Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2011, Article ID 941481, 9 pages
doi:10.1155/2011/941481

Research Article

Implementing Estimation of Capacity for Freeway Sections

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Received 3 December 2010; Accepted 19 January 2011

Academic Editor: Shuyu Sun

Based on the stochastic concept for freeway capacity, the procedure of capacity estimation is developed. Due to the fact that it is impossible to observe the value of the capacity and to obtain the probability distribution of the capacity, the product-limit method is used in this paper to estimate the capacity. In order to implement estimation of capacity using this technology, the lifetime table based on statistical methods for lifetime data analysis is introduced and the corresponding procedure is developed. Simulated data based on freeway sections in Beijing, China, were analyzed and the results indicate that the methodology and procedure are applicable and validated.

1. Introduction

In the last decades, the capacity of a freeway section is most taken as a constant value. For example, in the currently published Highway Capacity Manual (HCM2000) [1], it was still defined as the maximum hourly rate at which vehicles reasonably can be expected to traverse a point or a uniform section of a lane or roadway during a given time period under prevailing roadway, traffic, and control conditions. However, more and more investigations [2–11] showed that the traditional definition of capacity cannot explain the traffic phenomena such as traffic breakdown occurring at various flow rate levels. And it is shown that the freeway capacity is not a deterministic but a stochastic phenomenon.

Despite the fact that traditional definition of capacity has played an important role in traffic planning, design, and operational analysis, it was challenged due to the inadequacy and impracticality. As a result, a stochastic concept for freeway capacity was defined and new methods were suggested [6–8]. But how to estimate capacity based on the new concept and methods was not discussed in detail.

The aim of this paper is to develop a procedure for estimating the capacity of a freeway section based on the stochastic concept for freeway capacity. To fulfill this purpose, the traffic flow rate is divided into several intervals according to the surveyed data and requirement of capacity estimation precision. Based on the data, the distribution of traffic breakdown flow was estimated using maximum likelihood estimation. Then, the definition of capacity based on the probability of breakdown was used and the capacity compute model was developed. In order to illustrate how to use the method and traffic breakdown data to estimate the capacity of a freeway section, the simulation result was provided.

2. Literature Review

In order to understand the freeway capacity, researchers have made great efforts to model and describe stochastic nature of freeway capacity and breakdown phenomenon. Polus and Pollatschek [2] studied the stochastic nature of freeway capacity and used Gamma distribution to describe the capacity. But the definition of capacity is still based on the maximum flow rates. Lorenz and Elefteriadou [6, 7] examined the process of breakdown in detail and noted that the current HCM freeway capacity definition does not accurately reflect the relationship between breakdown and flow rate. They also suggested that freeway capacity should be described by incorporating a probability of breakdown component in the definition. Brilon et al. [8] introduced a new understanding of freeway capacity as the traffic flow below which traffic still flows and above which the traffic breaks down. In their study, a traffic breakdown is regarded as a failure event and the Product Limit Method for lifetime data analysis was used to estimate the capacity. Elefteriadou et al. [9] studied the breakdown flow rates. It was found that breakdown may occur at flows lower than the capacity and that breakdown is a probabilistic event. A linear model for describing the process of breakdown at ramp-freeway junction was developed. Elefteriadou and Ponlathep [10] examined the breakdown flow, prebreakdown flow, and maximum discharge flow to determine which one would be more appropriate for use in defining the “capacity” of a freeway. It was recommended that the breakdown flow be used. But they also pointed out that the numerical value of each of these three parameters varies and their range is relatively large, in the order of several hundred veh/h/lanes. Ozbay and Ozguven [11] provided the parameter estimating method for capacity probability distribution. It was concluded that including random component in the model results in better representation of observed data and thus improves understanding of real-life situation.

From the efforts of these researches, it is concluded that there is variability in the maximum flow rates and that the breakdown flows vary greatly. The breakdown phenomenon and breakdown flow rate should be considered in the model. So, the traditional definition of freeway capacity is not proper, and the definition based on probability distribution should be the better one.

3. Methodology

According to the definition of traffic breakdown, when the traffic flow exceeds the capacity, the traffic will break down. Hence, the capacity cannot be observed directly. The breakdown flow rate which is defined as the flow rate [7] (expressed as a per-lane, equivalent hourly rate) observed immediately prior to breakdown is used to estimate the capacity. Due to the fact that the breakdown flow rate is not the real observed value of capacity, the lifetime analysis method was used to model the capacity distribution and based on it a procedure is suggested to estimate the capacity.

