Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2016, Article ID 1967875, 7 pages
http://dx.doi.org/10.1155/2016/1967875

Research Article

Positive Filtering with ℓ_1-Gain for Discrete-Time Positive Systems

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Received 7 March 2016; Revised 30 March 2016; Accepted 7 April 2016

Academic Editor: Carmen Coll

This paper is concerned with the positive filtering problem for discrete-time positive systems under the ℓ_1-induced performance. We aim to propose a pair of positive filters with error-bounding features to estimate the output of positive systems. A novel characterization is first constructed so that the filtering error system is asymptotically stable with a prescribed ℓ_1-induced performance. Then, necessary and sufficient conditions for the existence of required filters are presented, and the obtained results are expressed as linear programming problems. Moreover, it is pointed out that the results can be easily checked by standard software. In addition, a numerical example is given to show the effectiveness of the proposed design procedures.

1. Introduction

In real world, many dynamical systems involve variables which are always confined to the positive orthant. This special category of systems is generally referred to as positive systems in the literature. Positive systems arise in different application fields such as physics, engineering, and social sciences [1, 2]. Since positive systems possess many unique features and have special structures, a lot of methods established for general systems cannot be used for positive systems. Due to their numerous applications and unique features, positive systems have received ever-increasing research interest in recent years [3–12]. For instance, the problem of reachability and controllability has been studied for positive systems in [13–15]. In [16, 17], the state-feedback controller synthesis results have been expressed as linear matrix inequality problems and linear programming problems. In [18, 19], a stability analysis method for compartmental dynamic systems has been proposed. An interesting result on the positive observer design problem has been given for positive systems in [20]. The positive realization problem has been discussed in [21]. In [22], a solution has been proposed to the model reduction problem for positive systems. Furthermore, the analysis and synthesis problems have been addressed for special classes of positive systems such as 2-D positive systems [4, 23] and time-delay positive systems [24–29].

It is remarkable that since many previous approaches used for the filtering problem of general systems fail to ensure the positivity of the filter, existing approaches cannot be directly applied for positive systems. Therefore, it is necessary to develop new techniques for positive systems. Moreover, differently from most existing results on stability and stabilizability of positive systems which were derived by resorting to the quadratic Lyapunov functions, the applications of the linear copositive Lyapunov functions led to many novel results in recent years [1, 16, 30–36]. In addition, linear copositive Lyapunov functions stimulate the use of ℓ_1-gain as a performance index for positive systems. In some situations, one might be interested in the sum of quantities for positive systems from a practical viewpoint. Therefore, some frequently used performance measures induced by ℓ_2 signals such as H_∞ norm are not very natural to describe the features of practical positive systems. On the other hand, 1-norm is more appropriate for positive systems because it represents the sum of the values of the components. For example, it is usually desirable to analyze the total mass of material in all the compartments for compartmental networks.

In this paper, the problem of ℓ_1-induced filtering is studied for positive systems with the positivity preserved in the filters. The main contributions of this paper are as follows. First, we present the ℓ_1-induced performance index for positive systems and characterize it analytically. Then, we propose a pair of ℓ_1-induced positive filters to estimate the output of positive systems at all times. Finally, we establish necessary and sufficient conditions to obtain the desired positive filters in terms of linear programming.

The layout of the paper is as follows. In Section 2, the problem addressed in this paper is formulated and the positive filters are introduced. The positive filter design procedure for positive systems is proposed in Section 3, followed by a numerical example in Section 4 to show the application of the theoretical results. Finally, we draw conclusions in Section 5.

