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Research Article

The MLE of the Parameters of a Discrete Competitive System Subject to Environmental Noise

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A randomized discrete competitive system is investigated and the maximum likelihood estimation (MLE) of the parameters of the system is obtained. Also, a corresponding numerical simulation is offered to support our theoretical results.

1. Introduction

It is well known that various discrete competitive systems have received great attention owing to their theoretical and practical significance and there is a large volume of literature relevant to many good results (see [1–6]). Note that the population systems, in the real world, are often perturbed by various types of environmental noises. May [7] also pointed out that due to environmental fluctuation, the birth rate, the death rate, and other parameters usually show random fluctuation to a certain extent. To accurately describe such systems, it is necessary to use stochastic difference equations. In the present contribution, we will consider the maximum likelihood estimation (MLE) of the parameters of a discrete competitive system with environmental noise and our motivation comes from the works in [8–10]. Let us first introduce the following competitive system governed by differential equations:

$$\begin{aligned} x_1'(t) &= x_1(t)\left(b_1 - a_{11}x_1(t) - \frac{c_2x_2(t)}{1+x_2(t)}\right),\\ x_2'(t) &= x_2(t)\left(b_2 - a_{22}x_2(t) - \frac{c_1x_1(t)}{1+x_1(t)}\right), \end{aligned} \tag{1}$$

where b_i, $i = 1,2$, represents the intrinsic growth rate; a_{ii} and c_i, $i = 1,2$, stand for intraspecific competing rate and interspecific competing rate, respectively. For the relevant ecology of model (1) we refer to the readers to [11]. Obviously, by a computation, system (1) becomes

$$\begin{aligned} \frac{d\log x_1(t)}{dt} &= b_1 - a_{11}x_1(t) - \frac{c_2x_2(t)}{1+x_2(t)},\\ \frac{d\log x_2(t)}{dt} &= b_2 - a_{22}x_2(t) - \frac{c_1x_1(t)}{1+x_1(t)}. \end{aligned} \tag{2}$$

Considering system (2) with the equidistant points in time $0, \Delta t, 2\Delta t, \ldots, n\Delta t$ for some $\Delta t > 0$, we obtain

$$\begin{aligned} \log x_{1,k} &= \log x_{1,k-1} + \left(b_1 - a_{11}x_{1,k-1} - \frac{c_2x_{2,k-1}}{1+x_{2,k-1}}\right)\Delta t,\\ \log x_{2,k} &= \log x_{2,k-1} + \left(b_2 - a_{22}x_{2,k-1} - \frac{c_1x_{1,k-1}}{1+x_{1,k-1}}\right)\Delta t. \end{aligned} \tag{3}$$

Suppose that the parameter b_i, $i = 1,2$, is stochastically perturbed in the following way:

$$b_i \longrightarrow b_i + \sigma_i\varepsilon_{i,k}, \tag{4}$$

where $\varepsilon_{i,k} \sim N(0,1)$ and σ_i^2 stands for the noise intensity. Then system (3) can be described by the randomized equations:

$$\begin{aligned}\log x_{1,k} &= \log x_{1,k-1} + \left(b_1 - a_{11}x_{1,k-1} - \frac{c_2 x_{2,k-1}}{1+x_{2,k-1}}\right)\Delta t \\ &\quad + \sigma_1\sqrt{\Delta t}\varepsilon_{1,k}, \\ \log x_{2,k} &= \log x_{2,k-1} + \left(b_2 - a_{22}x_{2,k-1} - \frac{c_1 x_{1,k-1}}{1+x_{1,k-1}}\right)\Delta t \\ &\quad + \sigma_2\sqrt{\Delta t}\varepsilon_{2,k}.\end{aligned} \tag{5}$$

Here we choose initial value $x(0) = (x_1(0), x_2(0)) \in R_+^2$.

The main aim of this paper is to investigate the MLE of the parameters of system (5). To the best of our knowledge, there are few published papers concerned with system (5). The rest of this paper is organized as follows. Section 2 focuses on the maximum likelihood estimation of the parameters of system (5). Numerical simulation and discussion are presented in Section 3.

2. The MLE of the Parameters

In this section, we focus on the MLE of the parameters of system (5). Suppose that $\{(x_{1,k}, x_{2,k}),\ k = 1,2,\ldots,n\}$ are the real observed values from system (5). For the sake of simplicity, let $u_{i,k} = \log x_{i,k}$ $(i = 1,2;\ k = 1,2,\ldots,n)$. Suppose that $(U_{1,0},U_{2,0}), (U_{1,1},U_{2,1}),\ldots,(U_{1,n},U_{2,n})$ are the corresponding observed quantity and ρ is the correlation coefficient of $U_{1,l}$ and $U_{2,l}$, $l = 1,2,\ldots,n$. Then the following notations are used throughout this paper:

$$\alpha_1 = \frac{\sigma_2\Delta t}{n}\left[\sum_{k=1}^{n}\exp(u_{1,k-1})\right]^2 - \sigma_2\Delta t\sum_{k=1}^{n}\exp\left(2u_{1,k-1}\right),$$

