

Research Article

Computation of Stability Delay Margin of Time-Delayed Generator Excitation Control System with a Stabilizing Transformer

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This paper investigates the effect of time delays on the stability of a generator excitation control system compensated with a stabilizing transformer known as rate feedback stabilizer to damp out oscillations. The time delays are due to the use of measurement devices and communication links for data transfer. An analytical method is presented to compute the delay margin for stability. The delay margin is the maximum amount of time delay that the system can tolerate before it becomes unstable. First, without using any approximation, the transcendental characteristic equation is converted into a polynomial without the transcendentality such that its real roots coincide with the imaginary roots of the characteristic equation exactly. The resulting polynomial also enables us to easily determine the delay dependency of the system stability and the sensitivities of crossing roots with respect to the time delay. Then, an expression in terms of system parameters and imaginary root of the characteristic equation is derived for computing the delay margin. Theoretical delay margins are computed for a wide range of controller gains and their accuracy is verified by performing simulation studies. Results indicate that the addition of a stabilizing transformer to the excitation system increases the delay margin and improves the system damping significantly.

1. Introduction

In electrical power systems, load frequency control (LFC) and excitation control system also known as automatic voltage regulator (AVR) equipment are installed for each generator to maintain the system frequency and generator output voltage magnitude within the specified limits when changes in real and reactive power demand occur [1]. This paper investigates the effect of the time delay on the stability of the generator excitation control system that includes a stabilizing transformer. Figure 1 shows the schematic block diagram of a typical excitation control system for a large synchronous generator. It consists of an exciter, a phasor measurement unit (PMU), a rectifier, a stabilizing transformer (rate feedback stabilizer), and a regulator [1]. The exciter provides DC power to the synchronous generator's field winding constituting power stage of the excitation system. Regulator consists of a proportional-integral (PI) controller and an amplifier [1, 2].

The regulator processes and amplifies input control signals to a level and form appropriate for control of the exciter. The PI controller is used to improve the dynamic response as well as reducing or eliminating the steady-state error. The amplifier may be magnetic amplifier, rotating amplifier, or modern power electronic amplifier. The PMU derives its input from the secondary sides of the three phases of the potential transformer (voltage transducer) and outputs the corresponding positive sequence voltage phasor. The rectifier rectifies the generator terminal voltage and filters it to a DC quantity. The stabilizing transformer provides an additional input signal to the regulator to damp power system oscillations [2].

Time delays have become an important issue in power system control and dynamic analysis since the use of phasor measurement units (PMUs) and open and distributed communication networks for transferring measured signals and data to the controller has introduced significant amount

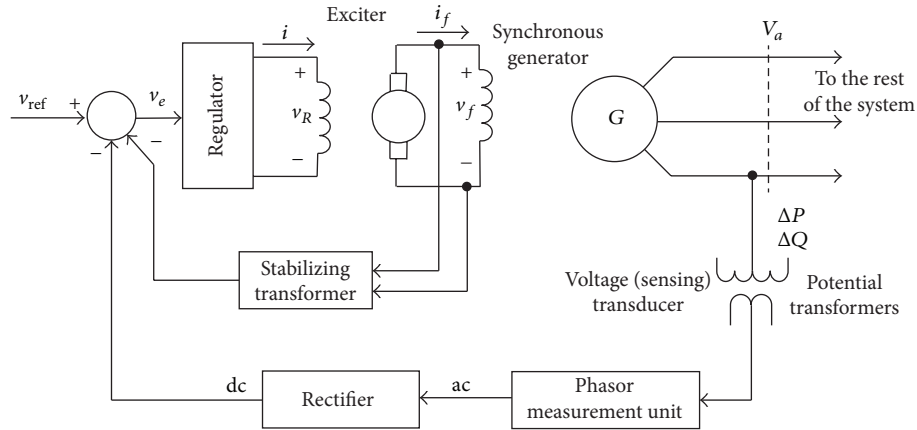


FIGURE 1: The schematic block diagram of the generator excitation control system.

of time delays. The PMUs are units that measure dynamic data of power systems, such as voltage, current, angle, and frequency using the discrete Fourier transform (DFT) [3]. The use of PMUs introduces the measurement delays that consist of voltage transducer delay and processing delay. The processing delay is the amount of time required in converting transducer data into phasor information with the help of DFT.

In power systems, various communication links used for data transfer include both wired options, such as telephone lines, fiber-optic cables, and power lines, and wireless options such as satellites [4]. In power system control, the total measurement delay is reported to be in the order of milliseconds. Depending on the communication link used, the total communication delay is considered to be in the range of 100–700 ms. Measurement and communication delays involved between the instant of measurement and that of signal being available to the controller are the major problem in the power system control. This delay can typically be in the range of 0.5–1.0 s [4–6]. Another processing delay in the order of milliseconds is observed when a digital PI controller is used in the regulator located in the feed-forward section of the AVR shown in Figure 1.

The inevitable time delays in power systems have a destabilizing impact, reduce the effectiveness of control system damping, and lead to unacceptable performance such as loss of synchronism and instability. Therefore, stability analysis and controller design methods must take into account large time delays and practical tools should be developed to study the complicated dynamic behavior of time-delayed power systems. Specifically, such tools should estimate the maximum amount of time delay that the system could tolerate without becoming unstable. Such knowledge on the delay margin (upper bound in the time delay) could also be helpful in the controller design for cases where uncertainty in the delay is unavoidable.

The previous studies on the dynamics of time-delayed power systems have mainly focused on the following issues: (i) to investigate the time-delay influence on the controller design for power system stabilizers (PSSs) [7], for load frequency control (LFC) [8, 9], and for thyristor-controlled

series compensator (TCSC) [10]; (ii) to determine and analyze the cause of time delays and to find appropriate methods to reduce their adverse effects [11–13]; (iii) to eliminate periodic and chaotic oscillations in power systems by applying time-delayed feedback control [14, 15]; (iv) to estimate the delay margin for small-signal stability of time-delayed power systems [16, 17]. However, less attention has been paid to the effects of time delays on the stability of generator excitation control systems including a stabilizing transformer.

