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Research Article

Robust Quadratic Stabilizability and H_∞ Control of Uncertain Linear Discrete-Time Stochastic Systems with State Delay

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This paper mainly discusses the robust quadratic stability and stabilization of linear discrete-time stochastic systems with state delay and uncertain parameters. By means of the linear matrix inequality (LMI) method, a sufficient condition is, respectively, obtained for the stability and stabilizability of the considered system. Moreover, we design the robust H_∞ state feedback controllers such that the system with admissible uncertainties is not only quadratically internally stable but also robust H_∞ controllable. A sufficient condition for the existence of the desired robust H_∞ controller is obtained. Finally, an example with simulations is given to verify the effectiveness of our theoretical results.

1. Introduction

It is well known that stability and stabilization are very important concepts in linear system theory. Due to a great number of applications of stochastic systems in the realistic world, the studies of stability and stabilization for stochastic systems attract lots of researchers' attention in recent years; we refer the reader to the classic book [1] and the follow-up books [2, 3], together with references [4–11] and the references therein, which include robust stochastic stability [4], exponential stabilization [6], mean-square stability, and $\mathscr{D}$-stability and $\mathscr{D}_R$-stability [8]. The stabilization of various systems, including impulsive Markovian jump delay systems [4], stochastic singular systems [10, 12, 13], uncertain stochastic T-S fuzzy systems [14], and time-delay systems [6, 11, 15–17], has been studied extensively. H_∞ control is one of the most important robust control approaches when the system is subject to the influence of external disturbance, which has been shown to be effective in attenuating the disturbance. The objective of standard H_∞ control requires designing a controller to attenuate l^2-gain from the external disturbance to controlled output below a given level $\gamma > 0$; see [18]. The study of H_∞ control of general linear discrete-time stochastic systems with multiplicative noise seems to be first initiated by [19]. Then, stochastic H_∞ control and its applications have been investigated extensively; see [14, 16, 20–24].

Because time-delay exists widely in practice and affects the system stability, there have been many works concerning the study in stability or H_∞ control of stochastic systems [4, 6, 9, 11, 14–16, 22, 25]. Due to limitations of measurement technique and tools, it is not easy to construct exact mathematical models. Compared with the nominal stochastic systems without uncertain terms investigated in [2, 5, 24], our considered system allows the coefficient matrix to vary in a certain range.

Discrete-time stochastic difference systems have attracted a great deal of attention with the development of computer technology in recent years. In our viewpoint, there are at least two motivations to study discrete-time stochastic systems, Firstly, discrete-time stochastic systems are ideal mathematical models in practical modeling such as genetic regulatory networks [23]. Secondly, discrete-time stochastic systems provide a better approach to understand extensively continuous-time stochastic Itô systems [2, 3, 26]. Therefore, it is of significance to study the stabilization and H_∞ control of discrete-time stochastic time-delay uncertain systems.

This paper will study quadratic stability, stabilization, and robust state feedback H_∞ control for uncertain discrete-time stochastic systems with state delay. The parameter uncertainties are time varying and norm bounded. It can be found that, up to now, many criteria for testing quadratic stabilization and H_∞ control have been given in terms of LMIs and algebraic Riccati equations by applying Lyapunov function approach. One of our main contributions is to study quadratic stability and stabilization via LMIs instead of algebraic Riccati equations which is hardly solved. What we have obtained extended the work of [15] about the quadratic stability and stabilization of deterministic uncertain systems. Another contribution is to solve the state feedback H_∞ control and present a state feedback H_∞ controller design.

The paper is organized as follows. In Section 2 we give some adequate preliminaries and useful definitions. In Section 3, sufficient conditions for quadratic stability and stabilization are given in terms of LMIs which is convenient to compute by the MATLAB LMI toolbox. Section 4 designs a state feedback H_∞ controller. Two numerical examples with simulations are given in Section 5 to verify the efficiency of the proposed results. Finally, we end this paper in Section 6 with a brief conclusion.

For convenience, the notations in this paper are quite standard such as the following: we let $\mathscr{R}^n$ and $\mathscr{R}^{m\times n}$ represent the set of all real n-dimensional vectors and $m\times n$ real matrices. For symmetric matrices X and Y, $X \geq Y$ (resp., $X > Y$) stands for the idea that the matrix $X-Y$ is positive semidefinite (resp., positive definite). I denotes the identity matrix of appropriate dimensions and X^T denotes the matrix transpose of X. $\|x\| = \sqrt{\sum_{k=0}^{\infty}|x_k|^2}$ represents the Euclidean norm or spectral norm of the vector x. $\mathscr{N}_{k_0} := \{k_0, k_0+1, k_0+2,\ldots\}$, especially, $\mathscr{N}_1 := \{1,2,\ldots\}$, $\mathscr{N}_0 := \{0,1,2,\ldots\}$, and $[\tau_1,\tau_2]$, represents the set of integers between τ_1 and τ_2 (inclusive). In symmetric block matrices, the symbol "$*$" is used as an ellipsis for terms induced by symmetry. $\mathscr{E}(\cdot)$ is the expectation operator.

2. Preliminaries

Consider a class of uncertain linear discrete-time stochastic systems with state delay described by

$$\begin{aligned} x(k+1) &= \left(A_0+\Delta A_0(k)\right)x(k)+\left(A_{0d}+\Delta A_{0d}(k)\right)\\ &\quad\cdot x(k-d)+\left(B_0+\Delta B_0(k)\right)u(k)\\ &\quad+\sum_{i=1}^{s}\{\left[C_0+\Delta C_0(k)\right]x(k)\\ &\quad+\left[C_{0d}+\Delta C_{0d}(k)\right]x(k-d)\\ &\quad+\left[D_0+\Delta D_0(k)\right]u(k)\}\,w_i(k),\\ x(j) &= \phi(j)\in\mathscr{R}^n,\\ &\qquad j\in\{-d,-d+1,\ldots,0\},\ k\in\mathscr{N}_0, \end{aligned} \tag{1}$$

where $x(k)\in\mathscr{R}^n$ is the system state and $u(k)\in\mathscr{R}^m$ is the control input, and $\{w(k)\}_{k\geq 0}$ are independent white noise process satisfying the following assumptions:

(H_1) $\mathscr{E}[w_k]=0$, $\mathscr{E}[w_kw_j]=\delta_{kj}$, where δ_{kj} is a Kronecker function defined by $\delta_{kj}=0$ for $k\neq j$ while $\delta_{kj}=1$ for $k=j$.