3.1. The Theory of Lifetime Data Analysis and Product Limit Method

In order to capture the real-life situation of the freeway capacity, a lifetime analysis application was proposed by Brilon et al. [8]. The capacity was regarded as a random variable C having its distribution as

$$F_c(q) = P(C < q), \tag{3.1}$$

where $F_c(q)$ is the probability function of capacity C, C is the freeway capacity, and q is the flow rate.

According to the definition of breakdown, when the traffic flow rate q reaches the capacity C, breakdown will occur. Therefore, the $F_c(q)$ is also the probability that the breakdown occurs at the flow rate q.

Analogically, the capacity survival function was defined to describe the probability that when traffic flow is above capacity, the traffic breakdown does not occur:

$$S_c(q) = P(C > q), \tag{3.2}$$

where $S_c(q)$ is the survival function.

In addition, the "product-limit method" [8] can be used to estimate the capacity survival function and capacity distribution function as

$$\begin{aligned} \widehat{S}_c(q) &= \prod_{i:q_i \le q} \frac{n_i - d_i}{n_i}, \\ \widehat{F}_c(q) &= 1 - \prod_{i:q_i \le q} \frac{n_i - d_i}{n_i}, \end{aligned} \tag{3.3}$$

where $\widehat{S}_c(q)$ is the estimate of survival function, q is the flow rate, q_i is the traffic flow rate in interval i, n_i is the number of intervals with traffic flow rate $q \le q_i$ and breakdown does not occur, $i = 1, \ldots, k+1$, d_i is the observed number of breakdown at a flow rate of q_i, $i = 1, \ldots, k+1$, and $\widehat{F}_c(q)$ is the estimated probability function of capacity C.

Equation (3.3) is the basis of estimating freeway capacity distribution, and it is directly transplanted from the lifetime data analysis. However, there is some difference between traffic data and lifetime data. First, to the lifetime data, the real individual death time can be observed in theory, but to the traffic flow, breakdown often lags traffic flow rate (traffic breakdown does not occur immediately after traffic flow overruns the capacity), and the observed time interval has a great effect on the analysis result. Second, there are "censored" observations which are used to explain that some individuals' explicit lifetimes are not observed in the lifetime data. In this sense, all traffic flow rates are "censored" data due to the reason that the capacity is not deterministic and it cannot be observed directly. If all the flow rates are taken into account, the capacity will be underestimated because that breakdown only occurred at few time intervals and it even can be taken as a small probability event. For example, for some days, the breakdown only occurred during morning and afternoon peak hours. If the time interval is 5 minutes, the probability of breakdown is only 1/144. Thereby, it is not appropriate to use observed traffic flow rate for all time intervals to estimate the capacity. In other words, only breakdown flow rates should be used.

3.2. Estimation of the Freeway Capacity Distribution

Based on the above analysis, only the breakdown flow rates are used to estimate the capacity. A procedure is developed as follows by using lifetime table method and theory in survival analysis [12].

Step 1. The breakdown traffic flow rates are divided into $k+1$ intervals and the jth interval is noted as $I_j = [a_{j-1}, a_j)$, $(j = 1, \dots, k+1)$, where a_0 can be determined by $a_0 = \min\{q_i,\ i = 1, \dots, N\} - q_0$, where q_i is the observed traffic breakdown flow rate at which the ith traffic breakdown occurs, N is the number of observed traffic breakdowns, q_0 is a constant greater than 0; $a_j = a_0 + h \times j$, $j = 1, \dots, k$, where h is the interval length (h can be determined according to the requirement of estimation precision), $a_{k+1} = +\infty$.

Step 2. Calculate the statistics. Let N_j be the number of traffic breakdowns with a traffic flow rate $q > a_{j-1}$, d_j the number of traffic breakdowns with a traffic flow at the interval of $I_j = [a_{j-1}, a_j)$, $P(j)$ the probability that traffic breakdown does not occur with traffic flow rate $q < a_j$, p_j the probability that traffic breakdown does not occur with flow rate of $q < a_j$ while on the condition that traffic breakdown does not occur with flow rate of $q < a_{j-1}$, and $q(j)$ the probability that traffic breakdown occurs with a traffic volume at the interval of $I_j = [a_{j-1}, a_j)$ while on the condition that traffic breakdown does not occur with flow rate of $q < a_{j-1}$.