There are several methods in the literature to compute delay margins of general time-delayed systems. The common starting point of them is the determination of all the imaginary roots of the characteristic equation. The existing procedures can be classified into the following five distinguishable approaches: (i) Schur-Cohn (Hermite matrix formation) [18–20]; (ii) elimination of transcendental terms in the characteristic equation [21]; (iii) matrix pencil, Kronecker sum method [18–20, 22]; (iv) Kronecker multiplication and elementary transformation [23]; (v) Rekasius substitution [24–26]. These methods demand numerical procedures of different complexity and they may result in different precisions in computing imaginary roots. A detailed comparison of these methods, demonstrating their strengths and weakness, can be found in [27].

This paper presents a direct approach based on the method reported in [21] to compute the delay margin for stability of excitation control system including a stabilizing transformer. The proposed method is an analytically elegant procedure that first converts the transcendental characteristic equation into a polynomial without the transcendentality. This procedure does not use any approximation or substitution to eliminate the transcendentality of the characteristic equation. Therefore, it is exact and the real roots of the new polynomial coincide with the imaginary roots of the characteristic equation exactly. The resulting polynomial without the transcendentality also enables us to easily determine the delay dependency of the system stability and the sensitivities of crossing roots (root tendency) with respect to the time delay. This is a remarkable feature of the proposed method.

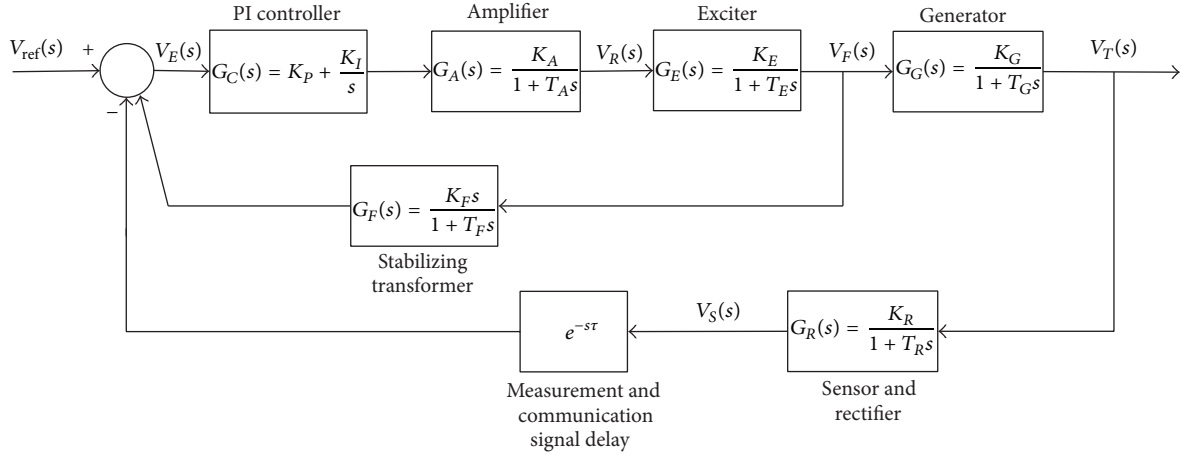


FIGURE 2: Block diagram of the excitation control system with a stabilizing transformer and time delay.

Then, an easy-to-use formula is derived to determine the delay margin in terms of system parameters and imaginary roots of the characteristic equation, which is the main contribution of the paper.

In this work, delay margins are first theoretically determined for a wide range of PI controller gains. Then, theoretical delay margin results are verified by using time-domain simulation capabilities of Matlab/Simulink [28]. Moreover, delay margin results of the excitation control system with a stabilizing transformer are compared with those of the excitation control system not including a stabilizing transformer. It is observed that the compensation of the excitation system with a stabilizing transformer increases the delay margin of the system (thus, the stability margin) and provides significant amount of damping to system oscillations enhancing the closed-loop stability of the time-delayed excitation control system.

2. AVR System Model with Time Delay and Stability

2.1. AVR System Model with Time Delay. For load frequency control and excitation control systems, linear or linearized models are commonly used to analyze the system dynamics and to design a controller. Figure 2 shows the block diagram of a generator excitation control system including a time delay. Note that each component of the system, namely, amplifier, exciter, generator, sensor, and rectifier, is modeled by a first-order transfer function [1, 2]. The transfer function of each component is given below:

$$\begin{aligned} G_A(s) &= \frac{K_A}{1 + T_A s}; & G_E(s) &= \frac{K_E}{1 + T_E s}; \\ G_G(s) &= \frac{K_G}{1 + T_G s}; & G_R(s) &= \frac{K_R}{1 + T_R s}, \end{aligned} \quad (1)$$

where K_A , K_E , K_G , and K_R are the gains of amplifier, exciter, generator, and sensor, respectively, and T_A , T_E , T_G , and T_R are the corresponding time constants.

Note that, as illustrated in Figure 2, using an exponential term, the total of measurement and communication delays (τ) is placed in the feedback part of the excitation control system. Moreover, a stabilizing transformer is introduced in the system by adding a derivative feedback to the control system to improve the dynamic performance [2]. The stabilizing transformer will add a zero to the AVR open-loop transfer function and, thus, will increase the relative stability of the closed-loop system. The transfer function of the stabilizing transformer is given as follows:

$$G_F(s) = \frac{K_F s}{1 + s T_F}, \quad (2)$$

where K_F and T_F are the gain and time constant, respectively. The transfer function of the PI controller is described as

$$G_C(s) = K_P + \frac{K_I}{s}, \quad (3)$$

where K_P and K_I are the proportional and integral gains, respectively. The proportional term affects the rate of voltage rise after a step change. The integral term affects the generator voltage settling time after initial voltage overshoot. The integral controller adds a pole at origin and increases the system type by one and reduces the steady-state error. The combined effect of the PI controller will shape the response of the generator excitation system to reach the desired performance.

2.2. Stability. The characteristic equation of the excitation control system can be easily obtained as

$$\Delta(s, \tau) = P(s) + Q(s) e^{-s\tau} = 0, \quad (4)$$

where $P(s)$ and $Q(s)$ are polynomials in s with real coefficients given below:

$$\begin{aligned} P(s) &= p_6 s^6 + p_5 s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s, \\ Q(s) &= q_2 s^2 + q_1 s + q_0. \end{aligned} \quad (5)$$

The coefficients of these polynomials in terms of gains and time constants are given in the Appendix.

