem which was introduced by Noor et al. [10]. The convergence analysis of the proposed method is considered under some suitable conditions. We show that the absolute complementarity problems are equivalent to variational inequalities. Results are very encouraging. The ideas and the technique of this paper may stimulate further research in these areas.

Let R^n be the finite dimension Euclidean space, whose the inner product and norm are denoted by $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$, respectively. For a given matrix $A \in R^{n\times n}$, a vector $b \in R^n$, we consider the problem of finding $x \in K^*$, such that

$$x \in K^*, \quad Ax - |x| - b \in K^*, \\ \langle Ax - |x| - b, x \rangle = 0, \quad (2)$$

where $K^* = \{x \in R^n : \langle x, y \rangle \geq 0, y \in K\}$ is the polar cone of a closed convex cone K in R^n and $|x|$ will denote the vector in R^n with absolute values of components of $x \in R^n$. We remark that the absolute value complementarity problem (2) can be viewed as an extension of the complementarity problem considered by Karamardian [6].

Let K be a closed and convex set in the inner product space R^n. We consider the problem of finding $x \in K$ such that

$$\langle Ax - |x| - b, y - x \rangle \geq 0, \quad \forall y \in K. \quad (3)$$

The problem (3) is called the absolute value variational inequality, which is a special form of the mildly nonlinear variational inequalities [11]. If $K = R^n$, then the problem (3) is equivalent to find $x \in R^n$ such that

$$Ax - |x| - b = 0. \quad (4)$$

To propose and analyze algorithms for absolute complementarity problems, we need the following definitions.

Definition 1. $B \in R^{n\times n}$ is called an L-matrix if $b_{ii} > 0$ for $i = 1, 2, \ldots, n$, and $b_{ij} \leq 0$ for $i \neq j$, $i, j = 1, 2, \ldots, n$.

Definition 2. If $A \in R^{n\times n}$ is positive definite, then

(i) there exists a constant $\gamma > 0$, such that

$$\langle Ax, x\rangle \geq \gamma\|x\|^2, \quad \forall x \in R^n; \tag{5}$$

(ii) there exists a constant $\beta > 0$ such that

$$\|Ax\| \leq \beta\|x\|, \quad \forall x \in R^n. \tag{6}$$

2. Absolute Complementarity Problems

To propose and analyze algorithm for absolute complementarity problems, we need the following results.

Lemma 3 (see [12]). *Let K be a nonempty closed convex set in R^n. For a given $z \in R^n$, $u \in K$ satisfies the inequality*

$$\langle u - z, u - v\rangle \geq 0, \quad v \in K, \tag{7}$$

if and only if

$$u = P_K z, \tag{8}$$

where P_K is the projection of R^n onto the closed convex set K.

Lemma 4 (see [10]). *If K is the positive cone in R^n, then $x \in K$ is a solution of absolute variational inequality* (3) *if and only if $x \in K$ is the solution of the complementarity problem* (2).

The next result proves the equivalence between variational inequality (3) and the fixed point.

Lemma 5 (see [10]). *If K is closed convex set in R^n, then $\rho > 0$; $x \in K$ satisfies* (3) *if and only if $x \in K$ satisfies the relation*

$$x = P_K(x - \rho[Ax - |x| - b]), \tag{9}$$

where P_K is the projection of R^n onto the closed convex set K.

Now using Lemmas 4 and 5, the absolute complementarity problem (2) can be transformed to fixed-point problem as

$$x = P_K(x - \rho[Ax - |x| - b]). \tag{10}$$

Theorem 6 (see [10]). *Let $A \in R^{n\times n}$ be a positive definite matrix with constant $\alpha > 0$ and continuous with constant $\beta > 0$. If $0 < \rho < 2(\gamma - 1)/(\beta^2 - 1)$, $\beta > 1$, $\gamma > 1$, then there exists a unique solution $x \in K$, such that*

$$\langle Ax - |x| - b, y - x\rangle \geq 0 \quad \forall y \in K, \tag{11}$$

where K is a closed convex set in R^n.

To define the projection operator P_K, we consider the special case when $K = [0, c]$ is a closed convex set in R^n, as follows.

