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Research Article

On the Solutions of a General System of Difference Equations

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We deal with the solutions of the systems of the difference equations $x_{n+1} = 1/x_{n-p}y_{n-p}$, $y_{n+1} = x_{n-p}y_{n-p}/x_{n-q}y_{n-q}$, and $x_{n+1} = 1/x_{n-p}y_{n-p}z_{n-p}$, $y_{n+1} = x_{n-p}y_{n-p}z_{n-p}/x_{n-q}y_{n-q}z_{n-q}$, $z_{n+1} = x_{n-q}y_{n-q}z_{n-q}/x_{n-r}y_{n-r}z_{n-r}$, with a nonzero real numbers initial conditions. Also, the periodicity of the general system of k variables will be considered.

1. Introduction

In this paper, we deal with the solutions of the systems of the difference equations

$$x_{n+1} = \frac{1}{x_{n-p}y_{n-p}}, \qquad y_{n+1} = \frac{x_{n-p}y_{n-p}}{x_{n-q}y_{n-q}},$$
$$x_{n+1} = \frac{1}{x_{n-p}y_{n-p}z_{n-p}}, \qquad y_{n+1} = \frac{x_{n-p}y_{n-p}z_{n-p}}{x_{n-q}y_{n-q}z_{n-q}}, \qquad z_{n+1} = \frac{x_{n-q}y_{n-q}z_{n-q}}{x_{n-r}y_{n-r}z_{n-r}}, \tag{1.1}$$

with a nonzero real numbers initial conditions. Also, the periodicity of the general system of k variables will be considered.

Recently, there has been a great interest in studying nonlinear difference equations and systems (cf. [1–14] and the references therein). One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real-life situations in population biology, economy, probability theory, genetics, psychology, sociology, and so forth. Such equations also appear naturally as discrete analogues of differential equations which model various biological and economical systems.

Cinar [3] has obtained the positive solution of the difference equation system:

$$x_{n+1} = \frac{1}{y_n}, \qquad y_{n+1} = \frac{y_n}{x_{n-1}y_{n-1}}. \tag{1.2}$$

Also, Çinar and Yalçinkaya [4] have obtained the positive solution of the difference equation system:

$$x_{n+1} = \frac{1}{z_n}, \qquad y_{n+1} = \frac{x_n}{x_{n-1}}, \qquad z_{n+1} = \frac{1}{x_{n-1}}. \tag{1.3}$$

Clark and Kulenović [5] have investigated the global stability properties and asymptotic behavior of solutions of the system

$$x_{n+1} = \frac{x_n}{a + cy_n}, \qquad y_{n+1} = \frac{y_n}{b + dx_n}. \tag{1.4}$$

Elabbasy et al. [6] have obtained the solution of particular cases of the following general system of difference equations:

$$\begin{gathered} x_{n+1} = \frac{a_1 + a_2 y_n}{a_3 z_n + a_4 x_{n-1} z_n}, \qquad y_{n+1} = \frac{b_1 z_{n-1} + b_2 z_n}{b_3 x_n y_n + b_4 x_n y_{n-1}}, \\ z_{n+1} = \frac{c_1 z_{n-1} + c_2 z_n}{c_3 x_{n-1} y_{n-1} + c_4 x_{n-1} y_n + c_5 x_n y_n}. \end{gathered} \tag{1.5}$$

Elsayed [10] has obtained the solution of systems of difference equations of rational form.

Also, the behavior of the solutions of the following systems:

$$x_{n+1} = \frac{x_{n-1}}{\pm 1 + x_{n-1} y_n}, \qquad y_{n+1} = \frac{y_{n-1}}{\mp 1 + y_{n-1} x_n}, \tag{1.6}$$

has been studied by Elsayed [15].

Özban [16] has investigated the positive solutions of the system of rational difference equations:

$$x_{n+1} = \frac{1}{y_{n-k}}, \qquad y_{n+1} = \frac{y_n}{x_{n-m} y_{n-m-k}}. \tag{1.7}$$

Özban [17] has investigated the solutions of the following system:

$$x_{n+1} = \frac{a}{y_{n-3}}, \qquad y_{n+1} = \frac{b y_{n-3}}{x_{n-q} y_{n-q}}. \tag{1.8}$$

Yang et al. [18] have investigated the positive solutions of the systems:

$$x_n = \frac{a}{y_{n-p}}, \qquad y_n = \frac{b y_{n-p}}{x_{n-q} y_{n-q}}. \tag{1.9}$$

Similar nonlinear systems of rational difference equations were investigated see [15–32].

Definition 1.1 (Periodicity). A sequence $\{x_n\}_{n=-k}^{\infty}$ is said to be periodic with period p if $x_{n+p} = x_n$ for all $n \geq -k$.

2. Main Results

2.1. The First System

In this section, we deal with the solutions of the system of the difference equations

$$x_{n+1} = \frac{1}{x_{n-p} y_{n-p}}, \qquad y_{n+1} = \frac{x_{n-p} y_{n-p}}{x_{n-q} y_{n-q}}, \tag{2.1}$$

with a nonzero real numbers initial conditions and $p \neq q$.

