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Research Article

Vibration Characteristics for Moving Printing Membrane with Variable Density along the Lateral Direction

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The vibration model of moving membrane with variable density distribution is established, and the density distribution of the moving membrane varies along the lateral direction. The transverse vibration differential equations of moving membrane are established based on D'Alembert's principle and discretized by using the differential quadrature method (DQM). The relationships of the first three dimensionless complex frequencies between dimensionless speed, density coefficient, and tension ratio of the membrane are analyzed by numerical calculation. The effects of the density coefficient and the tension ratio on transverse vibration characteristics of the membrane are investigated. The relationship between density coefficient and critical speed is obtained. The numerical results show that the density coefficient and the tension ratio have important influence on the stability of moving membrane. So the study provides a theoretical basis for improving the working stability of the membrane in the high-speed printing process.

1. Introduction

The membrane including plastic film, paper web, cloth, metal foil, and other types of film-like material is widely used to make printing and packaging products. The processing schematic of axially moving membrane is shown in Figure 1. The membrane deformation, the membrane folding, the surface scratches, and other processing defects can be caused by the transverse vibrations in printing process, which will affect the printing accuracy and printing quality seriously [1]. Therefore, the systematic study of transverse vibration characteristics of the membrane is important, which has caught the attention of many scholars.

Kulachenko and his coworkers [2, 3] studied nonlinear dynamics problem; the stability of transverse vibration of the web was studied by finite element method. The element-free Galerkin method was used to analyze free vibration of thin plates resting on Pasternak elastic foundations with all possible types of classical boundary conditions by Bahmyari et al. [4]. Nguyen et al. [5, 6] analyzed the stability and the control of transverse vibration of web by changing axial velocity and axial tension, respectively. Banichuk and his coinvestigators [7] considered the dynamic characteristics of the moving web with nonhomogeneous tension, and the analytical approaches method was used. Vedrines et al. [8] measured the vibrations of the printing web with a laser sensor and determined the vibration in plane. Wang et al. [9] applied finite difference method to acquire natural vibration frequency of the annular membrane with wrinkle. The results showed that the frequency increased with the increase of wrinkling level. Wu et al. [10, 11] established the vibration equations and analyzed the stability of the membrane with an intermediate elastic support and variable speed.

In these literatures above, the membrane chosen as sample was all supposed with constant density. But the membrane density is changing in many situations. In printing process, the plate is wetted and inked based on the image distribution, and then the ink is transferred to the substrate through rolling of the plate cylinder and impression cylinder. The ink and fountain solution which are absorbed into

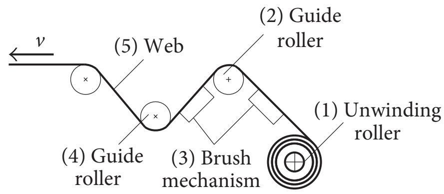


FIGURE 1: The axially moving web.

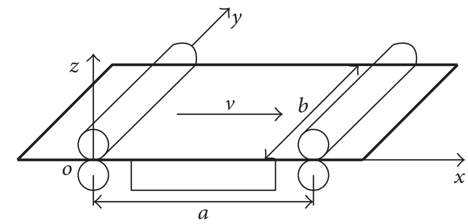


FIGURE 2: The model of printing moving membrane under the four edges fixed.

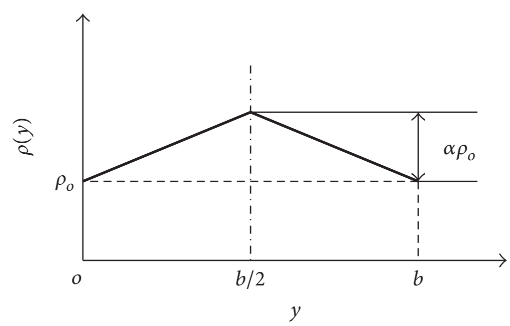


FIGURE 3: Density function of printing moving membrane along the lateral direction.

the paper web or membrane cause the change of density. Besides, the nonuniform thickness of membrane also can change the surface density [12]. Nonuniform density will affect the vibration characteristics and stability of membrane. Therefore, the study of the transverse vibration characteristics of the moving membrane with variable density has a great significance on controlling the membrane vibration during the printing process and improving overprint precision of printed products.

Transverse natural vibration of the annular membrane with variable surface density and without axial velocity was studied by using modified perturbation method in [13]. A numerical analysis of axisymmetric transverse-free vibration of the circular membrane with nonhomogeneous changing density was analyzed by Li [14, 15], and the first-order natural frequencies were given. Buchanan [16] examined the circular membrane dynamic properties, and the density distribution of the circular membrane was a linear variation along the diameter direction. Gupta and Khanna [17] and Zhou and Wang [18] studied the vibration characteristics of the viscoelastic plate with parabolic variable thickness. The suboptimal control method was applied to control the variable density of the moving web by Ma and her coworkers [19]. Sheet flutter and the interaction between sheet and air were analyzed by Pramila [20]; the results showed that surrounding air had an impact on the critical velocities and the eigenfrequencies. He [21] also studied the natural frequencies of an axially moving band vibrating in an ideal fluid. And the influence of the surrounding fluid was taken into account by using hydrodynamic added mass. Niemi and Pramila [22] researched transverse vibrations of an axially moving membrane submerged in ideal fluid by using the FEM. The effect of the density of the element mesh, the truncation distance, and the various lumping techniques on the accuracy of the results was also analyzed. Gutierrez et al. [23] employed a series of numerical experiments to deal with the transverse vibration of annular membranes where the density varies with the radial variable linearly, quadratically, and cubically. A general quasi-analytical model based on the Frobenius power series expansion method was described so as to handle vibrations of solid circular and annular membranes with continuously varying density by Willatzen [24]. Bala Subrahmanyam and Sujith [25] also studied the traverse vibration of annular membranes with continuously varying densities.