Definition 7 (see [3]). Let $K = [0, c]$ is a closed convex set in R^n. Then, the projection operator $P_K x$ is defined as

$$(P_K x)_i = \min\{\max(0, x_i), c_i\}, \quad i = 1, 2, \ldots, n. \tag{12}$$

Lemma 8 (see [3]). *For any x and y in R^n, the following facts hold:*

(i) $P_K(x + y) \leq P_K x + P_K y$;

(ii) $P_K x - P_K y \leq P_K(x - y)$;

(iii) $x \leq y \Rightarrow P_K x \leq P_K y$;

(iv) $P_K x + P_K(-x) \leq |x|$, *with equality, if and only if* $-c \leq x \leq c$.

Now one splits the matrix A as

$$A = D - L - U, \tag{13}$$

where D is the diagonal matrix and L and U are strictly lower and strictly upper triangular matrices, respectively. Let $0 < \omega < 2$; using (13), *one suggests the SSOR method for solving* (3) *as follows.*

Algorithm 9. Consider the following.

Step 1. Choose an initial vector $x_0 \in R^n$ and a parameter $\omega \in R_+$. Set $k = 0$.

Step 2. Calculate

$$\begin{aligned} x_{k+1} = P_K \Big(x_k - D^{-1} & \\ \times \big[-\omega L x_{k+1} + (\omega(2-\omega)A + \omega L) x_k & \\ - \omega(2-\omega)(|x_k| + b) \big] \Big). & \end{aligned} \tag{14}$$

Step 3. If $x_{k+1} = x_k$, then stop; else, set $k = k + 1$ and go to step 2.

Algorithm 10. Consider the following.

Step 1. Choose an initial vector $x_0 \in R^n$ and a parameter $\omega \in R_+$. Set $k = 0$.

Step 2. Calculate

$$\begin{aligned} x_{k+1} = P_K \Big(x_k - D^{-1} & \\ \times \big[-\omega U x_{k+1} + (\omega(2-\omega)A + \omega U) x_k & \\ - \omega(2-\omega)(|x_k| + b) \big] \Big). & \end{aligned} \tag{15}$$

Step 3. If $x_{k+1} = x_k$, then stop; else, set $k = k + 1$ and go to step 2.

Now we define an operator $g : R^n \to R^n$ such that $g(x) = \xi$, where ξ is the fixed point of the system

$$\begin{aligned} \xi = P_K \Big(x - D^{-1} & \\ \times \big[-\omega L \xi + (\omega(2-\omega)A + \omega L) x & \\ - \omega(2-\omega)(|x| + b) \big] \Big). & \end{aligned} \tag{16}$$

We also assume that the set

$$\varphi = \{x \in R^n : x \geq 0, Ax - |x| - b \geq 0\} \tag{17}$$

of the absolute complementarity problem is nonempty. To prove the convergence of Algorithm 9, we need the following result.

Theorem 11. *Consider the operator $g: R^n \to R^n$ as defined in* (16). *Assume that $A \in R^{n\times n}$ is an L-matrix. Also assume that $0 < \omega \leq 1$. Then for any $x \in \varphi$, it holds that.*

(i) $g(x) \leq x$;

(ii) $x \leq y \Rightarrow g(x) \leq g(y)$;

(iii) $\xi = g(x) \in \varphi$.

Proof. To prove (i), we need to prove that

$$\xi_i \leq x_i, \quad i = 1, 2, \ldots, n \tag{18}$$

with ξ_i satisfying

$$\xi_i = P_K\Bigg(x_i - a_{ii}^{-1} \times \Bigg[-\omega\sum_{j=1}^{i-1} L_{ij}\left(\xi_j - x_j\right) + \omega(2-\omega) \times (Ax - |x| - b)_i\Bigg]\Bigg). \tag{19}$$

To prove the required result, we use mathematical induction. For this, let $i = 1$:

$$\xi_1 = P_K\left(x_1 - a_{11}^{-1}\omega(2-\omega)(Ax - |x| - b)_1\right). \tag{20}$$

Since $Ax - |x| - b \geq 0$, $0 < \omega \leq 1$; therefore, $\xi_1 \leq x_1$.

For $i = 2$, we have

$$\xi_2 = P_K\left(x_2 - a_{22}^{-1} \times \left[-\omega L_{21}(\xi_1 - x_1) + \omega(2-\omega) \times (Ax - |x| - b)_2\right]\right). \tag{21}$$

Here, $Ax - |x| - b \geq 0$, $0 < \omega \leq 2$, $L_{21} \geq 0$ and $\xi_1 - x_1 \leq 0$. This implies that $\xi_2 \leq x_2$.