Theorem 2.1. *Suppose that* $\{x_n, y_n\}$ *are solutions of system* (2.1). *Also, assume that the initial conditions are arbitrary nonzero real numbers. Then all solutions of equation system* (2.1) *are eventually periodic with period* $(2q+2)$.

Proof. From (2.1), we see that

$$\begin{aligned}
x_{n+1} &= \frac{1}{x_{n-p} y_{n-p}}, \qquad y_{n+1} = \frac{x_{n-p} y_{n-p}}{x_{n-q} y_{n-q}}, \\
x_{n+2} &= \frac{1}{x_{n-p+1} y_{n-p+1}}, \qquad y_{n+2} = \frac{x_{n-p+1} y_{n-p+1}}{x_{n-q+1} y_{n-q+1}}, \\
x_{n+3} &= \frac{1}{x_{n-p+2} y_{n-p+2}}, \qquad y_{n+3} = \frac{x_{n-p+2} y_{n-p+2}}{x_{n-q+2} y_{n-q+2}}, \\
&\vdots \\
x_{n-p+q-2} &= \frac{1}{x_{n-2p+q-3} y_{n-2p+q-3}}, \qquad y_{n-p+q-2} = \frac{x_{n-2p+q-3} y_{n-2p+q-3}}{x_{n-p-3} y_{n-p-3}}, \\
x_{n-p+q-1} &= \frac{1}{x_{n-2p+q-2} y_{n-2p+q-2}}, \qquad y_{n-p+q-1} = \frac{x_{n-2p+q-2} y_{n-2p+q-2}}{x_{n-p-2} y_{n-p-2}}, \\
x_{n-p+q} &= \frac{1}{x_{n-2p+q-1} y_{n-2p+q-1}}, \qquad y_{n-p+q} = \frac{x_{n-2p+q-1} y_{n-2p+q-1}}{x_{n-p-1} y_{n-p-1}},
\end{aligned}$$

$$x_{n-p+q+1}=\frac{1}{x_{n-2p+q}y_{n-2p+q}},\qquad y_{n-p+q+1}=\frac{x_{n-2p+q}y_{n-2p+q}}{x_{n-p}y_{n-p}},$$

$$x_{n-p+q+2}=\frac{1}{x_{n-2p+q+1}y_{n-2p+q+1}},\qquad y_{n-p+q+2}=\frac{x_{n-2p+q+1}y_{n-2p+q+1}}{x_{n-p+1}y_{n-p+1}},$$

$$\vdots$$

$$x_{n-p+2q-1}=\frac{1}{x_{n-2p+2q-2}y_{n-2p+2q-2}},\qquad y_{n-p+2q-1}=x_{n-2p+2q-2}y_{n-2p+2q-2}x_{n-p-3}y_{n-p-3},$$

$$x_{n-p+2q}=\frac{1}{x_{n-2p+2q-1}y_{n-2p+2q-1}},\qquad y_{n-p+2q}=x_{n-2p+2q-1}y_{n-2p+2q-1}x_{n-p-2}y_{n-p-2},$$

$$x_{n-p+2q+1}=\frac{1}{x_{n-2p+q}y_{n-2p+q}},\qquad y_{n-p+2q+1}=x_{n-2p+2q}y_{n-2p+2q}x_{n-p-1}y_{n-p-1},$$

$$x_{n-p+2q+2}=\frac{1}{x_{n-2p+q+1}y_{n-2p+q+1}},\qquad y_{n-p+2q+2}=x_{n-2p+2q+1}y_{n-2p+2q+1}x_{n-p}y_{n-p},$$

$$\vdots$$

$$x_{n+q-1}=\frac{1}{x_{n-p+q-2}y_{n-p+q-2}},\qquad y_{n+q-1}=\frac{x_{n-p+q-2}y_{n-p+q-2}}{x_{n-2}y_{n-2}},$$

$$x_{n+q}=\frac{1}{x_{n-p+q-1}y_{n-p+q-1}},\qquad y_{n+q}=\frac{x_{n-p+q-1}y_{n-p+q-1}}{x_{n-1}y_{n-1}},$$

$$x_{n+q+1}=\frac{1}{\mathrm{x}_{n-p+q}y_{n-p+q}},\qquad y_{n+q+1}=\frac{x_{n-p+q}y_{n-p+q}}{x_ny_n},$$

$$x_{n+q+2}=\frac{1}{x_{n-p+q+1}y_{n-p+q+1}},\qquad y_{n+q+2}=\frac{x_{n-p+q+1}y_{n-p+q+1}}{x_{n+1}y_{n+1}}=x_{n-p+q+1}y_{n-p+q+1}x_{n-q}y_{n-q},$$

$$x_{n+q+3}=\frac{1}{x_{n-p+q+2}y_{n-p+q+2}},\qquad y_{n+q+3}=\frac{x_{n-\mathrm{p}+q+2}y_{n-p+q+2}}{x_{n+2}y_{n+2}}=x_{n-p+q+2}y_{n-p+q+2}x_{n-q+1}y_{n-q+1},$$

$$x_{n+q+4}=\frac{1}{x_{n-p+q+3}y_{n-p+q+3}},\qquad y_{n+q+4}=\frac{x_{n-p+q+3}y_{n-p+q+3}}{x_{n+3}y_{n+3}}=x_{n-p+q+3}y_{n-p+q+3}x_{n-q+2}y_{n-q+2},$$