(H_2) $\{w(k)\}_{k\geq 0}$ are defined on the filtered probability space $(\Omega,\mathscr{F},\mathscr{F}_k,\mathscr{P})$ with $\mathscr{F}_k = \sigma\{w(0),\ldots,w(k)\}$. In addition, $\{\mathscr{F}_k\}_{k\in\mathscr{N}_0}$ is an increasing sequence of σ-algebras with $\mathscr{F}_t\subset\mathscr{F}$.

A_0, A_{0d}, B_0, C_0, C_{0d}, D_0 are known real constant matrices with compatible dimensions. $\Delta A_0(k)$, $\Delta A_{0d}(k)$, $\Delta B_0(k)$, $\Delta C_0(k)$, $\Delta D_0(k)$, $\Delta C_{0d}(k)$ are norm bounded and time-varying uncertain parameter which are assumed to have the following form:

$$\begin{aligned} &\left[\Delta A_0(k)\ \ \Delta A_{0d}(k)\ \ \Delta B_0(k)\ \ \Delta C_0(k)\ \ \Delta C_{0d}(k)\ \ \Delta D_0(k)\right]\\ &\quad= EF(k)\left[G_{A_0}\ \ G_{A_{0d}}\ \ G_{B_0}\ \ G_{C_0}\ \ G_{C_{0d}}\ \ G_{D_0}\right], \end{aligned} \tag{2}$$

where $E, G_{A_0}, G_{A_{0d}}, G_{B_0}, G_{C_0}, G_{C_{0d}}, G_{D_0}$ are constant matrices and $F(k)\in\mathscr{R}^{m\times n}$ is the uncertain matrix satisfying

$$F(k)^T F(k)\leq I,\quad k\in\mathscr{N}_0. \tag{3}$$

For the purpose of simplicity, throughout this paper, we write system (1) in the following form:

$$\begin{aligned} &x(k+1)\\ &\quad= A_{0\Delta}x(k)+A_{0d\Delta}x(k-d)+B_{0\Delta}u(k)\\ &\qquad+\sum_{i=1}^{s}\left[C_{0\Delta}x(k)+C_{0d\Delta}x(k-d)+D_{0\Delta}u(k)\right]w_i(k),\\ &x(j)=\phi(j)\in\mathscr{R}^n,\quad j\in[-d,0],\ k\in\mathscr{N}_0, \end{aligned} \tag{4}$$

where $A_{0\Delta}$, $A_{0d\Delta}$, $B_{0\Delta}$, $C_{0\Delta}$, $C_{0d\Delta}$ are bounded uncertain system matrices with

$$\begin{aligned} A_{0\Delta} &= A_0+\Delta A_0(k) = A_0+EF(k)G_{A_0},\\ A_{0d\Delta} &= A_{0d}+\Delta A_{0d}(k) = A_{0d}+EF(k)G_{A_{0d}},\\ B_{0\Delta} &= B_0+\Delta B_0(k) = B_0+EF(k)G_{B_0},\\ C_{0\Delta} &= C_0+\Delta C_0(k) = C_0+EF(k)G_{C_0},\\ C_{0d\Delta} &= C_{0d}+\Delta C_{0d}(k) = C_{0d}+EF(k)G_{C_{0d}},\\ D_{0\Delta} &= D_0+\Delta D_0(k) = D_0+EF(k)G_{D_0}. \end{aligned} \tag{5}$$

Below, we define robust quadratic stability and robust quadratic stabilizability for the uncertain time-delay discrete-time system (1), which generalize Definition 1 of [15] to stochastic systems.

Definition 1. Uncertain discrete time-delay system (1) is said to be robustly quadratically stable, if there exist matrices $P>0$, $Q>0$ and a scalar $\omega>0$ such that, for all admissible uncertain terms and given initial condition $x(j)=\phi(j)\in\mathscr{R}^n$ for $j=0,-1,\ldots,-d$, the unforced system of (1) (with $u(k)\equiv 0$) satisfies

$$\mathscr{E}\left(\Delta V_k\right)=\mathscr{E}V_{k+1}-\mathscr{E}V_k\le -\omega\mathscr{E}\left\|\widehat{x}(k)\right\|^2 \tag{6}$$

for $\widehat{x}(k)\in\mathscr{R}^{2n}$ with $\widehat{x}(k)=(x(k)^T,x(k-d)^T)^T$ and

$$V_k=x(k)^T Px(k)+\sum_{j=1}^{d}x(k-j)^T Qx(k-j). \tag{7}$$

Definition 2. Uncertain discrete time-delay system (1) is said to be robustly quadratically stabilizable if there exists a matrix $K\in\mathscr{R}^{m\times n}$ such that closed-loop system (1) with $u(k)=Kx(k)$, that is,

$$\begin{aligned}x(k+1)&\\&=\left(A_{0\Delta}+B_{0\Delta}K\right)x(k)+A_{0d\Delta}(k)\,x(k-d)\\&\quad+\sum_{i=1}^{s}\left[\left(C_{0\Delta}+D_{0\Delta}K\right)x(k)+C_{0d\Delta}x(k-d)\right]\omega_i(k),\end{aligned} \tag{8}$$

is robustly quadratically stable for given $x(j)=\phi(j)\in\mathscr{R}^n$ for $j=0,-1,\ldots,-d$.

3. Robust Quadratic Stabilization

In this section, a sufficient condition about robust quadratic stability and robust quadratic stabilization will be presented via LMIs, respectively. First, we cite the following lemma which is essential in proving our main results.