Then the following results can be obtained:

$$P(0) = 1, \tag{3.4}$$

$$P(k+1) = 0, \tag{3.5}$$

$$N_j = N_{j-1} - d_{j-1}, \tag{3.6}$$

$$p_j = \frac{P(j)}{P(j-1)}, \tag{3.7}$$

$$q(j) = 1 - p_j, \tag{3.8}$$

$$P(j) = p_1 p_2 \cdots p_j. \tag{3.9}$$

Step 3. Estimate the distribution of freeway capacity.

According to the knowledge of the probability theory [12], the number of traffic breakdowns $d_1, d_2, \dots, d_k$ which are observed on the volume interval $I_1, I_2, \dots, I_k$ has a multinomial distribution

$$P(d_1, d_2, \dots, d_k) = \frac{N!}{d_1! \cdots d_k!} \prod_{i=1}^{k+1} \left(P_{j-1} - P_j\right)^{d_i}. \tag{3.10}$$

Inputing (3.7) and (3.8) into (3.10), it is derived that

$$P(d_1, d_2, \dots, d_k) = \frac{N!}{d_1! \cdots d_k!} \prod_{i=1}^{k+1} q(j)^{d_j} \left[1 - q(j)\right]^{N_j - d_j}, \tag{3.11}$$

Table 1: Lifetime table for capacity analysis.

$I_j = [a_{j-1}, a_j)$	d_j traffic breakdown occurred in I_j	$N_j = n - d_1 - \cdots - d_{j-1}$	$\hat{q}(j) = d_j/N_j$	$\hat{P}_j = 1 - \hat{q}(j)$	$\hat{P}(j)$
$[a_0, a_1)$	d_1	N_1	$\hat{q}(1)$	$\hat{P}_1$	$\hat{P}(1)$
$[a_1, a_2)$	d_2	N_2	$\hat{q}(2)$	$\hat{P}_2$	$\hat{P}(2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

where $N_j = n - d_1 - \cdots - d_{j-1}$. Using the maximum likelihood estimation (MLE) method, the MLE of $q(j)$ and $P(j)$ can be obtained:

$$\hat{q}(j) = \frac{d_j}{N_j}, \tag{3.12}$$

$$\hat{P}(j) = \hat{p}_1 \hat{p}_2 \cdots \hat{p}_j, \tag{3.13}$$

where $\hat{p}_j = 1 - \hat{q}(j)$. And freeway capacity can be estimated by (3.13).

In order to use the procedure simply, the "lifetime table" for capacity can be made referring to lifetime table methods used in the survival analysis [12]. Table 1 provides the lifetime table for capacity analysis, and the distribution of capacity can be estimated using it.

3.3. *The Capacity Definition and Calculation Based on Traffic Breakdown Probability*

According to the definition of freeway capacity based on the breakdown probability, "the rate of flow along a uniform freeway segment corresponding to the expected probability of breakdown was deemed acceptable under prevailing traffic and roadway conditions in a specified direction" [6]. The principle and method of estimating the freeway capacity can be illustrated as follows for the given acceptable breakdown probability $\alpha (0 < \alpha < 1)$, there exists a value of "j" and flow rate "q" which satisfies $\hat{P}_j < 1 - \alpha \le \hat{P}_{j-1}$, and $a_{j-1} \le q < a_j$ ($\hat{P}(j) = \hat{p}_1 \hat{p}_2 \cdots \hat{p}_j$ can be derived from formula (3.13) or Table 1. Then the value of "q" is one of the estimates of capacity C and it can be computed as follows.

(1) If $1 - \alpha = \hat{P}_j$, the capacity C can be estimated by

$$\hat{C} = \frac{[a_j + a_{j-1}]}{2}. \tag{3.14}$$

(2) If $\hat{P}_j < 1 - \alpha < \hat{P}_{j-1}$, the capacity C can be estimated as

$$\hat{C} = a_{j-1}. \tag{3.15}$$

Applying (3.14) or (3.15) to estimate capacity, the accuracy is closely related to the division of interval $I_j = [a_{j-1}, a_j)$ $(j = 1, \ldots, k)$. In order to improve the accuracy, one way is to divide more intervals.