$$\begin{aligned}\alpha_2 &= \frac{\sigma_2\Delta t}{n}\sum_{k=1}^{n}\exp\left(u_{1,k-1}\right)\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)} \\ &\quad - \sigma_2\Delta t\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}+u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)},\end{aligned}$$

$$\begin{aligned}\alpha_3 &= \rho\sigma_1\Delta t\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)} \\ &\quad - \frac{\rho\sigma_1\Delta t}{n}\sum_{k=1}^{n}\exp\left(u_{1,k-1}\right)\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)},\end{aligned}$$

$$\begin{aligned}\alpha_4 &= \rho\sigma_1\Delta t\sum_{k=1}^{n}\exp\left(u_{1,k-1}+u_{2,k-1}\right) \\ &\quad - \frac{\rho\sigma_1\Delta t}{n}\sum_{k=1}^{n}\exp\left(u_{1,k-1}\right)\sum_{k=1}^{n}\exp\left(u_{2,k-1}\right),\end{aligned}$$

$$\begin{aligned}\beta_1 &= \frac{\sigma_2\Delta t}{n}\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)}\sum_{k=1}^{n}\exp\left(u_{1,k}\right) \\ &\quad - \sigma_2\Delta t\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}+u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)},\end{aligned}$$

$$\begin{aligned}\beta_2 &= \frac{\sigma_2\Delta t}{n}\left[\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)}\right]^2 \\ &\quad - \sigma_2\Delta t\sum_{k=1}^{n}\frac{\exp\left(2u_{2,k-1}\right)}{\left[1+\exp\left(u_{2,k-1}\right)\right]^2},\end{aligned}$$

$$\begin{aligned}\beta_3 &= \rho\sigma_1\Delta t\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}+u_{2,k-1}\right)}{\left(1+\exp\left(u_{1,k-1}\right)\right)\left(1+\exp\left(u_{2,k-1}\right)\right)} \\ &\quad - \frac{\rho\sigma_1\Delta t}{n}\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)}\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)},\end{aligned}$$

$$\begin{aligned}\beta_4 &= \rho\sigma_1\Delta t\sum_{k=1}^{n}\frac{\exp\left(2u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)} \\ &\quad - \frac{\rho\sigma_1\Delta t}{n}\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)}\sum_{k=1}^{n}\exp\left(u_{2,k-1}\right),\end{aligned}$$

$$\begin{aligned}\gamma_1 &= \rho\sigma_2\Delta t\sum_{k=1}^{n}\frac{\exp\left(2u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)} \\ &\quad - \frac{\rho\sigma_2\Delta t}{n}\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)}\sum_{k=1}^{n}\exp\left(u_{1,k-1}\right),\end{aligned}$$

$$\begin{aligned}\gamma_2 &= \rho\sigma_2\Delta t\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}+u_{2,k-1}\right)}{\left(1+\exp\left(u_{1,k-1}\right)\right)\left(1+\exp\left(u_{2,k-1}\right)\right)} \\ &\quad - \frac{\rho\sigma_2\Delta t}{n}\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)}\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)},\end{aligned}$$

$$\begin{aligned}\gamma_3 &= \frac{\sigma_1\Delta t}{n}\left[\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)}\right]^2 \\ &\quad - \sigma_1\Delta t\sum_{k=1}^{n}\frac{\exp\left(2u_{1,k-1}\right)}{\left[1+\exp\left(u_{1,k-1}\right)\right]^2},\end{aligned}$$

$$\begin{aligned}\gamma_4 &= \frac{\sigma_1\Delta t}{n}\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)}\sum_{k=1}^{n}\exp\left(u_{2,k}\right) \\ &\quad - \sigma_1\Delta t\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}+u_{2,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)},\end{aligned}$$

$$\begin{aligned}\eta_1 &= \rho\sigma_2\Delta t\sum_{k=1}^{n}\exp\left(u_{1,k-1}+u_{2,k-1}\right) \\ &\quad - \frac{\rho\sigma_2\Delta t}{n}\sum_{k=1}^{n}\exp\left(u_{1,k-1}\right)\sum_{k=1}^{n}\exp\left(u_{2,k-1}\right),\end{aligned}$$

$$\begin{aligned}\eta_2 &= \rho\sigma_2\Delta t\sum_{k=1}^{n}\frac{\exp\left(2u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)} \\ &\quad - \frac{\rho\sigma_2\Delta t}{n}\sum_{k=1}^{n}\exp\left(u_{2,k-1}\right)\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)},\end{aligned}$$

$$\eta_3 = \frac{\sigma_1 \Delta t}{n} \sum_{k=1}^{n} \exp(u_{2,k-1}) \sum_{k=1}^{n} \frac{\exp(u_{1,k-1})}{1+\exp(u_{1,k-1})} - \sigma_1 \Delta t \sum_{k=1}^{n} \frac{\exp(u_{1,k-1}+u_{2,k-1})}{1+\exp(u_{1,k-1})},$$

$$\eta_4 = \frac{\sigma_1 \Delta t}{n} \left[\sum_{k=1}^{n} \exp(u_{2,k-1})\right]^2 - \sigma_1 \Delta t \sum_{k=1}^{n} \exp(2u_{2,k-1}),$$