Lemma 3 (see [27]). *Suppose that $W=W^T$, $F(k)$ satisfies (2), and then for any real matrices W, M, and N of suitable dimensions we have*

$$W+MF(k)N+N^TF(k)^TM^T<0 \tag{9}$$

if and only if (iff), for some $\alpha>0$,

$$W+\alpha MM^T+\alpha^{-1}N^TN<0. \tag{10}$$

Theorem 4. *Consider uncertain discrete-time stochastic delay system (1) with $u(k)=0$. This system is robustly quadratically stable if there exist positive definite matrices $X>0$, $Y>0$ such that the following LMI holds.*

$$\begin{bmatrix}\Delta_{11}&\Delta_{12}&A_0^TX&s^{1/2}C_0^TX&0&0\\ *&\Delta_{22}&A_{0d}^TX&s^{1/2}C_{0d}^TX&0&0\\ *&*&-X&0&XE&0\\ *&*&*&-X&0&XE\\ *&*&*&*&-I&0\\ *&*&*&*&*&-I\end{bmatrix}<0, \tag{11}$$

where

$$\begin{aligned}\Delta_{11}&=Y-X+G_{A_0}^TG_{A_0}+sG_{C_0}^TG_{C_0},\\ \Delta_{12}&=G_{A_{0d}}^TG_{A_{0d}}+sG_{C_{0d}}^TG_{C_{0d}},\\ \Delta_{22}&=-Y+G_{A_{0d}}^TG_{A_{0d}}+sG_{C_{0d}}^TG_{C_{0d}}.\end{aligned} \tag{12}$$

Proof. From Definition 1, taking a Lyapunov function V_k as in the form of (7), if uncertain discrete time-delay stochastic system (1) is quadratically stable, then, for all admissible uncertainties of (1), there exist matrices $P>0$, $Q>0$ and a scalar $\alpha>0$ such that $\mathscr{E}(\Delta V_k)$ associated with unforced system (8) satisfies (6). In view of the assumption (H_1), it is easy to compute

$$\begin{aligned}&\mathscr{E}V_{k+1}-\mathscr{E}V_k=\mathscr{E}\Big\{x(k)^T\Big[A_{0\Delta}(k)^TPA_{0\Delta}(k)\\&\quad+sC_{0\Delta}(k)^TPC_{0\Delta}(k)+Q-P\Big]x(k)+x(k)^T\\&\quad\cdot\Big[A_{0\Delta}(k)^TPA_{0d\Delta}(k)+sC_{0\Delta}(k)^TPC_{0d\Delta}(k)\Big]x(k\\&\quad-d)+x(k-d)^T\Big[A_{0d\Delta}(k)^TPA_{0\Delta}(k)\\&\quad+sC_{0d\Delta}(k)^TPC_{0\Delta}(k)\Big]x(k)+x(k-d)^T\\&\quad\cdot\Big[A_{0d\Delta}(k)^TPA_{0d\Delta}(k)+sC_{0d\Delta}(k)^TPC_{0d\Delta}(k)\\&\quad-Q\Big]x(k-d)\Big\}=\begin{bmatrix}x(k)\\x(k-d)\end{bmatrix}^T\Pi\begin{bmatrix}x(k)\\x(k-d)\end{bmatrix},\end{aligned} \tag{13}$$

where $A_{0\Delta}$, $A_{0d\Delta}$, $C_{0\Delta}$, and $C_{0d\Delta}$ are given in (5) and Π is shown as

$$\Pi=\begin{bmatrix}A_{0\Delta}(k)^TPA_{0\Delta}(k)+sC_{0\Delta}(k)^TPC_{0\Delta}(k)+Q-P & A_{0\Delta}(k)^TPA_{0d\Delta}(k)+sC_{0\Delta}(k)^TPC_{0d\Delta}(k)\\ A_{0d\Delta}(k)^TPA_{0\Delta}(k)+sC_{0d\Delta}(k)^TPC_{0\Delta}(k) & A_{0d\Delta}(k)^TPA_{0d\Delta}(k)+sC_{0d\Delta}(k)^TPC_{0d\Delta}(k)-Q\end{bmatrix}. \tag{14}$$

By Definition 1, system (1) with $u(k)=0$ is robustly quadratically stable, only if

$$\Pi<0 \tag{15}$$

which is equivalent to

$$\begin{aligned}\Pi&=\Pi_1+\Pi_2\\&=\begin{bmatrix}sC_{0\Delta}(k)^TPC_{0\Delta}(k)+Q-P & sC_{0\Delta}(k)^TPC_{0d\Delta}(k)\\ sC_{0d\Delta}(k)^TPC_{0\Delta}(k) & sC_{0d\Delta}(k)^TPC_{0d\Delta}(k)-Q\end{bmatrix}\\&\quad+\begin{bmatrix}A_{0\Delta}(k)^TPA_{0\Delta}(k) & A_{0\Delta}(k)^TPA_{0d\Delta}(k)\\ A_{0d\Delta}(k)^TPA_{0\Delta}(k) & A_{0d\Delta}(k)^TPA_{0d\Delta}(k)\end{bmatrix}<0.\end{aligned} \tag{16}$$

Note that Π_2 can be rewritten as

$$\Pi_2 = \begin{bmatrix} A_{0\Delta}(k)^T P \\ A_{0d\Delta}(k)^T P \end{bmatrix} P^{-1} \left[PA_{0\Delta}(k) \;\; PA_{0d\Delta}(k) \right]. \tag{17}$$

By Schur's complement, it is easy to derive that $\Pi < 0$ is equivalent to

$$\widehat{\Pi} = \begin{bmatrix} \pi_{11} & sC_{0\Delta}(k)^T PC_{0d\Delta}(k) & A_{0\Delta}(k)^T P \\ * & \pi_{22} & A_{0d\Delta}(k)^T P \\ * & * & -P \end{bmatrix}, \tag{18}$$

where

$$\begin{aligned} \pi_{11} &= sC_{0\Delta}(k)^T PC_{0\Delta}(k) + Q - P, \\ \pi_{22} &= sC_{0d\Delta}(k)^T PC_{0d\Delta}(k) - Q. \end{aligned} \tag{19}$$

Then, using the same way as in (16)–(19) yields

$$\widehat{\Pi} = \begin{bmatrix} Q-P & 0 & A_{0\Delta}(k)^T P & s^{1/2}C_{0\Delta}(k)^T P \\ * & -Q & A_{0d\Delta}(k)^T P & s^{1/2}C_{0d\Delta}(k)^T P \\ * & * & -P & 0 \\ * & * & * & -P \end{bmatrix} < 0. \tag{20}$$