$$\vdots$$

$$x_{n+2q}=\frac{1}{x_{n-p-3}y_{n-p-3}},\qquad y_{n+2q}=x_{n-p-3}y_{n-p-3}x_{n-2}y_{n-2},$$

$$x_{n+2q+1}=\frac{1}{x_{n-p-2}y_{n-p-2}},\qquad y_{n+2q+1}=x_{n-p-2}y_{n-p-2}x_{n-1}y_{n-1},$$

$$x_{n+2q+2}=\frac{1}{x_{n-p-1}y_{n-p-1}},\qquad y_{n+2q+2}=x_{n-p-1}y_{n-p-1}x_ny_n,$$

$$x_{n+2q+3}=\frac{1}{x_{n-p+2q+2}y_{n-p+2q+2}}=x_{n+1},\qquad y_{n+2q+3}=\frac{x_{n-p+2q+2}y_{n-p+2q+2}}{x_{n+q+2}y_{n+q+2}}=y_{n+1},$$

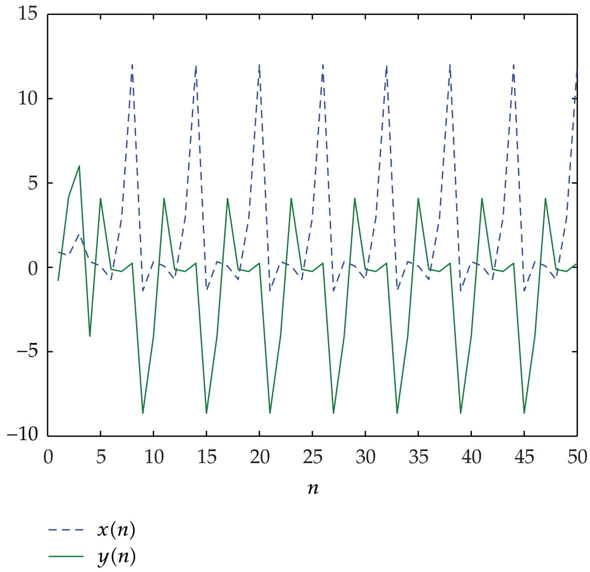

Figure 1

$$x_{n+2q+4} = \frac{1}{x_{n-p+2q+3}y_{n-p+2q+3}} = x_{n+2},$$
$$y_{n+2q+4} = \frac{x_{n-p+2q+3}y_{n-p+2q+3}}{x_{n+q+3}y_{n+q+3}} = y_{n+2}. \tag{2.2}$$

Hence, the proof is completed. □

Numerical Example

In order to illustrate the results of this section and to support our theoretical discussions, we consider the following numerical example.

Example 2.2. Consider the difference system (2.1) with $p = 1$, $q = 2$, and the initial conditions $x_{-2} = 0.9$, $x_{-1} = 0.7$, $x_0 = 2$, $y_{-2} = -0.8$, $y_{-1} = 4.2$, and $y_0 = 6$. (See Figure 1).

2.2. The Second System

In this section, we deal with the solutions of the system of the difference equations

$$x_{n+1} = \frac{1}{x_{n-p}y_{n-p}z_{n-p}}, \qquad y_{n+1} = \frac{x_{n-p}y_{n-p}z_{n-p}}{x_{n-q}y_{n-q}z_{n-q}}, \qquad z_{n+1} = \frac{x_{n-q}y_{n-q}z_{n-q}}{x_{n-r}y_{n-r}z_{n-r}}, \tag{2.3}$$

with a nonzero real numbers initial conditions and $p \neq q$, $q \neq r$.

Theorem 2.3. *Suppose that* $\{x_n, y_n, z_n\}$ *are solutions of system* (2.3). *Also, assume that the initial conditions are arbitrary nonzero real numbers. Then all solutions of equation system* (2.3) *are eventually periodic with period* $(2r+2)$.

Proof. From (2.3), we see that

$$x_{n+1} = \frac{1}{x_{n-p}y_{n-p}z_{n-p}}, \qquad y_{n+1} = \frac{x_{n-p}y_{n-p}z_{n-p}}{x_{n-q}y_{n-q}z_{n-q}}, \qquad z_{n+1} = \frac{x_{n-q}y_{n-q}z_{n-q}}{x_{n-r}y_{n-r}z_{n-r}},$$

$$x_{n+2} = \frac{1}{x_{n-p+1}y_{n-p+1}z_{n-p+1}}, \qquad y_{n+2} = \frac{x_{n-p+1}y_{n-p+1}z_{n-p+1}}{x_{n-q+1}y_{n-q+1}z_{n-q+1}}, \qquad z_{n+2} = \frac{x_{n-q+1}y_{n-q+1}z_{n-q+1}}{x_{n-r+1}y_{n-r+1}z_{n-r+1}},$$

$$x_{n+3} = \frac{1}{x_{n-p+2}y_{n-p+2}z_{n-p+2}}, \qquad y_{n+3} = \frac{x_{n-p+2}y_{n-p+2}z_{n-p+2}}{x_{n-q+2}y_{n-q+2}z_{n-q+2}}, \qquad z_{n+3} = \frac{x_{n-q+2}y_{n-q+2}z_{n-q+2}}{x_{n-r+2}y_{n-r+2}z_{n-r+2}},$$