Notation 1. All the matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations. Let $\mathbb{R}$ denote the set of real numbers; $\mathbb{R}^n$ is the n-column real vectors; $\mathbb{R}^{n\times m}$ stands for the set of real matrices of dimension $n \times m$. For a vector $x(k) = (x_1(k), x_2(k), \ldots, x_n(k))$, the 1-norm is defined as $\|x(k)\|_1 \triangleq \sum_{i=1}^{n} |x_i(k)|$. Let $\mathbb{R}_+^n$ denote the nonnegative orthant of $\mathbb{R}^n$. For a matrix $A \in \mathbb{R}^{m\times n}$, the element located at the ith row and the jth column is denoted by $[A]_{ij}$; $[A]_{r,i}$ and $[A]_{c,j}$ denote the ith row and the jth column, respectively. The ℓ_1-norm of an infinite sequence x is defined as $\|x\|_{\ell_1} \triangleq \sum_{k=0}^{\infty} \|x(k)\|_1$. $A \geq 0$ (resp., $A > 0$) means that, for all i and j, $[A]_{ij} \geq 0$ (resp., $[A]_{ij} > 0$). The induced 1-norm of a matrix $Q \triangleq [q_{ij}] \in \mathbb{R}^{m\times n}$ is denoted by $\|Q\|_1 \triangleq \max_{1\leq j\leq n}(\sum_{i=1}^{m} |q_{ij}|)$. The notation $A \geq B$ means that the matrix $A - B \geq 0$ and $A > B$ denotes $A - B > 0$. The Euclidean norm for vectors can be expressed as $\|\cdot\|$. The space of all vector-valued functions defined on $\overline{\mathbb{R}}_+^n$ with finite ℓ_1 norm is denoted by $\ell_1(\overline{\mathbb{R}}_+^n)$. $\mathbf{1}$ stands for a column vector with each entry equal to 1.

2. Problem Formulation

In this section, the ℓ_1-induced performance is first introduced for discrete-time positive systems. Moreover, the ℓ_1-induced filtering problem for a stable positive system is formulated.

Consider the following discrete-time linear system:

$$\begin{aligned} x(k+1) &= Ax(k) + Bw(k), \\ y(k) &= Cx(k) + Dw(k), \\ z(k) &= Lx(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $w(k) \in \mathbb{R}^l$, $y(k) \in \mathbb{R}^q$, and $z(k) \in \mathbb{R}^q$ denote the state vector, disturbance signal, measurement, and the signal to be determined, respectively.

Next, the following definition is presented below, which will be used in the sequel.

Definition 1. System (1) is positive if $x(k) \geq 0$, $y(k) \geq 0$, and $z(k) \geq 0$ always hold for all $x(0) \geq 0$ and all input $w(k) \geq 0$.

Before moving on, some useful results are introduced and the following lemmas are needed.

Lemma 2 (see [37]). *The discrete-time system (1) is positive if and only if*

$$\begin{aligned} A &\geq 0, \\ B &\geq 0, \\ C &\geq 0, \\ D &\geq 0, \\ L &\geq 0, \\ i &= 1, 2, \ldots, r. \end{aligned} \tag{2}$$

Lemma 3. *System (1) with input $w(k) = 0$ is asymptotically stable if and only if there exists a vector $p \geq 0$ (or $p > 0$) satisfying*

$$p^T A - p^T \ll 0. \tag{3}$$

Next, the definition of ℓ_1-induced performance is introduced. A stable positive system (1) is said to have ℓ_1-induced performance at the level γ if, under zero initial conditions,

$$\sup_{w\neq 0,\, w\in \ell_1(\overline{\mathbb{R}}_+^n)} \frac{\|z\|_{\ell_1}}{\|w\|_{\ell_1}} < \gamma, \tag{4}$$

where $\gamma > 0$ is a given scalar.

Now we are in the position to introduce the following result which serves as a characterization on the asymptotic stability of system (1) with the ℓ_1-induced performance in (4) via linear programming. The performance characterization result is a theoretical basis for further development.

