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CP violation in minimal supersymmetric standard model *

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CP violating phenomena predicted by the minimal supersymmetric standard model are discussed in a case where the CP violating phases in SUSY sector are not suppressed. The electric dipole moments of the neutron and the electron are large, but can be smaller than their experimental upper bounds if the scalar quarks and leptons are heavier than a few TeV. T violating asymmetries in the production processes of the different neutralino pair and the different chargino pair emerge at the tree level. They could be as large as of order 10^{-2} in unpolarized electron beam experiments and 10^{-1} in polarized electron beam experiments. In a pair production of the charginos of the same mass, the asymmetry emerges through the electric and the weak "electric" dipole moments of the charginos at the loop level, but its magnitude is at most of order 10^{-4} .

1. INTRODUCTION

Cabibbo-Kobayashi-Maskawa (CKM) matrix is a standard candidate for the origins of CP violation within a framework of the standard electroweak theory. In the minimal supersymmetric standard model (MSSM) [1] there appear some complex parameters if the supersymmetry (SUSY) is not an exact symmetry but is broken. So in addition to the phase of CKM matrix we have other sources of CP violation in MSSM. It is well-known that these new CP violating phases cause large electric dipole moments (EDMs) [2–4] which might give a denial or a signal of the existence of SUSY.

Not only EDMs are the CP violating phenomena expected in MSSM, but also T or CP -odd asymmetries in the production of SUSY particles, such as neutralinos and charginos [7–9]. These asymmetries are caused by tree diagrams so that they would not be too small to be tested by the future collider experiments in which the SUSY particles might be eventually produced. In this

report we shall discuss first EDMs and show that SUSY particles, especially squarks and sleptons, are as heavy as a few TeV if the phases of the SUSY parameters are of order unity. Secondly we shall discuss the T -odd asymmetries and show that it is not impossible to observe the asymmetries as the manifestation of CP violation originating from SUSY.

Before closing this section, for the definiteness we show the MSSM Lagrangian,

$$\begin{aligned}
 L &= L_{kin} + L_{gauge} + L_F + L_S; \\
 L_F &= [(E^c Y_E L) H_1 + (D^c Y_D Q) H_1 \\
 &\quad + (U^c Y_U Q) H_2 + m_H H_1 \times H_2]_F, \\
 -L_S &= (\tilde{E}^c \eta_E \tilde{L}) \tilde{H}_1 + (\tilde{D}^c \eta_D \tilde{Q}) \tilde{H}_1 \\
 &\quad + (\tilde{U}^c \eta_U \tilde{Q}) \tilde{H}_2 + \tilde{M}_H^2 \tilde{H}_1 \times \tilde{H}_2 \\
 &\quad + \frac{1}{2} \sum_{i=1}^3 \tilde{m}_i \lambda_i \lambda_i + \sum_{a,b} \tilde{M}_{ab}^2 \phi_a^* \phi_b, \quad (1)
 \end{aligned}$$

to fix our notation of the relevant parameters. We assume the grand unification of the fundamental interactions, and relate some of these parameters

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in (1) each other as

$$\begin{aligned}
\tilde{M}_{ab}^2 &= |m_{3/2}|^2 \delta_{ab}, \\
\eta_f &= Am_{3/2} Y_f \quad (f = E, D, U), \\
\tilde{M}_H^2 &= Bm_{3/2} m_H, \\
\tilde{m}_i &= M_\lambda \quad (i = 1, 2, 3),
\end{aligned} \tag{2}$$

to reduce the number of the parameters. The parameters $Am_{3/2}$, $Bm_{3/2}$, m_H , and M_λ in (2) are complex in general, and can become sources of CP violation. However, all the phases are not physical, but two of them are physical. So in this report we take $Am_{3/2}$ and m_H to be complex, and $Bm_{3/2}$ and M_λ to be real, for simplicity.