Suppose that

$$\xi_i \leq x_i \quad \text{for } i = 1, 2, \ldots, k-1, \tag{22}$$

we have to prove that the statement is true for $i = k$; that is,

$$\xi_k \leq x_k. \tag{23}$$

Consider

$$\begin{aligned}\xi_k &= P_K\Bigg(x_k - a_{kk}^{-1} \times \Bigg[-\omega\sum_{j=1}^{k-1} L_{kj}\left(\xi_j - x_j\right) + \omega(2-\omega) \times (Ax - |x| - b)_k\Bigg]\Bigg) \\ &= P_K\big(x_k - a_{kk}^{-1} \times [-\omega(L_{k1}(\xi_1 - x_1) + L_{k2}(\xi_2 - x_2) + \cdots + L_{kk-1}(\xi_{k-1} - x_{k-1})) + \omega(2-\omega)(Ax - |x| - b)_k]\big).\end{aligned} \tag{24}$$

Since $Ax - |x| - b \geq 0$, $0 < \omega_k \leq 2$, $L_{k1}, L_{k2}, \ldots, L_{kk-1} \geq 0$ and $\xi_i \leq x_i$ for $i = 1, 2, \ldots, k-1$; from (24), we can write

$$\xi_k \leq x_k. \tag{25}$$

Hence, (i) is proved.

Now we prove (ii), for this let us suppose that $\xi = g(x)$ and $\phi = g(y)$. We will prove that

$$x \leq y \Longrightarrow \xi \leq \phi. \tag{26}$$

As

$$\xi = P_K\big(x - D^{-1} \times [-\omega L\xi + (\omega(2-\omega)A + \omega L)x - \omega(2-\omega)(|x| + b)]\big), \tag{27}$$

so ξ_i can be written as

$$\xi_i = P_K\Bigg(-a_{ii}^{-1} \times \Bigg[-\omega\sum_{j=1}^{i-1} L_{ij}\xi_j + \omega a_{ii}x_i + (\omega - \omega(2-\omega)) \times \sum_{j=1}^{i-1} L_{ij}x_j - \omega(2-\omega) \times \sum_{j=1\, j\neq i}^{n} U_{ij}x_j - \omega(2-\omega)|x_i| - \omega(2-\omega)b_i\Bigg]\Bigg)$$

$$= P_K\Bigg((1-\omega)\, x_i - a_{ii}^{-1} \times \Bigg[-\omega \sum_{j=1}^{i-1} L_{ij}\xi_j + (\omega - \omega(2-\omega)) \times \sum_{j=1}^{i-1} L_{ij}x_j - \omega(2-\omega) \times \sum_{j=1\, j\neq i}^{n} U_{ij}x_j - \omega(2-\omega)\left|x_i\right| - \omega(2-\omega)\, b_i \Bigg]\Bigg). \tag{28}$$

Similarly, for ϕ_i we have

$$\phi_i = P_K\Bigg((1-\omega)\, y_i - a_{ii}^{-1} \times \Bigg[-\omega \sum_{j=1}^{i-1} L_{ij}\phi_j + (\omega - \omega(2-\omega)) \times \sum_{j=1}^{i-1} L_{ij}y_j - \omega(2-\omega) \times \sum_{j=1\, j\neq i}^{n} U_{ij}y_j - \omega(2-\omega)\left|y_i\right| - \omega(2-\omega)\, b_i \Bigg]\Bigg). \tag{29}$$

For $i = 1$, we have

$$\begin{aligned}\phi_1 &= P_K\Bigg((1-\omega)\, y_1 - a_{11}^{-1}\omega(2-\omega) \times \Bigg[-\sum_{j=1\, j\neq i}^{n} U_{1j}y_j - \left|y_1\right| - b_1 \Bigg]\Bigg) \\ &\geq P_K\Bigg((1-\omega)\, x_1 - a_{11}^{-1}\omega(2-\omega) \times \Bigg[-\sum_{j=1\, j\neq i}^{n} U_{1j}x_j - \left|x_1\right| - b_1 \Bigg]\Bigg) \\ &= \xi_1.\end{aligned} \tag{30}$$