Lemma 4 (see [32]). *The positive system (1) is asymptotically stable and satisfies $\|z\|_{\ell_1} < \gamma \|w\|_{\ell_1}$ if and only if there exists a vector $p \geq 0$ satisfying*

$$\begin{aligned} \mathbf{1}^T L + p^T A - p^T &\ll 0, \\ p^T B - \gamma \mathbf{1}^T &\ll 0. \end{aligned} \tag{5}$$

It is easy to see that the transient output cannot be estimated via conventional filters, which can only give an estimate of the output asymptotically. In order to design a filter which can be used to give the information of the transient output at all times, we intend to find a lower-bounding estimate $\check{z}(k)$ and an upper-bounding one $\widehat{z}(k)$. More specifically, the signal $z(k)$ can be encapsulated between the two estimates at all times. A pair of filters is proposed as follows:

$$\begin{aligned} \check{x}(k+1) &= \underline{A}_f \check{x}(k) + \underline{B}_f y(k), \\ \check{z}(k) &= \underline{C}_f \check{x}(k), \end{aligned} \tag{6}$$

$$\begin{aligned} \widehat{x}(k+1) &= \overline{A}_f \widehat{x}(k) + \overline{B}_f y(k), \\ \widehat{z}(k) &= \overline{C}_f \widehat{x}(k), \end{aligned} \tag{7}$$

where $\widehat{x}(k) \in \mathbb{R}^n$, $\check{x}(k) \in \mathbb{R}^n$, $\widehat{z}(k) \in \mathbb{R}^q$, and $\check{z}(k) \in \mathbb{R}^q$. $\underline{A}_f, \overline{A}_f, \underline{B}_f, \overline{B}_f, \underline{C}_f$, and $\overline{C}_f$ are filtering parameters.

First, the lower-bounding case is considered. Set $\check{x}_e(k) = x(k) - \check{x}(k)$, $\check{\xi}(k) = \left[x^T(k),\ \check{x}_e^T(k)\right]^T$, and $\check{e}(k) = z(k) - \check{z}(k)$. From (1) and (6), the augmented system is described as follows:

$$\begin{aligned}\check{\xi}(k+1) &= \underline{A}_\xi \check{\xi}(k) + \underline{B}_\xi w(k),\\ \check{e}(k) &= \underline{C}_\xi \check{\xi}(k),\end{aligned} \tag{8}$$

where

$$\begin{aligned}\underline{A}_\xi &= \begin{bmatrix} A & 0 \\ A - \underline{B}_f C - \underline{A}_f & \underline{A}_f \end{bmatrix},\\ \underline{B}_\xi &= \begin{bmatrix} B \\ B - \underline{B}_f D \end{bmatrix},\\ \underline{C}_\xi &= \begin{bmatrix} L - \underline{C}_f & \underline{C}_f \end{bmatrix}.\end{aligned} \tag{9}$$

Since, the lower-bounding filter (6) is designed to approximate $z(k)$ with $\check{z}(k)$, it is natural to require that the estimate $\check{z}(k)$ is also positive, which means that the filter (6) should be a positive system. From Lemma 2, it can be seen that $\underline{A}_f \geq 0$, $\underline{B}_f \geq 0$, and $\underline{C}_f \geq 0$ are required. Based on the above discussion, the positive lower-bounding filtering (PLBF) problem is formulated as folows.

Positive Lower-Bounding Filtering (PLBF). Given a stable positive system (1), find a positive filter (6) with $\underline{A}_f \geq 0$, $\underline{B}_f \geq 0$, and $\underline{C}_f \geq 0$ that ensures that the filtering error system (8) is positive, asymptotically stable, with performance $\|\check{e}\|_{\ell_1} < \gamma_l \|w\|_{\ell_1}$ under zero initial conditions.

Similarly, the second filtering error system can be obtained by defining $\hat{x}_e(k) = \hat{x}(k) - x(k)$, $\hat{\xi}(k) = \left[x^T(k),\ \hat{x}_e^T(k)\right]^T$, and $\hat{e}(k) = \hat{z}(k) - z(k)$, and it can be formulated as follows:

$$\begin{aligned}\hat{\xi}(k+1) &= \overline{A}_\xi \hat{\xi}(k) + \overline{B}_\xi w(k),\\ \hat{e}(k) &= \overline{C}_\xi \hat{\xi}(k),\end{aligned} \tag{10}$$

where

$$\begin{aligned}\overline{A}_\xi &= \begin{bmatrix} A & 0 \\ \overline{A}_f + \overline{B}_f C - A & \overline{A}_f \end{bmatrix},\\ \overline{B}_\xi &= \begin{bmatrix} B \\ \overline{B}_f D - B \end{bmatrix},\\ \overline{C}_\xi &= \begin{bmatrix} \overline{C}_f - L & \overline{C}_f \end{bmatrix}.\end{aligned} \tag{11}$$

Then, the positive upper-bounding filtering (PUBF) problem is proposed below.