$$\begin{aligned}\zeta_1 = {} & \sigma_2 \sum_{k=1}^{n} (u_{1,k}-u_{1,k-1}) \exp(u_{1,k-1}) - \frac{\sigma_2 (u_{1,n}-u_{1,0})}{n} \sum_{k=1}^{n} \exp(u_{1,k-1}) \\ & + \frac{\rho\sigma_1 (u_{2,n}-u_{2,0})}{n} \sum_{k=1}^{n} \exp(u_{1,k-1}) - \rho\sigma_1 \sum_{k=1}^{n} (u_{2,k}-u_{2,k-1}) \exp(u_{1,k-1}),\end{aligned}$$

$$\begin{aligned}\zeta_2 = {} & \sigma_2 \sum_{k=1}^{n} (u_{1,k}-u_{1,k-1}) \frac{\exp(u_{2,k-1})}{1+\exp(u_{2,k-1})} - \frac{\sigma_2 (u_{2,n}-u_{2,0})}{n} \sum_{k=1}^{n} \frac{\exp(u_{2,k-1})}{1+\exp(u_{2,k-1})} \\ & + \frac{\rho\sigma_1 (u_{2,n}-u_{2,0})}{n} \sum_{k=1}^{n} \frac{\exp(u_{2,k-1})}{1+\exp(u_{2,k-1})} - \rho\sigma_1 \sum_{k=1}^{n} (u_{2,k}-u_{2,k-1}) \frac{\exp(u_{2,k-1})}{1+\exp(u_{2,k-1})},\end{aligned}$$

$$\begin{aligned}\zeta_3 = {} & \sigma_1 \sum_{k=1}^{n} (u_{2,k}-u_{2,k-1}) \frac{\exp(u_{1,k-1})}{1+\exp(u_{1,k-1})} - \frac{\sigma_1 (u_{2,n}-u_{2,0})}{n} \sum_{k=1}^{n} \frac{\exp(u_{1,k-1})}{1+\exp(u_{1,k-1})} \\ & + \frac{\rho\sigma_2 (u_{1,n}-u_{1,0})}{n} \sum_{k=1}^{n} \frac{\exp(u_{1,k-1})}{1+\exp(u_{1,k-1})} - \rho\sigma_2 \sum_{k=1}^{n} (u_{1,k}-u_{1,k-1}) \frac{\exp(u_{1,k-1})}{1+\exp(u_{1,k-1})},\end{aligned}$$

$$\begin{aligned}\zeta_4 = {} & \sigma_1 \sum_{k=1}^{n} (u_{2,k}-u_{2,k-1}) \exp(u_{2,k-1}) - \frac{\sigma_1 (u_{2,n}-u_{2,0})}{n} \sum_{k=1}^{n} \exp(u_{2,k-1}) \\ & + \frac{\rho\sigma_2 (u_{1,n}-u_{1,0})}{n} \sum_{k=1}^{n} \exp(u_{2,k-1}) - \rho\sigma_2 \sum_{k=1}^{n} (u_{1,k}-u_{1,k-1}) \exp(u_{2,k-1}),\end{aligned}$$

$$A_1 = \frac{1}{\Delta t (1-\rho^2)} \sum_{k=1}^{n} [u_{1,k}-u_{1,k-1}-M_{1,2}(k)\Delta t]^2,$$

$$A_2 = \frac{1}{\Delta t (1-\rho^2)} \sum_{k=1}^{n} [u_{2,k}-u_{2,k-1}-M_{2,1}(k)\Delta t]^2,$$

$$B = \frac{\rho}{\Delta t (1-\rho^2)} \sum_{k=1}^{n} [u_{1,k}-u_{1,k-1}-M_{1,2}(k)\Delta t] \cdot [u_{2,k}-u_{2,k-1}-M_{2,1}(k)\Delta t],$$

$$M_{1,2}(k) = b_1 - a_{11}\exp(u_{1,k-1}) - \frac{c_2 \exp(u_{2,k-1})}{1+\exp(u_{2,k-1})},$$

$$M_{2,1}(k) = b_2 - a_{22}\exp(u_{2,k-1}) - \frac{c_1 \exp(u_{1,k-1})}{1+\exp(u_{1,k-1})}. \qquad (6)$$

Then we have the following main result.

Theorem 1. *The maximum likelihood estimation of the parameters of system (5) can be expressed as*

$$\begin{aligned}\widehat{a_{ii}} &= \frac{D_{ii}}{D}, \quad \widehat{c_i} = \frac{D_i}{D}, \quad (i=1,2), \\ \widehat{\sigma_1}^2 &= \frac{A_1A_2 - B\sqrt{A_1A_2}}{nA_2}, \\ \widehat{\sigma_2}^2 &= \frac{A_1A_2 - B\sqrt{A_1A_2}}{nA_1}, \\ \widehat{b_1} &= \frac{1}{n\Delta t}\left[u_{1,n}-u_{1,0} + \frac{D_{11}}{D}\Delta t \sum_{k=1}^{n}\exp(u_{1,k-1}) + \frac{D_2}{D}\Delta t \sum_{k=1}^{n}\frac{\exp(u_{2,k-1})}{1+\exp(u_{2,k-1})}\right], \\ \widehat{b_2} &= \frac{1}{n\Delta t}\left[u_{2,n}-u_{2,0} + \frac{D_{22}}{D}\Delta t \sum_{k=1}^{n}\exp(u_{2,k-1}) + \frac{D_1}{D}\Delta t \sum_{k=1}^{n}\frac{\exp(u_{1,k-1})}{1+\exp(u_{1,k-1})}\right],\end{aligned} \qquad (7)$$