In this study, the stability of the moving membrane with variable density in the lateral direction is studied. The transverse vibration differential equation of the moving membrane with variable density is established and discretized by using DQM. The relationships between the dimensionless vibration frequency and the dimensionless velocity, the density coefficient, and the tension ratio of the printing membrane are studied. The influence of the density coefficient and the tension ratio on vibration characteristics of the membrane is discussed.

2. The Transverse Vibration Model of Moving Membrane

The membrane between two guide rollers can be simplified to a moving membrane model shown in Figure 2. The rollers and the brush mechanisms (in Figure 1) which support the membrane can be regarded as the boundary condition with the four edges fixed. Here, the membrane is soft, homogeneous, and inextensible. The thickness of the membrane is neglected. The translating direction of membrane is x direction. The membrane lateral or width direction is y direction. The displacement direction of transverse vibration is the z direction. The membrane length is a, the width is b, and v is axially translating velocity. The function $\overline{w}(x, y, t)$ denotes the transverse displacement of the printing membrane, where t is time. T_x and T_y are uniform tension per unit length in the x, y direction.

The density function of membrane is $\rho(y)$, as is shown in Figure 3. The membrane surface density is varied along y direction that can be expressed as follows:

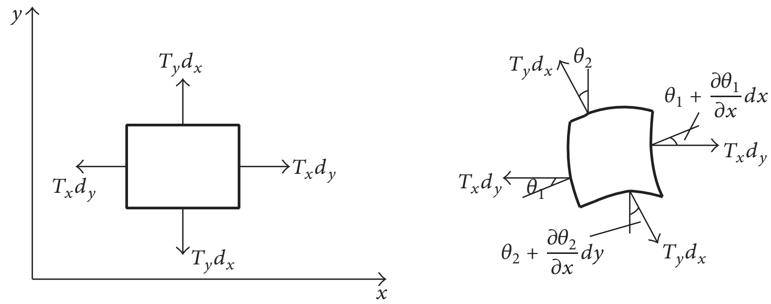


Figure 4: The force on an elemental area of the membrane.

$$\rho(y) = \begin{cases} \rho_0 \left(1 + 2\alpha \frac{y}{b}\right) & \left(0 \le y \le \frac{b}{2}\right) \\ \rho_0 \left(1 + 2\alpha - 2\alpha \frac{y}{b}\right) & \left(\frac{b}{2} \le y \le b\right), \end{cases} \quad (1)$$

where α denotes the density coefficient.

The speed $\overline{v}(t)$ in the transverse displacement direction can be given as follows:

$$\overline{v}(t) = \frac{d\overline{w}(x, y, t)}{dt} = \frac{\partial \overline{w}}{\partial t} + v \frac{\partial \overline{w}}{\partial x}. \quad (2)$$

Then the transverse acceleration $\overline{a}$ can be expressed as follows:

$$\begin{aligned} \overline{a} &= \frac{d\overline{v}(t)}{dt} = \frac{d(\partial \overline{w}/\partial t + v(\partial \overline{w}/\partial x))}{dt} \\ &= \frac{\partial^2 \overline{w}}{\partial t^2} + 2v \frac{\partial^2 \overline{w}}{\partial x \partial t} + v^2 \frac{\partial^2 \overline{w}}{\partial x^2}. \end{aligned} \quad (3)$$

As shown in Figure 4, consider an elemental area of the membrane $dxdy$ and analyze the force on them. The composition force in z direction caused by the membrane tension on dy and dx can be derived as follows:

$$\begin{aligned} T_x dy \left(\theta_1 + \frac{\partial \theta_1}{\partial x} dx\right) - T_x dy \theta_1 &= T_x \frac{\partial \theta_1}{\partial x} dxdy \\ &= T_x \frac{\partial (\partial \overline{w}/\partial x)}{\partial x} dxdy = T_x \frac{\partial^2 \overline{w}}{\partial x^2} dxdy, \\ T_y dy \left(\theta_2 + \frac{\partial \theta_2}{\partial x} dy\right) - T_y dx \theta_2 &= T_y \frac{\partial \theta_2}{\partial y} dxdy \\ &= T_y \frac{\partial (\partial \overline{w}/\partial y)}{\partial y} dxdy = T_y \frac{\partial^2 \overline{w}}{\partial y^2} dxdy. \end{aligned} \quad (4)$$

It is assumed that the membrane is subjected to a transverse load $F(x, y, t)$ in z direction; here, t represents time. The motion equation of vibration system can be given based on D'Alembert's principle [26]:

$$\begin{aligned} &T_x \frac{\partial^2 \overline{w}}{\partial x^2} + T_y \frac{\partial^2 \overline{w}}{\partial y^2} + F(x, y, t) \\ &\quad - \left[\rho(y) \left(\frac{\partial^2 \overline{w}}{\partial t^2} + 2v \frac{\partial^2 \overline{w}}{\partial x \partial t} + v^2 \frac{\partial^2 \overline{w}}{\partial x^2}\right)\right] = 0. \end{aligned} \quad (5)$$