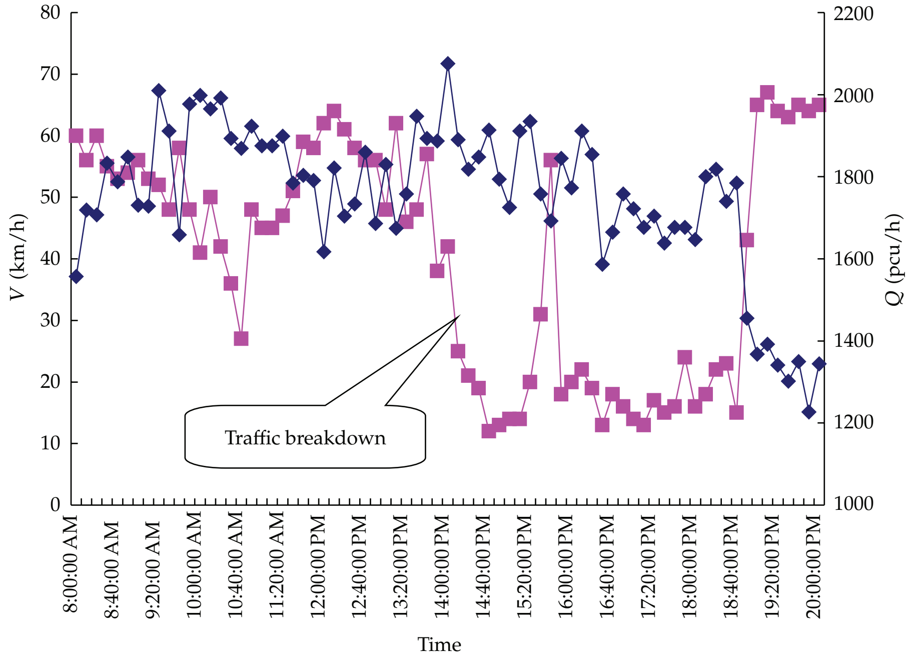

Figure 1: Time serial of traffic flow rates and speed.

Table 2: The statistical result of speed before breakdown.

Lane location	25% percentile	Median (km/h)	Mean (km/h)	75% percentile	Std. deviation	Minimum (km/h)	Maximum (km/h)
First lane inside	46.5	53.0	51.7	56.0	5.39	44.0	61.0
First lane outside	46.0	54.0	52.4	57.0	6.45	41.0	61.0

4. Empirical Analysis

4.1. Field Data Collection and Data Analysis

The field data were collected on the 3rd ring in Beijing City, China, by using microwave detectors. The posted speed limit on the 3rd ring is 80 km/h. Traffic volume is generally heavy and breakdowns are typical during morning and afternoon peak periods. One-month-period data are used in this paper for both traffic directions.

Figure 1 illustrates volume and speed changes on time-series plot. It is found from the figure that when traffic breakdown occurs, the speed of traffic flow will drop suddenly and the average speed will be less than a given threshold. Tables 2 and 3 represent the statistical results of the speed before and after breakdown in 10-minute analysis intervals. According to the analysis results, it is found that the speed value is lower than 35 km/h after breakdown and that the average speed is higher than 40 km/h.

Tables 4 and 5 are the statistical results of the flow rates before and after breakdown in 10-minute analysis intervals. It can be found that the average of flow rates after breakdown is higher than the one before breakdown and that the flow rates after breakdown are more stable then breakdown flow rates.

Table 3: The statistical result of speed after breakdown.

Lane location	25% percentile	Median (km/h)	Mean (km/h)	75% percentile	Std. deviation	Minimum (km/h)	Maximum (km/h)
First lane inside	20.5	27.0	25.5	30.0	5.35	18.0	33.0
First lane outside	21.3	24.0	24.0	28.0	4.50	17.0	31

Table 4: The statistics of flow rate before breakdown.

Lane location	25% percentile	Median (pcu/h/lane)	Mean (pcu/h/lane)	75% percentile	Std. deviation	Minimum (pcu/h/lane)	Maximum (pcu/h/lane)
First lane inside	1716	1878	1822	1966	190	1453	2067
First lane outside	1754	1857	1770	1943	279	1053	2052

Table 5: The statistics of flow rate after breakdown.

Lane location	25% percentile	Median (pcu/h/lane)	Mean (pcu/h/lane)	75% percentile	Std. deviation	Minimum (pcu/h/lane)	Maximum (pcu/h/lane)
First lane inside	1834	1863	1875	1987	132	1611	2085
First lane outside	1832	1893	1879	1992	139	1611	2085

4.2. Simulation Analysis Based on Field Data

Based on data collection and data analysis, the traffic flow is simulated by Monte-Carlo method and simulated data were used to validate the method and to demonstrate how to estimate the freeway capacity using the procedure. Tables 6, 7, and 8 list the estimated distributions of the freeway capacity for the inside lane, median lane, and shoulder lane, respectively.

