Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2012, Article ID 863707, 22 pages
doi:10.1155/2012/863707

Research Article

Hölder Scales of Sea Level

Ming Li,[1,2] YangQuan Chen,[3] Jia-Yue Li,[4] and Wei Zhao[1]

[1] *Department of Computer and Information Science, University of Macau, Avenida Padre Tomas Pereira, Taipa, Macau, Macau*
[2] *School of Information Science and Technology, East China Normal University, Shanghai 200241, China*
[3] *School of Engineering, University of California, Merced, CA 95343, USA*
[4] *College of Resources and Environmental Science, East China Normal University, Shanghai 200241, China*

Correspondence should be addressed to Ming Li, ming_lihk@yahoo.com

Received 2 November 2012; Accepted 21 November 2012

Academic Editor: Sheng-yong Chen

The statistics of sea level is essential in the field of geosciences, ranging from ocean dynamics to climates. The fractal properties of sea level, such as long-range dependence (LRD) or long memory, $1/f$ noise behavior, and self-similarity (SS), are known. However, the description of its multiscale behavior as well as local roughness with the Hölder exponent $h(t)$ from a view of multifractional Brownian motion (mBm) is rarely reported, to the best of our knowledge. In this research, we will exhibit that there is the multiscale property of sea level based on $h(t)$s of sea level data recorded by the National Data Buoy Center (NDBC) at six stations in the Florida and Eastern Gulf of Mexico. The contributions of this paper are twofold as follows. (i) Hölder exponent of sea level may not change with time considerably at small time scale, for example, daily time scale, but it varies significantly at large time scale, such as at monthly time scale. (ii) The dispersion of the Hölder exponents of sea level may be different at different stations. This implies that the Hölder roughness of sea level may be spatial dependent.

1. Introduction

The study of sea level fluctuations plays a role in geosciences [1–3]. There are two categories of time scales of sea level. One is for yearly data with time scales in one yr, or 10 yr, or more; see, for example, [4–16]. The other is about data with time scales hourly, daily, weekly, or monthly; see, for example, [17–39]. The former generally relates to the study of trend of relative mean sea level with respect to global and Earth or planetary changes, for example, in the filed of climates, while the latter is usually associated with the research of local dynamics of sea level in the aspects of navigations, coastal engineering, tide power production, ship design, and so forth. Our research uses the hourly sea level data recorded by NDBC [40].

Since the pioneering work of Hurst on time series with long-range dependence (LRD) is observed in the Nile Basin [41], the LRD property of time series in geosciences has been widely observed; see, for example, [42–59]. By LRD, one means that the covariance function $C(\tau)$ of time series $x(t)$ decays so slowly such that

$$\int_0^\infty C(\tau)d\tau = \infty, \tag{1.1}$$

where τ is time lag and $C(\tau) = E[x(t+\tau)x(t)]$. Therefore, LRD is a global property of time series [60–66].

In addition to LRD, there is another essential property of processes in geosciences, called self-similarity (SS); see, for example, [67–84]. By SS, we mean that a random function $x(t)$ satisfies the property given by

$$x(t) \triangleq a^H x(at), \quad \forall a > 0,\ t > 0, \tag{1.2}$$

where $\triangleq$ is the equality in distribution, $H \in (0,1)$ is the Hurst parameter that measures SS, and a is a scale [61, 83–86]. Note that the term SS implies the roughness or irregularity of a random function [86]. If $x(t)$ satisfies (1.2), it is globally self-similar. That is, its irregularity characterized by H keeps the same for all $t > 0$ [87], corresponding the case of monofractal [88, 89].

Since the global SS implies the same value of H for all t, it may be too restrictive to describe real data in engineering and sciences to use a monofractal model. Therefore, multifractal models are desired in various fields of sciences and engineering; see, for example, [85–92] and references therein, including those in geosciences; see, for example, [93–110], just citing a few. From a view of multifractal, a random function that is not self-similar may be of local self-similarity (LSS).

There are several ways of describing multifractality of a random function based on various definitions of dimensions, such as the Minkowski dimension, the Rényi dimension, the Hausdorff dimension, the packing dimension, the box-counting dimension, and the correlation dimension [86, 89, 90, 111–114]. In this paper, we adopt the Hölder exponent $0 < h(t) < 1$ in multifractional Brownian motion (mBm) introduced by Peltier and Levy-Vehel [115]. Taking into account $h(t)$ in mBm, therefore, one may use the following:

$$x(t) \triangleq a^{h(t)} x(at), \quad \forall a > 0,\ t > 0, \tag{1.3}$$

to characterize the LSS property of a locally self-similar random function $x(t)$ on a point-by-point basis. We call the LSS or local roughness characterized by $h(t)$ the Hölder roughness in this paper. The applications of $h(t)$ attract increasing interests of researchers in sciences and technologies, ranging from teletraffic to geophysics; see, for example, [116–134], simply mentioning a few.

This paper aims at investigating the Hölder multiscales (Hölder scales for short) of sea level. By Hölder scales, we mean the time scales described by the Hölder exponents in mBm. The contributions of this paper are in two aspects. On the one hand, we will reveal that variations of $h(t)$ of sea level may be indistinctively at small time scale, for example, daily time scale, but $h(t)$ of sea level varies significantly at large time scale, such as at monthly time scale. On the other hand, we will exhibit that the dispersion of the Hölder exponents of sea level may usually be spatial dependent.

Table 1: Measured data at LKWF1.

Series name	Record date and time	L (record length)
x_lkwf1_1996(t)	0:00, 1 Jan.–23:00, 31 Dec. 1996	8208
x_lkwf1_1997(t)	0:00, 1 Jan.–23:00, 31 Dec. 1997	7776
x_lkwf1_1998(t)	0:00, 1 Jan.–23:00, 31 Dec. 1998	8736
x_lkwf1_1999(t)	0:00, 1 Jan.–23:00, 31 Dec. 1999	8760
x_lkwf1_2000(t)	0:00, 1 Jan.–17:00, 26 Feb. 2000	1362
x_lkwf1_2001(t)	17:00, 8 Aug.–23:00, 31 Dec. 2001	2972
x_lkwf1_2002(t)	0:00, 1 Jan.–23:00, 31 Dec. 2002	8740
x_lkwf1_2003(t)	0:00, 1 Jan.–23:00, 31 Dec. 2003	8582
x_lkwf1_2004(t)	0:00, 1 Jan.–14:00, 5 Oct. 2004	6655

The remaining paper is organized as follows. Data used in this research are briefed in Section 2. The method for describing the Hölder exponent in mBm is explained in Section 3. Results of data processing and discussions are given in Section 4, which is followed by conclusions.

2. Data

NDBC is a part of the US National Weather Service (NWS) [135]. It provides scientists with data for their scientific research, including significant wave height and water level [136]. We use the data measured at stations named LKWF1, LONF1, SAUF1, SMKUF1, SPGF1, and VENF1, respectively. In terms of the names of measurement stations, LKWF1 implies the station at Lake Worth, FL [137]; the station LONF1 is the one at Long Key, FL [138]; the station SAUF1 is at St. Augustine, FL [139]; SMKUF1 is the station at Sombrero Key, FL [140]; SPGF1 is at Settlement Point, GBI [141]; and VENF1 is at Venice, FL [142]. They are located in the Florida and Eastern Gulf of Mexico. The data are under the directory of Water Level, which are publicly accessible [143], referring Gilhousen [144] as an instance of research using the data by NDBC.

All data were hourly recorded with ten separate devices indexed by TGn ($n = 01, 02, \ldots, 10$). Without losing generality, this research utilizes the data from the device *TG01*. Denote the data series by $x\_s\_yyyy(t)$, where s is the name of the measurement station and $yyyy$ stands for the index of year. Denote by $h\_s\_yyyy(t)$ its corresponding $h(t)$ at the station s in the year of $yyyy$. For example, x_lkwf1l_2002(t) and h_lkwf1l_2002(t), respectively, represent the measured sea level time series and its $h(t)$ at the station LKWF1 in 2002.

If the recorded data are labeled by 99, they are taken as outliers, which are not involved in the computations. In this case, they are replaced with the mean of that series. NDBC suggests that 10 ft should be subtracted from every level series $x\_s\_yyyy(t)$ [145]. By taking into account this suggestion in the computation of $h(t)$, we modify $x\_s\_yyyy(t)$ by subtracting 10 ft and denote $y\_s\_yyyy(t)$ modified data of sea level. That is,

$$y\_s\_yyyy(t) = x\_s\_yyyy(t) - 10. \tag{2.1}$$

Tables 1, 2, 3, 4, 5 and 6 list those data.

Table 2: Measured data at LONF1.

Series name	Record date and time	L (record length)
x_lonf11_1998(t)	0:00, 3 Nov.–23:00, 31 Dec. 1998	1416
x_lonf1_1999(t)	0:00, 1 Jan.–21:00, 31 Dec. 1999	8757
x_lonf1_2000(t)	0:00, 1 Jan.–23:00, 31 Dec. 2000	8484
x_lonf1_2001(t)	0:00, 1 Jan.–23:00, 31 Dec. 2001	8760
x_lonf1_2002(t)	0:00, 1 Jan.–23:00, 31 Dec. 2002	8760
x_lonf1_2003(t)	0:00, 1 Jan.–23:00, 31 Dec. 2003	8697
x_lonf1_2004(t)	0:00, 1 Jan.–23:00, 31 Dec. 2004	8758
x_lonf1_2005(t)	0:00, 1 Jan.–23:00, 31 Dec. 2005	8750
x_lonf1_2006(t)	0:00, 1 Jan.–23:00, 31 Dec. 2006	8735
x_lonf1_2007(t)	0:00, 1 Jan.–23:00, 31 Dec. 2007	8692
x_lonf1_2008(t)	0:00, 1 Jan.–21:00, 19 Jan. 2008	444

Table 3: Measured data at SAUF1.

Series name	Record date and time	L (record length)
x_sauf1_1996(t)	0:00, 1 Jan.–14:00, 10 Aug. 1996	5511
x_sauf1_1997(t)	0:00, 25 Feb.–23:00, 31 Dec. 1997	6240
x_sauf1_1998(t)	0:00, 1 Jan.–23:00, 31 Dec. 1998	8736
x_sauf1_1999(t)	0:00, 1 Jan.–23:00, 31 Dec. 1999	8136
x_sauf1_2000(t)	0:00, 1 Jan.–23:00, 31 Dec. 2000	8715
x_sauf1_2001(t)	0:00, 1 Jan.–21:00, 31 Dec. 2001	8758
x_sauf1_2002(t)	20:00, 6 Feb.–23:00, 20 Aug. 2002	4684

Table 4: Measured data at SMKF1.