where

$$D = \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \\ \eta_1 & \eta_2 & \eta_3 & \eta_4 \end{vmatrix}, \qquad D_{11} = \begin{vmatrix} \zeta_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \zeta_1 & \beta_2 & \beta_3 & \beta_4 \\ \zeta_1 & \gamma_2 & \gamma_3 & \gamma_4 \\ \zeta_1 & \eta_2 & \eta_3 & \eta_4 \end{vmatrix},$$

$$D_{22} = \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \zeta_1 \\ \beta_1 & \beta_2 & \beta_3 & \zeta_2 \\ \gamma_1 & \gamma_2 & \gamma_3 & \zeta_3 \\ \eta_1 & \eta_2 & \eta_3 & \zeta_4 \end{vmatrix}, \qquad (8)$$

$$D_1 = \begin{vmatrix} \alpha_1 & \alpha_2 & \zeta_1 & \alpha_4 \\ \beta_1 & \beta_2 & \zeta_2 & \beta_4 \\ \gamma_1 & \gamma_2 & \zeta_3 & \gamma_4 \\ \eta_1 & \eta_2 & \zeta_4 & \eta_4 \end{vmatrix}, \qquad D_2 = \begin{vmatrix} \alpha_1 & \zeta_1 & \alpha_3 & \alpha_4 \\ \beta_1 & \zeta_2 & \beta_3 & \beta_4 \\ \gamma_1 & \zeta_3 & \gamma_3 & \gamma_4 \\ \eta_1 & \zeta_4 & \eta_3 & \eta_4 \end{vmatrix}.$$

Proof. Making the change of variable $u_{i,k} = \log x_{i,k}$, $(i = 1, 2)$, we can rewrite system (5) as

$$u_{1,k} = u_{1,k-1} + \left(b_1 - a_{11}\exp u_{1,k-1} - \frac{c_2 \exp(u_{2,k-1})}{1 + \exp(u_{2,k-1})}\right)\Delta t + \sigma_1\sqrt{\Delta t}\varepsilon_{1,k},$$
$$u_{2,k} = u_{2,k-1} + \left(b_2 - a_{22}\exp u_{2,k-1} - \frac{c_1 \exp(u_{1,k-1})}{1 + \exp(u_{1,k-1})}\right)\Delta t + \sigma_1\sqrt{\Delta t}\varepsilon_{1,k}. \quad (9)$$

Suppose that $(U_{1,0}, U_{2,0}), (U_{1,1}, U_{2,1}), \ldots, (U_{1,n}, U_{2,n})$ are the observed quantity and $(u_{1,0}, u_{2,0}), (u_{1,1}, u_{2,1}), \ldots, (u_{1,n}, u_{2,n})$ are the corresponding observed values of system (9). The information flow is given by $\mathscr{F}_{k-1} = \sigma((U_{1,l}, U_{2,l}), l \le k-1)$. Denote by ρ the correlation coefficient of $U_{1,l}$ and $U_{2,l}$. Then, for given $\mathscr{F}_{k-1}$, the conditional density function of $(U_{1,k}, U_{2,k})$ can be expressed as

$$f((u_{1,k}, u_{2,k}) \mid \mathscr{F}_{k-1}) = \frac{1}{2\pi\sigma_1\sigma_2\Delta t\sqrt{1-\rho^2}} \cdot \exp\left\{-\frac{1}{2(1-\rho^2)} \cdot \left[\left(\frac{u_{1,k} - u_{1,k-1} - M_{1,2}(k)\Delta t}{\sigma_1\sqrt{\Delta t}}\right)^2 - 2\rho\left(\frac{u_{1,k} - u_{1,k-1} - M_{1,2}(k)\Delta t}{\sigma_1\sqrt{\Delta t}}\right) \cdot \left(\frac{u_{2,k} - u_{2,k-1} - M_{2,1}(k)\Delta t}{\sigma_2\sqrt{\Delta t}}\right) + \left(\frac{u_{2,k} - u_{2,k-1} - M_{2,1}(k)\Delta t}{\sigma_2\sqrt{\Delta t}}\right)^2\right]\right\}, \quad (10)$$

where

$$M_{1,2}(k) = b_1 - a_{11}\exp(u_{1,k-1}) - \frac{c_2\exp(u_{2,k-1})}{1 + \exp(u_{2,k-1})},$$
$$M_{2,1}(k) = b_2 - a_{22}\exp(u_{2,k-1}) - \frac{c_1\exp(u_{1,k-1})}{1 + \exp(u_{1,k-1})}. \quad (11)$$