2. ELECTRIC DIPOLE MOMENTS

EDMs of the quarks (leptons) are given by the one-loop diagrams by exchange of the squarks \tilde{q} (sleptons $\tilde{\ell}$) and the charginos ω_i , the neutralinos χ_j , or the gluinos \tilde{g} (not for the lepton EDM). People think that the gluino loop gives the largest contribution to EDMs of the quarks because the strong coupling constant is much larger than the electroweak coupling constant. Its typical estimate commonly made for the u -quark is like

$$\begin{aligned}
d_u^G/e &\simeq \frac{2\alpha_s}{9\pi\tilde{m}_3} \frac{\text{Im}(Am_{3/2})m_u}{\tilde{m}_3^2} I'(\tilde{M}_q^2/\tilde{m}_3^2), \\
I(x) &= \frac{1}{(1-x)^2} \left(1+x + \frac{2x}{1-x} \ln x\right).
\end{aligned} \tag{3}$$

Assuming that the gluino mass (\tilde{m}_3) and the squark masses (\tilde{M}_q) are 100 GeV, we would get the u -quark EDM of $10^{-22}e\cdot\text{cm}$. This is much larger than the present experimental upper bound of the neutron EDM, $10^{-25}e\cdot\text{cm}$ [5], so we might have to assume that the SUSY parameters are almost real. However fortunately or unfortunately, the SUSY particles such as squarks and gluinos have not yet been discovered, and their masses are larger than 100 GeV. Since, the heavier they are, the smaller EDM becomes, even if the SUSY phases are of order unity, sufficiently large SUSY particle masses can reduce EDM. Thus it is not a bad question to ask how heavy the SUSY particles should be to reduce EDM to the value smaller than its experimental bound.

To answer this question we calculate the EDM of the neutron arising from the gluino, chargino, and neutralino loops taking maximal CP violating phases. The results are shown in Figs. 1–4.

Figure 1. (i) The chargino, (ii) neutralino, and (iii) gluino contributions to the neutron EDM for $\tan\beta = 2$, $\tilde{m}_2 = |m_H| = 0.5\text{TeV}$.

Table 1
The values of \tilde{m}_2 and $|m_H|$ in Figs. 2 and 5

	(i)	(ii)	(iii)	(iv)
\tilde{m}_2 (TeV)	0.2	1.0	0.2	1.0
$ m_H $ (TeV)	0.2	0.2	1.0	1.0

The Fig. 1 shows as a function of the squark mass the absolute values of EDMs by the chargino, neutralino, and gluino loops which contribute to the neutron EDM. It should be noted that EDM from the chargino is larger than EDM from the gluino in this case, which would be contrary to the naive expectation. The reason of this comes mainly from the GUT mass relation $(\alpha_2/\alpha_3)\tilde{m}_3 = \tilde{m}_2$ which we assumed in

Figure 2. The neutron EDM from the chargino contribution. The values of the parameters are shown in Table 1, and $\tan\beta = 2$ for (a) and $\tan\beta = 10$ for (b).

the calculation. The large $SU(3)$ coupling α_3 , which could make (3) large, leads to the large gluino mass, which almost cancels the enhancement by the strong coupling. In addition to this, since the mixing between left-handed and right-handed squarks, which is needed for the EDM operator by the gluino loop, is small, the magnitude of the gluino contribution to the neutron EDM is suppressed to be smaller than that of the chargino.

The Fig. 2 shows the chargino contribution to the neutron EDM for the different values of the parameters, $\tan\beta$, \tilde{m}_2 , and $|m_H|$, as a function of the squark mass. In this parameter region the chargino contributes to the neutron EDM the most, and therefore its contribution can be taken to be approximately equal to the whole neutron EDM. From the figure it can be seen that, if the squark mass is larger than about 3 TeV for $\tan\beta = 2$ and 7 TeV for $\tan\beta = 10$, the predicted value can be smaller than the present experimental upper bound of $10^{-25}e\cdot\text{cm}$, even though the SUSY CP phase is of order one. This is the light gaugino case.

Can the squarks be lighter than 3 TeV? Figs. 3

Figure 3. The neutron EDM as a function of $|m_H|$ for $\tan\beta = 2$. The gaugino mass \tilde{m}_2 is 1 TeV for (a) and 3 TeV for (b). The squark mass \tilde{M}_q is 0.2 TeV for (i) and 1 TeV for (ii).

and 4 try to answer the question, and show the squarks can be really as light as 0.2 TeV if \tilde{m}_2 and $|m_H|$ are heavier than about 2 TeV. However, since EDM does not decrease as rapidly as in Fig. 2 (even begins to increase when $|m_H|$ becomes large!), the light squark possibility would be denied if the experimental bound on the neutron EDM is reduced by one order of magnitude.