Series name	Record date and time	L (record length)
x_smkf1_1998(t)	0:00, 3 Nov.–23:00, 31 Dec. 1998	1416
x_smkf1_1999(t)	0:00, 1 Jan.–23:00, 31 Dec. 1999	7775
x_smkf1_2000(t)	0:00, 1 Aug.–23:00, 31 Dec. 2000	3542
x_smkf1_2001(t)	0:00, 1 Jan.–23:00, 31 Dec. 2001	5776
x_smkf1_2002(t)	0:00, 1 Jan.–23:00, 31 Dec. 2002	8742
x_smkf1_2003(t)	0:00, 1 Jan.–23:00, 31 Dec. 2003	5851
x_smkf1_2004(t)	0:00, 1 Jan.–23:00, 31 Dec. 2004	8439
x_smkf1_2005(t)	0:00, 1 Jan.–23:00, 31 Dec. 2005	8667
x_smkf1_2006(t)	0:00, 1 Jan.–23:00, 31 Dec. 2006	8623
x_smkf1_2007(t)	0:00, 1 Jan.–23:00, 31 Dec. 2007	8702
x_smkf1_2008(t)	0:00, 1 Jan.–23:00, 31 Dec. 2008	8679
x_smkf1_2009(t)	0:00, 1 Jan.–23:00, 31 Dec. 2009	8109
x_smkf1_2010(t)	0:00, 1 Jan.–23:00, 31 July 2010	5074
x_smkf1_2011(t)	0:00, 1 Jan.–23:00, 31 Dec. 2011	8759

Table 5: Measured data at SPGF1.

Series name	Record date and time	L (record length)
x_spgf1_1996(t)	0:00, 1 Jan.–23:00, 15 Dec. 1996	8616
x_spg1_1997(t)	0:00, 6 Mar.–23:00, 15 Dec. 1997	7080
x_spg1_1998(t)	0:00, 1 Jan.–23:00, 7 Jan. 1998	168

Table 6: Measured data at VENF1.

Series name	Record date and time	L (record length)
x_venf1_2002(t)	0:00, 1 Oct.–23:00, 31 Dec. 2002	2208
x_ven1_2003(t)	0:00, 1 Jan.–23:00, 31 Dec. 2003	8760
x_ven1_2004(t)	0:00, 1 Jan.–16:00, 7 Jan. 2004	634
x_ven1_2006(t)	14:00, 22 July–23:00, 31 Dec. 2006	3882
x_ven1_2007(t)	0:00, 1 Jan.–23:00, 31 Dec. 2007	8663
x_ven1_2008(t)	0:00, 1 Jan.–23:00, 31 Oct. 2008	7189

3. Methodology

Let $B(t)$ be the standard Brownian motion. Then, $B(t)$ satisfies the following properties.

(i) The increments $B(\tau + t) - B(t)$ are Gaussian.

(ii) $E[B(\tau + t) - B(t)] = 0$ and

$$\mathrm{Var}[B(t + \tau) - B(t)] = \sigma^2\tau, \tag{3.1}$$

where $\sigma^2 = E\{[B(t + 1) - B(t)]^2\} = E\{[B(1) - B(0)]^2\} = E\{[B(1)]^2\}$.

(iii) In nonoverlapping intervals $[t_1, t_2]$ and $[t_3, t_4]$, the increments $B(t_4)$-$B(t_3)$ and $B(t_2)$-$B(t_1)$ are independent.

(iv) $B(0) = 0$ and $B(t)$ is continuous at $t = 0$.

Kolmogorov introduced a class of random functions the covariance function of which is now recognized as the one of fractional Brownian motion (fBm) [146, Theorem 6]. Note that, for a random function $x(t)$, the function $f(\tau)$ expressed by

$$f(\tau) = \mathrm{Var}[x(t + \tau) - x(t)] = E\left\{[x(t + \tau) - x(t)]^2\right\} \tag{3.2}$$

is termed serial variation function; see, for example, Matérn [147, page 51]. It is usually called variogram in geosciences [148–157]. In the field of fluid mechanics, it is named structure function [158–161]. Yaglom derived fBm based on the theory of structure functions [162]. In this paper, we use the fBm introduced by Bandelbrot and van Ness based on fractional calculus [163].

It is well known that $B(t)$ is nondifferentiable in the domain of ordinary functions [164–166]. In the domain of generalized functions, however, it is differentiable [167, 168].

Denote the fBm by $B_H(t)$. Based on the Weyl's fractional derivative or integral [163], it is expressed by

$$B_H(t) - B_H(0) = \frac{1}{\Gamma(H + 1/2)}\left\{\begin{aligned}&\int_{-\infty}^{0}\left[(t-u)^{H-0.5} - (-u)^{H-0.5}\right]dB(u)\\ &+\int_0^t (t-u)^{H-0.5}dB(u)\end{aligned}\right\}. \tag{3.3}$$

If the first item on the right hand of (3.3) is taken as the zero-input response of the system that generates $B_H(t)$ for $t > 0$, we may regard the fBm as the convolution of the impulse function $t^{H-1/2}/\Gamma(H+1/2)$ and $dB(t)/dt$ [169]. Therefore, (3.3) may be rewritten by

$$B_H(t) - B_H(0) = B^0(u) + \frac{t^{H-0.5}}{\Gamma(H+1/2)} * \frac{dB(t)}{dt}, \tag{3.4}$$

where $*$ is the operator of convolution and

$$B^0(u) = \frac{1}{\Gamma(H+1/2)} \int_{-\infty}^{0} \left[(t-u)^{H-0.5} - (-u)^{H-0.5}\right] dB(t). \tag{3.5}$$

It may be interesting to note that $t^{H-1/2}/\Gamma(H+1/2)$ is a special case of the operators of fractional order discussed by Mikusinski [170, Equation (59.1)].

The function $B_H(t)$ has the following properties.

(i) $B_H(0) = 0$.

(ii) The increments $B_H(t+t_0) - B_H(t_0)$ are Gaussian.

(iii) Its structure function is given by

$$\operatorname{Var}[B_H(t+\tau) - B_H(t)] = \sigma^2 \tau^{2H}, \tag{3.6}$$

where $\sigma^2 = E\{[B_H(t+1) - B_H(t)]^2\} = E\{[B_H(1) - B_H(0)]^2\} = E\{[B_H(1)]^2\}$.

In addition, it satisfies the self-similarity expressed by (1.2), which implies that $B_H(t)$ is globally self-similar. Consequently, there is a limitation that its self-similarity or roughness keeps the same for all $t > 0$. To release such a limitation, one may adopt the tool of the mBm equipped with the Hölder exponent $h(t)$; see, for example, [115, 119, 133]. In fact, the mBm is a generalization of fBm by replacing the Hurst parameter H in (3.3) with a continuous function $h(t)$ that satisfies $H : [0, \infty) \to (0, 1)$; see [87, 115–134, 171–182]. Denote the mBm by $X(t)$. Then,

$$X(t) = \frac{1}{\Gamma(h(t)+1/2)} \left\{ \begin{array}{c} \int_{-\infty}^{0} \left[(t-u)^{h(t)-0.5} - (-u)^{h(t)-0.5}\right] dB(u) \\ + \int_{0}^{t} (t-u)^{h(t)-0.5} dB(u) \end{array} \right\}. \tag{3.7}$$

Considering the local growth of the increment process of $X(t)$, one may write a sequence given by

$$S_k(j) = \frac{m}{N-1} \sum_{j=0}^{j+k} \left| X(i+1) - X(i) \right|, \quad 1 < k < N, \tag{3.8}$$

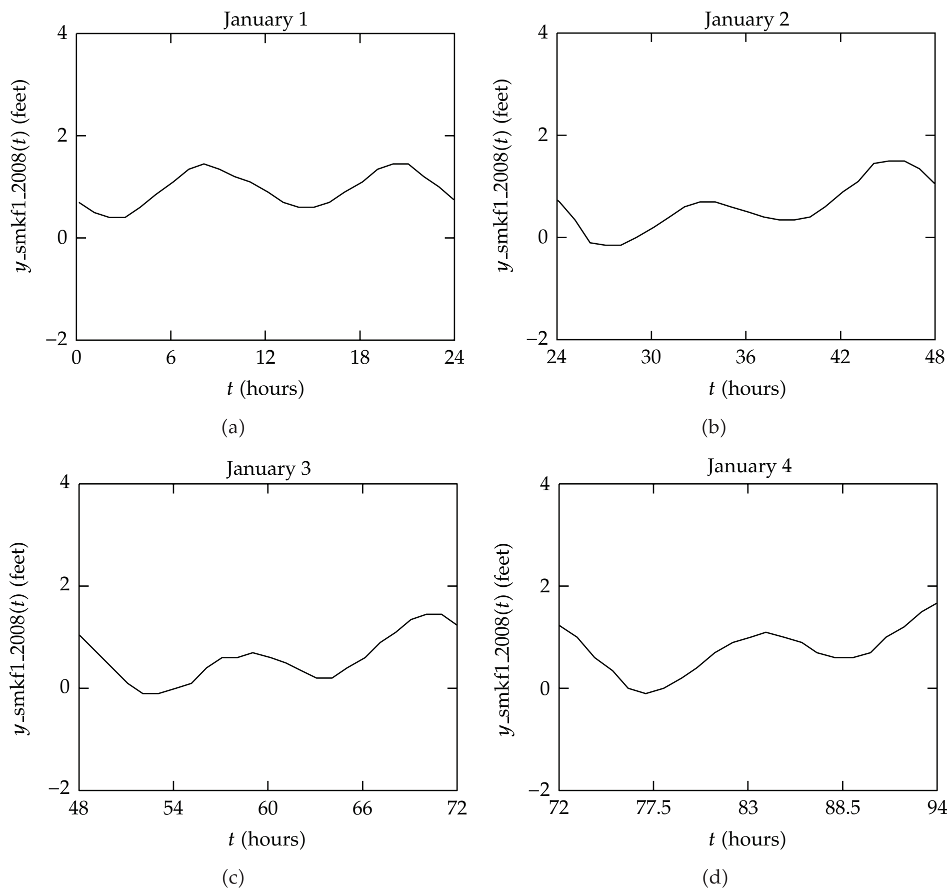

Figure 1: Daily sea level at the station SMKUF1 from January 1 to Jan. 4 in 2008. (a). x_smkf1_2008(t) on Jan. 1, 2008. (b). x_smkf1_2008(t) on Jan. 2, 2008. (c). x_smkf1_2008(t) on Jan. 3, 2008. (d). x_smkf1_2008(t) on Jan. 4, 2008.

where m is the largest integer not exceeding N/k. Then, $h(t)$ at point $t = j/(N-1)$ is given by

$$h(t) = -\frac{\log\left(\sqrt{\pi/2}S_k(j)\right)}{\log(N-1)}. \tag{3.9}$$

The above is the expression of applying mBm to investigate $h(t)$ of sea level time series, which measures the Hölder roughness of sea level on a point-by-point basis.