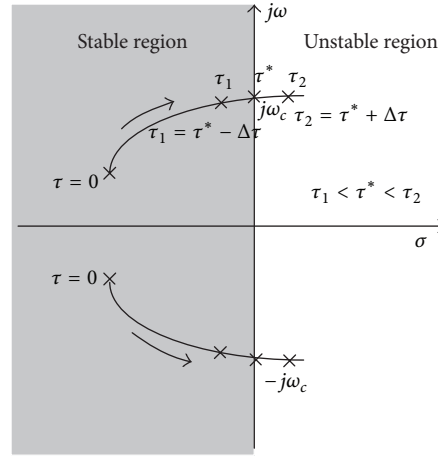


FIGURE 3: Illustration of the movement of the characteristic roots with respect to time delay.

The main goal of the stability studies of time-delayed systems is to determine conditions on the delay for any given set of system parameters that will guarantee the stability of the system. As with the delay-free system (i.e., $\tau = 0$), the stability of the AVR system depends on the locations of the roots of system's characteristic equation defined by (4). It is obvious that the roots of (4) are a function of the time delay τ . As τ changes, location of some of the roots may change. For the system to be asymptotically stable, all the roots of the characteristic equation of (4) must lie in the left half of the complex plane. That is,

$$\Delta(s, \tau) \neq 0, \quad \forall s \in C^+, \quad (6)$$

where C^+ represents the right half plane of the complex plane. Depending on system parameters, there are two different possible types of asymptotic stability situations due to the time delay τ [19, 21]:

- (i) *Delay-independent stability*: the characteristic equation of (4) is said to be *delay-independent stable* if the stability condition of (6) holds for all positive and finite values of the delay; $\tau \in [0, \infty)$.
- (ii) *Delay-dependent stability*: the characteristic equation of (4) is said to be *delay-dependent stable* if the condition of (6) holds for some values of delays belonging in the delay interval, $\tau \in [0, \tau^*)$, and is violated for other values of delay $\tau \geq \tau^*$.

In the delay-dependent case, the roots of the characteristic equations move as the time delay τ increases starting from $\tau = 0$. Figure 3 illustrates the movement of the roots. Note that the delay-free system ($\tau = 0$) is assumed to be stable. This is a realistic assumption since for the practical values of system parameters the excitation control system is stable when the total delay is neglected. Observe that, as the time delay τ is increased, a pair of complex eigenvalues moves in the left half of the complex plane. For a finite value of $\tau > 0$, they cross the imaginary axis and pass to the right half plane. The time delay value τ^* at which the characteristic equation has purely imaginary eigenvalues is the upper bound on the

delay size or the delay margin for which the system will be stable for any given delay less or equal to this bound, $\tau \leq \tau^*$. In order to characterize the stability property of (4) completely, we first need to determine whether the system for any given set of parameters is delay-independent stable or not and, if not, to determine the delay margin τ^* for a wide range of system parameters. In the following section, we present a practical approach that gives a criterion for evaluating the delay dependency of stability and an analytical formula to compute the delay margin for the delay-dependent case.

3. Delay Margin Computation

A necessary and sufficient condition for the system to be asymptotically stable is that all the roots of the characteristic equation of (4) lie in the left half of the complex plane. In the single delay case, the problem is to find values of τ^* for which the characteristic equation of (4) has roots (if any) on the imaginary axis of the s -plane. Clearly, $\Delta(s, \tau) = 0$ is an implicit function of s and τ which may, or may not, cross the imaginary axis. Assume for simplicity that $\Delta(s, 0) = 0$ has all its roots in the left half plane. That is, the delay-free system is stable. If, for some τ , $\Delta(s, \tau) = 0$ has a root on the imaginary axis at $s = j\omega_c$, so does $\Delta(-s, \tau) = 0$, for the same value of τ and ω_c because of the complex conjugate symmetry of complex roots. Therefore, looking for roots on the imaginary axis reduces to finding values of τ for which $\Delta(s, \tau) = 0$ and $\Delta(-s, \tau) = 0$ have a common root [21]. That is,

$$P(s) + Q(s)e^{-s\tau} = 0, \quad (7)$$

$$P(-s) + Q(-s)e^{s\tau} = 0.$$

By eliminating exponential terms in (7), we get the following polynomial:

$$P(s)P(-s) - Q(s)Q(-s) = 0. \quad (8)$$

If we replace s by $j\omega_c$ in (8), we have the polynomial in ω_c^2 given below:

$$W(\omega_c^2) = P(j\omega_c)P(-j\omega_c) - Q(j\omega_c)Q(-j\omega_c) = 0. \quad (9)$$

Substituting $P(s)$ and $Q(s)$ polynomials given in (5) into (9), we could obtain the following 12th order augmented characteristic equation or polynomial:

$$W(\omega_c^2) = t_{12}\omega_c^{12} + t_{10}\omega_c^{10} + t_8\omega_c^8 + t_6\omega_c^6 + t_4\omega_c^4 + t_2\omega_c^2 + t_0 = 0, \quad (10)$$

where the coefficients $t_0, t_2, t_4, t_6, t_8, t_{10}, t_{12}$ are real-valued and given in the Appendix.

Please note that transcendental characteristic equation with single delay given in (4) is now converted into a polynomial without the transcendentality given by (10), and its positive real roots coincide with the imaginary roots of (4) exactly. The roots of this polynomial might easily be determined by standard methods. Depending on the roots of (10), the following situation may occur.

- (i) The polynomial of (10) may not have any positive real roots, which implies that the characteristic equation of (4) does not have any roots on the $j\omega$ -axis. In that case, the system is stable for all $\tau \geq 0$, indicating that the system is *delay-independent stable*.
- (ii) The polynomial of (10) may have at least one positive real root, which implies that the characteristic equation of (4) has at least a pair of complex eigenvalues on the $j\omega$ -axis. In that case, the system is *delay-dependent stable*.