Irrespective of the transverse load, that is, $F(x, y, t) = 0$, then the transverse vibration differential equation of membrane can be expressed as follows:

$$\begin{aligned} &\rho(y) \left(\frac{\partial^2 \overline{w}}{\partial t^2} + 2v \frac{\partial^2 \overline{w}}{\partial x \partial t} + v^2 \frac{\partial^2 \overline{w}}{\partial x^2}\right) - T_x \frac{\partial^2 \overline{w}}{\partial x^2} - T_y \frac{\partial^2 \overline{w}}{\partial y^2} \\ &\quad = 0. \end{aligned} \quad (6)$$

The piecewise functions $\rho(y)$ can be expressed as follows:

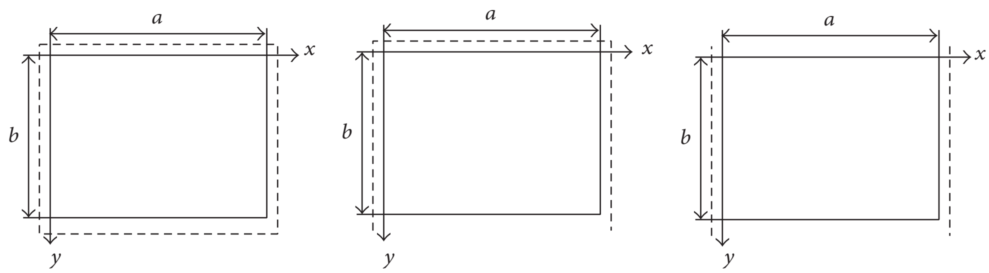
$$\rho(y) = \rho_0 (1 + \alpha) - 2\alpha \rho_0 \left|\frac{y}{b} - \frac{1}{2}\right|. \quad (7)$$

Substituting (4) into (3) yields

$$\begin{aligned} &\left[(1 + \alpha) \rho_0 - 2\alpha \rho_0 \left|\frac{y}{b} - \frac{1}{2}\right|\right] \\ &\quad \cdot \left(\frac{\partial^2 \overline{w}}{\partial t^2} + 2v \frac{\partial^2 \overline{w}}{\partial x \partial y} + v^2 \frac{\partial^2 \overline{w}}{\partial x^2}\right) - T_x \frac{\partial^2 \overline{w}}{\partial x^2} - T_y \frac{\partial^2 \overline{w}}{\partial y^2} \\ &\quad = 0. \end{aligned} \quad (8)$$

Introduce the dimensionless quantities as follows:

$$\begin{aligned} \zeta &= \frac{x}{a}, \\ \eta &= \frac{y}{b}, \\ w &= \frac{\overline{w}}{a}, \\ c &= v \sqrt{\frac{\rho_0}{T_x}}, \\ \tau &= t \sqrt{\frac{T_x}{a^2 \rho_0}}, \end{aligned}$$



(a) Four sides fixed (b) Three sides fixed and one free (c) Two sides fixed and two free

FIGURE 5: The boundary conditions of the printing moving membrane.

$$\lambda = \frac{T_y}{T_x},$$
$$\mu = \frac{a}{b}. \tag{9}$$

Substituting (9) into (8) yields

$$\left[(1+\alpha)\rho_0 - 2\alpha\rho_0\left|\eta - \frac{1}{2}\right|\right] \cdot \left(\frac{\partial^2 w}{\partial\tau^2} + 2c\frac{\partial^2 w}{\partial\zeta\partial\tau} + c^2\frac{\partial^2 \overline{w}}{\partial\zeta^2}\right) - \frac{\partial^2 w}{\partial\zeta^2} - \lambda\mu^2\frac{\partial^2 w}{\partial\eta^2} = 0. \tag{10}$$

Supposes the solution to (10) is

$$w(\zeta,\eta,\tau) = W(\zeta,\eta)\, e^{j\omega\tau}. \tag{11}$$

Substituting (11) into (10) obtains the transverse vibration differential equations of moving membrane with variable density:

$$\begin{aligned}
&(c^2-1)\frac{\partial^2 W}{\partial\zeta^2} - \lambda\mu^2\frac{\partial^2 W}{\partial\eta^2} + 2cj_1\omega\frac{\partial W}{\partial\zeta} - \omega^2 W \\
&\quad + 2c^2\alpha\eta\frac{\partial^2 W}{\partial\zeta^2} + 4c\alpha\eta j_1\omega\frac{\partial W}{\partial\zeta} - 2\alpha\eta\omega^2 W = 0 \quad \left(0 \le \eta \le \frac{1}{2}\right), \\
&(c^2-1)\frac{\partial^2 W}{\partial\zeta^2} - \lambda\mu^2\frac{\partial^2 W}{\partial\eta^2} + 2cj_1\omega\frac{\partial W}{\partial\zeta} - \omega^2 W \\
&\quad + 2c^2\alpha\frac{\partial^2 W}{\partial\zeta^2} + 4c\alpha j_1\omega\frac{\partial W}{\partial\zeta} - 2\alpha\omega^2 W - 2c^2\alpha\eta\frac{\partial^2 W}{\partial\zeta^2} \\
&\quad - 4c\alpha\eta j_1\omega\frac{\partial W}{\partial\zeta} + 2\alpha\eta\omega^2 W = 0 \quad \left(\frac{1}{2} \le \eta \le 1\right),
\end{aligned} \tag{12}$$

where $j_1 = \sqrt{-1}$, τ is dimensionless time, ω is dimensionless natural frequency, c is dimensionless speed, λ is tension ratio, and μ is aspect ratio.