The above inequality can be rewritten as

$$\begin{aligned} \widehat{\Pi} = \Pi_3 &+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ PE & 0 \\ 0 & PE \end{bmatrix} \operatorname{diag}(F(k), F(k)) \\ &\cdot \begin{bmatrix} G_{A0} & G_{A0d} & 0 & 0 \\ s^{1/2}G_{C0} & s^{1/2}G_{C0d} & 0 & 0 \end{bmatrix} + \begin{bmatrix} G_{A0}^T & s^{1/2}G_{C0}^T \\ G_{A0d}^T & s^{1/2}G_{C0d}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &\cdot \operatorname{diag}\left(F(k)^T, F(k)^T\right) \begin{bmatrix} 0 & 0 & E^T P & 0 \\ 0 & 0 & 0 & E^T P \end{bmatrix} < 0, \end{aligned} \tag{21}$$

where

$$\Pi_3 = \begin{bmatrix} Q-P & 0 & A_0^T P & s^{1/2}C_0^T P \\ * & -Q & A_{0d}^T P & s^{1/2}C_{0d}^T P \\ * & * & -P & 0 \\ * & * & * & -P \end{bmatrix}. \tag{22}$$

Because Π_3 is a symmetric matrix, applying Lemma 3, (21) holds iff the following inequality holds:

$$\begin{aligned} \Pi_3 &+ \alpha \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ PE & 0 \\ 0 & PE \end{bmatrix} \begin{bmatrix} 0 & 0 & E^T P & 0 \\ 0 & 0 & 0 & E^T P \end{bmatrix} \\ &+ \alpha^{-1} \begin{bmatrix} G_{A0}^T & s^{1/2}G_{C0}^T \\ G_{A0d}^T & s^{1/2}G_{C0d}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} G_{A0} & G_{A0d} & 0 & 0 \\ s^{1/2}G_{C0} & s^{1/2}G_{C0d} & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & A_0^T P & s^{1/2}C_0^T P \\ * & \Lambda_{22} & A_{0d}^T P & s^{1/2}C_{0d}^T P \\ * & * & \Lambda_{33} & 0 \\ * & * & * & \Lambda_{44} \end{bmatrix} < 0, \end{aligned} \tag{23}$$

where

$$\begin{aligned} \Lambda_{11} &= Q - P + \alpha^{-1}G_{A0}^T G_{A0} + s\alpha^{-1}G_{C0}^T G_{C0}, \\ \Lambda_{12} &= \alpha^{-1}G_{A0}^T G_{A0d} + s\alpha^{-1}G_{C0}^T G_{C0d}, \\ \Lambda_{22} &= -Q + \alpha^{-1}G_{A0d}^T G_{A0d} + s\alpha^{-1}G_{C0d}^T G_{C0d}, \\ \Lambda_{33} &= \Lambda_{44} = -P + \alpha PEE^T P. \end{aligned} \tag{24}$$

Take

$$\begin{aligned} P &= \alpha^{-1}X, \\ Q &= \alpha^{-1}Y \end{aligned} \tag{25}$$

and then by substituting (25) into (23), for $\alpha > 0$, we get

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & A_0^T X & s^{1/2}C_0^T X \\ * & \Delta_{22} & A_{0d}^T X & s^{1/2}C_{0d}^T X \\ * & * & -X + XEE^T X & 0 \\ * & * & * & -X + XEE^T X \end{bmatrix} < 0, \tag{26}$$

where $\Delta_{11}, \Delta_{12}, \Delta_{22}$ are shown in (12).

Using the same method as in (16)–(20), (11)-(12) follow immediately from the above inequality. □

Theorem 5. *System (1) is robustly quadratically stabilizable if there exist positive matrices $X > 0$, $Y > 0$, $K \in \mathscr{R}^{m\times n}$ and a scalar $\varepsilon > 0$ with $\varepsilon I - X^{-1} < 0$ such that the following LMI holds.*

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & (A_0+B_0K)^T & s^{1/2}(C_0+D_0K)^T & 0 & 0 & (G_{A0}+G_{B0}K)^T & s^{1/2}(G_{C0}+G_{D0}K)^T \\ * & \Theta_{22} & A_{0d}^T & s^{1/2}C_{0d}^T & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & E & 0 & 0 & 0 \\ * & * & * & -\varepsilon I & 0 & E & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (27)$$

where

$$\Theta_{11} = Y - X,$$
$$\Theta_{12} = (G_{A0}+G_{B0}K)^T G_{A0d} + (G_{C0}+G_{D0}K)^T G_{C0d}, \quad (28)$$
$$\Theta_{22} = -Y + G_{A0d}^T G_{A0d} + G_{C0d}^T G_{C0d}.$$

Moreover, a quadratically stabilizing state feedback controller is given by

$$u(k) = Kx(k). \quad (29)$$

Proof. By Definition 2, using the same way as in the proof of Theorem 4, the following inequality which has a similar form to (11)-(12) can be obtained by taking $u(k) = Kx(k)$

$$\begin{bmatrix} \widehat{\Theta}_{11} & \Theta_{12} & (A_0+B_0K)^T X & s^{1/2}(C_0+D_0K)^T X & 0 & 0 \\ * & \Theta_{22} & A_{0d}^T X & s^{1/2}C_{0d}^T X & 0 & 0 \\ * & * & -X & 0 & XE & 0 \\ * & * & * & -X & 0 & XE \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \quad (30)$$

where

$$\widehat{\Theta}_{11} = Y - X + (G_{A0}+G_{B0}K)^T (G_{A0}+G_{B0}K) + (G_{C0}+G_{D0}K)^T (G_{C0}+G_{D0}K). \quad (31)$$

In order to eliminate the nonlinear quadratic terms

$$(A_0+B_0K)^T X,$$
$$(C_0+D_0K)^T X, \quad (32)$$

pre- and postmultiplying

$$\operatorname{diag}(I, I, X^{-1}, I, I, I) \quad (33)$$

on both sides of (30) and considering $X^{-1} > \varepsilon I$, (27)-(28) can be obtained easily. This theorem is proved. □

Remark 6. Compared with the results about quadratic stability and quadratic stabilizability of deterministic systems given in [14], our two theorems not only extend the results of [14] to stochastic systems, but also provide the corresponding LMI criteria which can be easily tested by MATLAB LMI toolbox.