For a positive real root (ω_c) of (10) the corresponding value of delay margin τ^* can be easily obtained using (7) as [21]

$$\tau^* = \frac{1}{\omega_c} \text{Tan}^{-1} \left(\frac{\text{Im} \{P(j\omega_c)/Q(j\omega_c)\}}{\text{Re} \{-P(j\omega_c)/Q(j\omega_c)\}} \right) + \frac{2r\pi}{\omega_c}; \quad (11)$$

$r = 0, 1, 2, \dots, \infty.$

By substituting the polynomial of $P(s)$ and $Q(s)$ given in (5), the analytical formula for computing the delay margin of AVR system is determined as

$$\tau^* = \frac{1}{\omega_c} \text{Tan}^{-1} \left(\frac{a_7\omega_c^7 + a_5\omega_c^5 + a_3\omega_c^3 + a_1\omega_c}{a_8\omega_c^8 + a_6\omega_c^6 + a_4\omega_c^4 + a_2\omega_c^2 + a_0} \right) + \frac{2r\pi}{\omega_c}; \quad r = 0, 1, 2, \dots, \infty, \quad (12)$$

where the coefficients $a_i; i = 0, 1, \dots, 8$ are real-valued and given in the Appendix.

For the positive roots of (10), we also need to check if, at $s = j\omega_c$, the root of (4) crosses the imaginary axis with increasing τ . This can be determined by the sign of $\text{Re}[ds/d\tau]$. The necessary condition for the existence of roots crossing the imaginary axis is that the critical characteristic roots cross the imaginary axis with nonzero velocity (transversality condition); that is,

$$\text{Re} \left[\frac{ds}{d\tau} \right]_{s=j\omega_c} \neq 0, \quad (13)$$

where $\text{Re}(\cdot)$ denotes the real part of a complex variable. The sign of root sensitivity is defined as root tendency (RT):

$$\text{RT}|_{s=j\omega_c} = \text{sgn} \left\{ \text{Re} \left[\frac{ds}{d\tau} \right]_{s=j\omega_c} \right\}. \quad (14)$$

By taking the derivative of (4) with respect to τ and noticing that s is an explicit function of τ we obtain

$$\frac{ds}{d\tau} = \frac{Q(s)se^{-s\tau}}{P'(s) + Q'(s)e^{-s\tau} - Q(s)\tau e^{-s\tau}}, \quad (15)$$

where $P'(s)$ and $Q'(s)$ denote the first-order derivatives of $P(s)$ and $Q(s)$ with respect to s , respectively. Using (4) we can rewrite this expression as

$$\frac{ds}{d\tau} = -s \left[\frac{P'(s)}{P(s)} - \frac{Q'(s)}{Q(s)} + \tau \right]^{-1}. \quad (16)$$

We can get the corresponding root tendency by evaluating (16) at $s = j\omega_c$:

$$\begin{aligned} \text{RT}|_{s=j\omega_c} &= -\text{sgn} \left[\text{Re} \left(j\omega_c \left(\frac{P'(j\omega_c)}{P(j\omega_c)} - \frac{Q'(j\omega_c)}{Q(j\omega_c)} + \tau \right)^{-1} \right) \right] \\ &= -\text{sgn} \left[\text{Re} \left(\frac{1}{j\omega_c} \left(\frac{P'(j\omega_c)}{P(j\omega_c)} - \frac{Q'(j\omega_c)}{Q(j\omega_c)} + \tau \right) \right) \right] \\ &= \text{sgn} \left[\text{Im} \left(\frac{1}{\omega_c} \left(\frac{Q'(j\omega_c)}{Q(j\omega_c)} - \frac{P'(j\omega_c)}{P(j\omega_c)} \right) \right) \right]. \end{aligned} \quad (17)$$

It must be noted that the root tendency is independent of time delay τ . This implies that even though there are an infinite number of values of τ associated with each value of ω_c that makes $\Delta(j\omega_c, \tau) = 0$, the behavior of the roots at these points will always be the same. It can be easily shown by further simplifications of (17) that RT information can be deduced trivially from the polynomial $W(\omega_c^2)$ in (10). Recalling that $s = j\omega_c$, we have $W(\omega_c^2) = 0$. Then, from (9) we have $Q(j\omega_c) = P(j\omega_c)P(-j\omega_c)/Q(-j\omega_c)$. Thus,

$$\begin{aligned} \text{RT}|_{s=j\omega_c} &= \text{sgn} \left[\text{Im} \left(\frac{1}{\omega_c} \left(\frac{Q'(j\omega_c)Q(-j\omega_c)}{P(j\omega_c)P(-j\omega_c)} - \frac{P'(j\omega_c)}{P(j\omega_c)} \right) \right) \right] \\ &= \text{sgn} \left[\text{Im} \left(\frac{1}{\omega_c} \left(\frac{Q'(j\omega_c)Q(-j\omega_c) - P'(j\omega_c)P(-j\omega_c)}{P(j\omega_c)P(-j\omega_c)} \right) \right) \right] \\ &= \text{sgn} \left[\text{Im} \left(\frac{1}{\omega_c} (Q'(j\omega_c)Q(-j\omega_c) - P'(j\omega_c)P(-j\omega_c)) \right) \right], \end{aligned} \quad (18)$$

since $P(j\omega_c)P(-j\omega_c) = |P(j\omega_c)|^2 > 0$. Finally using the property $\text{Im}(z) = (z - \bar{z})/2j$, for any complex number z , we have

$$\begin{aligned} \text{RT}|_{s=j\omega_c} &= \text{sgn} \frac{1}{2j\omega_c} \\ &\times \left[\left(Q'(j\omega_c)Q(-j\omega_c) - Q(j\omega_c)Q'(-j\omega_c) \right. \right. \\ &\quad \left. \left. - P'(j\omega_c)P(-j\omega_c) + P(j\omega_c)P'(-j\omega_c) \right) \right], \end{aligned} \quad (19)$$

which finally leads us to

$$\text{RT}|_{s=j\omega_c} = \text{sgn} \left[W'(\omega_c^2) \right], \quad (20)$$

where the prime represents differentiation with respect to ω_c^2 .

The evaluation of the root sensitivities with respect to the time delay is unique feature of the proposed method. This expression gives a simple criterion to determine the direction of transition of the roots at $s = j\omega_c$ as τ increases from $\tau_1 = \tau^* - \Delta\tau$ to $\tau_2 = \tau^* + \Delta\tau$, $0 < \Delta\tau \ll 1$ as shown in Figure 3. The root $s = j\omega_c$ crosses the imaginary axis either to unstable right half plane when $\text{RT} = +1$ or to stable left half plane when $\text{RT} = -1$.