The electron EDM is shown in Fig. 5. Its present experimental upper bound is $10^{-26}e\cdot\text{cm}$ [6]. If the slepton is heavier than about 1 TeV for $\tan\beta = 2$ and 4 TeV for $\tan\beta = 10$, the predicted value can be smaller than the present experimental limit.

3. T(CP)–ODD ASYMMETRY

In the previous section it was realized that the present experimental bounds on the neutron and electron EDMs do not exclude the possibility that the SUSY CP phases are of order unity. What CP violating phenomena will be expected to come out from these new phases in high energy collider experiments? One possibility is the T -odd asymmetry [7–9] in the production and

Figure 4. The neutron EDM as a function of \tilde{m}_2 for $\tan\beta = 2$. The higgsino mass $|m_H|$ is 1 TeV for (a) and 3 TeV for (b). The squark mass \tilde{M}_q is 0.2 TeV for (i) and 1 TeV for (ii).

decay processes of the SUSY particles. In MSSM the asymmetry can appear at the tree level. To be definite, let us consider the production processes of two different charginos mediated by Z -boson,

$$e^+e^- \rightarrow \omega_2^+\omega_1^-, \quad (4)$$

and two different neutralinos,

$$e^+e^- \rightarrow \chi_i\chi_j. \quad (5)$$

The charginos and the neutralinos are the mixed states of the $SU(2) \times U(1)$ gauginos and the higgsinos. To get them as the mass eigenstates, we have to diagonalize their mass matrices by unitary matrices which contain imaginary phases originating from the complex SUSY parameters. When we rewrite the gaugino and higgsino couplings to Z by the mass eigenstates through the unitary transformations, there appear off-diagonal couplings of the charginos and neutralinos to Z . Their coupling constants are made from the complex elements of the unitary matrices. Moreover their relative phases can not be absorbed, and remain physical. Thus these off-diagonal couplings to Z cause $T(CP)$ violation at

Figure 5. The electron EDM from the chargino contribution. The parameters are the same as in Fig. 2.

the tree level, and the violation can be expected to be large enough to be observed.

First we discuss the T -odd asymmetry in the chargino production process (4). When the spin of ω_2 , s_2 , is summed, but the spin of ω_1 , s_1 (which will be chosen such as it is perpendicular to the interaction plane), is not summed, the cross section becomes in the CM system of e^+e^- as

$$\begin{aligned} & \sum_{s_2} d\sigma(e^+e^- \rightarrow \omega_2^+\omega_1^-) \\ &= \frac{G_F^2}{2\pi} \frac{p}{\sqrt{S}} \frac{M_Z^4}{(S - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \times \\ & \times [(f_L^2 + f_R^2)((|G_L^{12}|^2 + |G_R^{12}|^2)(E_1 E_2 + p^2/3) \\ & + 2\Re(G_L^{12} G_R^{12*})m_{\omega_1} m_{\omega_2}) \\ & + \text{sign}(s_1)(f_L^2 - f_R^2)\Im(G_L^{12} G_R^{12*})\frac{\pi}{2}m_{\omega_2} p], \quad (6) \end{aligned}$$

where p is the magnitude of the momentum of the chargino, $E_{1,2}$ are the energies of $\omega_{1,2}$, $G_{L,R}^{12}$ are the coupling constants of the charginos to Z defined as

$$\begin{aligned} J_\mu^Z &= e\bar{\omega}_1\gamma_\mu[G_L^{11}P_- + G_R^{11}P_+]\omega_1 \\ & + e\bar{\omega}_2\gamma_\mu[G_L^{22}P_- + G_R^{22}P_+]\omega_2 \\ & + e\bar{\omega}_1\gamma_\mu[G_L^{12}P_- + G_R^{12}P_+]\omega_2 \end{aligned}$$

$$+e\bar{\omega}_2\gamma_\mu[G_L^{21}P_- + G_R^{21}P_+]\omega_1, \quad (7)$$

$f_{L,R}$ are the coupling constants of the electron to Z , and $\text{sign}(s_1)$ is defined by the three momentum of the incident electron, \mathbf{p}^- , and the three momentum of ω_1 , \mathbf{p}_1 , as $\text{sign}(s_1) = \text{sign}(\mathbf{s}_1 \cdot (\mathbf{p}^- \times \mathbf{p}_1))$.