4. Observations and Discussions

We demonstrate $h(t)s$ of sea level series x_smkf1_2008(t) at the time scales of day, week, and month, respectively.

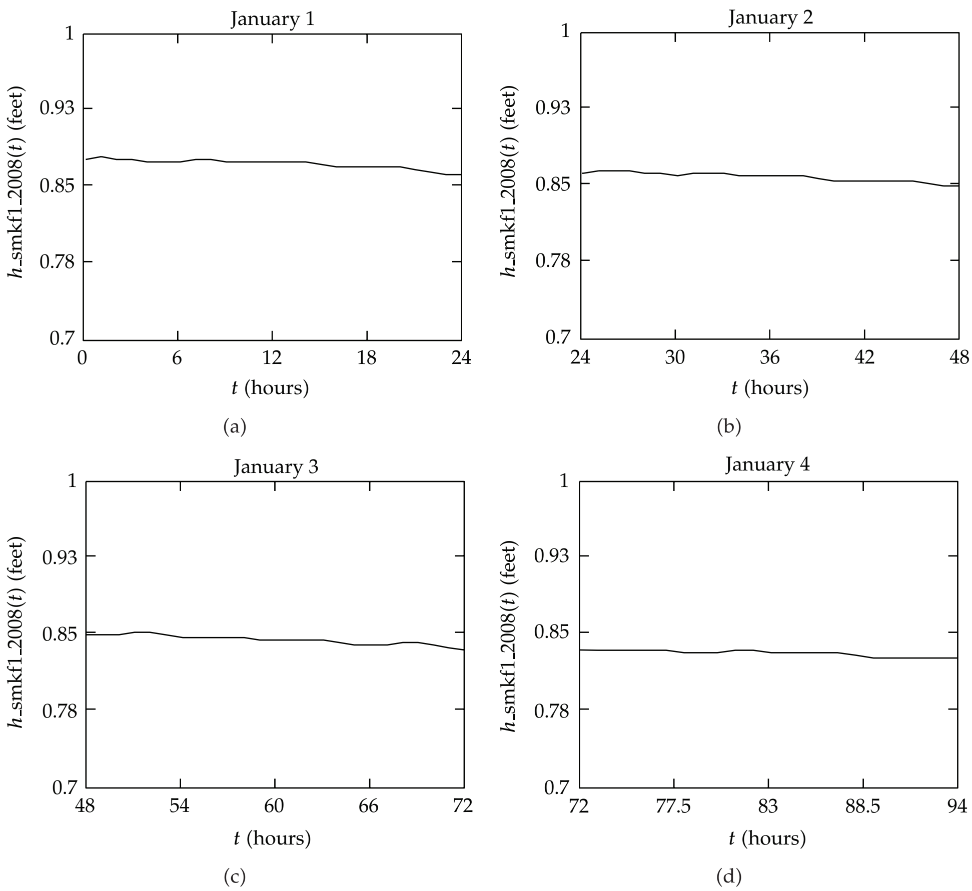

Figure 2: Hölder exponents of daily sea level at the station SMKUF1 from Jan. 1 to Jan. 4 in 2008. (a). h_smkf1_2008(t) on Jan. 1, 2008. (b). h_smkf1_2008(t) on Jan. 2, 2008. (c). h_smkf1_2008(t) on Jan. 3, 2008. (d). h_smkf1_2008(t) on Jan. 4, 2008.

4.1. Hölder Roughness at Daily Time Scale

Figure 1 indicates 4 daily series of sea level at the station SMKUF1 from Jan. 1 to Jan. 4 in 2008. Figure 2 demonstrates their corresponding Hölder exponents. From Figure 2, we see that $4h(t)$s of daily series of sea level vary with time insignificantly. Therefore, we obtain the remark below.

Remark 4.1. The Hölder exponents of sea level at the daily time scale, that is, 24 hours, may not vary significantly. This may imply that $h(t) \approx h(t+\tau)$ if $\tau \leq 24$ hours.

4.2. Hölder Roughness at Weekly Time Scale

Four weekly series of sea level at the station SMKUF1 in Jan. 2008 are shown in Figure 3. Their corresponding Hölder exponents are plotted in Figure 4. They appear monotonically increase, see Figures 4(b) and 4(d), or decrease, see Figures 4(a) and 4(c). In general, they imply the following remark.

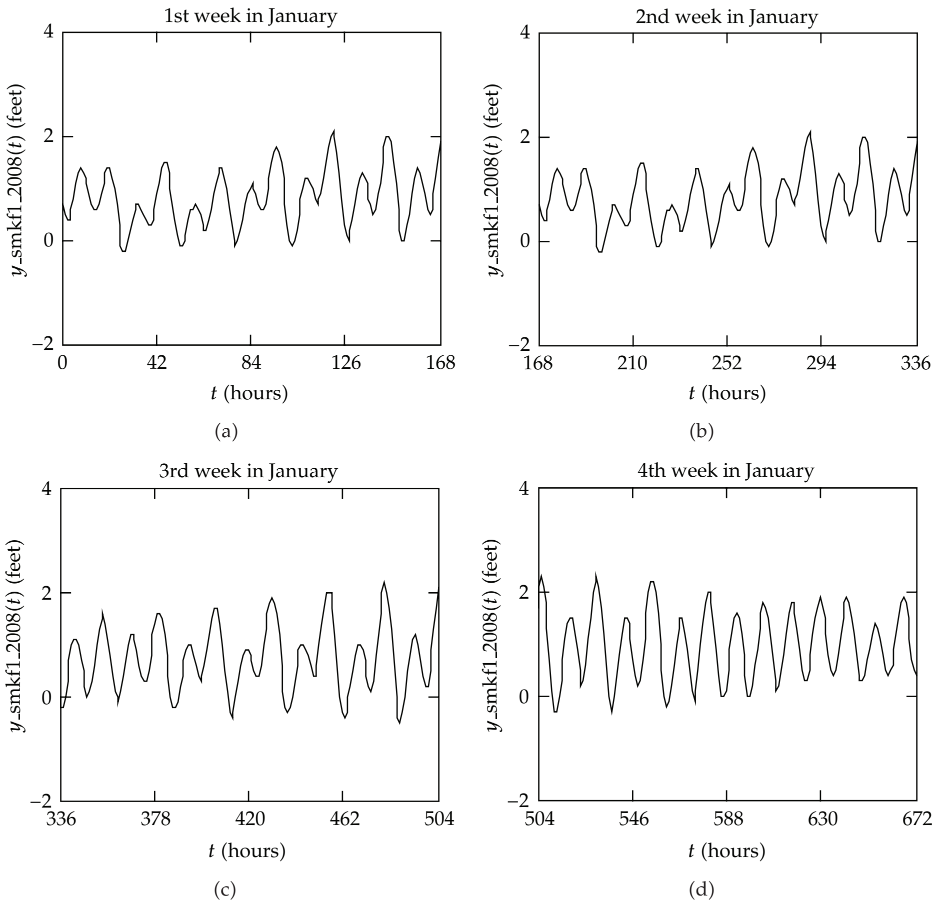

Figure 3: Weekly sea level at the station SMKUF1 on January 2008. (a). x_smkf1_2008(t) in the 1st week in Jan. 2008. (b). x_smkf1_2008(t) in the 2nd week in Jan. 2008. (c). x_smkf1_2008(t) in the 3rd week on January 2008. (d). x_smkf1_2008(t) in the 4th week in Jan. 2008.

Remark 4.2. The Hölder exponents of sea level at the weekly time scale, that is, 168 hours, may not vary considerably enough.

4.3. Hölder Roughness at Monthly Time Scale

Figure 5 illustrates 4 monthly series of sea level at the station SMKUF1 in 2008. Their corresponding Hölder exponents are indicated in Figure 6. From Figure 6, we see the following.

Remark 4.3. The Hölder exponents of sea level at the monthly time scale vary with time significantly.

4.4. Variation of Hölder Roughness at Large Time Scale

We now investigate the Hölder exponents of sea level at large time scale. By large time scale, we mean that the scale is around month or larger. Figure 7(a) indicates the sea level series

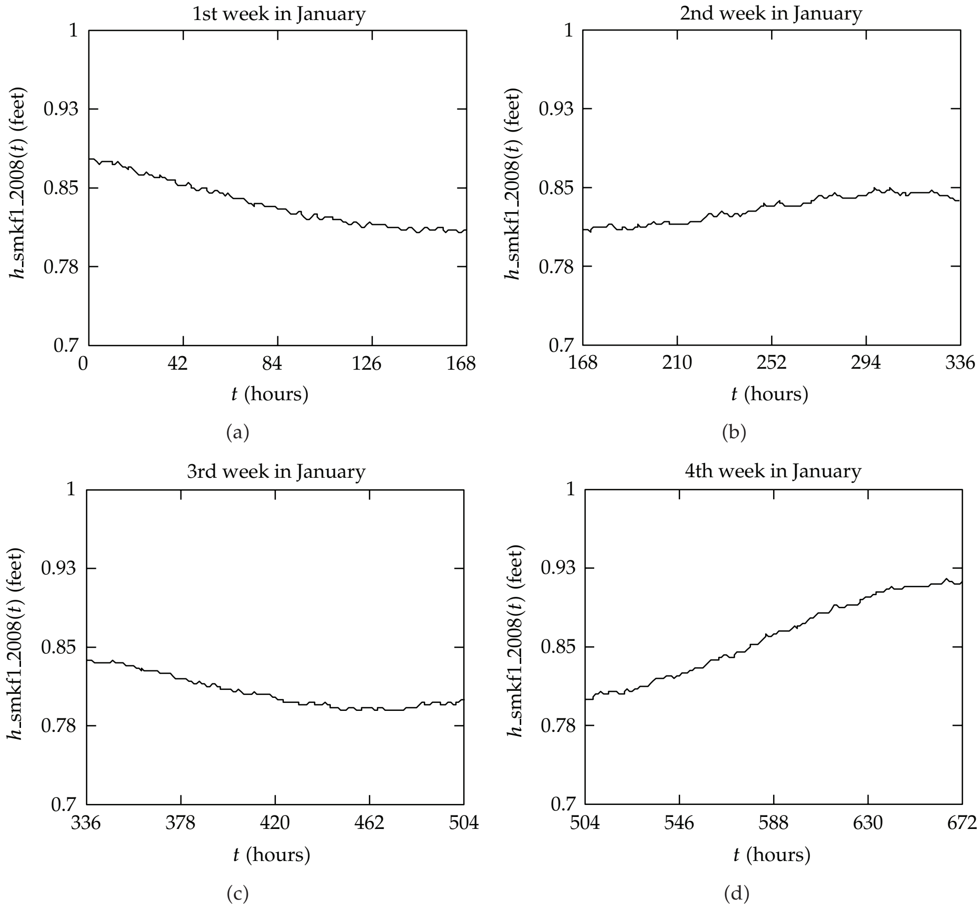

Figure 4: Hölder exponents of weekly sea level at the station SMKUF1 in January 2008. (a). h_smkf1_2008(t) in the 1st week on January 2008. (b). h_smkf1_2008(t) in the 2nd week in Jan. 2008. (c). h_smkf1_2008(t) in the 3rd week in Jan. 2008. (d). h_smkf1_2008(t) in the 4th week in Jan. 2008.