$$\vdots$$

$$x_{n-p+r-1} = \frac{1}{x_{n-2p+r-2}y_{n-2p+r-2}z_{n-2p+r-2}},$$

$$y_{n-p+r-1} = \frac{x_{n-2p+r-2}y_{n-2p+r-2}z_{n-2p+r-2}}{x_{n-q-p+r-2}y_{n-q-p+r-2}z_{n-q-p+r-2}},$$

$$z_{n-p+r-1} = \frac{x_{n-q-p+r-2}y_{n-q-p+r-2}z_{n-q-p+r-2}}{x_{n-p-2}y_{n-p-2}z_{n-p-2}},$$

$$x_{n-p+r} = \frac{1}{x_{n-2p+r-1}y_{n-2p+r-1}z_{n-2p+r-1}},$$

$$y_{n-p+r} = \frac{x_{n-2p+r-1}y_{n-2p+r-1}z_{n-2p+r-1}}{x_{n-q-p+r-1}y_{n-q-p+r-1}z_{n-q-p+r-1}},$$

$$z_{n-p+r} = \frac{x_{n-q-p+r-1}y_{n-q-p+r-1}z_{n-q-p+r-1}}{x_{n-p-1}y_{n-p-1}z_{n-p-1}},$$

$$x_{n-p+r+1} = \frac{1}{x_{n-2p+r}y_{n-2p+r}z_{n-2p+r}},$$

$$y_{n-p+r+1} = \frac{x_{n-2p+r}y_{n-2p+r}z_{n-2p+r}}{x_{n-q-p+r}y_{n-q-p+r}z_{n-q-p+r}},$$

$$z_{n-p+r+1} = \frac{x_{n-q-p+r}y_{n-q-p+r}z_{n-q-p+r}}{x_{n-p}y_{n-p}z_{n-p}},$$

$$x_{n-p+r+2} = \frac{1}{x_{n-2p+r+1}y_{n-2p+r+1}z_{n-2p+r+1}},$$

$$y_{n-p+r+2} = \frac{x_{n-p+r+1}y_{n-p+r+1}z_{n-p+r+1}}{x_{n-q-p+r+1}y_{n-q-p+r+1}z_{n-q-p+r+1}},$$

$$z_{n-p+r+2} = \frac{x_{n-q-p+r+1}y_{n-q-p+r+1}z_{n-q-p+r+1}}{x_{n-p+1}y_{n-p+1}z_{n-p+1}},$$

$$
\begin{aligned}
x_{n-p+r+3} &= \frac{1}{x_{n-2p+r+2}y_{n-2p+r+2}z_{n-2p+r+2}},\\
y_{n-p+r+3} &= \frac{x_{n-p+r+2}y_{n-p+r+2}z_{n-p+r+2}}{x_{n-q-p+r+2}y_{n-q-p+r+2}z_{n-q-p+r+2}},\\
z_{n-p+r+3} &= \frac{x_{n-q-p+r+2}y_{n-q-p+r+2}z_{n-q-p+r+2}}{x_{n-p+2}y_{n-p+2}z_{n-p+2}},\\
&\ \vdots\\
x_{n-p+2r} &= \frac{1}{x_{n-2p+2r-1}y_{n-2p+2r-1}z_{n-2p+2r-1}},\\
y_{n-p+2r} &= \frac{x_{n-2p+2r-1}y_{n-2p+2r-1}z_{n-2p+2r-1}}{x_{n-q-p+2r-1}y_{n-q-p+2r-1}z_{n-q-p+2r-1}},\\
z_{n-p+2r} &= \frac{x_{n-q-p+2r-1}y_{n-q-p+2r-1}z_{n-q-p+2r-1}}{x_{n-p+r-1}y_{n-p+r-1}z_{n-p+r-1}}\\
&= x_{n-q-p+2r}y_{n-q-p+2r}z_{n-q-p+2r}x_{n-p-2}y_{n-p-2}z_{n-p-2},\\
x_{n-p+2r+1} &= \frac{1}{x_{n-2p+2r}y_{n-2p+2r}z_{n-2p+2r}},\\
y_{n-p+2r+1} &= \frac{x_{n-2p+2r}y_{n-2p+2r}z_{n-2p+2r}}{x_{n-q-p+2r}y_{n-q-p+2r}z_{n-q-p+2r}},\\
z_{n-p+2r+1} &= \frac{x_{n-q-p+2r}y_{n-q-p+2r}z_{n-q-p+2r}}{x_{n-p+r}y_{n-p+r}z_{n-p+r}}\\
&= x_{n-q-p+2r}y_{n-q-p+2r}z_{n-q-p+2r}x_{n-p-1}y_{n-p-1}z_{n-p-1},\\
x_{n-p+2r+2} &= \frac{1}{x_{n-2p+2r+1}y_{n-2p+2r+1}z_{n-2p+2r+1}},\\
y_{n-p+2r+2} &= \frac{x_{n-p+2r+1}y_{n-p+2r+1}z_{n-p+2r+1}}{x_{n-q-p+2r+1}y_{n-q-p+2r+1}z_{n-q-p+2r+1}},\\
z_{n-p+2r+2} &= \frac{x_{n-q-p+2r+1}y_{n-q-p+2r+1}z_{n-q-p+2r+1}}{x_{n-p+r+1}y_{n-p+r+1}z_{n-p+r+1}}\\
&= x_{n-q-p+2r+1}y_{n-q-p+2r+1}z_{n-q-p+2r+1}x_{n-p}y_{n-p}z_{n-p},\\
x_{n-p+2r+3} &= \frac{1}{x_{n-2p+2r+2}y_{n-2p+2r+2}z_{n-2p+2r+2}},\\
y_{n-p+2r+3} &= \frac{x_{n-p+2r+2}y_{n-p+2r+2}z_{n-p+2r+2}}{x_{n-q-p+2r+2}y_{n-q-p+2r+2}z_{n-q-p+2r+2}},\\
z_{n-p+2r+3} &= \frac{x_{n-q-p+2r+2}y_{n-q-p+2r+2}z_{n-q-p+2r+2}}{x_{n-p+r+2}y_{n-p+r+2}z_{n-p+r+2}}\\
&= x_{n-q-p+2r+2}y_{n-q-p+2r+2}z_{n-q-p+2r+2}x_{n-p+1}y_{n-p+1}z_{n-p+1},\\
&\ \vdots
\end{aligned}
$$