For the excitation control system, the root tendency for each crossing frequency could be easily determined by the following expression obtained by taking the derivative of the polynomial given in (10) with respect to ω_c^2 :

$$\begin{aligned} \text{RT}|_{s=j\omega_c} &= 6t_{12}\omega_c^{10} + 5t_{10}\omega_c^8 + 4t_8\omega_c^6 \\ &\quad + 3t_6\omega_c^4 + 2t_4\omega_c^2 + t_2. \end{aligned} \quad (21)$$

It must be noted here that the polynomial of (10) may have multiple but finite number of positive real roots for all $\tau \in \mathfrak{R}^+$. Let us call this set

$$\{\omega_c\} = \{\omega_{c1}, \omega_{c2}, \dots, \omega_{cq}\}. \quad (22)$$

This finite number q is influenced not only by the system order, but also by the coefficients of polynomials $P(s)$ and $Q(s)$. Furthermore, for each ω_{cm} , $m = 1, 2, \dots, q$, we can get infinitely many periodically spaced τ_m^* values by using (12). Let us call this set

$$\{\tau_m^*\} = \{\tau_{m1}^*, \tau_{m2}^*, \dots, \tau_{m,\infty}^*\} \quad m = 1, 2, \dots, q, \quad (23)$$

where $\tau_{m,r+1} - \tau_{m,r} = 2\pi/\omega_c$ is the apparent period of repetition. According to the definition of delay margin, the minimum of τ_m^* , $m = 1, 2, \dots, q$, will be the system delay margin:

$$\tau^* = \min(\tau_m^*). \quad (24)$$

4. Results and Discussion

4.1. Theoretical Results. In this section, the delay margin τ^* for stability for a wide range of PI controller gains is computed using the expression given in (12). Theoretical delay margin

results are verified by using Matlab/Simulink. The gains and time constants of the exciter control system used in the analysis are as follows:

$$\begin{aligned} T_A &= 0.1 \text{ s}, & T_E &= 0.4 \text{ s}, & T_G &= 1.0 \text{ s}, \\ T_R &= 0.05 \text{ s}, & T_F &= 0.04 \text{ s}, \\ K_A &= 5, & K_E &= K_G = K_R = 1.0, & K_F &= 2.0. \end{aligned} \quad (25)$$

First, we choose typical PI controller gains $K_P = 0.7$; $K_I = 0.8 \text{ s}^{-1}$ to demonstrate the delay margin computation. The process of the delay margin computation consists of the following four steps.

Step 1. Determine the characteristic equation of time-delayed excitation control system using (4) and (5). This equation is found to be

$$\begin{aligned} \Delta(s, \tau) &= \left(8 \times 10^{-5} s^6 + 0.0047 s^5 + 0.4416 s^4 \right. \\ &\quad \left. + 7.817 s^3 + 16.99 s^2 + 9.0 s \right) \\ &\quad + \left(0.14 s^2 + 3.66 s + 4 \right) e^{-s\tau} = 0. \end{aligned} \quad (26)$$

Note that for $\tau = 0$ the characteristic equation of the delay-free system has roots at $s_{1,2} = -19.035 \pm j65.838$; $s_{3,4} = -0.501 \pm j0.4095$; $s_5 = -18.0155$; $s_6 = -1.4121$, which indicates that the delay-free system is stable.

Step 2. Construct the $W(\omega_c^2)$ polynomial using (10), and compute its real positive roots ω_{cm} , if they exist. The polynomial is computed as

$$\begin{aligned} W(\omega_c^2) &= 6.4 \times 10^{-9} \omega_c^{12} - 4.8754 \times 10^{-5} \omega_c^{10} + 0.1189 \omega_c^8 \\ &\quad + 56.0930 \omega_c^6 + 136.9545 \omega_c^4 + 68.7244 \omega_c^2 \\ &\quad - 16 = 0. \end{aligned} \quad (27)$$

It is found that this polynomial has only one positive real root. This positive real root is found to be $\omega_c = 0.4131 \text{ rad/s}$.

Step 3. Compute the delay margin for each positive root found in Step 2 using (12) and select the minimum of those as the system delay margin for this PI controller gains. The delay margin is computed as $\tau^* = 2.8760 \text{ s}$.

Step 4. Determine the root tendency (RT) for each positive root ω_{cm} using (21). The RT for $\omega_c = 0.4131 \text{ rad/s}$ is computed as $\text{RT} = +1$. This RT indicates that a pair of complex roots passes from stable half plane to unstable half plane crossing $j\omega$ -axis $s = \pm j0.4131 \text{ rad/s}$ for $\tau^* = 2.8760 \text{ s}$ and the system becomes unstable.

For the theoretical analysis, the effect of PI controller gains on the delay margin is also investigated. Table 1 shows delay margins of the AVR system with a stabilizing transformer for the range of $K_P = 0.3$ – 0.7 and $K_I = 0.1$ – 1.0 s^{-1} . It is clear from Table 1 that the delay margin decreases

TABLE 1: Delay margins τ^* for different values of K_I and K_P of the AVR system with a stabilizing transformer.

K_I (s^{-1})	τ^* (s)		
	$K_P = 0.3$	$K_P = 0.5$	$K_P = 0.7$
0.1	5.4172	4.4662	3.8108
0.2	4.0532	3.9038	3.6515
0.3	3.5181	3.5078	3.4234
0.4	3.2443	3.2645	3.2403
0.5	3.0813	3.1073	3.1059
0.6	2.9745	2.9996	3.0071
0.7	2.8997	2.9223	2.9330
0.8	2.8446	2.8645	2.8760
0.9	2.8026	2.8199	2.8312
1.0	2.7694	2.7846	2.7952

as the integral gain increases when K_P is fixed. In order to find out the quantitative impact of the stabilizing transformer on the delay margin, delay margins are also obtained for the case in which a stabilizing transformer is not used in the AVR system [29]. Table 2 shows delay margins of the AVR system without a stabilizing transformer. It is clear from Tables 1 and 2 that compensation of the AVR system by a stabilizing transformer significantly increases the delay margins for all values of PI controller gains, which makes the AVR system more stable. For example, for $K_P = 0.7$ and $K_I = 0.8 s^{-1}$, the delay margin when a stabilizing transformer is not included is found to be $\tau^* = 0.1554 s$ while it is $\tau^* = 2.8760 s$ when a stabilizing transformer is included. This is obviously a significant improvement in the stability performance of the AVR system. This observation is valid for all values of PI controller gains as indicated in Tables 1 and 2.