Remark 7. From these two theorems, we also can get the result about quadratic stability with the given decay rate. Take the function

$$x_\lambda(k) = x(k)e^{k\lambda}, \quad (34)$$

and then, substituting (34) into (8), we obtain the following new system:

$$x_\lambda(k+1) = (\widetilde{A}_{0\Delta} + \widetilde{B}_{0\Delta}K)x_\lambda(k) + \widetilde{A}_{0d\Delta}x_\lambda(k-d) + \sum_{i=1}^{s}\left[(\widetilde{C}_{0\Delta} + \widetilde{D}_{0\Delta}K)x_\lambda(k) + \widetilde{C}_{0d\Delta}x_\lambda(k-d)\right] \cdot w_i(k), \quad (35)$$

where

$$\widetilde{A}_{0\Delta} = e^\lambda A_{0\Delta},$$
$$\widetilde{B}_{0\Delta} = e^\lambda B_{0\Delta},$$
$$\widetilde{C}_{0\Delta} = e^\lambda C_{0\Delta},$$
$$\widetilde{D}_{0\Delta} = e^\lambda D_{0\Delta}, \quad (36)$$
$$\widetilde{A}_{0d\Delta} = e^{(d+1)\lambda} A_{0d\Delta},$$
$$\widetilde{C}_{0d\Delta} = e^{(d+1)\lambda} C_{0d\Delta}.$$

So system (1) is quadratically stabilizable with decay rate λ if system (35) is quadratically stabilizable.

4. State Feedback H_∞ Control

In this section we consider the state feedback discrete-time H_∞ control problem for the following uncertain linear stochastic system with state delay:

$$\begin{aligned} x(k+1) &= \left[A_0 + \Delta A_0(k)\right] x(k) + \left[A_{0d} + \Delta A_{0d}(k)\right] \\ &\quad \cdot x(k-d) + B\xi(k) + \left[B_0 + \Delta B_0(k)\right] u(k) \\ &\quad + \sum_{i=1}^{s} \{\left[C_0 + \Delta C_0(k)\right] x(k) \\ &\quad + \left[C_{0d} + \Delta C_{0d}(k)\right] x(k-d) \\ &\quad + \left[D_0 + \Delta D_0(k)\right] u(k)\} w_i(k), \\ x(j) &= \phi(j) \in \mathscr{R}^n, \\ &\qquad j \in \{-d, -d+1, \ldots, 0\}, \ k \in \mathcal{N}_0 \\ z(k) &= Cx(k) + Du(k), \end{aligned} \tag{37}$$

where $z(k) \in \mathscr{R}^{n_z}$ and $\xi(k) \in \mathscr{R}^q$ are called the controlled output and external disturbance, respectively. In addition, the effect of the disturbance $\xi(k)$ on the controlled output $z(k)$ is described by a perturbation operator $\mathscr{G}_{z\xi} : \xi \mapsto z$, which maps any finite energy disturbance signal ξ into the corresponding finite energy output signal z of the closed-loop system. The size of this linear operator, that is, $\|\mathscr{G}_{z\xi}\|$, measures the influence of the disturbances in the worst case. We denote by $l_w^2(\mathcal{N}_0, \mathscr{R}^l)$ the set of all nonanticipative square summable $\mathscr{R}^l$-valued stochastic processes

$$\begin{aligned} y &= \{y_k : y_k \\ &\in L^2(\Omega, \mathscr{R}^l), \ y_k \text{ is } \mathscr{F}_{k-1} \text{ measurable}\}_{k \in \mathcal{N}_0}. \end{aligned} \tag{38}$$

l^2-norm of $y \in l_w^2(\mathcal{N}_0, \mathscr{R}^l)$ is defined by

$$\|y\|_{l_w^2(\mathcal{N}_0, \mathscr{R}^l)} = \left(\sum_{k=0}^{\infty} \mathscr{E}\|y_k\|^2\right)^{1/2}. \tag{39}$$

Firstly, for system (37), we define the perturbed operator $\mathscr{G}_{z\xi}$ and its norm as follows.

Definition 8. The perturbed operator of system (37), $\mathscr{G}_{z\xi} : l_w^2(\mathcal{N}_0, \mathscr{R}^q) \mapsto l_w^2(\mathcal{N}_0, \mathscr{R}^{n_z})$, is defined as

$$\begin{aligned} \mathscr{G}_{z\xi} : \xi(k) \in l_w^2(\mathcal{N}_0, \mathscr{R}^q) &\longmapsto \\ Cx(k) + Du(k)&, \\ x(j) = 0, \quad j = 0, -1, -2, &\ldots, -d \end{aligned} \tag{40}$$

with its norm

$$\begin{aligned} \|\mathscr{G}_{z\xi}\| &= \sup_{\xi(k) \in l_w^2(\mathcal{N}_0, \mathscr{R}^q), \xi(k) \neq 0, x(j)=0, j \in [-d,0]} \frac{\|z(k)\|_{l_w^2(\mathcal{N}_0, \mathscr{R}^{n_z})}}{\|\xi(k)\|_{l_w^2(\mathcal{N}_0, \mathscr{R}^q)}} \\ &= \sup_{\xi(k) \in l_w^2(\mathcal{N}_0, \mathscr{R}^q), \xi(k) \neq 0, x(j)=0, j \in [-d,0]} \frac{\left(\sum_{k=0}^{\infty} E\|Cx(k) + Du(k)\|^2\right)^{1/2}}{\left(\sum_{k=0}^{\infty} E\|\xi(k)\|^2\right)^{1/2}}. \end{aligned} \tag{41}$$

Next, we present the definition about stochastic robust H_∞ control.

Definition 9. For a certain level $\gamma > 0$, $u^*(k) = Kx(k)$ is the H_∞ control of the system (37), if

(i) system (37) is internally stabilizable when $\xi(k) \equiv 0$;

(ii) the norm of the perturbed operator of system (37) satisfies $\|\mathscr{G}_{z\xi}\| < \gamma$ for all external disturbance $\xi(k) \in l_w^2(\mathcal{N}_0, \mathscr{R}^q)$.