x_smkf1_2008(t), Figure 7(b) shows its Hölder exponent, and Figure 7(c) the histogram of its Hölder exponent.

One thing worth noting is that variances of Hölder exponents of sea level at different stations may be considerably different. For instance,

$$\begin{aligned} \mathrm{Var}[h\_\mathrm{smkf1\_2008}(t)] &= 1.203 \times 10^{-3}, \\ \mathrm{Var}[h\_\mathrm{lonf1l2005}(t)] &= 6.425 \times 10^{-4}. \end{aligned} \tag{4.1}$$

The above implies that the variance of h_smkf1_2008(t) is larger than that of h_lonf1l2005(t) in one magnitude of order. Consequently, comes the following remark.

Remark 4.4. The variances of the Hölder exponents of sea level at different observation stations may be considerably different.

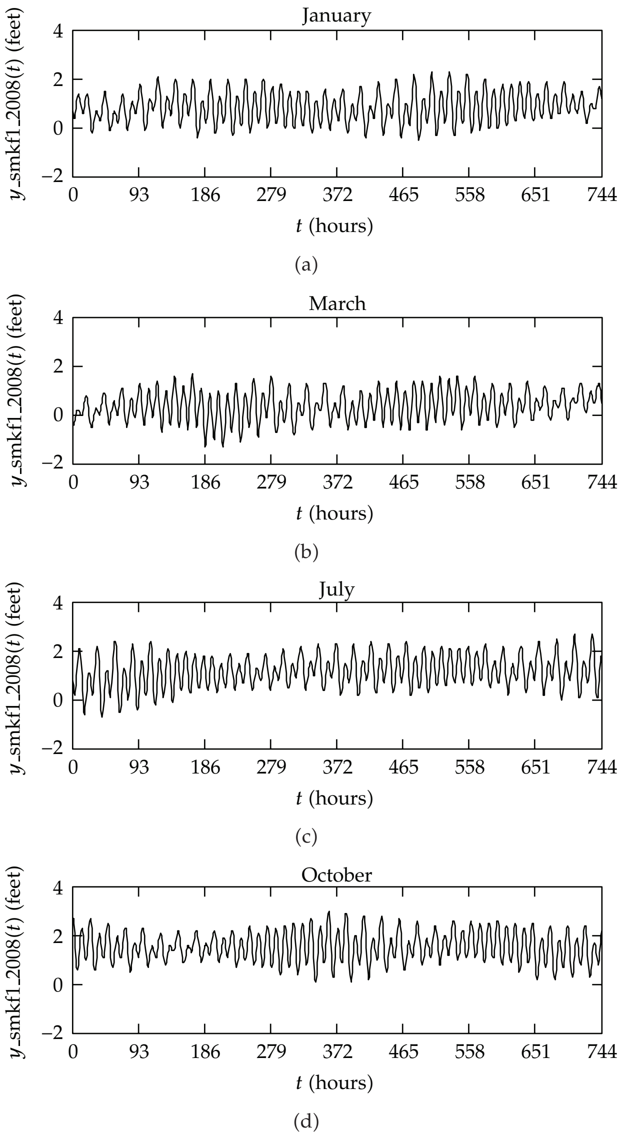

Figure 5: Monthly sea level at the station SMKUF1 in 2008. (a). x_smkf1_2008(t) on January 2008. (b). x_smkf1_2008(t) in March 2008. (c). x_smkf1_2008(t) on July 2008. (d). x_smkf1_2008(t) on October 2008.

We summarize the variances of the Hölder exponents of test data in Tables 7, 8, 9, 10, 11 and 12.

4.5. Discussions

Generally, the Hölder exponents of sea level series are time varying. They are considerably at large time scales but insignificantly at small time scales. In addition, their variations are in general spatial dependent as the Tables 7–12 exhibit. For instance, in 2002, Var$[h(t)]$ varies, in the form of magnitude of order, from 10^{-3} to 10^{-4} at different stations. This motivates us to

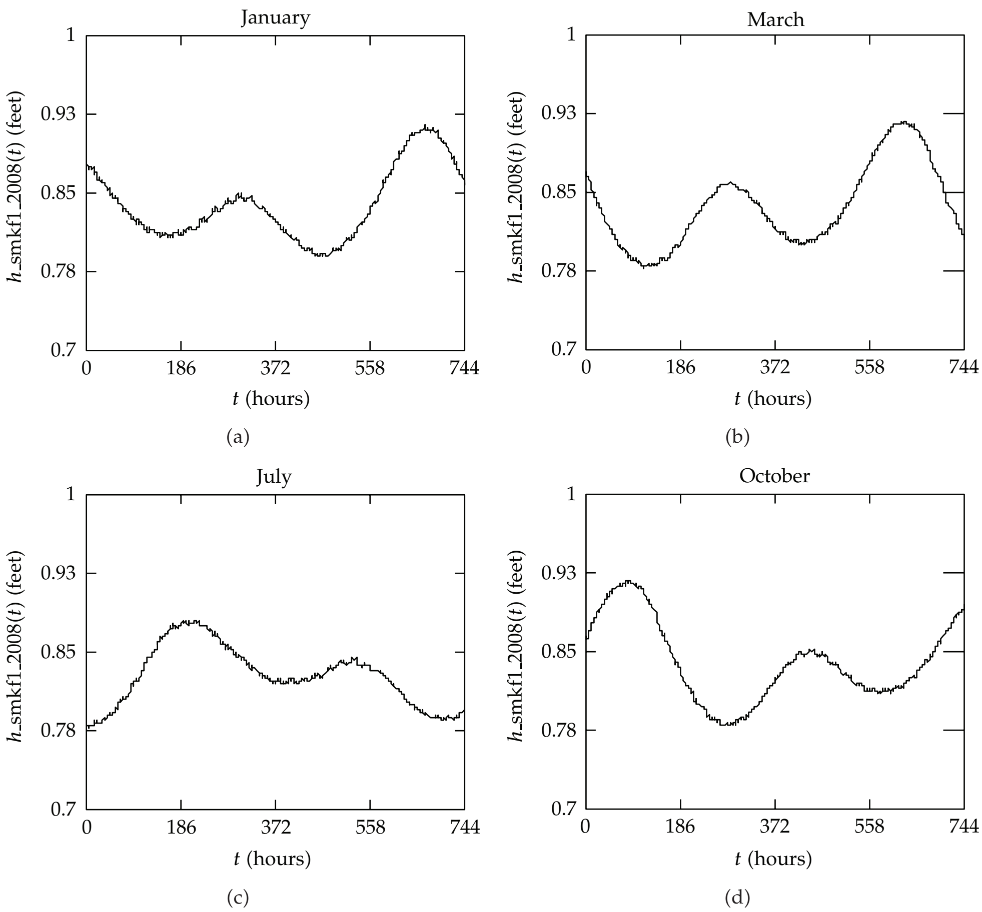

Figure 6: Hölder exponents of month sea level at the station SMKUF1 in 2008. (a). $h\_smkf1\_2008(t)$ on Jan. 2008. (b). $h\_smkf1\_2008(t)$ on March 2008. (c). $h\_smkf1\_2008(t)$ in July 2008. (d). $h\_smkf1\_2008(t)$ in October 2008.

take the spatial-time modeling of Hölder roughness of sea level as our possible future work. Finally, we note that the meaning of the term of local roughness of a random function is the same as that of local self-similarity [60, 65, 86]. Thus, according to (1.2), Remarks 4.1–4.3 exhibit the self-similarity of sea level at small and large time scales, respectively.

5. Conclusions

We have presented our results in the Hölder exponents of sea level in the Florida and Eastern Gulf of Mexico. The present results reveal an interesting phenomenon of time scales of sea level. To be precise, the Hölder exponents of sea level may not vary considerably at small time scales, such as daily time scale, but vary with time significantly at large time scale, such as monthly time scale. Moreover, our research exhibits that variations of the Hölder exponents of sea levels may be spatial dependent. Though the research is with the data in Florida and Eastern Gulf of Mexico, the results may be useful for further exploring general properties of the Hölder scales and roughness of sea level.