The term breaking T invariance is the last term in (6). Because it depends on s_1 , we must observe at least a spin state of either chargino to detect the T violation effect. The positive sign of s_1 means a vector product $\mathbf{s}_1 \cdot (\mathbf{p}^- \times \mathbf{p}_1)$ is positive, and the negative sign of s_1 means $\mathbf{s}_1 \cdot (\mathbf{p}^- \times \mathbf{p}_1) < 0$. Since these two states are transformed to each other by time reversal, the difference of the cross section between these two states manifests T violation at the first-order perturbation. The asymmetry of these two states

$$A_T = \frac{d\sigma(\mathbf{s}_1 \cdot (\mathbf{p}^- \times \mathbf{p}_1) > 0) - d\sigma(\mathbf{s}_1 \cdot (\mathbf{p}^- \times \mathbf{p}_1) < 0)}{d\sigma(\mathbf{s}_1 \cdot (\mathbf{p}^- \times \mathbf{p}_1) > 0) + d\sigma(\mathbf{s}_1 \cdot (\mathbf{p}^- \times \mathbf{p}_1) < 0)}, \quad (8)$$

may be a good observable to quantify how large the T violation is. As a numerical example, we show the result of a parameter set of $\tan\beta = 2$, $\tilde{m}_2 = 200$ GeV, and $m_H = 200e^{i\pi/4}$ GeV, which leads to the chargino masses 133, 275 GeV, and the neutralino masses 83, 145, 203, 278 GeV. The neutron and electron EDMs for this parameter set is shown in Table 2 by taking the squark and slepton masses for 3 TeV. Their values are smaller than the present experimental bounds [5,6].

Table 2
Electric dipole moments

	Neutron [$10^{-25}e \cdot \text{cm}$]	Electron [$10^{-27}e \cdot \text{cm}$]
Gluino	0.2	/
Chargino	-1.0	-4.2
Neutralino	0.003	0.04
Exp.	< 1.2	-2.7 ± 8.3

The calculated asymmetry is shown in Fig. 6 as a function of the CM energy (\sqrt{S}). When the incident electron beam is unpolarized, the magnitude of the asymmetry is smaller than 10^{-2}

Figure 6. The T -odd asymmetry in the chargino production. The solid line corresponds to unpolarized electron beam, and the dashed line corresponds to left-handedly polarized electron beam.

despite the tree level effect. This is due to the almost pure-axial coupling of the electron to Z . When the incident electron beam is polarized, the asymmetry is enhanced by about 6 times as large as the unpolarized case.

Secondly we discuss the T -odd asymmetry in the neutralino production process (5). As in the chargino production, by summing the spin of χ_j , s_j , but the spin of χ_i , s_i , is not summed, we get the cross section in the CM system of e^+e^- as

$$\begin{aligned} & \sum_{s_j} d\sigma(e^+e^- \rightarrow \chi_i\chi_j) \\ &= \frac{G_F^2 p}{4\pi \sqrt{S}} \frac{M_Z^4}{(S - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} |O_Z|^2 \times \\ & \times [(f_L^2 + f_R^2)(E_i E_j + p^2/3 - \cos 2\delta_Z m_{\chi_i} m_{\chi_j}) \\ & + \text{sign}(s_i)(f_L^2 - f_R^2)\frac{\pi}{4} \sin 2\delta_Z m_{\chi_j} p], \quad (9) \end{aligned}$$

where p is the magnitude of the momentum of the neutralino, $E_{i,j}$ are the energies of $\chi_{i,j}$, and $\text{sign}(s_i) = \text{sign}(\mathbf{s}_i \cdot (\mathbf{p}^- \times \mathbf{p}_i))$. O_Z is the coupling constant of the neutralinos to Z defined as

$$\begin{aligned} \mathcal{L}_{\text{int}}^Z &= \frac{e}{\sin 2\theta_W} \bar{\chi}_i \gamma_\mu (O_Z P_- - O_Z^* P_+) \chi_j Z^\mu; \\ P_\pm &= (1 \pm \gamma_5)/2, \quad (10) \end{aligned}$$