Figure 5 shows the classical boundary conditions for an axially moving membrane, which are four sides fixed and three sides fixed and one free and two sides fixed and two free, and we only consider the boundary condition with four sides fixed.

The boundary conditions of moving membrane with four sides fixed are expressed as follows:

$$\begin{aligned}
W(0,\eta) &= 0, \\
W(1,\eta) &= 0, \\
W(\zeta,0) &= 0, \\
W(\zeta,1) &= 0.
\end{aligned} \tag{13}$$

3. Establishment of Complex Characteristic Equation

The principle of DQM [27] is using the weighted sum of the function value of all nodes on the whole domain to replace the value of the function and its derivative at a given node, so the differential equations are turned into algebraic equations which regard the function value at the node as unknown. Introduce $N \times N$ grid points according to DQM. They are

$$\begin{aligned}
\zeta_1 &= 0, \\
\zeta_2 &= \delta, \\
\zeta_{N-1} &= 1-\delta, \\
\zeta_N &= 1, \\
\zeta_i &= \frac{1}{2}\left[1 - \cos\frac{(i-1)\pi}{N-1}\right] \quad (i = 3, 4, \ldots, N-2), \\
\eta_1 &= 0, \\
\eta_2 &= \delta, \\
\eta_{N-1} &= 1-\delta,
\end{aligned}$$

$$\eta_N = 1,$$
$$\eta_j = \frac{1}{2}\left[1 - \cos\frac{(2j-3)\pi}{2N-4}\right] \quad (j = 3, 4, \ldots, N-2). \tag{14}$$

The value of each of the order partial derivatives of the unknown function can be described as

$$\begin{aligned}
&(c^2-1)\sum_{k=1}^{N} A_{ik}^{[2]} W_{kj} - \lambda\mu^2 \sum_{k=1}^{N} B_{jk}^{[2]} W_{ik} \\
&+ 2cj_1\omega \sum_{k=1}^{N} A_{ik}^{[1]} W_{kj} - \omega^2 W_{ij} + 2c^2\alpha\eta \sum_{k=1}^{N} A_{ik}^{[2]} W_{kj} \\
&+ 4c\alpha\eta j_1\omega \sum_{k=1}^{N} A_{ik}^{[1]} W_{kj} - 2\alpha\eta\omega^2 W_{ij} = 0 \quad \left(0 \le \eta \le \frac{1}{2}\right), \\
&(c^2-1)\sum_{l=1}^{M} A_{il}^{[2]} W_{lj} - \lambda\mu^2 \sum_{l=1}^{M} B_{jl}^{[2]} W_{il} \\
&+ 2cj_1\omega \sum_{l=1}^{M} A_{il}^{[1]} W_{lj} - \omega^2 W_{ij} + 2c^2\alpha \sum_{l=1}^{M} A_{il}^{[2]} W_{lj} \\
&+ 4c\alpha j_1\omega \sum_{l=1}^{M} A_{il}^{[1]} W_{lj} - 2\alpha\omega^2 W_{ij} \\
&- 2c^2\alpha\eta \sum_{l=1}^{M} A_{il}^{[2]} W_{lj} - 4c\alpha\eta j_1\omega \sum_{l=1}^{M} A_{il}^{[1]} W_{lj} \\
&+ 2\alpha\eta\omega^2 W_{ij} = 0 \quad \left(\frac{1}{2} \le \eta \le 1\right).
\end{aligned} \tag{15}$$

The weight coefficient of each order can be acquired by

$$A_{ij} = l_j'(x_i) = \begin{cases} \dfrac{\prod_{k=1,k\ne i,j}^{N}(x_i - x_k)}{\prod_{k=1,k\ne j}^{N}(x_j - x_k)} & (i \ne j) \\ \displaystyle\prod_{\substack{k=1 \\ k\ne i}}^{N} \frac{1}{x_i - x_k} & (i = j) \end{cases} \tag{16}$$

$$A_{ij}^{[k]} = l_j^{[k]}(x_i) = \begin{cases} k\left[l_i^{[k-1]}(x_i)\, l_j'(x_i) - \dfrac{l_i^{[k-1]}(x_i)}{x_i - x_j}\right] & (i \ne j) \\ -\displaystyle\sum_{\substack{m=1 \\ m\ne i}}^{N} l_m^{[k]}(x_i) & (i = j) \end{cases} \quad (2 \le k \le N-1). \tag{17}$$