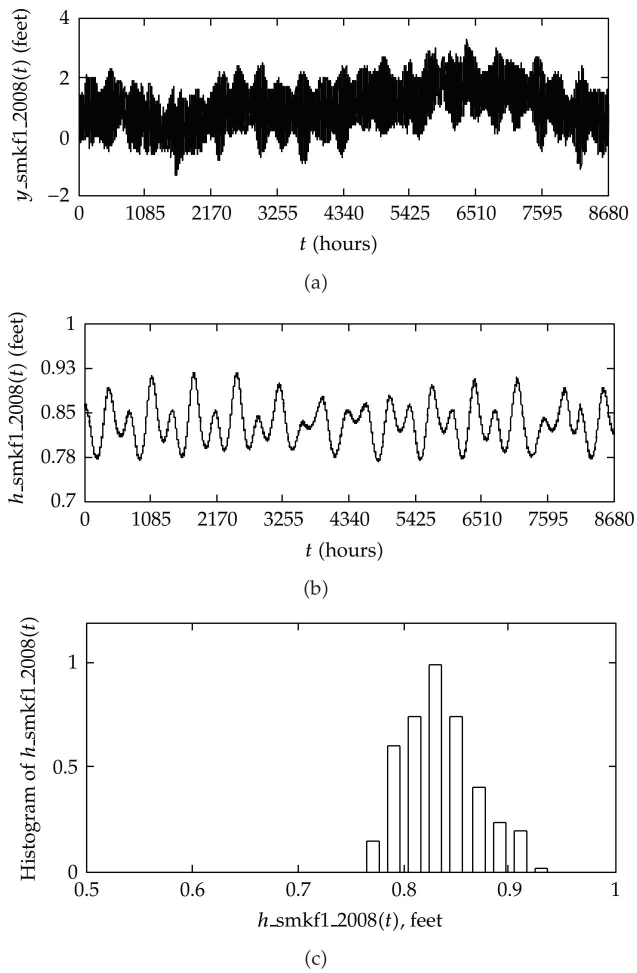

Figure 7: Illustrations of h_smkf1_2008(t) and its Hölder exponent. (a). x_smkf1_2008(t). (b) Hölder exponent h_smkf1_2008(t). (c). Histogram of h_smkf1_2008(t).

Table 7: Variances of the Hölder exponents at LKWF1.

Series name	Var[$h(t)$]
x_lkwf1_1996(t)	1.217×10^{-3}
x_lkwf1_1997(t)	1.006×10^{-3}
x_lkwf1_1998(t)	9.499×10^{-4}
x_lkwf1_1999(t)	1.164×10^{-3}
x_lkwf1_2000(t)	5.901×10^{-4}
x_lkwf1_2001(t)	1.169×10^{-3}
x_lkwf1_2002(t)	8.939×10^{-4}
x_lkwf1_2003(t)	9.710×10^{-4}
x_lkwf1_2004(t)	9.361×10^{-4}

Table 8: Variances of the Hölder exponents at LONF1.

Series name	Var[$h(t)$]
h_lonf11_1998(t)	3.978×10^{-3}
h_lonf1_1999(t)	4.123×10^{-4}
h_lonf1_2000(t)	1.570×10^{-3}
h_lonf1_2001(t)	1.135×10^{-3}
h_lonf1_2002(t)	1.407×10^{-3}
h_lonf1_2003(t)	2.359×10^{-3}
h_lonf1_2004(t)	9.493×10^{-4}
h_lonf1_2005(t)	6.425×10^{-4}
h_lonf1_2006(t)	1.245×10^{-3}
h_lonf1_2007(t)	2.142×10^{-3}
h_lonf1_2008(t)	8.245×10^{-5}

Table 9: Variances of the Hölder exponents at SAUF1.

Series name	Var[$h(t)$]
x_sauf1_1996(t)	1.083×10^{-3}
x_sauf1_1997(t)	1.355×10^{-3}
x_sauf1_1998(t)	8.766×10^{-4}
x_sauf1_1999(t)	1.324×10^{-3}
x_sauf1_2000(t)	7.272×10^{-4}
x_sauf1_2001(t)	6.961×10^{-4}
x_sauf1_2002(t)	3.992×10^{-3}

Table 10: Variances of the Hölder exponents at SMKF1.

Series name	Var[$h(t)$]
x_smkf1_1998(t)	9.501×10^{-4}
x_smkf1_1999(t)	1.144×10^{-3}
x_smkf1_2000(t)	1.310×10^{-3}
x_smkf1_2001(t)	1.520×10^{-3}
x_smkf1_2002(t)	1.181×10^{-3}
x_smkf1_2003(t)	1.176×10^{-3}
x_smkf1_2004(t)	1.243×10^{-3}
x_smkf1_2005(t)	1.210×10^{-3}
h_smkf1_2006(t)	1.101×10^{-3}
h_smkf1_2007(t)	1.164×10^{-3}
h_smkf1_2008(t)	1.203×10^{-3}
h_smkf1_2009(t)	1.242×10^{-3}
h_smkf1_2010(t)	1.084×10^{-3}
h_smkf1_2011(t)	1.176×10^{-3}

Table 11: Variances of the Hölder exponents at SPGF1.

Series name	Var[$h(t)$]
x_spgf1_1996(t)	1.018×10^{-3}
x_spgf1_1997(t)	8.803×10^{-4}
x_spgf1_1998(t)	2.659×10^{-4}

Table 12: Variances of the Hölder exponents at VENF1.

Series name	Var$[h(t)]$
x_venf1_2002(t)	1.069×10^{-3}
x_venf1_2003(t)	1.271×10^{-3}
x_venf1_2004(t)	8.863×10^{-4}
x_venf1_2006(t)	2.268×10^{-3}
x_venf1_2007(t)	2.454×10^{-3}
x_venf1_2008(t)	2.930×10^{-3}

Acknowledgments

This work was supported in part by the 973 Plan under the Project no. 2011CB302800, and the National Natural Science Foundation of China under the project Grants nos. 61272402, 61070214, and 60873264. The National Data Buoy Center is highly appreciated for its measured data that makes our research possible.

References

[1] Geophysics Study Committee and National Research Council, *Sea-Level Change*, National Academic Press, 1990.

[2] Committee on Engineering Implications of Changes in Relative Mean Sea Level, Marine Board, and National Research Council, *Responding To Changes in Sea Level: Engineering Implications*, National Academic Press, 1987.

[3] V. Gornitz, S. Couch, and E. K. Hartig, “Impacts of sea level rise in the New York City metropolitan area,” *Global and Planetary Change*, vol. 32, no. 1, pp. 61–88, 2001.

[4] S. Jevrejeva, J. C. Moore, and A. Grinsted, “Sea level projections to AD2500 with a new generation of climate change scenarios,” *Global and Planetary Changes*, vol. 80-81, pp. 14–20, 2012.

[5] M. Becker, B. Meyssignac, C. Letetrel, W. Llovel, A. Cazenave, and T. Delcroix, “Sea level variations at tropical Pacific islands since 1950,” *Global and Planetary Change*, vol. 80-81, pp. 85–98, 2012.

[6] B. Meyssignac and A. Cazenave, “Sea level: a review of present-day and recent-past changes and variability,” *Journal of Geodynamics*, vol. 58, pp. 96–109, 2012.

[7] A. J. Long, S. A. Woodroffe, G. A. Milne, C. L. Bryant, M. J. R. Simpson, and L. M. Wake, “Relative sea-level change in Greenland during the last 700 yrs and ice sheet response to the Little Ice Age,” *Earth and Planetary Science Letters*, vol. 315-316, pp. 76–85, 2012.

[8] M. Eliot, “Sea level variability influencing coastal flooding in the Swan River region, Western Australia,” *Continental Shelf Research*, vol. 33, pp. 14–28, 2012.

[9] N. N. Warner and P. E. Tissot, “Storm flooding sensitivity to sea level rise for Galveston Bay, Texas,” *Ocean Engineering*, vol. 44, pp. 23–32, 2012.

[10] A. Ozyavas and S. D. Khan, “The driving forces behind the Caspian Sea mean water level oscillations,” *Environmental Earth Sciences*, vol. 65, no. 6, pp. 1821–1830, 2012.

[11] J. Hinkel, S. Brown, L. Exner, R. J. Nicholls, A. T. Vafeidis, and A. S. Kebede, “Sea-level rise impacts on Africa and the effects of mitigation and adaptation: an application of DIVA,” *Regional Environmental Change*, vol. 12, no. 1, pp. 207–224, 2012.

[12] E. Gasson, M. Siddall, D. J. Lunt, O. J. L. Rackham, C. H. Lear, and D. Pollard, “Exploring uncertainties in the relationship between temperature, ice volume, and sea level over the past 50 million years,” *Reviews of Geophysics*, vol. 50, no. 1, Article ID RG1005, 2012.

[13] G. Wopelmann and M. Marcos, “Coastal sea level rise in southern Europe and the nonclimate contribution of vertical land motion,” *Journal of Geophysical Research*, vol. 117, no. 1, Article ID C01007, 2012.

[14] J. M. Gregory, J. A. Church, G. J. Boer et al., "Comparison of results from several AOGCMs for global and regional sea-level change 1900—2100," *Climate Dynamics*, vol. 18, no. 3-4, pp. 223–240, 2001.

[15] F. Albrecht, T. Wahl, J. Jensen, and R. Weisse, "Determining sea level change in the German Bight," *Ocean Dynamics*, vol. 61, no. 12, pp. 2037–2050, 2011.

[16] R. D. Ray and B. C. Douglas, "Experiments in reconstructing twentieth-century sea levels," *Progress in Oceanography*, vol. 91, no. 4, pp. 496–515, 2011.

[17] N. F. Barber and F. Ursell, "The generation and propagation of ocean waves and swell," *Philosophical Transactions A*, vol. 240, no. 824, pp. 527–560, 1948.

[18] E. M. Bitner-Gregersen and S. Gran, "Local properties of sea waves derived from a wave record," *Applied Ocean Research*, vol. 5, no. 4, pp. 210–214, 1983.

[19] A. Veltcheva, P. Cavaco, and C. G. Soares, "Comparison of methods for calculation of the wave envelope," *Ocean Engineering*, vol. 30, no. 7, pp. 937–948, 2003.

[20] C. G. Soares and Z. Cherneva, "Spectrogram analysis of the time-frequency characteristics of ocean wind waves," *Ocean Engineering*, vol. 32, no. 14-15, pp. 1643–1663, 2005.

[21] H. B. Bluestein, "A review of ground-based, mobile, W-band Doppler-radar observations of tornadoes and dust devils," *Dynamics of Atmospheres and Oceans*, vol. 40, no. 3, pp. 163–188, 2005.

[22] A. Baxevani, I. Rychlik, and R. J. Wilson, "A new method for modelling the space variability of significant wave height," *Extremes*, vol. 8, no. 4, pp. 267–294, 2005.

[23] L. C. Breaker, "Nonlinear aspects of sea surface temperature in Monterey Bay," *Progress in Oceanography*, vol. 69, no. 1, pp. 61–89, 2006.

[24] J. Gower, C. Hu, G. Borstad, and S. King, "Ocean color satellites show extensive lines of floating sargassum in the gulf of Mexico," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 44, no. 12, pp. 3619–3625, 2006.