Besides, if $u^*(k)$ exists, then system (37) is called H_∞ controllable in the disturbance attenuation. Furthermore, it is called strongly robust H_∞ controllable if $\gamma = 1$.

Theorem 10. *Consider system (37). For the given $\gamma > 0$ and some $\beta > 0$ with $P < \beta^{-1}I$ and $\alpha > 0$ if there exist $P > 0$, $Q > 0$, and $K \in \mathscr{R}^{m\times n}$ satisfying the following LMI*

$$\begin{bmatrix} -2P + Q + C^TC & \hbar_{12} & 0 & s^{1/2}(C_0 + D_0K)^T & (A_0 + B_0K) & K^TD^T & \hbar_{17} & \hbar_{18} \\ * & \hbar_{22} & 0 & s^{1/2}C_{0d}^T & A_{0d}^T & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & B^T & 0 & 0 & 0 \\ * & * & * & -\beta I + \alpha EE^T & 0 & 0 & 0 & 0 \\ * & * & * & * & -\beta I + \alpha EE^T & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{42}$$

where

$$\hbar_{12} = \alpha^{-1}\left[s^{1/4}\left(G_{C0}+G_{D0}K\right)^{T}G_{C0d} + \left(G_{A0}+G_{B0}K\right)^{T}G_{A0d}\right],$$
$$\hbar_{22} = -Q+\alpha^{-1}\left(s^{1/4}G_{C0d}^{T}G_{C0d}+G_{A0d}^{T}G_{A0d}\right), \quad (43)$$
$$\hbar_{17} = \alpha^{-1}s^{1/2}\left(G_{C0}+G_{D0}K\right)^{T},$$
$$\hbar_{18} = \alpha^{-1}\left(G_{A0}+G_{B0}K\right)^{T},$$

then system (37) is robustly H_∞ controllable with a control law $u(k) = Kx(k)$.

Proof. By Theorem 5, when disturbance $\xi(k) = 0$, it is easy to test that system (37) is internally stabilizable with $u^*(k) = Kx(k)$. Now we only need to show $\|\mathcal{G}_{z\xi}\| < \gamma$. By Definition 1, choose the Lyapunov function $V_k = x(k)^T Px(k) + \sum_{j=1}^{d} x(k-j)^T Qx(k-j)$ with $P > 0$ and $Q > 0$ to be determined, and then

$$\mathcal{E}\Delta V_k = \mathcal{E}V_{k+1} - \mathcal{E}V_k = \mathcal{E}\left[x^T(k+1)Px(k+1) + \sum_{j=1}^{d} x(k+1-j)^T Qx(k+1-j) - x^T(k)Px(k) - \sum_{j=1}^{d} x^T(k-j)Qx(k-j)\right]$$
$$= \mathcal{E}\left[x^T(k+1)Px(k+1) + x^T(k)(Q-P)x(k) - x^T(k-d)Qx(k-d)\right]. \quad (44)$$

So in the case of $x(j) = 0,\ j = 0, -1, \ldots, -d$, we have

$$\begin{aligned}
&\|z(k)\|_{l_w^2(\mathcal{N}_0,\mathcal{R}^{n_z})} - \gamma^2\|\xi(k)\|_{l_w^2(\mathcal{N}_0,\mathcal{R}^{q})} = \mathcal{E}\sum_{k=0}^{\infty}\left\{x^T(k)\cdot\left(C^TC+K^TD^TDK\right)x(k) + \Delta V - \gamma^2\xi^T(k)\xi(k)\right\} \\
&+ V(x(0)) - \liminf_{t\to\infty}\mathcal{E}V(x(k)) \le \mathcal{E}\sum_{k=0}^{\infty}\left\{x(k)^T\cdot\left(A_{0\Delta}+B_{0\Delta}K\right)^T P\left(A_{0\Delta}+B_{0\Delta}K\right)x(k) + x^T(k)\right. \\
&\cdot\left(A_{0\Delta}+B_{0\Delta}K\right)^T PA_{0d\Delta}x(k-d) + x^T(k)\cdot\left(A_{0\Delta}+B_{0\Delta}K\right)^T PB\xi(k) + x^T(k-d) \\
&\cdot A_{0d\Delta}^T P\left(A_{0\Delta}+B_{0\Delta}K\right)x(k) + x^T(k-d)\cdot A_{0d\Delta}^T PA_{0d\Delta}x(k-d) + x^T(k-d)A_{0d\Delta}^T PB\xi(k) \\
&+ \xi^T(k)B^TP\left(A_{0\Delta}+B_{0\Delta}K\right)x(k) + \xi^T(k)\cdot B^TPA_{0d\Delta}x(k-d) + \xi^T(k)B^TPB\xi(k) + sx^T(k) \\
&\cdot\left(C_{0\Delta}K\right)^T P\left(C_{0\Delta}+D_{0\Delta}K\right)x(k) + sx^T(k)\left(C_{0\Delta}+D_{0\Delta}K\right)^T PC_{0d\Delta}x(k-d) \\
&+ sx^T(k-d)C_{0d\Delta}^T P\left(C_{0\Delta}+D_{0\Delta}K\right)x(k) + sx^T(k-d)C_{0d\Delta}PC_{0d\Delta}x(k-d) - x^T(k)Px(k) \\
&+ x^T(k)(Q-P)x(k) - x^T(k-d)Qx(k-d) + x^T(k)\left(C^TC+K^TD^TDK\right)x(k) - \gamma^2\xi^T(k) \\
&\left.\cdot\,\xi(k)\right\} = \mathcal{E}\sum_{k=0}^{\infty}\begin{bmatrix} x(k) \\ x(k-d) \\ \xi(k)\end{bmatrix}^T \Xi \begin{bmatrix} x(k) \\ x(k-d) \\ \xi(k)\end{bmatrix},
\end{aligned} \quad (45)$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & \left(A_{0\Delta}+B_{0\Delta}K\right)^T PA_{0d\Delta} & \left(A_{0\Delta}+B_{0\Delta}K\right)^T PB & s^{1/2}\left(C_{0\Delta}+D_{0\Delta}K\right)^T P \\ * & A_{0d\Delta}^T PA_{0d\Delta} - Q & A_{0d\Delta}^T PB & s^{1/2}C_{0d\Delta}^T P \\ * & * & B^TPB-\gamma^2 I & 0 \\ * & * & * & -P \end{bmatrix}, \quad (46)$$
$$\Xi_{11} = \left(A_{0\Delta}+B_{0\Delta}K\right)^T P\left(A_{0\Delta}+B_{0\Delta}K\right) - 2P + Q + C^TC + K^TD^TDK.$$

Obviously, it is easy to get that $\|\mathcal{G}_{z\xi}\| < \gamma$ if $\Xi < 0$. Then, we need to eliminate the uncertainties. Using the same method as in the proof of Theorem 4, we know that, for some $\alpha > 0$, a sufficient condition for $\Xi < 0$ can be got from the following matrix inequality.