$$x_{n-q+r-1} = \frac{1}{x_{n-p-q+r-2}y_{n-p-q+r-2}z_{n-p-q+r-2}},$$

$$y_{n-q+r-1} = \frac{x_{n-p-q+r-2}y_{n-p-q+r-2}z_{n-p-q+r-2}}{x_{n-2q+r-2}y_{n-2q+r-2}z_{n-q-p+r-2}},$$

$$z_{n-q+r-1} = \frac{x_{n-2q+r-2}y_{n-2q+r-2}z_{n-q-p+r-2}}{x_{n-q-2}y_{n-q-2}z_{n-q-2}},$$

$$x_{n-q+r} = \frac{1}{x_{n-p-q+r-1}y_{n-p-q+r-1}z_{n-p-q+r-1}},$$

$$y_{n-q+r} = \frac{x_{n-p-q+r-1}y_{n-p-q+r-1}z_{n-p-q+r-1}}{x_{n-2q+r-1}y_{n-2q+r-1}z_{n-2q+r-1}},$$

$$z_{n-q+r} = \frac{x_{n-2q+r-1}y_{n-2q+r-1}z_{n-2q+r-1}}{x_{n-q-1}y_{n-q-1}z_{n-q-1}},$$

$$x_{n-q+r+1} = \frac{1}{x_{n-p-q+r}y_{n-p-q+r}z_{n-p-q+r}},$$

$$y_{n-q+r+1} = \frac{x_{n-p-q+r}y_{n-p-q+r}z_{n-p-q+r}}{x_{n-2q+r}y_{n-2q+r}z_{n-2q+r}},$$

$$z_{n-q+r+1} = \frac{x_{n-2q+r}y_{n-2q+r}z_{n-2q+r}}{x_{n-q}y_{n-q}z_{n-q}},$$

$$x_{n-q+r+2} = \frac{1}{x_{n-p-q+r+1}y_{n-p-q+r+1}z_{n-p-q+r+1}},$$

$$y_{n-q+r+2} = \frac{x_{n-p-q+r+1}y_{n-p-q+r+1}z_{n-p-q+r+1}}{x_{n-2q+r+1}y_{n-2q+r+1}z_{n-2q+r+1}},$$

$$z_{n-q+r+2} = \frac{x_{n-2q+r+1}y_{n-2q+r+1}z_{n-2q+r+1}}{x_{n-q+1}y_{n-q+1}z_{n-q+1}},$$

$$x_{n-q+r+3} = \frac{1}{x_{n-p-q+r+2}y_{n-p-q+r+2}z_{n-p-q+r+2}},$$

$$y_{n-q+r+3} = \frac{x_{n-p-q+r+2}y_{n-p-q+r+2}z_{n-p-q+r+2}}{x_{n-2q+r+2}y_{n-2q+r+2}z_{n-2q+r+2}},$$

$$z_{n-q+r+3} = \frac{x_{n-2q+r+2}y_{n-2q+r+2}z_{n-2q+r+2}}{x_{n-q+2}y_{n-q+2}z_{n-q+2}},$$

$$\vdots$$

$$x_{n-q+2r} = \frac{1}{x_{n-p-q+2r-1}y_{n-p-q+2r-1}z_{n-p-q+2r-1}},$$

$$y_{n-q+2r} = \frac{x_{n-p-q+2r-1}y_{n-p-q+2r-1}z_{n-p-q+2r-1}}{x_{n-2q+2r-1}y_{n-2q+2r-1}z_{n-2q+2r-1}},$$

$$z_{n-q+2r} = \frac{x_{n-2q+2r-1}y_{n-2q+2r-1}z_{n-2q+2r-1}}{x_{n-q+r-1}y_{n-q+r-1}z_{n-q+r-1}}$$