[25] R. Schumann, H. Baudler, Ä. Glass, K. Dümcke, and U. Karsten, "Long-term observations on salinity dynamics in a tideless shallow coastal lagoon of the Southern Baltic Sea coast and their biological relevance," *Journal of Marine Systems*, vol. 60, no. 3-4, pp. 330–344, 2006.

[26] A. Sarkar, J. Kshatriya, and K. Satheesan, "Auto-correlation analysis of wave heights in the Bay of Bengal," *Journal of Earth System Science*, vol. 115, no. 2, pp. 235–237, 2006.

[27] S. Caires, V. R. Swail, and X. L. Wang, "Projection and analysis of extreme wave climate," *Journal of Climate*, vol. 19, no. 21, pp. 5581–5605, 2006.

[28] T. L. Walton Jr., "Projected sea level rise in Florida," *Ocean Engineering*, vol. 34, no. 13, pp. 1832–1840, 2007.

[29] P. A. Pirazzoli and A. Tomasin, "Estimation of return periods for extreme sea levels: a simplified empirical correction of the joint probabilities method with examples from the French Atlantic coast and three ports in the southwest of the UK," *Ocean Dynamics*, vol. 57, no. 2, pp. 91–107, 2007.

[30] R. J. Romanowicz, P. C. Young, K. J. Beven, and F. Pappenberger, "A data based mechanistic approach to nonlinear flood routing and adaptive flood level forecasting," *Advances in Water Resources*, vol. 31, no. 8, pp. 1048–1056, 2008.

[31] S. Castanedo, F. J. Mendez, R. Medina, and A. J. Abascal, "Long-term tidal level distribution using a wave-by-wave approach," *Advances in Water Resources*, vol. 30, no. 11, pp. 2271–2282, 2007.

[32] R. Kalra and M. C. Deo, "Derivation of coastal wind and wave parameters from offshore measurements of TOPEX satellite using ANN," *Coastal Engineering*, vol. 54, no. 3, pp. 187–196, 2007.

[33] P. K. Tonnon, L. C. van Rijn, and D. J. R. Walstra, "The morphodynamic modelling of tidal sand waves on the shoreface," *Coastal Engineering*, vol. 54, no. 4, pp. 279–296, 2007.

[34] H. Bazargan, H. Bahai, and A. Aminzadeh-Gohari, "Calculating the return value using a mathematical model of significant wave height," *Journal of Marine Science and Technology*, vol. 12, no. 1, pp. 34–42, 2007.

[35] K. Günaydin, "The estimation of monthly mean significant wave heights by using artificial neural network and regression methods," *Ocean Engineering*, vol. 35, no. 14-15, pp. 1406–1415, 2008.

[36] J. W. Kang, S.-R. Moon, S.-J. Park, and K.-H. Lee, "Analyzing sea level rise and tide characteristics change driven by coastal construction at Mokpo Coastal Zone in Korea," *Ocean Engineering*, vol. 36, no. 6-7, pp. 415–425, 2009.

[37] T. Ito, M. Okubo, and T. Sagiya, "High resolution mapping of Earth tide response based on GPS data in Japan," *Journal of Geodynamics*, vol. 48, no. 3–5, pp. 253–259, 2009.

[38] Y. Lin, R. J. Greatbatch, and J. Sheng, "The influence of Gulf of Mexico loop current intrusion on the transport of the Florida Current," *Ocean Dynamics*, vol. 60, no. 5, pp. 1075–1084, 2010.

[39] P. Xiao, Z. Yu, and C. S. Li, "Compressive sensing SAR range compression with chirp scaling principle," *Science China Information Sciences*, vol. 54, no. 1, pp. 2292–2300, 2012.

[40] http://www.ndbc.noaa.gov/historical_data.shtml.

[41] H. E. Hurst, "Long-term storage capacity of reservoirs," *Transactions of the American Society of Civil Engineers*, vol. 116, pp. 770–799, 1951.

[42] B. B. Mandelbrot and J. R. Wallis, "Robustness of the rescaled range R/S in the measurement of noncyclic long run statistical dependence," *Water Resources Research*, vol. 5, no. 5, pp. 967–988, 1969.

[43] B. B. Mandelbrot and J. R. Wallis, "Some long-run properties of geophysical records," *Water Resources Research*, vol. 5, no. 2, pp. 967–988, 1969.

[44] W. Rea, M. Reale, J. Brown, and L. Oxley, "Long memory or shifting means in geophysical time series?" *Mathematics and Computers in Simulation*, vol. 81, no. 7, pp. 1441–1453, 2011.

[45] J. D. Pelletier and D. L. Turcotte, "Long-range persistence in climatological and hydrological time series: analysis, modeling and application to drought hazard assessment," *Journal of Hydrology*, vol. 203, no. 1–4, pp. 198–208, 1997.

[46] Q. Shao and M. Li, "A new trend analysis for seasonal time series with consideration of data dependence," *Journal of Hydrology*, vol. 396, no. 1-2, pp. 104–112, 2011.

[47] B. M. Dolgonosov, K. A. Korchagin, and N. V. Kirpichnikova, "Modeling of annual oscillations and 1/f-noise of daily river discharges," *Journal of Hydrology*, vol. 357, no. 3-4, pp. 174–187, 2008.

[48] V. Ganti, K. M. Straub, E. Foufoula-Georgiou, and C. Paola, "Space-time dynamics of depositional systems: experimental evidence and theoretical modeling of heavy-tailed statistics," *Journal of Geophysical Research F*, vol. 116, no. 2, Article ID F02011, 2011.

[49] J. Alvarez-Ramirez, J. Alvarez, L. Dagdug, E. Rodriguez, and J. C. Echeverria, "Long-term memory dynamics of continental and oceanic monthly temperatures in the recent 125 years," *Physica A*, vol. 387, no. 14, pp. 3629–3640, 2008.

[50] A. Berrones, "Persistence in a simple model for the Earth's atmosphere temperature fluctuations," *Fluctuation and Noise Letters*, vol. 5, no. 3, pp. L365–L374, 2005.

[51] J. Haslett and A. E. Raftery, "Space-time modelling with long-memory dependence: assessing Ireland's wind power resource," *Journal of the Royal Statistical Society C*, vol. 38, no. 1, pp. 1–50, 1989.

[52] K. Fraedrich, U. Luksch, and R. Blender, "$1/f$ model for long-time memory of the ocean surface temperature," *Physical Review E*, vol. 70, no. 3, Article ID 037301, 4 pages, 2004.

[53] M. S. Santhanam and H. Kantz, "Long-range correlations and rare events in boundary layer wind fields," *Physica A*, vol. 345, no. 3-4, pp. 713–721, 2005.

[54] J.-C. Bouette, J.-F. Chassagneux, D. Sibai, R. Terron, and A. Charpentier, "Wind in Ireland: long memory or seasonal effect?" *Stochastic Environmental Research and Risk Assessment*, vol. 20, no. 3, pp. 141–151, 2006.

[55] R. A. Monetti, S. Havlin, and A. Bunde, "Long-term persistence in the sea surface temperature fluctuations," *Physica A*, vol. 320, pp. 581–589, 2003.

[56] R. Blender, K. Fraedrich, and F. Sienz, "Extreme event return times in long-term memory processes near $1/f$," *Nonlinear Processes in Geophysics*, vol. 15, no. 4, pp. 557–565, 2008.

[57] T. S. Prass, J. M. Bravo, R. T. Clarke, W. Collischonn, and S. R. C. Lopes, "Comparison of forecasts of mean monthly water level in the Paraguay River, Brazil, from two fractionally differenced models," *Water Resources Research*, vol. 48, no. 5, Article ID W05502, 13 pages, 2012.

[58] S. M. Barbosa, M. J. Fernandes, and M. E. Silva, "Long-range dependence in North Atlantic sea level," *Physica A*, vol. 371, no. 2, pp. 725–731, 2006.

[59] M. Li, C. Cattani, and S. Y. Chen, "Viewing sea level by a one-dimensional random function with long memory," *Mathematical Problems in Engineering*, vol. 2011, Article ID 654284, 13 pages, 2011.

[60] B. B. Mandelbrot, *The Fractal Geometry of Nature*, W. H. Freeman, New York, NY, USA, 1982.

[61] R. J. Adler, R. E. Feldman, and M. S. Taqqu, Eds., *A Practical Guide To Heavy Tails: Statistical Techniques and Applications*, Birkhäuser, Boston, Mass, USA, 1998.

[62] P. Abry, P. Borgnat, F. Ricciato, A. Scherrer, and D. Veitch, "Revisiting an old friend: on the observability of the relation between long range dependence and heavy tail," *Telecommunication Systems*, vol. 43, no. 3-4, pp. 147–165, 2010.

[63] T. Gneiting and M. Schlather, "Stochastic models that separate fractal dimension and the hurst effect," *SIAM Review*, vol. 46, no. 2, pp. 269–282, 2004.