Then, for given $\mathscr{F}_0$, the joint conditional density function of $((U_{1,1}, U_{2,1}), (U_{1,2}, U_{2,2}), \ldots, (U_{1,n}, U_{2,n}))$ is

$$f(((u_{1,1}, u_{2,1}), (u_{1,2}, u_{2,2}), \ldots, (u_{1,n}, u_{2,n})) \mid \mathscr{F}_0) = \left(\frac{1}{2\pi\Delta t\sqrt{1-\rho^2}\sigma_1\sigma_2}\right)^n \cdot \prod_{k=1}^{n}\exp\left\{-\frac{1}{2(1-\rho^2)} \cdot \left[\left(\frac{u_{1,k} - u_{1,k-1} - M_{1,2}(k)\Delta t}{\sigma_1\sqrt{\Delta t}}\right)^2 - 2\rho\left(\frac{u_{1,k} - u_{1,k-1} - M_{1,2}(k)\Delta t}{\sigma_1\sqrt{\Delta t}}\right) \cdot \left(\frac{u_{2,k} - u_{2,k-1} - M_{2,1}(k)\Delta t}{\sigma_2\sqrt{\Delta t}}\right) + \left(\frac{u_{2,k} - u_{2,k-1} - M_{2,1}(k)\Delta t}{\sigma_2\sqrt{\Delta t}}\right)^2\right]\right\}. \quad (12)$$

With the constants omitted, the logarithmic likelihood function can be written as

$$L_n(b_1, b_2, a_{11}, a_{22}, c_1, c_2, \sigma_1^2, \sigma_2^2) = -\frac{n}{2}(\log\sigma_1^2 + \log\sigma_2^2) - \frac{1}{2\sigma_1^2\Delta t(1-\rho^2)} \cdot \sum_{k=1}^{n}[u_{1,k} - u_{1,k-1} - M_{1,2}(k)\Delta t]^2 + \frac{\rho}{\Delta t(1-\rho^2)}\sum_{k=1}^{n}\left[\frac{u_{1,k} - u_{1,k-1} - M_{1,2}(k)\Delta t}{\sigma_1} \times \frac{u_{2,k} - u_{2,k-1} - M_{2,1}(k)\Delta t}{\sigma_2}\right] - \frac{1}{2\sigma_2^2\Delta t(1-\rho^2)}\sum_{k=1}^{n}[u_{2,k} - u_{2,k-1} - M_{2,1}(k)\Delta t]^2. \quad (13)$$

Taking partial derivative with respect to parameters $b_1, b_2, a_{11}, a_{22}, c_1, c_2, \sigma_1, \sigma_2$ in (13), respectively, one obtains the following likelihood equations:

$$n\rho\Delta t\sigma_1 b_2 - n\sigma_2 b_1\Delta t = \rho\sigma_1(u_{2,n} - u_{2,0}) - \sigma_2(u_{1,n} - u_{1,0}) + c_1\rho\Delta t\sigma_1\sum_{k=1}^{n}\frac{\exp(u_{1,k})}{1 + \exp(u_{1,k})} - c_2\Delta t\sigma_2\sum_{k=1}^{n}\frac{\exp(u_{2,k})}{1 + \exp(u_{2,k})} + a_{22}\rho\Delta t\sigma_1\sum_{k=1}^{n}\exp(u_{2,k-1}) - a_{11}\Delta t\sigma_2\sum_{k=1}^{n}\exp(u_{1,k-1}), \quad (14)$$

$$\begin{aligned} & n\rho\Delta t\left(\sigma_2 b_1-\sigma_1 b_2\right) \\ & \quad =\rho\sigma_2\left(u_{1,n}-u_{1,0}\right)-\sigma_1\left(u_{2,n}-u_{2,0}\right) \\ & \quad\quad +c_2\rho\Delta t\sigma_2\sum_{k=1}^{n}\frac{\exp\left(u_{2,k}\right)}{1+\exp\left(u_{2,k}\right)} \\ & \quad\quad -c_1\Delta t\sigma_1\sum_{k=1}^{n}\frac{\exp\left(u_{1,k}\right)}{1+\exp\left(u_{1,k}\right)}+a_{11}\rho\Delta t\sigma_2\sum_{k=1}^{n}\exp\left(u_{1,k-1}\right) \\ & \quad\quad -a_{22}\Delta t\sigma_1\sum_{k=1}^{n}\exp\left(u_{2,k-1}\right), \end{aligned} \tag{15}$$

$$\begin{aligned} & \sigma_2\sum_{k=1}^{n}\left[u_{1,k}-u_{1,k-1}-M_{1,2}(k)\Delta t\right]\exp\left(u_{1,k-1}\right) \\ & \quad =\rho\sigma_1\sum_{k=1}^{n}\left[u_{2,k}-u_{2,k-1}-M_{2,1}(k)\Delta t\right]\exp\left(u_{1,k-1}\right), \end{aligned} \tag{16}$$

$$\begin{aligned} & \sigma_2\sum_{k=1}^{n}\left[u_{1,k}-u_{1,k-1}-M_{1,2}(k)\Delta t\right]\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)} \\ & \quad =\rho\sigma_1\sum_{k=1}^{n}\left[u_{2,k}-u_{2,k-1}-M_{2,1}(k)\Delta t\right]\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)}, \end{aligned} \tag{17}$$