and $\delta_Z = \arg(O_Z)$. Again we have the T violating term in (9), and we can define an asymmetry

$$A_T = \frac{d\sigma(\mathbf{s}_i \cdot (\mathbf{p}^- \times \mathbf{p}_i) > 0) - d\sigma(\mathbf{s}_i \cdot (\mathbf{p}^- \times \mathbf{p}_i) < 0)}{d\sigma(\mathbf{s}_i \cdot (\mathbf{p}^- \times \mathbf{p}_i) > 0) + d\sigma(\mathbf{s}_i \cdot (\mathbf{p}^- \times \mathbf{p}_i) < 0)}, \quad (11)$$

which quantifies how large the T violation is. It should be noted that, differently from the chargino production, the final state of the process (5) is an eigenstate of CP because of the neutralinos being the Majorana particles, so that the asymmetry (11) is a CP -odd asymmetry. Thus a non-vanishing value of (11) directly means the violation of CP . The calculated asymmetry is shown in Fig. 7 as a function of \sqrt{S} . The magni-

Figure 7. The T -odd asymmetry in the neutralino production. The solid line corresponds to unpolarized electron beam, and the dashed line corresponds to left-handedly polarized beam.

tude of the asymmetry is approximately the same as the chargino one. We also have calculated an asymmetry of the lepton pair produced by the decay of χ_j , A'_T , which is more realistic than (8) and (11) because we only have to measure the momenta of the final state leptons, \mathbf{p}_D^\pm , instead of the spin of χ_j being experimentally difficult to be measured. In Fig. 7 the result in the left-handedly polarized electron beam case is shown by dots.

The magnitude of A'_T is the same order as A_T near the threshold, but it rapidly decreases as \sqrt{S} becomes large. This is because, as \sqrt{S} becomes large, E_i also becomes large so that \mathbf{p}_D^+ and \mathbf{p}_D^- come to orient to the same direction. Since the T violation term is proportional to $\mathbf{p}^- \cdot (\mathbf{p}_D^+ \times \mathbf{p}_D^-)$, A'_T decreases as \sqrt{S} gets large. Therefore, if one wishes to get a large asymmetry, the asymmetry should be measured near the threshold.

4. DISCUSSION

In this report we have viewed $CP(T)$ violation in MSSM. The present experimental bounds on the neutron and electron EDMs do not immediately rule out the possibility that the imaginary phases of the SUSY parameters have their natural values of $\mathcal{O}(1)$. If this is the case, the CP violation originating from MSSM could lead to $T(CP)$ -odd phenomena in high energy experiments. If the charginos and neutralinos are produced in e^+e^- collisions, T -odd asymmetry would be observed in the angular distribution of the final decay products.

In the $T(CP)$ -odd asymmetries discussed here two different mass eigenstates of the charginos or neutralinos are involved. In the pair production of the charginos or neutralinos of the same mass eigenstate, does the asymmetry appear? The answer is 'yes' for the charginos. The electric and the weak "electric" dipole moments of the charginos, D_ω , are generated at one-loop level, which break T invariance. These dipole moment terms give rise to T -odd asymmetry in the same chargino pair-production processes. However, from the dimensional grounds, A_T in this case can be roughly estimated as $A_T \sim \sqrt{S}D_\omega/e$. The main contributions to D_ω are given by the loop diagrams involving the top quark, the top squarks, the W bosons, and/or the Higgs bosons. The top or stop contribution to D_ω is proportional to the large top-quark mass. However, within MSSM the magnitude of D_ω is strictly restricted by the upper bounds of the neutron and electron EDMs, and it would be at most $10^{-20}e\text{-cm}$, if $m_t \sim 150$ GeV and unless one of the scalar quarks is accidentally much lighter than the other scalar quarks and leptons. The contri-

butions from the W bosons, and the Higgs bosons are roughly estimated to be also $10^{-20}e\cdot\text{cm}$. Thus we could expect in $e^+e^- \rightarrow \omega_i\bar{\omega}_i$ $A_T \sim 10^{-4}$ which would be too small for detection in the future e^+e^- experiments.

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