Positive Upper-Bounding Filtering (PUBF). Given a stable positive system (1), find a positive filter (7) with $\overline{A}_f \geq 0$, $\overline{B}_f \geq 0$, and $\overline{C}_f \geq 0$ that ensures that the filtering error system (10) is positive, asymptotically stable, with performance $\|\hat{e}\|_{\ell_1} < \gamma_u \|w\|_{\ell_1}$ under zero initial conditions.

3. Main Results

In this section, a pair of positive error-bounding filters is obtained which can bound the signal $z(k)$ at all times with the ℓ_1-induced performance satisfied. Necessary and sufficient conditions in terms of linear programming are presented to design a lower-bounding filter, which is followed by parallel results obtained for the upper-bounding case.

Now, we are in a position to provide conditions to design the desired lower-bounding filter for system (1) in the following theorem.

Theorem 5. *Given a stable discrete-time positive system (1), a lower-bounding filter (6) exists such that the filtering error system (8) is positive, asymptotically stable with $\|\check{e}\|_{\ell_1} < \gamma_l \|w\|_{\ell_1}$ if and only if there exist vectors $p_1 \geq 0$, $p_2 \geq 0$ and matrices $\underline{M}_{A_f}$, $\underline{M}_{B_f}$, $\underline{C}_f \geq 0$ such that the following LMIs are feasible:*

$$\left[\underline{M}_{A_f}\right]_{gv} \geq 0, \tag{12}$$

$$\left[\underline{M}_{B_f}\right]_{gs} \geq 0, \tag{13}$$

$$L - \underline{C}_f \geq 0, \tag{14}$$

$$p_{2g}^T [A]_{gv} - \left[\underline{M}_{B_f}\right]_{r,g} [C]_{c,v} - \left[\underline{M}_{A_f}\right]_{gv} \geq 0, \tag{15}$$

$$p_{2g}^T [B]_{gs} - \left[\underline{M}_{B_f}\right]_{r,g} [D]_{c,s} \geq 0, \tag{16}$$

$$\begin{aligned}&\mathbf{1}^T \left(L - \underline{C}_f\right) + p_1^T A + p_2^T A - \sum_{g=1}^{n} \left[\underline{M}_{B_f}\right]_{r,g} C\\ &\quad - \sum_{g=1}^{n} \left[\underline{M}_{A_f}\right]_{r,g} - p_1^T \ll 0,\end{aligned} \tag{17}$$

$$\mathbf{1}^T \underline{C}_f + \sum_{g=1}^{n} \left[\underline{M}_{A_f}\right]_{r,g} - p_2^T \ll 0, \tag{18}$$

$$p_1^T B + p_2^T B - \sum_{g=1}^{n} \left[\underline{M}_{B_f}\right]_{r,g} D - \gamma_l \mathbf{1}^T \ll 0, \tag{19}$$

where $g, v = 1, \ldots, n$; $s = 1, \ldots, m$. Then, a possible choice for $\underline{A}_f$ and $\underline{B}_f$ is given by

$$\begin{aligned}\left[\underline{A}_f\right]_{gv} &= p_{2g}^{-1} \left[\underline{M}_{A_f}\right]_{gv},\\ \left[\underline{B}_f\right]_{gs} &= p_{2g}^{-1} \left[\underline{M}_{B_f}\right]_{gs}.\end{aligned} \tag{20}$$

Proof .

Sufficiency. From $p_2 \geq 0$, (12), (13), and (20), we have that $\underline{A}_f \geq 0$ and $\underline{B}_f \geq 0$, which together with $\underline{C}_f \geq 0$ guarantees that the lower-bounding filter (6) is a positive system.