4.2. Verification of Theoretical Delay Margin Results. Matlab/Simulink is used to verify the theoretical results on the delay margin and to illustrate how the stabilizing transformer damps the oscillations in the presence of time delays. For the illustrative purpose, PI controller gains are chosen as $K_P = 0.7$ and $K_I = 0.8 s^{-1}$. From Table 1, for these gains, the delay margin is found to be $\tau^* = 2.8760 s$ when the stabilizing transformer is included. Simulation result for this delay value is shown in Figure 4(a). It is clear that sustained oscillations are observed indicating a marginally stable operation. When the time delay is less than the delay margin, it is expected that the exciter system will be stable. Figure 4(a) also shows such a simulation result for $\tau = 2.74 s$. Similarly, when the time delay is larger than the delay margin, the system will have growing oscillations indicating an unstable operation, as illustrated in Figure 4(a) for $\tau = 3.0 s$. These simulation results show that the theoretical method correctly estimates the delay margin of the AVR system compensated by a stabilizing transformer. Moreover, the voltage response of the AVR system not including a stabilizing transformer is presented in Figure 4(b), indicating the same type of dynamic behavior

TABLE 2: Delay margins τ^* for different values of K_I and K_P of the AVR system without a stabilizing transformer.

K_I (s^{-1})	τ^* (s)		
	$K_P = 0.3$	$K_P = 0.5$	$K_P = 0.7$
0.1	1.4164	0.6275	0.3652
0.2	0.9700	0.5471	0.3349
0.3	0.6781	0.4665	0.3037
0.4	0.4831	0.3912	0.2722
0.5	0.3459	0.3235	0.2411
0.6	0.2447	0.2639	0.2111
0.7	0.1674	0.2118	0.1825
0.8	0.1066	0.1664	0.1554
0.9	0.0576	0.1267	0.1300
1.0	0.0174	0.0920	0.1063

as the AVR system including a stabilizing transformer with a smaller delay margin ($\tau^* = 0.1554 s$).

The damping effect of the stabilizing transformer could be easily illustrated by time-domain simulations. Figure 5(a) compares the voltage response of the AVR system with and without a stabilizing transformer for $\tau^* = 0.1554 s$. It is clear that the AVR system not including the stabilizing transformer is marginally stable or on the stability boundary having sustained oscillations while the AVR system compensated by a stabilizing transformer has a response with quickly damped oscillations. Similarly, Figure 5(b) shows the voltage response of the AVR system with and without the stabilizing transformer for $\tau = 0.17 s$. When the stabilizing transformer is not considered, the system is unstable since $\tau = 0.17 s > \tau^* = 0.1554 s$ and growing oscillations are observed. However, the addition of the stabilizing transformer damps the oscillations and makes the excitation control system stable.

5. Conclusions

This paper has proposed an exact method to compute the delay margin for stability of the AVR system including a stabilizing transformer. The method first eliminates the transcendental term in the characteristic equation without using any approximation. The resulting augmented equation has become a regular polynomial whose real roots coincide with the imaginary roots of the characteristic equation exactly. An expression in terms of system parameters and imaginary roots of the characteristic equation has been derived for computing the delay margin. Moreover, using the augmented equation, a simple root sensitivity test has been developed to determine the direction of the root transition.

The effect of PI controller gains on the delay margin has been investigated. The theoretical results indicate that the delay margin decreases as the integral gain increases for a given proportional gain. Such a decrease in delay margin implies a less stable AVR system. Moreover, it has been observed that the compensation of the AVR system with a stabilizing transformer remarkably increases the delay

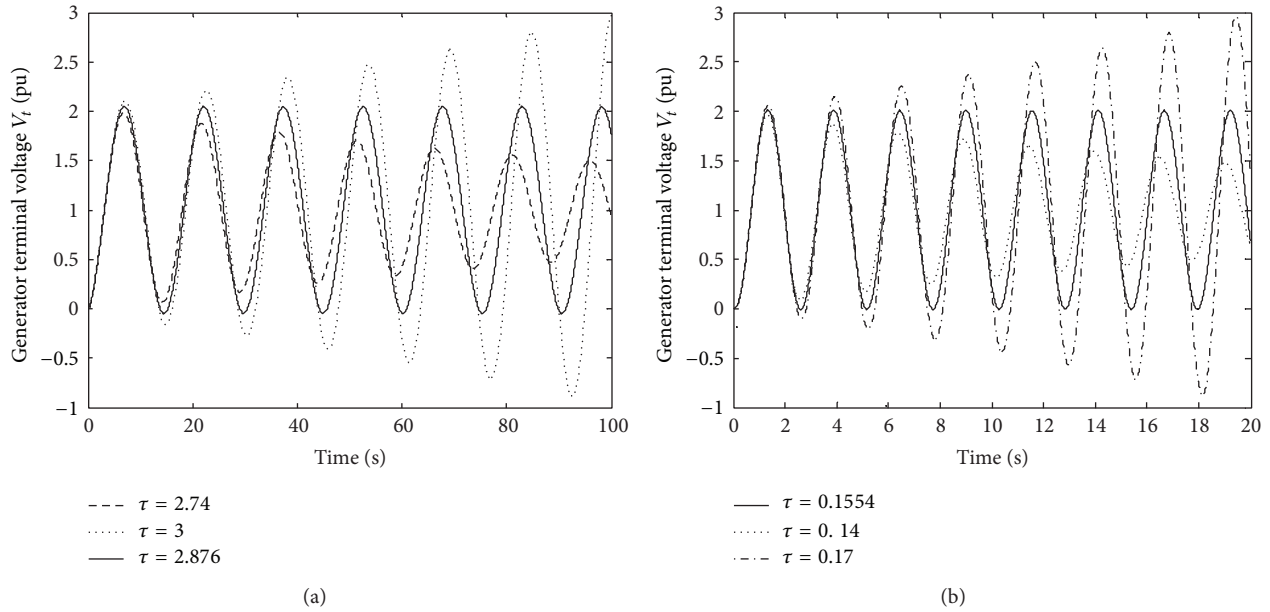


FIGURE 4: Voltage response of the AVR system for $K_p = 0.7$ and $K_I = 0.8 \text{ s}^{-1}$: (a) with a stabilizing transformer and $\tau = 2.74, 2.876,$ and 3.0 s and (b) without a stabilizing transformer and $\tau = 0.14, 0.1554,$ and 0.17 s .