$$\begin{aligned} & \sigma_1\sum_{k=1}^{n}\left[u_{2,k}-u_{2,k-1}-M_{2,1}(k)\Delta t\right]\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)} \\ & \quad =\rho\sigma_2\sum_{k=1}^{n}\left[u_{1,k}-u_{1,k-1}-M_{1,2}(k)\Delta t\right]\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)}, \end{aligned} \tag{18}$$

$$\begin{aligned} & \sigma_1\sum_{k=1}^{n}\left[u_{2,k}-u_{2,k-1}-M_{2,1}(k)\Delta t\right]\exp\left(u_{2,k-1}\right) \\ & \quad =\rho\sigma_2\sum_{k=1}^{n}\left[u_{1,k}-u_{1,k-1}-M_{1,2}(k)\Delta t\right]\exp\left(u_{2,k-1}\right), \end{aligned} \tag{19}$$

$$\begin{aligned} & -\frac{n}{2\sigma_1^2}+\frac{1}{2\sigma_1^4\Delta t\left(1-\rho^2\right)}\sum_{k=1}^{n}\left[u_{1,k}-u_{1,k-1}-M_{1,2}(k)\Delta t\right]^2 \\ & \quad =\frac{\rho}{2\sigma_1^3\sigma_2\Delta t\left(1-\rho^2\right)}\sum_{k=1}^{n}\left[u_{1,k}-u_{1,k-1}-M_{1,2}(k)\Delta t\right] \\ & \quad\quad \cdot\left[u_{2,k}-u_{2,k-1}-M_{2,1}(k)\Delta t\right], \end{aligned} \tag{20}$$

$$\begin{aligned} & -\frac{n}{2\sigma_2^2}+\frac{1}{2\sigma_2^4\Delta t\left(1-\rho^2\right)}\sum_{k=1}^{n}\left[u_{2,k}-u_{2,k-1}-M_{2,1}(k)\Delta t\right]^2 \\ & \quad =\frac{\rho}{2\sigma_1\sigma_2^3\Delta t\left(1-\rho^2\right)}\sum_{k=1}^{n}\left[u_{1,k}-u_{1,k-1}-M_{1,2}(k)\Delta t\right] \\ & \quad\quad \cdot\left[u_{2,k}-u_{2,k-1}-M_{2,1}(k)\Delta t\right]. \end{aligned} \tag{21}$$

By (14) and (15), one has

$$\begin{aligned} b_1= & \frac{1}{n\Delta t}\left[u_{1,n}-u_{1,0}+a_{11}\Delta t\sum_{k=1}^{n}\exp\left(u_{1,k-1}\right)\right. \\ & \left.+c_2\Delta t\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)}\right], \end{aligned} \tag{22}$$

$$\begin{aligned} b_2= & \frac{1}{n\Delta t}\left[u_{2,n}-u_{2,0}+a_{22}\Delta t\sum_{k=1}^{n}\exp\left(u_{2,k-1}\right)\right. \\ & \left.+c_1\Delta t\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)}\right]. \end{aligned} \tag{23}$$

It then follows from (22), (23), and (16)–(19) that

$$\begin{aligned} & \alpha_1 a_{11}+\alpha_2 c_2+\alpha_3 c_1+\alpha_4 a_{22} \\ & \quad =\sigma_2\sum_{k=1}^{n}\left(u_{1,k}-u_{1,k-1}\right)\exp\left(u_{1,k-1}\right) \\ & \quad\quad -\frac{\sigma_2\left(u_{1,n}-u_{1,0}\right)}{n}\sum_{k=1}^{n}\exp\left(u_{1,k-1}\right) \\ & \quad\quad +\frac{\rho\sigma_1\left(u_{1,n}-u_{1,0}\right)}{n}\sum_{k=1}^{n}\exp\left(u_{1,k-1}\right) \\ & \quad\quad -\rho\sigma_1\sum_{k=1}^{n}\left(u_{2,k}-u_{2,k-1}\right)\exp\left(u_{1,k-1}\right), \end{aligned} \tag{24}$$

$$\begin{aligned} & \beta_1 a_{11}+\beta_2 c_2+\beta_3 c_1+\beta_4 a_{22} \\ & \quad =\sigma_2\sum_{k=1}^{n}\left(u_{1,k}-u_{1,k-1}\right)\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)} \\ & \quad\quad -\frac{\sigma_2\left(u_{2,n}-u_{2,0}\right)}{n}\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)} \\ & \quad\quad +\frac{\rho\sigma_1\left(u_{2,n}-u_{2,0}\right)}{n}\sum_{k=1}^{n}\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)} \\ & \quad\quad -\rho\sigma_1\sum_{k=1}^{n}\left(u_{2,k}-u_{2,k-1}\right)\frac{\exp\left(u_{2,k-1}\right)}{1+\exp\left(u_{2,k-1}\right)}, \end{aligned} \tag{25}$$

$$\begin{aligned} & \gamma_1 a_{11}+\gamma_2 c_2+\gamma_3 c_1+\gamma_4 a_{22} \\ & \quad =\sigma_1\sum_{k=1}^{n}\left(u_{2,k}-u_{2,k-1}\right)\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)} \\ & \quad\quad -\frac{\sigma_1\left(u_{2,n}-u_{2,0}\right)}{n}\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)} \\ & \quad\quad +\frac{\rho\sigma_2\left(u_{1,n}-u_{1,0}\right)}{n}\sum_{k=1}^{n}\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)} \\ & \quad\quad -\rho\sigma_2\sum_{k=1}^{n}\left(u_{1,k}-u_{1,k-1}\right)\frac{\exp\left(u_{1,k-1}\right)}{1+\exp\left(u_{1,k-1}\right)}, \end{aligned} \tag{26}$$