Since $y_1 \geq x_1$, therefore $-|y_1| \leq -|x_1|$. Hence, it is true for $i = 1$. Suppose it is true for $i = 1, 2, \ldots, k-1$; we will prove it for $i = k$; for this consider

$$\begin{aligned}\phi_k &= P_K\Bigg((1-\omega)\, y_k - a_{kk}^{-1} \times \Bigg[\sum_{j=1}^{k-1} L_{kj}\phi_j + (\omega - \omega(2-\omega)) \times \sum_{j=1}^{k-1} L_{kj}y_j - \omega(2-\omega) \times \sum_{j=1\, j\neq i}^{n} U_{kj}y_j - \omega(2-\omega)\left|y_k\right| - \omega(2-\omega)\, b_k \Bigg]\Bigg) \\ &\geq P_K\Bigg((1-\omega)\, x_k - a_{kk}^{-1} \times \Bigg[\sum_{j=1}^{k-1} L_{kj}\xi_j + (\omega - \omega(2-\omega)) \times \sum_{j=1}^{k-1} L_{kj}x_j - \omega(2-\omega) \times \sum_{j=1\, j\neq i}^{n} U_{kj}x_j - \omega(2-\omega)\left|x_k\right| - \omega(2-\omega)\, b_k \Bigg]\Bigg) \\ &= \xi_k.\end{aligned} \tag{31}$$

Since $x \leq y$ and $\xi_i \leq \phi_i$ for $i = 1, 2, \ldots, k-1$, hence it is true for k and (ii) is verified.

Next we prove (iii); that is,

$$\xi = g(x) \in \varphi. \tag{32}$$

Let $\lambda = g(\xi) = P_K(\xi - D^{-1}[\omega L(\lambda - \xi) + \omega(2-\omega)(A\xi - |\xi| - b)])$ from (i) $g(\xi) = \lambda \leq \xi$. Also by definition of $g, \xi = g(x) \geq 0$ and $\lambda = g(\xi) \geq 0$.

Now

$$\lambda_i = P_K\Bigg(\xi_i - a_{ii}^{-1} \times \Bigg[-\omega \sum_{j=1}^{i-1} L_{ij}\left(\lambda_j - \xi_j\right) + \omega(2-\omega) \times \left(A\xi - |\xi| - b\right)_i \Bigg]\Bigg) \tag{33}$$

For $i = 1, \xi_1 \geq 0$ by definition of g. Suppose that $(A\xi - |\xi| - b)_i < 0$, so

$$\lambda_1 = P_K\left(\xi_1 - a_{11}^{-1}\omega(2-\omega)\left(A\xi - |\xi| - b\right)_1\right) > P_K\left(\xi_1\right) = \xi_1, \tag{34}$$

which contradicts the fact that $\lambda \leq \xi$. Therefore, $(A\xi - |\xi| - b)_i \geq 0$.

Now we prove it for any k in $i = 1, 2, \ldots, n$. Suppose the contrary $(A\xi - |\xi| - b)_i < 0$; then

$$\lambda_k = P_K\left(\xi_k - a_{kk}^{-1} \times \left[-\omega\sum_{j=1}^{k-1} L_{kj}\left(\lambda_j - \xi_j\right) + \omega(2-\omega) \times \left(A\xi - |\xi| - b\right)_k\right]\right). \tag{35}$$

As it is true for all $\alpha \in [0, 1]$, it should be true for $\alpha = 0$. That is,

$$\lambda_k = P_K\left(\xi_k - a_{kk}^{-1}\omega(2-\omega)\left(A\xi - |\xi| - b\right)_k\right) > P_K\left(\xi_k\right) = \xi_k, \tag{36}$$

which contradicts the fact that $\lambda \leq \xi$. So, $(A\xi - |\xi| - b)_k \geq 0$, for any k in $i = 1, 2, \ldots, n$.

Hence, $\xi = f(x) \in \varphi$. □

Now we prove the convergence criteria of Algorithm 9 when the matrix A is an L-matrix as stated in the next result.

Theorem 12. *Assume that $A \in R^{n\times n}$ is an L-matrix and $0 < \omega \leq 1$. Then for any initial vector $x_0 \in \varphi$, the sequence $\{x_k\}$, $k = 0, 1, 2, \ldots$, defined by Algorithm 9 has the following properties:*

(i) $0 \leq x_{k+1} \leq x_k \leq x_0$; $k = 0, 1, 2, \ldots$;

(ii) $\lim_{k\to\infty} x_k = x^*$ *is the unique solution of the absolute complementarity problem.*

Proof. Since $x_0 \in \varphi$, by (i) of Theorem 11, we have $x_1 \leq x_0$ and $x_1 \in \varphi$. Recursively using Theorem 11, we obtain

$$0 \leq x_{k+1} \leq x_k \leq x_0; \quad k = 0, 1, 2, \ldots. \tag{37}$$

From (i), we observe that the sequence $\{x_k\}$ is monotone bounded; therefore, it converges to some $x^* \in R^n_+$ satisfying

$$\begin{aligned} x^* &= P_K\left(x^* - D^{-1} \times \left[-\omega L x^* + \omega(2-\omega)(A + \omega L)x^* - \omega(2-\omega)\left(\left|x^*\right| + b\right)\right]\right) \\ &= P_K\left(x^* - D^{-1}\omega(2-\omega) \times \left[Ax^* - \left|x^*\right| - b\right]\right). \end{aligned} \tag{38}$$

Hence, x^* is the solution of the absolute complementarity problem (2). □

Note. The convergence of Algorithm 10 has the same steps as given in Theorems 11 and 12.