Together with (20) and $p_2 \geq 0$, (15)-(16) imply

$$\begin{aligned}[A]_{gv} - \left[\underline{B}_f\right]_{r,g} [C]_{c,v} - \left[\underline{A}_f\right]_{gv} &\geq 0,\\ [B]_{gs} - \left[\underline{B}_f\right]_{r,g} [D]_{c,s} &\geq 0,\end{aligned} \tag{21}$$

and we have that

$$\begin{gathered} A-\underline{B}_f C-\underline{A}_f \geq 0, \\ B-\underline{B}_f D \geq 0. \end{gathered} \tag{22}$$

From (10), $\underline{A}_f \geq 0$, $\underline{B}_f \geq 0$, $\underline{C}_f \geq 0$, and (14), it can be easily seen that the filtering error system (8) is positive.

Moreover, from (20), we obtain that

$$\begin{gathered} \sum_{g=1}^{n}\left[\underline{M}_{A_f}\right]_{r,g} = p_2^T \underline{A}_f, \\ \sum_{g=1}^{n}\left[\underline{M}_{B_f}\right]_{r,g} = p_2^T \underline{B}_f. \end{gathered} \tag{23}$$

Inequalities (17)–(19) together with (23) imply that

$$\begin{aligned} &\mathbf{1}^T\left(L-\underline{C}_f\right)+p_1^T A+p_2^T A-p_2^T\underline{B}_f C-p_2^T\underline{A}_f-p_1^T \\ &\quad \ll 0, \\ &\mathbf{1}^T\underline{C}_f+p_2^T\underline{A}_f-p_2^T \ll 0, \\ &p_1^T B+p_2^T B-p_2^T\underline{B}_f D-\gamma_l\mathbf{1}^T \ll 0, \end{aligned} \tag{24}$$

and we have

$$\begin{aligned} &\mathbf{1}^T\left[L-\underline{C}_f \quad \underline{C}_f\right]+p^T\begin{bmatrix} A & 0 \\ A-\underline{B}_f C-\underline{A}_f & \underline{A}_f \end{bmatrix}-p^T \\ &\quad \ll 0, \\ &p^T\begin{bmatrix} B \\ B-\underline{B}_f D \end{bmatrix}-\gamma_l\mathbf{1}^T \ll 0, \end{aligned} \tag{25}$$

where $p^T = \left[p_1^T \quad p_2^T\right]$.

By Lemma 4, the filtering error system (8) is asymptotically stable with $\|\check{e}\|_{\ell_1} < \gamma_l\|w\|_{\ell_1}$.

Necessity. By Lemma 4, we can conclude that there exists $p \geq 0$ such that inequality (25) holds. Set $p^T \triangleq \left[p_1^T \quad p_2^T\right]$. We can deduce that the following inequalities hold:

$$\begin{aligned} &\mathbf{1}^T\left(L-\underline{C}_f\right)+p_1^T A+p_2^T A-p_2^T\underline{B}_f C-p_2^T\underline{A}_f-p_1^T \\ &\quad \ll 0, \\ &\mathbf{1}^T\underline{C}_f+p_2^T\underline{A}_f-p_2^T \ll 0, \\ &p_1^T B+p_2^T B-p_2^T\underline{B}_f D-\gamma_l\mathbf{1}^T \ll 0. \end{aligned} \tag{26}$$

From $p_2^T\underline{A}_f = \sum_{i=1}^{n} p_{2g}\left[\underline{A}_f\right]_{r,g}$ and $p_2^T\underline{B}_f = \sum_{i=1}^{n} p_{2g}\left[\underline{B}_f\right]_{r,g}$, it follows that the change of variables

$$\begin{gathered} \left[\underline{M}_{A_f}\right]_{gv} = p_{2g}\left[\underline{A}_f\right]_{gv}, \\ \left[\underline{M}_{B_f}\right]_{gs} = p_{2g}\left[\underline{B}_f\right]_{gs} \end{gathered} \tag{27}$$

makes the problem linear and yields (17)–(20).