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & 0 & s^{1/2}(C_0+D_0K)^T P & (A_0+B_0K)^T P \\ * & \Gamma_{22} & 0 & s^{1/2}C_{0d}^T P & A_{0d}^T P \\ * & * & -\gamma^2 I & 0 & B^T P \\ * & * & * & -P+\alpha PEE^T P & 0 \\ * & * & * & * & -P+\alpha PEE^T P \end{bmatrix} < 0, \tag{47}$$

where

$$\begin{aligned} \Gamma_{11} &= -2P + Q + C^T C + K^T D^T DK + \alpha^{-1} s^{1/4} (G_{C0} \\ &\quad + G_{D0}K)^T (G_{C0} + G_{D0}K) + \alpha^{-1} (G_{A0} + G_{B0}K)^T \\ &\quad \cdot (G_{A0} + G_{B0}K), \\ \Gamma_{12} &= \alpha^{-1} \left[s^{1/4} (G_{C0} + G_{D0}K)^T G_{C0d} \right. \\ &\quad \left. + (G_{A0} + G_{B0}K)^T G_{A0d} \right], \\ \Gamma_{22} &= -Q + \alpha^{-1} \left(s^{1/4} G_{C0d}^T G_{C0d} + G_{A0d}^T G_{A0d} \right). \end{aligned} \tag{48}$$

Then, by pre- and postmultiplying

$$\operatorname{diag}\left[I \quad I \quad I \quad P^{-1} \quad P^{-1} \right] \tag{49}$$

on both sides of (47), we have

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & 0 & s^{1/2}(C_0+D_0K)^T & (A_0+B_0K)^T \\ * & \Gamma_{22} & 0 & s^{1/2}C_{0d}^T & A_{0d}^T \\ * & * & -\gamma^2 I & 0 & B^T \\ * & * & * & -P^{-1}+\alpha EE^T & 0 \\ * & * & * & * & -P^{-1}+\alpha EE^T \end{bmatrix} < 0. \tag{50}$$

For some constant $\beta > 0$ with $P^{-1} > \beta I$, Theorem 10 is concluded; that is, an H_∞ control of system (37) is obtained by solving LMIs (42)-(43). This completes the proof. □

5. Simulation Example

In this section, we consider two simple examples with simulations to illustrate the effectiveness of the proposed approach.

Example 11. Consider discrete-time stochastic system (1) with the following parameters:

$$\begin{aligned} A_0 &= \begin{bmatrix} 1 & 0 \\ 0 & 0.8 \end{bmatrix}, \\ A_{0d} &= \begin{bmatrix} 0.02 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ B_0 &= \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \\ C_0 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \\ C_{0d} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ D_0 &= \begin{bmatrix} 0.2 \\ 0.7 \end{bmatrix}, \\ E &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \\ G_{A_0} &= \begin{bmatrix} 0.01 & 0 \\ 0 & 0.03 \end{bmatrix}, \\ G_{A_{0d}} &= \begin{bmatrix} 0.04 & 0 \\ 0 & 0.05 \end{bmatrix}, \\ G_{B_0} &= \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \\ G_{C_0} &= \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ G_{D_0} &= \begin{bmatrix} 0.1 \\ 0.7 \end{bmatrix}, \\ G_{C_{0d}} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.01 \end{bmatrix}, \\ F(k) &= \begin{bmatrix} \cos(w(k)) & 0 \\ 0 & \sin(w(k)) \end{bmatrix}, \\ s &= 1. \end{aligned} \tag{51}$$

Using LMI toolbox to solve (11)-(12) in Theorem 4, we find out that $t_{\min} = 0.0086 > 0$ which means that there is no feasible solution and indicates that system (1) with $u \equiv 0$ is unstable. Figure 1 verifies the result. By solving LMI (27), a group of feasible solutions with $t_{\min} = -0.9649 < 0$ are shown as $\varepsilon = 16.7436$ and

$$\begin{aligned} X &= \begin{bmatrix} 22.1026 & 0.6519 \\ 0.6519 & 20.0007 \end{bmatrix}, \\ Y &= \begin{bmatrix} 11.3608 & -0.6710 \\ -0.6710 & 13.2231 \end{bmatrix}. \end{aligned} \tag{52}$$

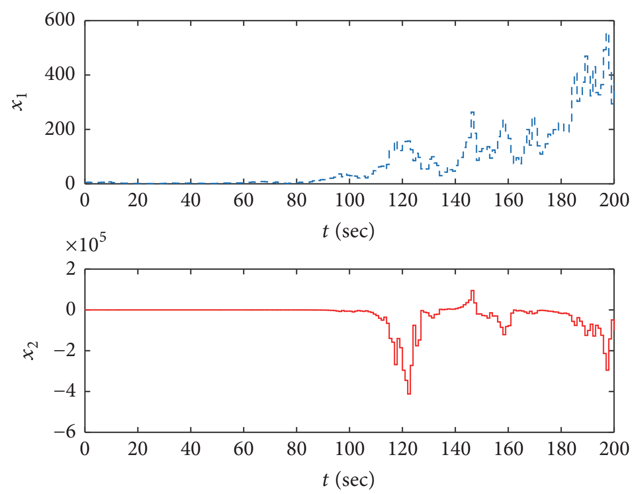

Figure 1: State trajectories of the autonomous system.