The complex characteristic equation is established by using DQM. Then (15) is turned into differential quadrature form:

$$\begin{aligned}
&(c^2-1)\sum_{k=1}^{N} A_{ik}^{[2]} W_{kj} - \lambda\mu^2 \sum_{k=1}^{N} B_{jk}^{[2]} W_{ik} \\
&+ 2cj_1\omega \sum_{k=1}^{N} A_{ik}^{[1]} W_{kj} - \omega^2 W_{ij} + 2c^2\alpha\eta \sum_{k=1}^{N} A_{ik}^{[2]} W_{kj} \\
&+ 4c\alpha\eta j_1\omega \sum_{k=1}^{N} A_{ik}^{[1]} W_{kj} - 2\alpha\eta\omega^2 W_{ij} = 0 \quad \left(0 \le \eta \le \frac{1}{2}\right), \\
&(c^2-1)\sum_{l=1}^{M} A_{il}^{[2]} W_{lj} - \lambda\mu^2 \sum_{l=1}^{M} B_{jl}^{[2]} W_{il} \\
&+ 2cj_1\omega \sum_{l=1}^{M} A_{il}^{[1]} W_{lj} - \omega^2 W_{ij} + 2c^2\alpha \sum_{l=1}^{M} A_{il}^{[2]} W_{lj} \\
&+ 4c\alpha j_1\omega \sum_{l=1}^{M} A_{il}^{[1]} W_{lj} - 2\alpha\omega^2 W_{ij} \\
&- 2c^2\alpha\eta \sum_{l=1}^{M} A_{il}^{[2]} W_{lj} - 4c\alpha\eta j_1\omega \sum_{l=1}^{M} A_{il}^{[1]} W_{lj} \\
&+ 2\alpha\eta\omega^2 W_{ij} = 0 \quad \left(\frac{1}{2} \le \eta \le 1\right).
\end{aligned} \tag{18}$$

The boundary conditions are

$$\begin{aligned} W_{1j} &= W_{Nj} = 0 \quad (j = 1, 2, \ldots, N), \\ W_{i1} &= W_{iN} = 0 \quad (i = 1, 2, \ldots, N). \end{aligned} \tag{19}$$

Converting (18) and (19) to matrix form obtains

$$\left|\omega^2\mathbf{R} + \omega\mathbf{G} + \mathbf{K}\right| = 0. \tag{20}$$

Matrix **R**, **G**, **K** contains tension ratio λ, dimensionless velocity c, density coefficient α, aspect ratio μ, and other parameters.

4. Numerical Results and Analysis

When the density coefficient $\alpha = 0$, the vibration equation of moving membrane degenerates the transverse vibration equation in which density distribution is uniform. To illustrate the effectiveness of the differential quadrature method, assuming density coefficient $\alpha = 0$, dimensionless speed $c = 0$, tension ratio $\lambda = 1$, and aspect ratio $\mu = 1$, $\mu = 2$, respectively, the results comparison between this study and [28] is shown in Table 1. Then assuming density coefficient $\alpha = 0$, dimensionless velocity $c = 0$, aspect ratio $\mu = 2$, and tension ratio $\lambda = 0.5$, $\lambda = 0.8$, respectively, the results

TABLE 1: Comparisons of the transverse vibration frequency with those in [28] ($\lambda = 1$).

Aspect ratio μ	1.0		2.0	
Frequency order	This study	Ref. [28]	This study	Ref. [28]
1	4.4429	4.4429	7.0248	7.0248
2	7.0251	7.0248	8.8896	8.8858
3	8.8862	8.8858	12.9636	12.9271

TABLE 2: Comparisons of the transverse vibration frequency with those in [28] ($\mu = 2$).

Tension ratio λ	0.5		0.8	
Frequency order	This study	Ref. [28]	This study	Ref. [28]
1	5.4414	5.4414	6.4383	6.4383
2	7.6997	7.6953	8.4338	8.4298
3	9.4319	9.4247	10.9043	10.9731