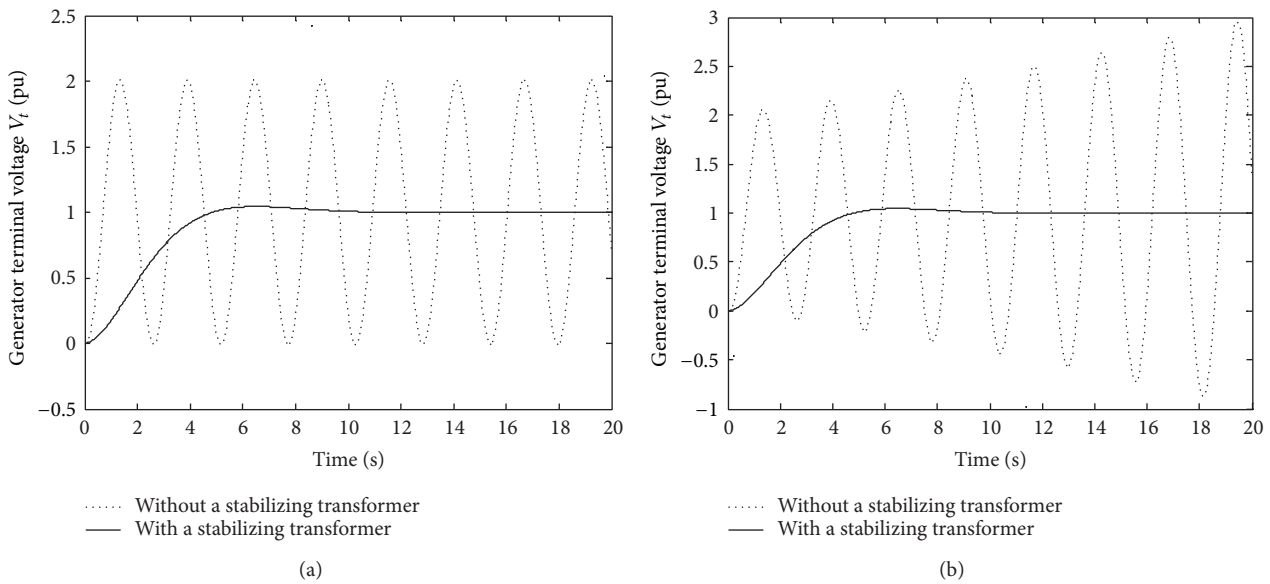


FIGURE 5: Voltage response of the AVR system with and without a stabilizing transformer for $K_p = 0.7$ and $K_I = 0.8 \text{ s}^{-1}$: (a) $\tau = 0.1554 \text{ s}$; (b) $\tau = 0.17 \text{ s}$.

margin of the system, which indicates a more stable AVR system.

Theoretical delay margin results have been verified by carrying out simulation studies. It has been observed that the proposed method correctly estimates the delay margin of the AVR system. Additionally, with the help of simulations, it has been shown that the stabilizing transformer significantly improves the system dynamic performance by damping the oscillations.

With the help of the results presented, controller gains could be properly selected such that the excitation control system will be stable and will have a desired damping performance even if certain amount of time delays exists in the system.

The following studies have been put in perspective as future work: (i) the extension of the proposed method into multimachine power systems with commensurate time delays, (ii) the influence of power system stabilizer (PSS)

on the delay margin, and (iii) the probabilistic evaluation of delay margin as to take into account random nature of time delays.

Appendix

The coefficients of polynomials $P(s)$ and $Q(s)$ given in (5) in terms of gains and time constants of the AVR system:

$$\begin{aligned}
p_6 &= T_A T_E T_F T_G T_R, \\
p_5 &= T_A T_E T_F T_G \\
&\quad + T_R (T_A T_E T_F + T_A T_E T_G + T_A T_F T_G + T_E T_F T_G), \\
p_4 &= T_A T_E T_F + T_A T_E T_G + T_A T_F T_G + T_E T_F T_G \\
&\quad + T_R (T_A T_E + T_A T_F + T_E T_F + T_A T_G + T_E T_G \\
&\quad\quad + T_F T_G + K_A K_E K_P K_F T_G), \\
p_3 &= T_A T_E + T_A T_F + T_E T_F + T_A T_G + T_E T_G \\
&\quad + T_F T_G + K_A K_E K_P K_F T_G \\
&\quad + T_R (T_A + T_E + T_F + K_A K_E K_P K_F \\
&\quad\quad + K_A K_E K_I K_F T_G + T_G), \\
p_2 &= T_A + T_E + T_F + K_A K_E K_P K_F + K_A K_E K_I K_F T_G + T_G \\
&\quad + T_R (K_A K_E K_I K_F + 1), \\
p_1 &= 1 + K_A K_E K_I K_F, \\
q_2 &= K_A K_E K_P K_G K_R T_F, \\
q_1 &= K_A K_E K_G K_R (K_P + K_I T_F), \\
q_0 &= K_A K_E K_I K_G K_R.
\end{aligned} \tag{A.1}$$

The coefficients of the polynomial $W(\omega_c^2)$ given in (10):

$$\begin{aligned}
t_{12} &= p_6^2, & t_{10} &= p_5^2 - 2p_4 p_6, \\
t_8 &= p_4^2 - 2p_3 p_5 + 2p_2 p_6, \\
t_6 &= p_3^2 - 2p_2 p_4 + 2p_1 p_5, \\
t_4 &= p_2^2 - 2p_1 p_3 - q_2^2, \\
t_2 &= p_1^2 - q_1^2 + 2q_0 q_2, & t_0 &= -q_0^2.
\end{aligned} \tag{A.2}$$

The coefficients of the analytical formula given in (12):

$$\begin{aligned}
a_8 &= -p_6 q_2, & a_7 &= p_6 q_1 - p_5 q_2, \\
a_6 &= p_4 q_2 + p_6 q_0 - p_5 q_1,
\end{aligned}$$

$$a_5 = -p_4 q_1 + p_3 q_2 + p_5 q_0,$$

$$a_4 = -p_2 q_2 - p_4 q_0 + p_3 q_1,$$

$$a_3 = p_2 q_1 - p_1 q_2 - p_3 q_0,$$

$$a_2 = p_2 q_0 - p_1 q_1,$$

$$a_1 = p_1 q_0, \quad a_0 = 0.$$

(A.3)