If the acceptable breakdown probability is taken as $\alpha = 20\%$, then the estimated capacity is 1950 veh/h/lane, 1870 veh/h/lane, and 1850 veh/h/lane, respectively, for the three lanes. Also, the capacities for the three lanes are calculated using speed-volume relationship and the results are 2060 veh/h/lane, 2040 veh/h/lane, and 1990 veh/h/lane (The probability that breakdown occurs with the flow rates 2060 veh/h/lane, 2040 veh/h/lane, and 1990 veh/h/lane is also obtained as 0.63, 0.77, and 0.64), which is more than the expectation values of the breakdown flow rates. Therefore, the capacity based on the breakdown probability is more moderate than the traditional defined capacity comparatively.

5. Conclusion and Future Research

Based on the exiting research, the breakdown flow rate is determined to be used to study the capacity of the freeway sections in this paper. Due to the fact that, when the traffic flow approaches the level of capacity, breakdown will occur, it is impossible to observe the value of the capacity. The product-limit method is suggested to estimate the capacity. In order to complete this method, the lifetime table method and the estimate procedure are suggested. The results indicate that the lifetime table method and the suggested procedure are feasible and reasonable.

Table 6: Estimation result of the capacity distribution for the inside lane.

$I_j = [a_{j-1}, a_j)$	d_j	N_j	$\hat{q}(j)$	$\hat{p}_j = 1 - \hat{q}(j)$	$\hat{P}(j)$
[1740, 1790)	2	200	0.010	0.990	0.990
[1790, 1840)	1	198	0.005	0.995	0.985
[1840, 1890)	9	197	0.046	0.954	0.940
[1890, 1940)	11	188	0.059	0.941	0.885
[1940, 1990)	22	177	0.124	0.876	0.775
[1990, 2040)	38	155	0.245	0.755	0.585
[2040, 2090)	43	117	0.368	0.632	0.370
[2090, 2140)	28	74	0.378	0.622	0.230
[2140, 2190)	22	46	0.478	0.522	0.120
[2190, 2240)	15	24	0.625	0.375	0.045
[2240, 2290)	5	9	0.556	0.444	0.020
[2290, 2340)	3	4	0.750	0.250	0.005
[2340, 2390)	1	1	1.000	0.000	0.000
>2390		0			

Table 7: Estimation result of the capacity distribution for the median lane.

$I_j = [a_{j-1}, a_j)$	d_j	N_j	$\hat{q}(j)$	$\hat{p}_j = 1 - \hat{q}(j)$	$\hat{P}(j)$
[1720, 1770)	1	203	0.005	0.995	0.995
[1700, 1750)	3	202	0.015	0.985	0.980
[1750, 1800)	7	199	0.035	0.965	0.946
[1800, 1850)	11	192	0.057	0.943	0.891
[1850, 1900)	24	181	0.133	0.867	0.773
[1900, 1950)	39	157	0.248	0.752	0.581
[1950, 2000)	41	118	0.347	0.653	0.379
[2000, 2050)	30	77	0.390	0.610	0.232
[2050, 2100)	29	47	0.617	0.383	0.089
[2100, 2150)	8	18	0.444	0.556	0.049
[2150, 2200)	3	10	0.300	0.700	0.034
[2200, 2250)	6	7	0.857	0.1423	0.005
[2250, 2300)	1	1	1.000	0.000	0.000
>2300	0	0			

Table 8: Estimation result of the capacity distribution for the shoulder lane.

$I_j = [a_{j-1}, a_j)$	d_j	N_j	$\hat{q}(j)$	$\hat{p}_j = 1 - \hat{q}(j)$	$\hat{P}(j)$
[1700, 1840)	11	198	0.056	0.944	0.944
[1840, 1890)	28	187	0.150	0.850	0.803
[1890, 1940)	28	159	0.176	0.824	0.662
[1940, 1990)	59	131	0.450	0.550	0.364
[1990, 2040)	59	72	0.819	0.181	0.066
[2040, 2090)	9	13	0.692	0.308	0.020
[2090, 2140)	4	4	1.000	0.000	0.000
>2140		0			

However, there are still some problems to be solved in the future.

(1) The effect of geometric characteristics and traffic condition on the breakdown probability should be studied. Only by this, the research result may be applied generally.

(2) An acceptable breakdown probability value should be determined. This value reflects the balance between risk of breakdown and higher traffic volume. So, it is determined by the operating agency and questionnaires should be conducted.

(3) The definition of breakdown flow rate should be studied. Although, in this paper, the breakdown flow rate is defined as the flow rate observed immediately prior to breakdown, there are still some questions need to be answered, such as should the traffic flow be obtained for 5-minute, 10-minute, or 15-minute time interval prior to breakdown?

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