3. Numerical Results

In this section, we consider several examples to show the efficiency of the proposed methods. The convergence of SSOR method is guaranteed for L-matrices only, but it is also possible to solve different type of systems. All the experiments are performed with Intel(R) Core 2×2.1 GHz, 1 GB RAM, and the codes are written in MATLAB 7.

Example 13 (see [10]). Consider the ordinary differential equation

$$\frac{d^2x}{dt^2} - |x| = \left(1 - t^2\right), \qquad 0 \leq x \leq 1, \qquad x(0) = 0 \ \ x(1) = 1. \tag{39}$$

The exact solution is

$$x(t) = \begin{cases} 0.7378827425\sin(t) - 3\cos(t) + 3 - t^2 & x < 0, \\ -0.7310585786e^{-t} - 0.2689414214e^{t} + 1 + t^2 & x > 0. \end{cases} \tag{40}$$

We take $n = 10$; the matrix A is given by

$$a_{i,j} = \begin{cases} -242, & \text{for } j = i \\ 121 & \text{for } \begin{cases} j = i + 1, & i = 1, 2, \ldots, n-1 \\ j = i - 1, & i = 2, 3, \ldots, n \end{cases} \\ 0, & \text{otherwise.} \end{cases} \tag{41}$$

The constant vector b is given by

$$b = \left(\frac{120}{121}, \frac{117}{121}, \frac{112}{121}, \frac{105}{121}, \frac{96}{121}, \frac{85}{121}, \frac{72}{121}, \frac{57}{121}, \frac{40}{121}, \frac{-14620}{121}\right)^T. \tag{42}$$

Here, A is not an L-matrix. The comparison between the exact solution and the approximate solutions is given in Figure 1.

In Figure 1, we see that the SSOR method converges rapidly to the approximate solution of absolute complementarity problem (2) as compared to GAOR method.

In the next example, we compare SSOR method with iterative method by Noor et al. [13].

Example 14 (see [13]). Let the matrix A be given by

$$a_{i,j} = \begin{cases} 8, & \text{for } j = i \\ -1 & \text{for } \begin{cases} j = i + 1, & i = 1, 2, \ldots, n-1 \\ j = i - 1, & i = 2, 3, \ldots, n \end{cases} \\ 0, & \text{otherwise.} \end{cases} \tag{43}$$

Let $b = (6, 5, 5, \ldots, 5, 6)^T$, the problem size n, ranging from 4 to 1024. The stopping criteria are $\|Ax - |x| - b\| < 10^{-6}$.

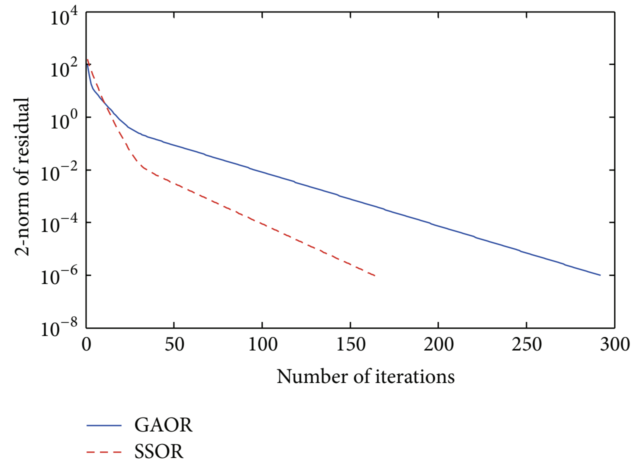

FIGURE 1: Comparison between GAOR method and SSOR method.

TABLE 1

n	Iterative method		SSOR method	
	Number of iterations	TOC	Number of iterations	TOC
4	10	0.0168	8	0.001
8	11	0.018	8	0.001
16	11	0.143	9	0.001
32	12	3.319	9	0.001
64	12	7.145	9	0.015
128	12	11.342	9	0.020
256	12	25.014	10	1.051
512	12	98.317	10	6.130
1024	13	534.903	10	126.242

We choose initial guess x_0 as $x_0 = (0, 0, \ldots, 0)^T$. The computational results are shown in Table 1.