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

- [1] P. Kundur, *Power System Stability and Control*, McGraw-Hill, New York, NY, USA, 1994.
- [2] IEEE Standard 421.5, *IEEE Recommended Practice for Excitation System Models for Power System Stability Studies*, IEEE Power Engineering Society, 2005.
- [3] A. G. Phadke, "Synchronized phasor measurements in power systems," *IEEE Computer Applications in Power*, vol. 6, no. 2, pp. 10–15, 1993.
- [4] B. Naduvathuparambil, M. C. Venti, and A. Feliachi, "Communication delays in wide area measurement systems," in *Proceedings of the 34th Southeastern Symposium on System Theory*, vol. 1, pp. 118–122, University of Alabama, March 2002.
- [5] S. Bhowmik, K. Tomsovic, and A. Bose, "Communication models for third party load frequency control," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 543–548, 2004.
- [6] M. Liu, L. Yang, D. Gan, D. Wang, F. Gao, and Y. Chen, "The stability of AGC systems with commensurate delays," *European Transactions on Electrical Power*, vol. 17, no. 6, pp. 615–627, 2007.
- [7] H. Wu, K. S. Tsakalis, and G. T. Heydt, "Evaluation of time delay effects to wide area power system stabilizer design," *IEEE Transactions on Power Systems*, vol. 19, no. 4, pp. 1935–1941, 2004.
- [8] X. Yu and K. Tomsovic, "Application of linear matrix inequalities for load frequency control with communication delays," *IEEE Transactions on Power Systems*, vol. 19, no. 3, pp. 1508–1515, 2004.
- [9] L. Jiang, W. Yao, Q. H. Wu, J. Y. Wen, and S. J. Cheng, "Delay-dependent stability for load frequency control with constant and time-varying delays," *IEEE Transactions on Power Systems*, vol. 27, no. 2, pp. 932–941, 2012.
- [10] J. Quanyuan, Z. Zhenyu, and C. Yijia, "Wide-area TCSC controller design in consideration of feedback signals' time delays," in *Proceedings of the IEEE Power Engineering Society General Meeting*, pp. 1676–1680, San Francisco, Calif, USA, June 2005.
- [11] J. Luque, J. I. Escudero, and F. Pérez, "Analytic model of the measurement errors caused by communications delay," *IEEE Transactions on Power Delivery*, vol. 17, no. 2, pp. 334–337, 2002.
- [12] S. P. Carullo and C. O. Nwankpa, "Experimental validation of a model for an information-embedded power system," *IEEE Transactions on Power Delivery*, vol. 20, no. 3, pp. 1853–1863, 2005.

- [13] C.-W. Park and W.-H. Kwon, "Time-delay compensation for induction motor vector control system," *Electric Power Systems Research*, vol. 68, no. 3, pp. 238–247, 2004.
- [14] H. Okuno and T. Fujii, "Delayed feedback controlled power system," in *Proceedings of the SICE Annual Conference*, pp. 2659–2663, August 2005.
- [15] H.-K. Chen, T.-N. Lin, and J.-H. Chen, "Dynamic analysis, controlling chaos and chaotification of a SMIB power system," *Chaos, Solitons and Fractals*, vol. 24, no. 5, pp. 1307–1315, 2005.
- [16] H. Jia, X. Yu, Y. Yu, and C. Wang, "Power system small signal stability region with time delay," *International Journal of Electrical Power and Energy Systems*, vol. 30, no. 1, pp. 16–22, 2008.
- [17] S. Ayasun, "Computation of time delay margin for power system small-signal stability," *European Transactions on Electrical Power*, vol. 19, no. 7, pp. 949–968, 2009.
- [18] J. Chen, G. Gu, and C. N. Nett, "A new method for computing delay margins for stability of linear delay systems," *Systems and Control Letters*, vol. 26, no. 2, pp. 107–117, 1995.
- [19] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time Delay Systems*, Birkhäuser, Boston, Mass, USA, 2003.
- [20] P. Fu, S.-I. Niculescu, and J. Chen, "Stability of linear neutral time-delay systems: exact conditions via matrix pencil solutions," *IEEE Transactions on Automatic Control*, vol. 51, no. 6, pp. 1063–1069, 2006.
- [21] K. Walton and J. E. Marshall, "Direct method for TDS stability analysis," *IEE Proceedings D: Control Theory and Applications*, vol. 134, no. 2, pp. 101–107, 1987.
- [22] J.-H. Su, "Asymptotic stability of linear autonomous systems with commensurate time delays," *IEEE Transactions on Automatic Control*, vol. 40, no. 6, pp. 1114–1117, 1995.
- [23] J. Louisell, "A matrix method for determining the imaginary axis eigenvalues of a delay system," *IEEE Transactions on Automatic Control*, vol. 46, no. 12, pp. 2008–2012, 2001.
- [24] Z. V. Rekasius, "A stability test for systems with delays," in *Proceedings of Joint Automatic Control Conference*, paper no. TP9-A, 1980.
- [25] N. Olgac and R. Sipahi, "An exact method for the stability analysis of time-delayed linear time-invariant (LTI) systems," *IEEE Transactions on Automatic Control*, vol. 47, no. 5, pp. 793–797, 2002.
- [26] H. Fazelinia, R. Sipahi, and N. Olgac, "Stability robustness analysis of multiple time-delayed systems using "building block" concept," *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 799–810, 2007.
- [27] R. Sipahi and N. Olgac, "A comparative survey in determining the imaginary characteristic roots of LTI time delayed systems," in *Proceedings of the 16th IFAC World Congress*, vol. 16, p. 635, Prague, Czech Republic, July 2005.
- [28] *MATLAB: High-Performance Numeric Computation and Visualization Software*, The Mathworks, Natick, Mass, USA, 2001.
- [29] S. Ayasun and A. Gelen, "Stability analysis of a generator excitation control system with time delays," *Electrical Engineering*, vol. 91, no. 6, pp. 347–355, 2010.



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