Next, the lower-bounding filter (6) is positive and we have $\underline{A}_f \geq 0$, $\underline{B}_f \geq 0$, and $\underline{C}_f \geq 0$. Because of (27) and $p_2 \geq 0$, $\underline{A}_f \geq 0$ and $\underline{B}_f \geq 0$ imply $\underline{M}_{A_f} \geq 0$ and $\underline{M}_{B_f} \geq 0$.

Moreover, if the filtering error system (8) is positive, the following inequalities hold:

$$\begin{gathered} L-\underline{C}_f \geq 0, \\ [A]_{gv}-\left[\underline{B}_f\right]_{r,g}[C]_{c,v}-\left[\underline{A}_f\right]_{gv} \geq 0, \\ [B]_{gs}-\left[\underline{B}_f\right]_{r,g}[D]_{c,s} \geq 0, \end{gathered} \tag{28}$$

and this implies (14)–(16). The whole proof is completed. □

The parallel result is presented in the following for the upper-bounding case. We propose the following theorem to design the upper-bounding filter for positive systems. The proof is similar to the lower-bounding case and thus is omitted here.

Theorem 6. *Given a stable discrete-time positive system (1), an upper-bounding filter (7) exists such that the closed-loop system (10) is positive, asymptotically stable with $\|\widehat{e}\|_{\ell_1} < \gamma_u\|w\|_{\ell_1}$ if and only if there exist vectors $p_1 \geq 0$, $p_2 \geq 0$ and matrices $\overline{M}_{A_f}$, $\overline{M}_{B_f}$, $\overline{C}_f \geq 0$ such that the following LMIs are feasible:*

$$\begin{aligned} &\left[\overline{M}_{A_f}\right]_{gv} \geq 0, \\ &\left[\overline{M}_{B_f}\right]_{gs} \geq 0, \\ &\overline{C}_f-L \geq 0, \\ &\left[\overline{M}_{A_f}\right]_{gv}+\left[\overline{M}_{B_f}\right]_{r,g}[C]_{c,v}-p_{2g}^T[A]_{gv} \geq 0, \\ &\left[\overline{M}_{B_f}\right]_{r,g}[D]_{c,s}-p_{2g}^T[B]_{gs} \geq 0, \\ &\mathbf{1}^T\left(\overline{C}_f-L\right)+p_1^T A-p_2^T A+\sum_{g=1}^{n}\left[\overline{M}_{B_f}\right]_{r,g}C \\ &\quad +\sum_{g=1}^{n}\left[\overline{M}_{A_f}\right]_{r,g}-p_1^T \ll 0, \\ &\mathbf{1}^T\overline{C}_f+\sum_{g=1}^{n}\left[\overline{M}_{A_f}\right]_{r,g}-p_2^T \ll 0, \\ &p_1^T B-p_2^T B+\sum_{g=1}^{n}\left[\overline{M}_{B_f}\right]_{r,g}D-\gamma_u\mathbf{1}^T \ll 0, \end{aligned} \tag{29}$$

where $g, v = 1, \ldots, n$; $s = 1, \ldots, m$. Then, a possible choice for $\overline{A}_f$ and $\overline{B}_f$ is given by

$$\begin{gathered} \left[\overline{A}_f\right]_{gv} = p_{2g}^{-1}\left[\overline{M}_{A_f}\right]_{gv}, \\ \left[\overline{B}_f\right]_{gs} = p_{2g}^{-1}\left[\overline{M}_{B_f}\right]_{gs}. \end{gathered} \tag{30}$$

Remark 7. Conditions obtained in Theorems 5 and 6 can be easily solved by linear programming. Moreover, it is noted that the ℓ_1-induced error-bounding filters can be designed by combining Theorems 5 and 6.

4. Illustrative Example

In this section, an illustrative example is given to illustrate the effectiveness of the theoretical results.