$$
\begin{aligned}
&\eta_1 a_{11} + \eta_2 c_2 + \eta_3 c_1 + \eta_4 a_{22} \\
&\quad = \sigma_1 \sum_{k=1}^{n} \left(u_{2,k} - u_{2,k-1}\right) \exp\left(u_{2,k-1}\right) \\
&\qquad - \frac{\sigma_1 \left(u_{2,n} - u_{2,0}\right)}{n} \sum_{k=1}^{n} \exp\left(u_{2,k-1}\right) \\
&\qquad + \frac{\rho \sigma_2 \left(u_{1,n} - u_{1,0}\right)}{n} \sum_{k=1}^{n} \exp\left(u_{2,k-1}\right) \\
&\qquad - \rho \sigma_2 \sum_{k=1}^{n} \left(u_{1,k} - u_{1,k-1}\right) \exp\left(u_{2,k-1}\right).
\end{aligned} \tag{27}
$$

So, the Gramer rule implies that

$$
\widehat{a_{ii}} = \frac{D_{ii}}{D}, \quad \widehat{c_i} = \frac{D_i}{D}, \quad (i = 1, 2). \tag{28}
$$

Further, from (22) and (23), one achieves the fact that

$$
\begin{aligned}
\widehat{b_1} &= \frac{1}{n\Delta t} \left[u_{1,n} - u_{1,0} + \frac{D_{11}}{D} \Delta t \sum_{k=1}^{n} \exp\left(u_{1,k-1}\right) \right. \\
&\qquad \left. + \frac{D_2}{D} \Delta t \sum_{k=1}^{n} \frac{\exp\left(u_{2,k-1}\right)}{1 + \exp\left(u_{2,k-1}\right)} \right], \\
\widehat{b_2} &= \frac{1}{n\Delta t} \left[u_{2,n} - u_{2,0} + \frac{D_{22}}{D} \Delta t \sum_{k=1}^{n} \exp\left(u_{2,k-1}\right) \right. \\
&\qquad \left. + \frac{D_1}{D} \Delta t \sum_{k=1}^{n} \frac{\exp\left(u_{1,k-1}\right)}{1 + \exp\left(u_{1,k-1}\right)} \right].
\end{aligned} \tag{29}
$$

It is easy to see from (20), (21), (23), and (24) that

$$
\begin{aligned}
-\frac{n}{\sigma_1^2} + \frac{A_1}{\sigma_1^4} - \frac{B}{\sigma_1^3 \sigma_2} &= 0, \\
-\frac{n}{\sigma_2^2} + \frac{A_2}{\sigma_2^4} - \frac{B}{\sigma_2^3 \sigma_1} &= 0.
\end{aligned} \tag{30}
$$

Then one achieves the fact that

$$
\begin{aligned}
\widehat{\sigma_1}^2 &= \frac{A_1 A_2 - B\sqrt{A_1 A_2}}{n A_2}, \\
\widehat{\sigma_2}^2 &= \frac{A_1 A_2 - B\sqrt{A_1 A_2}}{n A_1}.
\end{aligned} \tag{31}
$$

This completes the proof. □

3. Numerical Simulations

In this section, we give some simulation results of the above estimators to compare the true values and the estimators. In Tables 1–4, we fix the true values of parameters as follows: $a_{11} = 2$, $a_{22} = 1$, $b_1 = 1$, $b_2 = 1.5$, $c_1 = 1.2$, $c_2 = 1.8$, $\rho = 0.65$, $\sigma_1^2 = 0.09$, and $\sigma_2^2 = 0.16$, the number of the sample "size n" increases from 500 to 2000, the data of the columns named Average are obtained by the average of 10 MLEs from the data coming from (5) with $x_1(0) = 0.5$, $x_2(0) = 0.44$, $a_{11} = 2$, $a_{22} = 1$, $b_1 = 1$, $b_2 = 1.5$, $c_1 = 1.2$, $c_2 = 1.8$, $\rho = 0.65$, $\sigma_1^2 = 0.09$, $\sigma_2^2 = 0.16$, and $\Delta t = 0.1$. The columns named AE show the absolute error of MLE. In Tables 1–4, it is easy to see that there is a correlation between the absolute error of MLE and the sample size. Generally speaking, with the increase in the number of the sample, the absolute error of MLE will decrease. Therefore, it is reasonable to estimate the parameters of system (5) by MLE.

Table 1: The simulation values of the MLE of (a_{11}, a_{22}) when $\Delta t = 0.1$.

True (a_{11}, a_{22})	Size n	Average a_{11}-MLE	Average a_{22}-MLE	AE a_{11}	AE a_{22}
(2, 1)	500	1.991630	1.014662	0.008370	0.014662
	1000	2.004029	0.992371	0.004029	0.007629
	2000	2.006013	1.005748	0.006013	0.005748

Table 2: The simulation values of the MLE of (b_1, b_2) when $\Delta t = 0.1$.