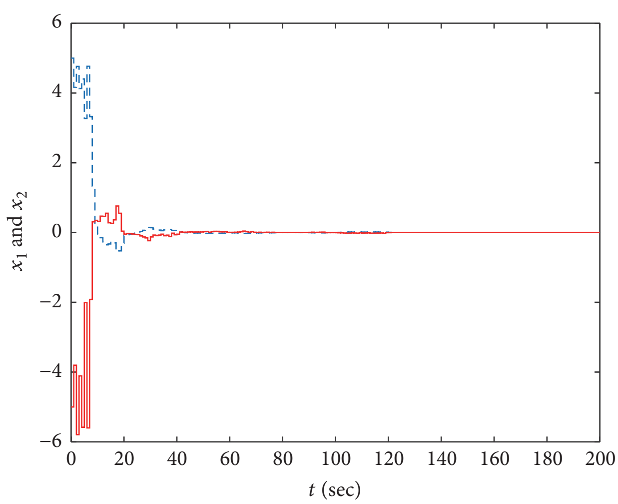

Figure 2: State trajectories of the closed-loop system.

By Theorem 5, the system is mean-square stabilizable which is verified by Figure 2. A robust stabilizing controller is given by

$$u(k) = Kx(k) = \begin{bmatrix} -0.092 & -0.1344 \end{bmatrix} x(k). \qquad (53)$$

Example 12. Consider system (42) with the following parameters:

$$\begin{aligned}
A_0 &= \begin{bmatrix} 1.2 & 0 \\ 0 & 1.1 \end{bmatrix}, \\
A_{0d} &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
B_0 &= \begin{bmatrix} 1.3 \\ 1 \end{bmatrix}, \\
C_0 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \\
B &= \begin{bmatrix} 1 \\ 0.65 \end{bmatrix}, \\
C &= \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}, \\
D &= \begin{bmatrix} 0.8 \\ 1 \end{bmatrix}, \\
C_{0d} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \\
D_0 &= \begin{bmatrix} 0.2 \\ 0.7 \end{bmatrix}, \\
E &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \\
G_{A_0} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \\
G_{A_{0d}} &= \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \\
G_{B_0} &= \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \\
G_{C_0} &= \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
G_{D_0} &= \begin{bmatrix} 0.1 \\ 0.7 \end{bmatrix}, \\
G_{C_{0d}} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
F(k) &= \begin{bmatrix} \cos(w(k)) & 0 \\ 0 & \sin(w(k)) \end{bmatrix}, \\
s &= 1.
\end{aligned} \qquad (54)$$

For perturbed system (42), we take the external disturbance as $\xi(k) = e^{-k}$ and the certain level as $\gamma = 0.8$. In addition, according to Lemma 3, an appropriate α is given as $\alpha = 4.9$. Then, by the result of Theorem 10, using LMI toolbox to solve (43) and (47), we find that $t_{\min} = -0.1046$, which means we have got a group of feasible solutions with

$$P = \begin{bmatrix} 1.7258 & 0.0320 \\ 0.0320 & 1.8314 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1.6581 & -0.0480 \\ -0.0480 & 1.5180 \end{bmatrix},$$

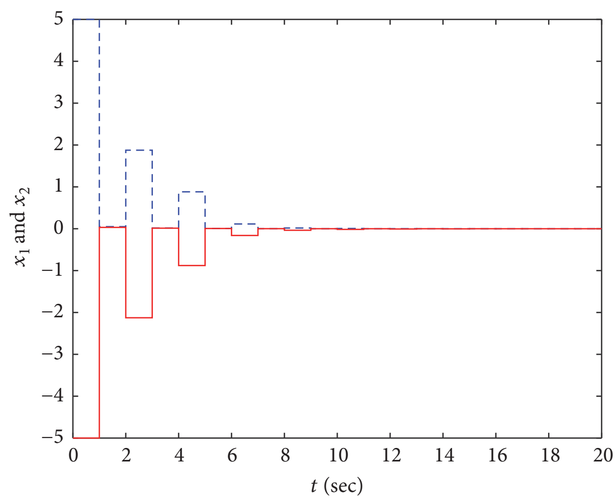

FIGURE 3: State trajectories of the closed-loop system.

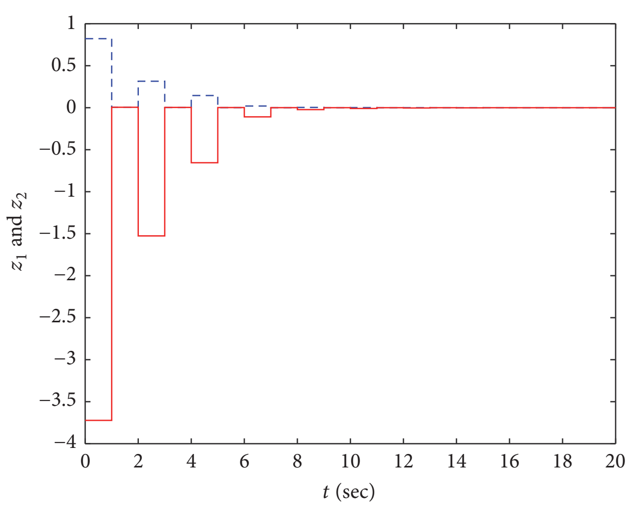

FIGURE 4: Controlled output trajectories of the closed-loop system.

$$K = \begin{bmatrix} -0.3281 & -0.2836 \end{bmatrix},$$
$$\beta = 3.0531. \tag{55}$$

The simulation results of state trajectories and controlled output trajectories of system (42) are given in Figures 3 and 4 with the H_∞ controller

$$u(k) = Kx(k) = \begin{bmatrix} -0.3281 & -0.2836 \end{bmatrix} x(k). \tag{56}$$

This further verifies the effectiveness of Theorem 10.

6. Conclusion

In this paper, we have studied the robust quadratic stability, quadratic stabilization, and robust H_∞ state feedback control of discrete-time stochastic systems with state delay and uncertain parameters. Based on LMI technique, a sufficient condition about quadratic stability and quadratic stabilization of our considered system is, respectively, given. Moreover, an H_∞ state feedback controller is obtained by solving two LMIs. Finally, we supply two simulation examples to show the validity of the proposed results. It is expected to solve the H_∞ output feedback control and H_∞ filtering in our forthcoming work.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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