$$= x_{n-2q+2r-1}y_{n-2q+2r-1}z_{n-2q+2r-1}x_{n-q-2}y_{n-q-2}z_{n-q-2},$$

$$x_{n-q+2r+1} = \frac{1}{x_{n-p-q+2r}y_{n-p-q+2r}z_{n-p-q+2r}},$$

$$y_{n-q+2r+1} = \frac{x_{n-p-q+2r}y_{n-p-q+2r}z_{n-p-q+2r}}{x_{n-2q+2r}y_{n-2q+2r}z_{n-2q+2r}},$$

$$z_{n-q+2r+1} = \frac{x_{n-2q+2r}y_{n-2q+2r}z_{n-2q+2r}}{x_{n-q+r}y_{\mathrm{n}-q+r}z_{n-q+r}},$$

$$= x_{n-2q+2r}y_{n-2q+2r}z_{n-2q+2r}x_{n-q-1}y_{n-q-1}z_{n-q-1},$$

$$x_{n-q+2r+2} = \frac{1}{x_{n-p-q+2r+1}y_{n-p-q+2r+1}z_{n-p-q+2r+1}},$$

$$y_{n-q+2r+2} = \frac{x_{n-p-q+2r+1}y_{n-p-q+2r+1}z_{n-p-q+2r+1}}{x_{n-2q+2r+1}y_{n-2q+2r+1}z_{n-2q+2r+1}},$$

$$z_{n-q+2r+2} = x_{n-2q+2r+1}y_{n-2q+2r+1}z_{n-2q+2r+1}x_{n-q}y_{n-q}z_{n-q},$$

$$x_{n-q+2r+3} = \frac{1}{x_{n-p-q+2r+2}y_{n-p-q+2r+2}z_{n-p-q+2r+2}},$$

$$y_{n-q+2r+3} = \frac{x_{n-p-q+2r+2}y_{n-p-q+2r+2}z_{n-p-q+2r+2}}{x_{n-2q+2r+2}y_{n-2q+2r+2}z_{n-2\mathrm{q}+2r+2}},$$

$$z_{n-q+2r+3} = x_{n-2q+2r+2}y_{n-2q+2r+2}z_{n-2q+2r+2}x_{n-p+1}y_{n-p+1}z_{n-p+1},$$

$$\vdots$$

$$x_{n+r} = x_{n-p-2}y_{n-p-2}z_{n-p-2}, \qquad y_{n+r} = \frac{x_{n-q-2}y_{n-q-2}z_{n-q-2}}{x_{n-p-2}y_{n-p-2}z_{n-p-2}},$$

$$z_{n+r} = \frac{1}{x_{n-q-2}y_{n-q-2}z_{n-q-2}x_{n-1}y_{n-1}z_{n-1}},$$

$$x_{n+r+1} = x_{n-p-1}y_{n-p-1}z_{n-p-1}, \qquad y_{n+r+1} = \frac{x_{n-q-1}y_{n-q-1}z_{n-q-1}}{x_{n-p-1}y_{n-p-1}z_{n-p-1}},$$

$$z_{n+r+1} = \frac{1}{x_{n-q-1}y_{n-q-1}z_{n-q-1}x_ny_nz_n},$$

$$x_{n+r+2} = x_{n-p}y_{n-p}z_{n-p}, \qquad y_{n+r+2} = \frac{x_{n-q+1}y_{n-q+1}z_{n-q+1}}{x_{n-p}y_{n-p}z_{n-p}},$$

$$z_{n+r+2} = \frac{x_{n-r}y_{n-r}z_{n-r}}{x_{n-q+1}y_{n-q+1}z_{n-q+1}},$$

$$x_{n+r+3} = x_{n-p+1}y_{n-p+1}z_{n-p+1}, \qquad y_{n+r+3} = \frac{x_{n-q+2}y_{n-q+2}z_{n-q+2}}{x_{n-p+1}y_{n-p+1}z_{n-p+1}},$$

$$z_{n+r+3} = \frac{x_{n-r+1}y_{n-r+1}z_{n-r+1}}{x_{n-q+2}y_{n-q+2}z_{n-q+2}},$$

$$x_{n+r+4} = x_{n-p+2}y_{n-p+2}z_{n-p+2}, \qquad y_{n+r+4} = \frac{x_{n-q+3}y_{n-q+3}z_{n-q+3}}{x_{n-p+2}y_{n-p+2}z_{n-p+2}},$$