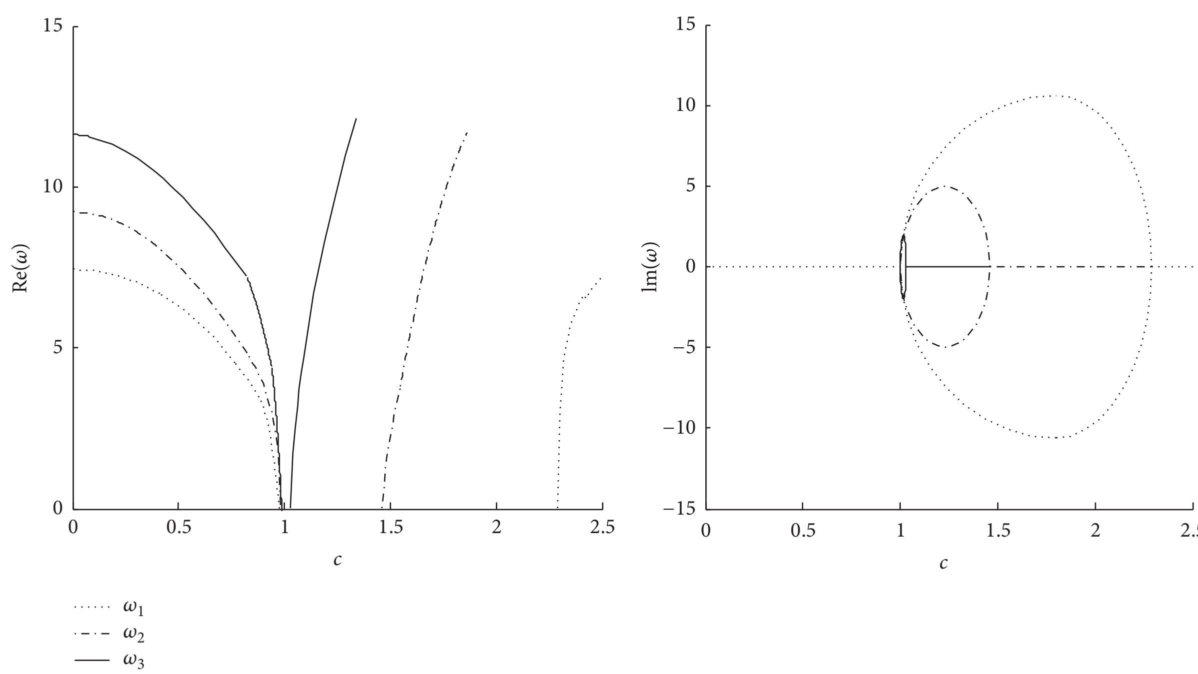


FIGURE 6: Dimensionless complex frequency varied with dimensionless velocity ($\lambda = 1$, $\mu = 2$, $\alpha = 0$).

comparison between this study and [28] is shown in Table 2. It can be seen from Tables 1 and 2 that the degenerated results in this study are in good consistency with the results in [28]. Therefore, it proves that the method used in this paper is feasible and believable.

Figures 6 and 7 show the relationship between first three dimensionless natural frequencies and the dimensionless speed, when the density coefficient $\alpha = 0$, the aspect ratio $\mu = 2$, and the tension ratio $\lambda = 1$ and $\lambda = 0.5$, respectively. As illustrated in Figure 6, the dimensionless speed increases, the real part $\mathrm{Re}(\omega)$ of the first three order dimensionless complex frequencies tends to decrease gradually. When $0 < c < 1$, the first three dimensionless complex frequencies diminish with the increase of dimensionless speed, and the imaginary part of the complex frequencies is zero consistently, so the membrane works in steady state. When the dimensionless speed $c = 1$, the real part of the complex frequencies becomes zero simultaneously; here, the dimensionless speed $c = 1$ is called the critical speed c_r. When $c > 1$, the imaginary part of the complex frequencies is not zero anymore, and the

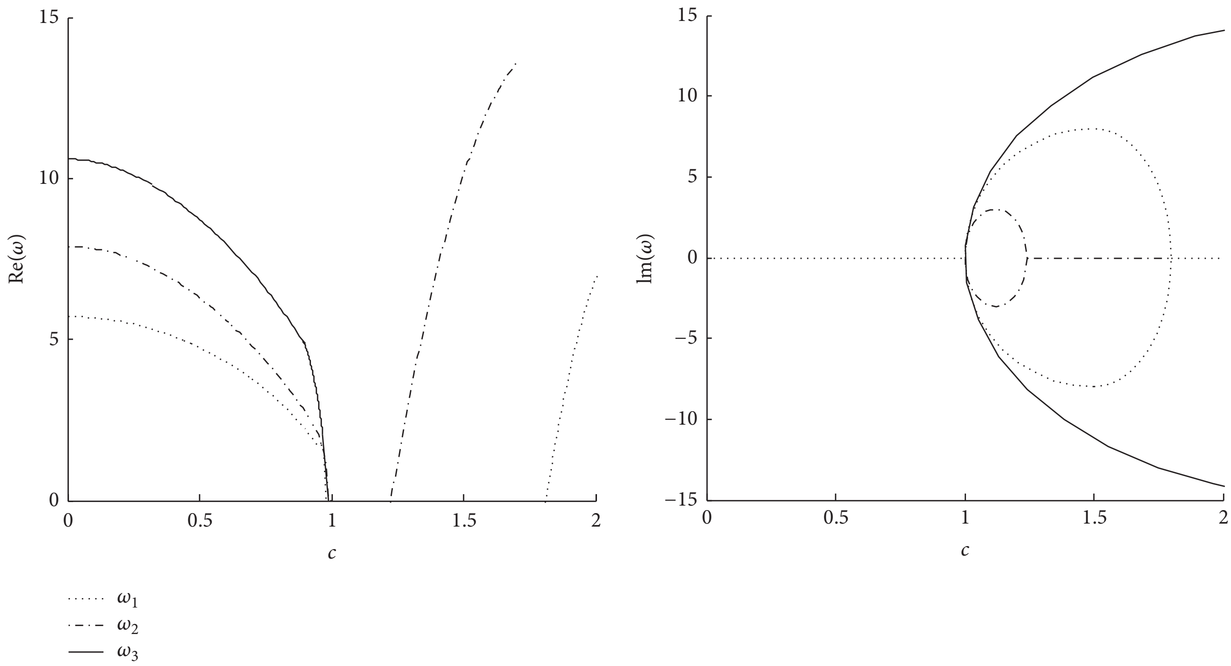


Figure 7: Dimensionless complex frequency varied with dimensionless velocity ($\lambda = 0.5$, $\mu = 2$, $\alpha = 0$).