Consider system (1) with system matrices given by

$$\begin{aligned} A &= \begin{bmatrix} 0.05 & 0.5 \\ 0.35 & 0.15 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \\ C &= \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}, \\ D &= 0.8, \\ L &= \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}. \end{aligned} \quad (31)$$

For $\gamma_l = 0.5$, by implementing the linear program in Theorem 5, we obtain the feasible solution

$$\begin{aligned} p_1 &= \begin{bmatrix} 1.0152 & 1.0167 \end{bmatrix}^T, \\ p_2 &= \begin{bmatrix} 1.3744 & 1.3168 \end{bmatrix}^T, \end{aligned} \quad (32)$$

which yields

$$\begin{aligned} \underline{A}_f &= \begin{bmatrix} 0.0023 & 0.4206 \\ 0.2381 & 0.0254 \end{bmatrix}, \\ \underline{B}_f &= \begin{bmatrix} 0.0796 \\ 0.1965 \end{bmatrix}, \\ \underline{C}_f &= \begin{bmatrix} 0.0578 & 0.0078 \end{bmatrix}. \end{aligned} \quad (33)$$

Moreover, for $\gamma_u = 0.5$, by implementing the linear program described in Theorem 6, we obtain the feasible solution

$$\begin{aligned} p_1 &= \begin{bmatrix} 1.3264 & 1.7831 \end{bmatrix}^T, \\ p_2 &= \begin{bmatrix} 1.6229 & 1.7213 \end{bmatrix}^T, \end{aligned} \quad (34)$$

which further yields

$$\begin{aligned} \overline{A}_f &= \begin{bmatrix} 0.0722 & 0.3832 \\ 0.2673 & 0.0639 \end{bmatrix}, \\ \overline{B}_f &= \begin{bmatrix} 0.1625 \\ 0.2633 \end{bmatrix}, \\ \overline{C}_f &= \begin{bmatrix} 0.1625 & 0.0263 \end{bmatrix}. \end{aligned} \quad (35)$$

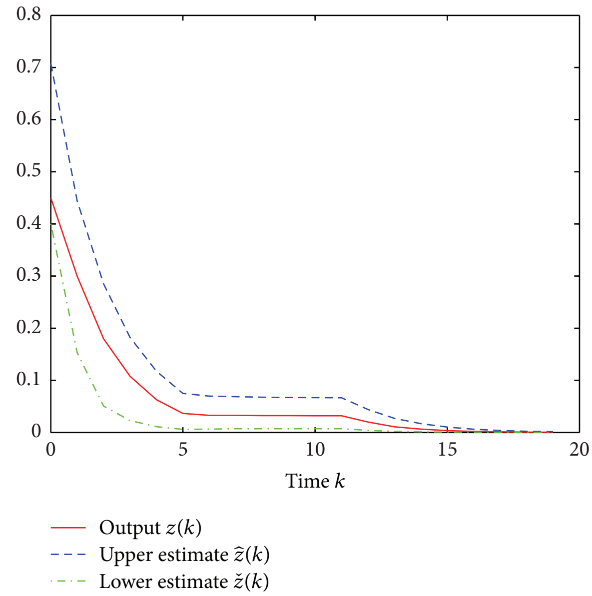

FIGURE 1: Output $z(k)$ and its estimates.

Next, the following disturbance is used in this example:

$$w(k) = \begin{cases} 0.4, & 5 \leq k \leq 10, \\ 0, & \text{otherwise}. \end{cases} \quad (36)$$

Figure 1 shows the lower estimate $\check{z}(k)$, the output $z(k)$, and the upper estimate $\hat{z}(k)$. From Figure 1, we see that the signal $z(k)$ can be encapsulated at all times with the lower-bounding estimate $\check{z}(k)$ and the upper-bounding one $\hat{z}(k)$. In other words, by the filters designed through Theorem 6 and Theorem 5, the output of the original positive system can be estimated at all times.

5. Conclusion

This paper has addressed the problem of positive filtering for positive systems under ℓ_1 performance. Based on linear programming, we have established a novel performance characterization of the filtering error system. Moreover, necessary and sufficient conditions have been developed such that the error-bounding system is positive, asymptotically stable with ℓ_1 performance. Finally, a numerical example is presented to verify the theoretical findings. As a future research direction, it would be of interest to consider the filtering problem for positive systems with time-varying or distributed delays.

Competing Interests

The author declares that he has no competing interests.

Acknowledgments

This work is partially supported by NSFC 61503184, NSFC 61573184, and NSFC 61503037.

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