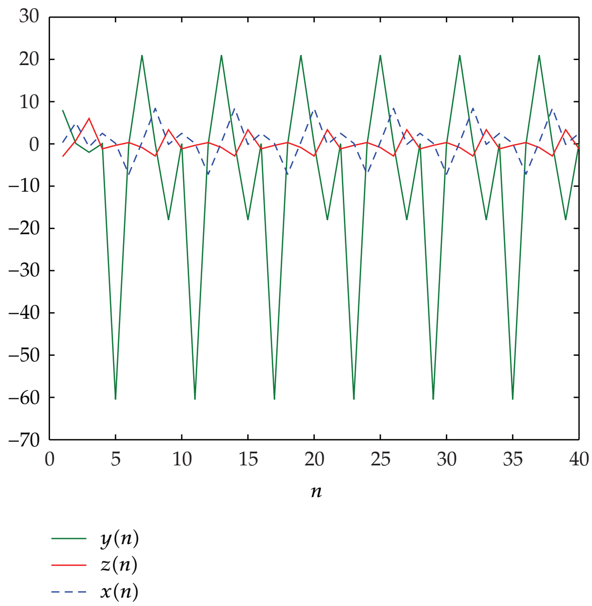

Figure 2

$$
\begin{aligned}
z_{n+r+4} &= \frac{x_{n-r+2}y_{n-r+2}z_{n-r+2}}{x_{n-q+3}y_{n-q+3}z_{n-q+3}}, \\
&\vdots \\
x_{n+2r+1} &= \frac{1}{x_{n-p-2}y_{n-p-2}z_{n-p-2}}, \qquad y_{n+2r+1} = \frac{x_{n-p-2}y_{n-p-2}z_{n-p-2}}{x_{n-q-2}y_{n-q-2}z_{n-q-2}}, \\
z_{n+2r+1} &= \frac{x_{n-q-2}y_{n-q-2}z_{n-q-2}}{x_{n-1}y_{n-1}z_{n-1}}, \\
x_{n+2r+2} &= \frac{1}{x_{n-p-1}y_{n-p-1}z_{n-p-1}}, \qquad y_{n+2r+2} = \frac{x_{n-p-1}y_{n-p-1}z_{n-p-1}}{x_{n-q-1}y_{n-q-1}z_{n-q-1}}, \\
z_{n+2r+2} &= \frac{x_{n-q-1}y_{n-q-1}z_{n-q-1}}{x_n y_n z_n}, \\
x_{n+2r+3} &= x_{n+1}, \qquad y_{n+2r+3} = y_{n+1}, \qquad z_{n+2r+3} = y_{n+1}.
\end{aligned}
\tag{2.4}
$$

Hence, the proof is completed. □

Example 2.4. Consider the difference system (2.3) with $p = 1$, $q = 0$, $r = 2$, and the initial conditions $x_{-2} = 0.3$, $x_{-1} = 5$, $x_0 = -0.7$, $y_{-2} = 8$, $y_{-1} = 0.1$, $y_0 = 2$, $z_{-2} = 3$, $z_{-1} = 0.8$, and $z_0 = 6$ (See Figure 2).

Proposition 2.5. *It is easy to see by induction that the following general system is periodic with period* $(2p_k + 2)$*:*

$$x_{n+1}^{1} = \frac{1}{\prod_{i=1}^{k} x_{n-p_1}^{i}}, \qquad x_{n+1}^{2} = \frac{\prod_{i=1}^{k} x_{n-p_1}^{i}}{\prod_{i=1}^{k} x_{n-p_2}^{i}}, \qquad x_{n+1}^{3} = \frac{\prod_{i=1}^{k} x_{n-p_2}^{i}}{\prod_{i=1}^{k} x_{n-p_3}^{i}}, \ldots,$$
$$x_{n+1}^{j} = \frac{\prod_{i=1}^{k} x_{n-p_{j-1}}^{i}}{\prod_{i=1}^{k} x_{n-p_j}^{i}}, \ldots, x_{n+1}^{k} = \frac{\prod_{i=1}^{k} x_{n-p_{k-1}}^{i}}{\prod_{i=1}^{k} x_{n-p_k}^{i}}. \tag{2.5}$$

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References

[1] N. Battaloglu, C. Cinar, and I. Yalçınkaya, "The dynamics of the difference equation," *Ars Combinatoria*, vol. 97, pp. 281–288, 2010.

[2] E. Camouzis and G. Ladas, *Dynamics of Third-Order Rational Difference Equations with Open Problems and Conjectures*, vol. 5 of *Advances in Discrete Mathematics and Applications*, Chapman & Hall/CRC, Boca Raton, Fla, USA, 2008.

[3] C. Çinar, "On the positive solutions of the difference equation system $x_{n+1} = 1/y_n, y_{n+1} = y_n/x_{n-1}y_{n-1}$," *Applied Mathematics and Computation*, vol. 158, no. 2, pp. 303–305, 2004.

[4] C. Çinar and I. Yalçınkaya, "On the positive solutions of the difference equation system $x_{n+1} = 1/z_n$, $y_{n+1} = x_n/x_{n-1}$, $z_{n+1} = 1/x_{n-1}$," *International Mathematical Journal*, vol. 5, pp. 525–527, 2004.

[5] D. Clark and M. R. S. Kulenović, "A coupled system of rational difference equations," *Computers & Mathematics with Applications*, vol. 43, no. 6-7, pp. 849–867, 2002.

[6] E. M. Elabbasy, H. El-Metwally, and E. M. Elsayed, "On the solutions of a class of difference equations systems," *Demonstratio Mathematica*, vol. 41, no. 1, pp. 109–122, 2008.

[7] E. M. Elabbasy, H. El-Metwally, and E. M. Elsayed, "Global behavior of the solutions of difference equation," *Advances in Difference Equations*, vol. 2011, article 28, 2011.

[8] E. M. Elabbasy and E. M. Elsayed, "Global attractivity and periodic nature of a difference equation," *World Applied Sciences Journal*, vol. 12, no. 1, pp. 39–47, 2011.

[9] S. N. Elaydi, *An Introduction to Difference Equations*, Springer, New York, NY, USA, 2nd edition, 1999.