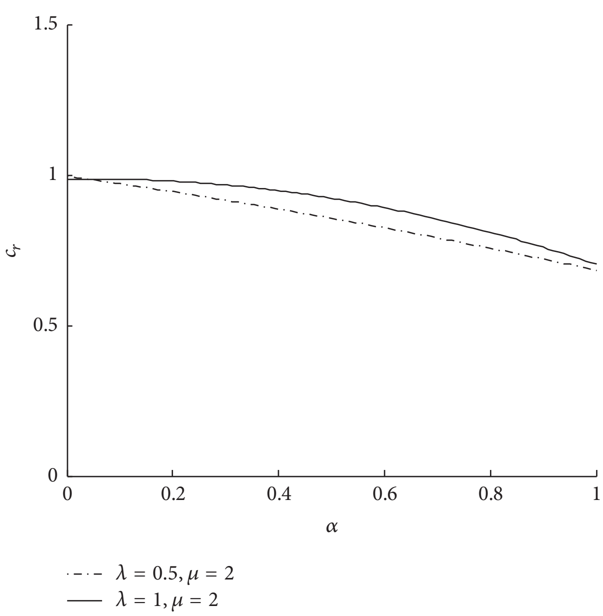


Figure 8: Relationship between density coefficient and critical speed.

membrane begins to be in a divergent instability state. Compared with Figure 7, for any same dimensionless speed in stable work region, the dimensionless complex frequency decreases gradually with the decrease of tension ratio.

Figure 8 shows the relationship between density coefficient α and critical speed c_r, when $\lambda = 1$, $\mu = 2$ and $\lambda = 0.5$, and $\mu = 2$, respectively. It is found from the figure that the critical speed diminishes with the increase of density coefficient. The critical speed is $c_r = 1$ for the different tension ratio when $\alpha = 0$. When $\alpha > 0$, the density coefficient is fixed, and the smaller the tension ratio, the smaller the critical speed.

Figure 9 shows the relationship between the first three dimensionless natural frequencies and the density coefficient when $\lambda = 1$, $\mu = 2$ and $c = 0$, $c = 0.5$, respectively. The results show that the real part of the dimensionless

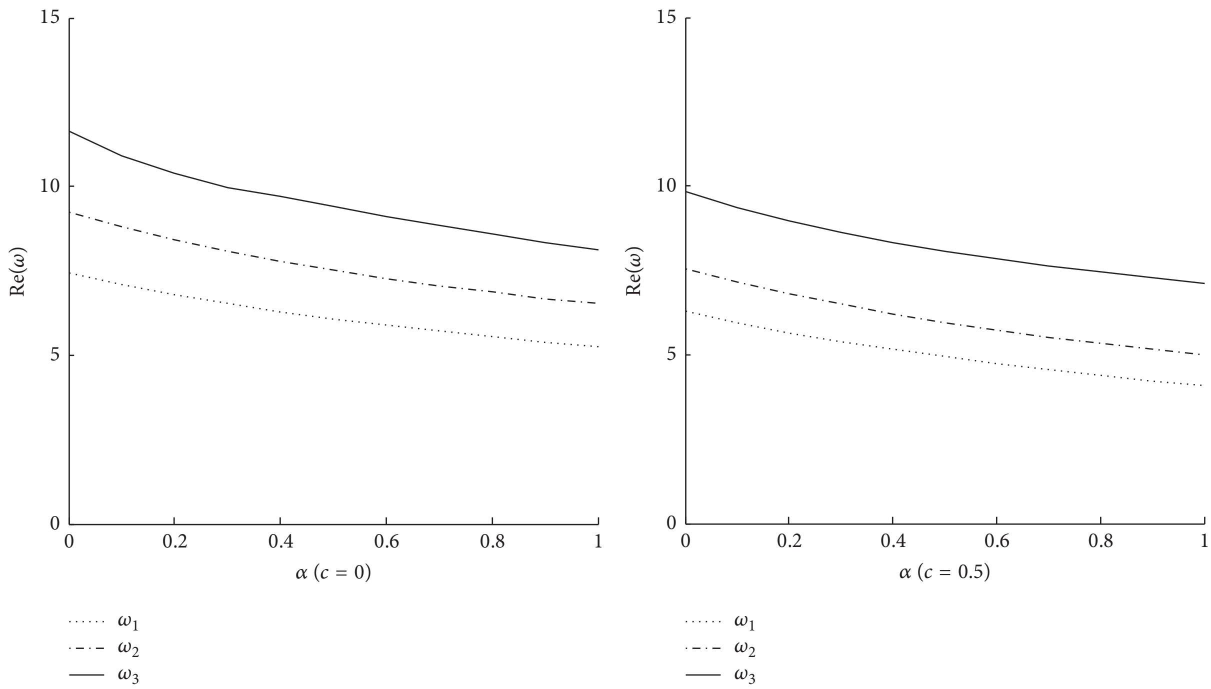


FIGURE 9: Relationship between density coefficient and dimensionless complex frequency ($\lambda = 1$, $\mu = 2$).

complex frequency is decreasing with the increasing density coefficient and the real part of the dimensionless complex frequency is decreasing overall when the dimensionless velocity increases.

Figures 10(a)–10(d) show the relationship between the first three dimensionless natural frequencies and the tension ratio when ($\mu = 2, c = 0, \alpha = 0$), ($\mu = 2, c = 0.5, \alpha = 0$), ($\mu = 2, c = 0, \alpha = 0.8$), and ($\mu = 2, c = 0.5, \alpha = 0.8$), respectively. Comparative analysis shows that, for any order dimensionless complex frequency when dimensionless speed $c = 0$, the vibration frequency of the moving printing membrane increases with the increase of the tension ratio; the vibration frequency is downward with increasing dimensionless velocity; the real part of dimensionless complex frequency decreases with the increasing of density coefficient when the dimensionless velocity is the same. In this process, the imaginary part of complex frequency $\text{Im}(\omega) = 0$, so the motion system is in a stable state.