[64] M. Li, "Generation of teletraffic of generalized Cauchy type," *Physica Scripta*, vol. 81, no. 2, Article ID 025007, 10 pages, 2010.
[65] S. C. Lim and M. Li, "A generalized Cauchy process and its application to relaxation phenomena," *Journal of Physics A*, vol. 39, no. 12, pp. 2935–2951, 2006.
[66] S. V. Muniandy and J. Stanslas, "Modelling of chromatin morphologies in breast cancer cells undergoing apoptosis using generalized Cauchy field," *Computerized Medical Imaging and Graphics*, vol. 32, no. 7, pp. 631–637, 2008.
[67] D. Ceresetti, G. Molinié, and J.-D. Creutin, "Scaling properties of heavy rainfall at short duration: a regional analysis," *Water Resources Research*, vol. 46, no. 9, Article ID W09531, 12 pages, 2010.
[68] M. Radziejewski and Z. W. Kundzewicz, "Fractal analysis of flow of the river Warta," *Journal of Hydrology*, vol. 200, no. 1–4, pp. 280–294, 1997.
[69] S. Hirabayashi and T. Sato, "Scaling of mixing parameters in stationary, homogeneous, and stratified turbulence," *Journal of Geophysical Research C*, vol. 115, no. 9, Article ID C09023, 9 pages, 2010.
[70] R. Mantilla, B. M. Troutman, and V. K. Gupta, "Testing statistical self-similarity in the topology of river networks," *Journal of Geophysical Research F*, vol. 115, no. 3, Article ID F03038, 12 pages, 2010.
[71] A. A. Suleymanov, A. A. Abbasov, and A. J. Ismaylov, "Fractal analysis of time series in oil and gas production," *Chaos, Solitons & Fractals*, vol. 41, no. 5, pp. 2474–2483, 2009.
[72] S. Rehman and A. H. Siddiqi, "Wavelet based hurst exponent and fractal dimensional analysis of Saudi climatic dynamics," *Chaos, Solitons & Fractals*, vol. 40, no. 3, pp. 1081–1090, 2009.
[73] R. Zuo, Q. Cheng, Q. Xia, and F. P. Agterberg, "Application of fractal models to distinguish between different mineral phases," *Mathematical Geosciences*, vol. 41, no. 1, pp. 71–80, 2009.
[74] R. C. García, A. S. Galán, J. R. Castrejón Pita, and A. A. C. Pita, "The fractal dimension of an oil spray," *Fractals*, vol. 11, no. 2, pp. 155–161, 2003.
[75] S. Benmehdi, N. Makarava, N. Benhamidouche, and M. Holschneider, "Bayesian estimation of the self-similarity exponent of the Nile River fluctuation," *Nonlinear Processes in Geophysics*, vol. 18, no. 3, pp. 441–446, 2011.
[76] S. I. Badulin, A. N. Pushkarev, D. Resio, and V. E. Zakharov, "Self-similarity of wind-driven seas," *Nonlinear Processes in Geophysics*, vol. 12, no. 6, pp. 891–945, 2005.
[77] V. Carbone, P. Veltri, and R. Bruno, "Solar wind low-frequency magnetohydrodynamic turbulence: extended self-similarity and scaling laws," *Nonlinear Processes in Geophysics*, vol. 3, no. 4, pp. 247–261, 1996.
[78] T. B. Sangoyomi, U. Lall, and H. D. I. Abarbanel, "Nonlinear dynamics of the Great Salt Lake: dimension estimation," *Water Resources Research*, vol. 32, no. 1, pp. 149–159, 1996.
[79] M. Ozger, "Scaling characteristics of ocean wave height time series," *Physica A*, vol. 390, no. 6, pp. 981–989, 2011.
[80] A. Chmel, V. N. Smirnov, and M. P. Astakhov, "The Arctic sea-ice cover: fractal space-time domain," *Physica A*, vol. 357, no. 3-4, pp. 556–564, 2005.
[81] F. Berizzi, G. Bertini, M. Martorella, and M. Bertacca, "Two-dimensional variation algorithm for fractal analysis of sea SAR images," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 44, no. 9, pp. 2361–2373, 2006.
[82] K. Fraedrich and R. Blender, "Scaling of atmosphere and ocean temperature correlations in observations and climate models," *Physical Review Letters*, vol. 90, no. 10, Article ID 108501, 4 pages, 2003.
[83] T. Xu, I. D. Moore, and J. C. Gallant, "Fractals, fractal dimensions and landscapes—a review," *Geomorphology*, vol. 8, no. 4, pp. 245–262, 1993.
[84] G. Korvin, *Fractal Models in the Earth Science*, Elsevier, 1992.
[85] J. Levy-Vehel, E. Lutton, and C. Tricot, Eds., *Fractals in Engineering*, Springer, 1997.
[86] B. B. Mandelbrot, *Gaussian Self-Affinity and Fractals*, Springer, 2001.
[87] S. C. Lim and S. V. Muniandy, "On some possible generalizations of fractional Brownian motion," *Physics Letters A*, vol. 266, no. 2-3, pp. 140–145, 2000.
[88] H. E. Stanley, L. A. N. Amaral, A. L. Goldberger, S. Havlin, P. C. Ivanov, and C.-K. Peng, "Statistical physics and physiology: monofractal and multifractal approaches," *Physica A*, vol. 270, no. 1-2, pp. 309–324, 1999.
[89] B. B. Mandelbrot, *Multifractals and 1/f Noise*, Springer, 1998.
[90] D. Harte, *Multifractals: Theory and Applications*, Chapman & Hall, 2001.

[91] D. L. Turcotte, *Fractals and Chaos in Geology and Geophysics*, Cambridge University Press, Cambridge, UK, 2nd edition, 1997.

[92] J. Lévy-Véhel and E. Lutton, Eds., *Fractals in Engineering: New Trends in Theory and Applications*, Springer, 2005.

[93] S. Gaci, N. Zaourar, L. Briqueu, and M. Hamoudi, "Regularity analysis of airborne natural Gamma ray data measured in the Hoggar area (Algeria)," in *Advances in Data, Methods, Models and Their Applications in Geoscience*, D. Chen, Ed., pp. 93–108, InTech, 2011.

[94] S. Gaci and N. Zaourar, "Two-dimensional multifractional Brownian motion-based investigation of heterogeneities from a core image," in *Advances in Data, Methods, Models and Their Applications in Geoscience*, D. Chen, Ed., pp. 109–124, InTech, 2011.

[95] P. Afzal, A. Z. Zarifi, and A. B. Yasrebi, "Identification of uranium targets based on airborne radiometric data analysis by using multifractal modeling, Tark and Avanligh 1:50 000 sheets, NW Iran," *Nonlinear Processes in Geophysics*, vol. 19, no. 2, pp. 283–289, 2012.

[96] M. S. Jouini, S. Vega, and E. A. Mokhtar, "Multiscale characterization of pore spaces using multifractals analysis of scanning electronic microscopy images of carbonates," *Nonlinear Processes in Geophysics*, vol. 18, no. 6, pp. 941–953, 2011.

[97] S. S. Teotia and D. Kumar, "Role of multifractal analysis in understanding the preparation zone for large size earthquake in the North-Western Himalaya region," *Nonlinear Processes in Geophysics*, vol. 18, no. 1, pp. 111–118, 2011.

[98] F. Serinaldi, "Multifractality, imperfect scaling and hydrological properties of rainfall time series simulated by continuous universal multifractal and discrete random cascade models," *Nonlinear Processes in Geophysics*, vol. 17, no. 6, pp. 697–714, 2010.

[99] W. M. Macek, "Multifractality and intermittency in the solar wind," *Nonlinear Processes in Geophysics*, vol. 14, no. 6, pp. 695–700, 2007.

[100] S. I. Badulin, A. N. Pushkarev, D. Resio, and V. E. Zakharov, "Self-similarity of wind-driven seas," *Nonlinear Processes in Geophysics*, vol. 12, no. 6, pp. 891–945, 2005.

[101] Y. Ida, M. Hayakawa, A. Adalev, and K. Gotoh, "Multifractal analysis for the ULF geomagnetic data during the 1993 Guam earthquake," *Nonlinear Processes in Geophysics*, vol. 12, no. 2, pp. 157–162, 2005.

[102] L. Seuront, F. Schmitt, D. Shertzer, Y. Lagadette, and S. Lovejoy, "Multifractal intermittency of Eulerian and Lagrangian turbulence of ocean temperature and plankton fields," *Nonlinear Processes in Geophysics*, vol. 3, no. 4, pp. 236–246, 1996.

[103] M. Arias, P. Gumiel, D. J. Sanderson, and A. Martin-Izard, "A multifractal simulation model for the distribution of VMS deposits in the Spanish segment of the Iberian Pyrite Belt," *Computers & Geosciences*, vol. 37, no. 12, pp. 1917–1927, 2011.

[104] J. Paz-Ferreiro, E. V. Vázquez, and J. G. V. Miranda, "Assessing soil particle-size distribution on experimental plots with similar texture under different management systems using multifractal parameters," *Geoderma*, vol. 160, no. 1, pp. 47–56, 2010.

[105] P. C. Chu, "Multi-fractal thermal characteristics of the southwestern GIN sea upper layer," *Chaos, Solitons & Fractals*, vol. 19, no. 2, pp. 275–284, 2004.

[106] N. Su, Z.-G. Yu, V. Anh, and K. Bajracharya, "Fractal tidal waves in coastal aquifers induced both anthropogenically and naturally," *Environmental Modelling & Software*, vol. 19, no. 12, pp. 1125–1130, 2004.

[107] X. S. Liang and A. R. Robinson, "Localized multiscale energy and vorticity analysis. I. Fundamentals," *Dynamics of Atmospheres and Oceans*, vol. 38, no. 3-4, pp. 195–230, 2005.

[108] J. D. Pelletier, "Natural variability of atmospheric temperatures and geomagnetic intensity over a wide range of time scales," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 99, supplement 1, pp. 2546–2553, 2002.

[109] K. Daoudi and J. Lévy-Véhel, "Signal representation and segmentation based on multifractal stationarity," *Signal Processing*, vol. 82, no. 12, pp. 2015–2024, 2002.

[110] T. Bedford, "Hölder exponents and box dimension for self-affine fractal functions," *Constructive Approximation*, vol. 5, no. 1, pp. 33–48, 1989.

[111] K. J. Falconer, *Fractal Geometry, Mathematical Foundations and Applications*, John Wiley & Sons, 2003.

[112] J. W. Kantelhardt, S. A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde, and H. E. Stanley, "Multifractal detrended fluctuation analysis of nonstationary time series," *Physica A*, vol. 316, no. 1–4, pp. 87–114, 2002.

[113] K. Daoudi, J. Lévy-Véhel, and Y. Meyer, "Construction of continuous functions with prescribed local regularity," *Constructive Approximation*, vol. 14, no. 3, pp. 349–385, 1998.

[114] B. J. West, "Fractal physiology and the fractional calculus: a perspective," *Frontiers in Fractal Physiology*, vol. 1, article 12, 2010.

[115] R. F. Peltier and J. Levy-Vehel, *Multifractional Brownian Motion: Definition and Preliminaries Results*, 1995, INRIA RR, 2645.

[116] A. Ayache, S. Cohen, and J. Levy Vehel, "Covariance structure of Multifractional Brownian motion, with application to long range dependence," in *Proceedings of the IEEE Interntional Conference on Acoustics, Speech, and Signal Processing (ICASSP '00)*, pp. 3810–3813, June 2000.

[117] A. Ayache, "The generalized multifractional field: a nice tool for the study of the generalized multifractional brownian motion," *Journal of Fourier Analysis and Applications*, vol. 8, no. 6, pp. 581–601, 2002.

[118] A. Ayache and J. Levy-Vehel, "The generalized multifractional Brownian motion," *Statistical Inference For Stochastic Processes*, vol. 3, no. 1-2, pp. 7–18, 2000.

[119] R. L. Guevel and J. Levy-Vehel, *Incremental Moments and Hölderexponents of Multifractional Multistable Processes*, INRIA, 2010.

[120] K. J. Falconer, R. Le Guével, and J. Lévy-Véhel, "Localizable moving average symmetric stable and multistable processes," *Stochastic Models*, vol. 25, no. 4, pp. 648–672, 2009.