[10] E. M. Elsayed, "On the solutions of higher order rational system of recursive sequences," *Mathematica Balkanica*, vol. 22, no. 3-4, pp. 287–296, 2008.

[11] E. M. Elsayed, "On the solutions of a rational system of difference equations," *Fasciculi Mathematici*, no. 45, pp. 25–36, 2010.

[12] E. M. Elsayed, "Dynamics of recursive sequence of order two," *Kyungpook Mathematical Journal*, vol. 50, no. 4, pp. 483–497, 2010.

[13] E. M. M. Elsayed, "Behavior of a rational recursive sequences," *Studia. Universitatis Babeş-Bolyai Mathematica*, vol. 56, no. 1, pp. 27–42, 2011.

[14] E. M. Elsayed, "Solution of a recursive sequence of order ten," *General Mathematics*, vol. 19, no. 1, pp. 145–162, 2011.

[15] E. M. Elsayed, "Solutions of rational difference systems of order two," *Mathematical and Computer Modelling*, vol. 55, no. 3-4, pp. 378–384, 2012.

[16] A. Y. Özban, "On the positive solutions of the system of rational difference equations, $x_{n+1} = 1/y_{n-k}, y_{n+1} = y_n/x_{n-m}y_{n-m-k}$," *Journal of Mathematical Analysis and Applications*, vol. 323, no. 1, pp. 26–32, 2006.

[17] A. Y. Özban, "On the system of rational difference equations $x_{n+1} = a/y_{n-3}, y_{n+1} = by_{n-3}/x_{n-q}y_{n-q}$," *Applied Mathematics and Computation*, vol. 188, no. 1, pp. 833–837, 2007.

[18] X. Yang, Y. Liu, and S. Bai, "On the system of high order rational difference equations $x_n = a/y_{n-p}$, $y_n = by_{n-p}/x_{n-q}y_{n-q}$," *Applied Mathematics and Computation*, vol. 171, no. 2, pp. 853–856, 2005.

[19] E. M. Elsayed, "Solution and attractivity for a rational recursive sequence," *Discrete Dynamics in Nature and Society*, vol. 2011, Article ID 982309, 17 pages, 2011.

[20] E. M. Elsayed, "On the solution of some difference equations," *European Journal of Pure and Applied Mathematics*, vol. 4, no. 3, pp. 287–303, 2011.

[21] E. M. Elsayed, "On the dynamics of a higher-order rational recursive sequence," *Communications in Mathematical Analysis*, vol. 12, no. 1, pp. 117–133, 2012.

[22] E. A. Grove and G. Ladas, *Periodicities in Nonlinear Difference Equations*, vol. 4, Chapman & Hall/CRC, Boca Raton, Fla, USA, 2005.

[23] E. A. Grove, G. Ladas, L. C. McGrath, and C. T. Teixeira, "Existence and behavior of solutions of a rational system," *Communications on Applied Nonlinear Analysis*, vol. 8, no. 1, pp. 1–25, 2001.

[24] A. S. Kurbanlı, C. Çinar, and I. Yalçinkaya, "On the behavior of positive solutions of the system of rational difference equations $x_{n+1} = x_{n-1}/y_nx_{n-1} + 1$, $y_{n+1} = y_{n-1}/x_ny_{n-1} + 1$," *Mathematical and Computer Modelling*, vol. 53, no. 5-6, pp. 1261–1267, 2011.

[25] N. Touafek and E. M. Elsayed, "On the solutions of systems of rational difference equations," *Mathematical and Computer Modelling*, vol. 55, no. 7-8, pp. 1987–1997, 2012.

[26] I. Yalcinkaya, "On the global asymptotic stability of a second-order system of difference equations," *Discrete Dynamics in Nature and Society*, vol. 2008, Article ID 860152, 12 pages, 2008.

[27] I. Yalçinkaya, "On the difference equation $x_{n+1} = \alpha + x_{n-m}/x_n^k$," *Discrete Dynamics in Nature and Society*, vol. 2008, Article ID 805460, 8 pages, 2008.

[28] I. Yalcinkaya, "On the global asymptotic stability of a second-order system of difference equations," *Discrete Dynamics in Nature and Society*, vol. 2008, Article ID 860152, 12 pages, 2008.

[29] I. Yalcinkaya, "On the global asymptotic behavior of a system of two nonlinear difference equations," *ARS Combinatoria*, vol. 95, pp. 151–159, 2010.

[30] I. Yalçinkaya, C. Çinar, and M. Atalay, "On the solutions of systems of difference equations," *Advances in Difference Equations*, vol. 2008, Article ID 143943, 9 pages, 2008.

[31] X. Yang, "On the system of rational difference equations $x_n = A + y_{n-1}/x_{n-p}y_{n-q}$, $y_n = A + x_{n-1}/x_{n-r}y_{n-s}$," *Journal of Mathematical Analysis and Applications*, vol. 307, no. 1, pp. 305–311, 2005.

[32] Y. Zhang, X. Yang, G. M. Megson, and D. J. Evans, "On the system of rational difference equations $x_n = A + 1/y_{n-p}$, $y_n = A + y_{n-1}/x_{n-t}y_{n-s}$," *Applied Mathematics and Computation*, vol. 176, no. 2, pp. 403–408, 2006.


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