True (b_1, b_2)	Size n	Average b_1-MLE	Average b_2-MLE	AE b_1	AE b_2
(1, 1.5)	500	1.013570	1.508662	0.013570	0.008662
	1000	0.993755	1.504625	0.006245	0.004625
	2000	1.004742	1.496252	0.004742	0.003748

Table 3: The simulation values of the MLE of (c_1, c_2) when $\Delta t = 0.1$.

True (c_1, c_2)	Size n	Average c_1-MLE	Average c_2-MLE	AE c_1	AE c_2
(1.2, 1.8)	500	1.225380	1.785378	0.025380	0.014622
	1000	1.209025	1.808455	0.009025	0.008455
	2000	1.207742	1.854748	0.007742	0.054748

Table 4: The simulation values of the MLE of (σ_1^2, σ_2^2) when $\Delta t = 0.1$.

True (σ_1^2, σ_2^2)	Size n	Average σ_1^2-MLE	Average σ_2^2-MLE	AE σ_1^2	AE σ_2^2
(0.09, 0.16)	500	0.093120	0.164635	0.003120	0.004635
	1000	0.086975	0.163322	0.003025	0.003322
	2000	0.093742	0.163342	0.003742	0.003342

Conflict of Interests

The authors declare that they have no conflict of interests.

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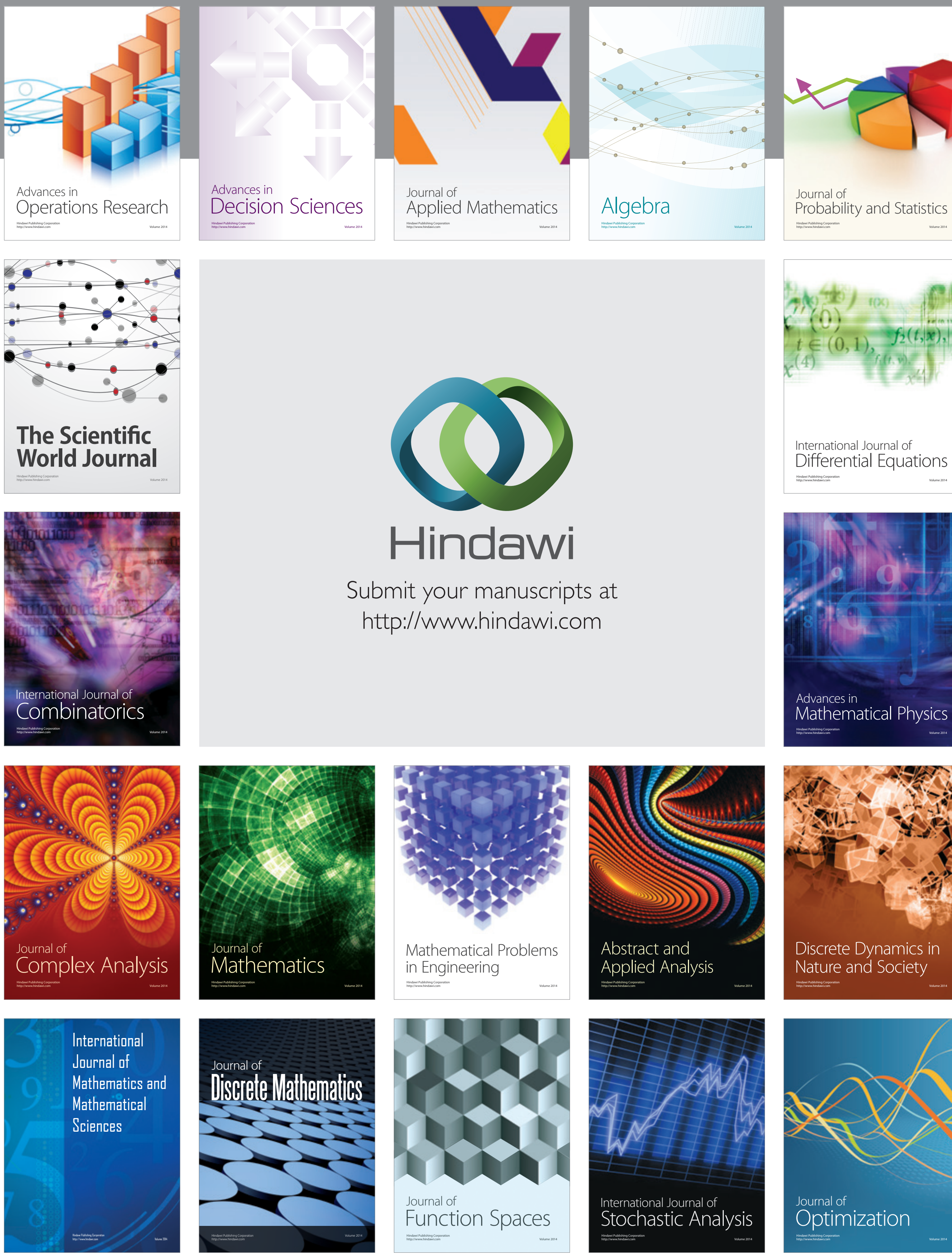
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