In Table 1, TOC denotes the total time taken by CPU. The rate of convergence of SSOR method is better than that of iterative method [13].

4. Conclusion

In this paper, we have discussed symmetric SOR method for solving absolute complementarity problem. The comparison with other methods showed the efficiency of the method. The results and ideas of this paper may be used to solve the variational inequalities and related optimization problems.

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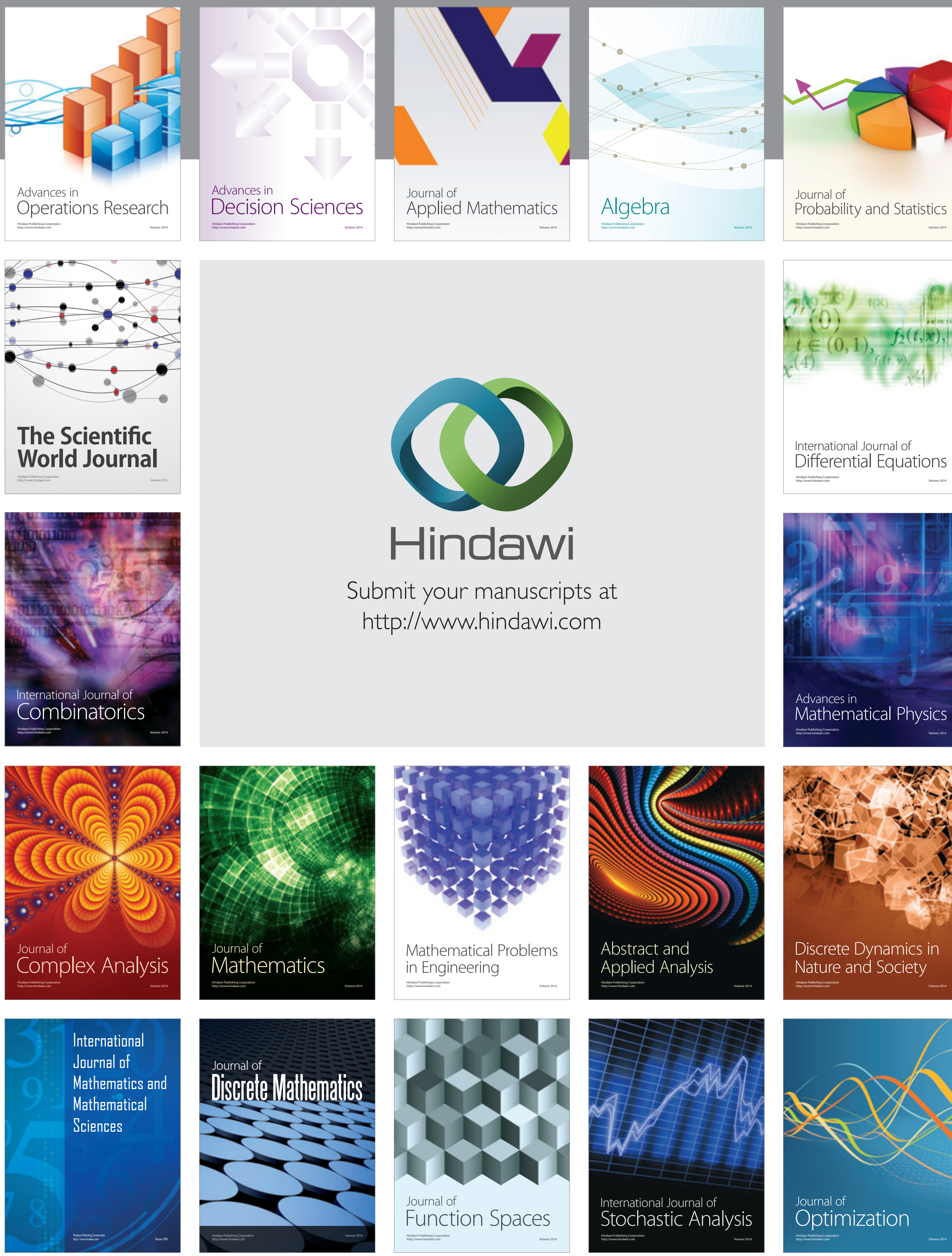
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