[121] K. J. Falconer and J. Lévy Véhel, "Multifractional, multistable, and other processes with prescribed local form," *Journal of Theoretical Probability*, vol. 22, no. 2, pp. 375–401, 2009.

[122] K. J. Falconer, "The local structure of random processes," *Journal of the London Mathematical Society A*, vol. 67, no. 3, pp. 657–672, 2003.

[123] K. Dębicki and P. Kisowski, "Asymptotics of supremum distribution of α (t)-locally stationary Gaussian processes," *Stochastic Processes and Their Applications*, vol. 118, no. 11, pp. 2022–2037, 2008.

[124] C. Cattani, G. Pierro, and G. Altieri, "Entropy and multifractality for the myeloma multiple TET 2 gene," *Mathematical Problems in Engineering*, vol. 2012, Article ID 193761, 14 pages, 2012.

[125] S. V. Muniandy, S. C. Lim, and R. Murugan, "Inhomogeneous scaling behaviors in Malaysian foreign currency exchange rates," *Physica A*, vol. 301, no. 1–4, pp. 407–428, 2001.

[126] P. Shang, Y. Lu, and S. Kama, "The application of Hölder exponent to traffic congestion warning," *Physica A*, vol. 370, no. 2, pp. 769–776, 2006.

[127] T. Jin and H. Zhang, "Statistical approach to weak signal detection and estimation using Duffing chaotic oscillators," *Science China Information Sciences*, vol. 54, no. 11, pp. 2324–2337, 2011.

[128] M. Li, W. Zhao, and S. Y. Chen, "MBm-based scalings of traffic propagated in internet," *Mathematical Problems in Engineering*, vol. 2011, Article ID 389803, 21 pages, 2011.

[129] H. Sheng, Y.-Q. Chen, and T.-S. Qiu, "Heavy-tailed distribution and local long memory in time series of molecular motion on the cell membrane," *Fluctuation and Noise Letters*, vol. 10, no. 1, pp. 93–119, 2011.

[130] H. Sheng, H. Sun, Y.-Q. Chen, and T.-S. Qiu, "Synthesis of multifractional Gaussian noises based on variable-order fractional operators," *Signal Processing*, vol. 91, no. 7, pp. 1645–1650, 2011.

[131] H. Sheng, Y. Q. Chen, and T. S. Qiu, *Fractional Processes and Fractional Order Signal Processing*, Springer, 2012.

[132] J. F. Muzy, E. Bacry, and A. Arneodo, "Multifractal formalism for fractal signals: the structure-function approach versus the wavelet-transform modulus-maxima method," *Physical Review E*, vol. 47, no. 2, pp. 875–884, 1993.

[133] O. Barrière and J. Lévy-Véhel, "Local Hölder regularity-based modeling of RR intervals," INRIA-00539046, Version 1, 2010.

[134] S. Gaci, N. Zaourar, M. Hamoudi, and M. Holschneider, "Local regularity analysis of strata heterogeneities from sonic logs," *Nonlinear Processes in Geophysics*, vol. 17, no. 5, pp. 455–466, 2010.

[135] http://www.ndbc.noaa.gov/index.shtml.

[136] http://www.ndbc.noaa.gov/historical_data.shtml.

[137] http://www.ndbc.noaa.gov/station_page.php?station=lkwf1.

[138] http://www.ndbc.noaa.gov/station_page.php?station=lonf1.

[139] http://www.ndbc.noaa.gov/station_page.php?station=sauf1.

[140] http://www.ndbc.noaa.gov/station_page.php?station=smkf1.

[141] http://www.ndbc.noaa.gov/station_page.php?station=spgf1.

[142] http://www.ndbc.noaa.gov/station_page.php?station=venf1.
[143] http://www.ndbc.noaa.gov/historical_data.shtml#wlevel.
[144] D. B. Gilhousen, "A field evaluation of NDBC moored buoy winds," *Journal of Atmospheric and Oceanic Technology*, vol. 4, no. 1, pp. 94–104, 1987.
[145] http://www.ndbc.noaa.gov/measdes.shtml#wlevel.
[146] A. N. Kolmogorov, "Wienersche Spiralen und einige andere interessante Kurven im. Hilbertschen Raum," *Comptes Rendus de l'Académie des Sciences de l'URSS*, vol. 26, no. 2, pp. 115–118, 1940.
[147] B. Matérn, *Spatial Variation*, Springer, 2nd edition, 1986.
[148] J.-P. Chiles and P. Delfiner, *Geostatistics, Modeling Spatial Uncertainty*, Wiley, New York, NY, USA, 1999.
[149] R. Webster and M. A. Oliver, *Geostatistics For Environmental Scientists*, Wiley, Chichester, UK, 2007.
[150] H. Wackernagel, *Multivariate Geostatistics: An Introduction With Applications*, Springer, Dordrecht, The Netherlands, 2005.
[151] O. Schabenberger and C. A. Gotway, *Statistical Methods For Spatial Data Analysis*, Chapman & Hall, CRC, Boca Raton, Fla, USA, 2005.
[152] B. D. Ripley, *Spatial Statistics*, Wiley-Interscience, Hoboken, NJ, USA, 2004.
[153] M. Schlather and T. Gneiting, "Local approximation of variograms by covariance functions," *Statistics & Probability Letters*, vol. 76, no. 12, pp. 1303–1304, 2006.
[154] B. Minasny and A. B. McBratney, "The Matérn function as a general model for soil variograms," *Geoderma*, vol. 128, no. 3-4, pp. 192–207, 2005.
[155] D. J. Gorsich and M. G. Genton, "Variogram model selection via nonparametric derivative estimation," *Mathematical Geology*, vol. 32, no. 3, pp. 249–270, 2000.
[156] M. G. Genton, "The correlation structure of Matheron's classical variogram estimator under elliptically contoured distributions," *Mathematical Geology*, vol. 32, no. 1, pp. 127–137, 2000.
[157] D. Marcotte, "Fast variogram computation with FFT," *Computers & Geosciences*, vol. 22, no. 10, pp. 1175–1186, 1996.
[158] A. S. Monin and A. M. Yaglom, *Statistical Fluid Mechanics: Mechanics of Turbulence*, vol. 2, The MIT Press, Cambridge, Mass, USA, 1971.
[159] E. I. Kaganov and A. M. Yaglom, "Errors in wind-speed measurements by rotation anemometers," *Boundary-Layer Meteorology*, vol. 10, no. 1, pp. 15–34, 1976.
[160] E. O. Schulz-DuBois and I. Rehberg, "Structure function in lieu of correlation function," *Applied Physics*, vol. 24, no. 4, pp. 323–329, 1981.
[161] Z. Warhaft, "Turbulence in nature and in the laboratory," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 99, 1, pp. 2481–2486, 2002.
[162] A. M. Yaglom, *Correlation Theory of Stationaryand Related Random Functions, Vol. I: Basic Results*, Springer, 1987.
[163] B. B. Bandelbrot and J. W. van Ness, "Fractional Brownian motions, fractional noises and applications," *SIAM Review*, vol. 10, no. 4, pp. 422–437, 1968.
[164] T. Hida, *Brownian Motion*, Springer, 1980.
[165] N. C. Nigam, *Introduction To Random Vibrations*, The MIT Press, 1983.
[166] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill, 2nd edition, 1984.
[167] I. M. Gelfand and K. Vilenkin, *Generalized Functions*, vol. 1, Academic Press, New York, NY, USA, 1964.
[168] F. Biagini, Y. Hu, B. Øksendal, and T. Zhang, *Stochastic Calculus For Fractional Brownian Motion and Applications*, Springer, 2008.
[169] M. Li, "Fractal time series—a tutorial review," *Mathematical Problems in Engineering*, vol. 2010, Article ID 157264, 26 pages, 2010.
[170] J. Mikusinski, *Operational Calculus*, Pergamon Press, 1959.
[171] A. Ayache, S. Jaffard, and M. S. Taqqu, "Wavelet construction of generalized multifractional processes," *Revista Matematica Iberoamericana*, vol. 23, no. 1, pp. 327–370, 2007.
[172] A. Ayache and J. L. Véhel, "On the identification of the pointwise Hölder exponent of the generalized multifractional Brownian motion," *Stochastic Processes and their Applications*, vol. 111, no. 1, pp. 119–156, 2004.
[173] P. R. Bertrand, A. Hamdouni, and S. Khadhraoui, "Modelling NASDAQ series by sparse multifractional Brownian motion," *Methodology and Computing in Applied Probability*, vol. 14, no. 1, pp. 107–124, 2012.
[174] Z. Lin and J. Zheng, "Some properties of a multifractional Brownian motion," *Statistics and Probability Letters*, vol. 77, no. 7, pp. 687–692, 2007.

[175] S. C. Lim and L. P. Teo, "Weyl and Riemann-Liouville multifractional Ornstein-Uhlenbeck processes," *Journal of Physics A*, vol. 40, no. 23, pp. 6035–6060, 2007.
[176] S. V. Muniandy and S. C. Lim, "Modeling of locally self-similar processes using multifractional Brownian motion of Riemann-Liouville type," *Physical Review E*, vol. 63, no. 4, 7 pages, 2001.
[177] S. C. Lim, "Fractional Brownian motion and multifractional Brownian motion of Riemann-Liouville type," *Journal of Physics A*, vol. 34, no. 7, pp. 1301–1310, 2001.
[178] S. A. Stoev and M. S. Taqqu, "How rich is the class of multifractional Brownian motions?" *Stochastic Processes and Their Applications*, vol. 116, no. 2, pp. 200–221, 2006.
[179] J.-F. Coeurjolly, "Identification of multifractional Brownian motion," *Bernoulli*, vol. 11, no. 6, pp. 987–1008, 2005.
[180] S. Stoev and M. S. Taqqu, "Path properties of the linear multifractional stable motion," *Fractals*, vol. 13, no. 2, pp. 157–178, 2005.
[181] A. Benassi, S. Cohen, and J. Istas, "Identifying the multifractional function of a Gaussian process," *Statistics and Probability Letters*, vol. 39, no. 4, pp. 337–345, 1998.
[182] R. Navarro Jr., R. Tamangan, N. Guba-Natan, E. Ramos, and A. D. Guzman, "The identification of long memory process in the Asean-4 stock markets by fractional and multifractional Brownian motion," *The Philippine Statistician*, vol. 55, no. 1-2, pp. 65–83, 2006.