5. Conclusions

The DQM is used to analyze the transverse vibration characteristics of moving membrane with variable density distribution along the lateral direction. The results show that the tension ratio and the density coefficient have important impacts on the stability of moving membrane. The conclusions are as follows:

(1) When $\alpha = 0$, for the same aspect ratio, with the increase of tension ratio, the motion system is in a stable state in region $0 < c < 1$. When the actual speed reaches the critical speed ($c_r = 1$), the motion system comes into a divergent instability state. The tension ratio of the membrane has no influence on the critical speed.

(2) When density coefficient $\alpha > 0$, for the same aspect ratio, with the decrease of tension ratio, the critical speed is becoming smaller, the stable work region of motion system is smaller too. For any same dimensionless speed of stable work region, with the decrease of tension ratio, the dimensionless complex frequencies tend to be smaller and smaller. The tension ratio has important influence on the stability of moving membrane.

(3) The density coefficient α has an influence on the critical speed and the dimensionless natural frequencies. The critical speed and the natural frequencies diminish with the increasing density coefficient. Tension ratio does not affect the change trends of critical speed with density coefficient, but it affects the critical speed value.

The study provides a theoretical basis for improving the working stability of the membrane in the high-speed printing process.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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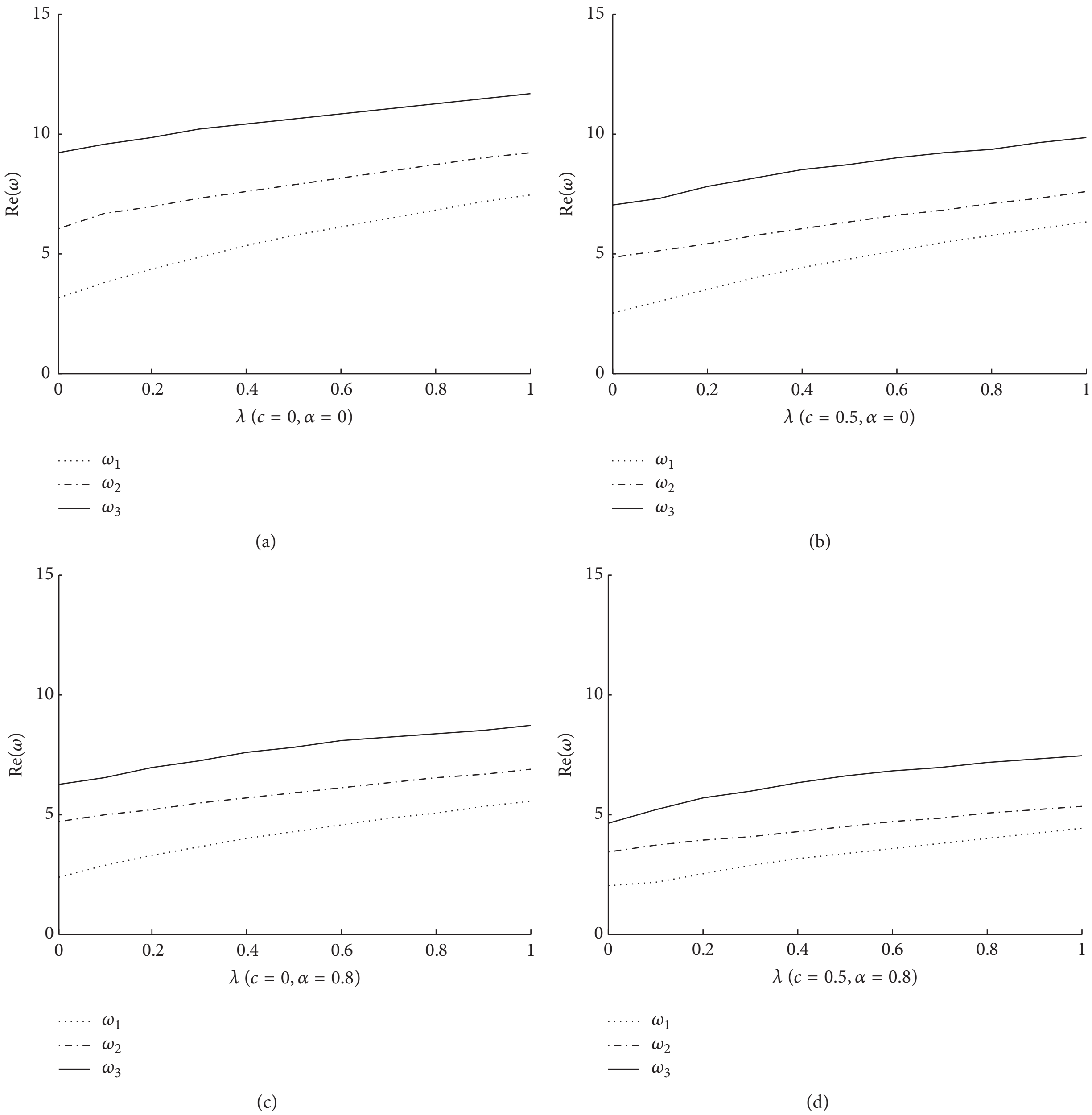

Figure 10: Relationship between tension ratio and dimensionless complex frequency ($\mu = 2$).

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